



MATHS

BOOKS - RD SHARMA MATHS (HINGLISH)

MAXIMA AND MINIMA

Solved Examples And Exercises

1. Find the coordinates of a point on the parabola $y = x^2 + 7x + 2$ which is closest to

the straight line y = 3x - 3.

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2. Find the points of local maxima or local minima, if any, of the following function, using the first derivative test. Also, find the local maximum or local minimum values, as the case may be: $f(x) = \frac{x}{2} + \frac{2}{x}, x > 0$

3. An open tank is to be constructed with square base and vertical sides so as to contain a given quantity of water. Show that the expenses of lining with lead will be least, if depth is made half of width.



4. Find the points of local maxima or local minima, if any, of the following function, using the first derivative test. Also, find the local



day is $Rsrac{x^2}{4}35x+25$ and the price per set at

which they may be sold is $Rs50-rac{x}{2}$. Find the

daily output to maximize the total profit.



7. A box of constant volume *c* is to be twice as long as it is wide. The material on the top and four sides cost three times as much per square metre as that in the bottom. What are the most economic dimensions?



8. The sum of the surface areas of a sphere and a cube is given. Show that when the sum of their volumes is least, the diameter of the sphere is equal to the edge of the cube.

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9. Find the maximum and the minimum values, if any, without using derivatives of the following functions: $f(x) = -(x-1)^2 + 2$ on R



10. Find the maximum and the minimum values, if any, without using derivatives of the following functions: $f(x) = 4x^2 - 4x + 4$ on R.



11. Show that the maximum volume of the cylinder which can be inscribed in a sphere of radius $5\sqrt{3}cm$ is $500\pi cm^3$.



12. Find the maximum and the minimum values, if any, without using derivatives of the following functions: $f(x) = 2x^3 + 5$ on R

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13. Find the points of local maxima or local minima and corresponding local maximum and local minimum values of each of the

following functions. Also, find the points of

inflection, if any : $f(x) = xe^x$



14. Find the points of local maxima or local minima and corresponding local maximum and local minimum values of each of the following functions. Also, find the points of inflection, if any : $f(x) = x^4 - 62x^2 + 120x + 9$

15. The function $y = a \log x + bx^2 + x$ has

extreme values at x=1,2.Find a and b

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16. The maximum value of $[x(x - 1) + 1]^{\frac{1}{3}}$ in $0 \le x \le 1$ is:

A. 0

$$\mathsf{B.}\;\frac{1}{2}$$

C. 1



Answer: B

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17. A rectangle is inscribed in a semi-circle of radius r with one of its sides on diameter of semi-circle. Find the dimensions of the rectangle so that its area is maximum. Find also the area.

18. A large window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 2 metres find the dimensions of the rectangle that will produce the largest area of the window.

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19. Find the largest possible area of a right angled triangle whose hypotenuse is 5cm



20. A closed cylinder has volume $2156cm^3$. What will be the radius of its base so that its total surface area is minimum?

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21. Find the points of local maxima or local minima, if any, of the following function, using

the first derivative test. Also, find the local maximum or local minimum values, as the case

may be: $f(x)=x^3(2x-1)^3$

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22. Show that among all positive numbers x and y with $x^2 + y^2 = r^2,$ the sum x + y is largest when $x = y = rac{r}{\sqrt{2}}$.

23. Find the points of local maxima or local minima, if any, of the following function, using the first derivative test. Also, find the local maximum or local minimum values, as the case may be: $f(x) = (x - 5)^4$

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24. Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface is $\cot^{-1}(\sqrt{2})$.



25. Show that the cone of the greatest volume which can be inscribed in a given sphere has an altitude equal to 2/3 of the diameter of the sphere.

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26. Find the points of local maxima or local minima, if any, of the following function, using the first derivative test. Also, find the local

maximum or local minimum values, as the case

may be:
$$f(x) = (x-1)(x+2)^2$$

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27. Find the points of local maxima or local minima, if any, of the following function, using the first derivative test. Also, find the local maximum or local minimum values, as the case may be: $f(x) = \cos x$, $0 < x < \pi$

28. Determine the points on the curve $x^2 = 4y$

which are nearest to the point (0,5).



29. The total area of a page is $150cm^2$. The combined width of the margin at the top and bottom is 3cm and the side 2cm. What must be the dimensions of the page in order that the area of the printed matter may be maximum?



30. A straight line is drawn through a given point P(1, 4). Determine the least value of the sum of the intercepts on the coordinate axes.

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31. The strength of a beam varies as the product of its breadth and square of its depth. Find the dimensions of the strongest beam

which can be cut from a circular log of radius

 $a \cdot$



32. The given quantity of metal is to be cost into a half cylinder with a rectangular base and semicircular ends. Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its semi-circular ends is $\pi: (\pi + 2)$.



