doubtnut
India's Number 1 Education App

MATHS

BOOKS - RD SHARMA MATHS (HINGLISH)

SCALAR TRIPLE PRODUCT

Solved Examples And Exercises

1. Find $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$, when (ii) $\vec{a}=2\hat{i}-3\hat{j},\ \vec{b}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=3\hat{i}-\hat{k}$ (i) $\vec{a}=\hat{i}-2\hat{j}+3\hat{k},\ \vec{b}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=\hat{j}+\hat{k}$

Watch Video Solution

2. Find the volume of the parallelepiped whose coterminous edges are represented by the vectors: $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k},\ \vec{b}=\hat{i}+2\hat{j}-\hat{k},\ \vec{c}=3\hat{i}-\hat{j}+2\hat{k}$

$\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k},\ \vec{b} = \hat{i} + 2\hat{j} - \hat{k},\ \vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$

$\vec{a} = 11\hat{i},\ \vec{b} = 2\hat{j} - \hat{k},\ \vec{c} = 13\hat{k}$

$\vec{a} = \hat{i} + \hat{j} + \hat{k},\ \vec{b} = \hat{i} - \hat{j} + \hat{k},\ \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

Watch Video Solution

3. Evaluate : $\left[\hat{i}\hat{j}\hat{k}\right] + \left[\hat{j}\hat{k}\hat{i}\right] + \left[\hat{k}\hat{i}\hat{j}\right]\ \left[2\hat{i}\hat{j}\hat{k}\right] + \left[\hat{i}\hat{k}\hat{j}\right] + \left[\hat{k}\hat{j}2\hat{i}\right]$

Watch Video Solution

4. Show that the four points having position vectors $6\hat{i} - 7\hat{j}, 16\hat{i} - 19\hat{j} - 4\hat{k}, 3\hat{j} - 6\hat{k}, 2\hat{i} + 5\hat{j} + 10\hat{k}$ are not coplanar.

Watch Video Solution

5. Prove that: $\left(\vec{a} - \vec{b}\right) \cdot \left\{\left(\vec{b} - \vec{c}\right) \times \left(\vec{c} - \vec{a}\right)\right\} = 0$

Watch Video Solution

6. Show that each of the following triads of vectors are coplanar:

$\overrightarrow{a} = \hat{i} + 2\hat{j} - \hat{k}, \overrightarrow{b} = 3\hat{i} + 2\hat{j} + 7\hat{k}, \overrightarrow{c} = 5\hat{i} - 6\hat{j} + 5\hat{k}$

$\overrightarrow{a} = -4\hat{i} - 6\hat{j} - 2\hat{k}, \overrightarrow{b} = -\hat{i} + 4\hat{j} + 3\hat{k}, \overrightarrow{c} = 8\hat{i} - \hat{j} + 3\hat{k}$

$\hat{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \overrightarrow{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \overrightarrow{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

Watch Video Solution

7. Find the value of λ so that the following vectors are coplanar:

$\overrightarrow{a} = \hat{i} - \hat{j} + \hat{k}, \overrightarrow{b} = 2\hat{i} + \hat{j} - \hat{k}, \overrightarrow{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

Watch Video Solution

8. Find the value of λ for which the four points with position vectors $-\hat{j} - \hat{k}, 4\hat{i} + 5\hat{j} + \lambda\hat{k}, 3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4\hat{i} + 4\hat{j} + 4\hat{k}$ are coplanar.

Watch Video Solution

9. $\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of points A, B and C respectively, prove that : $\vec{a}\,x\,\vec{b} + \vec{b}\,x\,\vec{c} + \vec{c}\,x\,\vec{a}$ is vector perpendicular to the plane of triangle ABC.

Watch Video Solution

10. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i}$ and $\hat{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then, (1)If $c_1 = 1$ and $c_2 = 2$, find c_3 which makes $\vec{a}, \vec{b}$ and $\vec{c}$ coplanar. (2)If $c_2 = -1$ and $c_3 = 1$, show that no value of c_1 can make $\vec{a}, \vec{b}$ and $\vec{c}$ coplanar.

