

# MATHS

# **BOOKS - RD SHARMA MATHS (HINGLISH)**

# DIFFERENTIABILITY

Solved Examples And Exercises

1. Find the values of a and b so that the function  $f(x)=ig\{x^2+3x+a,bx2\ ,$  if  $x\leq 1$  if x>1 is differentiable at each xR.

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2. Show that  $f(x)=ig\{12x-13,2x^2+5,$  if  $x\leq 3$  if x>3 is

differentiable at x=3 . Also, find  $f^{\,\prime}(3)_{\cdot}$ 



**3.** Show that  $f(x) = x^{rac{1}{3}}$  is not differentiable at x = 0.



**4.** Show that f(x) = |x - 3| is continuous at x = 3.

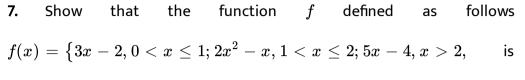
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5. Show that the function 
$$f(x)=iggl\{x^m\siniggl(rac{1}{x}iggr),0,x
eq 0,x=0$$
 is

differentiable at x=0,if m>1

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6. Discuss the continuity and differentiability of the function f(x) = |x| + |x-1| in the interval (-1,2).



continous at x=2 but not differentiable.

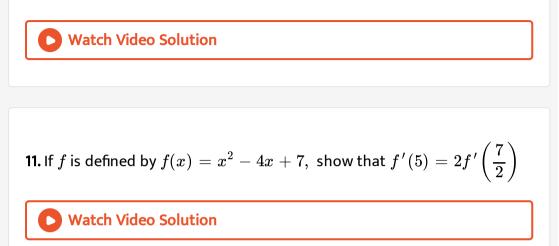
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8. Discuss the continuity and differentiability of 
$$f(x) = \left\{ (x-c) \cos\left(rac{1}{x-c}\right), x 
eq c ext{ and } 0, x = c 
ight.$$

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**9.** Show that f(x) = |x-3| is not differentiable at x = 3.

10. Discuss the continuity and differentiability of  $f(x) = |\mathrm{log}| |x|$  .



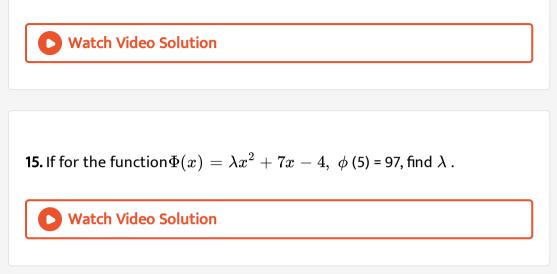
12. If 
$$f$$
 is defined by  $f(x)=x^2, ext{ find } f'(2).$ 

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13. Find the derivative of the function f defined by f(x) = mx + c at

x = 0.

14. Discuss the continuity and differentiability of  $f(x) = e^{|x|}$ .



16. Examine the continuity f(x) ={(3x-2,xlt=0),

(x+1,x >0)}atx=0`

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17. The setoff points where the function f(x) given by  $f(x)=|x-3|\cos x$  is differentiable, is R (b)  $R-\{3\}$  (c)  $(0,\infty)$  (d) none of these



**18.** If  $f(x) = \begin{cases} \frac{1 - \cos x}{x \sin x}, & x \neq 0 \text{ and } \frac{1}{2}, x = 0 \text{ then at } x = 0, f(x) \text{ is} \end{cases}$ (a)continuous and differentiable (b)differentiable but not continuous (c)continuous but not differentiable (d)neither continuous nor differentiable

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19. If 
$$f(x)=iggl\{rac{1}{1+e^{rac{1}{x}}},x
eq 00,x=0,thenf(x) ext{ is continuous as well }$$

as differentiable at x=0 continuous but not differentiable at x=0

differentiable but not continuous at x=0 none of these

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**20.** If f(x) = |3 - x| + (3 + x), where (x) denotes the least integer greater than or equal to x,

then f(x) is continuous and differentiable at x=3continuous but not differentiable at x=3differentiable but not continuous at x=3neither differentiable nor continuous at x=3



21. Let  $f(x) = a + b|x| + c|x|^4$ , where a, bandc are real constants. Then, f(x) is differentiable at x = 0, if a = 0 (b) b = 0 (c) c = 0 (d) none of these

