MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Solved Examples And Exercises

1. Show that the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has no real solution.

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2. Solve for $x : 4^x - 3^{x - 1/2} = 3^{x + 1/2} - 2^{2x - 1}$.

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3. Solve for $x: \sqrt{x+1} - \sqrt{x-1} = 1$.

4. If $x, y \in R$ and $2x^2 + 6xy + 5y^2 = 1$,then $|x| \leq \sqrt{5}$ b. $|x| \geq \sqrt{5}$ c. $y^2 \leq 2$ d. $y^2 \leq 4$

5. If the roots $x^5 - 40x^4 + Px^3 + Qx^2 + Rx + S = 0$ are n G.P. and the sum of their reciprocals is 10, then $|S|$ is 4 b. 6 c. 8 d. none of these

6. Show that for any triangle with sides $a, b,$ and $c 3(ab + bc + ca) < (a + b + c)^2 < 4(bc + ca + ab)$. When are the first two expressions equal ?

7. For what value of m does the system of equations $3x + my = m, 2x - 5y = 20$ has solution satisfying the conditions $x > 0, y > 0$?

8. Show that the square to $\left(\sqrt{26 - 15\sqrt{3}} \right) / \left(5\sqrt{2} - \sqrt{38 + 5\sqrt{3}} \right)$ is a rational number.

9. If α, β are the roots of $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + rx + s = 0$, evaluate $(\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta)$ in terms of p, q, r, and s. Deduce the condition that the equation has a common root.

10. Let $f(x) = x^2 + bx + c, where b, c \in R.$ If $f(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, then the least value of $f(x)$ is: (a.) 2 (b.) 3 (c.) $5/2$ (d.) 4

11. If the equation $ax^2 + bx + c = x$ has no real roots, then the equation $a(ax^2 + bx + c)^2 + b(ax^2 + bx + c) + c = x$ will have a. four real roots b. no real root c. at least two least roots d. none of these

12. The value of expression $x^4 - 8x^3 + 18x^2 - 8x + 2$ when $x = 2 + \sqrt{3}$

a. 2 b. 1 c. 0 d. 3

13. The exhaustive set of values of a for which inequation $(a-1)x^2 - (a+1)x + a - 1 \geq 0$ is true $\forall x > 2$ $\quad (a)(-\infty, 1)$ $(b)\left[\frac{7}{3}, \infty\right)$ $(c)\left[\frac{3}{7}, \infty\right)$ (d) none of these

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14. If p, q, r, s are rational numbers and the roots of $f(x) = 0$ are eccentricities of a parabola and a rectangular hyperbola, where $f(x) = px^3 + qx^2 + rx + s$,then $p + q + r + s =$ a. p b. $-p$ c. $2p$ d. 0

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15. If a_0, a_1, a_2, a_3 are all the positive, then $4a_0x^3 + 3a_1x^2 + 2a_2x + a_3 = 0$ has least one root in $(-1, 0)$ if (a) $a_0 + a_2 = a_1 + a_3$ and $4a_0 + 2a_2 > 3a_1 + a_3$ (b) $4a_0 + 2a_2 < 3a_1 + a_3$ (c) $4a_0 + 2a_2 = 3a_1 + a_0$and $4a_0 + a_2 < a_1 + a_3$ (d) none of these

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16. If $ax^2 + bx + c = 0$ has imaginary roots and $a - b + c > 0$ then the set of points (x, y) satisfying the equation $\left| a\left(x^2 + \dfrac{y}{a}\right) + (b+1)x + c \right| = |ax^2 + bx + c| + |x + y|$ consists of the region in the $xy - plane$ which is on or above the bisector of I and III quadrant on or above the bisector of II and IV quadrant on or below the bisector of I and III quadrant on or below the bisector of II and IV quadrant

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17. All the values of $'a'$ for which the quadratic expression $ax^2 + (a - 2)x - 2$ is negative for exactly two integral values of x may lie in $\left[1, \dfrac{3}{2}\right]$ (b) $\left[\dfrac{3}{2}, 2\right)$ [1, 2) (d) $[-1, 2)$

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18. If $a + b + c = 0, a^2 + b^2 + c^2 = 4, then a^4 + b^4 + c^4$ is_____.

19. Find the solution set of the system $x + 2y + z = 1 \; 2x - 3y - w = 2$

$x \geq 0, y \geq 0, z \geq 0, w \geq 0$

20. mn squares of equal size are arranged to form a rectangle of dimension m by n, where m and n are natural numbers. Two square will be called neighbors if they have exactly one common side. A number is written in each square such that the number written in any square is the arithmetic mean of the numbers written in its neighboring squares. Show that this is possible only if all the numbers used are equal.
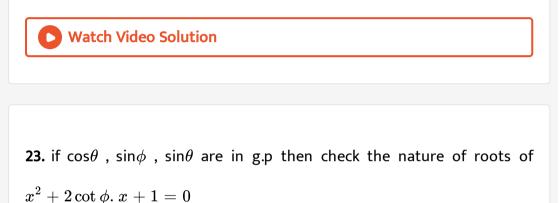
21. Form a quadratic equation whose roots are -4 and 6

22. If a, b, c are odd integere then about that $ax^2 + bx + c = 0$, does not have rational roots

23. if $\cos\theta$, $\sin\phi$, $\sin\theta$ are in g.p then check the nature of roots of $x^2 + 2\cot\phi.\ x + 1 = 0$

24. If a ,b ,c are non zero rational no then prove roots of equation $\left(abc^2\right)x^2 + 3a^2cx + b^2cx - 6a^2 - ab + 2b^2 = 0$ are rational.

25. If $ab + bc + ca = 0$, then solve $a(b - 2c)x^2 + b(c - 2a)x + c(a - 2b) = 0$.

26. If roots of equation $3x^2 + 5x + 1 = 0 \, are \, (\sec\theta_1 - tan\theta_1) and (\cos ec\theta_2 - \cot\theta_2)$. Then find the equation whose roots are $(\sec\theta_1 + tan\theta_1) and (\cos ec\theta_2 + \cot\theta_2)$.

27. If roots of the equation $ax^2 + bx + c = 0$ are $\alpha and \beta$, find the equation whose roots are $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$ (ii) $-\alpha, \, -\beta$ (iii) $\dfrac{1-\alpha}{1+\alpha}, \dfrac{1-\beta}{1+\beta}$

28. Form a quadratic equation with real coefficients whose one root is $3 - 2i$.

29. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^0 and \tan 15^0$, respectively, then find the value of $2 + q - p$.

30. If a, b and c are in A.P. and one root of the equation $ax^2 + bc + c = 0$ is 2, the find the other root.

31. If a is the root (having the least absolute value) or the equation $x^2 - bx - 1 = 0 (b \in R^+)$, then prove that `-1

32. If α, β are roots of $x^2 - 3x + a = 0$, $a \in R$ and $\alpha < 1 < \beta$ then find the value of a.

33. If $a < b < c < d$, then for any real non-zero λ, the quadratic equation $(x - a)(x - c) + \lambda(x - b)(x - d) = 0$,has

34. The quadratic $x^2 + ax + b + 1 = 0$ has roots which are positive integers, then $(a^2 + b^2)$ can be equal to a.50 b. 37 c. 61 d. 19

35. The sum of values of x satisfying the equation $\left(31 + 8\sqrt{15}\right)^{x^2-3} + 1 = \left(32 + 8\sqrt{15}\right)^{x^2-3}$ is a 3 b. 0 c. 2 d. none of these

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36. If α, β are real and distinct roots of $ax^2 + bx - c = 0 and p, q$ are real and distinct roots of $ax^2 + bx - |c| = 0, where (a > 0),$ then

A. $\alpha, \beta \in (p, q)$

B. $\alpha, \beta \in [p, q]$

C. $p, q \in (\alpha, \beta)$

D. None

Answer: B

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37. Let $a \neq 0$ and $p(x)$ be a polynomial of degree greater than 2. If $p(x)$ leaves reminders a and a when divided respectively, by $x + a$ and $x - a$, the remainder when $p(x)$ is divided by $x^2 - a^2$ is $2x$ b. $-2x$ c. x d. x

38. A quadratic equation with integral coefficients has two different prime numbers as its roots. If the sum of the coefficients of the equation is prime, then the sum of the roots is 2 b. 5 c. 7 d. 11

39. If a, b, c are three distinct positive real numbers, the number of real and distinct roots of $ax^2 + 2b|x| - c = 0$ is 0 b. 4 c. 2 d. none of these

40. Let a, b and c be real numbers such that $4a + 2b + c = 0$ and $ab < 0$. Then the equation $ax^2 + bx + c = 0$.

41. If α, β are the roots of the equation $x^2 - 2x + 3 = 0$ obtain the equation whose roots are $\alpha^3 - 3\alpha^2 + 5\alpha - 2$ and $\beta^3 - \beta^2 + \beta = 5$

42. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the value of $a\alpha^2 + c/a\alpha + b + (a\beta^2 + c)/(a\beta + b)$ is $\dfrac{b(b^2 - 2ac)}{4a}$ b. $\dfrac{b^2 - 4ac}{2a}$ c. $\dfrac{b(b^2 - 2ac)}{a^2c}$ d. none of these

43. If $a \in (-1, 1)$, then roots of the quadratic equation $(a-1)x^2 + ax + \sqrt{1-a^2} = 0$ are a. real b. imaginary c. both equal d. none of these

44. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is $p^3 - q(3p-1) + q^2 = 0$ $p^3 - q(3p+1) + q^2 = 0$ $p^3 + q(3p-1) + q^2 = 0$ $p^3 + q(3p+1) + q^2 = 0$

45. Let α, β be the roots of the quadratic equation $ax^2 + bx + c = 0$ and $\triangle = b^2 - 4a$. If $\alpha + \beta, \alpha^2 + \beta^2 \alpha^3 + \beta^3$ are in G.P. Then a. $\triangle = 0$ b. $\triangle \neq 0$ c. $b = 0$ d. $c = 0$

46. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ are roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral value of p and q, respectively, are $-2, -32$ b. $-2, 3$ c. $-6, 3$ d. $-6, -32$

    A. $-2, -32$

    B. $-2, 3$

    C. $-6, 3$

    D. $-6, -32$

Answer: A

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47. If $f(x) = x^2 + 2bc + 2c^2 \, and \, g(x) = -x^2 - 2cx + b^2$ are such that $min \, f(x) > max \, f(x)$, ten he relation between $b \, and \, c$ is a. no relation b. `0

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48. For the equation $3x^2 + px + 3 = 0, p > 0$, if one of the root is square of the other, then p is equal to $1/3$ b. 1 c. 3 d. $2/3$
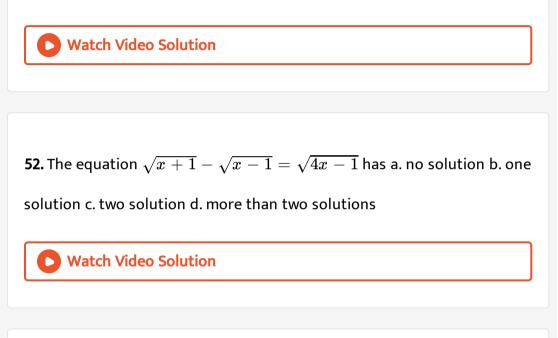
49. Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and let $m(b)$ be the minimum value of $f(x)$. As b varies, the range of $m(b)$ is $[0, \}$ b. $\left(0, \dfrac{1}{2}\right)$ c. $\dfrac{1}{2}, 1$ d. $(0, 1]$

50. If $\alpha and \beta$ are the roots of the equation $x^2 + bc + c = 0$,where $c < a < b$ then a. $0 < \alpha < \beta$ b. $\alpha < 0 < \beta < |\alpha|$ c. $\alpha < \beta < 0$ d. $\alpha < 0 < |\alpha| < \beta$

51. If $b > a$, then the equation $(x - a)(x - b) - 1 = 0$ has (a) both roots in (a, b) (b) both roots in $(-\infty, a)$ (c) both roots in $(b, +\infty)$ (d)one root in $(-\infty, a)$ and the other in $(b, +\infty)$

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52. The equation $\sqrt{x + 1} - \sqrt{x - 1} = \sqrt{4x - 1}$ has a. no solution b. one solution c. two solution d. more than two solutions

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53. If the roots of the equation $x^2 - 2ax + a^2 - a - 3 = 0$ are real and less than 3, then (a)$a < 2$ b. $2 < -a \le 3$ c. `34`

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54. A value of b for which the equation $x^2 + bx - 1 = 0$, $x^2 + x + b = 0$ have one root in common is $-\sqrt{2}$ b. $-i\sqrt{3}$ c. $\sqrt{2}$ d. $\sqrt{3}$

55. Let $p and q$ be real numbers such that $p \neq 0, p^3 \neq q, and p^3 \neq -q$. If $\alpha and \beta$ are nonzero complex numbers satisfying $\alpha + \beta = -p and \alpha^2 + \beta^2 = q$, then a quadratic equation having $\alpha/\beta and \beta/\alpha$ as its roots is A. $\left(p^3 + q\right)x^2 - \left(p^3 + 2q\right)x + \left(p^3 + q\right) = 0$

B. $\left(p^3 + q\right)x^2 - \left(p^3 - 2q\right)x + \left(p^3 + q\right) = 0$ C.

$\left(p^3 + q\right)x^2 - \left(5p^3 - 2q\right)x + \left(p^3 - q\right) = 0$ D.

