



MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

BINOMIAL THEOREM

Solved Examples And Exercises

1. Prove that

$$({}^{2n}C_0)^2 + ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 + \dots + ({}^{2n}C_{2n})^2 = (-1)^n$$

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2. Find the largest term in the expansion of $(3 + 2x)^{50}$, where $x = 1/5$.

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3. Find the following sum: $\frac{1}{n!} + \frac{1}{2!(n-2)!} + \frac{1}{4!(n-4)!} + \dots$

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4. Find the sum of the last 30 coefficients in the expansion of $(1+x)^{59}$, when expanded in ascending powers of x .

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5. If $x = 1/3$, find the greatest term in the expansion of $(1+4x)^8$.

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6. If the sum of coefficients in the expansion of $(x-2y+3z)^n$ is 128, then find the greatest coefficients in the expansion of $(1+x)^n$.

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7. Find the sum of the coefficients in the expansion of $(1 + 2x + 3x^2 + nx^n)^2$.

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8. The number of terms in the expansion of $(1 + x)^{101}(1 + x^2 - x)^{100}$ in powers of x is

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9. Find the sum of coefficients in $(1 + x - 3x^2)^{4163}$.

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10. Find the middle term in the expansion of $\left(x^2 + \frac{1}{x^2} + 2\right)^n$.

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11. In the expansion of $(1 + x)^{50}$, find the sum of coefficients of odd powers of x .

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12. If $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^{12} + \dots + a_{12}x^{12}$, then find the value of $a_2 + a_4 + a_6 + \dots + a_{12}$.

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13. If the middle term in the binomial expansion of $\left(\frac{1}{x} + x \sin x^{10}\right)$ is equal to $\frac{63}{8}$, find the value of x .

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14. Find the sum $C_0 + 3C_1 + 3^2C_2 + \dots + 3^nC_n$.

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15. If $(1 + x)^n = \sum_{r=0}^n C_r x^r$, then prove that $C_1 + 2C_2 + 3C_3 + \dots + nC_n = n2^{n-1}$.

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16. If $T_0, T_1, T_2, \dots, T_n$ represent the terms in the expansion of $(x + a)^n$, then find the value of $(T_0 - T_2 + T_4 - \dots)^2 + (T_1 - T_3 + T_5 - \dots)^2 n \in N$.

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17. If $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \dots + a_{2n}x^{2n}$, find the value of $a_0 + a_3 + a_6 + \dots, n \in N$.

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18. Find the sum $C_0 - C_2 + C_4 - C_6 + \dots$, where $C_r = {}^n C_r$.

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19. Prove that ${}^n C_0 + {}^n C_3 + {}^n C_6 + \dots = \frac{1}{3} \left(2^n + 2 \frac{\cos(n\pi)}{3} \right)$.

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20. Given that the 4th term in the expansion of $[2 + (3/8x)]^{10}$ has the maximum numerical value. Then find the range of value of x .

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21. Find the greatest coefficient in the expansion of $(1 + 2x/3)^{15}$.

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22. Find the greatest term in the expansion of $\sqrt{3} \left(1 + \frac{1}{\sqrt{3}} \right)^{20}$.

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23. Find the numerically greatest term in the expansion of $(3 - 5x)^{15}$ when $x = 1/5$.

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24. Let n be an odd natural number greater than 1. Then, find the number of zeros at the end of the sum $99^n + 1$.

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25. Find the remainder when 27^{40} is divided by 12.

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26. In the expansion of $(1 + x)^n$, 7th and 8th terms are equal. Find the value of $(7/x + 6)^2$.

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27. Find the sum $\sum_{j=0}^n \binom{4n+1}{j} C_j + \binom{4n+1}{2n-j} C_{2n-j}$.

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28. Show that no three consecutive binomial coefficients can be in (i) G.P.,
(ii) H.P.

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29. Find the sum $\sum_{r=1}^n r^2 \frac{\binom{n}{r}}{\binom{n}{r-1}}$.

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30. Show that $9^{n+1} - 8n - 9$ is divisible by 64, where n is a positive integer.

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31. If the 3rd, 4th, 5th and 6th term in the expansion of $(x + \alpha)^n$ be, respectively, a, b, c and d , prove that $\frac{b^2 - ac}{c^2 - bd} = \frac{5a}{3c}$.

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32. Find the remainder when 7^{98} is divided by 5.

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33. Show that $2^{4n+4} - 5n - 15$, where $n \in \mathbb{N}$ is divisible by 225.

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34. If $(2 + \sqrt{3})^n = I + f$, where I and n are positive integers and $f > 0$

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35. Find the degree of the polynomial

$$\frac{1}{\sqrt{4x+1}} \left\{ \left(\frac{1 + \sqrt{4x+1}}{2} \right)^7 - \left(\frac{1 - \sqrt{4x+1}}{2} \right)^7 \right\}$$

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36. If $9^7 + 7^9$ is divisible by 2^n , then find the greatest value of n , where $n \in \mathbb{N}$.

A. 5

B. 6

C. 7

D. None of these

Answer: B

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37. Prove that $\sqrt{10}[(\sqrt{10} + 1)^{100} - (\sqrt{10} - 1)^{100}]$ is an even integer .

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38. Find the remainder when $x = 5^{5^{5^{\dots}}}$ (24 times 5) is divided by 24.

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39. Find the remainder when $1690^{2608} + 2608^{1690}$ is divided by 7.

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40. Find the value of $\{3^{2003}/28\}$, where $\{.\}$ denotes the fractional part.

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41. Find the remainder when 5^{99} is divided by 13.



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42. Find the remainder when 7^{103} is divided by 25.



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43. Using binomial theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25.



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44. If the coefficient of the middle term in the expansion of $(1+x)^{2n+2}$ is α and the coefficients of middle terms in the expansion of $(1+x)^{2n+1}$ are β and γ then relate α , β and γ .



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45. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1:7:42, then find the value of n .

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46. In the coefficients of r th, $(r + 1)$ th, and $(r + 2)$ th terms in the binomial expansion of $(1 + y)^m$ are in A.P., then prove that $m^2 - m(4r + 1) + 4r^2 - 2 = 0$.

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47.

$$(C_0 + C_1)(C_1 + C_2)(C_2 + C_3)(C_3 + C_4)\dots\dots\dots (C_{n-1} + C_n) = \frac{C_0 C_1 C_2 \dots C_n}{n!}$$

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48. If a_1, a_2, a_3, a_4 be the coefficient of four consecutive terms in the expansion of $(1 + x)^n$, then prove that:

$$\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}.$$

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49. Find the sum of $\sum_{r=1}^n \frac{r^n C_r}{n C_{r-1}}$.

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50. Find the positive integer just greater than $(1 + 0.0001)^{10000}$.

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51. Find (i) the last digit, (ii) the last two digits, and (iii) the last three digits of 17^{256} .

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52. If 10^m divides the number $101^{100} - 1$ then, find the greatest value of m .

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53. Using the principle of mathematical induction, prove that $(2^{3n} - 1)$ is divisible by 7 for all $n \in \mathbb{N}$

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54. If x is very large as compare to y , then prove that

$$\sqrt{\frac{x}{x+y}} \sqrt{\frac{x}{x-y}} = 1 + \frac{y^2}{2x^2}.$$

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55. Find the coefficient of x^n in the expansion of $(1 - 9x + 20x^2)^{-1}$.



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56. Prove that the coefficient of x^r in the expansion of $(1 - 2x)^{-\frac{1}{2}}$ is $\frac{2r!}{(2^r)(r!)^2}$



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57. Find the sum $1 - \frac{1}{8} + \frac{1}{8} \times \frac{3}{16} - \frac{1 \times 3 \times 5}{8 \times 16 \times 24} +$



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58. Show that $\sqrt{3} = 1 + \frac{1}{3} + \left(\frac{1}{3}\right) \left(\frac{3}{6}\right) + \left(\frac{1}{3}\right) \left(\frac{3}{6}\right) \left(\frac{5}{9}\right) + \left(\frac{7}{12}\right) + \dots$



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59. Assuming x to be so small that x^2 and higher power of x can be

neglected, prove that
$$\frac{\left(1 + \frac{3x}{4}\right)^{-4} (16 - 3x)^{\frac{1}{2}}}{(8 + x)^{\frac{2}{3}}} = 1 - \left(\frac{305}{96}\right)x$$



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60. Find the sum $\sum_{i=0}^n (i+1)2^i$



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61. Find the condition for which the formula

$$(a + b)^m = a^m + ma^{m-1}b + \frac{m(m-1)}{1 \cdot 2}a^{m-2}b^2 + \dots \text{ Holds when}$$



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62. Find the value of x , for which $1/\sqrt{5+4x}$ can be expanded as infinite series.



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63. Find the fourth term in the expansion of $(1 - 2x)^{3/2}$.

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64. Prove that $\sum_{r=0}^n \binom{2n}{r} \binom{n}{r} = 2^{2n}$.

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65. Prove that $\sum_{r=0}^n \binom{n}{r} \binom{n}{r} = \sum_{r=0}^n \binom{n}{r}^2 = \binom{2n}{n}$.