Watch Video Solution

11. Find $\left[\vec{a}, \vec{b}, \vec{c}\right]$,when $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

Watch Video Solution

12. Find the volume of a parallelepiped whose edges are given by $3\hat{i}+7\hat{j}-5\hat{k}, 5\hat{i}+7\hat{j}-3\hat{k}$ and $7\hat{i}-5\hat{j}-3\hat{k}$

Watch Video Solution

13. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and the vectors $\vec{A}=\hat{i}+a\hat{j}+a^2\hat{k}, \vec{B}=\hat{i}+b\hat{j}+b^2\hat{k}, \vec{C}=\hat{i}+c\hat{j}+c^2\hat{k}$ are non-coplanar, then prove that $abc=-1$.

Watch Video Solution

14. Evaluate $\left[\hat{i}\hat{j}\hat{k}\right]\left[\hat{i}\hat{k}\hat{j}\right]$

Watch Video Solution

15. Simplify: $\left[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}\right]$

Watch Video Solution

16. If the vectors $\overrightarrow{\alpha} = a\hat{i} + a\hat{j} + c\hat{k}, \overrightarrow{\beta} = \hat{i} + \hat{k}$ and $\overrightarrow{\gamma} = c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar, then prove that c is the geometric mean of a and b

Watch Video Solution

17. Determine α such that a vector $\overrightarrow{r}$ is at right angles to each of the vectors $\overrightarrow{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}, \overrightarrow{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}, \overrightarrow{c} = -2\hat{i} + \alpha\hat{j} + 3\hat{k}$

Watch Video Solution

18. Find the range of f(x)= $\sin^{-1}x + \tan^{-1}x + \cos^{-1}x$

Watch Video Solution

19. For any three vectors a b,c, show that $\overrightarrow{a} \times \left(\overrightarrow{b} + \overrightarrow{c}\right) + \overrightarrow{b} \times \left(\overrightarrow{c} + \overrightarrow{a}\right) + \overrightarrow{c} \times \left(\overrightarrow{a} + \overrightarrow{b}\right) = 0$

Watch Video Solution

20. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a} \, and \, \vec{b}$. If the between $\vec{a} \, and \, \vec{b}$ is $\pi/6$, prove that $\left[\vec{a} \vec{b} \vec{c}\right]^2 = \frac{1}{4}\left|\vec{a}\right|^2\left|\vec{b}\right|^2.$

Watch Video Solution

21. Let $\vec{a}, \vec{b} \, and \, \vec{c}$, be non-zero non-coplanar vectors. Prove that: $\vec{a} - 2\vec{b} + 3\vec{c}, -2\vec{a} + 3\vec{b} - 4\vec{c} \, and \, \vec{c} - 3\vec{b} + 5\vec{c}$ are coplanar vectors. $2\vec{a} - \vec{b} + 3\vec{c}, \vec{a} + \vec{b} - 2\vec{c} \, and \, \vec{a} + \vec{b} - 3\vec{c}$ are non-coplanar vectors.

Watch Video Solution

22. Find the altitude of a parallelepiped determined by the vectors $\vec{a}, \vec{b} \, and \, \vec{c}$, if the base is taken as the parallelogram determined by

$\vec{a}$ and $\vec{b}$, and if

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + 3\hat{k}$.

Watch Video Solution

23. Let $\vec{a}, \vec{b}, \vec{c}$, be three non-zero vectors. If $\vec{a} \cdot \vec{b} \times \vec{c} = 0$ and $\vec{b}$ and $\vec{c}$ are not parallel, then prove that $\vec{a} = \lambda\vec{b} + \mu\vec{c}$, where λ are some scalars.

Watch Video Solution

24. If a is a non-zero real number, then prove that the vectors $\vec{\alpha} = a\hat{i} + 2a\hat{j} - 3a\hat{k}$, $\vec{\beta} = (2a+1)\hat{i} + (2a+3)\hat{j} + (a+1)\hat{k}$ and, $\vec{\gamma} = ($

are never coplanar.

Watch Video Solution

25. If $\vec{a} + \vec{b} + \vec{c} = 0$, prove that $\left(\vec{a} \times \vec{b}\right) = \left(\vec{b} \times \vec{c}\right) = \left(\vec{c} \times \vec{a}\right)$

Watch Video Solution

26. Show that the vectors $\vec{a} = -2\hat{i} - 2\hat{j} + 4\hat{k}, \ \vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k} \ and \ \vec{c} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ are coplanar.

Watch Video Solution

27. Find λ so that the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \ \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k} \ and \ \vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ re coplanar.