$\left(p^3 + q\right)x^2 - \left(5p^3 + 2q\right)x + \left(p^3 + q\right) = 0$

56. Let α, β be the roots of the equation $x^2 - px + r = 0 and \alpha/2, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is

$\frac{2}{9}(p-q)(2q-p)$ b. $\frac{2}{9}(q-p)(2q-p)$ c. $\frac{2}{9}(q-2p)(2q-p)$ d. $\frac{2}{9}(2p-q)(2q-p)$

57. Let a, b, c be the sides of a triangle, where $a \neq b \neq c$ and $\lambda \in R$. If the roots of the equation $x^2 + 2(a+b+c)x + 3\lambda(ab+bc+ca) = 0$ are real. Then a.$\lambda < \frac{4}{3}$ b. $\lambda > \frac{5}{3}$ c. $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$ d. $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$

58. Let S be the set of all non-zero real numbers such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots $x_1 and x_2$ satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is (are) a subset (s) of S? $\left(\frac{1}{2}, \frac{1}{\sqrt{5}}\right)$ b. $\left(\frac{1}{\sqrt{5}}, 0\right)$ c. $\left(0, \frac{1}{\sqrt{5}}\right)$ d. $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

59. For real x, the function $\dfrac{(x-a)(x-b)}{x-c}$ will assume all real values provided a)$a > b > c$ b)$a < b < c$ c) $a > c < b$ d) $a \le c \le b$

60. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has

A. only purely imaginary roots

B. all real roots

C. two real and purely imaginary roots

D. neither real nor purely imaginary roots

61. Let α and β be the roots $x^2 - 6x - 2 = 0$, with $\alpha > \beta$ If $a_n - \beta^n$ for or $n \ge 1$ then the value of $\dfrac{a_{10} - 2a_8}{2a_9}$ is (a) 1 (b) 2 (c) 3 (d) 4

62. For the following question, choose the correct answer from the codes (a), (b), (c) and (d) defined as follows: Statement I is true, Statement II is also true; Statement II is the correct explanation of Statement I. Statement I is true, Statement II is also true; Statement II is not the correct explanation of Statement I. Statement I is true; Statement II is false Statement I is false; Statement II is true. Let a, b, c, p, q be the real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \dfrac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$. Statement I $(p^2 - q)(b^2 - ac) \geq 0$ and Statement II $b \notin pa$ or $c \notin qa$.

63. All the values of m for whilch both the roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4 lie in the interval

A `-2 B. $m > 3$

C. `-1 D. $1 < m < 4$

64. If the roots of the quadratic equation $(4p - p^2 - 5)x^2 - (2p - 1)x + 3p = 0$ lie on either side of unit, then the number of integer values of p is a.1 b. 2 c. 3 d. 4
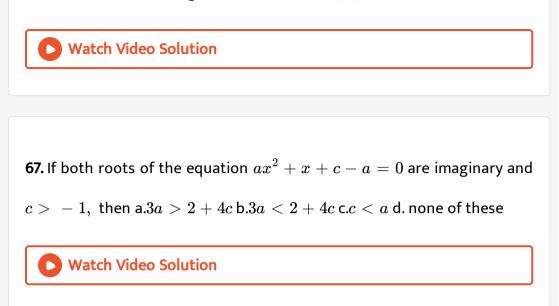
65. If roots of $x^2 - (a - 3)x + a = 0$ are such that at least one of them is greater than 2, then $a \in [7, 9]$ b. $a \in [7, \infty]$ c. $a \in [9, \infty]$ d. $a \in [7, 9]$

66. Let $f(x) = ax^2 + bx + a, b, c \in R$. If $f(x)$ takes real values for real values of x and non-real values for non-real values of x , then $a = 0$ b.

$b = 0$ c. $c = 0$ d. nothing can be said about $a, b,$ ·

67. If both roots of the equation $ax^2 + x + c - a = 0$ are imaginary and $c > -1,$ then a.$3a > 2 + 4c$ b.$3a < 2 + 4c$ c.$c < a$ d. none of these

68. The set of all possible real values of a such that the inequality $(x - (a - 1))(x - (a^2 - 1)) < 0$ holds for all $x \in (-1, 3)$ is $(0, 1)$ b. $(\infty, -1]$ c. $(-\infty, -1)$ d. $(1, \infty)$

69. The interval of a for which the equation $tan^2 x - (a - 4)\tan x + 4 - 2a = 0$ has at least one solution $\forall x \in [0, \pi/4]$ $a \in (2, 3)$ b. $a \in [2, 3]$ c. $a \in (1, 4)$ d. $a \in [1, 4]$

70. The range of a for which the equation $x^2 + ax - 4 = 0$ has its smaller root in the interval $(-1, 2)$ is a. $(-\infty, -3)$ b. $(0, 3)$ c. $(0, \infty)$ d. $(-\infty, -3) \cup (0, \infty)$

71. Consider the equation $x^2 + 2x - n = 0$ where $n \in N$ and $n \in [5, 100]$ The total number of different values of n so that the given equation has integral roots is a.8 b. 3 c. 6 d. 4

72. The total number of values a so that $x^2 - x - a = 0$ has integral roots, where $a \in N and 6 \le a \le 100$, is equal to a.2 b. 4 c. 6 d. 8

73. Let $P(x) = x^3 - 8x^2 + cx - d$ be a polynomial with real coefficients and with all it roots being distinct positive integers. Then number of possible value of c is_____.

74. Let $P(x) = \dfrac{5}{3} - 6x - 9x^2 \, and \, Q(y) = -4y^2 + 4y + \dfrac{13}{2}$. if there exists unique pair of real numbers (x, y) such that $P(x)Q(y) = 20$, then the value of $(6x + 10y)$ is ____.

75. if a $< c < b$, then check the nature of roots of the equation
$(a - b)^2x^2 + 2(a + b - 2c)x + 1 = 0$

76. If $a + b + c = 0$ then check the nature of roots of the equation $4ax^2 + 3bx + 2c = 0$ where $a, b, c \in R$.

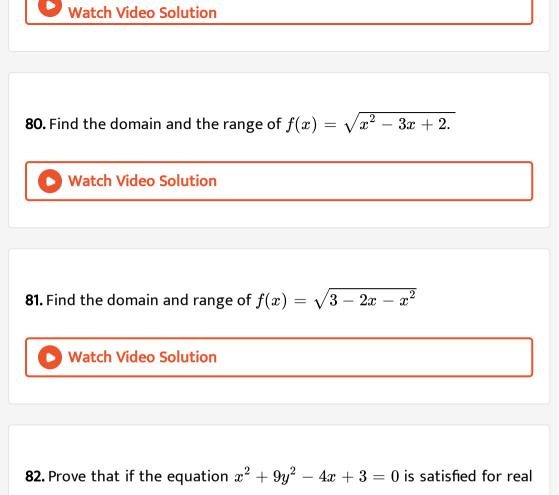**77.** Find the value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assumes the least value.

78. If $x_1, and x_2$ are the roots of $x^2 + (\sin\theta - 1)x - \dfrac{1}{2}(\cos^2\theta) = 0$, then find the maximum value of $x_1^2 + x_2^2$

79. If $p, q \in \{1, 2, 3, 4, 5\}$, then find the number of equations of form $p^2x^2 + q^2x + 1 = 0$ having real roots.
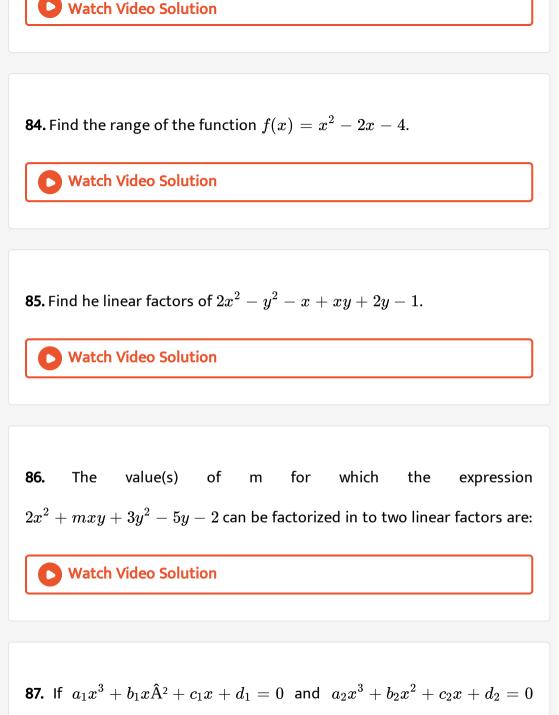
80. Find the domain and the range of $f(x) = \sqrt{x^2 - 3x + 2}.$

81. Find the domain and range of $f(x) = \sqrt{3 - 2x - x^2}$

82. Prove that if the equation $x^2 + 9y^2 - 4x + 3 = 0$ is satisfied for real values of $x\, and\, y,\, then\, x$ must lie between 1 and 3 and y must lie between-1/3 and 1/3.

83. Find the least value of $\dfrac{\left(6x^2 - 22x + 21\right)}{\left(5x^2 - 18 + 17\right)}$ for real $x.$

84. Find the range of the function $f(x) = x^2 - 2x - 4$.

85. Find he linear factors of $2x^2 - y^2 - x + xy + 2y - 1$.

86. The value(s) of m for which the expression $2x^2 + mxy + 3y^2 - 5y - 2$ can be factorized in to two linear factors are:

87. If $a_1x^3 + b_1x² + c_1x + d_1 = 0$ and $a_2x^3 + b_2x^2 + c_2x + d_2 = 0$ have a pair of repeated roots common, then prove that

$$\begin{vmatrix} 3a_1 & 2b_1 & c_1 \\ 3a_2 & 2b_2 & c_2 \\ a_2b_1 - a_1b_2 & c_1a_2 - c_2a_1 & d_1a_2 - d_2a_1 \end{vmatrix} = 0$$

88. If $x - c$ is a factor of order m of the polynomial $f(x)$ of degree `n(1

89. Solve the equation $x^3 - 13x^2 + 15x + 189 = 0$ if one root exceeds the other by 2.

90. If $\tan\theta \, and \sec\theta$ are the roots of $ax^2 + bx + c = 0,$ then prove that $a^4 = b^2(b^2 - 4ac).$

91. If the roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then find the value of $b^2 - 4 \cdot$

92. For what real values of a do the roots of the equation $x^2 - 2x - (a^2 - 1) = 0$ lie between the roots of the equation $x^2 - 2(a + 1)x + a(a - 1) = 0$.

93. Find the value of a for which the equation $\sin\left(x + \dfrac{\pi}{4}\right) = \sin 2x + 9$ will have real solution.

94. Let a, b and c be real numbers such that $a + 2b + c = 4$. Find the maximum value of $(ab + bc + ca)$.

95. Prove that for real values of x, $(ax^2 + 3x - 4)/(3x - 4x^2 + a)$ may have any value provided a lies between 1 and 7.

