MATHS

BOOKS - RD SHARMA MATHS (HINGLISH)

MEAN VALUE THEOREMS

Solved Examples And Exercises

1. Using Lagranges mean value theorem, show that `sin<>0.`

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2. Using mean value theorem, prove that $\tan x > x$ for all $x\left(0, \frac{\pi}{2}\right)$.

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3. Using Lagranges mean value theorem, find a point on the curve $y = \sqrt{x-2}$ defined on the interval [2,3], where the tangent is parallel to the chord joining the end points of the curve.

A. x= $\frac{9}{2}$

B. x= $\frac{9}{4}$

C. x= $\frac{3}{2}$

D. None of these

Answer: C

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4. Verify Lagranges mean value theorem for the following functions on the indicated intervals. $f(x)=x-2\sin x on[-\pi,\pi]$

$f(x)=2\sin x+\sin 2x on[0,\pi]$ $f(x)=(\log)_e x on[1,2]$

$f(x)=\{2+x^3, \quad \text{if} \quad x\le 13x, x>0 on[-1,2]$

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5. Verify lagranges mean value theorem for the function $f(x)=(x-3)(x-6)(x-9) on[3,5]$

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6. Verify lagranges mean value theorem for the following functions on the indicated intervals. Also, find a point c in the indicated interval: $f(x) = x(x-2)on[1, 3]$ $f(x) = x(x-1)(x-2)on\left[0, \frac{1}{2}\right]$.

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7. Find the point on the curve $y = \cos x - 1, x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ at which the tangent in parallel to the x-axis.

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8. It is given that the Rolles theorem holds for the function $f(x) = x^3 + bx^2 + cx, x \in [1, 2]$ at the point $x = \frac{4}{3}$.

Find the values of b and c dot

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9. Using Lagranges mean value theorem, prove that $\frac{b-a}{b} < \log\left(\frac{b}{a}\right) < \frac{b-a}{a}$,where $0 < a < b$

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10. Let $fandg$ be differentiable on [0,1] such that $f(0) = 2, g(0) = 0, f(1) = 6andg(1) = 2.$ Show that there exists $c \in (0, 1)$ such that $f'(c) = 2g'(c)$.

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11. If the value of c prescribed inRolles theorem for the function $f(x) = 2x(x-3)^n$ on the interval $\left[0, 2\sqrt{3}\right] is \frac{3}{4}$, write the value of n (a positive integer).

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12. Using Lagranges mean value theorem, prove that $(b-a)\sec^2 a < (\tan b - \tan a) < (b-a)\sec^2 b$, where $0 < a < b < \frac{\pi}{2}$

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13. It is given that for the function $f(x) = x^3 - 6x^2 + ax + bon[1, 3]$

, Rolles theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$.

Find the values of $aandb$, if $f(1) = f(3) = 0.$

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14. Verify Rolles theorem for each of the following functions on the indicated intervals:

$f(x) = x(x+3)e^{-\frac{x}{2}}on[-3, 0]$

$f(x) = e^x(\sin x - \cos x)on\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

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15. Find a point on the curve $y = x^2 + x,$ where the tangent is parallel to the chord joining (0,0) and (1,2).

A. $x = 2$

B. $x = 1$

C. $x = \frac{1}{2}$

D. $x = \frac{2}{3}$

Answer: C

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16. Let f be a twice differentiable function such that $f(a) = f(b) = 0$ and $f(c) > 0$ for $a < c < b$. Prove that there exists at least one value λ between $a and b$ fow which $f^{\lambda} < 0.$

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17. find the percentage error in calculating the volume of the cubical box if an error of 1% is made in measuring the length of the edges of the cube.

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18. Verify Rolles theorem for the function $f(x) = x^2 - 5x + 6$ on the interval $[2, 3]$.

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19. Verify Rolles theorem for the function $f(x) = (x - a)^m (x - b)^n$ on the interval $[a, b]$, where m, n are positive integers.

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20. Rolls theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ is applicable in the interval .

