



## MATHS

### BOOKS - RD SHARMA MATHS (HINGLISH)

#### CONGRUENT TRIANGLE

Others

1.  $BD$  and  $CE$  are bisectors of  $\angle B$  and  $\angle C$  of an isosceles  $ABC$  with  $AB = AC$ . Prove that  $BD = CE$ .

Since  $AB = AC$   $\angle ABC = \angle ACB$ . . . . . (i) [Angles opposite to equal sides are equal]



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2. Two lines  $AB$  and  $CD$  intersect at  $O$  such that  $BC$  is equal and parallel to  $AD$ . Prove that the lines  $AB$  and  $CD$  bisect at  $O$ .

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3. In Figure,  $AC = BC$ ,  $\angle DCA = \angle ECB$  and  $\angle DBC = \angle EAC$ . Prove that triangles  $DBC$  and  $EAC$  are congruent, and hence  $DC = EC$  and  $BD = AE$

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4. If the bisector of the vertical angle of a triangle bisects the base of the triangle. then the triangle is isosceles. GIVEN : A  $ABC$  in which  $AD$  is the bisector of  $\angle A$  meeting  $BC$  in  $D$  such that  $BD = DC$ .

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5. In an isosceles triangle altitude from the vertex bisects the base. GIVEN : An isosceles triangle  $ABC$  such that  $AB = AC$  and an altitude  $AD$  from  $A$  on side  $BC$ . TO PROVE :  $D$  bisects  $BC$  i.e.  $BD = DC$ . Figure

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6. If the altitude from one vertex of a triangle bisects the opposite side, then the triangle is isosceles. GIVEN : A  $ABC$  such that the altitude  $AD$  from  $A$  on the opposite side  $BC$  bisects  $BC$  i.e.,  $BD = DC$ . TO PROVE :  $AB = AC$  i.e. the triangle  $ABC$  is isosceles.

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7. Prove that the perimeter of a triangle is greater than the sum of its altitudes.

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8. In  $ABC$ , side  $AB$  is produced to  $D$  so that  $BD = b$ . If  $\angle B = 60^\circ$  and  $\angle A = 70^\circ$ , prove that : (i)  $AD > CD$  (ii)  $AD > AC$

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9. Prove that in a quadrilateral the sum of all the sides is greater than the sum of its diagonals.

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10. GIVEN :  $PQRS$  is a quadrilateral.  $PQ$  is its longest side and  $RS$  is its shortest side. TO PROVE : (i)  $\angle R > \angle P$  (ii)  $\angle S > \angle Q$  CONSTRUCTION : Join  $PR$  and  $QS$ . Figure

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11. Of all the line segments drawn from a point  $P$  to a line  $m$  not containing  $P$ , let  $PD$  be the shortest. If  $B$  and  $C$  are points on  $m$  such

that  $D$  is the mid-point of  $BC$ , prove that  $PB = PC$ .

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12. If  $ABC$  is an isosceles triangle with  $AB = AC$ . Prove that the perpendiculars from the vertices  $B$  and  $C$  to their opposite sides are equal.

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13. If the altitudes from two vertices of a triangle to the opposite sides are equal, prove that the triangle is isosceles.

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14.  $AD$  and  $BC$  are equal perpendiculars to a line segment  $AB$ . Show that  $CD$  bisects  $AB$ .

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15. In  $ABC$ ,  $AB = AC$ , and the bisectors of angles  $B$  and  $C$  intersect at point  $O$ . Prove that  $BO = CO$  and the ray  $AO$  is the bisector of angles  $BAC$ .

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16. In Figure, line  $l$  is the bisector of angle  $A$  and  $B$  is any point on  $l$ .  $BP$  and  $BQ$  are perpendiculars from  $B$  to the arms of  $A$ . Show that :  
 $APB \cong AQB$   $BP = BQ$  or  $B$  is equidistant from the arms of  $\angle A$ .

Figure

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17. Prove that measure of each angle of an equilateral triangle is  $60^\circ$ .

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**18.** Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angle of the other triangle. GIVEN : Two triangles  $ABC$  and  $DEF$  such that  $AB = DE$ ,  $AC = DF$  and  $\angle A = \angle D$  PROVE :  $ABC \cong DEF$  Figure

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**19.** In  $\triangle ABC$ ,  $\angle A = 100^\circ$  and  $AB = AC$ . Find  $\angle B$  and  $\angle C$

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**20.** Angles opposite to two equal sides of a triangle are equal. GIVEN :  $ABC$  in which  $AB = AC$  TO PROVE :  $\angle C = \angle B$  CONSTRUCTION : Draw the bisector  $AD$  of  $\angle A$  which meets  $BC$  in  $D$  Figure

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21. In Figure,  $X$  and  $Y$  are two points on equal sides  $AB$  and  $AC$  of a  $\triangle ABC$  such that  $AX = AY$ . Prove that  $XC = YB$ .



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22. Prove that the angle between internal bisector of one base angle and the external bisector of the other base angle of a triangle is equal to one half of the vertical angle.