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66. Find the sum of the coefficients of all the integral powers of x in the expansion of $(1 + 2\sqrt{x})^{40}$.

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67. If the sum of the coefficient in the expansion of $(\alpha^2 x^2 - 2\alpha x + 1)^{51}$ vanishes, then find the value of α

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68. Prove that
$$\sum_{\alpha + \beta + \gamma = 10} \frac{10!}{\alpha! \beta! \gamma!} = 3^{10}.$$

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69. If $(1 + x - 2x^2)^{20} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_{40} x^{40}$, then find the value of $a_1 + a_3 + a_5 + \dots + a_{39}$.

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70. Find the sum of the series ${}^{15}C_0 + {}^{15}C_1 + {}^{15}C_2 + \dots + {}^{15}C_7$.

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71. Find the sum $\sum_{k=0}^{10} {}^{20}C_k =$

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72. Find the sum of all the coefficients in the binomial expansion of $(x^2 + x - 3)^{319}$.

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73. If the sum of coefficient of first half terms in the expansion of $(x + y)^n$ is 256, then find the greatest coefficient in the expansion.

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74. Find the value of $\sum_{p=1}^n \left(\sum_{m=p}^n {}^n C_m {}^m C_p \right)$. And hence, find the value of $\lim_{n \rightarrow \infty} \frac{1}{3^n} \sum_{p=1}^n \left(\sum_{m=p}^n {}^n C_m {}^m C_p \right)$.



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75. Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{(1 \cdot 3 \cdot 5 \cdots (2n-1))}{n!} 2^n x^n$, where n is a positive integer.

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76. If the middle term in the expansion of $(x^2 + 1/x)^n$ is $924x^6$, then find the value of n .

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77. The first three terms in the expansion of $(1+ax)^n$ ($n \neq 0$) are 1 , $6x$ and $16x^2$. Then find the value of a and n .

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78. If x^4 occurs in the r th term in the expansion of $\left(x^4 + \frac{1}{x^3}\right)^{15}$, then find the value of r .

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79. Find the coefficient of x^{-10} in the expansion of $\left(\frac{a}{x} + bx\right)^{12}$.

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80. Find the constant term in the expansion of $(x - 1/x)^6$.

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81. If the coefficients of $(r - 5)$ th and $(2r - 1)$ th terms in the expansion of $(1 + x)^{34}$ are equal, find r .

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82. In $\left(2^{\frac{1}{3}} + \frac{1}{3^{\frac{1}{3}}}\right)^n$ if the ratio of 7th term from the beginning to the 7th term from the end is $1/6$, then find the value of n .

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83. If the coefficient of 4th term in the expansion of $(a + b)^n$ is 56, then n is

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84. If p and q are positive, then prove that the coefficients of x^p and x^q in the expansion of $(1 + x)^{p+q}$ will be equal.

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85. Find the number of irrational terms in the expansion of $\left(5^{1/6} + 2^{1/8}\right)^{100}$.



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86. If x^p occurs in the expansion of $(x^2 + 1/x)^{2n}$, prove that its coefficient is $\frac{(2n)!}{\left[\frac{1}{3}(4n - p)\right]! \left[\frac{1}{3}(2n + p)\right]!}$.



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87. Find the coefficient of $a^3b^4c^5$ in the expansion of $(bc + ca + ab)^6$.



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88. Find the coefficient of x^7 in the expansion of $(1 + 3x - 2x^3)^{10}$.



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89. If the number of terms in the expansion of $(x + y + z)^n$ are 36, then find the value of n .



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90. Find the coefficient of a^3b^4c in the expansion of $(1 + a - b + c)^9$.



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91. Find the coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^{11}$.



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92. Find the number of terms which are free from radical signs in the expansion of $(y^{1/5} + x^{1/10})^{55}$.



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93. Find the coefficient of x^5 in the expansion of $(1 + x^2)^5(1 + x^4)^6$.



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94. Find the coefficient of x^{13} in the expansion of $(1 - x)^5 \times (1 + x + x^2 + x^2)^4$.

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95. Find the sum ${}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9$.

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96. Find the sum of $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \dots$

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97. If n is an even positive integer, then find the value of x if the greatest term in the expansion of $(1 + x)^n$ may have the greatest coefficient also.

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98. If $|x| < 1$, then find the coefficient of x^n in the expansion of $(1 + 2x + 3x^2 + 4x^3 + \dots)^{1/2}$.

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99. If $(r + 1)$ th term is the first negative term in the expansion of $(1 + x)^{7/2}$, then find the value of r .

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100. If $|x| < 1$, then find the coefficient of x^n in the expansion of $(1 + x + x^2 + \dots)^2$.

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101. If $|x| > 1$, then expand $(1 + x)^{-2}$.



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102. Find the cube root of 217, correct to two decimal places.

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103. Find the coefficient of $x^2 \in \left(\frac{a}{a+x}\right)^{1/2} + \left(\frac{a}{a-x}\right)^{1/2}$

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104. Prove that ${}^{10}C_1(x-1)^2 - {}^{10}C_2(x-2)^2 + {}^{10}C_3(x-3)^2 \pm \dots \pm {}^{10}C_{10}(x-10)^2 = x^2$

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105. If the third term in the expansion of $(1+x)^m$ is $-\frac{1}{8}x^2$, then find the value of m .



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106. Prove that $\sum_{r=0}^n r(n-r)C_r 2 = n^2 \binom{n}{2} C_n$.

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107. Prove that $1 - {}^n C_1 \frac{1+x}{1+nx} + {}^n C_2 \frac{1+2x}{(1+nx)^2} - {}^n C_3 \frac{1+3x}{(1+nx)^3} + \dots$

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108. Find the coefficient of x^{20} in $\left(x^2 + 2 + \frac{1}{x^2}\right)^{-5} (1+x^2)^{40}$.

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109. The number of terms in the expansion of $(a+b+c)^n$, where $n \in \mathbb{N}$.

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110. Find the coefficient of x^{50} in the expansion of $(1+x)^{101} \times (1-x+x^2)^{100}$.

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111. Find the coefficient of x^4 in the expansion of $(2-x+3x^2)^6$.

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112. Find the coefficient of x^k in the expansion of $1 + (1+x) + (1+x)^2 + \dots + (1+x)^n$ ($0 \leq k \leq n$).

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113. Find the term independent of x in the expansion of $(1+x+2x^3) \left[\left(\frac{3x^2}{2} \right) - \left(\frac{1}{3x} \right) \right]^9$

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114. If a and b are distinct integers, prove that $a - b$ is a factor of $a^n - b^n$, wherever n is a positive integer.

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115. Find the a , b , and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729, 7290, and 30375, respectively.

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116. Find the coefficient of x^{25} in expansion of expression

$$\sum_{r=0}^{50} {}^{50}C_r (2x - 3)^r (2 - x)^{50-r}.$$

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117. If the sum of the coefficients of the first, second, and third terms of the expansion of $\left(x^2 + \frac{1}{x}\right)^m$ is 46, then find the coefficient of the term that does not contain x .

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118. If $p + q = 1$, then show that $\sum_{r=0}^n r^2 \binom{n}{r} p^r q^{n-r} = npq + n^2 p^2$.

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119. If $(18x^2 + 12x + 4)^n = a_0 + a_1x + a_2x^2 + \dots + a_{2n}x^{2n}$, prove that $a_r = 2^n 3^r \left(\binom{2n}{r} C_1^{2n-2} C_r + \binom{2n}{r} C_2^{2n-4} C_r + \dots \right)$.

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120. Prove that $\binom{m}{1} C_1^n C_m - \binom{m}{2} C_2^{2n} C_m + \binom{m}{3} C_3^{3n} C_m \equiv (-1)^{m-1} n^m$.

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121. Prove that

$${}^nC_0 {}^{2n}C_n - {}^nC_1 {}^{2n-1}C_n + {}^nC_2 {}^{2n-2}C_n + \dots + (-1)^n {}^nC_n {}^nC_n = 1.$$

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122. Find the sum $\sum_{r=0}^n (n+r)C_r$.

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123. Find the value of $\sum_{0 \leq i \leq j \leq n} (i+j)(nC_i + nC_j)$.

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124. Find the value of $\sum_{1 \leq i \leq j \leq n-1} (ij)^n c_i^n c_j$.

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125. Find the value of $\sum_{i=0}^n \sum_{j=0}^i 1$

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126. Find the sum $\sum_{0 \leq i < j \leq n} {}^n C_i {}^n C_j$

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127. Prove that $\sum_{r=0}^s \sum_{s=1}^n {}^n C_s {}^n C_r = 3^n - 1$.

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128. Find the sum $\sum_{0 \leq i < j \leq n} {}^n C_i$

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129. Find the coefficient of x^4 in the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$.

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130. Find the term in $\left(\frac{a}{\sqrt{b}}3 + \sqrt{\frac{b}{a3}}\right)^{21}$ which has the same power of a and b .

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131. Using the binomial theorem, evaluate $(102)^5$.

