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## MATHS

## BOOKS - CENGAGE MATHS (HINGLISH)

## APPLICATIONS OF DERIVATIVES

## Multiple Correct Answer Type

1. Equation of a line which is tangent to both the curve $y=x^{2}+1$ and $y=x^{2} \quad$ is $\quad y=\sqrt{2} x+\frac{1}{2} \quad$ (b) $\quad y=\sqrt{2} x-\frac{1}{2}$
$y=-\sqrt{2} x+\frac{1}{2}$ (d) $y=-\sqrt{2} x-\frac{1}{2}$
A. $y=\sqrt{2} x-\frac{1}{2}$
B. $y=\sqrt{2} x+\frac{1}{2}$
C. $y=-\sqrt{2} x+\frac{1}{2}$
D. $y=-\sqrt{2} x-\frac{1}{2}$

## D Watch Video Solution

2. For the functions defined parametrically by the equations
$f(t)=x=\left\{\begin{array}{ll}2 t+t^{2} \sin . \frac{1}{t} & t \neq 0 \\ 0 & t=0\end{array}\right.$ and
$g(t)=y= \begin{cases}\frac{1}{t} \operatorname{sint}^{2} & t \neq 0 \\ 0 & t=0\end{cases}$
A. equation of tangent at $\mathrm{t}=0$ is $x-2 y=0$
B. equation of normal at $\mathrm{t}=0$ is $2 x+y=0$
C. tangent does not exist at $\mathrm{t}=0$
D. normal does not exist at $t=0$

## Answer: A: B

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3. Prove that the segment of the normal to the curve $x=2 a \sin t+a \sin t \cos ^{2} t ; y=-a \cos ^{3} t$ contained between the coordinate axes is equal to $2 a$.
A. normal is inclined at an angle $\frac{\pi}{2}+t$ with $x$-axis.
B. normal is inclined at an angle $t$ with $x$-axis.
C. portion of normal contained between the co-ordinate axes is equal to 2a.
D. portion of normal containned between the co-ordinate axes is equal to 4 a .

## Answer: A: C

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4. The curve $y=a x^{3}+b x^{2}+c x$ is inclined at $45^{\circ}$ to $x$-axis at $(0,0)$ but it touches $x$-axis at $(1,0)$, then
A. $f^{\prime}(1)=0$
B. $\mathrm{f}^{\prime \prime}(1)=2$
C. $\mathrm{f}^{\prime \prime}(2)=12$
D. $f(2)=2$

## Answer: A::B::D

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5. If $L_{T}, L_{N}, L_{S T}$ and $L_{S N}$ denote the lengths of tangent, normal subtangent and sub-normal, respectively, of a curve $y=f(x)$ at a point $P(2009$, 2010) on it, then
A. $\frac{L_{S T}}{2010}=\frac{2010}{L_{S N}}$
B. $\left|\frac{L_{T}}{L_{N}} \sqrt{\frac{L_{S N}}{L_{S T}}}\right|=\mathrm{constant}$
C. $1-L_{S T} L_{S N}=\frac{2000}{2010}$
D. $\left(\frac{L_{T}+L_{N}}{L_{T}-L_{N}}\right)^{2}=\frac{L_{S T}}{L_{S N}}$

## D View Text Solution

6. Which of the following pair(s) of family is/are orthogonl?
where c and k are arbitrary constant.
A. $16 x^{2}+y^{2}=c$ and $y^{16}=k x$
B. $y=x+c e^{-x}$ and $x+2=y+k e^{-y}$
C. $y=c x^{2}$ and $x^{2}+2 y^{2}=k$
D. $x^{2}-y^{2}=c$ and $x y=k$

## Answer: A::B::C::D

## D Watch Video Solution

7. Let $f(x)=\left|\begin{array}{ccc}1 & 1 & 1 \\ 3-x & 5-3 x^{2} & 3 x^{3}-1 \\ 2 x^{2}-1 & 3 x^{5}-1 & 7 x^{8}-1\end{array}\right|$ then the equation of
$f(x)=0$ has
A. $f(x)=0$ has at least two real roots
B. $f^{\prime}(x)=0$ has at least one real root.
C. $f(x)$ is many-one function
D. none of these