96. Let $x^2 - (m - 3)x + m = 0(m\varepsilon R)$ be a quadratic equation . Find the values of m for which the roots are (ix)one root is smaller than 2 & other root is greater than 2 (x) both the roots are greater than 2 (xi) both the roots are smaller than 2 (xii)exactly one root lies in the interval (1;2) (xiii) both the roots lies in the interval (1;2) (xiv) atleast one root lies in the interval (1;2) (xv) one root is greater than 2 and the other root is smaller than 1

97. Prove that for all real values of x and y, $x^2 + 2xy + 3y^2 - 6x - 2y \geq -11$.

98. The values of 'a' for which the equation

$$\left(x^2 + x + 2\right)^2 - (a - 3)\left(x^2 + x + 2\right)\left(x^2 + x + 1\right) + (a - 4)\left(x^2 + x + 1\right)$$

has atlesast one real root is:

99. Find the values of a for whilch the equation $\sin^4 x + a \sin^2 x + 1 = 0$ will have a solution.

100. Find all the value of m for which the equation $\sin^2 x - (m - 3)\sin x + m = 0$ has real roots.

101. If α is a real root of the quadratic equation $ax^2 + bx + c = 0 \, and \, \beta$ ils a real root of $-ax^2 + bx + c = 0$, then show that there is a root γ of equation $(a/2)x^2 + bx + c = 0$ whilch lies between $\alpha \& \beta$

102. Find the condition if the roots of $ax^2 + 2bx + c = 0 \, and \, bx^2 - 2\sqrt{ac}x + b = 0$ are simultaneously real.

103. Solve $\left(x^2 - 5x + 7\right)^2 - (x - 2)(x - 3) = 1$.

104. Solve the equation $x^4 - 5x^2 - 6x - 5 = 0$.

105. Solve $\dfrac{x^2 - 2x - 3}{x + 1} = 0$.

106. Solve $\left(x^3 - 4x\right)\sqrt{x^2 - 1} = 0$.

107. Solve $\dfrac{2x - 3}{x - 1} + 1 = \dfrac{6x - x^2 - 6}{x - 1}$.

108. Using differentiation method check how many roots of the equation $x^3 - x^2 + x - 2 = 0$ are real?

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109. Analyze the roots of the following equations: $2x^3 - 9x2 + 12 - (9/2) = 0$. $2x^3 - 9x2 + 12x - 3 = 0$.

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110. Find how many roots of the equations $x^4 + 2x^2 - 8x + 3 = 0$.

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111. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?

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112. Solve $\sqrt{5x^2 - 6x + 8} - \sqrt{5x^2 - 6x - 7} = 1$.

113. Solve $\sqrt{3x^2 - 7x - 30} + \sqrt{2x^2 - 7x - 5} = x + 5$.

114. If $x = \sqrt{7 + 4\sqrt{3}}$, prove that $x + 1/x = 4$

115. Solve $\sqrt{5x^2 - 6x + 8} - \sqrt{5x^2 - 6x - 7} = 1$.

116. Solve $\sqrt{x^2 + 4x - 21} + \sqrt{x^2 - x - 6} = \sqrt{6x^2 - 5x - 39}.$

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117. Solve $4^x + 6^x = 9^x.$

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118. Solve $3^{2x^2 - 7x + 7} = 9.$

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119. How many solutions does the equation $\dfrac{8^x + 27^x}{12^x + 18^x} = \dfrac{7}{6}$ have? (A) Exactly one (B) Exactly two (C) Finitely many (D) Infinitely many

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120. Find the number of real roots of the equation $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$.

A. 0

B. 1

C. 2

D. 3

Answer: A

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121. How many roots of the equation $3x^4 + 6x^3 + x^2 + 6x + 3 = 0$ are real ?

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122. Find the value of k if $x^3 - 3x + k = 0$ has three real distinct roots.

123. $a, b,$ and c are all different and non-zero real numbers on arithmetic progression. If the roots of quadratic equation $ax^2 + bx + c = 0$ are α and β such that $\dfrac{1}{\alpha} + \dfrac{1}{\beta}, \alpha + \beta,$ and $\alpha^2 + \beta^2$ are in geometric progression the value of a/c will be____.

124. Let $x^2 + y^2 + xy + 1 \geq a(x + y) \ \forall x, y \in R,$ then the number of possible integer (s) in the range of a is_____.

125. The quadratic equation $x^2 + mx + n = 0$ has roots which are twice those of $x^2 + px + m = 0$ and m, n and $p \neq 0.$ The n the value of n/p is ____.

126. All he value of k for which the quadratic polynomial

$f(x) = 2x^2 + kx + k^2 + 5$ has two distinct zeroes and only one of them

satisfying `0

127. Let $a, b, and c$ be rel numbers which satisfy the equation

$a + \dfrac{1}{bc} = \dfrac{1}{5}, b + \dfrac{1}{ac} = \dfrac{-1}{15}, and c + \dfrac{1}{ab} = \dfrac{1}{3}$. The value of $\dfrac{c - b}{c - a}$ is

equal to _____.

128. If α, β are the roots of the quadratic equation $ax^2 + bx = c = 0$,

then which of the following expression will be the symmetric function of

roots   a. $\left| \log\left(\dfrac{\alpha}{\beta}\right) \right|$ b. $\alpha^2 \beta^5 + \beta^2 \alpha^5$ c. $\tan(\alpha - \beta)$ d.

$\left(\log\left(\dfrac{1}{\alpha}\right) \right)^2 + (\log \beta)^2$

129. If a, b, c are non-zero real numbers, then the minimum value of the expression $\left(\dfrac{\left(a^4 + 3a^2 + 1\right)\left(b^4 + 5b^2 + 1\right)\left(c^4 + 7c^2 + 1\right)}{a^2 b^2 c^2} \right)$ is not divisible by prime number.

130. if diagonals of a parallelogram bisect each other,prove that its a rhombus

131. Referred to the principal axes as the axes of co ordinates find the equation of hyperbola whose focii are at $\left(0, \pm \sqrt{10}\right)$ and which passes through the point $(2, 3)$

132. If the equation $ax^2 + bx + c = 0 (a > 0)$ has two real roots α and β such that $\alpha < -2$ and $\beta > 2$, then which of the following statements is/are true? (a) $a - |b| + c < 0$ (b) $c < 0, b^2 - 4ac > 0$ (c) $4a - 2|b| + c < 0$ (d) $9a - 3|b| + c < 0$

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133. If fig shows the graph of $f(x) = ax^2 + bx + c$, then Fig $ac < 0$ b. $bc > 0$ c. $ab > 0$ d. $abc < 0$

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134. If $c \neq 0$ and the equation $p/(2x) = a/(x+c) + b/(x-c)$ has two equal roots, then p can be $\left(\sqrt{a} - \sqrt{b}\right)^2$ b. $\left(\sqrt{a} + \sqrt{b}\right)^2$ c. $a + b$ d. $a - b$

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135. If the equation $4x^2 - x - 1 = 0$ and $3x^2 + (\lambda + \mu)x + \lambda - \mu = 0$ have a root common, then he rational values of λ and μ are $\lambda = \dfrac{-3}{4}$ b. $\lambda = 0$ c. $\mu = \dfrac{3}{4}$ b. $\mu = 0$

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136. If the equation whose roots are the squares of the roots of the cubic $x^3 - ax^2 + bx - 1 = 0$ is identical with the given cubic equation, then $a = 0, b = 3$ b. $a = b = 0$ c. $a = b = 3$ d. $a, b,$ are roots of $x^2 + x + 2 = 0$

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137. If the equation $ax^2 + bx + c = 0, a, b, c, \in R$ have none-real roots, then $c(a - b + c) > 0$ b. $c(a + b + c) > 0$ c. $c(4a - 2b + c) > 0$ d. none of these

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138. If $p(q - r)x^2 + q(r - p)x + r(p - q) = 0$ has equal roots, then prove that $\dfrac{2}{q} = \dfrac{1}{p} + \dfrac{1}{r}$.

139. Let $\alpha, \beta \in R$. If α, β^2 are the roots of quadratic equation $x^2 - px + 1 = 0.$ $and \alpha^2, \beta$ are the roots of quadratic equation $x^2 - qx + 8 = 0$, then find p, q, α, β.

140. If the ratio of the roots of the equation $x^2 + px + q = 0$ are equal to ratio of the roots of the equation $x^2 + bx + c = 0$, then prove that $p^{2c} = b^2 q$.

141. If $s\int h\eta, \cos\theta$ be the roots of $ax^2 + bx + c = 0$, then prove that $b^2 = a^2 + 2a$.

142. If the sum of the roots of the equation $\dfrac{1}{x+a} + \dfrac{1}{x+b} = 1/c$ is zero, the prove that the product of the root is $\left(-\dfrac{1}{2}\right)(a^2 + b^2)$.

143. Solve the equation $x^2 + px + 45 = 0$. it is given that the squared difference of its roots is equal to 144

144. If α, β are the roots of the equation $2x^2 - 35x + 2 = 0$, the find the value of $(2\alpha - 35)^3(2\beta - 35)^3$.

145. Find a quadratic equation whose product of roots $x_1 and x_2$ is equal to 4 an satisfying the relation $\dfrac{x_1}{x_1 - 1} + \dfrac{x_2}{x_2 - 1} = 2.$

146. If a and $b (\neq 0)$ are the roots of the equation $x^2 + ax + b = 0$, then find the least value of $x^2 + ax + b (x \in R)$.

147. If the sum of the roots of the equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0$ is -1, then find the product of the roots.

A. 1

B. -1

C. 2

D. -2

Answer: C

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148. The quadratic polynomial $p(x)$ has the following properties: $p(x) \geq 0$ for all real numbers, $p(1) = 0 \, and \, p(2) = 2$. Find the value of $p(3)$ is_____.

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149. function f , R $\to$ R , $f(x) = \dfrac{3x^2 + mx + n}{x^2 + 2}$, if the range of function is [-4,3), find the value of |m+n| is ....
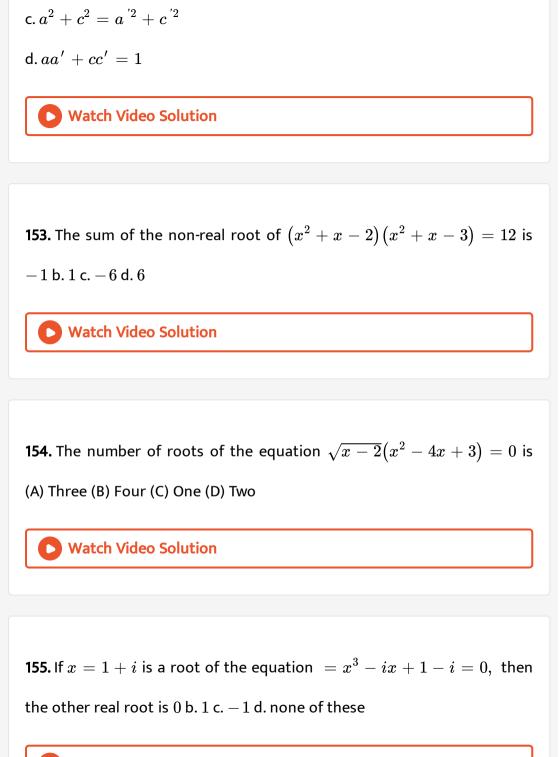
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150. If $x^2 + px = 1$ is a factor of the expression $ax^3 + bx = c$, then $a^2 - c^2 = ab$ b. $a^2 + c^2 = -ab$ c. $a^2 - c^2 = -ab$ d. none of these

151. If α, β are the roots of $x^2 - px + q = 0 \, and \, \alpha', \beta'$ are the roots of $x^2 - p'x + q' = 0$, then the value of

$(\alpha - \alpha')^2 + (\beta - \alpha')^2 + (\alpha - \beta')^2 + (\beta - \beta')^2$ is