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132. Find the 6th term in expansion of $(2x^2 - 1/3x^2)^{10}$.

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133. Find a if the 7th and 18th terms of the expansion $(2 + a)^{50}$ are equal.

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134. Find n , if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$.

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135. Simplify: $x^5 + 10x^4a + 40x^3a^2 + 80x^2a^3 + 80xa^4 + 32a^5$.

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136. Find the value of

$$\frac{18^3 + 7^3 + 3 \times 18 \times 7 \times 25}{3^6 + 6 \times 243 \times 2 + 15 \times 18 \times 4 + 20 \times 27 \times 8 + 15 \times 9 \times 16 + 6 \times 3 \times \dots}$$

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137. Find the approximation of $(0.99)^5$ using the first three terms of its expansion.

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138. There are two bags each of which contains n balls. A man has to select an equal number of balls from both the bags. Prove that the number of ways in which a man can choose at least one ball from each bag is $2^{2n}C_n - 1$.

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139. Find the sum $\sum_{i=0}^r \binom{n_1}{r-i} \binom{n_2}{i}$.

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140. Prove that $\sum_{r=0}^{2n} \binom{2n}{r}^2 = n^{4n}C_{2n}$.

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141. If k and n are positive integers and $s_k = 1^k + 2^k + 3^k + \dots + n^k$, then prove that $\sum_{r=1}^m \binom{m+1}{r} s_r = (n+1)^{m+1} - (n+1)$.

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142. Prove that $\sum_{r=1}^n (-1)^{r-1} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{r}\right)^n C_r = \frac{1}{n}$.

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143. Prove that

$$\frac{C_1}{1} - \frac{C_2}{2} + \frac{C_3}{3} - \frac{C_4}{4} + \dots + \frac{(-1)^{n-1}}{n} C_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}.$$

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144. Prove that $\sum_{r=0}^n {}^n C_r \sin rx \cos(n-r)x = 2^{n-1} \sin(nx)$.

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145. Find the last two digits of the number $(23)^{14}$.

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146. Find the last three digits of the number 27^{27} .

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147. Find the number of nonzero terms in the expansion of $(1 + 3\sqrt{2}x)^9 + (1 - 3\sqrt{2}x)^9$.

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148. Find the value of $(\sqrt{2} + 1)^6 - (\sqrt{2} - 1)^6$.

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149. Using binomial theorem (without using the formula for ${}^n C_r$),
prove that

$${}^n C_4 + {}^m C_2 - {}^{m-n} C_1 {}^n C_2 = {}^m C_4 - {}^{m+n} C_1 {}^m C_3 + {}^{m+n} C_2 {}^m C_2 - {}^{m+n} C_3 {}^m C_1 + {}^{m+n} C_4$$

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150. Prove that

$$(r+1)^n {}^n C_r - r^n {}^n C_r + (r-1)^n {}^n C_2 - {}^n C_3 + \dots + (-1)^r {}^n C_r = (-1)^r {}^n C_0$$

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151. Find the sum ${}^n C_0 + {}^n C_4 + {}^n C_8 + \dots$

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152. Find the value of ${}^{4n}C_0 + {}^{4n}C_4 + {}^{4n}C_8 + \dots + {}^{4n}C_{4n}$.



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153. If

$$\frac{1}{\sqrt{4x+1}} \left\{ \left(\frac{1 + \sqrt{4x+1}}{2} \right)^n - \left(\frac{1 - \sqrt{4x+1}}{2} \right)^n \right\} = a_0 + a_1x$$

then find the possible value of n .



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154. Find the coefficient of x^n in the polynomial

$$(x + {}^nC_0)(x + 3^n C_1) \times (x + 5^n C_2) [x + (2n + 1)^n C_n].$$



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155. If $(1 + x)^{15} = C_0 + C_1x + C_2x^2 + \dots + C_{15}x^{15}$, then find the value of $C_2 + 2C_3 + 3C_4 + \dots + 14C_{15}$.

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156. Prove that $\frac{{}^nC_0}{1} + \frac{{}^nC_2}{3} + \frac{{}^nC_4}{5} + \frac{{}^nC_6}{7} + \dots = \frac{2^n}{n+1}$.

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157. Find the sum $\sum_{0 \leq i \leq j \leq n} {}^nC_i {}^nC_j$

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158. Find the sum $\sum_{i \neq j} {}^nC_i {}^nC_j$

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159. Show that the integer next above $(\sqrt{3} + 1)^{2m}$ contains 2^{m+1} as a factor.



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160. Prove that

$$\frac{1^2}{3} {}^n C_1 + \frac{1^2 + 2^2}{5^n} C_2 + \frac{1^1 + 2^2 + 3^2}{7^n} C_3 + \dots + \frac{1^2 + 2^2 + \dots + n^2}{(2n + 1)^n} C_n = \frac{n(n + 3)}{62^{n-2}}.$$



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161. Prove that

$$\frac{1}{n + 1} = \frac{{}^n C_1}{2} - \frac{2({}^n C_2)}{3} + \frac{3({}^n C_3)}{4} - \dots + (-1)^{n+1} \frac{n({}^n C_n)}{n + 1}.$$



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162. Find the sum $2C_0 \frac{2^3}{2} C_1 + \frac{2^3}{3} C_2 + \frac{2^4}{4} C_3 + \dots + \frac{2^{11}}{11} C_{10}$.



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163. If in the expansion of $(2x + 5)^{10}$, the numerically greatest term is equal to the middle term, then find the values of x .

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164. Find the value of

$$\frac{1}{81^n} - \frac{10}{(81^n)^{2n}} C_1 + \frac{10^2}{(81^n)^{2n}} C_2 - \frac{10^3}{(81^n)^{2n}} C_3 + \dots + \frac{10^{2n}}{81^n}.$$

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165. Find the value of

$${}^{20}C_0 - \frac{{}^{20}C_1}{2} + \frac{{}^{20}C_2}{3} - \frac{{}^{20}C_3}{4} + \dots$$

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166. Find the sum $1C_0 + 2C_1 + 3C_2 + \dots + (n + 1)C_n$, where $C_r = {}^n C_r$.

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167. If $(1 + x + x^2 + \dots + x^p)^n = a_0 + a_1x + a_2x^2 + \dots + a_{np}x^{np}$, then find the value of $a_1 + 2a_2 + 3a_3 + \dots + np a_{np}$.

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168. If $n > 2$, then prove that

$C_1(a - 1) - C_2 \times (a - 2) + \dots + (-1)^{n-1} C_n(a - n) = a$, where $C_r = {}^n C_r$.

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169. Find the sum $1 \times 2 \times C_1 + 2 \times 3C_2 + \dots + n(n + 1)C_n$, where $C_r = {}^n C_r$.

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170. If $x + y = 1$, prove that $\sum_{r=0}^n {}^n C_r x^r y^{n-r}$.

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171. Find the sum $3^n C_0 - 8^n C_1 + 13^n C_2 - 18^n C_3 + \dots$

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172. Prove that $\frac{{}^n C_1}{2} + \frac{{}^n C_3}{4} + \frac{{}^n C_5}{6} + \dots = \frac{2^n - 1}{n + 1}$.

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173. If $(1 + x)^n = \sum_{r=0}^n {}^n C_r x^r$, show that $C_0 + \frac{C_1}{2} + \dots + \frac{C_n}{n + 1} = \frac{2^{n+1} - 1}{n + 1}$.

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174. If $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r$ and $a_k = 1$ for all $k \geq n$, then show that $b_n = {}^{2n+1}C_{n+1}$.

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175. Statement 1: $3^{2n+2} - 8n - 9$ is divisible by 64, $\forall n \in \mathbb{N}$. Statement 2: $(1+x)^n - nx - 1$ is divisible by x^2 , $\forall n \in \mathbb{N}$.

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176. Statement 1: The number of distinct terms in $(1+x+x^2+x^3+x^4)^{1000}$ is 4001. Statement 2: The number of distinct terms in expansion $(a_1 + a_2 + \dots + a_m)^n$ is ${}^{n+m-1}C_{m-1}$.

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177. Statement 1: if $n \in \mathbb{N}$ and n is not a multiple of 3 and

$(1 + x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then the value of $\sum_{r=0}^n (-1)^r a_r C_r$ is zero

Statement 2: The coefficient of x^n in the expansion of $(1 - x^3)^n$ is zero, if

$n = 3k + 1$ or $n = 3k + 2$.



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178. Statement 1: Three consecutive binomial coefficients are always in A.P.

Statement 2: Three consecutive binomial coefficients are not in H.P. or G.P.