Watch Video Solution

28. The four points whose position vector are $6\hat{i} - 7\hat{j}, 16\hat{i} - 29\hat{j} - 4\hat{k}, 3\hat{i} - 6\hat{k}, 2\hat{i} + 5\hat{j} + 10\hat{k}$ are

Watch Video Solution

29. For any three vectors $a,\ b,\ c$ prove that $\left[\vec{a} + \vec{b}\ \vec{b} + \vec{c}\ \vec{c} + \vec{a}\right] = 2\left[\vec{a}\ \vec{b}\ \vec{c}\right].$

Watch Video Solution

30. Show that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$ are coplanar.

Watch Video Solution

31. Show that the vectors $\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}$ are coplaner.

Watch Video Solution

32. Evaluate the following: $\left[\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{j}\ \hat{k}\ \hat{i}\right]+\left[\hat{k}\ \hat{i}\ \hat{j}\right]$.

Watch Video Solution

33. Evaluate : $\left[\hat{i}\hat{j}\hat{k}\right]+\left[\hat{j}\hat{k}\hat{i}\right]+\left[\hat{k}\hat{i}\hat{j}\right]\left[2\hat{i}\hat{j}\hat{k}\right]+\left[\hat{i}\hat{k}\hat{j}\right]+\left[\hat{k}\hat{j}2\hat{i}\right]$

Watch Video Solution

34. Find $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$, when : $\vec{a}=2\hat{i}-3\hat{j},\ \vec{b}=\hat{i}+\hat{j}-\hat{k}\ and\ \vec{c}=3\hat{i}-\hat{k}.$

Watch Video Solution

35. Find $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$, when : $\vec{a}=\hat{i}-2\hat{j}+3\hat{k},\ \vec{b}=2\hat{i}+\hat{j}-\hat{k}\ and\ \vec{c}=\hat{j}+\hat{k}$.

Watch Video Solution

36. Find the volume of the parallelepiped whose coterminous edges are represented by the vector: $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \ \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \ \vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

Watch Video Solution

37. Find the volume of the parallelepiped whose coterminous edges are represented by the vector: $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \ \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \ \vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$.

Watch Video Solution

38. Find the volume of the parallelepiped whose coterminous edges are represented by the vector: $\vec{a} = 11\hat{i}, \ \vec{b} = 2\hat{j}, \ \vec{c} = 13\hat{k}$.

Watch Video Solution

39. Find the volume of the parallelepiped whose coterminous edges are represented by the vector: $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \ \vec{b} = \hat{i} - \hat{j} + \hat{k}, \ \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$.

Watch Video Solution

40. Show that each of the following triads of vectors are coplanar: $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \ \vec{b} = 3\hat{i} + 2\hat{j} + 7\hat{k}, \ \vec{c} = 5\hat{i} + 6\hat{j} + 5\hat{k}$.

Watch Video Solution

41. Show that each of the following triads of vectors are coplanar: $\vec{a} = -4\hat{i} - 6\hat{j} - 2\hat{k}, \ \vec{b} = -\hat{i} + 4\hat{j} + 3\hat{k}, \ \vec{c} = -8\hat{i} - \hat{j} + 3\hat{k}$.

Watch Video Solution

42. Show that each of the following triads of vectors are coplanar: $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \ \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \ \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$.

Watch Video Solution

43. Find the value of λ so that the following vectorts are coplanar: $\overrightarrow{a}=\hat{i}-\hat{j}+\hat{k},\ \overrightarrow{b}=2\hat{i}+\hat{j}-\hat{k},\ \overrightarrow{c}=\lambda\hat{i}-\hat{j}+\lambda\hat{k}$

Watch Video Solution

44. Find the value of λ so that the following vectorts are coplanar: $\overrightarrow{a}=2\hat{i}-\hat{j}+\hat{k},\ \overrightarrow{b}=\hat{i}+2\hat{j}-3\hat{k},\ \overrightarrow{c}=\lambda\hat{i}+\lambda\hat{j}+5\hat{k}.$

Watch Video Solution

45. Find the value of λ so that the following vectorts are coplanar: $\overrightarrow{a}=\hat{i}+2\hat{j}-3\hat{k},\ \overrightarrow{b}=3\hat{i}+\lambda\hat{j}+\hat{k},\ \overrightarrow{c}=\hat{i}+2\hat{j}+2\hat{k}$

Watch Video Solution

46. Find the value of λ so that the following vectorts are coplanar: $\overrightarrow{a} = \hat{i} + 3\hat{j}, \ \overrightarrow{b} = 5\hat{k}, \ \overrightarrow{c} = \lambda\hat{i} - \hat{j}$

Watch Video Solution

47. Show that the points having position vectors $6\hat{i} - 7\hat{j}, \ 16\hat{i} - 19\hat{j} - 4\hat{k}, \ 3\hat{j} - 6\hat{k}, \ 2\hat{i} + 5\hat{j} + 10\hat{k}$ are not coplanar.