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23. In  $\triangle ABC$  and  $\triangle PQR$  Figure,  $AB = PQ$ ,  $BC = QR$  and  $CB$  and  $RQ$  are extended to  $X$  and  $Y$  respectively and  $\angle ABX = \angle PQY$ . Prove that  $\triangle ABC \cong \triangle PQR$ . Figure



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**24.** In Figure, the side  $BC$  of  $ABC$  is produced to form ray  $BD$  as shown. Ray  $CE$  is drawn parallel to  $BA$ . Show directly, without using the angle sum property of a triangle that  $\angle ACD = \angle A + \angle B$  and deduced that  $\angle A + \angle B + \angle C = 180^\circ$ .

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**25.** In a triangle the greater angle has the longer side opposite to it.

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**26.** If the bisectors of the base angles of a triangle enclose an angle of  $135^\circ$ , prove that the triangle is a right triangle.

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27. If two sides of a triangle are unequal, the longer side has greater angle opposite to it. GIVEN : A  $ABC$  in which  $AC > AB$ . TO PROVE :  $\angle ABC > \angle ACB$

CONSTRUCTION : Mark a point  $D$  on  $AC$  such that  $AB = AD$ . Joint  $BD$ .

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28. The bisectors of base angles of a triangle cannot enclose a right angle in any case.

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29.  $ABCD$  is a square,  $X$  and  $Y$  are points on sides  $AD$  and  $BC$  respectively such that  $AY = BX$ . Prove that  $BY = AX$  and  $\angle BAY = \angle ABX$ .

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**30.** If perpendiculars from any point with an angle on its arms are congruent, prove that it lies on the bisector of that angle.

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**31.**  $ABC$  is a triangle in which  $BE$  and  $CF$  are, respectively, the perpendiculars to the sides  $AC$  and  $AB$ . If  $BE = CF$ , prove that  $ABC$  is isosceles.

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**32.**  $ABC$  is a triangle and  $D$  is the mid-point of  $BC$ . The perpendiculars from  $D$  to  $AB$  and  $AC$  are equal. Prove that the triangle is isosceles.

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**33.** The side  $BC$  of a  $ABC$  is produced on both sides. Show that the sum of the exterior angles so formed is greater than  $\angle A$  by two right angles.

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**34.**  $P$  is a point equidistant from two lines  $l$  and  $m$  intersecting at a point  $A$  see in figure, Show that  $AP$  bisects the angle between them. Figure

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**35.** If two parallel lines are intersected by a transversal, prove that the bisectors of the interior angles on the same side of transversal intersect each other at right angles.

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**36.** If  $ABC$  is an isosceles triangle such that  $AB = AC$  and  $AD$  is an altitude from  $A$  on  $BC$ . Prove that (i)  $\angle B = \angle C$  (ii)  $AD$  bisects  $BC$  (iii)  $AD$  bisects  $\angle A$



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**37.** A triangle  $ABC$  is right angled at  $A$ .  $AL$  is drawn perpendicular to  $BC$ . Prove that  $\angle BAL = \angle ACB$ .



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**38.** The sum of any two sides of a triangle is greater than the third side.  
GIVE : A  $ABC$



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39.  $AB$  is a line segment and line  $l$  is its perpendicular bisector. If a point  $P$  lies on  $l$ , show that  $P$  is equidistant from  $A$  and  $B$ .

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40.  $ABC$  is a triangle in which  $\angle A = 72^\circ$ , the internal bisectors of angles  $B$  and  $C$  meet in  $O$ . Find the magnitude of  $\angle BOC$ .

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41. If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right triangle.

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42. Suppose line segments  $AB$  and  $CD$  intersect at  $O$  in such a way that  $AO = OD$  and  $OB = OC$ . Prove that  $AC = BD$  but  $AC$  may not be

parallel to  $BD$ .



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43. Two angles of a triangle are equal and the third angle is greater than each of those angles by  $30^\circ$ . Determine all the angle of the triangle.



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44. If  $D$  is the mid-point of the hypotenuse  $AC$  of a right triangle  $ABC$ , prove that  $BD = \frac{1}{2}AC$ . GIVEN : A  $ABC$  in which  $\angle B = 90^\circ$  and  $D$  is the mid-point of  $AC$ . TO PROVE :  $BD = \frac{1}{2}AC$  CONSTRUCTION Produce  $BD$  to  $E$  so that  $BD = DE$ . Join  $EC$ .



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45. In triangle  $ABC$ ,  $\angle B = 45^\circ$ ,  $\angle C = 55^\circ$  and bisector of  $\angle A$  meets  $BC$  at a point  $D$ . Find  $\angle ADB$  and  $\angle ADC$ .



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46.  $AB$  is a line segment,  $AX$  and  $BY$  are two equal line segments drawn on opposite sides of line  $AB$  such that  $AXBY$ . If  $AB$  and  $XY$  intersect each other  $P$ , prove that  $APX \cong BPY$   $AB$  and  $XY$  bisect each other.



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47. (Exterior Angle Theorem): If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles. GIVEN : A triangle  $ABC$  is a point of  $BC$  produced, forming exterior angle  $\angle 4$ . TO PROVE :  $\angle 4 = \angle 1 + \angle 2$  i.e. ,  $\angle ACD = \angle CAB + \angle CBA$ .