$2\{p^2 - 2q + p'^2 - 2q' - pp'\}$ $2\{p^2 - 2q + p'^2 - 2q' - qq'\}$

$2\{p^2 - 2q - p'^2 - 2q' - pp'\} \, 2\{p^2 - 2q - p'^2 - 2q' - qq'\}$

152. If $(ax^2 + c)y + (a'x^2 + c') = 0$ and x is a rational function of y and ac is negative, then
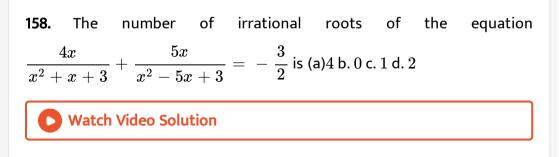
a. $ac' + c'c = 0$

b. $a/a' = c/c'$

c. $a^2 + c^2 = a'^2 + c'^2$

d. $aa' + cc' = 1$

---

153. The sum of the non-real root of $(x^2 + x - 2)(x^2 + x - 3) = 12$ is

-1 b. 1 c. -6 d. 6

---

154. The number of roots of the equation $\sqrt{x - 2}(x^2 - 4x + 3) = 0$ is

(A) Three (B) Four (C) One (D) Two

---

155. If $x = 1 + i$ is a root of the equation $= x^3 - ix + 1 - i = 0,$ then

the other real root is 0 b. 1 c. -1 d. none of these

156. If the expression $x^2 + 2(a + b + c) + 3(bc + c + ab)$ is a perfect square, then a = b = c b. a = ±b = ±c c. a = b ≠ c d. *none of these*

157. The curve $y = (\lambda + 1)x^2 + 2$ intersects the curve $y = \lambda x + 3$ in exactly one point, if λ equals $\{-2, 2\}$ b. $\{1\}$ c. $\{-2\}$ d. $\{2\}$

158. The number of irrational roots of the equation
$$\frac{4x}{x^2 + x + 3} + \frac{5x}{x^2 - 5x + 3} = -\frac{3}{2}$$ is (a)4 b. 0 c. 1 d. 2

159. If the equation $x^2 - 3px + 2q = 0$ and $x^2 - 3ax + 2b = 0$ have a common roots and the other roots of the second equation is the reciprocal of the other roots of the first, then $(2 - 2b)^2$. a. $36pa(q - b)^2$ b. $18pa(q - b)^2$ c. $36bq(p - a)^2$ d. $18bq(p - a)^2$

160. Solve the equation $3^{x^2 - x} + 4^{x^2 - x} = 25$.

161. Solve the equation $12x^4 - 56x^3 + 89x^2 - 56x + 12 = 0$.

162. Solve the equation $(x + 2)(x + 3)(x + 8) \times (x + 12) = 4x^2$.

163. Solve the equation $(x-1)^4 + (x-5)^4 = 82$.

164. Solve $\sqrt{x+5} + \sqrt{x+21} = \sqrt{6x+40}$.

165. Evaluate $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \infty}}}$.

166. If the expression $ax^4 + bx^3 - x^2 + 2x + 3$ has remainder $4x + 3$ when divided by $x^2 + x - 2$, find the value of a and b.

167. Find the remainder when $x^3 + 4x^2 - 7x + 6$ is diided by $x - 1$.

168. Use the factor theorem to find the value of k for which $(a + 2b), where a, b \neq 0$ is a factor of $a^4 + 32b^4 + a^3b(k + 3)$.

169. Given that $x^2 + x - 6$ is a factor of $2x^4 + x^3 - ax^2 + bx + a + b - 1,$ find the value of $a and b$.

170. If p, q, r are positive and are in A.P., the roots of quadratic equation $px^2 + qx + r = 0$ are all real for a. $\left|\dfrac{r}{p} - 7\right| \geq 4\sqrt{3}$ b. $\left|\dfrac{p}{r} - 7\right| \geq 4\sqrt{3}$ c. all p and r d. no p and r

171. The number of points of intersection of two curves $y = 2\sin x$ and $y = 5x^2 + 2x + 3$ is 0 b. 1 c. 2 d. ∞

172. If α and β are the roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always. A. one positive and one negative root B . two positive roots C . two negative roots D . cannot say anything

173. If $a^2 + b^2 + c^2 = 1, then ab + bc + ca$ lie in the interval $\left[\dfrac{1}{3}, 2\right]$ b. $[-1, 2]$ c. $\left[-\dfrac{1}{2}, 1\right]$ d. $\left[-1, \dfrac{1}{2},\right]$

174. Let α, β be the roots of the equation $(x - a)(x - b) = c, c \neq 0$. Then the roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are a, c b. b, c c. a, b d. $a + c, b + c$

175. Let a, b, c be real numbers, $a \neq 0$. If α is a zero of $a^2x^2 + bx + c = 0$, β is the zero of $a^2x^2 - bx - c = 0$ and $0, \alpha < \beta$ then prove that the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies $\alpha < \gamma < \beta$.

176. If $(x^2 + px + 1)$ is a factor of $(ax^3 + bx + c)$, then $a^2 + c^2 = -ab$ b. $a^2 - c^2 = -ab$ c. $a^2 - c^2 = ab$ d. none of these

177. Both the roots of the equation

$(x - b)(x - c) + (x - a)(x - c) + (x - a)(x - b) = 0$ are always a.

positive b. real c. negative d. none of these

178. The equation $x - \dfrac{2}{x - 1} = 1 - \dfrac{2}{x - 1}$ has a. no root b. one root c.

two equals roots d. Infinitely many roots

179. Two towns A and B are 60 km a part. A school is to be built to serve

150 students in town A and 50 students in town B. If the total distance to

be travelled by all 200 students is to be as small as possible, then the

school be built be a. town B   b. 45 km from town A c. town

A   d. 45 km from town B

180. Find the condition on a, b, c, d such that equations $2ax^3 + bx^2 + cx + d = 0$ and $2ax^2 + 3bx + 4c = 0$ have a common root.

181. Let $f(x), g(x)$, and $h(x)$ be the quadratic polynomials having positive leading coefficients and real and distinct roots. If each pair of them has a common root, then find the roots of $f(x) + g(x) + h(x) = 0$.

182. If a, b, c be the sides of ABC and equations $ax^2 + bx + c = 0$ and $5x^2 + 12 + 13 = 0$ have a common root, then find $\angle C$.

183. If $b^2 < 2ac$, then prove that $ax^3 + bx^2 + cx + d = 0$ has exactly one real root.

184. If two roots of $x^3 - ax^2 + bx - c = 0$ are equal inn magnitude but opposite in signs, then prove that $ab = \cdot$

185. If $\alpha, \beta and \gamma$ are the roots of $x^3 + 8 = 0$ then find the equation whose roots are $\alpha^2, \beta^2 and \gamma^2$.

186. If α, β, γ are the roots of the equation $x^3 - px + q = 0,$ then find the cubic equation whose roots are $\dfrac{\alpha}{1+\alpha}, \dfrac{\beta}{1+\beta}, \dfrac{\gamma}{1+\gamma}$.

187. If the roots of equation $x^3 + ax^2 + b = 0$ are α_1, α_2 and $\alpha_3 (a, b \neq 0)$, then find the equation whose roots are $\dfrac{\alpha_1\alpha_2 + \alpha_2\alpha_3}{\alpha_1\alpha_2\alpha_3}, \dfrac{\alpha_2\alpha_3 + \alpha_3\alpha_1}{\alpha_1\alpha_2\alpha_3}, \dfrac{\alpha_1\alpha_3 + \alpha_1\alpha_2}{\alpha_1\alpha_2\alpha_3}$

188. If α, β and γ are roots of $2x^3 + x^2 - 7 = 0$, then find the value of $\sum \left(\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} \right)$.

189. Let $r, s,$ and t be the roots of equation $8x^3 + 1001x + 2008 = 0$. Then find the value of $(r + s)^3 + (s + t)^3 + (t + r)^3$.

190. The number of value of k for which $\left[x^2 - (k-2)x + k^2\right] \times \left[x^2 + kx + (2k-1)\right]$ is a perfect square is a.2 b. 1 c. 0 d. none of these

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191. The total number of integral values of a so that $x^2 - (a+1)x + a - 1 = 0$ ha integral roots is equal to 1 b. 2 c. 4 d. none of these
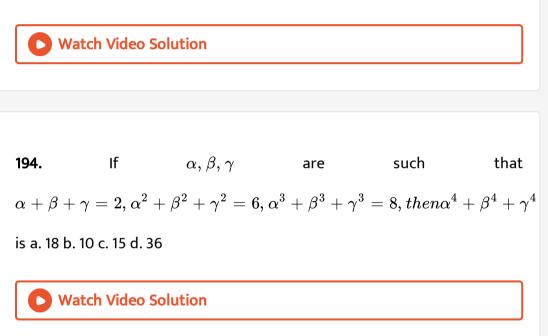
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192. The number of positive integral solutions of $x^4 - y^4 = 3789108$ is a. 0 b. 1 c. 2 d. 4

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193. If α, β are the roots of $x^2 + px + q = 0$ and $nx^{2n} + p^n x^n + q^n = 0$ and il $f(\alpha/\beta), (\beta/\alpha)$ are the roots of $x^n + 1 + (x + 1)^n = 0$, the $\cap$ $(\in N)$ a. must be an odd integer b. may be any integer c. must be an even integer d. cannot say anything

194. If α, β, γ are such that $\alpha + \beta + \gamma = 2, \alpha^2 + \beta^2 + \gamma^2 = 6, \alpha^3 + \beta^3 + \gamma^3 = 8, then \alpha^4 + \beta^4 + \gamma^4$ is a. 18 b. 10 c. 15 d. 36

195. If $xy = 2(x + y), x \leq y and x, y \in N,$ then the number of solutions of the equation are a. two b. three c. no solution d. infinitely many solutions

196. The number of real solutions of the equation $(9/10)^x = -3 + x - x^2$ is a. 2 b. 0 c. 1 d. none of these

197. The number of integral values of a for which the quadratic equation $(x + a)(x + 1991) + 1 = 0$ has integral roots are a. 3 b.0 c.1 d. 2

198. If the equation $\cot^4 x - 2 \cos ec^2 x + a^2 = 0$ has at least one solution, then the sum of all possible integral values of a is equal to a. 4 b. 3 c. 2 d. 0

199. The number of real solutions of $|x| + 2\sqrt{5 - 4x - x^2} = 16$ is/are a.

6 b. 1 c. 0 d. 4

200. If the quadratic equation $ax^2 + bx + 6 = 0$ does not have real roots

and $b \in R^+$, then prove that $a > max\left\{\dfrac{b^2}{24}, b - 6\right\}$

201. What is the minimum height of any point on the curve

$y = x^2 - 4x + 6$ above the x-axis?

202. What is the minimum height of any point on the curve

$y = -x^2 + 6x - 5$ above the x-axisdv?

203. Find the largest natural number a for which the maximum value of $f(x) = a - 1 + 2x - x^2$ is smaller than the minimum value of $g(x) = x^2 - 2ax + 10 - 2a$.

204. Let $f(x) = ax^2 + bx + c$ be a quadratic expression having its vertex at (3, -2) and value of $f(0) = 10$. $F \in df(x)$.

205. Find the least value of n such that $(n2)x^2 + 8x + n + 4 > 0$, $\forall x \in R$,where $n \in N$.

206. If the inequality $(mx^2 + 3x + 4) / (x^2 + 2x + 2) < 5$ is satisfied for all $x \in R$, then find the value of m.

207. If $f(x) = (a_1x + b_1)^2 + (a_2x + b_2)^2 + ... + (a_nx + b_n)^2$, then prove that $(a_1b_1 + a_2b_2 + + a_nb_n)^2 \le (a12 + a22 + + an2)^{b12 + b22 + + bn2}$.

208. If c is positive and $2ax^2 + 3bx + 5c = 0$ does not have aby real roots, then prove that $2a - 3b + 5b > 0$.

209. If $ax^2 + bx = 6 = 0$ does not have distinct real roots, then find the least value of $3a + b$.

210. A quadratic trinomial $P(x) = ax^2 + bx + c$ is such that the equation $P(x) = x$ has no real roots. Prove that in this case equation $P(P(x)) = x$ has no real roots either.

