

MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

APPLICATIONS OF DERIVATIVES

Multiple Correct Answer Type

1. Equation of a line which is tangent to both the curve $y = x^2 + 1 \text{ and } y = x^2$ is $y = \sqrt{2}x + \frac{1}{2}$ (b) $y = \sqrt{2}x - \frac{1}{2}$ $y = -\sqrt{2}x + \frac{1}{2}$ (d) $y = -\sqrt{2}x - \frac{1}{2}$ A. $y = \sqrt{2}x - \frac{1}{2}$ B. $y = \sqrt{2}x + \frac{1}{2}$ C. $y = -\sqrt{2}x + \frac{1}{2}$ D. $y = -\sqrt{2}x - \frac{1}{2}$

Answer: B::C::D



2. For the functions defined parametrically by the equations

$$f(t) = x = egin{cases} 2t + t^2 \sin rac{1}{t} & t
eq 0 \ 0 & t = 0 \ \end{bmatrix}$$
 and $g(t) = y = egin{cases} rac{1}{t} \sin t^2 & t
eq 0 \ 0 & t = 0 \ \end{bmatrix}$

A. equation of tangent at t = 0 is x-2y=0

B. equation of normal at t = 0 is 2x + y = 0

C. tangent does not exist at t = 0

D. normal does not exist at t=0

Answer: A::B

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3. Prove that the segment of the normal to the curve $x = 2a \sin t + a \sin t \cos^2 t$; $y = -a \cos^3 t$ contained between the co-ordinate axes is equal to 2a.

A. normal is inclined at an angle $rac{\pi}{2}+t$ with x-axis.

B. normal is inclined at an angle t with x-axis.

C. portion of normal contained between the co-ordinate axes is equal

to 2a.

D. portion of normal containned between the co-ordinate axes is equal to 4a.

Answer: A::C

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4. The curve $y = ax^3 + bx^2 + cx$ is inclined at 45° to x-axis at (0,0) but

it touches x-axis at (1, 0), then

A. f'(1) = 0 B. f''(1) = 2 C. f'''(2) = 12 D. f(2) = 2

Answer: A::B::D

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5. If $L_T \, L_N \, L_{ST}$ and L_{SN} denote the lengths of tangent, normal subtangent and sub-normal, respectively, of a curve y = f(x) at a point P(2009, 2010) on it, then

A.
$$rac{L_{ST}}{2010} = rac{2010}{L_{SN}}$$

B. $\left| rac{L_T}{L_N} \sqrt{rac{L_{SN}}{L_{ST}}} \right| = ext{constant}$
C. $1 - L_{ST}L_{SN} = rac{2000}{2010}$
D. $\left(rac{L_T + L_N}{L_T - L_N}
ight)^2 = rac{L_{ST}}{L_{SN}}$

Answer: A::B



6. Which of the following pair(s) of family is/are orthogonl? where c and k are arbitrary constant.

A.
$$16x^{2} + y^{2} = c$$
 and $y^{16} = kx$
B. $y = x + ce^{-x}$ and $x + 2 = y + ke^{-y}$
C. $y = cx^{2}$ and $x^{2} + 2y^{2} = k$
D. $x^{2} - y^{2} = c$ and $xy = k$

Answer: A::B::C::D



7. Let
$$f(x)=egin{bmatrix} 1&1&1\ 3-x&5-3x^2&3x^3-1\ 2x^2-1&3x^5-1&7x^8-1 \end{bmatrix}$$
 then the equation of

f(x)=0 has

A. f(x) = 0 has at least two real roots

B. f'(x) =0 has at least one real root.

C. f(x) is many-one function

D. none of these

Answer: A::B::C

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$$\begin{array}{l} \mathsf{A}.\, \frac{\tan^{-1}x - \tan^{-1}y}{x - y} \leq 1\,\forall x, y \in R, \, (x \neq y) \\ \mathsf{B}.\, \frac{\sin^{-1}x - \sin^{-1}y}{x - y} > 1\,\forall x, y \in [\,-1,1], x \neq y \\ \mathsf{C}.\, \frac{\cos^{-1}x - \cos^{-1}y}{x - y} < 1\,\forall x, y \in [\,-1,1], x \neq y \end{array}$$

D.
$$rac{\cot^{-1}x-\cot^{-1}y}{x-y} < 1 \, orall x, y \in R, x
eq y$$

Answer: A::B

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Comprehension Type

1. A lamp post of length 10 meter placed at the end A of a ladder AB of length 13 meters, which is leaning against a vertical wall as shown in figure and its base slides away from the wall. At the instant base B is 12 m from the vertical wall, the base B is moving at the rate of 5 m/sec. A man (M) of height 1.5 meter standing at a distance of 15 m from the vertical wall.