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22. The function  $f(x) = rac{\sin(\pi [x-\pi])}{4+{[x]}^2}$  , where [] denotes the greatest

integer function, is continuous as well as differentiable for all  $x \in R$ 

(b) continuous for all x but not differentiable at some x

(c) differentiable for all x but not continuous at some x

.(d) none of these



23. Let 
$$f(x)=\left\{ax^2+1,x>1;
ight.$$
  
 $x+rac{1}{2},x\leq 1.$  then,  $f(x)$  is derivable at  $x=1,$  if  $a=2$   
(b)  $a=1$   
(c)  $a=0$   
(d)  $a=rac{1}{2}$ 

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24. Show that the function defined by  $g_{-}(x) = x_{-}[x]$  is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.

25. Find all points of discontinuity of f, where f is defined by  $f(x)=\{2x+3, ext{ if } x\leq 2\ 2x-3, ext{ if } x>2$ 

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26. If 
$$f(x) = \{x^2 + 3x + a, x \le 1bx + 2, f \text{ or } x > 1 \text{ is everywhere} \}$$

differentiable, find the values of aandb-

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27. Let 
$$f(x) = |x|$$
 and  $g(x) = \left|x^3
ight|, ext{ then }$ 

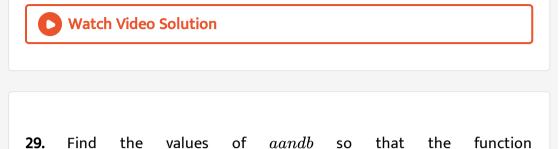
(a) f(x)andg(x) both are continuous at x=0

(b)f(x)andg(x) both are differentiable at x=0

(c)f(x) is differentiable but g(x) is not differentiable at x=0

(d)f(x) and g(x) both are not differentiable at x=0

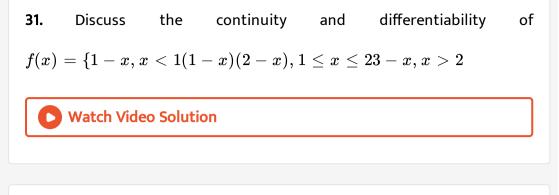
#### **28.** Is $|\sin x|$ differentiable? What about $\cos |x|$ ?



 $f(x)=ig\{x^2+3x+a, ext{ if } x\leq 1bx+2, ext{ if } x>1 ext{ is differentiable at}$ each  $x\in R$ 

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**30.** The function  $f(x) = e^{-|x|}$  is continuous everywhere but not differentiable at x = 0 continuous and differentiable everywhere not continuous at x = 0 none of these



**32.** Show that the function  $f(x) = \{|2x-3| [x] | x \leq 0 \text{ and } \sin\left(rac{\pi x}{2}
ight),$ 

x>0 is continuous but not differentiable at x=0

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**33.** The set of points where the function f(x) = x |x| is differentiable is

$$(\,-\infty,\infty)$$
 (b)  $(\,-\infty,0)\cup(0,\infty)$   $(0,\infty)$  (d)  $[0,\infty)$ 

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**34.** The function  $f(x) = \sin^{-1}(\cos x)$  is (a) . discontinuous at x = 0 (b).

continuous at x = 0 (c) . differentiable at x = 0 (d) . non of these



**35.** If  $f(x) = |\ln|x||$ , then (a)f(x) is continuous and differentiable for all x in its domain (b)f(x) is continuous for all for all x in its domain but not differentiable at  $x = \pm 1$  (c)f(x) is neither continuous nor differentiable at  $x = \pm 1$  (d)none of these

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**36.** Let 
$$f(x) = \left\{ \frac{1}{|x|} f \text{ or } |x| \ge 1ax^2 + bf \text{ or } |x| < 1 \Leftrightarrow (x) \text{ is continuous and differentiable at any point, then  $a = \frac{1}{2}, b = -\frac{3}{2}$  (b)$$

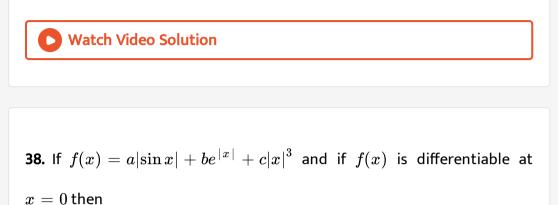
$$a = -rac{1}{2}, b = rac{3}{2} \, a = 1, b = -1$$
 (d) none of these