211. Let $a, b, c \in Q^+$ satisfying $a > b > \cdot$ Which of the following statements (s) hold true of the quadratic polynomial $f(x) = (a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b)$? The mouth of the parabola $y = f(x)$ opens upwards Both roots of the equation $f(x) = 0$ are rational The x-coordinate of vertex of the graph is positive The product of the roots is always negative

212. If $x, y \in R$ satify the equation $x^2 + y^2 - 4x - 2y + 5 = 0$, then the value of the expression $\left[\left(\sqrt{x} - \sqrt{y} \right)^2 + \sqrt{xy} \right] / \left(x + 4\sqrt{xy} \right)$ is

$\sqrt{2} + 1$ b. $\dfrac{\sqrt{2} + 1}{2}$ c. $\dfrac{\sqrt{2} - 1}{2}$ d. $\dfrac{\sqrt{2} + 1}{\sqrt{2}}$

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213. If $x = 1 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{3 + \frac{1}{2\infty}}}}$ a $\sqrt{\dfrac{5}{2}}$ b. $\sqrt{\dfrac{3}{2}}$ c. $\sqrt{\dfrac{7}{3}}$ d. $\sqrt{\dfrac{5}{3}}$

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214. Find the values of a for which all the roots of the euation $x^4 - 4x^3 - 8x^2 + a = 0$ are real.

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215. If $x = 2 + 2^{2/3} + 2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is (a)3 b. 2 c. 1 d. -2

216. The least value of the expression $x^2 + 4y^2 + 3z^2 - 2x - 12y - 6z + 14$ is a.1 b. no least value c. 0 d. none of these

217. If $x = 2 + 2^{2/3} + 2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is (a)3 b. 2 c. 1 d. -2

218. Find the values of k for which $\left| \dfrac{x^2 + kx + 1}{x^2 + x + 1} \right| < 2, \ \forall x \in R$

219. Solve the equation $\sqrt{a(2^x - 2) + 1} = 1 - 2^x, x \in R$.

220. For $a < 0$, deterine all real roots of the equation $x^2 - 2a|x - a| - 3a^2 = 0$.

221. If $ax^2 + bx + c = 0 and bx^2 + cx + a = 0$ have a common root and a, b, and c are nonzero real numbers, then find the value of $(a^3 + b^3 + c^3)/abc$

222. If $x^2 + 3x + 5 = 0$ and $ax^2 + bx + c = 0$ have common root/roots and $a, b, c \in N,$ then find the minimum value of $a + b + \cdot$

223. If $\alpha \neq \beta$ and $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3.$ find the equation whose roots are α / β and $\beta / \alpha.$

224. If α, β are the roots of lthe equation $2x^2 - 3x - 6 = 0,$ find the equation whose roots are $\alpha^2 + 2$ and $\beta^2 + 2.$

225. Determine the values o m for which equations $3x^2 + 4mx + 2 = 0$ and $2x^2 + 3x - 2 = 0$ may have a common root.

226. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then find the roots of the equation $ax^2 - bx(x-1) + c(x-1)^2 = 0$ in term of α and β.
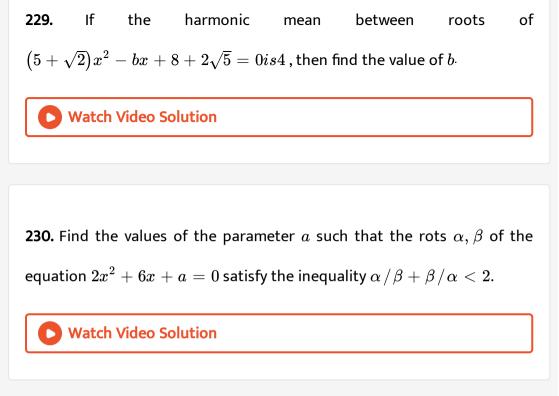
227. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less then $\sqrt{5}$, then find the set of possible value of a.

228. Find the value of $3a$ for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other.

229. If the harmonic mean between roots of $(5 + \sqrt{2})x^2 - bx + 8 + 2\sqrt{5} = 0$ is 4, then find the value of b.

230. Find the values of the parameter a such that the rots α, β of the equation $2x^2 + 6x + a = 0$ satisfy the inequality $\alpha/\beta + \beta/\alpha < 2$.

231. Let $\alpha \, and \, \beta$ be the solutions of the quadratic equation $x^2 - 1154x + 1 = 0$, then the value of $\alpha^{\frac{1}{4}} + \beta^{\frac{1}{4}}$ is equal to _____.

232. If $a, b, c \in R^+ \, and \, 2b = a + c$, then check the nature of roots of equation $ax^2 + 2bx + c = 0$.

233. Determine the value of k for which $x + 2$ is a factor of $(x + 1)^7 + (2x + k)^3$.

234. Given that the expression $2x^3 + 3px^2 - 4x + p$ hs a remainder of 5 when divided by $x + 2$, find the value of p.

235. In how many points the graph of $f(x) = x^3 + 2x^2 + 3x + 4$ meets the x-axis ?

236. Analyze the roots of the equation $(x-1)^3 + (x-2)^3 + (x-4)^3 + (x-5)^3 = 0$ by differentiation method.

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237. Find the values of a for which the roots of the equation $x^2 + a^2 = 8x + 6a$ are real.

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238. If $f(x) = x^3 - x^2 + ax + b$ is divisible by $x^2 - x$, then find the value of $f(2)$.

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239. If $f(x) = x^3 - 3x^2 + 2x + a$ is divisible by $x - 1$, then find the remainder when $f(x)$ is divided by $x - 2$.

240. Find the value of p for which $x + 1$ is a factor of $x^4 + (p - 3)x^3 - (3p - 5)x^2 + (2p - 9)x + 6$. Find the remaining factor for this value of p.

241. Given that $x^2 - 3x + 1 = 0$, then the value of the expression $y = x^9 + x^7 + x^{-9} + x^{-7}$ is divisible by prime number.

242. Suppose $a, b, c \in I$ such that the greatest common divisor of $x^2 + ax + b$ and $x^2 + bx + cis(x + 1)$ and the least common multiple of $x^2 + ax + b$ and $x^2 + bx + c$ is $(x^3 - 4x^2 + x + 6)$. Then the value of $|a + b + c|$ is equal to _____.

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243. If the roots of ht cubic, $x^3 + ax^2 + bx + c = 0$ are three consecutive positive integers, then the value of $(a^2 / b + 1)$ is equal to _____.

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244. If $x + y + z = 12 and x^2 + y^2 + z^2 = 96 and \dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 36$, then the value $x^3 + y^3 + z^3$ divisible by prime number is_____.

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245. If the equation $x^2 + bx - a = 0$ and $x^2 - ax + b = 0$ have a common root, then a. $a + b = 0$ b. $a = b$ c. $a - b = 1$ d. $a + b = 1$

246. If $x^3 + 3x^2 - 9x + c$ is of the form $(x - \alpha)^2(x - \beta)$, then c is equal to a.27 b. -27 c. 5 d. -5

247. If $a and b$ are positive numbers and eah of the equations $x^2 + ax + 2b = 0$ and $x^2 + 2bx + a = 0$ has real roots, then the smallest possible value of $(a + b)$ is_____.

248. Suppose a, b, c are the roots of the cubic $x^3 - x^2 - 2 = 0$. Then the value of $a^3 + b^3 + c^3$ is ____.

249. If $ax^2 + (b - c)x + a - b - c = 0$ has unequal real roots for all $c \in R, then$

250. If a, b, c real in G.P., then the roots of the equation $ax^2 + bx + c = 0$ are in the ratio $\frac{1}{2}(-1 + \sqrt{3})$ b. $\frac{1}{2}(1 - i\sqrt{3})$ $\cdot \frac{1}{2}(-1 - i\sqrt{3})$ d. $\frac{1}{2}(1 + i\sqrt{3})$

251. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, then it must be equal to a. $\dfrac{p' - p'q}{q - q'}$ b. $\dfrac{q - q'}{p' - p}$ c. $\dfrac{p' - p}{q - q'}$ d. $\dfrac{pq' - p'q}{p - p'}$

252. Given that α, γ are roots of the equation $Ax^2 - 4x + 1 = 0$, and β, δ the roots of the equation of $Bx^2 - 6x + 1 = 0$, such that α, β, γ, and δ are in H.P., then a. $A = 3$ b. $A = 4$ B $= 2$ d. $B = 8$

253. The graph of the quadratic trinomial $u = ax^2 + bx + c$ has its vertex at (4, -5) and two x-intercepts, one positive and one negative. Which of the following holds good? $a > 0$ b. $b < 0$ c. < 0 d. $8a = b$

254. The function $kf(x) = ax^2 + bx^2 + cx + d$ has three positive roots. If the sum of the roots of $f(x)$ is 4, the larget possible inegal values of c/a is _____.

255. If $\left(18x^2 + 12x + 4\right)^n = a_0 + a_{1x} + a_{2x}^2 + \ldots\ldots + a_{2n}x^{2n}$, prove that

$$a_r = 2^n 3^r \left(^{2n}C_r + {}^nC_1 {}^{2n-2}C_r + {}^nC_2 {}^{2n-4}C_r + \ldots.\right)$$

256. If $(\sin\alpha)x^2 - 2x + b \geq 2$, for all real values of $x \leq 1$ and $\alpha \in \left(0, \dfrac{\pi}{2}\right) \cup (\pi/2, \pi)$, then possible real value of b is /are

a2 b. 3 c. 4 d. 5

257. If one root $x^2 - x - k = 0$ is square of the other, then $k = $ a. $2 \pm \sqrt{5}$ b. $2 \pm \sqrt{3}$ c. $3 \pm \sqrt{2}$ d. $5 \pm \sqrt{2}$

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258. If $\alpha, and \beta$ be t roots of the equation $x^2 + px - 1/2p^2 = 0, where p \in R.$ Then the minimum value of $\alpha^4 + \beta^4$ is $2\sqrt{2}$ b. $2 - \sqrt{2}$ c. 2 d. $2 + \sqrt{2}$

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259. Find the rang of $f(x) = \dfrac{x^2 + 34x - 71}{x^2 + 2x - 7}$ $f(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

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260. $x^2 - xy + y^2 - 4x - 4y + 16 = 0$ represents a. a point b. a circle c. a pair of straight line d. none of these

261. If α, β are the nonzero roots of $ax^2 + bx + c = 0$ and α^2, β^2 are the roots of $a^2x^2 + b^2x + c^2 = 0$, then a, b, c are in (A) G.P. (B) H.P. (C) A.P. (D) none of these

262. If the roots of the equation $ax^2 + bx + c = 0$ are of the form $(k+1)/k$ and $(k+2)/(k+1)$, $then(a + b + c)^2$ is equal to $2b^2 - ac$ b. $a62$ c. $b^2 - 4ac$ d. $b^2 - 2ac$

263. If α, β are the roots of $ax^2 + bx + c = 0 and \alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0 then h = $ $-\dfrac{1}{2}\left(\dfrac{a}{b} - \dfrac{p}{q}\right)$ b. $\left(\dfrac{b}{a} - \dfrac{q}{p}\right)$ c. $\dfrac{1}{2}\left(\dfrac{b}{q} - \dfrac{q}{p}\right)$ d. none of these

264. The equation $(x^2 + x + 1)^2 + 1 = (x^2 + x + 1)(x^2 - x - 5)$ for $x \in (-2, 3)$ will have number of solutions. 1 b. 2 c. 3 d. 0

265. If α, β re the roots of $ax^2 + c = bx$, then the equation $(a + cy)^2 = b^2 y$ in y has the roots a.$\alpha\beta^{-1}, \alpha^{-1}\beta$ b. α^{-2}, β^{-2} c. α^{-1}, β^{-1} d. α^2, β^2

266. If the roots of the equation $x^2 + 2ax + b = 0$ are real and distinct and they differ by at most $2m$,then b lies in the interval a.$(a^2, a^2, + m^2)$ b. $(a^2 - m^2, a62)$ c. $[a^2 - m^2, a^2)$ d. none of these

267. If the ratio of the roots of $ax^2 + 2bx + c = 0$ is same as the ratios of roots of $px^2 + 2qx + r = 0$, then a. $\dfrac{2b}{ac} = \dfrac{q^2}{pr}$ b. $\dfrac{b}{ac} = \dfrac{q}{pr}$ c. $\dfrac{b^2}{ac} = \dfrac{q^2}{pr}$

d. none of these

268. Show that

$$\frac{(x+b)(x+c)}{(b-a)(c-a)} + \frac{(x+c)(x+a)}{(c-b)(a-b)} + \frac{(x+a)(x+b)}{(a-c)(b-c)} = 1 \quad \text{is} \quad \text{an}$$

identity.