A. $[1, 4]$

B. $[1, 2]$

C. $[1, 3]$

D. Cant say

Answer: C

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21. Verify Rolle's theorem for each of the following functions on indicated intervals; $f(x)=\sin^2 x$ on $0 \leq x \leq \pi$ $f(x)=\sin x+\cos x-1$ on $\left[0, \frac{\pi}{2}\right]$ $f(x)=\sin x-\sin 2x$ on $[0, \pi]$

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22. Verify Rolle theorem for the function $f(x)=\log\left\{\frac{x^2+ab}{x(a+b)}\right\} on[a, b],$ where `0

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23. Rolles theorem is applicable for

A. $f(x) = |x|$ on $[-1, 1]$

B. f(x) = [x] for x ∈ [5, 9]

C. $f(x) = x^2 - 1x \in [1, 2]$

D. NOne of these

Answer: D

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24. Discuss the applicability of Rolles theorem for the function $f(x) = 3 + (x - 2)^{2/3}$ on $[1, 3]$

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25. Discuss the applicability of Rolles theorem for $f(x) = \tan x$ on $[0, \pi]$

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26. Discuss the applicability of Rolle's theorem on the function

$f(x) = \{x^2 + 1$, when $0 \leq x \leq 1$

$\{3 - x$, when $1 < x \leq 2$

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27. Verify Rolles theorem for the function $f(x) = x^2 - 5x + 6$ on the interval [2, 3].

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28. Verify Rolles theorem for the function $f(x) = x(x - 3)^2, \ 0 \leq x \leq 3$.

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29. Verify Rolles theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ on the interval [1, 3].

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30. Verify Rolles theorem for the function $f(x) = (x-a)^m (x-b)^n$ on the interval $[a, b]$, where m, n are positive integers.

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31. Verify Rolles theorem for the function $f(x) = \sqrt{4-x^2}$ on $[-2, 2]$.

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32. Verify Rolles theorem for the function $f(x) = \log\left\{\frac{x^2+ab}{x(a+b)}\right\}$ on $[a, b]$, where `0

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33. Verify Rolles theorem for the function: $f(x) = s \in^2 x$ on $0 \leq x \leq \pi$

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34. Verify Rolles theorem for the function: $f(x) = \sin x + \cos x - 1$ on $[0, \ \pi/2]$.

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35. Verify Rolles theorem for the function: $f(x) = \sin x - \sin 2x$ on $[0, \ \pi]$

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36. Verify Rolles theorem for the function: $f(x) = x(x+3)e^{-x/2}$ on $[-3, 0]$.

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37. Verify Rolles theorem for the function: $f(x) = e^{x}(\sin x - \cos x)$ on $[\pi/4, 5\pi/4]$.

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38. It is given that for the function $f(x) = x^3 - 6x^2 + ax + b$ on $[1, 3]$, Rolles theorem

holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b, if $f(1) = f(3) = 0$.

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39. It is given that for the function f given by $f(x) = x^3 + bx^2 + ax$, $x \in [1, 3]$. Rolles theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b.

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40. Find the point on the curve $y = \cos x - 1, \quad x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ at which the tangent is parallel to the x-axis.

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41. Discuss the applicability of Rolles theorem for the function $f(x) = 3 + (x-2)^{2/3}$ on $[1,\ 3]$

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42. Discuss the applicability of Rolles theorem for the function $f(x) = [x]$ for $-1 \leq x \leq 1$, where $[x]$ denotes the greatest integer not exceeding x

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43. Discuss the applicability of Rolles theorem for the function $f(x) = \dfrac{\sin 1}{x}$ for $-1 \leq x \leq 1$

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44. Discuss the applicability of Rolles theorem for the function $f(x) = 2x^2 - 5x + 3$ on $[1, 3]$

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45. Discuss the applicability of Rolles theorem for the function $f(x) = x^{2/3}$ on $[-1, 1]$

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46. Verify Rolles theorem for function $f(x) = x^2 - 8x + 12$ on $[2, 6]$

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47. Verify Rolles theorem for function $f(x) = x^2 - 4x + 3$ on $[1, 3]$

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48. Verify Rolles theorem for function $f(x) = (x-1)(x-2)^2$ on $[1, 2]$

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49. Verify Rolles theorem for function $f(x) = x(x-1)^2$ on $[0, 1]$

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50. Verify Rolles theorem for function $f(x)=(x^2-1)(x-2)$ on $[-1,\ 2]$

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51. Verify Rolles theorem for function $f(x)=x(x-4)^2$ on $[0,\ 4]$

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52. Verify Rolles theorem for function $f(x)=x(x-2)^2$ on $[0,\ 2]$

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53. Verify Rolles theorem for function $f(x) = x^2 + 5x + 6$ on $[-3, -2]$

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54. Verify Rolles theorem for function $f(x) = \cos 2(x - \pi/4)$ on $[0, \pi/2]$.