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179. The value of

$$\binom{30}{0} \binom{30}{10} - \binom{30}{1} \binom{30}{11} + \binom{30}{2} \binom{30}{12} + \dots + \binom{30}{20} \binom{30}{30} =$$

a. $60C_{20}$ b. $30C_{10}$ c. $60C_{30}$ d. $40C_{30}$



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180. If $f(x) = x^n, f(1) + \frac{f^1(1)}{1} + \frac{f^2(1)}{2!} + \frac{f^n(1)}{n!}$, where $f^r(x)$ denotes the r th order derivative of $f(x)$ with respect to x , is n b. 2^n c. 2^{n-1} d. none of these

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181. The fractional part of $\frac{2^{4n}}{15}$ is ($n \in N$) (A) $\frac{1}{15}$ (B) $\frac{2}{15}$ (C) $\frac{4}{15}$ (D) none of these

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182. The value of ${}^{15}C_0^2 - {}^{15}C_1^2 + {}^{15}C_2^2 - \dots - {}^{15}C_{15}^2$ is

- (a) 15
b. -15
c. 0
d. 51

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183. If the sum of the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ is a and if the sum of the coefficients in the expansion of $(1 + x^2)^n$ is b , then

a. $a = 3b$

b. $a = b^3$

c. $b = a^3$

d. none of these



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184. If $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + a_3x^3 + \dots$, then the value of $a_2 + a_4 + a_6 + \dots + a_{12}$ will be (a) 32 (b) 31 (c) 64 (d) 1024



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185. Maximum sum of coefficient in the expansion of $(1 - x \sin \theta + x^2)^n$ is 1 b. 2^n c. 3^n d. 0



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186. If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is 924 b. 792 c. 1594 d. none of these

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187. The number of distinct terms in the expansion of $\left(x + \frac{1}{x} + x^2 + \frac{1}{x^2}\right)^{15}$ is/are (with respect to different power of x) 255 b. 61 c. 127 d. none of these

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188. The sum of the coefficients of even power of x in the expansion of $(1 + x + x^2 + x^3)^5$ is 256 b. 128 c. 512 d. 64

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189. If the coefficient of x^7 in $\left[ax^2 - \left(\frac{1}{bx^2}\right)\right]^{11}$ equal the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation
 $a + b = 1$ b. $a - b = 1$ c. $b = 1$ d. $\frac{a}{b} = 1$

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190. If the coefficients of the $(2r + 4)$ th, $(r - 2)$ th term in the expansion of $(1 + x)^{18}$ are equal, then the value of r is.

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191. If the coefficients of the r th, $(r + 1)$ th, $(r + 2)$ th terms in the expansion of $(1 + x)^{14}$ are in A.P, then the largest value of r is.

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192. If the three consecutive coefficients in the expansion of $(1 + x)^n$ are 28, 56, and 70, then the value of n is.

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193. Degree of the polynomial

$$\left[\sqrt{x^2 + 1} + \sqrt{x^2 - 1} \right]^8 + \left[\frac{2}{\sqrt{x^2 + 1} + \sqrt{x^2 - 1}} \right]^8 \text{ is.}$$

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194. Least positive integer just greater than $(1 + 0.00002)^{50000}$ is.

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195. If $U_n = (\sqrt{3} + 1)^{2n} + (\sqrt{3} - 1)^{2n}$, then prove that

$$U_{n+1} = 8U_n - 4U_{n-1}.$$

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196. Prove that the coefficient of x^n in the expansion of

$$\frac{1}{(1-x)(1-2x)(1-3x)} \text{ is } \frac{1}{2}(3^{n+2} - 2^{n+3} + 1).$$

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197. The value of

$$(30, 0)(30, 10) - (30, 1)(30, 11) + (30, 2)(30, 11) - \dots + (30, 20)(30, 30)$$

, where $(n, r) = {}^nC_r$ is a. (30, 10) b. (30, 15) c. (60, 30) d. (31, 10)

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198. If $n = 12m(m \in N)$, prove that

$${}^nC_0 - \frac{{}^nC_2}{(2 + \sqrt{3})^2} + \frac{{}^nC_4}{(2 + \sqrt{3})^4} - \frac{{}^nC_6}{(2 + \sqrt{3})^6} + \dots = \left(\frac{2\sqrt{2}}{1 + \sqrt{3}} \right)^n.$$

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199. In the expansion of $(1 + x)^n(1 + y)^n(1 + z)^n$, the sum of the coefficients of the terms of degree 'r' is

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200. Prove that

$${}^{100}C_2 {}^{100}C_2 + {}^{100}C_2 {}^{100}C_4 + {}^{100}C_4 {}^{100}C_6 + \dots + {}^{100}C_{98} {}^{100}C_{100} = \frac{1}{2} [{}^{200}C_{99} - {}^{200}C_{100}]$$

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201. Prove that $\sum_{r=1}^{m-1} \frac{2r^2 - r(m-2) + 1}{(m-r)^m C_r} = -\frac{1}{m}$.

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202. Find the coefficients of x^{50} in the expression $(1 + x)^{1000} + 2x(1 + x)^{999} + 3x^2(1 + x)^{998} + \dots + 1001x^{1000}$.

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203. If b_1, b_2, \dots, b_n are the n th roots of unity, then prove that

$${}^n C_1 b_1 + {}^n C_2 b_2 + \dots + {}^n C_n b_n = \frac{b}{b-1} \{(1+b)^n - 1\}$$

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204. If ${}^{n+1}C_r : {}^n C_r : (n-1)C_{r-1} = 11:6:3$, then $nr =$ 20

b. 30 c. 40 d. 50

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205. If the last term in the binomial expansion of

$\left(2^{\frac{1}{3}} - \frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3^{\frac{5}{3}}}\right)^{\log_3 8}$, then 5th term from the beginning is 210 b.

420 c. 105 d. none of these

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206. The last two digits of the number $(23)^{14}$ are

A. 01

B. 03

C. 09

D. None of these

Answer: D

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207. The value of x for which the sixth term in the expansion of

$$\left[2^{\log_2} - 2\sqrt{9^{(x-1)+7}} + \frac{1}{\frac{2^1}{5}(\log)_2(3^{(x-1)+1})} \right]^7$$

is 84 is 4 b. 1 or 2 c.

0 or 1 d. 3

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208. If the 6th term in the expansion of $\left(\frac{1}{x^{\frac{8}{3}}} + x^2(\log)_{10}x\right)^8$ is 5600,

then x equals 1 b. $(\log)_e 10$ c. 10 d. x does not exist

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209. The total number of terms which are dependent on the value of x in

the expansion of $\left(x^2 - 2 + \frac{1}{x^2}\right)^n$ is equal to $2n + 1$ b. $2n$ c. n d. $n + 1$

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210. In the expansion of $\left(3^{-x/4} + 3^{5x/4}\right)^n$ the sum of binomial

coefficient is 64 and term with the greatest binomial coefficient exceeds

the third by $(n - 1)$, the value of x must be 0 b. 1 c. 2 d. 3

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211. If n is an integer between 0 and 21, then the minimum value of $n!(21 - n)!$ is attained for $n =$ 1 b. 10 c. 12 d. 20

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212. If R is remainder when $6^{83} + 8^{83}$ is divided by 49, then the value of $R/5$ is.

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213. Let a and b be the coefficients of x^3 in $(1 + x + 2x^2 + 3x^3)^4$ and $(1 + x + 2x^2 + 3x^3 + 4x^4)^4$, then respectively. Then the value of $4a/b$ is.

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214.

Let

$$1 + \sum_{r=1}^{10} \left(3^r \binom{10}{r} C_r + r \binom{10}{r} C_r \right) = 2^{10} (\alpha \cdot 4^5 + \beta) \text{ where } \alpha, \beta \in \mathbb{N}$$

If α, β lies between the roots of $f(x) = 0$, then find the smallest positive integral value of k .


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215. Let $a = 3^{\frac{1}{223}} + 1$ and for all

$$\geq 3, \text{ let } f(n) = {}^n C_0 a^{n-1} - {}^n C_1 a^{n-2} + {}^n C_2 a^{n-3} - \dots + (-1)^{n-1} {}^n C_{n-1}$$

. If the value of $f(2007) + f(2008) = 3^k$ where $k \in \mathbb{N}$, then the value of k is.


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216. If the constant term in the binomial expansion of

$$\left(x^2 - \frac{1}{x} \right)^n, n \in \mathbb{N} \text{ is 15, then the value of } n \text{ is equal to.}$$


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217. The largest value of x for which the fourth tem in the expansion

$$\left(5^{\left(\frac{2}{5}\right) (\log)_5 \sqrt{4^x + 44}} + \frac{1}{5^{\log_5 \left(2^{(x-1) + 7}\right)^{\frac{1}{3}}}} \right)^8 \text{ is } 336 \text{ is.}$$

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218. The number of values in set of values of r for which

$${}^{23}C_r + 2 \cdot {}^{23}C_{r+1} + {}^{23}C_{r+2} \geq {}^{25}C_{15} \text{ is}$$

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219. If the second term of the expansion $\left[a^{\frac{1}{13}} + \frac{a}{\sqrt{a^{-1}}} \right]^n$ is $14a^{5/2}$,

then the value of $\frac{{}^n C_3}{{}^n C_2}$ is.

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220. Given $(1 - 2x + 5x^2 - 10x^3)(1 + x)^n = 1 + a_1x + a_2x^2 + \dots$ and that $a_{12} = 2a_2$ then the value of n is.