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**48.** Two triangles are congruent if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle. GIVE : Two  $\triangle ABC$  and  $\triangle DEF$  such that  $\angle B = \angle E$ ,  $\angle C = \angle F$  and  $BC = EF$  TO PROVE :  $\triangle ABC \cong \triangle DEF$

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**49.**  $A, B, C$ , are three angles of a triangle. If  $A - B = 15^\circ$ ,  $B - C = 30^\circ$ , find  $\angle A$ ,  $\angle B$  and  $\angle C$

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**50.** In Figure, diagonal  $AC$  of a quadrilateral  $ABCD$  bisects the angles  $A$  and  $C$ . Prove that  $AB = AD$  and  $CB = CD$

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51. In a triangle  $ABC$ ,  $\angle ABC = \angle ACB$  and the bisectors of  $\angle ABC$  and  $\angle ACB$  intersect at  $O$  such that  $\angle BOC = 120^\circ$ . Show that  $\angle A = \angle B = \angle C = 60^\circ$



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52. If each angle of a triangle is less than the sum of the other two, show that the triangle is acute angled.



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53. In two right triangles, one side and an acute angle of one triangle are equal to one side and the corresponding acute angle of the other triangle. Prove that the two triangles are congruent.



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54. In figure, if  $QT \perp PR$ ,  $\angle TQR = 40^\circ$  AND  $\angle SPR = 30^\circ$ , find  $x$  and  $y$ . Figure

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55. If  $D$  is any point on the base  $BC$  produced, of an isosceles triangle  $ABC$ , prove that  $AD > AB$ .

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56. In figure, sides  $QP$  and  $RQ$  of  $PQR$  are produced to point  $S$  and  $T$  respectively. If  $\angle SPR = 135^\circ$  and  $\angle PQT = 110^\circ$ , find  $\angle PRQ$ . Figure

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57. Show that in a right triangle the hypotenuse is the longest side. GIVEN : A right triangle  $ABC$  in which  $\angle ABC = 90^\circ$ . TO PROVE : Hypotenuse

$AC$  is the longest side, i.e.  $AC > AB$  (ii)  $AC > BC$



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58. An exterior angle of a triangle is  $110^\circ$ , and one of the interior opposite angles is  $30^\circ$ . Find the other two angles of the triangle.



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59. Show that the sum of the three altitudes of a triangle is less than the sum of three sides of the triangle. GIVEN : A  $ABC$  in which  $AD \perp BC$ ,  $BE \perp AC$  and  $CF \perp AB$ . PROVE :  $AD + BE + CF$



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60. The sides  $BC$ ,  $CA$  and  $AB$  of a triangle  $ABC$ , are produced in order, forming exterior angles  $\angle ACD$ ,  $\angle BAE$  and  $\angle CBF$ . Show that  $\angle ACD + \angle BAE + \angle CBF = 360^\circ$

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**61.** Prove that any two sides of a triangle are together greater than twice the median drawn to the third side. GIVEN :  $ABC$  in which  $AD$  is a median. PROVE :  $AB + AC > 2AD$  CONSTRUCTION : Produce  $AD$  to  $E$  such that  $AD = DE$ . Join  $EC$ .

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**62.** Prove that the sum of the three angles of a triangle is  $180^\circ$ .

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**63.** In a  $ABC$ , if  $\angle A = 50^\circ$  and  $\angle B = 60^\circ$ , determine the shortest and largest sides of the triangle.

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64. In a triangle  $ABC$ ,  $\angle B = 115^\circ$ ,  $\angle C = 40^\circ$ , Find  $\angle A$



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65. The sum of two equal angles of a triangle is equal to its third angle.

Determine the measure of the third angle.



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66. Of the three angles of a triangle, one is twice the smallest and another is three times the smallest. Find the angles.

A. 30,40,110

B. 30,60,90

C. 45,45,90

D. none

Answer: B



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**67.** Prove that the perimeter of a triangle is greater than the sum of the three medians.



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**68.** If the angles of a triangle are in the ratio  $2:3:4$  . determine three angles.



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**69.** The sum of two angles of a triangle is  $80^\circ$  and their difference is  $20^\circ$  .  
Find all the angles.



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**70.** A triangle  $ABC$  is an isosceles triangle if any one of the following conditions hold: Altitude  $AD$  bisects  $\angle BAC$ . Bisector of  $\angle BAC$  is perpendicular to the base  $BC$ .

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**71.** In a  $ABC$ , if  $2\angle A = 3\angle B = 6\angle C$ , determine  $\angle A$ ,  $\angle B$  and  $\angle C$ .

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**72.** In a right angled triangle, one acute angle is double the other. Prove that the hypotenuse is double the smallest side. GIVEN : A  $ABC$  in which  $\angle B = 90^\circ$  and  $\angle ACB = 2\angle CAB$  . to prove :  $AC = 2BC$   
CONSTRUCTION : Produce  $CB$  to  $D$  such that  $BD = CB$ . Join  $AD$ .

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73. The sides  $AB$  and  $AC$  of a  $ABC$  are produced to  $P$  and  $Q$  respectively. If the bisectors of  $\angle PBC$  AND  $\angle QCB$  intersect at  $O$ , then  $\angle BOC = 90^\circ - \frac{1}{2}\angle A$  GIVEN : A  $ABC$  in which sides  $AB$  and  $AC$  are produced to  $P$  and  $Q$  respectively. The bisectors of  $\angle PBC$  and  $\angle QCB$  intersect at  $O$ .