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37. If  $f(x)=\sqrt{1-\left(\sqrt{1}-x^2
ight)}$  , then f(x) is (a)continuous on [-1, 1] and differentiable on (-1, 1) (b) continuous on [-1,1] and differentiable on

 $(\,-1,\,0)\cup(0,\,1)$  (C) continuous and differentiable on [-1, 1](d) none of

#### these



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**39.** If 
$$f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \frac{x^2}{(1+x^2)^n} +$$
, then at  $x = 0, f(x)$  has no limit (b) is discontinuous is continuous but not

differentiable (d) is differentiable

**40.** If 
$$f(x) = |(\log)_2 x|$$
, then  $f(1^+) = 1$  (b)  $f(1^-) = -1$   $f(1) = 1$  (c)  $f'(1) = -1$ 

**41.** If 
$$f(x) = \left\{ rac{|x+2|}{ anu{tan}^{-1}(x+2)}, x 
eq -2 ext{ and } 2, x = -2 ext{ then } f(x) ext{ is } 
ight.$$

continuous/discontinuous at x = -2?

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**42.** Let f(x) = (x+|x|)|x| then, for all x . f is continuous (b) f is

differentiable for some x f' is continuous (d) f'' is continuous

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**43.** The function  $f(x) = e^{-|x|}$  is continuous everywhere but not differentiable at x = 0 continuous and differentiable everywhere not continuous at x = 0 none of these



44. The function  $f(x) = |\cos x|$  is everywhere continuous and differentiable everywhere continuous but not differentiable at  $(2n+1)\frac{\pi}{2}, n \in Z$  . neither continuous not differentiable at  $(2x+1)\frac{\pi}{2}, n \in Z$ 

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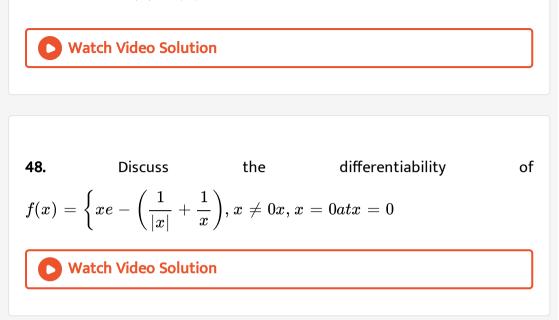
**45.** Discuss the continuity of the function  $f(x) = \{2x - 1 \ ext{ if } \ x < 2 \}$ 

$${3x\over 2}$$
 if  $x\geq 2$ 

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46. Show that the function 
$$f(x)=igg\{x^2\sinigg(rac{1}{x}igg), ext{ if } x
eq 00, ext{ if } x=0 ext{ is differentiable at } x=0 ext{ and } f'(0)=0$$

**47.** Show that f(x) = |x| is not differentiable at x = 0.



**49.** For what choice of a and b is the function  $f(x)=ig\{x^2,x\leq c ext{ and } ax+b,x>c ext{ is differentiable at }x=c$ 

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**50.** Discuss the differentiability of f(x) = x |x| at x = 0

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**52.** If f(2) = 2 and f'(2) = 1, then find  $(\lim_{x \to 2} \frac{xf(2) - 2f(x)}{x - 2})$ 

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53. A function  $f\colon R o R$  satisfies the equation f(x+y)=f(x)f(y) for all  $x,y\in R.f(x)
eq 0$  Suppose that the function is differentiable at x=0 and f'(0)=2. Prove that f'(x)=2f(x).

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**54.** Show that f(x) = |x| is not differentiable at x = 0 .

55. Show that the function 
$$f(x) = \{x - 1, \text{ if } x < 22x - 3, \text{ if } x \ge 2 \text{ is not}$$
 differentiable at  $x = 2$ .