Rate at which θ decreases, when the base B is 12 m from the vertical wall, is

A. 1 rad/sec

B. 2 rad/sec

C. 5 rad/sec

D. 1/2 rad/sec

Answer: A

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2. A lamp post of length 10 meter placed at the end A of a ladder AB of length 13 meters, which is leaning against a vertical wall as shown in figure and its base slides away from the wall. At the instant base B is 12 m from the vertical wall, the base B is moving at the rate of 5 m/sec. A man (M) of height 1.5 meter standing at a distance of 15 m from the vertical wall.

The rate at which the length of shadow of man increases, when the base B is 12 m from vertical wall, is

A. 15 m/sec

B. 40/27 m/sec

C. 15/2 m/sec

D. 5 m/sec

Answer: B

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3. Let f(x) be a function such that its derovative f'(x) is continuous in [a, b] and differentiable in (a, b). Consider a function $\phi(x) = f(b) - f(x) - (b - x)f'(x) - (b - x)^2 A$. If Rolle's theorem is applicable to $\phi(x)$ on, [a,b], answer following questions.

If there exists some unmber c(a lt c lt b) such that $\phi'(c)=0$ and $f(b)=f(a)+(b-a)f'(a)+\lambda(b-a)^2f''(c)$, then λ is

A. 1

B. 0

C.
$$\frac{1}{2}$$

D. $-\frac{1}{2}$

Answer: C

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4. Let f(x) be a function such that its derovative f'(x) is continuous in [a, b] and differentiable in (a, b). Consider function а $\phi(x)=f(b)-f(x)-(b-x)f'(x)-(b-x)^2$ A. If Rolle's theorem is applicable to $\phi(x)$ on, [a,b], answer following questions. $f(x) = x^3 - 3x + 3, a = 1 \, \, {
m and} \, \, b = 1 + h.$ If there Let exists $c\in (1,1+h)$ such that $\phi'(c)=0$ and $rac{f(1+h)-f(1)}{h^2}=\lambda c, ext{ then }\lambda$ = A. 1/2B. 2

C. 3

D. does not exist

Answer: C

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5. Let f(x) be a function such that its derovative f'(x) is continuous in [a, b] and differentiable b). Consider function in (a, а $\phi(x)=f(b)-f(x)-(b-x)f'(x)-(b-x)^2$ A. If Rolle's theorem is applicable to $\phi(x)$ on, [a,b], answer following questions. Let $f(x) = \sin x, a = \alpha$ and $b = \alpha + h$. If have exists a real number t $0 < t < 1, \phi'(\alpha + th) = 0$ such that and $rac{\sin(lpha+h)-\sinlpha-h\coslpha}{h^2}=\lambda\sin(lpha+th), ext{ then }\lambda=$ A. $\frac{1}{2}$ $\mathsf{B.}-\frac{1}{2}$ C. $\frac{1}{4}$ D. $\frac{1}{3}$

Answer: B



Subjective Type

1. Prove that for $\lambda>1$, the equation $x\log x+x=\lambda$ has least one solution in $[1,\lambda].$



2. If f(x) and g(x) are continuous and differentiable functions, then prove that there exists $c \in [a, b]$ such that $\frac{f'(c)}{f(a) - f(c)} + \frac{g'(c)}{g(b) - g(c)} = 1.$

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Single Correct Answer Type

1. The equation of the normal to the curve parametrically represented by $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$ at the point P(2, -1) is A. 2x + 3y - 1 = 0B. 6x - 7y - 11 = 0C. 7x + 6y - 8 = 0D. 3x + y - 1 = 0

Answer: C

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2. In the curve $y = x^3 + ax$ and $y = bx^2 + c$ pass through the point (-1, 0) and have a common tangent line at this point then the value of a + b + c is