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74.  $AD$  and  $BE$  are respectively altitudes of triangle  $ABC$  such that  $AE=BD$ .  
Prove that  $AD=BE$ .

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75. If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.

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76. Prove that Two right triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other triangle.



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77. In given figure the altitudes  $AD$ ,  $BE$  and  $CF$ , the altitudes of triangle  $ABC$  are equal. Prove that  $ABC$  is an equilateral triangle.



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78. A point  $O$  is taken inside an equilateral four sided figure  $ABCD$  such that its distances from the angular points  $D$  and  $B$  are equal. Show that  $AO$  and  $OC$  are in one and the same straight line. GIVEN : A point  $O$  inside an equilateral quadrilateral four sided figure  $ABCD$  such that  $BO = OD$ . TO PROVE :  $AO$  and  $OC$  are in one and the same straight line.



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79. If two isosceles triangles have a common base, prove that the line joining their vertices bisects them at right angles.

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80. Explain -Side Side Side(SSS) Congruence .

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81.  $ABCD$  is a parallelogram, if the two diagonals are equal, find the measure of  $\angle ABC$ .

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82. In a  $ABC$ , it is given that  $AB = AC$  and the bisectors of  $\angle B$  and  $\angle C$  intersect at  $O$ , If  $M$  is a point on  $BO$  produced, prove that

$$\angle MOC = \angle ABC.$$



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83.  $ABC$  is a triangle in which  $\angle B = 2 \angle C$ .  $D$  is a point on  $BC$  such that  $AD$  bisects  $\angle BAC$  and  $AB = CD$ . Prove that  $\angle BAC = 72^\circ$ .



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84. In  $ABC$ ,  $\angle A = 100^\circ$  and  $AB = AC$ . Find  $\angle B$  and  $\angle C$



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85. Prove that measure of each angle of an equilateral triangle is  $60^\circ$ .



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86. Given: In  $\triangle ABC$  and  $\triangle PQR$ ,  
 $AB = PQ$ ,  $BC = QR$  and  $CB$  and  $RQ$  are extended to  $X$  and  $Y$   
respectively and  $\angle ABX = \angle PQY$ . Prove that  $ABC \cong PQR$



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87. Suppose line segments  $AB$  and  $CD$  intersect at  $O$  in such a way that  
 $AO = OD$  and  $OB = OC$ . Prove that  $AC = BD$  but  $AC$  may not be  
parallel to  $BD$



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88. If  $D$  is the mid-point of the hypotenuse  $AC$  of a right triangle  $ABC$ ,  
prove that  $BD = \frac{1}{2}AC$



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89.  $AB$  is a line segment and line  $l$  is its perpendicular bisector. If a point  $P$  lies on  $l$ , show that  $P$  is equidistant from  $A$  and  $B$ .

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90. In quadrilateral  $ACBD$ ,  $AC = AD$  and  $AB$  bisects  $\angle A$ . Show that  $ABC \cong ABD$ . What can you say about  $BC$  and  $BD$ ?

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91. Prove that  $ABC$  is isosceles if any one of the following holds: (i) *Altitude*  $AD$  bisects  $BC$  (ii) *Median*  $AD$  is perpendicular to the base  $BC$

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92. In right triangle  $ABC$ , right angle at  $C$ ,  $M$  is the mid-point of the hypotenuse  $AB$ .  $C$  is joined to  $M$  and produced to a point  $D$  such that

$DM = CM$ . Point  $D$  is joined to point  $B$ . Show that  $AMC \cong BMD$  (ii)

$\angle DBC = 90^\circ$  (iii)  $DBC \cong ACB$  (iv)  $CM = \frac{1}{2}AB$



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93. Prove that the medians of an equilateral triangle are equal.



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94. In a  $ABC$ , if  $\angle A = 120^\circ$  and  $AB = AC$ . Find  $\angle B$  and  $\angle C$



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95. In  $ABC$ , if  $AB = AC$  and  $\angle B = 70^\circ$ , find  $\angle A$



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96. The vertical angle of an isosceles triangle is  $100^\circ$ . Find its base angles.



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97. Find the measure of each exterior angle of an equilateral triangle.



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98. If the base of an isosceles triangle is produced on both sides, prove that the exterior angles so formed are equal to each other.



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99. Determine the measure of each of the equal angles of a right-angled isosceles triangle.



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**100.**  $ABC$  is a right-angled triangle in which  $\angle A = 90^\circ$  and  $AB = AC$ .

Find  $\angle B$  and  $\angle C$

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**101.**  $AB$  is a line segment.  $AX$  and  $BY$  are two equal line segments drawn on opposite sides of line  $AB$  such that  $AX \parallel BY$ . If  $AB$  and  $XY$  intersect each other at  $P$ , prove that  $APX \cong BPY$  (ii)  $AB$  and  $XY$  bisect each other.

In the given figure, segment  $AX$  and  $BY$  are equal

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**102.** In two right triangles, one side and an acute angle of one triangle are equal to one side and the corresponding acute angle of the other triangle. Prove that the two triangles are congruent.