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55. Verify Rolles theorem for function $f(x) = \sin 2x$ on $[0, \pi/2]$

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56. Verify Rolles theorem for function $f(x) = \cos 2x$ on $[-\pi/4, \pi/4]$

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57. Verify Rolles theorem for function $f(x) = e^x \sin x$ on $[0, \pi]$

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58. Verify Rolles theorem for function $f(x) = e^x \cos x$ on $[-\pi/2, \pi/2]$

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59. Verify Rolles theorem for function $f(x) = \cos 2x$ on $[0, \ \pi]$

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60. Verify Rolles theorem for function $f(x) = \frac{\sin x}{e^x}$ on $0 \le x \le \pi$

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61. Verify Rolles theorem for function $f(x) = \sin 3x$ on $[0, \ \pi]$

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62. Verify Rolles theorem for function $f(x) = e^{1 - x^2}$ on $[-1, 1]$

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63. Verify Rolles theorem for function $f(x) = \log(x^2 + 2) - \log 3$ on $[-1, 1]$

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64. Verify Rolles theorem for function $f(x) = \sin x + \cos x$ on $[0, \pi/2]$

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65. Verify Rolles theorem for function $f(x) = 2\sin x + \sin 2x$ on $[0, \ \pi]$

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66. Verify Rolle's theorem for function $f(x) = \frac{x}{2} - \sin\left(\frac{\pi x}{6}\right)$ on $[-1, \ 0]$

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67. Verify Rolles theorem for function $f(x) = \frac{6x}{\pi} - 4\,s \in^2 x$ on $\left[0, \ \frac{\pi}{6}\right]$

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68. Verify Rolles theorem for function $f(x) = 4^{\sin x}$ on $[0, \pi]$

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69. Verify Rolles theorem for function $f(x) = x^2 - 5x + 4$ on $[1, 4]$

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70. Verify Rolles theorem for function $f(x) = \sin^4 x + \cos^4 x$ on $\left[0, \frac{\pi}{2}\right]$

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71. Verify Rolles theorem for the function: $f(x) = \sin x - \sin 2x$ on $[0, \ \pi]$

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72. Using Rolles theorem, find points on the curve $y = 16 - x^2$, $x \in [-1, \ 1]$, where tangent is parallel to x-axis

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73. At what points on the curve $y = x^2$ on $[-2, 2]$ is the tangent parallel to x-axis?

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74. At what points on the curve $y = e^1 - x^2$ on $[-1, 1]$ is the tangent parallel to x-axis?

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75. If $f : [-5, 5] \rightarrow R$ is differentiable and if $f'(x)$ doesnt vanish anywhere, then prove that $f(-5) \neq f(5)$.

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76. It is given that the Rolles theorem holds for the function $f(x) = x^3 + bx^2 + cx, \quad x \in [1, 2]$ at the point $x = \frac{4}{3}$. Find the values of b and c.

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77. Verify Lagrange's mean value theorem for the function $f(x) = (x - 3)(x - 6)(x - 9)$ on the interval $[3. 5]$

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78. Verify Lagranges mean value theorem for $f(x) = x(x - 2)$ on $[1, \ 3]$ on the indicated intervals. Also, find a point c in the indicated interval:

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79. Verify Lagranges mean value theorem for $f(x) = x - 2\sin x$ on $[-\pi, \pi]$

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80. Verify L.M.V. theorem for $f(x) = \{2 + x^3, x \leq 13x, x > 1$ on $[-1, 2]$