33. A particle is moving in a straight line such that its distance s at any time t is given by $s = \frac{t^4}{4} - 2t^3 + 4t^2 - 7$. Find when its velocity is maximum and acceleration minimum.

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34. A wire of length 20m is to be cut into two pieces. One of the places will be bent into

shape of a square and the other shape of an equilateral triangle. Where the wire should be cut so that the sum of the areas of the square and triangle is minimum?

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35. A beam is supported at the two ends and is uniformly loaded. The bending moment M at a distance x from one end is given by $M = \frac{WL}{2} \times -\frac{W}{2}x^2$ Find the point at which M is maximum in each case.



36. Determine two positive numbers whose sum is 15 and the sum of whose squares is minimum.

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37. Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.



39. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8:15 is converted into anopen rectangular box by folding after removing squares of equal area

from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the length of the sides of the rectangular sheet are 24 (b) 32 (c) 45 (d) 60

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40. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side

of the square to be cut off so that the volume

of the box is the maximum possible?



value of $3x^4 - 8x^3 + 12x^2 - 48x + 1$ on the interval [1,4] .

42. Show that $f(x) = \sin x (1 + \cos x)$ is maximum at $x = \frac{\pi}{3}$ in the interval $[0, \pi]$.

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43. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

44. The sum of the surface areas of a cuboid with sides x, 2x and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal to three times the radius of sphere. Also find the minimum value of the sum of their volumes.



45. Find the maximum and minimum value of
$$f(x) = \sin x + rac{1}{2} \cos 2x \in \left[0, rac{\pi}{2}
ight]$$
.





46. Prove that $f(x) = \sin x + \sqrt{3} \cos x$ has maximum value at $x = \frac{\pi}{6}$.

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47. Find the difference between the greatest and least values of the function

$$f(x)=\sin 2x-xon\Big[-rac{\pi}{2},rac{\pi}{2}\Big].$$

48. Find the maximum and minimum values of

$$f(x)=x^{50}-x^{20}$$
 in the interval $\left[0,1
ight]$.

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49. If $f(x) = a \log |x| + bx^2 + x$ has extreme values at x = -1 and at x = 2, then find a and b .





52. If $y = rac{ax-b}{(x-1)(x-4)}$ has a turning point

P(2, -1), find the value of aandb and show

that y is maximum at P_{\cdot}



53. A metal box with a square base and vertical sides is to contain 1024 cm3 of water, the material for the top and bottom costs Rs 5 per cm2 and the material for the sides costs Rs 2.50 per cm2. Find the least cost of the box.



54. OR An open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

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55. Find the point on the curve $y^2 = 4x$ which

is nearest to the point (2, 1).





57. The function $f(x) = \sum_{r=1}^{5} (x-r)^2$ assuming minimum value at x = (a)5 (b) $\frac{5}{2}$ (c)3 (d)2

58. The least value of the function $f(x) = x^3 - 18x^2 + 96x$ in the interval [0, 9] is 126 (b) 135 (c) 160 (d) 0

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59. The maximum value of $x^{rac{1}{x}}, x>0$ is (a) $e^{rac{1}{e}}$

(b)
$$\left(rac{1}{e}
ight)^e$$
 (c) 1 (d) none of these

60.

$$f(x) = (x - a)^{2} + (x - b)^{2} + (x - c)^{2}.$$

Then, $f(x)$ has a minimum at $x = \frac{a + b + c}{3}$
(b) $\frac{1}{2}$ (c) $\frac{1}{8}$ (d) none of these
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Let

61. Show that of all the rectangles inscribed in

a given fixed circle, the square has the maximum area.


62. AB is a diameter of a circle and C is any point on the circumference of the circle. Then the area of ABC is maximum when it is isosceles the area of ABC is minimum when it is isosceles the perimeter of ABC is minimum when it when it is isosceles

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63. Find the local maxima and local minima, if any, of the following functions. Find also the

local maximum and the local minimum values,



65. Amongst all pairs of positive numbers with

product 256, find those whose sum is the least.

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66. Find two positive numbers whose sum is 14

and the sum of whose squares in minimum.

67. A beam is supported at the two ends and is uniformly loaded. The bending moment M at a distance x from one end is given by $M = \frac{WL}{2}x - \frac{W}{2}x^2$ $M = \frac{Wx}{3} - \frac{W}{3}\frac{x^3}{L^2}$ Find the point at which M is maximum in each

case.

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68. Show that all the rectangles with a given

perimeter, the square has the largest area.



69. Find all the points of local maxima and local minima of the function $f(x) = x^3 - 6x^2 + 12x - 8.$ Watch Video Solution 70. Show that the function $f(x)=4x^3-18x^2+27x-7$ has neither

maxima nor minima.