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**103.** Two lines  $AB$  and  $CD$  intersect at  $O$  such that  $BC$  is equal and parallel to  $AD$ . Prove that the lines  $AB$  and  $CD$  bisect at  $O$

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**104.**  $BD$  and  $CE$  are bisectors of  $\angle B$  and  $\angle C$  of an isosceles  $ABC$  with  $AB = AC$ . Prove that  $BD = CE$

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**105.**  $abc$  is an isosceles triangle with  $AB = AC$ . Side  $BA$  is produced to  $D$  such that  $AB = AD$ . Prove that  $\angle BCD$  is a right angle.

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**106.** In Figure,  $AB = AC$ ,  $BE$  and  $CF$  are respectively the bisectors of  $\angle B$  and  $\angle C$ . Prove that  $ebc \cong FCB$



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**107.** If  $ABC$  is an isosceles triangle with  $AB = AC$ . Prove that the perpendiculars from the vertices  $B$  and  $C$  to their opposite sides are equal.

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**108.** If the altitudes from two vertices of a triangle to the opposite sides are equal, prove that the triangle is isosceles.

In  $\triangle ABE$  and  $\triangle ACF$   $\angle AEB = \angle AFC$  ( $90^\circ$  each)

$\angle BAE = \angle CAF$  (common angle)

$\angle ABE = \angle ACF$  using angle sum property

$BE = CF$ ,  $\triangle ABE \cong \triangle ACF$

$\Rightarrow AB = AC$

Hence triangle  $ABC$  is an isosceles triangle as two sides are equal to each other

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**109.**  $AD$  and  $BC$  are equal perpendiculars to a line segment  $AB$ . Show that  $CD$  bisects  $AB$ .



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**110.** In  $ABC$ ,  $AB = AC$ , and the bisectors of angles  $B$  and  $C$  intersect at point  $O$ . Prove that  $BO = CO$  and the ray  $AO$  is the bisector of angle  $BAC$

Since the angles opposite to equal sides are equal,

$$AB = AC$$

$$\Rightarrow \angle C = \angle B$$

$$\Rightarrow 2\angle B = 2\angle C$$

Since  $BO$  and  $CO$  are bisectors of  $\angle B$  and  $\angle C$ , we also have

$$\angle ABO = 2\angle B$$

$$\angle ACO = 2\angle C$$

$$\angle ABO = 2\angle B = 2\angle C = \angle ACO.$$

Consider  $\triangle BCO$ :  $\angle OBC = \angle OCB$

$BO = CO$  (Sides opposite to equal angles are equal)

Finally, consider triangles  $ABO$  and  $ACO$ .

$BA = CA$  (given)

$BO = CO$  (proved)

$\angle ABO = \angle ACO$  (proved)

Hence, by S.A.S postulate  $\triangle ABO \cong \triangle ACO$

$\angle BAO = \angle CAO$

$AO$  bisects  $\angle A$ .



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**111.** In a right angled triangle, one acute angle is double the other. Prove that the hypotenuse is double the smallest side.



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**112.** A triangle  $ABC$  is an isosceles triangle if any one of the following conditions hold: Altitude  $AD$  bisects  $\angle BAC$  Bisector of  $\angle BAC$  is perpendicular to the base  $BC$

In triangle,  $ABC$

let the altitude  $AD$  bisects  $\angle BAC$

Then we have to prove that the triangle  $ABC$ , is isosceles.

In  $\triangle ADB$  and  $ADC$ ,

$$\angle BAD = \angle CAD$$

( $AD$  is bisector of  $\angle BAC$ )

$$AD = AD(\text{common})$$

$$\angle ADB = \angle ADC \text{ (Each equal to } 90^\circ)$$

$$\Rightarrow \triangle ADB \cong \triangle ADC \text{ (by ASA congruence criterion)}$$

$$AB = AC(\text{cpct})$$

Hence, A triangle  $ABC$  is an isosceles



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**113.** A triangle  $ABC$  is an isosceles triangle if any one of the following conditions hold: Bisector of  $\angle BAC$  is perpendicular to the base  $BC$



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**114.** In two right triangles one side and an acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

Let  $ABC$  and  $DEF$  be two right triangles such that

$$\angle A = \angle D, BC = EF \text{ and } \angle B = \angle E = 90^\circ$$

Thus, in  $\triangle ABC$  and  $DEF$

$$\angle A = \angle D$$

$$\angle B = \angle E = 90^\circ$$

$BC = EF$  So, by AAS congruence criterion, we obtain

$$\triangle ABC \cong \triangle DEF$$



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**115.** Join  $A \rightarrow D$

Hence,  $AD \parallel BC$

$$\angle ACB = \angle CAD \text{ (alternate angle)}$$

$$\angle BAC = \angle ACD \text{ (alternate angle)}$$

Hence  $AB = AC$  (by ASA congruency) so, it is isosceles triangle.