269. A certain polynomial $P(x), x \in R$ when divided by $x - a, x - b$ and $x - c$ leaves remainders $a, b,$ and c, resepectively. Then find remainder when $P(x)$ is divided by $(x - a)(x - b)(x - c)$ where a, b, c are distinct.

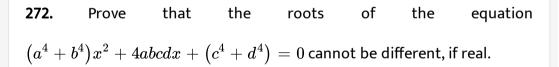**270.** If c, d are the roots of the equation $(x - a)(x - b) - k = 0$, prove that a, b are roots of the equation $(x - c)(x - d) + k = 0$.

271. If $(a^2 - 1)x^2 + (a - 1)x + a^2 - 4a + 3 = 0$ is identity in x , then find the value of a .

272. Prove that the roots of the equation $(a^4 + b^4)x^2 + 4abcdx + (c^4 + d^4) = 0$ cannot be different, if real.

273. If the roots of the equation $x^2 - 8x + a^2 - 6a = 0$ are real distinct, then find all possible value of a.

274. If roots of equation $x^2 - 2cx + ab = 0$ are real and unequal, then prove that the roots of $x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$ will be imaginary.

275. If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, show that $2/b = 1/a + 1/ \cdot$

276. Find the quadratic equation with rational coefficients whose one root is $1/(2 + \sqrt{5})$.

277. If $f(x) = ax^2 + bx + c$, $g(x) = -ax^2 + bx + c$, $where ac \neq 0$, then prove that $f(x)g(x) = 0$ has at least two real roots.

278. If x is real, then $x/(x^2 - 5x + 9)$ lies between $-1 and -1/11$ b. $1 and -1/11$ c. $1 and 1/11$ d. none of these

279. Set of all real value of a such that $f(x) = \dfrac{(2a - 1) + x^2 + 2(a + 1)x + (2a - 1)}{x^2 - 2x + 40}$ is always negative is $-\infty, 0$ b. $0, \infty$ c. $-\infty, 1/2$ d. none

280. If α, β and γ are the roots of $x^3 - x^2 - 1 = 0$, then value of $\dfrac{1+\alpha}{1-\alpha} + \dfrac{1+\beta}{1-\beta} + \dfrac{1+\gamma}{1-\gamma}$ is

281. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 - Kx^3 + Kx^2 + Lx + m = 0, where K, L, and M$ are real numbers, then the minimum value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is 0 b. -1 c. 1 d. 2

282. Suppose that $f(x)$ isa quadratic expresson positive for all real x. If $g(x) = f(x) + f'(x) + f^x$, then for any real $x (where f'(x) and f^x$ represent 1st and 2nd derivative, respectively). $g(x) < 0$ b. $g(x) > 0$ c. $g(x) = 0$ d. $g(x) \geq 0$

283. Let $f(x) = ax^2 - bx + c^2 \neq 0$ and $f(x) \neq 0$ for all $x \in R$. Then

(a) $a^2 + c^2 2$ (b) c (c) $a - 3b + c^2 < 0$ (d) non of these

284. If $a, b \in R, a \neq 0$ and the quadratic equation $ax^2 - bx + 1 = 0$ has imaginary roots, then $(a + b + 1)$ is a. positive b. negative c. zero d. Dependent on the sign of b

285. If he expression $[mx - 1 + (1/x)]$ is non-negative for all positive real x, then the minimum value of m must be $-1/2$ b. 0 c. $1/4$ d. $1/2$

286. $x_1 and x_2$ are the roots of $ax^2 + bx + c = 0 and x_1 x_2 < 0$. Roots of $x_1(x - x_2)^2 + x_2(x - x_1)^2() = 0$ are a. real and of opposite sign b. negative c. positive d. none real

287. If a, b, c, d are four consecutive terms of an increasing A.P., then the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are a. non-real complex b. real and equal c. integers d. real and distinct

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288. Let a, b and be the roots of the equation $x^2 - 10xc - 11d = 0$ and those roots of c and d of $x^2 - 10ax - 11b = 0$, · then find the value of `a+b+c+d

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289. Fill in the blanks The coefficient of x^{99} in the polynomial $(x - 1)(x - 2)\ldots\ldots\ldots(x - 100) is$_ __ _ ·

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290. Fill in the blanks If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, $where\, p\, and\, q$ are real, then $(p, q) = (\_\_ \quad \_\_ , \_ \quad \_\_)$.
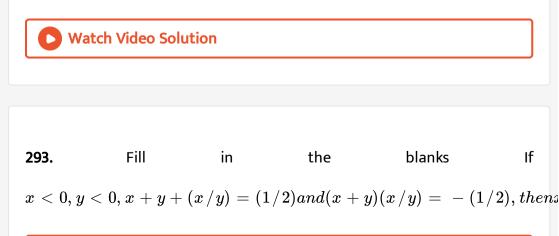
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291. Fill in the blanks If the product of the roots of the equation $x^2 - 3kx + 2e^{21nk} - 1 = 0$ is 7, then the roots are real for_____.

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292. If the equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ have one common root. Then find the numerical value of a+b.
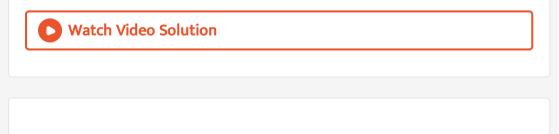
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293. Fill in the blanks If $x < 0, y < 0, x + y + (x/y) = (1/2)\, and\, (x + y)(x/y) = -(1/2)$, then x

294. True or false The equation $2x^2 + 3x + 1 = 0$ has an irrational root. If `a

295. If l, m, n are real $l \neq m$, then the roots of the equation $(l - m)x^2 - 5(l_+m)x - 2(l - m) = 0$ are a. real and equal b. Complex c. real and unequal d. none of these

296. If $x, y, and z$ are real and different and $u = x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy, the\nu$ is always a. non-negative b. zero c. non-positive d. none of these

297. Let $a > 0, b > 0$ and $c > 0$. Then, both the roots of the equation $ax^2 + bx + c = 0$. (1979, 1M) are real and negative have negative real parts have positive real parts None of the above

298. Let a is a real number satisfying $a^3 + \dfrac{1}{a^3} = 18$. Then the value of $a^4 + \dfrac{1}{a^4} - 39$ is ____.

299. If $ax^2 + bx + c = 0, a, b, c \in R$ has no real zeros, and if $c < 0$, then which of the following is true? $a < 0$ $a + b + c > 0$ $a > 0$

300. If $f(x) = \sqrt{x^2 + ax + 4}$ is defined for all x, then find the values of a.

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301. Find the domain and range of $f(x) = \sqrt{x^2 - 4x + 6}$

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302. Find the range of the function $f(x) = 6^x + 3^x + 6^{-x} + 3^{-x} + 2$.

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303. If α, β are the roots of the equation $2x^2 + 2(a + b)x + a^2 + b^2 = 0$ then find the equation whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$

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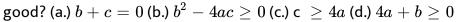**304.** If $a, b, \in R$ such that $a + b = 1$ and $(1 - 2ab)(a^3 + b^3) = 12$. The value of $(a^2 + b^2)$ is equal to____.
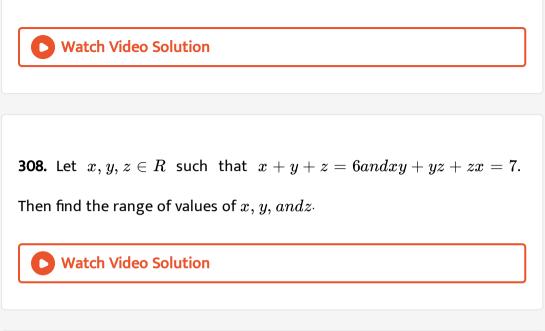
305. Find the range of $f(x) = x^2 - x - 3$.

306. The polynomial $f(x) = x^4 + ax^3 + bx^3 + cx + d$ has real coefficients and $f(2i) = f(2 + i) = 0$. Find the value of $(a + b + c + d)$.

307. If the quadratic equation $ax^2 + bx + c = 0(a > 0)$ has $\sec^2 \theta$ and $\cos ec^2 \theta$ as its roots, then which of the following must hold

good? (a.) $b + c = 0$ (b.) $b^2 - 4ac \geq 0$ (c.) $c \geq 4a$ (d.) $4a + b \geq 0$

308. Let $x, y, z \in R$ such that $x + y + z = 6 and xy + yz + zx = 7$. Then find the range of values of $x, y, and z$.

309. if $ax^2 + bx + c = 0$ has imaginary roots and $a + c < b$ then prove that $4a + c < 2b$

310. Let $a, b, and c$ be distinct nonzero real numbers such that
$$\frac{1 - a^3}{a} = \frac{1 - b^3}{b} = \frac{1 - c^3}{c}.$$ The value of $(a^3 + b^3 + c^3)$ is _____.

311. If the cubic $2x^3 - 9x^2 + 12x + k = 0$ has two equal roots then minimum value of $|k|$ is_____.

312. If the quadratic equation $4x^2 - 2(a + c - 1)x + ac - b = 0(a > b > c)$ (a)Both roots se greater than a (b)Both roots are less than c (c)Both roots lie between $\frac{c}{2}$ and $\frac{a}{2}$ (d)Exactly one of the roots lies between $\frac{c}{2}$ and $\frac{a}{2}$

313. If the equation $x^2 = ax + b = 0$ has distinct real roots and $x^2 + a|x| + b = 0$ has only one real root, then which of the following is true? $b = 0, a > 0$ b. $b = 0, a < 0$ c. $b > 0, a < 0$ d. $b\langle 0, a\rangle 0$

314. If the equation $\left|x^2 + bx + c\right| = k$ has four real roots, then

A. $b^2 - 4c > 0$ and $0 < k < \dfrac{4c - b^2}{4}$

B. $b^2 - 4c < 0$ and $0 < k < \dfrac{4c - b^2}{4}$

C. $b^2 - 4c > 0$ and $k > \dfrac{4c - b^2}{4}$

D. none of these

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315. If P(x) is a polynomial with integer coefficients such that for 4 distinct integers $a, b, c, d, P(a) = P(b) = P(c) = P(d) = 3$, if $P(e) = 5$, (e is an integer) then

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316. Let x, y, z, t be real numbers $x^2 + y^2 = 9, z^2 + t^2 = 4$, and $xt - yz = 6$ Then the greatest value of $P = xz$ is a. 2 b. 3 c. 4 d. 6

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317. If a, b, c are distinct positive numbers, then the nature of roots of the equation $1/(x-a) + 1/(x-b) + 1/(x-c) = 1/x$ is a. all real and is distinct b. all real and at least two are distinct c. at least two real d. all non-real

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318. If $\left(b^2 - 4ac\right)^2 \left(1 + 4a^2\right) < 64a^2$, $a < 0$, then maximum value of quadratic expression $ax^2 + bx + c$ is always less than a. 0 b. 2 c. -1 d. -2

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319. For $x^2 - (a+3)|x| + 4 = 0$ to have real solutions, the range of a is $(-\infty, -7] \cup [1, \infty)$ b. $(-3, \infty)$ c. $(-\infty, -7)$ d. $[1, \infty)$

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320. The number of integral value of x satistying $\sqrt{x^2 + 10x - 16} < x - 2$ is

321. If $x^2 + ax - 3x - (a + 2) = 0$ has real and distinct roots, then minimum value of $(a^2 + 1)/(a^2 + 2)$ is

322. Let $\alpha + i\beta; \alpha, \beta \in R$, be a root of the equation $x^3 + qx + r = 0; q, r \in R$. A real cubic equation, independent of $\alpha \& \beta$, whose one root is 2α is $x^3 + qx - 4 = 0$ (b) $x^3 - qx + 4 = 0$ $x^3 + 2qx + r = 0$ (d) None of these

323. In equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ if two its roots are equal in magnitude but opposite e in find the roots.

324. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then find he value of $\left(\alpha - \dfrac{1}{\beta\gamma}\right)\left(\beta - \dfrac{1}{\gamma\alpha}\right)\left(\gamma - \dfrac{1}{\alpha\beta}\right)$.

325. Equations $x^3 + 5x^2 + px + q = 0 \, and \, x^3 + 7x^2 + px + r = 0$ have two roots in common. If the third root of each equation is $x_1 \, and \, x_2$, respectively, then find the ordered pair (x_1, x_2).