## Answer: A: : $\mathrm{B}:: \mathrm{C}$

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8. Which of the following is correct ?
A. $\frac{\tan ^{-1} x-\tan ^{-1} y}{x-y} \leq 1 \forall x, y \in R,(x \neq y)$
B. $\frac{\sin ^{-1} x-\sin ^{-1} y}{x-y}>1 \forall x, y \in[-1,1], x \neq y$
C. $\frac{\cos ^{-1} x-\cos ^{-1} y}{x-y}<1 \forall x, y \in[-1,1], x \neq y$
D. $\frac{\cot ^{-1} x-\cot ^{-1} y}{x-y}<1 \forall x, y \in R, x \neq y$

## Answer: A::B

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## Comprehension Type

1. A lamp post of length 10 meter placed at the end $A$ of a ladder $A B$ of length 13 meters, which is leaning against a vertical wall as shown in figure and its base slides away from the wall. At the instant base B is 12 m from the vertical wall, the base $B$ is moving at the rate of $5 \mathrm{~m} / \mathrm{sec}$. A man (M) of height 1.5 meter standing at a distance of 15 m from the vertical wall.

Rate at which $\theta$ decreases, when the base B is 12 m from the vertical wall, is
A. $1 \mathrm{rad} / \mathrm{sec}$
B. $2 \mathrm{rad} / \mathrm{sec}$
C. $5 \mathrm{rad} / \mathrm{sec}$
D. 1/2 rad/sec

## Answer: A

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2. A lamp post of length 10 meter placed at the end $A$ of a ladder $A B$ of length 13 meters, which is leaning against a vertical wall as shown in figure and its base slides away from the wall. At the instant base B is 12 m from the vertical wall, the base $B$ is moving at the rate of $5 \mathrm{~m} / \mathrm{sec}$. A man (M) of height 1.5 meter standing at a distance of 15 m from the vertical wall.

The rate at which the length of shadow of man increases, when the base $B$ is 12 m from vertical wall, is

$$
\text { A. } 15 \mathrm{~m} / \mathrm{sec}
$$

B. $40 / 27 \mathrm{~m} / \mathrm{sec}$
C. $15 / 2 \mathrm{~m} / \mathrm{sec}$
D. $5 \mathrm{~m} / \mathrm{sec}$

## Answer: B

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3. Let $f(x)$ be a function such that its derovative $f^{\prime}(x)$ is continuous in $[a, b]$ and differentiable in (a, b). Consider a function $\phi(x)=f(b)-f(x)-(b-x) f^{\prime}(x)-(b-x)^{2} \mathrm{~A}$. If Rolle's theorem is applicable to $\phi(x)$ on, $[\mathrm{a}, \mathrm{b}]$, answer following questions.

If there exists some unmber $c(a$ lt $c$ lt b) such that $\phi^{\prime}(c)=0$ and $f(b)=f(a)+(b-a) f^{\prime}(a)+\lambda(b-a)^{2} f^{\prime \prime}(c)$, then $\lambda$ is
A. 1
B. 0
C. $\frac{1}{2}$
D. $-\frac{1}{2}$

## Answer: C

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4. Let $f(x)$ be a function such that its derovative $f^{\prime}(x)$ is continuous in $[a, b]$ and differentiable in (a, b). Consider a function $\phi(x)=f(b)-f(x)-(b-x) f^{\prime}(x)-(b-x)^{2}$ A. If Rolle's theorem is applicable to $\phi(x)$ on, [a,b], answer following questions.

Let $\quad f(x)=x^{3}-3 x+3, a=1$ and $b=1+h$. If there exists $c \in(1,1+h)$ such that $\phi^{\prime}(c)=0$ and $\frac{f(1+h)-f(1)}{h^{2}}=\lambda c$, then $\lambda=$
A. $1 / 2$
B. 2
C. 3
D. does not exist

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5. Let $f(x)$ be a function such that its derovative $f^{\prime}(x)$ is continuous in $[a, b]$ and differentiable in (a, b). Consider a function $\phi(x)=f(b)-f(x)-(b-x) f^{\prime}(x)-(b-x)^{2}$ A. If Rolle's theorem is applicable to $\phi(x)$ on, [a,b], answer following questions.