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**116.** In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Let each base angle be  $x$  in an isosceles  $\triangle ABC$

Then vertex angle be  $2(x + x) = 4x$

Since sum of angles of a triangle is  $180^\circ$

Hence,  $4x + x + x = 180^\circ$

$$6x = 180^\circ$$

$$\Rightarrow x = 30^\circ$$

Angles are  $30^\circ, 30^\circ, 120^\circ$



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**117.**  $PQR$  is a triangle in which  $PQ = PR$  and  $S$  is any point on the side  $PQ$ . Through  $S$ , a line is drawn parallel to  $QR$  and intersecting  $PR$  at  $T$ . Prove that  $PS = PT$

In  $\triangle PQR$ , we have  $PQ = PR$



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**118.** In a  $ABC$ , it is given that  $AB = AC$  and the bisectors of  $\angle B$  and  $C$  intersect at  $O$ . If  $M$  is a point on  $BO$  produced, prove that  $\angle MOC = \angle ABC$

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**119.**  $P$  is a point on the bisector of an angle  $\angle ABC$ . If the line through  $P$  parallel to  $AB$  meets  $BC$  at  $Q$ , prove that triangle  $BPQ$  is isosceles.

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**120.** Prove that each angle of an equilateral triangle is  $60^\circ$

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**121.** Angle  $A$ ,  $B$ ,  $C$  of a triangle  $ABC$  are equal to each other. Prove that  $ABC$  is equilateral.

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**122.**  $ABC$  is a triangle in which  $\angle B = 2\angle C$ .  $D$  is a point on  $BC$  such that  $AD$  bisects  $\angle BAC$  and  $AB = CD$ . Prove that  $\angle BAC = 72^\circ$

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**123.**  $ABC$  is a right angled triangle in which  $\angle A = 90^\circ$  and  $AB = AC$ .  
Find  $\angle B$  and  $\angle C$

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**124.**  $ABCD$  is a parallelogram, if the two diagonals are equal, find the measure of  $\angle ABC$ .



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**125.** If two isosceles triangles have a common base, prove that the line joining their vertices bisects them at right angles.

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**126.**  $ABC$  and  $DBC$  are two isosceles triangles on the same base  $BC$  and vertices  $A$  and  $D$  are on the same side of  $BC$ . If  $AD$  is extended to intersect  $BC$  at  $P$ , show that  $ABD \cong ACD$  (ii)  $ABP \cong ACP$

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**127.**  $ABC$  and  $DBC$  are two isosceles triangles on the same base  $BC$  and vertices  $A$  and  $D$  are on the same side of  $BC$ . If  $AD$  is extended to intersect  $BC$  at  $P$ , show that  $AP$  bisects  $\angle A$  as well as  $\angle D$ .  $AP$  is the perpendicular bisector of  $BC$

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**128.** A point  $O$  is taken inside an equilateral four sided figure  $ABCD$  such that its distances from the angular points  $D$  and  $B$  are equal. Show that  $AO$  and  $OC$  are in one and the same straight line. GIVEN : A point  $O$  inside an equilateral quadrilateral four sided figure  $ABCD$  such that  $BO = OD$ . TO PROVE :  $AO$  and  $OC$  are in one and the same straight line.

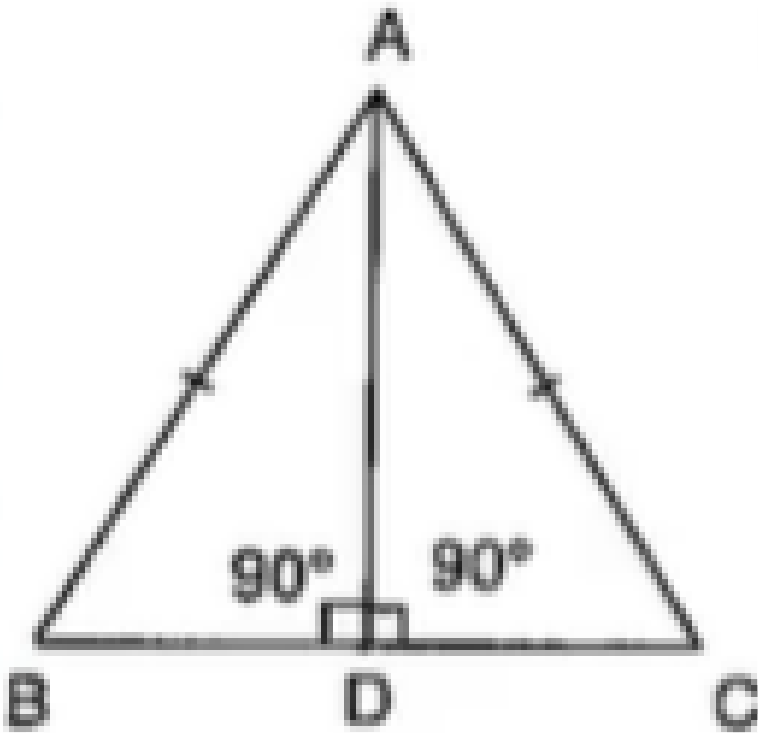
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**129.**  $AD$ ,  $BE$  and  $CF$ , the altitudes of  $ABC$  are equal. Prove that  $ABC$  is an equilateral triangle.

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**130.** If  $ABC$  is an isosceles triangle such that  $AB = AC$  and  $AD$  is an altitude from  $A$  on  $BC$ . Prove that (i)  $\angle B = \angle C$  (ii)  $AD$  bisects  $BC$  (iii)

$AD$  bisects  $\angle A$



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**131.**  $ABC$  is a triangle and  $D$  is the mid-point of  $BC$ . The perpendiculars from  $D$  to  $AB$  and  $AC$  are equal. Prove that the triangle is isosceles.