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**56.** Show that the function 
$$f(x) = \begin{cases} \begin{pmatrix} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$
 is

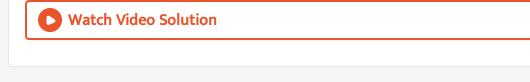
differentiable at x=0

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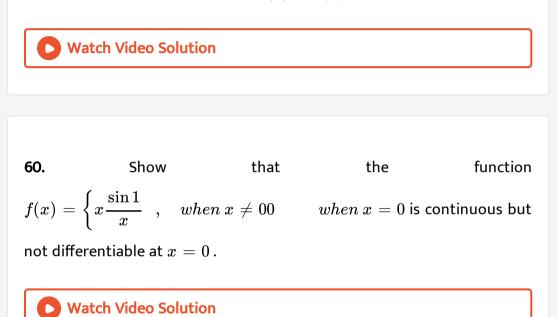
57. Show that  $f(x)=x^2$  is differentiable at x=1 and find  $f^{\,\prime}(1)$  .

58. Show that the function f(x) = |x+1| + |x-1| for all  $x \in R$  , is

not differentiable at x = -1 and x = 1 .



**59.** Discuss the differentiability of f(x) = x|x| at x = 0 .



61. Discuss the differentiability of
$$f(x) = \left\{ xe - \left(\frac{1}{|x|} + \frac{1}{x}\right), x \neq 0x, x = 0atx = 0 \right\}$$
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62. If f(x) is differentiable at x=a , find  $(\lim_{x
ightarrow a} rac{x^2 f(a)-a^2}{x-a} rac{f(x)}{x-a}$  . Watch Video Solution For what choice of *a* and *b* is the function 63.  $f(x)=ig\{x^2 \ , \ x\leq cax+b \ , \ x>c$  is differentiable at x=c . Watch Video Solution **64.** If f(2)=4 and  $f^{\,\prime}(2)=1$  , then find  $(\ \lim \ )_{x
ightarrow 2}rac{x\ f(2)-2f\left(x
ight)}{x-2}$  . Watch Video Solution

**65.** A function  $f: R \to R$  satisfies that equation f(x+y) = f(x)f(y)for all  $x, y \in R$ ,  $f(x) \neq 0$ . Suppose that the function f(x) is differentiable at x = 0 and f'(0) = 2. Prove that f'(x) = 2 f(x).

**66.** Show that f(x) = |x - 3| is continuous but not differentiable at x = 3.

**67.** Show that  $f(x) = x^{1/3}$  is not differentiable at x = 0 .

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68. Show that  $f(x) = \{12x - 13, \text{ if } x \leq 32x^2 + 5, \text{ if } x > 3\}$ 

is differentiable at x=3 . Also, find  $f^{\,\prime}(3)$  .

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**69.** Show that the function f defined as follows  $f(x)=\{3x-2, \setminus \setminus 02 \ iscont \in uousatx=2^{\circ}, but not differentiable thereat.$ 



70. Discuss the continuity and differentiability of the function f(x) = |x| + |x - 1| in the interval (-1, 2).

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**71.** Find whether the following function is differentiable at x = 1 and x = 2

or not : 
$$f(x) = ig\{x, x < 12 - x, 1 \leq x \leq 2 - 2 + 3x - x^2, x > 2ig\}$$

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72. Show that the function  $f(x) = \left\{x^m \sin\left(rac{1}{x}
ight), 0, \ x 
eq 0, \ x = 0
ight.$ 

is differentiable at x=0 , if m>1

73. Show that the function  $f(x) = \left\{ x^m \sin\left(rac{1}{x}
ight), \ x 
eq 00, \qquad x=0 ext{ is continuous but }$ 

not differentiable at  $x\,=\,0$  , if `0

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74. Show that the function 
$$f(x) = igg\{ x^m \sinigg(rac{1}{x}igg), x 
eq 00, x = 0 ext{ is neither } igg\}$$

continuous nor differentiable, if  $m \leq 0$ 

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75. Find the values of a and b so that the function  $f(x) = \left\{x^2 + 3x + a, ext{ if } x \leq 1bx + 2, ext{ if } x > 1 ext{ is } 
ight.$ 

differentiable at each  $x \in R$  .

76. Show that the function  $f(x) = \Big\{ |2x-3| \ [x], \ x \geq 1 \sin \Big( rac{\pi x}{2} \Big), \ x < 1$  is continuous but

not differentiable at x=1 .