71. Find all the points of local maxima and minima and the corresponding maximum and minimum values of the function $f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$. Watch Video Solution

72. Find all the points of local maxima and minima and the corresponding maximum and



$$f(x) = 2x^3 - 21x^2 + 36x - 20.$$

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73. Find the points of local maxima, local minima and the points of inflection of the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$. Also, find the corresponding local maximum and local minimum values.





75. Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.



76. Show that the total surface area of a closed

cuboid with square base and given volume is

minimum, when it is a cube.

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77. Find the maximum and the minimum values, if any, of the following functions $f(x)=3x^2+6x+8, x\in R$

78. Find the maximum and the minimum values, if any, of the following functions f(x) = -|x-1| + 5f or $allx \in R$

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79. Find the maximum and the minimum values, if any, of the following functions $f(x)=\sin 3x+4, x\in \left(-rac{\pi}{2},rac{\pi}{2}
ight)$

80. Find the maximum and the minimum values, if any, of the following functions f(x)=|x+3| for all $x\in R$.

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81. A rectangle is inscribed in a semi-circle of radius r with one of its sides on diameter of semi-circle. Find the dimensions of the rectangle so that its area is maximum. Find also the area.



82. A wire of length 36m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces, so that the combined area of the square and the circle is minimum?



83. Find the maximum and the minimum values of $f(x) = 3x^2 + 6x + 8, \ x \in R$, if any.



84. Find the maximum and the minimum values of f(x) = x + 1 for all $x \in (-1, 1)$, if any.



85. Find the maximum and the minimum values of $f(x) = |\sin 4x + 3|, \ x \in (R)$, if any.



86. Find the maximum and the minimum values of $f(x) = x^3 + 1$ for all $x \in R$, if any.

87. Find the maximum and the minimum values of $f(x) = \sin(\sin x)$ for all $x \in R$, if any.



88. Find the maximum and the minimum values of f(x) = |x+3| for all $x \in R$, if any.

89. Find the maximum and minimum values of

 $f(x)=4x^2-4x+4$ on R , if any, without

using derivatives.



90. Find the maximum and minimum values of

 $f(x)=\ -\left(x-1
ight)^{2}+2$ on R , if any, without

using derivatives.



91. Find the maximum and minimum values of f(x) = |x+2| on R , if any, without using derivatives.

92. Find the maximum and minimum values of $f(x) = \sin 2x + 5$ on R , if any, without using derivatives.

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93. Find the maximum and minimum values of $f(x) = |\sin 4x + 3|$ on R , if any, without

using derivatives.



94. Find the maximum and minimum values of $f(x) = 2x^3 + 5$ on R , if any, without using derivatives.

95. Find the maximum and minimum values of

 $f(x)=\,-\,|x+1|+3$ on R , if any, without

using derivatives.



96. Find the maximum and minimum values of $f(x) = 16x^2 - 16x + 28$ on R , if any,

without using derivatives.



97. Find the maximum and minimum values of $f(x) = x^3 - 1$ on R , if any, without using derivatives.



98. Find all the points of local maxima and

minima of the function

$$f(x) = x^3 - 6x^2 + 9x + 15$$
 .

99. Find all the points of local maxima and local minima as well as the corresponding local maximum and local minimum values for the function $f(x) = (x - 1)^3 (x + 1)^2$.

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local minima of the function

 $f(x) = x^3 - 6x^2 + 12x + 15$.

101. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither

maxima nor minima.



102. Find the points of local maxima, local minima and the points of inflection of the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$. Also, find the corresponding local maximum and local minimum values

103. Find the local maxima or local minima, if any, of the function $f(x) = \sin x + \cos x, \ 0 < x < \frac{\pi}{2}$ using the first derivative test.

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104. Find the local maximum or local minimum,

if any, of the function

$$f(x) = \sin^4 x + \cos^4 x, \hspace{0.2cm} 0 < x < rac{\pi}{2}$$
 using

the first derivative test.

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105. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has local maxima and local minima and points of inflexion.

106. Find the points of local maxima or local minima, if any, using first derivative test, and local maximum or local minimum of $f(x) = (x - 5)^4$

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107. Find the points of local maxima or local minima, if any, using first derivative test, and local maximum or local minimum of $f(x) = x^3 - 3x$



108. Find the points of local maxima or local minima, if any, using first derivative test, and local maximum or local minimum of $f(x) = x^3(2x-1)^3$

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109. Find the points of local maxima or local minima, if any, using first derivative test, and

local maximum or local minimum of

$$f(x) = (x-1)(x+2)^2$$

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111. Find the points of local maxima or local minima, if any, using first derivative test, and local maximum or local minimum of $f(x) = x^3 - 6x^2 + 9x + 15$

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112. Find the points of local maxima or local minima, if any, using first derivative test, and local maximum or local minimum of $f(x)=\sin 2x, 0 < x < \pi$



113. Find the points of local maxima or local minima, if any, using first derivative test, and local maximum or local minimum of $f(x) = \sin x - \cos x, \ 0 < x < 2\pi$