.Given :  $D$  is the mid-point of  $BC$  and  $PD=DQ$ .

To prove :  $\triangle ABC$  is isosceles.

Proof

Let  $DE$  and  $DF$  be the perpendiculars from  $D$  on  $AB$  and  $AC$  respectively.

In  $\triangle BDE$  and  $\triangle CDF$

$DE=DF$  ( given )

$\angle BED = \angle CFD = 90^\circ$

$BD=DC$  (  $D$  is the mid-point of  $BC$  )

$\therefore \triangle BDE \cong \triangle CDF$  ( By RHS )

$\angle B = \angle C$  ( by CPCT )

$AC=AB$  ( Sides opposite to equal angles are equal )

$\therefore \triangle ABC$  is isosceles.



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**132.**  $ABC$  is a triangle in which  $BE$  and  $CF$  are, respectively, the perpendiculars to the sides  $AC$  and  $AB$ . If  $BE = CF$ , prove that  $ABC$  is isosceles.



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**133.** In perpendiculars from any point within an angle on its arms are congruent, prove that it lies on the bisector of that angle.

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**134.**  $ABCD$  is a square,  $X$  and  $Y$  are points on sides  $AD$  and  $BC$  respectively such that  $AY = BX$ . Prove that  $BY = AX$  and  $\angle BAY = \angle ABX$

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**135.** Which of the following statements are true (T) and which are false (F): Side opposite to equal angles of a triangle may be unequal. Angle opposite to equal sides of a triangle are equal. The measure of each angle of an equilateral triangle is  $60^\circ$ . If the altitude from one vertex of a triangle bisects the opposite side, then the triangle may be isosceles. The bisectors of two equal angles of a triangle are equal. If the bisector of the vertical angle of a triangle bisects the base, then the triangle may be

isosceles. The two altitudes corresponding to two equal sides of a triangle need not be equal. If any two sides of a right triangle are respectively equal to two sides of other right triangle, then the two triangles are congruent. Two right triangles are congruent if hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.



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**136.** Fill the blanks in the following so that each of the following statements is true.

- (i) Sides opposite to equal angles of a triangle are .....
- (ii) Angle opposite to equal sides of a triangle are .....
- (iii) In an equilateral triangle all angles are .....
- (iv) In a  $\Delta ABC$  if  $\angle A = \angle C$  , then  $AB = \dots$
- (v) If altitudes  $CE$  and  $BF$  of a triangle  $ABC$  are equal, then  $AB = \dots$
- (vi) In an isosceles triangle  $ABC$  with  $AB = AC$ , if  $BD$  and  $CE$  are its altitudes, then  $BD$  is .....  $CE$ .



(vii) In right triangles  $ABC$  and  $DEF$ , if hypotenuse  $AB = EF$  and side  $AC = DE$ , then  $\triangle ABC \cong \triangle \dots$

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**137.** In a  $ABC$ , if  $\angle A = 45^0$  and  $\angle B = 70^0$ . Determine the shortest and largest sides of the triangle.

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**138.** In a  $ABC$ , if  $\angle A = 50^0$  and  $\angle B = 60^0$ , determine the shortest and largest sides of the triangle.

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**139.** If  $D$  is any point on the base  $BC$  produced, of an isosceles triangle  $ABC$ , prove that  $AD > AB$ .

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**140.** Prove that any two sides of a triangle are together greater than twice the median drawn to the third side.

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**141.** Prove that the perimeter of a triangle is greater than the sum of its three medians. In the  $\triangle ABC$ , D,E and F are the midpoints of sides BC,CA and AB respectively.

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**142.** In  $PQR$  ,  $S$  is any point on the side  $QR$  . Show that  $PQ + QR + RP > 2PS$

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**143.** Of all the line segments drawn from a point  $P$  to a line  $m$  not containing  $P$ , let  $PD$  be the shortest. If  $B$  and  $C$  are points on  $m$  such that  $D$  is the mid-point of  $BC$ , prove that  $PB = PC$



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**144.** In  $ABC$ , if  $\angle A = 40^\circ$  and  $\angle B = 60^\circ$ . Determine the longest and shortest sides of the triangle.



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**145.** In a  $ABC$ , if  $\angle B = \angle C = 45^\circ$ , which is the longest side?



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**146.** Is it possible to draw a triangle with sides of length  $2\text{cm}$ ,  $3\text{cm}$  and  $7\text{cm}$ ?

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147. In  $ABC$ ,  $\angle B = 35^\circ$ ,  $\angle C = 65^\circ$  and the bisector of  $\angle BAC$  meets  $BC$  in  $P$ . Arrange  $AP$ ,  $BP$  and  $CP$  in descending order.

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148. Prove that the perimeter of a triangle is greater than the sum of its altitudes.

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149. Prove that in a quadrilateral the sum of all the sides is greater than the sum of its diagonals.

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150. In Figure, prove that  $CD + DA + AB + BC > 2AC$

$$CD + DA + AB + BC > 2AC$$



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151. Which of the following statements are true (T) and which are false (F)? Sum of the three sides of a triangle is less than the sum of its three altitudes. Sum of any two sides of a triangle is greater than twice the median drawn to the third side. Sum of any two sides of a triangle is greater than the third side. Difference of any two sides of a triangle is equal to the third side. If two angles of a triangle are unequal, then the greater angle has the larger side opposite to it. Of all the line segments that can be drawn from a point to a line not containing it, the perpendicular line segment is the shortest one.