A. 0

B. 1

C. - 3

 $\mathsf{D.}-1$

Answer: D

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3. If the function $f(x) = x^4 + bx^2 + 8x + 1$ has a horizontal tangent and a point of inflection for the same value of x then the value of b is equal to -1 (b) 1 (c) 6 (d) -6

A. - 2

- B.-6
- C. 6

D. 3

Answer: B

4. Let $f(x) = x^3 + x + 1$ and let g(x) be its inverse function then equation of the tangent to y = g(x) at x = 3 is

A.
$$x-4y+1=0$$

B. x + 4y - 1 = 0

C. 4x - y + 1 = 0

D.
$$4x+y-1=0$$

Answer: A

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5. A curve is represented parametrically by the equations $x = t + e^{at}$ and $y = -t + e^{at}$ when $t \in R$ and a > 0. If the curve touches the axis of x at the point A, then the coordinates of the point A are

A. (1, 0)

B.(2e, 0)

 $\mathsf{C}.\,(e,0)$

D. (1/e, 0)

Answer: B

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6. The equation of the straight lines which are both tangent and normal

to the curve $27x^2=4y^3$ are

A.
$$x=~\pm\sqrt{2}(y-2)$$

B.
$$x=\pm\sqrt{3}(y-2)$$

C.
$$x=\pm\sqrt{2}(y-3)$$

D.
$$x=\pm\sqrt{3}(y-3)$$

Answer: A

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7. If the tangent at (1,1) on $y^2 = x(2-x)^2$ meets the curve again at P, then find coordinates of P

A. (4, 4)

B. (2, 0)

$$\begin{array}{l}\mathsf{C}.\left(\frac{9}{4},\frac{3}{8}\right)\\\\\mathsf{D}.\left(3,3^{1/2}\right)\end{array}$$

Answer: C

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8. A curve with equation of the form $y = ax^4 + bx^3 + cx + d$ has zero gradient at the point (0, 1) and also touches the x – axis at the point (-1, 0) then the value of x for which the curve has a negative gradient are:

A. x > -1B. x > 1C. x < -1D. $-1 \le x \le 1$

Answer: C

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9. Find Distance between the points for which lines that pass through the point (1,1) and are tangent to the curve represent parametrically as $x=2t-t^2$ and $y=t+t^2$

A.
$$\frac{2\sqrt{43}}{9}$$

B. 2

C. 3



Answer: D



10. The value of parameter t so that the line $(4-t)x + ty + \left(a^3 - 1\right) = 0$ is normal to the curve xy = 1 may lie in the interval

A. (1, 4)B. $(-\infty, 0) \cup (4, \infty)$ C. (-4, 4)

 $\mathsf{D}.\left[3,4\right]$

Answer: B

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11. The tangent at any point on the curve $x = at^3$. $y = at^4$ divides the abscissa of the point of contact in the ratio m:n, then |n + m| is equal to (m and n are co-prime)

A. 1/4

B. 3/4

C.3/2

D. 2/5

Answer: B

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12. The length of the sub-tangent to the hyperbola $x^2 - 4y^2 = 4$ corresponding to the normal having slope unity is $\frac{1}{\sqrt{k}}$, then the value

of k is

C. 3

D. 4

Answer: C

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13. Cosine of the acute angle between the curve $y = 3^{x-1}\log_e x$ and $y = x^x - 1$, at the point of intersection (1,0) is

A. 0

B. 1

$$\mathsf{C}.\,\frac{\sqrt{3}}{2}$$

 $D.\frac{1}{2}$

Answer: B

14. Acute angle between two curve $x^2+y^2=a^2\sqrt{2}$ and $x^2-y^2=a^2$ is

A.
$$\frac{\pi}{6}$$

B. $\frac{\pi}{3}$
C. $\frac{\pi}{4}$

D. none of these

Answer: C

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15. The minimum distance between a point on the curve $y=e^x$ and a point on the curve $y=\log_e x$ is

A.
$$\frac{1}{\sqrt{2}}$$

B. $\sqrt{2}$

C. 3

D. $2\sqrt{2}$

Answer: B



16. Tangents are drawn from origin to the curve $y = \sin + \cos x \hat{\mathsf{A}}$. Then their points of contact lie on the curve