Let $f(x)=\sin x, a=\alpha$ and $b=\alpha+h$. If have exists a real number t such that $0<t<1, \phi^{\prime}(\alpha+t h)=0$ and $\frac{\sin (\alpha+h)-\sin \alpha-h \cos \alpha}{h^{2}}=\lambda \sin (\alpha+t h)$, then $\lambda=$
A. $\frac{1}{2}$
B. $-\frac{1}{2}$
C. $\frac{1}{4}$
D. $\frac{1}{3}$

## Answer: B

## Subjective Type

1. Prove that for $\lambda>1$, the equation $x \log x+x=\lambda$ has least one solution in $[1, \lambda]$.

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2. If $f(x)$ and $g(x)$ are continuous and differentiable functions, then prove that there exists $c \in[a, b]$ such that $\frac{f^{\prime}(c)}{f(a)-f(c)}+\frac{g^{\prime}(c)}{g(b)-g(c)}=1$.

## D View Text Solution

## Single Correct Answer Type

1. The equation of the normal to the curve parametrically represented by $x=t^{2}+3 t-8$ and $y=2 t^{2}-2 t-5$ at the point $P(2,-1)$ is
A. $2 x+3 y-1=0$
B. $6 x-7 y-11=0$
C. $7 x+6 y-8=0$
D. $3 x+y-1=0$

## Answer: C

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2. In the curve $y=x^{3}+a x$ and $y=b x^{2}+c$ pass through the point ( $-1,0$ ) and have a common tangent line at this point then the value of $a+b+c$ is
A. 0
B. 1
C. -3
D. -1

## Answer: D

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3. If the function $f(x)=x^{4}+b x^{2}+8 x+1$ has a horizontal tangent and a point of inflection for the same value of $x$ then the value of $b$ is equal to -1 (b) 1 (c) 6 (d) -6
A. -2
B. -6
C. 6
D. 3

## Answer: B

4. Let $f(x)=x^{3}+x+1$ and let $\mathrm{g}(\mathrm{x})$ be its inverse function then equation of the tangent to $y=g(x)$ at $\mathrm{x}=3$ is
A. $x-4 y+1=0$
B. $x+4 y-1=0$
C. $4 x-y+1=0$
D. $4 x+y-1=0$

## Answer: A

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5. A curve is represented parametrically by the equations $x=t+e^{a t}$ and $y=-t+e^{a t}$ when $t \in R$ and $a>0$. If the curve touches the axis of $x$ at the point $A$, then the coordinates of the point $A$ are
A. $(1,0)$
B. $(2 e, 0)$
C. $(e, 0)$
D. $(1 / e, 0)$

## Answer: B

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6. The equation of the straight lines which are both tangent and normal to the curve $27 x^{2}=4 y^{3}$ are
A. $x= \pm \sqrt{2}(y-2)$
B. $x= \pm \sqrt{3}(y-2)$
C. $x= \pm \sqrt{2}(y-3)$
D. $x= \pm \sqrt{3}(y-3)$

## Answer: A

7. If the tangent at $(1,1)$ on $y^{2}=x(2-x)^{2}$ meets the curve again at $P$, then find coordinates of $P$
A. $(4,4)$
B. $(2,0)$
C. $\left(\frac{9}{4}, \frac{3}{8}\right)$
D. $\left(3,3^{1 / 2}\right)$

## Answer: C

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8. A curve with equation of the form $y=a x^{4}+b x^{3}+c x+d$ has zero gradient at the point $(0,1)$ and also touches the $x-$ axis at the point $(-1,0)$ then the value of $x$ for which the curve has a negative gradient are:
A. $x>-1$
B. $x>1$
C. $x<-1$
D. $-1 \leq x \leq 1$

## Answer: C

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9. Find Distance between the points for which lines that pass through the point $(1,1)$ and are tangent to the curve represent parametrically as $x=2 t-t^{2}$ and $y=t+t^{2}$
A. $\frac{2 \sqrt{43}}{9}$
B. 2
C. 3
D. $\frac{2 \sqrt{53}}{9}$