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**152.** Fill in the blanks to make the following statements true: In a right triangle the hypotenuse is the ..... side. The sum of three altitudes of a triangle is ..... than its perimeter. The sum of any two sides of a triangle is ..... than the third side. If two angles of a triangle are unequal, then the smaller angle has the ..... side opposite to it. Difference of any two sides of a triangle is ..... than the third side. If two sides of a triangle are unequal, then the larger side has ..... angle opposite to it.

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**153.** In two congruent triangles  $ABC$  and  $DEF$ , if  $AB = DE$  and  $BC = EF$ . Name the pairs of equal angles.

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**154.** In two triangles  $ABC$  and  $DEF$ , it is given that  $\angle A = \angle D$ ,  $\angle B = \angle E$  and  $\angle C = \angle F$ . Are the two triangles necessarily congruent?



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155. If  $ABC$  and  $DEF$  are two triangles such that  $AC = 2.5 \text{ cm}$ ,  $BC = 5 \text{ cm}$ ,  $\angle C = 75^\circ$ ,  $de = 25 \text{ cm}$ ,  $df = 5 \text{ cm}$  and  $\angle F = 75^\circ$ . Are two triangles congruent?



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156. In two triangles  $ABC$  and  $ADC$ , if  $AB = AD$  and  $BC = CD$ . Are they congruent?



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157. In triangle  $ABC$  and  $CDE$ , if  $AC = CE$ ,  $BC = CD$ ,  $\angle A = 60^\circ$ ,  $\angle C = 30^\circ$  and  $\angle D = 90^\circ$ . Are two triangles congruent?



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**158.**  $ABC$  is an isosceles triangle in which  $AB = AC$  and  $BE$  and  $CF$  are its two medians. Show that  $BE = CF$ .

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**159.** Find the measure of each angle of an equilateral triangle.

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**160.**  $CDE$  is an equilateral triangle formed on a side  $CD$  of a square  $ABCD$ . Show that  $ADE \cong BCE$ .

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**161.** Show that the sum of the three altitudes of a triangle is less than the sum of three sides of the triangle.

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162. If  $ABC \cong ACB$ , then  $ABC$  is isosceles with  $AB = AC$  (b)  $AB = BC$  (c)  $AC = BC$  (d) None of these

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163. If  $ABC \cong PQR$  and  $ABC$  is not congruent to  $RPQ$ , then which of the following is not true: (a)  $BC = PQ$  (b)  $AC = PR$  (c)  $AB = PQ$  (d)  $QR = BC$

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164. In triangles  $ABC$  and  $PQR$  three equality relations between some parts are as follows:  $AB = QP$ ,  $\angle B = \angle P$  and  $BC = PR$ . State which of the congruence conditions applies: (a)  $SAS$  (b)  $ASA$  (c)  $SSS$  (d)  $RHS$

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165. In triangles  $ABC$  and  $PQR$ , if  $\angle A = \angle R$ ,  $\angle B = \angle P$  and  $AB = RP$ , then which one of the following congruence conditions applies:

- (a)  $SAS$
- (b)  $ASA$
- (c)  $SSS$
- (d)  $RHS$

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166. If  $PQR \cong EFD$ , then  $ED =$  (a)  $PQ$  (b)  $QR$  (c)  $PR$  (d) None of these

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167. If  $PQR \cong EFD$ , then  $\angle E =$  (a)  $\angle P$  (b)  $\angle Q$  (c)  $\angle R$  (d) None of these

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**168.** In a  $ABC$ , if  $AB = AC$  and  $BC$  is produced to  $D$  such that  $\angle ACD = 100^\circ$ , then  $\angle A = 20^\circ$  (b)  $40^\circ$  (c)  $60^\circ$  (d)  $80^\circ$

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**169.** In an isosceles triangle, if the vertex angle is twice the sum of the base angles, then the measure of vertex angle of the triangle is  $100^\circ$  (b)  $120^\circ$  (c)  $110^\circ$  (d)  $130^\circ$

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**170.**  $D, E, F$  are the mid-point of the sides  $BC, CA$  and  $AB$  respectively of  $ABC$ . Then  $DEF$  is congruent to triangle.  $ABC$  (b)  $AEF$  (c)  $BFD, CDE$  (d)  $AFE, BFD, CDE$

Given :  $D, E, F$  are the mid-point of the sides  $BC, CA$  and  $AB$  .

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171. Which of the following is not a criterion for congruence of triangles?

*SAS* (b) *SSA* (c) *ASA* (d) *SSS*

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172. If  $ABC$  and  $DEF$  are two triangles such that  $ABC \cong FDE$  and  $AB = 5\text{cm}$ ,  $\angle B = 40^\circ$  and  $\angle A = 80^\circ$ . Then, which of the following is true?  $DF = 5\text{cm}$ ,  $\angle F = 60^\circ$  (b)

$DE = 5\text{cm}$ ,  $\angle E = 60^\circ$   $DF = 5\text{cm}$ ,  $\angle E = 60^\circ$  (d)

$DE = 5\text{cm}$ ,  $\angle D = 40^\circ$

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