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221. Sum of last three digits of the number $N = 7^{100} - 3^{100}$ is.

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222. Let n be a positive integer and $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \dots + a_{2n}x^{2n}$. Show that $a_0^2 - a_1^2 + a_2^2 - \dots + a_{2n}^2 = a_n$.

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223. Prove that $\sum_{r=1}^k (-3)^{r-1} \binom{3n}{2r-1} = 0$, where $k = 3n/2$ and n is an even integer.



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224. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same, if α equals

a. $-\frac{5}{3}$

b. $\frac{10}{3}$

c. $-\frac{3}{10}$

d. $\frac{3}{5}$



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225. If in the expansion of $(1 + x)^n$, a, b, c are three consecutive coefficients, then $n = \frac{ac + ab + bc}{b^2 + ac}$ b. $\frac{2ac + ab + bc}{b^2 - ac}$ c. $\frac{ab + ac}{b^2 - ac}$ d. none of these



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226. If n and k are positive integers, show that $2^k \binom{n}{0} \binom{n}{k} - 2^{k-1} \binom{n}{1} \binom{n-1}{k-1} + 2^{k-2} \binom{n}{2} \binom{n-2}{k-2} - \dots$ stands for $\binom{n}{k}$.

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227. Prove that $(25)^{n+1} - 24n + 5735$ is divisible by $(24)^2$ for all $n = 1, 2,$

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228. The coefficient of $1/x$ in the expansion of $(1+x)^n(1+1/x)^n$ is $\frac{n!}{(n-1)!(n+1)!}$ b. $\frac{(2n)!}{(n-1)!(n+1)!}$ c. $\frac{(2n)!}{(2n-1)!(2n+1)!}$ d. none of these

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229. The coefficient x^5 in the expansion of $(1+x)^{21} + (1+x)^{22} + \dots + (1+x)^{30}$ is a. ${}^{51}C_5$ b. 9C_5 c. ${}^{31}C_6 - {}^{21}C_6$ d. ${}^{30}C_5 + {}^{20}C_5$



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230. If x^m occurs in the expansion of $\left(x + \frac{1}{x^2}\right)^{2n}$, then the coefficient of x^m is

a. $\frac{(2n)!}{(m)!(2n-m)!}$

b. $\frac{(2n)!3!3!}{(2n-m)!}$

c. $\frac{(2n)!}{\left(\frac{2n-m}{3}\right)! \left(\frac{4n+m}{3}\right)!}$

d. none of these



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231. If the coefficients of 5th, 6th, and 7th terms in the expansion of $(1+x)^n$ are in A.P., then $n =$ a. 7 only b. 14 only c. 7 or 14 d. none of

these

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232. If $(1 + 2x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then $a =$ $\binom{n}{2}^2$ b.
 $\binom{n}{r} \binom{n}{r+1}$ c. $\binom{2n}{r}$ d. $\binom{2n}{r+1}$

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233. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$, $n \in N$ if sum of the coefficients of x^5 and x^{10} is 0 then n is

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234. If the coefficients of r th and $(r + 1)$ th terms in the expansion of $(3 + 7x)^{29}$ are equal, then r equals a. 15 b. 21 c. 14 d. none of these

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235. In the expansion of $(1 + 3x + 2x^2)^6$, the coefficient of x^{11} is

a. 144

b. 288

c. 216

d. 576



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236. If $n - 1C_r = (k^2 - 3)^n C_{r+1}$, then (a) $(-\infty, -2]$ (b) $[2, \infty)$ (c) $[-\sqrt{3}, \sqrt{3}]$ (d) $(\sqrt{3}, 2]$



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237. Prove that $\frac{3!}{2(n+3)} = \sum_{r=0}^n (-1)^r \binom{n}{r+3} C_r$



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238. If $a_n = \sum_{r=0}^n \frac{1}{nC_r}$, then $\sum_{r=0}^n \frac{r}{nC_r}$ equals

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239. The expression $\left(x + \frac{(x^3 - 1)^{\frac{1}{2}}}{2}\right)^5 + \left(x - \frac{(x^3 - 1)^{\frac{1}{2}}}{2}\right)^5$ is a polynomial of degree a. 5 b. 6 c. 7 d. 8

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240. For $2 \leq r$

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241. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of the 5th and 6th term is zero. Then a/b equals $(n - 5)/6$ b. $(n - 4)/5$ c. $n/(n - 4)$ d. $6/(n - 5)$





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242. Coefficient of x^{11} in the expansion of $(1 + x^2)^4 (1 + x^3)^7 (1 + x^4)^{12}$

is

- a. 1051
- b. 1106
- c. 1113
- d. 1120



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243. Given positive integers $r > 1, n > 2$ and that the coefficient of $(3rd)th$ and $(r + 2)th$ terms in the binomial expansion of $(1 + x)^{2n}$ are equal. Then (a) $n = 2r$ (b) $n = 2r + 1$ (c) $n = 3r$ (d) non of these



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244. The coefficient of x^4 in $(x/2 - 3/x^2)^{10}$ is $\frac{405}{256}$ b. $\frac{504}{259}$ c. $\frac{450}{263}$ d.

none of these

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245. If C_r stands for nC_r , then the sum of the series

$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!} [C_0^2 - 2C_1^2 + 3C_2^2 - \dots + (-1)^n(n+1)C_n^2], \text{ where}$$

n is an even positive integer, is

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246. The sum $\sum_{i=0}^m \binom{10}{i} \binom{20}{m-i}$, where $\binom{p}{q} = 0$ if $p < q$, is

maximum when m is equal to (A) 5 (B) 10 (C) 15 (D) 20

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247. The coefficient of X^{24} in the expansion of $(1 + X^2)^{12}(1 + X^{12})(1 + X^{24})$

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248. The term independent of a in the expansion of $\left(1 + \sqrt{a} + \frac{1}{\sqrt{a} - 1}\right)^{-30}$ is (a) $30C_{20}$ (b) 0 (c) $30C_{10}$ (d) non of these

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249. The coefficient of x^{53} in the following expansion

$$\sum_{m=0}^{100} {}^{100}C_m (x - 3)^{100-m} \cdot 2^m \text{ is}$$

(a) ${}^{100}C_{47}$

(b.) ${}^{100}C_{53}$

(c.) $-{}^{100}C_{53}$

(d.) none of these

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250. The coefficient of the term independent of x in the expansion of

$$\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}} \right)^{10}$$
 is 210 b. 105 c. 70 d. 112



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251. In the expansion of $(1 + x + x^3 + x^4)^{10}$, the coefficient of x^4 is

$${}^{10}C_4 \text{ b. } {}^{10}C_3 \text{ c. } 210 \text{ d. } 310$$



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252. If coefficient of $a^2b^3c^4 \in (a + b + c)^m$ (where $n \in N$) is L ($L \neq 0$),

then in same expansion coefficient of $a^4b^4c^1$ will be (A) L (B) $\frac{L}{3}$ (C) $\frac{mL}{4}$

(D) $\frac{L}{2}$



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253. The last two digits of the number 3^{400} are (A) 81 (B) 43 (C) 29 (D) 01

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254. The expression $\left(\sqrt{2x^2 + 1} + \sqrt{2x^2 - 1}\right)^6 + \left(\frac{2}{\left(\sqrt{2x^2 + 1} + \sqrt{2x^2 - 1}\right)^{\square}}\right)^6$ is polynomial of degree 6 b. 8 c. 10 d. 12

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255. The coefficient of x^r [$0 \leq r \leq (n - 1)$] in the expansion of $(x + 3)^{n-1} + (x + 3)^{n-2}(x + 2) + (x + 3)^{n-3}(x + 2)^2 + \dots + (x + 2)^{n-1}$ is ${}^n C_r(3^r - 2^n)$ b. ${}^n C_r(3^{n-r} - 2^{n-r})$ c. ${}^n C_r(3^r + 2^{n-r})$ d. none of these

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256. If $(1 + 2x + 3x^2)^{10} = a_0 + a_1x + a_2x^2 + \dots + a_{20}x^{20}$, then a_1 equals
10 b. 20 c. 210 d. none of these

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257. In the expansion of $(5^{1/2} + 7^{1/8})^{1024}$, the number of integral terms is 128 b. 129 c. 130 d. 131

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258. For which of the following values of x , 5th term is the numerically greatest term in the expansion of $(1 + x/3)^{10}$, a. -2 b. 1.8 c. 2 d. -1.9

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259. For natural numbers m, n , if $(1 - y)^m(1 + y)^n = 1 + a_1y + a_2y^2 + \dots$, and $a_1 = a_2 = 10$, then
a. $m+n=80$ b. $m-n=20$ c. $m+n=20$ d. $m-n=80$



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260. If the middle term in the expansion of $\left(\frac{x}{2} + 2\right)^8$ is 1120, then the sum of possible real values of x is.