A.
$$\frac{1}{x^2} + \frac{2}{y^2} = 1$$

B. $\frac{2}{x^2} - \frac{1}{y^2} = 1$
C. $\frac{2}{x^2} + \frac{1}{y^2} = 1$
D. $\frac{2}{y^2} - \frac{1}{x^2} = 1$

Answer: D

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17. If 3x + 2y = 1 is a tangent to y = f(x) at x = 1/2, then $\lim_{x \to 0} \frac{x(x-1)}{f(\frac{e^{2x}}{2}) - f(\frac{e^{-2x}}{2})}$ A. 1/3B. 1/2C. 1/6

Answer: A

D.1/7

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18. Distance of point P on the curve $y = x^{3/2}$ which is nearest to the

point M (4, 0) from origin is

A.
$$\sqrt{\frac{112}{27}}$$

B. $\sqrt{\frac{100}{27}}$

C.
$$\sqrt{\frac{101}{9}}$$

D. $\sqrt{\frac{112}{9}}$

Answer: A

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19. If the equation of the normal to the curve y = f(x)atx = 0 is 3x - y + 3 = 0 then the value of $\lim_{x \to 0} \frac{x^2}{\{f(x^2) - 5f(4x^2) + 4f(7x^2)\}}$ is A. -3 B. 1/3 C. 3 D. -1/3

Answer: D

20. The rate of change of $\sqrt{x^2+16}$ with respect to $rac{x}{x-1}$ at x=3 is

B.
$$\frac{11}{5}$$

C. $-\frac{12}{5}$
D. -3

A. 1

Answer: C



21. The eccentricity of the ellipse $3x^2 + 4y^2 = 12$ is decreasing at the rate

of 0.1 per sec.The time at which it will coincide with auxiliary circle is:

A. 2 seconds

B. 3 seconds

C. 5 seconds

Answer: C

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22. A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x-coordinate (measured in metres) increases at a rate of 10 m/sec. If the angle of inclination θ of the line joining the particle to the origin change, when x = 3 m, at the rate of k rad/sec., then the value of k is

A. 1

 $\mathsf{B.}\,2$

C.1/2

D. 1/3

Answer: A

23. The rate of change of volume of a sphere is equal to the rate of change of its radius, then its radius is equal to (a) 1 unit (b) units (c) unit(d) unit

A. 1

B. 2

C. 0.5

D. none of these

Answer: B

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24. Water is dropped at the rate of $2m^2/s$ into a cone of semivertical angel of 45° . The rate at which periphery of water surface changes when height of water in the cone is 2 m, is

A. 0.5m/s

B. 2m/s

 $\mathsf{C.}\,3m\,/\,s$

D. 1m/s

Answer: D



25. Suppose that water is emptied from a spherical tank of radius 10 cm. If the depth of the water in the tank is 4 cm and is decreasing at the rate of 2 cm/sec, then the radius of the top surface of water is decreasing at the rate of

A. 1

B. 2/3

C.3/2

D. 2

Answer: C



26. The altitude of a cone is 20 cm and its semi-vertical angle is 30° . If the semi-vertical angle is increasing at the rate of 2° per second, then the radius of the base is increasing at the rate of

A. 30 cm/sec

$$\mathsf{B.}\,\frac{160}{3}cm/\sec$$

- C. 10 cm/sec
- D. 160 cm/sec

Answer: B



27. Let the equation of a curve be $x = a(\theta + \sin \theta), y = a(1 - \cos \theta)$. If θ changes at a constant rate k then the rate of change of the slope of the tangent to the curve at $\theta = \frac{\pi}{3}$ is (a) $\frac{2k}{\sqrt{3}}$ (b) $\frac{k}{\sqrt{3}}$ (c) k (d) none of these A. $2k/\sqrt{3}$ B. $k/\sqrt{3}$ C. k D. none of these

Answer: D

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$$f(x) = |1-x|, 1 \leq x \leq 2 \, ext{ and } g(x) = f(x) + b \sin rac{\pi}{2} x, 1 \leq x \leq 2$$

then which of the following is correct?