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10. The value of parameter $t$ so that the line $(4-t) x+t y+\left(a^{3}-1\right)=0$ is normal to the curve $\mathrm{xy}=1$ may lie in the interval
A. $(1,4)$
B. $(-\infty, 0) \cup(4, \infty)$
C. $(-4,4)$
D. $[3,4]$

## Answer: B

11. The tangent at any point on the curve $x=a t^{3} . y=a t^{4}$ divides the abscissa of the point of contact in the ratio $m: n$, then $|n+m|$ is equal to ( m and n are co-prime)
A. $1 / 4$
B. $3 / 4$
C. $3 / 2$
D. $2 / 5$

## Answer: B

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12. The length of the sub-tangent to the hyperbola $x^{2}-4 y^{2}=4$ corresponding to the normal having slope unity is $\frac{1}{\sqrt{k}}$, then the value of $k$ is
A. 1
B. 2
C. 3
D. 4

## Answer: C

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13. Cosine of the acute angle between the curve $y=3^{x-1} \log _{e} x$ and $y=x^{x}-1$, at the point of intersection $(1,0)$ is
A. 0
B. 1
C. $\frac{\sqrt{3}}{2}$
D. $\frac{1}{2}$

## Answer: B

14. Acute angle between two curve $x^{2}+y^{2}=a^{2} \sqrt{2}$ and $x^{2}-y^{2}=a^{2}$ is
A. $\frac{\pi}{6}$
B. $\frac{\pi}{3}$
C. $\frac{\pi}{4}$
D. none of these

## Answer: C

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15. The minimum distance between a point on the curve $y=e^{x}$ and a point on the curve $y=\log _{e} x$ is
A. $\frac{1}{\sqrt{2}}$
B. $\sqrt{2}$
C. 3
D. $2 \sqrt{2}$

## Answer: B

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16. Tangents are drawn from origin to the curve $y=\sin +\cos x \hat{A} \cdot$ Then their points of contact lie on the curve
A. $\frac{1}{x^{2}}+\frac{2}{y^{2}}=1$
B. $\frac{2}{x^{2}}-\frac{1}{y^{2}}=1$
C. $\frac{2}{x^{2}}+\frac{1}{y^{2}}=1$
D. $\frac{2}{y^{2}}-\frac{1}{x^{2}}=1$

Answer: D

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17. If $3 x+2 y=1$ is a tangent to $y=f(x)$ at $x=1 / 2$, then $\lim _{x \rightarrow 0} \frac{x(x-1)}{f\left(\frac{e^{2 x}}{2}\right)-f\left(\frac{e^{-2 x}}{2}\right)}$
A. $1 / 3$
B. $1 / 2$
C. $1 / 6$
D. $1 / 7$

## Answer: A

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18. Distance of point P on the curve $y=x^{3 / 2}$ which is nearest to the point $M(4,0)$ from origin is
A. $\sqrt{\frac{112}{27}}$
B. $\sqrt{\frac{100}{27}}$
C. $\sqrt{\frac{101}{9}}$
D. $\sqrt{\frac{112}{9}}$

## Answer: A

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19. If the equation of the normal to the curve $y=f(x) a t x=0$ is $3 x-y+3=0$ then the value of
$\lim _{x \rightarrow 0} \frac{x^{2}}{\left\{f\left(x^{2}\right)-5 f\left(4 x^{2}\right)+4 f\left(7 x^{2}\right)\right\}}$ is
A. -3
B. $1 / 3$
C. 3
D. $-1 / 3$

## Answer: D

20. The rate of change of $\sqrt{x^{2}+16}$ with respect to $\frac{x}{x-1}$ at $x=3$ is
A. 1
B. $\frac{11}{5}$
C. $-\frac{12}{5}$
D. -3

## Answer: C

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21. The eccentricity of the ellipse $3 x^{2}+4 y^{2}=12$ is decreasing at the rate of 0.1 per sec.The time at which it will coincide with auxiliary circle is:
A. 2 seconds
B. 3 seconds
C. 5 seconds
D. 6 seconds