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77. If 
$$f(x)=igg\{ax^2-b, \quad ext{if} \ |x|<1rac{1}{|x|}, \quad ext{if} \ |x|\geq 1$$
 is

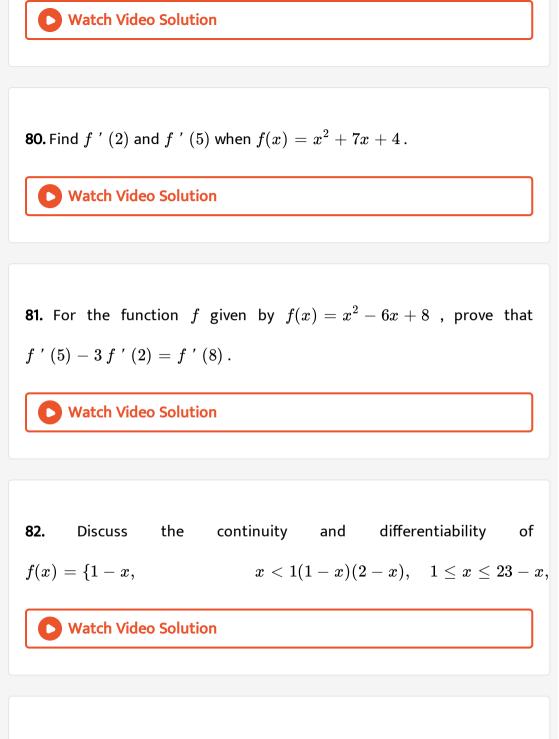
differentiable at x=1 , find  $a,\ b$ 

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78. Find the values of a and b , if the function f(x) defined by  $f(x)=ig\{x^2+3x+a,\ x\leq 1bx+2,\ x>1$  is differentiable at x=1 .

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**79.** If  $f(x) = x^2 + 2x + 7$  , find f ' (3) .



**83.** Discuss the differentiability of f(x) = |x-1| + |x-2|

84. If 
$$f(x) = \{x^2 + 3x + a, f \text{ or } x \leq 1bx + 2, f \text{ or } x > 1 \text{ is } \}$$

everywhere differentiable, find the values of a and b.

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**85.** Discuss the differentiability of  $f(x) = |(\log)_e x| x > 0$ .

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**86.** If f is defined by  $f(x) = x^2$  , find f ' (2) .

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87. If f is defined by  $f(x)=x^2-4x+7$  , show that f ' (5)=2f '  $\left(rac{7}{2}
ight)$ 

88. Show that the derivative of the function f given by  $f(x)=2x^3-9x^2+12x+9$  , at x=1 and x=2 are equal.

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**89.** If for the function  $\operatorname{Phi}(x) = \lambda x^2 + 7x - 4$ ,  $\operatorname{Phiprime}(5) = 97$  ,

find  $\lambda$  .

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90. If  $f(x) = x^3 + 7x^2 + 8x - 9$  , find f ' (4) .

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**91.** Find the derivative of the function f defined by f(x) = mx + c at

x = 0.

**92.** Examine the differentiability of the function f defined by

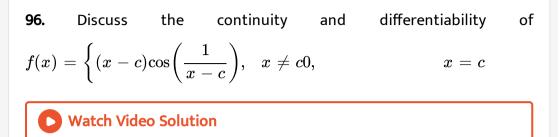
 $f(x) = \{2x+3, \quad ext{ if } -3 \leq x \leq -2x+1, \quad ext{ if } -2 \leq x < 0x+2,$ 

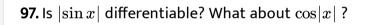
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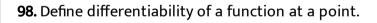
93. Write an example of a function which is everywhere continuous but

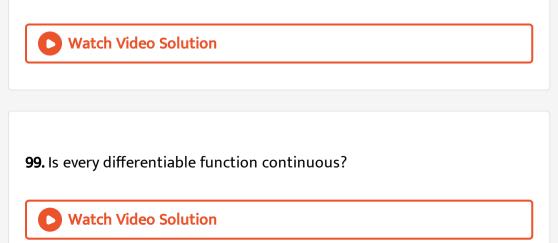
fails to be differentiable exactly at five points.