A. Rolle's theorem is applicable to both f and g with $b = \frac{3}{2}$.

B. LMVT is not applicable to f and Rolle's theorem is applicable to g

with
$$b=rac{1}{2}$$

C. LMVT is applicable to f and Rolle's theorem is applicable to g with b

= 1.

D. Rolle's theorem is not applicable to both f and g for any real b.

Answer: C

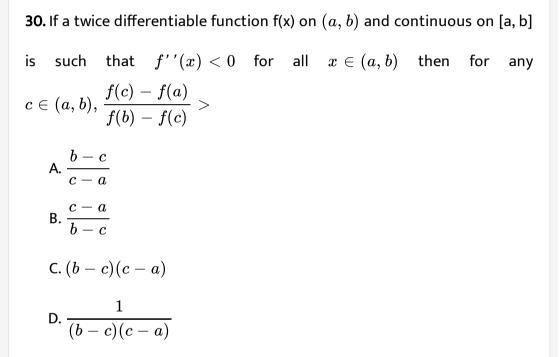
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29. If $c = \frac{1}{2}$ and $f(x) = 2x - x^2$, then interval of x in which LMVT is applicable, is

A. (1, 2)B. (-1, 1)C. (0, 1)D. (2, 1)

Answer: C





Answer: B

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31. Let $a,n\in N$ such that $a\geq n^3.$ Then $\sqrt[3]{a+1}-\sqrt[3]{a}$ is always

A. less than
$$\frac{1}{3n^2}$$

B. less than $\frac{1}{2n^3}$
C. more than $\frac{1}{n^3}$
D. more than $\frac{1}{4n^2}$

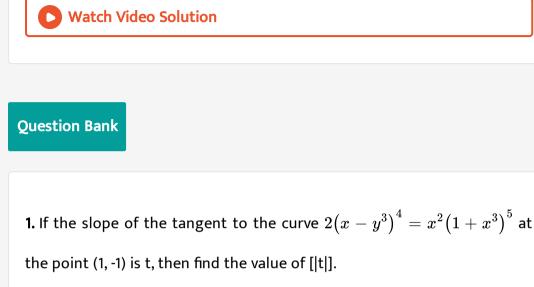
Answer: A

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32. Given
$$f'(1) = 1$$
 and $\frac{d}{dx}f(2x) = f'(x) \forall x > 0$. If $f'(x)$ is differentiable then there exists a numberd $x \in (2, 4)$ such that $f''(c)$ equals

- A. 1/4
- B. 1/2
- C. 1/4
- D. 1/8





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2. Let $P(x_0, y_0)$ be a point on the curve $C: (x^2 - 11)(y + 1) + 4 = 0 where x_0, y_0 \in N.$ If area of the triangle formed by the normal drawn to the curve 'C' at P and the coordinate axes is (a/b), $ab \in N$ then find the least value of (a-6b)



3. If the function $k(x) = \log_e \left(rac{x^2 - x + 1}{x^2 + x + 1}
ight.$ is strictly decreasing in `x in (-

t/7, t/7), then find the greatest integral value of t.



4. The line y = x is a tangent to the curve $y = px^2 + qx + r$ at the point x

=1. If the curve passes through the point (-1, 0) then the value of (p+r)/q is

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5. Let
$$f(x)=egin{cases} x+1 & x<1\ \lambda & x=1\ x=1\ be$$
 a strictly increasing function at x $x^2-x+3 & x>1 \end{cases}$

=1, then the number of integers in the range of λ , is

6. If the tangent at a point P on the curve $x^{7}(2) = sqrt^{2}(1/7)$

meets the co-ordinates axes A and B respectively then 2((BP)/(AP)) is

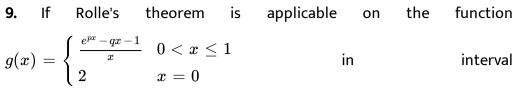


7. If $f(x) = 2x^3 + 9x^2 + px + 20$ is an increasing function of x in the

largest interval (-1,4) then p is equal to

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8. Let $f\colon [1,3[o [0,\infty))$ be continuous and differentiable function and if $(f(3)-f(1)).\ (f^2(3)+f^2(1)+f(3)f(1)=kf^2(c)f'(c)wherec\in(1,3)$, then find the value of k.