## Answer: C

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22. A particle moves along the parabola $y=x^{2}$ in the first quadrant in such a way that its $x$-coordinate (measured in metres) increases at a rate of $10 \mathrm{~m} / \mathrm{sec}$. If the angle of inclination $\theta$ of the line joining the particle to the origin change, when $x=3 \mathrm{~m}$, at the rate of $k \mathrm{rad} / \mathrm{sec}$., then the value of $k$ is
A. 1
B. 2
C. $1 / 2$
D. $1 / 3$

## Answer: A

23. The rate of change of volume of a sphere is equal to the rate of change of its radius, then its radius is equal to (a) 1 unit (b) units (c) unit (d) unit
A. 1
B. 2
C. 0.5
D. none of these

## Answer: B

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24. Water is dropped at the rate of $2 \mathrm{~m}^{2} / \mathrm{s}$ into a cone of semivertical angel of $45^{\circ}$. The rate at which periphery of water surface changes when height of water in the cone is 2 m , is
A. $0.5 \mathrm{~m} / \mathrm{s}$
B. $2 m / s$
C. $3 m / s$
D. $1 \mathrm{~m} / \mathrm{s}$

## Answer: D

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25. Suppose that water is emptied from a spherical tank of radius 10 cm . If the depth of the water in the tank is 4 cm and is decreasing at the rate of $2 \mathrm{~cm} / \mathrm{sec}$, then the radius of the top surface of water is decreasing at the rate of
A. 1
B. $2 / 3$
C. $3 / 2$
D. 2

## Answer: C

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26. The altitude of a cone is 20 cm and its semi-vertical angle is $30^{\circ}$. If the semi-vertical angle is increasing at the rate of $2^{\circ}$ per second, then the radius of the base is increasing at the rate of
A. $30 \mathrm{~cm} / \mathrm{sec}$
B. $\frac{160}{3} \mathrm{~cm} / \mathrm{sec}$
C. $10 \mathrm{~cm} / \mathrm{sec}$
D. $160 \mathrm{~cm} / \mathrm{sec}$

## Answer: B

27. Let the equation of a curve be $x=a(\theta+\sin \theta), y=a(1-\cos \theta)$. If $\theta$ changes at a constant rate $k$ then the rate of change of the slope of the tangent to the curve at $\theta=\frac{\pi}{3}$ is (a) $\frac{2 k}{\sqrt{3}}$ (b) $\frac{k}{\sqrt{3}}$ (c) k (d) none of these
A. $2 k / \sqrt{3}$
B. $k / \sqrt{3}$
C. k
D. none of these

## Answer: D

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28. 

$f(x)=|1-x|, 1 \leq x \leq 2$ and $g(x)=f(x)+b \sin . \frac{\pi}{2} x, 1 \leq x \leq 2$ then which of the following is correct?
A. Rolle's theorem is applicable to both f and g with $b=\frac{3}{2}$.
B. LMVT is not applicable to $f$ and Rolle's theorem is applicable to $g$ with $b=\frac{1}{2}$
C. LMVT is applicable to $f$ and Rolle's theorem is applicable to $g$ with $b$ $=1$.
D. Rolle's theorem is not applicable to both $f$ and $g$ for any real $b$.

## Answer: C

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29. If $c=\frac{1}{2}$ and $f(x)=2 x-x^{2}$, then interval of x in which LMVT is applicable, is
A. $(1,2)$
B. $(-1,1)$
C. $(0,1)$
D. $(2,1)$

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30. If a twice differentiable function $\mathrm{f}(\mathrm{x})$ on $(a, b)$ and continuous on $[\mathrm{a}, \mathrm{b}]$ is such that $f^{\prime \prime}(x)<0$ for all $x \in(a, b)$ then for any
$c \in(a, b), \frac{f(c)-f(a)}{f(b)-f(c)}>$
A. $\frac{b-c}{c-a}$
B. $\frac{c-a}{b-c}$
C. $(b-c)(c-a)$
D. $\frac{1}{(b-c)(c-a)}$

## Answer: B

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31. Let $a, n \in N$ such that $a \geq n^{3}$. Then $\sqrt[3]{a+1}-\sqrt[3]{a}$ is always
A. less than $\frac{1}{3 n^{2}}$
B. less than $\frac{1}{2 n^{3}}$
C. more than $\frac{1}{n^{3}}$
D. more than $\frac{1}{4 n^{2}}$