326. If α, β, γ are the roots of he euation $x^3 + 4x + 1 = 0$, then find the value of $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1}$.

327. If the roots of the equation $x^3 + Px^2 + Qx - 19 = 0$ are each one more that the roots of the equation $x^3 - Ax^2 + Bx - C = 0, where A, B, C, P, and Q$ are constants, then find the value of $A + B + C$.

328. If a, b, p, q are nonzero real numbers, then how many cooman roots would two equations $2a^2x^2 - 2abx + b^2 = 0 and p^2x^2 + 2pqx + q^2 = 0$ have?

329. If $x^2 + px + q = 0 and x^2 + qx + p = 0, (p \neq q)$ have a common roots, show that $1 + p + q = 0$. Also, show that their other roots are the roots of the equation $x^2 + x + pq = 0$.

330. a,b,c are positive real numbers forming a G.P. ILf $ax^2 + 2bx + c = 0$ and $x^2 + 2ex + f = 0$ have a common root, then prove that $\dfrac{d}{a}, \dfrac{e}{b}, \dfrac{f}{c}$ are in A.P.

331. If equations $x^2 + ax + 12 = 0. x^2 + bx + 15 = 0 and x^2 + (a + b)x + 36 = 0,$ have a common positive root, then find the values of $a and b$.

332. If x is real and the roots of the equation $ax^2 + bx + c = 0$ are imaginary, then prove tat $a^2x^2 + abx + ac$ is always positive.

333. Solve $\left(x^2 + 2\right)^2 + 8x^2 = 6x\left(x^2 + 2\right)$

334. Find the value of $2 + \dfrac{1}{2 + \dfrac{1}{2 + \frac{1}{2+\infty}}}$

335. If both the roots of $ax^2 + ax + 1 = 0$ are less than 1, then find the exhaustive range of values of a.

336. If both the roots of $x^2 + ax + 2 = 0$ lies in the interval (0, 3), then find the exhaustive range of value of a.

337. Solve $\dfrac{x^2 + 3x + 2}{x^2 - 6x - 7} = 0.$

338. Solve $\sqrt{x - 2} + \sqrt{4 - x} = 2.$

339. Solve $\sqrt{x - 2}\left(x^2 - 4x - 5\right) = 0.$

340. Solve the equation $x(x+2)(x^2-1) = -1$.

341. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is :

342. Prove that graphs of $y = x^2 + 2 and y = 3x - 4$ never intersect.

343. In how many points the line $y + 14 = 0$ cuts the curve whose equation is $-x(x^2 + x + 1) = 0$?

344. If $x^2 + px - 444p = 0$ has integral roots where p is prime number, then find the value $(s) of p$.

345. The equation $ax^2 + bx + c = 0$ has real and positive roots. Prove that the roots of the equation $a^2x^2 + a(3b - 2c)x + (2b - c)(b - c) + ac = 0$ re real and positive.

346. If the roots of the equation $x^2 - ax + b = 0$y are real and differ b a quantity which is less than $c(c > 0)$, then show that b lies between $\frac{a^2 - c^2}{4}$ and $\frac{a^2}{4}$.

347. Show that the minimum value of $(x + a)(x + b)/(x + c)$ where $a > c, b > c$, is $\left(\sqrt{a - c} + \sqrt{b - c}\right)^2$ for real values of $x > $.

348. Let $a, b \in n$ and $a > 1.$ Also p is a prime number. If $ax^2 + bx + c = p$ for any intergral values of $x,$ then prove that $a + bx + c \neq 2p$ for any integral value of $x.$

349. If $2x^2 - 3xy - 2y^2 = 7,$ then prove that there will be only two integral pairs (x, y) satisfying the above relation.

350. If a and c are odd prime numbers and $ax^2 + bx + c = 0$ has rational roots , where $b \in I$, prove that one root of the equation will be independent of a, b, c.

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351. If $f(x) = x^3 + bx^2 + cx + d$ and $f(0)$, $f(-1)$ are odd integers, prove that $f(x) = 0$ cannot have all integral roots.

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352. If x is real, then the maximum value of $y = 2(a - x)\left(x + \sqrt{x^2 + b^2}\right)$

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353. If equation $x^4 - (3m + 2)x^2 + m^2 = 0 (m > 0)$ has four real

solutions which are in A.P., then the value of m is_____.

354. Number of positive integers x for which

$f(x) = x^3 - 8x^2 + 20x - 13$ is a prime number is_____.

355. If set of values a for which $f(x) = ax^2 - (3 + 2a)x + 6, a \neq 0$ is

positive for exactly three distinct negative integral values of x is $(c, d]$,

then the value of $(c^2 + 4|d|)$ is equal to _____.

356. Polynomial $P(x)$ contains only terms of aodd degree. when $P(x)$ is divided by $(x - 3)$, the ramainder is 6. If $P(x)$ is divided by $(x^2 - 9)$ then remainder is $g(x)$. Then find the value of $g(2)$.

357. If the equation $2x^2 + 4xy + 7y^2 - 12x - 2y + t = 0,\ where\ t$ is a parameter has exactly one real solution of hte form (x, y) , then hte sum of $(x + y)$ is equal to _____.

358. Let α_1, β_1 be the roots $x^2 - 6x + p = 0\ and\ \alpha_2, \beta_2$ be the roots $x^2 - 54x + q = 0$. If $\alpha_1, \beta_1, \alpha_2, \beta_2$ form an increasing G.P., then sum of the digits of the value of $(q - p)$ is _____.

359. If $\sqrt{\sqrt{\sqrt{x}}} = x^4 + 4444,$ then the value of x^4 is____.

360. Let $P(x) = x^4 + ax^3 + bx^2 + cx + d$ be a polynomial such that $P(1) = 1, P(2) = 8, + P(3) = 27, P(4) = 64$ then the value of $152 - P(5)$ is_____.

361. If the equation $x^2 + 2(\lambda + 1)x + \lambda^2 + \lambda + 7 = 0$ has only negative roots, then the least value of λ equals_____.

362. Given $\alpha and \beta$ are the roots of the quadratic equation $x^2 - 4x + k = 0 (k \neq 0)$. If $\alpha\beta, \alpha\beta^2 + \alpha^2\beta, \alpha^3 + \beta^3$ are in geometric progression, then the value of $7k/2$ equals_____.

363. If $\left(x^2 + ax + 3\right) / \left(x^2 + x + a\right)$ takes all real values for possible real values of x, then a.$4a^2 + 39 < 0$ b. $4a^5 + 39 \succ 0$ c. $a \geq \dfrac{1}{4}$ d. $a < \dfrac{1}{4}$

364. If $\cos^4 \theta + \alpha$ and $\sin^4 \theta + \alpha$ are the roots of the equation $x^2 + 2bx + b = 0 and \cos^2 \theta + \beta$, $\sin^2 \theta + \beta are$ the roots of the equation $x^2 + 4x + 2 = 0$, then values of b are 2 b. -1 c. -2 d. 2

365. If the roots of the equation $x^2 + ax + b = 0 are c$ and d, then roots of the equation $x^2 + (2c + a)x + c^2 + ac + b = 0$ are a c b. $d - c$ c. $2c$ d. 0

366. If $a, b, c \in R$ and $abc < 0$, then equation $bcx^2 + (2b + c - a)x + a = 0$ has (a). both positive roots (b). both negative roots (c). real roots (d) one positive and one negative root

367. Let $P(x) = x^2 + bx + c$ $where$ b and c are integer. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5,$ $then$ a.$P(x) = 0$ has imaginary roots b.$P(x) = 0$ has roots of opposite c.$P(1) = 4$ d . $P(1) = 6$

368. If $\left|ax^2 + bx + c\right| \leq 1$ for all x in [0, 1], then $|a| \leq 8$ b. $|b| > 8$ c. $|c| \leq 2$ d. $|a| + |b| + |c| \leq 19$

369. Let $f(x) = ax^2 + bx + \cdot$ Consider the following diagram. Then Fig

$c < 0\ b > 0\ a + b - c > 0\ abc < 0$

370. If roots of $ax^2 + bx + c = 0$ are

$\alpha\ and\ \beta\ and\ 4a + 2b + c > 0, 4a, - 2b + c > 0, and\ c < 0,$ then possible

values /values of $[\alpha] + [\beta]$ is/are (where [.] represents greatest integer

function) a.-2 b.-1c. 0d. 1

371. The equation $\left(\dfrac{x}{x+1}\right)^2 + \left(\dfrac{x}{x-1}\right)^2 = a(a-1)$ has a. Four real

roots if $a > 2$ b.Four real roots if $a < -1$ c Two real roots if `1

372. Find the complete set of values of a such that $(x^2 - x)/(1 - ax)$ attains all real values.

373. If α, β are roots of $x^2 + px + 1 = 0$ $and \gamma, \delta$ are the roots of $x^2 + qx + 1 = 0$, then prove that $q^2 - p^2 = (\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$.

374. If he roots of the equation $12x^2 - mx + 5 = 0$ are in the ratio 2:3 then find the value of m.

375. If α and β are the roots of $x^2 - a(x - 1) + b = 0$ then find the value of $1/(\alpha^2 - a\alpha) + 1/(\beta^2 - \beta) + 2/a + b.$

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376. The equation formed by decreasing each root of $ax^2 + bx + c = 0$ by 1 is $2x^2 + 8x + 2 = 0$ then

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377. If the sum of the roots of an equation is 2 and the sum of their cubes is 98, then find the equation.

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378. If x is real and $(x^2 + 2x + c)/(x^2 + 4x + 3c)$ can take all real values, of then show that $0 \leq c \leq 1.$

379. If α, β are the roots of the equation $2x^2 + 2(a+b)x + a^2 + b^2 = 0$, then find the equation whose roots are $(\alpha + \beta)^2 and (\alpha - \beta)^2$

380. If $x^2 + ax + bc = 0 and x^2 + bx + ca = 0 (a \neq b)$ have a common root, then prove that their other roots satisfy the equation $x^2 + cx + ab = 0$.

381. Let α, β are the roots of $x^2 + bx + 1 = 0.$ Then find the equation whose roots are $(\alpha + 1/\beta) and (\beta + 1/\alpha)$.

382. Find the greatest value of a non-negative real number λ for which both the equations $2x^2 + (\lambda - 1)x + 8 = 0$ and $x^2 - 8x + \lambda + 4 = 0$ have real roots.

383. If $a, b, c \in R$ such that $a + b + c = 0$ and $a \neq c$, then prove that the roots of $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ are real and distinct.

384. If the fraction $\dfrac{x^3 + (a - 10)x^2 - x + a - 6}{x^3 + (a - 6)x^2 - x + a - 10}$ reduces to a quotient of two functions, then a equals_____.

385. If the equation $(a - 5)x^2 + 2(a - 10)x + a + 10 = 0$ has roots of opposite sign, then find the value of a.

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386. If $\alpha and \beta$ are the roots of $ax^2 + bx + c = 0 and S_n = \alpha^n + \beta^n$, then $aS_{n+1} + bS_n + cS_{n-1} = 0$ and hence find S_5.

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387. If α is a root of the equation $4x^2 + 2x - 1 = 0$, then prove that $4\alpha^3 - 3\alpha$ is the other root.

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388. If both the roots of $x^2 - ax + a = 0$ are greater than 2, then find the value of a.

389. If $\left(y^2 - 5y + 3\right)\left(x^2 + x + 1\right) < 2x$ for all $x \in R,$ then fin the interval in which y lies.

390. The values of 'a' for which $4^x - (a - 4)2^x + \dfrac{9a}{4} < 0 \, \forall x \in (1, 2)$ is

391. Find the number of positive integral values of k for which $kx^2 + (k - 3)x + 1 < 0$ for atleast one positive x.

392. If $x^2 + 2ax + a < 0 \, \forall x \in [1, 2]$ then find set of all possible values of a

393. Given that a, b, c are distinct real numbers such that expressions $ax^2 + bx + c, bx^2 + cx + a$ and $cx^2 + ax + b$ are always non-negative. Prove that the quantity $\left(a^2 + b^2 + c^2\right) / (ab + bc + ca)$ can never lie in $(-\infty, 1) \cup [4, \infty)$.
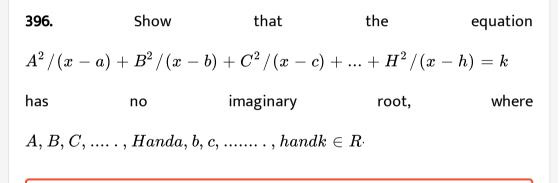
394. Find the number of quadratic equations, which are unchanged by squaring their roots.