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81. Verify Lagranges mean value theorem for function $f(x) = x^2 - 1$ on $[2, 3]$ and find a point $'c'$ in the

indicated interval

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82. Verify Lagranges mean value theorem for function $f(x) = x(x-1)$ on $[1, 2]$ and find a point $'c'$ in the indicated interval:

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83. Verify Lagranges mean value theorem for function $f(x) = x^2 - 3x + 2$ on $[-1, 2]$

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84. Verify Lagranges mean value theorem for function $f(x) = 2x^2 - 3x + 1$ on $[1, 3]$ and find a point $'c'$ in the indicated interval:

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85. Verify Lagranges mean value theorem for function $f(x) = x^2 - 2x + 4$ on $[1, 5]$ and find a point $'c'$ in the indicated interval:

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86. Verify Lagranges mean value theorem for function $f(x) = 2x - x^2$ on $[0, 1]$ and find a point $'c'$ in the indicated interval:

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87. Verify Lagranges mean value theorem for function $f(x) = \sqrt{25 - x^2}$ on $[-3, 4]$ and find a point $'c'$ in the indicated interval:

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88. Verify Lagranges mean value theorem for function $f(x) = \tan^{-1} x$ on $[0, 1]$ and find a point $'c'$ in the indicated interval:

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89. Verify Lagranges mean value theorem for function $f(x) = x + \frac{1}{x}$ on $[1,\ 3]$ and find a point $'c'$ in the indicated interval:

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90. Verify Lagranges mean value theorem for function $f(x) = \sqrt{x^2 - 4}$ on $[2,\ 4]$ and find a point $'c'$ in the indicated interval:

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91. Verify Lagranges mean value theorem for function $f(x) = x^2 + x - 1$ on $[0,\ 4]$ and find a point $'c'$ in the indicated interval:

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92. Discuss the applicability of Lagranges mean value theorem for the function $f(x) = |x|$ on $[-1, 1]$.

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93. Show that the lagranges mean value theorem is not applicable to the function $f(x) = \frac{1}{x}$ on $[-1, 1]$.

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94. Find a point on the parabola $y = (x-4)^2$, where the tangent is parallel to the chord joining (4, 0) and (5, 1).

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95. Find a point on the curve $y = x^2 + x$, where the tangent is parallel to the chord joining (0, 0) and (1, 2).

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96. Find a point on the parabola $y = (x-3)^2$, where the tangent is parallel to the chord joining (3, 0) and (4, 1).

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97. Find the points on the curve $y = x^3 - 3x$, where the tangent to the curve is parallel to the chord joining $(1, \ -2)$ and $(2, \ 2)$

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98. Find a point on the curve $y = x^3 + 1$ where the tangent is parallel to the chord joining $(1, \ 2)$ and $(3, \ 28)$.

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99. Let C be a curve defined parametrically as $x = a\cos^3\theta$, $y = a\sin^3\theta, \ \ 0 \leq \theta \leq \frac{\pi}{2}$. Determine a point P on C, where the tangent to C is parallel to the chord joining the points $(a,\ 0)$ and $(0,\ a)$.

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100. If $f(x) = Ax^2 + Bx + C$ is such that $f(a) = f(b)$, then write the value of c in Rolles theorem.

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101. State Rolle's theorem.

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102. Cauchys mean value theorem

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103. Find the value of c prescribed by Lagranges mean value theorem for the function $f(x)=\sqrt{x^2-4}$ defined on $[2, 3]$.

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104. For the function $f(x)=x+\frac{1}{x}, \quad x\in[1, 3]$, the value of c for the Lagranges mean value theorem is (a) 1 (b) $\sqrt{3}$ (c) 2 (d) none of these

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105. The value of c in Rolles theorem when $f(x) = 2x^3 - 5x^2 - 4x + 3$, is $x \in [1/3,\ 3]$

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106. When the tangent to the curve $y = x \log x$ is parallel to the chord joining the points (1, 0) and $(e,\ e)$, the value of x is $e^{1/1-e}$ (b) $e^{(e-1)(2e-1)}$ (c) $e^{\frac{2e-1}{e-1}}$ (d) $\frac{e-1}{e}$