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261. If

$$(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n, \text{ then } C_0 - (C_0 + C_1 + \dots) + (C_0 - C_1 + C_2 - \dots) + \dots$$

is even integer is a positive value a negative value divisible by 2^{n-1}

divisible by 2^n



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262. In the expansion of $\left(x^2 + 1 + \frac{1}{x^2}\right)^n, n \in N,$ (a) number of terms

is $2n + 1$ (b) coefficient of constant terms is 2^{n-1} (c) coefficient of

x^{2n-1} is n (d) coefficient of x^2 in n



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263. The value of ${}^nC_1 + {}^{n+1}C_2 + {}^{n+2}C_3 + \dots + {}^{n+m-1}C_m$ is equal to

(a) ${}^mC_{n-1}$ (b) mC_n (c) ${}^mC_{n-1}$

${}^mC_1 + {}^{m+1}C_2 + {}^{m+2}C_3 + \dots + {}^{m+n-1}C_n$ (d) ${}^mC_{m-1}$

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264. If

$(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n, n \in N$, then $C_0 - C_1 + C_2 - \dots + (-1)^m C_m$

is equal to ($m < n$)

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265. The 10th term of $\left(3 - \sqrt{\frac{17}{4} + 3\sqrt{2}}\right)^{20}$ is (a) an irrational number

(b) a rational number (c) a positive integer (d) a negative integer

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266. For the expansion $(x \sin p + x^{-1} \cos p)^{10}$, ($p \in R$), The greatest value of the term independent of x is (a) $10! / 2^5 (5!)^2$ (b) the least value of sum of coefficient is zero (c) the greatest value of sum of coefficient is 32 (d) the least value of the term independent of x occurs when $p = (2n + 1) \frac{\pi}{4}$, $n \in Z$

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267. Let $(1 + x^2)^2 (1 + x)^n = \sum_{k=0}^{n+4} a_k x^k$. If a_1, a_2 and a_3 are in arithmetic progression, then the possible value/values of n is/are a. 5 b. 4 c. 3 d. 2

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268. The middle term in the expansion of $(x/2 + 2)^8$ is 1120, then $x \in R$ is equal to a. -2 b. 3 c. -3 d. 2

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269. If $(1 - x^2)^n = \sum_{r=0}^n a_r x^r (1 - x)^{2n-r}$, then a_r is equal to $\binom{n}{r} 3^r$ b. $\binom{n}{r} 2^n$ c. $\binom{n}{r} 2^n 3^r$ d. $\binom{n}{r} 2^r$

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270.

$$\left[\left(\binom{n}{0} + \binom{n}{3} \right) \frac{1}{2} \left(\binom{n}{1} + \binom{n}{2} + \binom{n}{4} + \binom{n}{5} \right)^2 + \frac{3}{4} \left(\binom{n}{1} - \binom{n}{2} \right) \right]$$

3 b. 4 c. 2 d. 1

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271. If $\sum_{r=0}^{10} \binom{r+2}{r+1} \cdot \binom{n}{r} C_r = \frac{2^8 - 1}{6}$, then n is (A) 8 (B) 4 (C) 6 (D) 5

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272. Let $f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n + \dots$ and

$\frac{f(x)}{1-x} = b_0 + b_1 x + b_2 x^2 + \dots + b_n x^n + \dots$, then $b_n + b_{n-1} = a_n$ b.

$$b_n - b_{n-1} = a_n \text{ c. } b_n/b_{n-1} = a_n \text{ d. none of these}$$

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273. $\sum_{r=0}^{300} a_r x^r = (1 + x + x^2 + x^3)^{100}$. If $a = \sum_{r=0}^{300} a_r$, then $\sum_{r=0}^{300} r a_r$ is equal to 300a b. 100a c. 150a d. 75a

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274. The value of $\sum_{r=1}^{n+1} \left(\sum_{k=1}^n {}^k C_{r-1} \right)$ (where $r, k, n \in N$) is equal to a. $2^{n+1} - 2$ b. $2^{n+1} - 1$ c. 2^{n+1} d. none of these

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275. If $\frac{x^2 + x + 1}{1 - x} = a_0 + a_1 x + a_2 x^2 + \dots$, then $\sum_{r=1}^{50} a_r$ is equal to 148 b. 146 c. 149 d. none of these

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276. p is a prime number and n

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277. The coefficient of x^9 in the expansion of $(1+x)(1+x^2)(1+x^3)\dots(1+x^{100})$ is

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278. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5:10:14. Then $n =$ _____.

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279. If

$(1-x)^{-n} = a_0 + a_1x + a_2x^2 + \dots + a_r x^r + \dots$, then $a_0 + a_1 + a_2 + \dots + a_r =$ _____.

is equal to $\frac{n(n+1)(n+2)(n+r)}{r!}$ $\frac{(n+1)(n+2)(n+r)}{r!}$
 $\frac{n(n+1)(n+2)(n+r-1)}{r!}$ none of these

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280. The value of $\sum_{r=0}^{20} r(20-r)({}^{20}C_r)^2$ is equal to

a. $400^{39}C_{20}$ b. $400^{40}C_{19}$ c. $400^{39}C_{19}$ d. $400^{38}C_{20}$

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281. The coefficient of x^{10} in the expansion of $(1+x^2-x^3)^8$ is 476 b.

496 c. 506 d. 528

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282. If the term independent of x in the $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then k

equals 2, -2 b. 3, -3 c. 4, -4 d. 1, -1

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283. The coefficient of x^2y^3 in the expansion of $(1 - x + y)^{20}$ is $\frac{20!}{213!}$ b.
 $-\frac{20!}{213!}$ c. $\frac{20!}{5!2!3!}$ d. none of these

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284. The coefficient of x^5 in the expansion of $(x^2 - x - 2)^5$ is

– 83

b. – 82

c. – 86

d. – 81

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285. The coefficient of $a^8b^4c^9d^9$ in $(abc + abd + acd + bcd)^{10}$ is $10!$ b.

$\frac{10!}{8!4!9!9!}$ c. 2520 d. none of these

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286. If the coefficient of x^7 in $\left[ax^2 - \left(\frac{1}{bx^2}\right)\right]^{11}$ equal the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation

a. $a + b = 1$ b. $a - b = 1$ c. $b = 1$ d. $\frac{a}{b} = 1$

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287. If $(1 + x)^5 = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$, then the value of $(a_0 - a_2 + a_4)^2 + (a_1 - a_3 + a_5)^2$ is equal to

a. 243 b. 32 c. 1 d. 2^{10}

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288. The coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$ is

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289. The coefficient of x^{28} in the expansion of $(1 + x^3 - x^6)^{30}$ is 1 b. 0 c.

$\binom{30}{6}$ d. $\binom{30}{3}$

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290. The coefficient of x^n in $(1 + x)^{101}(1 - x + x^2)^{100}$ is nonzero, then

n cannot be of the form $3r + 1$ b. $3r$ c. $3r + 2$ d. none of these

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291. Prove that

$$\sum_{r=0}^n (-1)^r \binom{n}{r} \left[\frac{1}{2^r} + \frac{3}{2^{2r}} + \frac{7}{2^{3r}} + \frac{15}{2^{4r}} + \dots \rightarrow m \text{ terms} \right] = \frac{2^{mn}}{2^{mn}(2^n)}$$

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292. In the expansion of $(7^{1/3} + 11^{1/9})^{6561}$, (a) there are exactly 730 rational term (b) there are exactly 5831 irrational terms (c) the term which

involves greatest binomial coefficients is irrational (d) the term which involves greatest binomial coefficients is rational

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293. If for z as real or complex,

$$(1 + z^2 + z^4)^8 = C_0 + C_1z^2 + C_2z^4 + \dots + C_{16}z^{32} \text{ then} \quad (\text{a})$$

$$C_0 - C_1 + C_2 - C_3 + \dots + C_{16} = 1 \quad (\text{b})$$

$$C_0 + C_3 + C_6 + C_9 + C_{12} + C_{15} = 3^7 \quad (\text{c})$$

$$C_2 + C_5 + C_8 + C_{11} + C_{14} = 3^6 \quad (\text{d})$$

$$C_1 + C_4 + C_7 + C_{10} + C_{13} + C_{16} = 3^7$$

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294. The sum of coefficient in the expansion of $(1 + ax - 2x^2)^n$ is

(a) positive, when $a < 1$ and $n = 2k, k \in N$ (b) negative, when

$a < 1$ and $n = 2k + 1, k \in N$ (c) positive, when $a < 1$ and $n \in N$ (d) zero,

when $a = 1$

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295. If the 4th term in the expansion of $(ax + 1/x)^n$ is $5/2$, then $a = \frac{1}{2}$
b. $n = 8$ c. $a = \frac{2}{3}$ d. $n = 6$

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296. The number of values of r satisfying the equation
 ${}^{69}C_{3r-1} - {}^{69}C_{r^2} = {}^{69}C_{r^2-1} - 3r69$ is a. 1 b. 2 c. 3 d. 7

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297. If $(4 + \sqrt{15})^n = I + f$, where n is an odd natural number, I is an integer and $f > 0$