395. Solve the following:

$$\left(\sqrt{x^2 - 5x + 6} + \sqrt{x^2 - 5x + 4}\right)^{\frac{x}{2}} + \left(\sqrt{x^2 - 5x + 6}\right)$$

$$\sqrt{x^2 - 5x + 4}\Big)^{x/2} = 2^{\frac{x+4}{4}}$$ -

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396. Show that the equation

$$A^2 / (x - a) + B^2 / (x - b) + C^2 / (x - c) + \dots + H^2 / (x - h) = k$$

has no imaginary root, where

$A, B, C, \dots\dots, H and a, b, c, \dots\dots\dots, h and k \in R.$

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397. Find the values of a if $x^2 - 2(a - 1)x + (2a + 1) = 0$ has positive roots.

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398. If $\alpha and \beta, \alpha and \gamma, \alpha and \delta$ are the roots of the equations $ax^2 + 2bx + c = 0, 2bx^2 + cx + a = 0 and ncx^2 + ax + 2b = 0,$ respectively, where a, b, and c are positive real numbers, then $\alpha + \alpha^2 =$

a.*abc* b. $a + 2b + c$ c. -1 d. 0

399. If the roots of the equation $ax^2 - bx + c = 0 are \alpha, \beta,$ then the roots of the equation $b^2 cx^2 - ab^2 x + a^3 = 0$ are $\dfrac{1}{\alpha^3 + \alpha\beta}, \dfrac{1}{\beta^3 + \alpha\beta}$ b. $\dfrac{1}{\alpha^2 + \alpha\beta}, \dfrac{1}{\beta^2 + \alpha\beta}$ c. $\dfrac{1}{\alpha^4 + \alpha\beta}, \dfrac{1}{\beta^4 + \alpha\beta}$ d. none of these

400. If $a(p + q)^2 + 2bpq + c = 0 and a(p + r)^2 + 2bpr + c = 0 (a \neq 0)$, then $qr = p^2$ b. $qr = p^2 + \dfrac{c}{a}$ c. $qr = p^2$ d. none of these

401. If α_1, α_2 are the roots of equation $x^2 - px + 1 = 0$ and β_1, β_2 are those of equation $x^2 - qx + 1 = 0$ and vector $\alpha_1 \hat{i} + \beta_1 \hat{j}$ is parallel to $\alpha_2 \hat{i} + \beta_2 \hat{j}$, then $p =$ a. $\pm q$ b. $p = \pm 2q$ c. $p = 2q$ d. none of these

402. Suppose A, B, C are defined as $A = a^2 b + ab^2 - a^2 c - ac^2$, $B = b^2 c + bc^2 - a^2 b - ab^2$, and $C = a^2 c + ac^2$ and the equation $Ax^2 + Bx + C = 0$ has equal roots, then a, b, c are in $A \dot{P}$. b. $G \dot{P}$. c. $H \dot{P}$. d. $A \dot{G} \dot{P}$.

403. The integral value of m for which the root of the equation $mx^2 + (2m - 1)x + (m - 2) = 0$ are rational are given by the expression [where n is integer]

(A)n^2

(B) $n(n + 2)$

(C) $n(n + 1)$

(D) none of these

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404. IIf $b_1, b_2 = 2(c_1 + c_2,)$ then at least one of the equation $x^2 + b_1x + c_1 = 0 and x^2 + b_2x + c_2 = 0$ has a. imaginary roots b. real roots c. purely imaginary roots d. none of these

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405. If the root of the equation $(a - 1)(x^2 - x + 1)^2 = (a + 1)(x^4 + x^2 + 1)$ are real and distinct, then the value of $a \in$ a.$(- \infty, 3]$ b. $(- \infty, - 2) \cup (2, \infty)$ c. $[- 2, 2]$ d. $[- 3, \infty)$

406. If α and β are roots of the equation $ax^2 + bx + c = 0$, then the roots of the equation $a(2x + 1)^2 - b(2x + 1)(x - 3) + c(x - 3)^2 = 0$ are $\dfrac{2\alpha + 1}{\alpha - 3}, \dfrac{2\beta + 1}{\beta - 3}$ b. $\dfrac{3\alpha + 1}{\alpha - 2}, \dfrac{3\beta + 1}{\beta - 2}$ c. $\dfrac{2\alpha - 1}{\alpha - 2}, \dfrac{2\beta + 1}{\beta - 2}$ d. none of these

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407. If $a, b, c, d \in R$, then the equation $(x^2 + ax - 3b)(x^2 - cx + b)(x^2 - dx + 2b) = 0$ has a. 6 real roots b. at least 2 real roots c. 4 real roots d. none of these
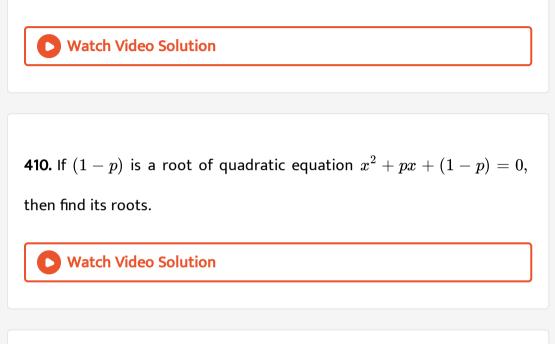
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408. In how many points graph of $y = x^3 - 3x2 + 5x - 3$ interest the x-axis?

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409. The quadratic polynomial $p(x)$ ha following properties $p(x)$ can be positive or zero for all real numbers $p(1) = 0 and p(2) = 2$. Then find the quadratic polynomial.

410. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$, then find its roots.

411. A polynomial in x of degree 3 vanishes when $x = 1$ and $x = -2$, ad has the values 4 and 28 when $x = -1$ and $x = 2$, respectively. Then find the value of polynomial when $x = 0$.

412. Let $f(x) = a^2 + bx + c$ where a ,b , c in R and $\neq 0$. It is known that $f(5) = -3f(2)$ and that 3 is a root of $f(x) = 0$. then find the other of $f(x) = 0$.

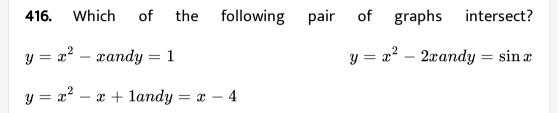**413.** If $x = 1$ and $x = 2$ are solutions of equations $x^3 + ax^2 + bx + c = 0$ and $a + b = 1$, then find the value of b.

414. If $x \in R$, and a, b, c are in ascending or descending order of magnitude, show that $(x-a)(x-c)/(x-b)$ (where $x \neq b$) can assume any real value.

415. Prove that graphs $y = 2x - 3$ and $y = x^2 - x$ never interest.

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416. Which of the following pair of graphs intersect?

$y = x^2 - x$ and $y = 1$

$y = x^2 - 2x$ and $y = \sin x$

$y = x^2 - x + 1$ and $y = x - 4$

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417. If α and β are the rootsof he equations $x^2 - ax + b = 0$ and $A_n = \alpha^n + \beta^n$, then which of the following is true?

a. $A_{n+1} = aA_n + bA_{n-1}$

b. $A_{n+1} = bA_{n-1} + aA_n$

c. $A_{n+1} = aA_n - bA_{n-1}$

d. $A_{n+1} = bA_{n-1} - aA_n$

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418. If α, β are the roots of $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px + r = 0$, then $\dfrac{(\alpha - \gamma)(\alpha - \delta)}{(\beta - \gamma)(\beta - \delta)} =$ a.1 b. q c. r d. $q + r$

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419. If the equations $ax^2 + bx + c = 0$ and $x^3 + 3x^2 + 3x + 2 = 0$ have two common roots, then $a = b = c$ b. $a = b \neq c$ c. $a = -b = c$ d. none of these

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420. The value m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is -2 b. 1 c. 2 d. none of these

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421. Let $p(x) = 0$ be a polynomial equation of the least possible degree, with rational coefficients having $\sqrt[3]{7} + \sqrt[3]{49}$ as one of its roots. Then product of all the roots of $p(x) = 0$ is 56 b. 63 c. 7 d. 49

422. The number of values of a for which equations $x^3 + ax + 1 = 0 \, and \, x^4 + ax^2 + 1 = -$ have a common root is a. 0 b. 1 c. 2 d. Infinite

423. If $(m_r, 1/m_r), r = 1, 2, 3, 4,$ are four pairs of values of $x \, and \, y$ that satisfy the equation $x^2 + y^2 + 2gx + 2fy + c = 0$, then the value of m_1, m_2, m_3, m_4 is 0 b. 1 c. -1 d. none of these

424. If $\alpha, \beta, \gamma, \sigma$ are the roots of the equation $x^4 + 4x^3 - 6x^2 + 7x - 9 = 0$, then he value of $(1 + \alpha^2)(1 + \beta^2)(1 + \gamma^2)(1 + \sigma^2)$ is 9 b. 11 c. 13 d. 5

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425. If $\tan\theta_1, \tan\theta_2, \tan\theta_3$ are the real roots of the $x^3 - (a+1)x^2 + (b-a)x - b = 0, where\theta_1 + \theta_2 + \theta_3 \in (0, \pi)$, then $\theta_1 + \theta_2 + \theta_3$, is equal to $\pi/2$ b. $\pi/4$ c. $3\pi/4$ d. π

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426. If roots of an equation $x^n - 1 = 0are1, a_1, a_2, ... a_{n-1}$, then the value of $(1 - a_1)(1 - a_2)(1 - a_3)(1 - a_{n-1})$ will be n b. n^2 c. n^n d. 0

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427. If α, β are the roots of $ax^2 + bx + c = 0$, $(a \neq 0)$ and $\alpha + \delta$, $\beta + \delta$ are the roots of $Ax^2 + Bx + C = 0$, $(A \neq 0)$ for some constant δ then prove that (2000, 4M) $\dfrac{b^2 - 4ac}{a^2} = \dfrac{B^2 - 4AC}{A^2}$

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428. Let $f(x) = Ax^2 + Bx + c$, $where A, B, C$ are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers $2A$, $A + B$, $and C$ are all integer. Conversely, prove that if the number $2A$, $A + B$, $and C$ are all integers, then $f(x)$ is an integer whenever x is integer.

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429. Let S be a square of nit area. Consider any quadrilateral, which has none vertex on each side of S. If $a, b, cand d$ denote the lengths of the sides of het quadrilateral, prove that $2 \leq a^2 + b^2 + c^2 + x^2 \leq 4$.

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430. The real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + bx + \gamma = 0$ are in A.P. Find the intervals in which β and γ lie.

431. Let a, b, c be real. If $ax^2 + bx + c = 0$ has two real roots α and β, where $\alpha \langle -1$ and $\beta \rangle 1$, then show that $1 + \dfrac{c}{a} + \left| \dfrac{b}{a} \right| < 0$

432. For a $a \leq 0$, determine all real roots of the equation $x^2 - 2a|x - a| - 3a^2 = 0$.

433. Solve for x: $\left(5 + 2\sqrt{6}\right)^{x} \char`\^ (2 - 3) + \left(5 - 2\sqrt{6}\right)^{x} \char`\^ (2 - 3) = 10.$

434. If one root of the equation $ax^2 + bx + c = 0$ is equal to the nth power of the other, then $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b$ is equal to

435. If $a, b, c \in R$ and equations $ax^2 + bx + c = 0 \, and \, x^2 + 2x + 3 = 0$ have a common a rot, then find $a : b : c$

436. Find the condition that the expressions $ax^2 - bxy + cy^2 \, and \, a_1 x^2 + b_1 xy + c_1 y^2$ may have factors $y - mx \, and \, my - x,$ respectively.

437. If $x^2 + (a - b)x = (1 - a - b) = 0$. $where a, b \in R$, then find the values of a for which equation has unequal real roots for all values of b.

438. Let a, b, c be real numbers with $a \neq 0 and let \alpha, \beta$ be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β.

439. If the product of the roots of the equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0 is 2$, then find the sum roots.