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114. Find the points of local maxima or local minima, if any, using first derivative test, and



115. Find the points of local maxima or local minima, if any, using first derivative test, and local maximum or local minimum of $f(x) = \sin 2x - x, \ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

116. Find the points of local maxima or local minima, if any, using first derivative test, and local maximum or local minimum of $f(x) = 2\sin x - x, \ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ Watch Video Solution

117. Find the points of local maxima or local minima, if any, using first derivative test, and local maximum or local minimum of $f(x) = x\sqrt{1-x}, \ x > 0$



118. Find the points of local maxima or local minima, if any, using first derivative test, and local maximum or local minimum of $f(x) = x^3(2x-1)^3$

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119. Find the points of local maxima or local minima, if any, using first derivative test, and



120. Find all the points of local maxima and minima and the corresponding maximum and minimum values of the function $f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$.

121. Find all the points of local maxima and minima and the corresponding maximum and minimum values of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$.

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122. Find the points of local maxima and local minima, if any, and local maximum and local minimum values of $f(x) = \sin 2x - x$, where `-pi/2



123. Find the points of local maxima and local minima, if any, and local maximum and local minimum values of $f(x)=\sin x+rac{1}{2}\cos 2x$, where `0

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124. Find the points of local maxima and local minima, if any, and local maximum and local minimum values of $f(x)=\sin^4x+\cos^4x$, `0



125. Find the points of local maxima or local minima, if any, and local maximum or local minimum values of $f(x)=\sin x+\cos x$, where `0

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126. Find the points of local maxima and local minima, if any, and local maximum and local
minimum values of $f(x) = \sin x - \cos x$,

where `0

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127. Find the points of local maxima and local minima, if any, and local maximum and local minimum values of $f(x)=\sin 2x$, where

 $0 < x < \pi$

128. Find the points of local maxima and local minima, if any, and local maximum and local minimum values of $f(x)=2\cos x+x$, where

 $0 < x < \pi$

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129. Find the points of local maxima and local minima, if any, and local maximum and local minimum values of $f(x)=2\sin x-x$, $-rac{\pi}{2}< x<rac{\pi}{2}$







131. Show that none of the following functions has a local maximum or a local minimum: $x^3 + x^2 + x + 1$ (ii) e^x

132. Show that none of the following functions

has a local maximum or a local minimum: $\log x$

(ii) $\cos x, \ \ 0 < x < \pi$

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133. Find the maximum profit that a company can make, if the profit function is given $P(x) = 41 + 24x - 18x^2$.

134. At what points, the slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ at point (x, y) is given by maximum slope.

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135. If $f(x) = a \log |x| + bx^2 + x$ has extreme

values at x = -1 and at x = 2 , then find a

and b.

136. It is given that at x=1 , the function x^4-62x^2+ax+9 attains its maximum value on the interval $[0,\ 2]$. Find the value of a .

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137. If
$$y = rac{ax-b}{(x-1)(x-4)}$$
 has a turning

point P(2, -1) , find the values of a and b

and show that y is maximum at P .



is $e^{1/e}$.



139. Show that
$$\sin^p \theta \cos^q \theta$$
 attains a maximum, when $heta= an^{-1}\sqrt{rac{p}{q}}$.

140. Find the points of local maxima or minima and corresponding local maximum and minimum values of

 $f(x)=x^4-62x^2+120x+9$. Also, find the

points of inflection, if any:

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141. Find the points of local maxima or minima

and corresponding local maximum and minimum values of

 $f(x)=x^3-6x^2+9x+15$. Also, find the

points of inflection, if any:



142. Find the points of local maxima or minima and corresponding local maximum and minimum values of $f(x) = (x - 1)(x + 2)^2$.

Also, find the points of inflection, if any:

143. Find the points of local maxima or minima and corresponding local maximum and minimum values of

 $f(x)=2/x-2/x^2,\,\,x>0$. Also, find the

points of inflection, if any:

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144. Find the points of local maxima or minima

and corresponding local maximum and

minimum values of $f(x) = x \ e^x$. Also, find the

points of inflection, if any:



145. Find the points of local maxima or minima and corresponding local maximum and minimum values of

 $f(x)=x\left/2+2\left/x,
ight. x>0$. Also, find the

points of inflection, if any:

146. Find the points of local maxima or minima and corresponding local maximum and minimum values of $f(x) = (x+1)(x+2)^{1/3}, \ x \ge -2$. Also,

find the points of inflection, if any:

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147. Find the points of local maxima or minima and corresponding local maximum and minimum values of

$$f(x)=x\sqrt{32-x^2}, \; -5\leq x\leq 5$$
 . Also, find

the points of inflection, if any:



148. Find the points of local maxima or minima and corresponding local maximum and minimum values of $f(x)=x^3-2ax^2+a^2\,x$, $a>0,\ x\in R$. Also, find the points of

inflection, if any:



149. Find the points of local maxima or minima and corresponding local maximum and minimum values of $f(x) = x + rac{a^2}{x}, \ a > 0, \ x
eq 0$. Also, find

the points of inflection, if any:

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150. Find the points of local maxima or minima and corresponding local maximum and minimum values of



find the points of inflection, if any:



152. The function $y = a \log x + bx^2 + x$ has extreme values at x = 1 and x = 2 . Find aand b.



153. Show that
$$\frac{\log x}{x}$$
 has a maximum value at $x = e$.

the function
$$f(x)=rac{4}{x+2}+x$$
 .