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298. In the expansion of $(x + a)^n$ if the sum of odd terms is P and the sum of even terms is Q , then (a) $P^2 - Q^2 = (x^2 - a^2)^n$ (b) $4PQ = (x + a)^{2n} - (x - a)^{2n}$ (c) $2(P^2 + Q^2) = (x + a)^{2n} + (x - a)^{2n}$ (d) none of these

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299. If the coefficients of r th, $(r + 1)$ th, and $(r + 2)$ th terms in the expansion of $(1 + x)^{14}$ are in A.P., then r is/are a. 5 b. 11 c. 10 d. 9

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300. The value of x in the expression $(x + x^{(\log)_{10}x})^5$ if third term in the expansion is 10,00,000 is/are a. 10 b. 100 c. $10^{-5/2}$ d. $10^{-3/2}$

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301. Let $R = (5\sqrt{5} + 11)^{2n+1}$ and $f = R - [R]$ where $[]$ denotes the greatest integer function, prove that $Rf = 4^{2n+1}$

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302. If $|x| < 1$, then the coefficient of x^n in expansion of $(1 + x + x^2 + x^3 + \dots)^2$ is n b. $n - 1$ c. $n + 2$ d. $n + 1$

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303. The coefficient of x^5 in $(1 + 2x + 3x^2 + \dots)^{-3/2}$ is ($|x| < 1$) 21 b. 25
c. 26 d. none of these

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304. If x is so small that x^3 and higher powers of x may be neglected, then

$$\frac{(1+x)^{3/2} - \left(1 + \frac{1}{2}x\right)^3}{(1-x)^{1/2}}$$
 may be approximated as $3x + \frac{3}{8}x^2$ b.

$$1 - \frac{3}{8}x^2 \quad \text{c.} \quad \frac{x}{2} - \frac{3}{x^2} \quad \text{d.} \quad -\frac{3}{8}x^2$$



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305. If x is positive, the first negative term in the expansion of

$$(1 + x)^{27/5} \text{ is}$$

a. 5th term

b. 8th term

c. 6th term

d. 7th term



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306. Value of $\sum_{k=1}^{\infty} \sum_{r=0}^k \frac{1}{3^k} ({}^k C_r)$ is $\frac{2}{3}$ b. $\frac{4}{3}$ c. 2 d. 1



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307. If the expansion in powers of x of the function $1/[(1-ax)(1-bx)]$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$, then a_n is

a. $\frac{b^n - a^n}{b - a}$ b. $\frac{a^n - b^n}{b - a}$ c. $\frac{b^{n+1} - a^{n+1}}{b - a}$ d. $\frac{a^{n+1} - b^{n+1}}{b - a}$

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308. If $f(x) = 1 - x + x^2 - x^3 + \dots + x^{15} + x^{16} - x^{17} + \dots$, then the coefficient of x^2 in $f(x-1)$ is 826 b. 816 c. 822 d. none of these

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309. The sum of rational term in $(\sqrt{2} + 33 + 56)^{10}$ is equal to 12632 b. 1260 c. 126 d. none of these

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310. The value of $\sum_{r=0}^{10} \binom{20}{r} C_r$ is equal to: a. $20(2^{18} + {}^{19}C_{10})$ b. $10(2^{18} + {}^{19}C_{10})$ c. $20(2^{18} + {}^{19}C_{11})$ d. $10(2^{18} + {}^{19}C_{11})$

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311. If $p = (8 + 3\sqrt{7})^n$ and $f = p - [p]$, where $[.]$ denotes the greatest integer function, then the value of $p(1 - f)$ is equal to a. 1 b. 2 c. 2^n d. 2^{2n}

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312. Statement 1: Greatest term in the expansion of $(1 + x)^{12}$, when $x = 11/10$ is 7th Statement 2: 7th term in the expansion of $(1 + x)^{12}$ has the factor ${}^{12}C_6$ which is greatest value of ${}^{12}C_r$.

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313. Statement 1: Remainder *when* $\widehat{34562222}$ is divided by 7 is 4.

Statement 2: Remainder *when* $\widehat{52222}$ is divided by 7 is 4.

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314. the value of x , for which the 6th term in the expansions of

$$\left[2^{\log_2 \left(\sqrt{9^{(x-1)+7}} \right) + \frac{1}{2^{\frac{1}{5}} (\log)_2 (3^{x-1} + 1)}} \right]^7 \text{ is } 84, \text{ is equal to a. 4 b. 3}$$

c. 2 d. 1

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315. Each question has four choices a, b, c and d, out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. Both the statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1. Both the statements are TRUE but STATEMENT 2 is NOT the correct explanation of STATEMENT 1. STATEMENT 1 is TRUE and STATEMENT 2 is FALSE. STATEMENT 1 is FALSE and STATEMENT 2 is TRUE. Statement 1:

The value of

$(\binom{10}{0}C_0) + (\binom{10}{0}C_0 + \binom{10}{1}C_1) + (\binom{10}{0}C_0 + \binom{10}{1}C_1 + \binom{10}{2}C_2) + \dots$
is 102^9 . Statement 2: $\binom{n}{1}C_1 + 2^n C_2 + 3^n C_3 + \dots + n^n C_n = n2^{n-1}$.

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316. The number $51^{49} + 51^{48} + 51^{47} + \dots + 51 + 1$ is divisible by a. 10 b. 20
c. 25 d. 50

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317. If $\sum_{r=0}^n \frac{r}{\binom{n}{r}} = \sum_{r=0}^n \frac{n^2 - 3n + 3}{2^n \binom{n}{r}}$, then $n = 1$ b. $n = 2$ c. $n = 3$ d.

none of these

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318. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, then show that the sum of the products of the coefficients taken two at a time,

represented by $\sum_{0 \leq i < j \leq n} {}^n c_i {}^n c_j$ is equal to $2^{2n-1} - \frac{(2n)!}{2(n!)^2}$

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319. For any positive integer (m, n) (with $n \geq m$), Let $\binom{n}{m} = {}^n C_m$

Prove that

$$\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + \dots + (n-m+1)\binom{m}{m}$$

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320. If $\sum_{r=0}^n \{a_r(x - \alpha + 2)^r - b_r(\alpha - x - 1)^r\} = 0$, then prove that $b_n - (-1)^n a_n = 0$.

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321. Let $a = \left(2^{1/401} - 1\right)$ and for each $n \geq 2$, let $b_n = {}^n C_1 + {}^n C_2 a + {}^n C_3 a^2 + \dots + {}^n C_n \cdot a^{n-1}$. Find the

value of $(b_{2006} - b_{2005})$.

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322. Prove that

$$\sum_{r=0}^n {}^n C_r (-1)^r [i + i^{2r} + i^{3r} + i^{4r}] = 2^n + 2^{\frac{n}{2}+1} \cos(n\pi/4), \text{ where } i = \sqrt{-1}$$

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323. Find the coefficient of x^n in $\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!}\right)^2$.

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324. Prove that

$$\frac{{}^n C_0}{x} - \frac{{}^n C_0}{x+1} + \frac{{}^n C_1}{x+2} - \dots + (-1)^n \frac{{}^n C_n}{x+n} = \frac{n!}{x(x+1)(x-n)},$$

where n is any positive integer and x is not a negative integer.

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325. If n is a positive integer, prove that

$$1 - 2n + \frac{2n(2n-1)}{2!} - \frac{2n(2n-1)(2n-2)}{3!} + \dots + (-1)^{n-1} \frac{2n(2n-1)\dots(2n-(n-1))}{(n-1)!}$$

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326. Given,

$$s_n = 1 + q + q^2 + \dots + q^n, S_n = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^2 + \dots + \left(\frac{q+1}{2}\right)^n$$

prove that ${}^{n+1}C_1 + {}^{n+1}C_2 s_1 + {}^{n+1}C_3 s_2 + \dots + {}^{n+1}C_{n+1} s_n = 2^n S_n$.

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327. The sum of $1 + n\left(1 - \frac{1}{x}\right) + \frac{n(n+1)}{2!}\left(1 - \frac{1}{x}\right)^2 + \dots + \infty$ will be x^n

b. x^{-n} c. $\left(1 - \frac{1}{x}\right)^n$ d. none of these

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328. $\sum_{k=1}^{\infty} k \left(1 - \frac{1}{n}\right)^{k-1} =$ a. $n(n-1)$ b. $n(n+1)$ c. n^2 d. $(n+1)^2$

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329. The coefficient of x^4 in the expansion of $\left\{\sqrt{1+x^2} - x\right\}^{-1}$ in ascending powers of x , when $|x| < 1$, is 0 b. $\frac{1}{2}$ c. $-\frac{1}{2}$ d. $-\frac{1}{8}$

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330. $1 + \frac{1}{3}x + \frac{1 \times 4}{3 \times 6}x^2 + \frac{1 \times 4 \times 7}{3 \times 6 \times 9}x^3 + \dots$ is equal to a. $(1+x)^{1/3}$ b. $(1+x)^{1/3}$ c. $(1-x)^{1/3}$ d. $(1-x)^{-1/3}$

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331. The value of $\sum_{r=1}^{15} \frac{r2^r}{(r+2)!}$ is a. $\frac{(17)! - 2^{16}}{(17)!}$ b. $\frac{(18)! - 2^{17}}{(18)!}$ c. $\frac{(16)! - 2^{15}}{(16)!}$ d. $\frac{(15)! - 2^{14}}{(15)!}$

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332. $(n + 2)C_0(2^{n+1}) - (n + 1)C_1(2^n) + (n)C_2(2^{n-1}) - \dots$ is equal to

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333. The value of $\sum_{r=0}^{50} (-1)^r \frac{(50)C_r}{r+2}$ is equal to a. $\frac{1}{50 \times 51}$ b. $\frac{1}{52 \times 50}$
c. $\frac{1}{52 \times 51}$ d. none of these

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334. In the expansion of $[(1 + x)/(1 - x)]^2$, the coefficient of x^n will be
 $4n$ b. $4n - 3$ c. $4n + 1$ d. none of these

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335. Statement 1: The sum of coefficient in the expansion of $(3^{-x/4} + 3^{5x/4})^n$ is 2^n . Statement 2: The sum of coefficient in the expansion of $(x + y)^n$ is 2^n .