Watch Video Solution

48. Show that the points $A(-1, \ 4, \ -3), \ B(3, \ 2, \ -5), \ C(-3, \ 8, \ -5) and \ D(-3, \ 2, \ 1)$ are coplanar.

Watch Video Solution

49. Show that four points whose position vectors are $6\hat{i}-7\hat{j},\ 16i-19\hat{j}-4\hat{k},\ 3\hat{i}-6\hat{k},\ 2\hat{i}-5\hat{j}+10\,\hat{k}$ are coplanar.

Watch Video Solution

50. Find the value of λ for which the four points with position vectors $\hat{j}-\hat{k},\ 4\hat{i}+5\hat{j}+\lambda\hat{k},\ 3\hat{i}+9\hat{j}+4\hat{k}\ and-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar.

Watch Video Solution

51. Prove that $\left(\vec{a}-\vec{b}\right)\dot{\left(\vec{b}-\vec{c}\right)\times}\left(\vec{c}-\vec{a}\right)=0$

Watch Video Solution

52. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=i$ and $\vec{c}=c_1\hat{i}+\hat{c}_2j+c_3\hat{k}$ Then (a) if $c_1=1$ and $c_2=2$, find c_3 which makes $\vec{a},\vec{b},\vec{c}$ coplanar (b) if

$c_2 = -1$ and $c_3 = 1$, show that no value of c_3 can makes $\vec{a}, \vec{b}, \vec{c}$

coplanar.

Watch Video Solution

53. Find λ for which the points $A(3, 2, 1), B(4, \lambda, 5), C(4, 2, -2)$ and $D(6, 5, -1)$ are coplanar.

Watch Video Solution

54. Write the value of $\left[2\hat{i}\ 3\hat{j}\ 4\hat{k}\right]$

Watch Video Solution

55. Write the value of $\left[\hat{i} + \hat{j}\ \hat{j} + \hat{k}\ \hat{k} + \hat{i}\right]$

Watch Video Solution

56. Write the value of $\left[\hat{i}-\hat{j}\,\hat{j}-\hat{k}\,\hat{k}-\hat{i}\right]$

Watch Video Solution

57. Find the values of a for which are vectors $\vec{\alpha}=\hat{i}+2\hat{j}+\hat{k},\ \vec{\beta}=a\hat{i}+\hat{j}+2\hat{k}\ and\ \vec{\gamma}=\hat{i}+2\hat{j}+a\hat{k}$ are coplanar.

Watch Video Solution

58. Find $\vec{a}.\left(\vec{b}\times\vec{c}\right)$ if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k},\ \vec{b}=-\hat{i}+2\hat{j}+\hat{k}\ and\ \vec{c}=3\hat{i}$ -

Watch Video Solution

59. If the vectors $4\hat{i}+11\hat{j}+m\hat{k},\ 7\hat{i}+2\hat{j}+6\hat{k}\ and\ \hat{i}+5\hat{j}+4\hat{k}$ are coplanar then $m=$ 38 b. \ 10\ c. -1 d. -10

Watch Video Solution

Others

1. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, prove that $\left[\vec{a}+\vec{b}+\vec{c}\,\vec{a}+\vec{b}\,\vec{a}+\vec{c}\right] = -\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

View Text Solution

2. If the vectors $\vec{\alpha} = a\hat{i}+\hat{j}+\hat{k}, \vec{\beta} = \hat{i}+b\hat{j}+\hat{k} and \vec{\gamma} = \hat{i}+\hat{j}+c\hat{k}$ are coplanar, then prove that $\frac{1}{1-a}+\frac{1}{1+b}+\frac{1}{1-c} = 1, where a \neq 1, b \neq 1 and c \neq 1.$

View Text Solution