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155. Find the maximum and minimum values of

the function f(x) = an x - 2x .



156. If $f(x) = x^3 + ax^2 + bx + c$ has a

maximum at x = -1 and minimum at x = 3

. Determine $a, \ b$ and c .





 $f(x)=2x^3-24x+107$ in the interval $[1,\ 3]$



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159. Find the maximum and minimum values of

 $f(x) = x + \sin 2x \, .$

$$f(x)=\sin x+rac{1}{2}{\cos 2x}$$
 in $\left[0,\;\pi/2
ight].$

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161. Find the maximum and minimum values of

$$f(x)=x^{50}-x^{20}$$
 in the interval $[0,\;1]$.

 $f(x)=x+\sin 2x$ in the interval $[0,\ 2\pi]$

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163. Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

164. Show that $f(x) = \sin x (1 + \cos x)$ is maximum at $x = \frac{\pi}{3}$ in the interval $[0, \pi]$.

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165. Find the absolute maximum value and the absolute minimum value of $f(x) = 2x^3 - 15x^2 + 36x + 1$ in [1, 5]

166. Find the absolute maximum value and the

absolute minimum value of

 $f(x) = \sin x + \cos x$ in $[0, \pi]$

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167. Find both the maximum and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval [0,3] .

168. Find the absolute maximum and absolute minimum values of $f(x) = 4x - \frac{x^2}{2}$ in [-2, 4.5] **Vatch Video Solution**

169. Find the absolute maximum and absolute minimum values of $f(x) = \left(x - 1\right)^2 + 3$ in $\left[-3, \ 1
ight]$

A. absolute maximum = 19 absolute minimum = 3

B. absolute	maximum	=	16	absolute
minimum = 3				
C. absolute	maximum	=	3	absolute
minimum = 19				
D. None of these				
•				

Answer: A

170. Find the absolute maximum and absolute

minimum values of
$$f(x)=3x^4-8x^3+12x^2-48x+25$$
 in $[0,\ 3]$

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171. Find the absolute maximum and the absolute minimum values of the following function in the given intervals: $f(x) = (x-2)\sqrt{x-1}$ in [1,9]



172. Find the maximum value of $2x^3 - 24x + 107$ in the interval [1, 3]. Find the maximum value of the same function in [-3, -1].

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173. Find the absolute maximum and minimum values of the function f given by $f(x)=\cos^2 x+\sin x$, $x\in [0,\ \pi]$.



175. Find the absolute maximum and minimum values of a function f given by





176. Find two numbers whose sum is 24 and

whose product is as large as possible.

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177. Find two positive numbers x and y such that x + y = 60 and xy^3 is maximum.



178. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is a maximum.



179. Amongst all pairs of positive numbers with product 256, find those whose sum is the least.





14 and the sum of whose squares is minimum.



181. A beam of length l is supported at one end. If W is the uniform load per unit length, the bending moment M at a distance x from the end is given by $M = \frac{1}{2}lx - \frac{1}{2}Wx^2$. Find the point on the beam at which the bending

moment has the maximum value.



183. Show that all the rectangles with a given perimeter, the square has the largest area.



185. Show that of all the rectangles inscribed

in a given circle, the square has the maximum

area.



186. Show that the rectangle of maximum perimeter which can be inscribed in a circle of

radius a is a square of side $\sqrt{2}a$.

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187. AB is a diameter of a circle and C is any point on the circle. Show that the area of ABC is maximum, when it is isosceles.

188. Tangent to the circle $x^2 + y^2 = a^2$ at any point on it in the first quadrant makes intercepts OA and OB on x and y axes respectively, O being the centre of the circle. Find the minimum value of OA + OB.

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189. If the sum of the lengths of the hypotenues and a side of a right angled triangle is given, show that the area of the

triangle is maximum when the angle between

them is $\pi/3$.



190. Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.
191. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.



192. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will

be least when depth of the tank is half of its

width.



193. A metal box with a square base and vertical sides is to contain 1024 cm3 of water, the material for the top and bottom costs Rs 5 per cm2 and the material for the sides costs Rs 2.50 per cm2. Find the least cost of the box.