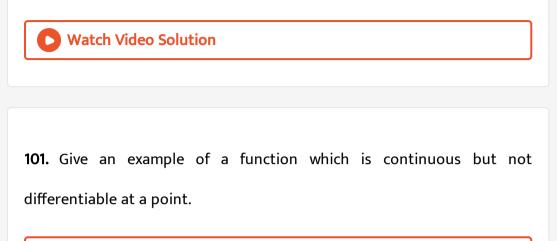








#### 100. Is every continuous function differentiable?

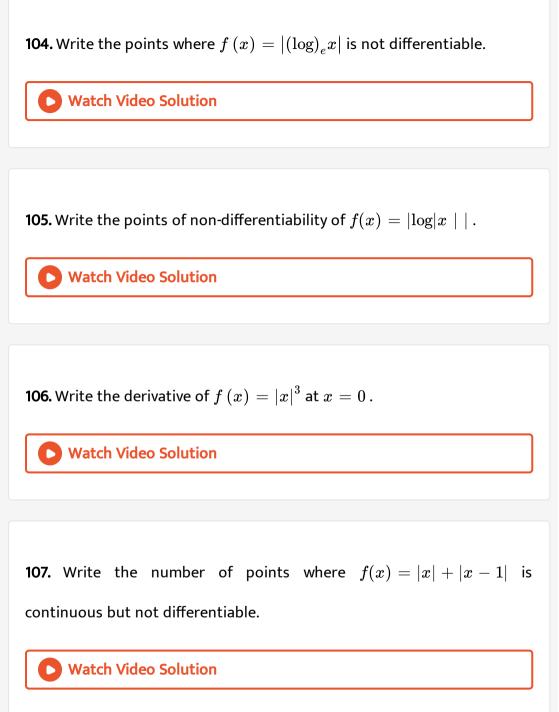


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102. If f(x) is differentiable at x=c , then write the value of  $(\ \lim \ )_{x o c} f(x)$  .

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**103.** If f(x) = |x - 2| write whether f ' (2) exists or not.



108. If  $(\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$  exists finitely, write the value of  $(\lim_{x \to c} f(x)$ .

109. Write the value of the derivative of f(x) = |x-1| + |x-3| at x=2 .

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110. If 
$$f(x)=\sqrt{x^2+9}$$
 , write the value of  $(\ \lim\ )_{x
ightarrow 4}rac{f(x)-f(4)}{x-4}$  .

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111. Let f(x)=|x| and  $g(x)=\left|x^3\right|$  , then f(x) and g(x) both are continuous at x=0 (b) f(x) and g(x) both are differentiable at x=0

(c) f(x) is differentiable but g(x) is not differentiable at x = 0 (d) f(x)and g(x) both are not differentiable at x = 0



112. The function  $f(x) = \sin^{-1}(\cos x)$  is discontinuous at x = 0 (b)

continuous at x=0 (c) differentiable at x=0 (d) none of these

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**113.** The set of points where the function f(x) = x|x| is differentiable is

(a) 
$$(-\infty, \ \infty)$$
 (b)  $(-\infty, \ 0) \cup (0, \ \infty)$  (c)  $(0, \ \infty)$  (d)  $[0, \ \infty]$ 

114. If 
$$f(x) = \begin{cases} \frac{|x+2|}{\tan^{-1}(x+2)}, & x \neq -22, \\ f(x) \text{ is continuous at } x = -2 \text{ (b) not continuous at } x = -2 \text{ (c)} \end{cases}$$
  
differentiable at  $x = -2$  (d) continuous but not derivative at  $x = -2$ 

115. Let f(x) = |x| . Then, for all  $x \; f$  is continuous (b) f is differentiable

for some x (c) f ' is continuous (d) f is continuous

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**116.** The function  $f(x) = e^{|x|}$  is (a) Continuous everywhere but not differentiable at x = 0 (b) Continuous and differentiable everywhere (c) Not continuous at x = 0 (d) None of the above

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117. The function  $f(x) = |\cos x|$  is (a) everywhere continuous and differentiable (b) everywhere continuous but not differentiable at  $(2n+1)\pi/2$ ,  $n \in Z$  (c) neither continuous nor differentiable at  $(2n+1)\pi/2$ ,  $n \in Z$  (d) none of these **118.** If  $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$ , then f(x) is (a) continuous on [-1, 1] and differentiable on (-1, 1) (b) continuous on [-1, 1] and differentiable on  $(-1, 0) \cup \varphi(0, 1)$  (c) continuous and differentiable on [-1, 1] (d) none of these

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119. If 
$$f(x)=a|\sin x|+b\,e^{|x|}+c\,|x|^3$$
 and if  $f(x)$  is differentiable at  $x=0$  , then  $a=b=c=0$  (b)  $a=0,\ b=0;\ c\in R$  (c)  $b=c=0,\ a\in R$  (d)  $c=0,\ a=0,\ b\in R$ 