## Answer: A

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32. Given $f^{\prime}(1)=1$ and $\left.\frac{d}{d x} f(2 x)\right)=f^{\prime}(x) \forall x>0$. If $f^{\prime}(x)$ is differentiable then there exists a numberd $x \in(2,4)$ such that $f^{\prime \prime}(c)$ equals
A. $1 / 4$
B. $-1 / 2$
C. $-1 / 4$
D. $-1 / 8$

## Answer: D

## D Watch Video Solution

## Question Bank

1. If the slope of the tangent to the curve $2\left(x-y^{3}\right)^{4}=x^{2}\left(1+x^{3}\right)^{5}$ at the point $(1,-1)$ is $t$, then find the value of $[|t|]$.

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2. Let $P\left(x_{0}, y_{0}\right)$ be a point on the curve $C:\left(x^{2}-11\right)(y+1)+4=0$ where $_{0}, y_{0} \in N$. If area of the triangle formed by the normal drawn to the curve ' C ' at P and the coordinate axes is ( $\mathrm{a} / \mathrm{b}$ ), $a b \in N$ then find the least value of ( $\mathrm{a}-\mathrm{bb}$ )

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3. If the function $k(x)=\log _{e}\left(\frac{x^{2}-x+1}{x^{2}+x+1}\right.$ is strictly decreasing in $\times \mathrm{x}$ in $(-$ $t / 7, t / 7)$, then find the greatest integral value of $t$.

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4. The line $\mathrm{y}=\mathrm{x}$ is a tangent to the curve $y=p x^{2}+q x+r$ at the point x $=1$. If the curve passes through the point $(-1,0)$ then the value of $(p+r) / q$ is

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5. Let $f(x)=\left\{\begin{array}{ll}x+1 & x<1 \\ \lambda & x=1 \\ x^{2}-x+3 & x>1\end{array}\right.$ be a strictly increasing function at x $=1$, then the number of integers in the range of $\lambda$, is

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6. If the tangent at a point $P$ on the curve ${ }^{\wedge} \wedge(7) \cdot y^{\wedge}(2)=\operatorname{sqrt7}+2^{\wedge}(1 / 7)$ meets the co-ordinates axes $A$ and $B$ respectively then $2((B P) /(A P))$ is

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7. If $f(x)=2 x^{3}+9 x^{2}+p x+20$ is an increasing function of x in the largest interval $(-1,4)$ then $p$ is equal to

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8. Let $f:[1,3[\rightarrow[0, \infty)$ be continuous and differentiable function and if $(f(3)-f(1)) \cdot\left(f^{2}(3)+f^{2}(1)+f(3) f(1)=k f^{2}(c) f^{\prime}(c)\right.$ wherec $\in(1,3)$ , then find the value of $k$.

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9. If Rolle's theorem is applicable on the function $g(x)= \begin{cases}\frac{e^{p x}-q x-1}{x} & 0<x \leq 1 \\ 2 & x=0\end{cases}$ in interval
$x \in[0,1]$, then $f \in$ dthevalueof $\left(\mathrm{p}^{\wedge}(2)+\mathrm{q}^{\wedge}(2)\right){ }^{\prime}$.

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10. Given the function $f(x)=2 x^{2}-4 x-5$. If $x_{1}$ and $\mathrm{x}_{-}(2)\left(\mathrm{x}_{-}(1)\right.$ gt $\left.\mathrm{x}_{-}(2)\right)$ arethe $|c|$ issaeofp $\oint s o n \$ f(x) \$$ sucht $\hat{t} h e \tan \geq n t s d r a w n a t t h e m p a s s t h r o u$ (3x_(1)-2x_(2))' equals

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11. Let $y=f(x)$ be an invertible function such that $x$-intercept of the tangent at any point $P(x, y)$ on $y=f(x)$ is equal to the square of abscissa of the point of tangency. If $\mathrm{f}(2)=1$, then $f^{-1}\left(\frac{5}{8}\right)$ equals
12. Let $f(x)$ be the curve passing through $(-2,1)$ such that slope of the normal line at the point $\$(\mathrm{x}, \mathrm{y}) \$$ on the curve is equal to $x^{2} y$. The area bounded by the curve $y=x f^{2}(x)$ and coordinates axes, is