194. An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units. Watch Video Solution

195. The sum of the surface areas of the cuboid with sides x, 2x and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of the volumes is minimum, if x is equal to three

times the radius of the sphere. Also, find the

minimum value of the sum of their volumes.



196. Show that the triangle of maximum area

that can be inscribed in a given circle is an equilateral triangle.



197. A wire of length 36m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces, so that the combined area of the square and the circle is minimum?

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198. A figure consists of a semi-circle with a rectangle on its diameter. Given the perimeter

of the figure, find its dimensions in order that

the area may be maximum.



199. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum? Also, find the maximum volume.



200. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8:15 is converted into anopen rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the length of the sides of the rectangular sheet are 24 (b) 32 (c) 45 (d) 60

201. Find the volume of the larges cylinder that can be inscribed in a sphere of radius r



202. Show that a right-circular cylinder of given volume, open at the top, has minimum total surface area, provided its height is equal to the radius of the base.



203. Show that the height of a closed right circular cylinder of given surface and maximum volume, is equal to the diameter of its base.

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204. Show that a cylinder of a given volume which is open at the top has minimum total surface area, when its height is equal to the radius of its base.



maximum volume that can be inscribed in a

sphere of radius R is $rac{2R}{\sqrt{3}}$.



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206. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.





207. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

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208. Prove that the volume of the largest cone

that can be inscribed in a sphere of radius R is

 $\frac{8}{27}$ of the volume of the sphere.





209. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

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210. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi

vertical angle is one-third that of the cone and

the greatest volume of cylinder is ${4\over 27}\pi h^3 an^2 lpha.$

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211. Let AP and BQ be two vertical poles at points A and B respectively. If AP = 16m, BQ = 22m and AB = 20m, then find the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum.





212. If the length of three sides of a trapezium

other than base are equal to 10 cm, then find

the area of trapezium when it is maximum.

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213. A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs 300 per subscriber. The company proposes to increase the annual subscription

and it is believed that every increase of Rs 1 one subscriber will discontinue the service. Find what increase will bring maximum revenue?



214. Find the point on the curve $y^2 = 4x$

which is nearest to the point (2, 1) .

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215. A jet of enemy is along the curve $y = x^2 + 2$ and a soldier is placed at (3,2).Find the minimum distance between the jet and soldier.



216. The shortest distance between line
$$y-x=1$$
 and curve $x=y^2$ is (a) $\frac{3\sqrt{2}}{8}$ (b) $\frac{8}{3\sqrt{2}}$ (c) $\frac{4}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{4}$

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217. Find the shortest distance of the point (0, c) from the parabola $y=x^2$, where $0\leq c\leq 5$.

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218. Find the area of the greatest isosceles triangle that can be inscribed in the ellipse $\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = 1$ having its vertex

coincident with one extremity of the major

axis.



220. A point on the hypotenuse of a right triangle is at distances a and b from the sides of the triangle. Show that the minimum length

of the hypotenuse is
$$\left(a^{rac{2}{3}}+b^{rac{2}{3}}
ight)^{rac{3}{2}}$$

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221. Determine two positive numbers whose sum is 15 and the sum of whose squares is minimum.

222. Divide 64 into two parts such that the sum of the cubes of two parts is minimum.

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223. How should we choose two numbers, each

greater than or equal to -2 , whose sum is

1/2 so that the sum of the first and the cube

of the second is minimum?

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224. Divide 15 into two parts such that product of square of one part and cube of other is maximum



225. Of all the closed cylindrical cans (right circular), which enclose a given volume of 100

cm3, which has the minimum surface area?



226. A beam is supported at the two ends and is uniformly loaded. The bending moment M at a distance x from one end is given by $M = \frac{WL}{2}x - \frac{W}{2}x^2$. Find the point at which M is maximum.

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227. A beam is supported at the two ends and is uniformly loaded. The bending moment M at a distance x from one end is given by

 $M=rac{Wx}{3}-rac{W}{3}rac{x^3}{L^2}$. Find the point at which

M is maximum.

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228. A wire of length 28m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces so that the combined area of the circle and the square is minimum?

229. A wire of length 20m is to be cut into two pieces. One of the places will be bent into shape of a square and the other shape of an equilateral triangle. Where the wire should be cut so that the sum of the areas of the square and triangle is minimum?

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230. Given the sum of the perimeters of a square and a circle, show that the sum of their

areas is least when one side of the square is

equal to diameter of the circle.



231. Find the largest possible area of a right angled triangle whose hypotenuse is 5 cm

long.



232. Two sides of a triangle have lengths 'a' and 'b' and the angle between them is θ . What value of θ will maximize the area of the triangle? Find the maximum area of the triangle also.

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233. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the

flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum? Also, find the maximum volume.



234. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off squares from each corners and folding up the flaps. What should be the side

of the square to be cut off so that the volume

of the box is maximum possible?



235. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m3. If building of tank costs Rs 70 per square metre for the base and Rs 45 per square metre for sides, what is the cost of least expensive tank?





236. A window in the form of a rectangle is surmounted by a semi-circular opening. The total perimeter of the window is 10m. Find the dimensions of the rectangular part of the window to admit maximum light through the whole opening.



237. A large window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 metres find the dimensions of the rectangle that will produce the largest area of the window.

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238. Show that the height of the cylinder of maximum volume that can be inscribed in a

sphere of radius R is $rac{2R}{\sqrt{3}}$.

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239. A rectangle is inscribed in a semi-circle of radius r with one of its sides on diameter of semi-circle. Find the dimensions of the rectangle so that its area is maximum. Find also the area.



240. Prove that a conical tent of given capacity

will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base.



241. Show that the cone of the greatest volume which can be inscribed in a given sphere has an altitude equal to 2/3 of the diameter of the sphere.

242. Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface is $\cot^{-1}(\sqrt{2})$.



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243. An isosceles triangle of vertical angle 2θ is

inscribed in a circle of radius a . Show that the

area of the triangle is maximum when $heta=rac{\pi}{6}$.



244. Prove that the least perimeter of an isosceles triangle in which a circle of radious r can be inscribed is $6\sqrt{3}r$



245. Find the dimensions of the rectangle of perimeter 36cm which will sweep out a volume as large as possible when revolved about one of its sides.



246. Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12cm is 16cm.

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247. A closed cylinder has volume 2156cm3.

What will be the radius of its base so that its

total surface area is minimum?

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248. Show that the maximum volume of the cylinder which can be inscribed in a sphere of radius $5\sqrt{3}cm$ is $500 \pi cm^3$.

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249. Show that among all positive numbers x and y with $x^2 + y^2 = r^2$, the sum x + y is

larger when $x=y=r/\sqrt{2}$.

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250. Determine the points on the curve $x^2 = 4y$ which are nearest to the point (0, 5)Watch Video Solution **251.** Find the point on the curve $y^2 = 4x$ which is nearest to the point (2, -8).

252. Find the point on the curve $x^2 = 8y$ which is nearest to the point (2, 4).

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253. Find the point on the parabolas $x^2 = 2y$

which is closest to the point (0, 5).

254. Find the coordinates of a point on the parabola $y = x^2 + 7x + 2$ which is closest to

the straight line y = 3x - 3.



255. Find the point on the curve $y^2 = 2x$ which is at a minimum distance from the point (1, 4).



256. Find the maximum slope of the curve
$$y = -x^3 + 3x^2 + 2x - 27$$
. Vatch Video Solution

257. The total cost of producing x radio sets per day is $Rs\frac{x^2}{4}35x + 25$ and the price per set at which they may be sold is $Rs50 - \frac{x}{2}$. Find the daily output to maximize the total profit. **258.** A manufacturer can sell x items at a price of Rs. $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5} + 500\right)$. Find the number of

items he should sell to earn maximum profit.

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259. An open tank is to be constructed with square base and vertical sides so as to contain a given quantity of water. Show that the

expenses of lining with lead will be least, if

depth is made half of width.



260. A box of constant volume *c* is to be twice as long as it is wide. The material on the top and four sides cost three times as much per square metre as that in the bottom. What are the most economic dimensions?



261. The sum of the surface areas of a sphere and a cube is given. Show that when the sum of their volumes is least, the diameter of the sphere is equal to the edge of the cube.

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262. The given quantity of metal is to be cost into a half cylinder with a rectangular base and semicircular ends. Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the

diameter of its semi-circular ends is $\pi : (\pi + 2).$

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263. The strength of a beam varies as the product of its breadth and square of its depth. Find the dimensions of the strongest beam which can be cut from a circular log of radius

 $a \cdot$



264. A straight line is drawn through a given point P(1, 4). Determine the least value of the sum of the intercepts on the coordinate axes.



265. The total area of a page is $150cm^2$. The combined width of the margin at the top and bottom is 3cm and the side 2cm. What must be the dimensions of the page in order that the area of the printed matter may be maximum?



266. The space s described in time t by a particle moving in a straight line is given by $s = t^5 - 40t^3 + 30t^2 + 80t - 250$. Find the minimum value of acceleration.

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267. A particle is moving in a straight line such that its distance s at any time t is given by



268. Write necessary condition for a point x = c to be an extreme point of the function f(x) .

269. Write sufficient conditions for a point

x = c to be a point of local maximum.

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270. If f(x) attains a local minimum at x=c ,

then write the values of f'(c) and f(c) .



271. Write the minimum value of
$$f(x) = x + \frac{1}{x}, x > 0$$



273. Write the point where $f(x) = x (\log)_e x$

attains minimum value.

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274. Find the least value of $f(x) = ax + rac{b}{x},$ where $a > 0, \ b > 0$ and x > 0.