 $x \in [0, 1], then f \in dthevalue of(p^(2)+q^(2))`.$



10. Given the function $f(x) = 2x^2 - 4x - 5$. If x_1 and x_2 (x(1) gt x(2)) $arethe|c|issaeofp \oint son f(x) such t \hat{t} he \tan \ge nts drawnat them pass throw (3x(1)-2x(2)) equals$

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11. Let y=f(x) be an invertible function such that x -intercept of the tangent

at any point P(x, y) on y=f(x) is equal to the square of abscissa of the point

of tangency. If f(2)=1, then
$$f^{-1}\left(rac{5}{8}
ight)$$
 equals

12. Let f(x) be the curve passing through (-2,1) such that slope of the normal line at the point (x, y) on the curve is equal to x^2y . The area bounded by the curve $y = xf^2(x)$ and coordinates axes, is



13. If y=4 x-5 is a tangent to the curve $C\!:\!y^2=px^3+q$ at M (2,3) then the

value of (p-q) is

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14. k is the least positive integer for which the function $f(x)=(2x+1)^{50}(3x-4)^{60}$ is increasing in [k, oo)`. The value of 'k' is

15. Number of integral values of a for which the function
$$f(x)=igg(rac{4a-7}{3}igg)x^3+(a-3)x^2+x+5$$
 is monotonic for every $x\in R,$ is



16. Let f(x) be a cubic polynomial which has local maximum at x=-1 and f'(x) has a local minimum at x=1. If f(-1) = 10 and f(3) = -22, then find the distance between its two horizontal tangents.

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17. Let
$$f^{\,\prime}(x)=e^{x^2}$$
 and f(0)=10. If A

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18. Minimum distance between the curves $f(x) = e^x$ and g(x0 = Inx is



19. f(x) is a polynomial of degree 6 which decreases in the interval $(0, \infty)$ and increases in the interval $(-\infty, 0)$. If f'(2) = 0, f'(0) = 0, f(0) = 0, f(0) = 1 and f(1)-f(-1) = 8/5', then -3(f(1)+f(-1)) equals

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20. The least positive integral value of λ for which $f(x) = \frac{3x^3}{2} + \frac{\lambda x^2}{3} + x + 7$ has a point of maxima is View Text Solution

21. Minimum positive integral value of : k for which $f(x) = k \cos 2x - 4 \cos^3 x$ has exactly one critical point in (0, pi), is

22. Let the radius and height of right circular cylinder is related as $r^2+h=5$. Let λ is maximum volume. Then $rac{16\lambda}{25\pi}$, is

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23. A solid box is formed by placing a cylinder, having equal height and radius on top of a cube such that, the circular base of cylinder is the inscribed circle for square top of the cube. If the radius of cytinder is changing at the rate $\frac{1}{2\pi + 16}c\frac{m}{s}$, then the rate of change of volume of the box when radius is 2 cm, is (Assuming that box always remain in the given shape)

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24. On the curve x^{m+n} . $y^n=a,$ $mn\in N,$ $a\in R^+$, if the ratio of slopes of tangent at any point \$P\$ and that of line segment OP(O being origin) is -4 , then the value of m/n is 25. If $a \in (a_1, a_2)$ is the complete set satisfying the condition that the. point of local minima and the point of local maxima is less than 4 and greater than -2, respectively for the function $f(x) = x^3 - 3ax^2 + 3(a^2 - 1)x + 1$, then $(a_2 - a_1)$ is

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26. x and y are sides of two squares sach that $y = x - x^2$. Let f(x) denote the rate of change of area of the second square with respect to the area of the first'square, then f(2) is

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27. If the tangent at a point P on the curve $x^{(7)}y^{(2)} = sqrt7+2^{(1/7)}$

meets the co-ordinates axes A and B respectively then 2((BP)/(AP)) is

28. Number of solutions of the equation $(x-k)e^{-x}=rac{1}{e^2}$, where k<1

is



29. The total number of local maxima and local minima of the function

$$f(x) = igg(rac{2-x}{\pi}igg) \mathrm{cos}(\pi x + 3\pi) + rac{1}{\pi}\mathrm{sin}(\pi x + 3\pi)$$
, where $0 < x < x$ is

equal is