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107. The value of c in Rolles theorem for the function $f(x) = \frac{x(x+1)}{e^x}$ defined on $[-1,\ 0]$ is 0.5 (b) $\frac{1+\sqrt{5}}{2}$ (c) $\frac{1-\sqrt{5}}{2}$ (d) -0.5

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108. The value of c in Largrange's mean value theorem for the function $f(x) = x(x-2)$ when $x \in [1, 2]$ is

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109. The value of c in Rolles theorem for the function $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$ is (a) 1 (b) -1 (c) $3/2$ (d) $1/3$

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110. If $f(x) = e^x \sin x$ in $[0, \ \pi]$, then c in Rolles theorem is

$\pi / 6$ (b) $\pi / 4$ (c) $\pi / 2$ (d) $3\pi / 4$

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Others

1. Discuss the applicability of Rolles theorem for the function

$$f(x) = \{ -4x + 5, \quad 0 \leq x \leq 1$$

$$\{2x - 3, \quad 1 \leq x \leq 2$$

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2. At what points on the curve $y = 12(x + 1)(x - 2)$ on $[-1, 2]$.

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3. Examine if Rolles theorem is applicable to any one of the following functions: $f(x) = [x]$ for $x \in [5, 9]$ Can you say something about the converse of Rolles Theorem from these functions?

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4. Verify Lagranges mean value theorem for $f(x) = x(x - 1)(x - 2)$ on $[0, 1/2]$

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5. Using Lagranges mean value theorem, find a point on the curve $y=\sqrt{x-2}$ defined on the interval $[2, 3]$, where the tangent is parallel to the chord joining the end points of the curve.

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6. Verify Lagranges mean value theorem for $f(x)=2\sin x+\sin 2x$ on $[0,\ \pi]$

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7. Using Langrange mean value theorem, show that $\sin x < x$ for $x > 0$

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8. Using mean value theorem, prove that $\tan x > x$ for all $x\ (0,\ \pi/2)$

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9. Let f and g be differentiable on $[0,\ 1]$ such that $f(0) = 2$, $g(0) = 0$, $f(1) = 6$ and $g(1) = 2$. Show that there exists $c \in (0,\ 1)$ such that $f'(c) = 2\,g'(c)$.

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10. Verify Lagranges mean value theorem for function $f(x) = x^3 - 2x^2 - x + 3$ on $[0, 1]$ and find a point $'c'$ in the indicated interval:

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11. Verify Lagranges mean value theorem for function $f(x) = (x-1)(x-2)(x-3)$ on $[0, 4]$ and find a point $'c'$ in the indicated interval:

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12. Verify Lagranges mean value theorem for function $f(x) = x(x+4)^2$ on $[0,\ 4]$ and find a point $'c'$ in the indicated interval:

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13. Verify Lagranges mean value theorem for function $f(x) = \sin x - \sin 2x - x$ on $[0,\ \pi]$

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14. Verify Lagranges mean value theorem for function $f(x) = x^3 - 5x^2 - 3x$ on $[1,\ 3]$

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15. Verify the hypothesis and conclusion of Lagranges mean value theorem for the function $f(x) = \frac{1}{4x-1}, \ 1 \leq x \leq 4$.

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16. If the value of c prescribed in Rolles theorem for the function $f(x) = 2x(x-3)^n$ on the interval $[0, \ 2\sqrt{3}]$ is $\frac{3}{4}$, write the value of n (a positive integer)

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17. If the polynomial equation $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dot{+} a_2x^2 + a_1x + a_0 = 0$ n being a positive integer, has two different real roots α and β, then between α and β, the equation $n\, a_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \dot{+} a_1 = 0$ has

A. (a) exactly one root

B. (b) almost one root

C. (c) at least one root

D. (d) no root

Answer: (c) at least one root

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18. If $4a+2b+c=0$, then the equation $3ax^2+2bx+c=0$ has at least one real lying in the interval $(0, 2)$

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19. Rolles theorem is applicable in case of $\varphi(x)=a^{\sin x},\ a>0$ in (a) any interval (b) the interval $[0,\ \pi]$ (c) the interval $(0,\ \pi/2)$ (d) none of these

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