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275. Write the minimum value of $f(x) = x^x$.



278. The maximum value of $x^{rac{1}{x}}, x>0$ is (a) $e^{rac{1}{e}}$

(b)
$$\left(\frac{1}{e}\right)^e$$
 (c) 1 (d) none of these

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279. If
$$ax+rac{b}{x}\geq c$$
 for all positive x where $a,\ b,\ >0$, then $ab<rac{c^2}{4}$ (b) $\geqrac{c^2}{4}$ (c) $ab\geqrac{c}{4}$ (d) none of these

280. The minimum value of $\frac{x}{\left(\log\right)_e x}$ is e (b)

1/e (c) 1 (d) none of these

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281. For the function $f(x) = x + \frac{1}{x} x = 1$ is a point of maximum (b) x = -1 is a point of minimum (c) maximum value > minimum value (d) maximum value < minimum value





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(d) none of these

284. The number which exceeds its square by the greatest possible quantity is $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) none of these



285. Let

$$f(x) = (x - a)^2 + (x - b)^2 + (x - c)^2$$
.
Then, $f(x)$ has a minimum at $x = \frac{a + b + c}{3}$.
(b) $3\sqrt{abc}$ (c) $\frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$ (d) none of these

286. The sum of two non-zero numbers is 8, the minimum value of the sum of their reciprocals is $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{8}$ (d) none of these

287. The function
$$f(x) = \sum_{r=1}^{5} (x-r)^2$$
 assuming minimum value at $x =$ (a)5 (b) $\frac{5}{2}$ (c)3 (d)2

288. At $x = \frac{5\pi}{6}$, $f(x) = 2\sin 3x + 3\cos 3x$ is (a) 0 (b) maximum (c) minimum (d) none of these



289. If x lies in the interval [0, 1] , then the

least value of x^2+x+1 is (a) 3 (b) 3/4 (c) 1

(d) none of these

290. The least value of the function $f(x) = x^3 - 18x^2 + 96x$ in the interval [0, 9] is 126 (b) 135 (c) 160 (d) 0

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292. Find the point on the curve $y^2 = 4x$ which is nearest to the point (2, 1).

293. If x + y = 8 , then the maximum value of

xy is (a) 8 (b) 16 (c) 20 (d) 24

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294. The least and greatest values of $f(x) = x^3 - 6x^2 + 9x$ in [0, 6], are 3, 4 (b) 0, 6 (c) 0, 3 (d) 3, 6

295.
$$f(x) = \sin + \sqrt{3} \cos x$$
 is maximum when

A.
$$x=rac{\pi}{3}$$

B. $x=rac{\pi}{4}$
C. $x=rac{\pi}{6}$

D. x = 0

Answer: C

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296. If a cone of maximum volume is inscribed in a given sphere, then the ratio of the height of the cone to the diameter of the sphere is 3/4 (b) 1/3 (c) 1/4 (d) 2/3

297. The minimum value of $\left(x^2 + rac{250}{x}
ight)$ is (a)

75 (b) 50 (c) 25 (d) 55

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298. If
$$f(x)=x+rac{1}{x},\ x>0$$
 , then its greatest value is -2 (b) 0 (c) 3 (d) none of

these

299. If
$$f(x) = \frac{1}{4x^2 + 2x + 1}$$
, then its maximum value is
(a) $\frac{4}{3}$
(b) $\frac{2}{3}$
(c) 1
(d) $\frac{3}{4}$
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300. Let x, y be two variables and $x>0, \ xy=1$, then minimum value of

$$x+y$$
 is (a) 1 (b) 2 (c) $2rac{1}{2}$ (d) $3rac{1}{3}$

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301.

 $f(x)=1+2\sin x+3\cos^2 x,\ 0\leq x\leq rac{2\pi}{3}$ is Minimum at $x=\pi/2$ (b) Maximum at $x=\sin^{-1}ig(1/\sqrt{3}ig)$ (c) Minimum at $x=\pi/6$ (d) Maximum at $\sin^{-1}(1/6)$





304. Let $f(x) = 2x^3 - 3x^2 - 12x + 5$ on [-2, 4]. The relative maximum occurs at x = -2 (b) -1 (c) 2 (d) 4 Watch Video Solution

305. The minimum value of $x (\log)_e x$ is equal

to e (b) 1/e (c) -1/e (d) 2e (e) e

306. The minimum value of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$ is (a) -128(b) -126(c) -120

(d) none of these



1. A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs. 300 per subscriber. The company proposes to increase the annual subscription and it is believed that every increase of Rs. 1 one subscriber will discontinue the services. Find what increase will bring maximum revenue?



2. The combined resistance R of two resistors

 R_1 and $R_2(R_1, R_2>0)$ is given by $rac{1}{R}=rac{1}{R_1}+rac{1}{R_2}\,.$

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