120. If 
$$f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \frac{x^2}{(1+x^2)^n} +$$
, then at  $x = 0, f(x)$  has no limit (b) is discontinuous is continuous but not differentiable (d) is differentiable

121. If  $f(x)=|(\log)_e x|$  , then (a)  $f'ig(1^+ig)=1$  (b)  $f'ig(1^-ig)=-1$  (c) f'(1)=1 (d) f'(1)=-1

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122. If  $f(x) = |(\log)_e |x||$ , then f(x) is continuous and differentiable for all x in its domain] (b) f(x) is continuous for all x in its domain but not differentiable at  $x = \pm 1$  (c) f(x) is neither continuous nor differentiable at  $x = \pm 1$  (d) none of these

123. Let 
$$f(x) = \begin{cases} \frac{1}{|x|} & f \text{ or } |x| \ge 1\\ ax^2 + b & f \text{ or } |x| < 1 \end{cases}$$
. If  $f(x)$  is continuous and differentiable at any point, then (A)  $a = \frac{1}{2}, b = -\frac{3}{2}$  (B)  $a = -\frac{1}{2}, b = \frac{3}{2}$  (C)  $a = 1, b = -1$  (D) none of these

124. The function f(x) = x - [x], where [] denotes the greatest integer function is (a) continuous everywhere (b) continuous at integer points only (c) continuous at non-integer points only (d) differentiable everywhere

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125. Let  $f(x)=ig\{ax^2+1,\quad x>1,\quad x+1/2,\quad x\leq 1$  Then, f(x) is derivable at x=1 , if a=2 (b) b=1 (c) a=0 (d) a=1/2

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126. Let  $f(x) = |\sin x|$ . Then, (a) f(x) is everywhere differentiable. (b) f(x) is everywhere continuous but not differentiable at  $x = n \pi$ ,  $n \in Z$ (c) f(x) is everywhere continuous but not differentiable at  $x = (2n+1)\frac{\pi}{2}$ ,  $n \in Z$ .(d) none of these 127. Let  $f(x)=|\cos x|$  . Then, f(x) is everywhere differentiable (b) f(x)is everywhere continuous but not differentiable at  $x=n\pi,\ n\in Z$  (c) f(x) is everywhere continuous but not differentiable at  $x=(2n+1)\ rac{\pi}{2},\ n\in Z$  (d) none of these

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128. The function  $f(x)=1+|\cos x|$  is (a) continuous no where (b) continuous everywhere (c) not differentiable at x=0 (d) not differentiable at  $x=n\pi,\ n\in Z$ 

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129. The function  $f(x)=|\cos x|$  is differentiable at  $x=(2n+1)\,\pi/2,\ n\in Z$  (b) continuous but not differentiable at

 $x=(2n+1)~\pi/2,~n\in Z$  (c) neither differentiable nor continuous at  $x=n\pi,~n\in Z$  (d) none of these

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130. The function  $f(x) = (\sin(\pi[x - \pi]))$ , where [] denotes the greatest integer function, is continuous as well as differentiable for all  $x \in R$  (b) continuous for all x but not differentiable at some x (c) differentiable for all x but not continuous at some x.(d) none of these

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131. Let  $f(x) = a + b|x| + c|x|^4$ , where a, b, and c are real constants.

Then, f(x) is differentiable at x=0 , if a=0 (b) b=0 (c) c=0 (d) none

of these

132. If f(x) = |3 - x| + (3 + x), where (x) denotes the least integer greater than or equal to x, then f(x) is continuous and differentiable at x = 3 (b) continuous but not differentiable at x = 3 (c) differentiable but not continuous at x = 3 (d) neither differentiable nor continuous at x = 3

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133. If 
$$f(x) = \left\{\frac{1}{1+e^{1/x}}, x \neq 00, x = 0$$
, then  $f(x)$  is continuous as well as differentiable at  $x = 0$  (b) continuous but not differentiable at  $x = 0$  (c) differentiable but not continuous at  $x = 0$  (d) none of these

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**134.** If  $f(x) = \left\{ \frac{1 - \cos x}{x \sin x}, x \neq 0 \text{ and } \frac{1}{2}, x = 0 \text{ then at } x = 0, f(x) \text{ is} \right.$ (a)continuous and differentiable (b)differentiable but not continuous