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336. Let n be a positive integer and k be a whole number, $k \leq 2n$.

Statement 1: The maximum value of $\binom{2n}{k}$ is 2^n . Statement 2:

$$\frac{\binom{2n}{k+1}}{\binom{2n}{k}} < 1, \text{ for } k = 0, 1, 2, \dots, n-1 \text{ and } \frac{\binom{2n}{k}}{\binom{2n}{k-1}} > 1, \text{ for } k = n, \dots, 2n.$$



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337. Statement 1: $\sum_{0 \leq i < j \leq n} \left(\frac{i}{\binom{n}{i}} + \frac{j}{\binom{n}{j}} \right)$ is equal to

$$\frac{n^2}{2} a, \text{ where } a = \sum_{r=0}^n \frac{r}{\binom{n}{r}} = n.$$

$$c_r = \sum_{r=0}^n \frac{(n-r)}{\binom{n}{r}} = n - a = 0.$$



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338. Statement 1: $\sum_{r=0}^m C_r + \sum_{r=1}^m C_{r-1} + \sum_{r=1}^m C_{r-2} + \dots + \sum_{r=0}^m C_r = m+1$

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339. $1 + \left(\frac{1}{4}\right) + \left(\frac{1 \cdot 3}{4 \cdot 8}\right) + \left(\frac{1 \cdot 4 \cdot 7}{4 \cdot 8 \cdot 12}\right) + \dots =$

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340. If $|x| < 1$, then $1 + n\left(\frac{2x}{1+x}\right) + \frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2 + \dots$ is equal to a. $\left(\frac{2x}{1+x}\right)^n$ b. $\left(\frac{1+x}{2x}\right)^n$ c. $\left(\frac{1-x}{1+x}\right)^n$ d. $\left(\frac{1+x}{1-x}\right)^n$

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341. Statement 1: If p is a prime number ($p \neq 2$), then $\left[(2 + \sqrt{5})^p\right] - 2^{p+1}$ is always divisible by p (where $[.]$ denotes the

greatest integer function). Statement 2: if n prime, then

${}^n C_1, {}^n C_2, {}^n C_3, \dots, {}^n C_{n-1}$ must be divisible by n .

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342. Statement 1: The total number of dissimilar terms in the expansion of $(x_1 + x_2 + \dots + x_n)^3$ is $\frac{n(n+1)(n+2)}{6}$. Statement 2: The total number of dissimilar terms in the expansion of $(x_1 + x_2 + x_3)^n$ is $\frac{n(n+1)(n+2)}{6}$.

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343. Statement 1: In the expansion of $(1+x)^{41}(1-x+x^2)^{40}$, the coefficient of x^{85} is zero. Statement 2: In the expansion of $(1+x)^{41}$ and $(1-x+x^2)^{40}$, x^{85} term does not occur.

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344. Statement 1: The coefficient of x^n is $\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}\right)^3$ is $\frac{3^n}{n!}$ Statement 2: The coefficient of x^n in e^{3x} is $\frac{3^n}{n!}$

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345. The value of $\sum_{r=0}^{40} rC(40, r)C(30, r)$

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346. $\sum_{r=0}^{10} r^{10} C_r, 3^r \cdot (-2)^{10-r}$ is -

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347. The value of $\sum_{r=1}^n (-1)^{r+1} \frac{{}^n C_r}{r+1}$ is equal to a. $-\frac{1}{n+1}$ b. $\frac{1}{n}$ c. $\frac{1}{n+1}$ d. $\frac{n}{n+1}$

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348. If ${}^nC_0, {}^nC_1, {}^nC_2, \dots, {}^nC_n$ are the binomial coefficient, then $2 \times {}^nC_1 + 2^3 \times {}^nC_3 + 2^5 \times {}^nC_5 + \dots$ equals $\frac{3^n + (-1)^n}{2}$ b. $\frac{3^n - (-1)^n}{2}$
 c. $\frac{3^n + 1}{2}$ d. $\frac{3^n - 1}{2}$

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349. The value of $\frac{{}^nC_0}{n} + \frac{{}^nC_1}{n+1} + \frac{{}^nC_2}{n+2} + \dots + \frac{{}^nC_n}{2n}$ is equal to a. $\int_0^1 x^{n-1}(1-x)^n dx$ b. $\int_1^2 x^n(x-1)^{n-1} dx$ c. $\int_1^2 x^{n-1}(1+x)^n dx$ d. $\int_0^1 (1-x)^{n-1} dx$

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350. The value of ${}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + {}^{20}C_3 + {}^{20}C_4 + {}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15}$ is

a. $2^{19} - \frac{\binom{20}{10} + \binom{20}{9}}{2}$ b. $2^{19} - \frac{\binom{20}{10} + 2 \times \binom{20}{9}}{2}$ c. $2^{19} - \frac{\binom{20}{10}}{2}$ d. none of these

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351. If $(3 + x^{2008} + x^{2009})^{2010} = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, then the value of $a_0 - \frac{1}{2}a_1 - \frac{1}{2}a_2 + a_3 - \frac{1}{2}a_4 - \frac{1}{2}a_5 + a_6 - \dots$ is 3^{2010} b. 1 c. 2^{2010} d. none of these

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352.

$$\binom{40}{4} + \binom{40}{1} + \binom{40}{3} + \binom{40}{2} + \binom{40}{4} + \binom{40}{3} + \binom{40}{2} + \binom{40}{1} + \binom{40}{4}$$

is equal to a. $(40)_4$ b. $(10)_4$ c. 0 d. $(20)_4$

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353. The sum of series

$${}^{\wedge}(20)C_0 - {}^{\wedge}(20)C_1 + {}^{\wedge}(20)C_2 - {}^{\wedge}(20)C_3 + \dots + {}^{\wedge}(20)C_{10} \text{ is } \frac{1}{2}$$

${}^{\wedge}(20)C_{10}$ b. 0 c. ${}^{\wedge}(20)C_{10}$ d. $-{}^{\wedge}(20)C_{10}$



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354. If

$$(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n, \text{ then } C_0C_2 + C_1C_3 + C_2C_4 + \dots +$$

$\frac{(2n)!}{(n!)^2}$ b. $\frac{(2n)!}{(n-1)!(n+1)!}$ c. $\frac{(2n)!}{(n-2)!(n+2)!}$ d. none of these



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355. The value of $\lim_{n \rightarrow \infty} \sum_{r=0}^n \left(\sum_{t=0}^{r-1} \frac{1}{5^n} \cdot {}^nC_r \cdot {}^rC_t \cdot (3^t) \right)$ is equal to



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356. Prove that

$$C_0 - 2^2 C_1 + 3^2 C_2 - 4^2 C_3 + \dots + (-1)^n (n+1)^2 \times C_n = 0 \text{ where } C_r = {}^n C_r$$



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357. Given that

$$C_1 + 2C_2x + 3C_3x^2 + \dots + 2nC_{2n}x^{2n-1} = 2n(1+x)^{2n-1}, \text{ where } C_r = {}^{2n}C_r$$

then prove that $C_1 - 2C_2 + 3C_3 - \dots - 2nC_{2n} = (-1)^n C_n$.



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358. The remainder, if $1 + 2 + 2^2 + \dots + 2^{1999}$ is divided by 5 is.



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359. The largest real value of x such that $\sum_{k=0}^4 \left(\frac{3^{4-k}}{(4-k)!} \right) \left(\frac{x^k}{k!} \right) = \frac{32}{3}$ is.

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360. If in the expansion of $(a - 2b)^n$, the sum of 5th and 6th terms is 0, then the values of $a/b = \frac{n-4}{5}$ b. $\frac{2(n-4)}{5}$ c. $\frac{5}{n-4}$ d. $\frac{5}{2(n-4)}$

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361. The number of real negative terms in the binomial expansion of $(1 + ix)^{4n-2}$, $n \in \mathbb{N}$, $n > 0$, $I = \sqrt{-1}$, is

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