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13. If $\mathrm{y}=4 \mathrm{x}-5$ is a tangent to the curve $C: y^{2}=p x^{3}+q$ at $\mathrm{M}(2,3)$ then the value of $(p-q)$ is

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14. $k$ is the least positive integer for which the function $f(x)=(2 x+1)^{50}(3 x-4)^{60}$ is increasing in $[\mathrm{k}$, oo ). The value of ' k ' is

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15. Number of integral values of $a$ for which the function $f(x)=\left(\frac{4 a-7}{3}\right) x^{3}+(a-3) x^{2}+x+5$ is monotonic for every $x \in R$, is

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16. Let $f(x)$ be a cubic polynomial which has local maximum at $x=-1$ and $f^{\prime}(x)$ has a local minimum at $\mathrm{x}=1$. If $\mathrm{f}(-1)=10$ and $\mathrm{f}(3)=-22$, then find the distance between its two horizontal tangents.

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17. Let $f^{\prime}(x)=e^{x^{2}}$ and $\mathrm{f}(0)=10$. If A

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18. Minimum distance between the curves $f(x)=e^{x}$ and $\mathrm{g}(\mathrm{xO}=\ln \mathrm{x}$ is
19. $f(x)$ is a polynomial of degree 6 which decreases in the interval $(0, \infty)$ and increases in the interval $(-\infty, 0)$. If $f^{\prime}(2)=0, f^{\prime}(0)=0, f(0)=0, f(0)=1$ and $f(1)-f(-1)=8 / 5^{\prime}$, then $-3(f(1)+f(-1))$ equals

## - View Text Solution

20. The least positive integral value of $\lambda$ for which $f(x)=\frac{3 x^{3}}{2}+\frac{\lambda x^{2}}{3}+x+7$ has a point of maxima is

## - View Text Solution

21. Minimum positive integral value of : $k$ for which $f(x)=k \cos 2 x-4 \cos ^{3} x$ has exactly one critical point in ( $0, \mathrm{pi}$ ), is
22. Let the radius and height of right circular cylinder is related as $r^{2}+h=5$. Let $\lambda$ is maximum volume. Then $\frac{16 \lambda}{25 \pi}$, is

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23. A solid box is formed by placing a cylinder, having equal height and radius on top of a cube such that, the circular base of cylinder is the inscribed circle for square top of the cube. If the radius of cytinder is changing at the rate $\frac{1}{2 \pi+16} c \frac{m}{s}$, then the rate of change of volume of the box when radius is 2 cm , is (Assuming that box always remain in the given shape)

## - View Text Solution

24. On the curve $x^{m+n} . y^{n}=a, m n \in N, a \in R^{+}$, if the ratio of slopes of tangent at any point \$P\$ and that of line segment OP( O being origin) is -4 , then the value of $m / n$ is
25. If $a \in\left(a_{1}, a_{2}\right)$ is the complete set satisfying the condition that the. point of local minima and the point of local maxima is less than 4 and greater than -2 , respectively for the function $f(x)=x^{3}-3 a x^{2}+3\left(a^{2}-1\right) x+1$, then $\left(a_{2}-a_{1}\right)$ is

## - View Text Solution

26. x and y are sides of two squares sach that $y=x-x^{2}$. Let $\mathrm{f}(\mathrm{x})$ denote the rate of change of area of the second square with respect to the area of the first'square, then $f(2)$ is

## D View Text Solution

27. If the tangent at a point $P$ on the curve ${ }^{\wedge} x^{\wedge}(7) \cdot y^{\wedge}(2)=\operatorname{sqrt7}+2^{\wedge}(1 / 7)$ meets the co-ordinates axes $A$ and $B$ respectively then $2((B P) /(A P))$ is
28. Number of solutions of the equation $(x-k) e^{-x}=\frac{1}{e^{2}}$, where $k<1$ is

## - View Text Solution

29. The total number of local maxima and local minima of the function $f(x)=\left(\frac{2-x}{\pi}\right) \cos (\pi x+3 \pi)+\frac{1}{\pi} \sin (\pi x+3 \pi)$, where $0<x<x$ is equal is
