MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

CONIC SECTIONS

Solved Examples And Exercises

1. Given that $A(1, 1)$ and $B(2, -3)$ are two points and D is a point on AB produced such that $AD = 3AB$. Find the coordinates of D.

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2. Find the coordinates of the point which divides the line segments joining the points $(6, 3)$ and $(-4, 5)$ in the ratio $3:2$ (i) internally and (ii) externally.

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3. Four points $A(6, 3)$, $B(-3, 5)$, $C(4, -2)$ and $D(x, 2x)$ are given in such a way that $\frac{(AreaofDBC)}{(AreaofABC)} = \frac{1}{2}$.

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4. If the points $(1, 1) : (0, \sec^2\theta)$; and $(\cos ec^2\theta, 0)$ are collinear, then find the value of θ

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5. Given that A_1, A_2, A_3, A_n are n points in a plane whose coordinates are $x_1, y_1), (x_2, y_2), (x_n, y_n)$, respectively. A_1A_2 is bisected at the point P_1, P_1A_3 is divided in the ratio $A : 2$ at P_2, P_2A_4 is divided in the ratio 1:3 at P_3, P_3A_5 is divided in the ratio $1 : 4$ at P_4 , and so on until all n points are exhausted. Find the final point so obtained.

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6. If P divides OA internally in the ratio $\lambda_1 : \lambda_2$ and Q divides OA externally in the ratio $\lambda_1 ; \lambda_2$, then prove that OA is the harmonic mean of OP and OQ.

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7. Prove that the point $(-2, -1), (1, 0), (4, 3) and (1, 2)$ are the vertices of parallel-gram. Is it a rectangle?

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8. Determine the ratio in which the line $3x + y - 9 = 0$ divides the segment joining the points (1,3) and $(2, 7)$.

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9. Find the orthocentre of the triangle whose vertices are $(0, 0), (3, 0)$, and $(0, 4)$.

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10. If a vertex of a triangle is $(1, 1)$, and the middle points of two sides passing through it are $-2, 3)$ and $(5, 2)$, then find the centroid and the incenter of the triangle.

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11. The vertices of a triangle are $A(-1, -7), B(5, 1) and C(1, 4)$. If the internal angle bisector of $\angle B$ meets the side AC in D, then find the length AD.

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12. If ABC having vertices $A(a\cos\theta_1, a\sin\theta_1), B(a\cos\theta_2 a\sin\theta_2), andC(a\cos\theta_3, a\sin\theta_3)$ is equilateral, then prove that $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = \sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 0.$

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13. If the point $(x, -1), (3, y), (-2, 3), and(-3, -2)$ taken in order are the vertices of a parallelogram, then find the values of $xandy\cdot$

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14. If the midpoints of the sides of a triangle are $(2, 1), (-1, -3), and(4, 5),$ then find the coordinates of its vertices.

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15. If the circumcenter of an acute-angled triangle lies at the origin and the centroid is the middle point of the line joining the points (a^2+1, a^2+1) and $(2a, -2a)$, then find the orthocentre.

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16. If a vertex, the circumcenter, and the centroid of a triangle are (0, 0), (3,4), and (6, 8), respectively, then the triangle must be (a) a right-angled triangle (b) an equilateral triangle (c) an isosceles triangle (d) a right-angled isosceles triangle

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17. Orthocenter and circumcenter of a $\mathrm{Delta}ABC$ are $(a, b)and(c, d)$, respectively. If the coordinates of the vertex A are (x_1, y_1), then find the coordinates of the middle point of BC.

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18. If $A(x_1, y_1), B(x_2, y_2),$ and $C(x_3, y_3)$ are three non-collinear points such that $x12 + y12 = x22 + y22 = x32 + y32,$ then prove that $x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C = y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C = 0.$

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19. The points $(a, b), (c, d),$ and $\left(\frac{kc + la}{k + l}, \frac{kd + lb}{k + l}\right)$ are (a) vertices of an equilateral triangle (b) vertices of an isosceles triangle (c) vertices of a right-angled triangle (d) collinear

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20. The line joining $A(b \cos \alpha b \sin \alpha)$ and $B(a \cos \beta, a \sin \beta)$ is produced to the point $M(x, y)$ so that AM and BM are in the ratio $b : a \cdot$ Then prove that $x + y \tan\left(\alpha + \frac{\beta}{2}\right) = 0.$

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21. If the middle points of the sides of a triangle are $(-2, 3), (4, -3), and (4, 5)$, then find the centroid of the triangle.

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22. In what ratio does the x=axis divide the line segment joining the points $(2, -3)$ and (5, 6)?

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23. If $(1, 4)$ is the centroid of a triangle and the coordinates of its any two vertices are $(4, -8)$ and $(-9, 7)$, find the area of the triangle.

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24. If $(x_i, y_i), i = 1, 2, 3,$ are the vertices of an equilateral triangle such that

$(x_1+2)^2+(y_1-3)^2=(x_2+2)^2+(y_2-3)^2=(x_3+2)^2+(y_3-3)^2$,

then find the value of $\dfrac{x_1+x_2+x_3}{y_1+y_2+y_3}$.

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25. The vertices of a triangle are $A(x_1, x_1, \tan\theta_1)$, $B(x_2, x_2, \tan\theta_2)$ and $C(x_3, x_3, \tan\theta_3)$. If the circumcentre coincides with origin then

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26. Find the locus of a point, so that the join of $(-5, 1)$ and $(3, 2)$ subtends a right angle at the moving point.

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27. The sum of the squares of the distances of a moving point from two fixed points (a,0) and $(-a, 0)$ is equal to a constant quantity $2c^2$. Find the equation to its locus.

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28. AB is a variable line sliding between the coordinate axes in such a way that A lies on the x-axis and B lies on the y-axis. If P is a variable point on AB such that $PA = b, Pb = a$, and $AB = a + b,$ find the equation of the locus of P.

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29. A rod of length l slides with its ends on two perpendicular lines. Find the locus of its midpoint.

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30. Find the locus of the point $\left(t^2 - t + 1, t^2 + t + 1\right), t \in R$.

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31. Find the locus of a point such that the sum of its distance from the points (2, 2) and $(2, -2)$ is 6.

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32. Two points P(a,0) and Q(-a,0) are given. R is a variable point on one side of the line PQ such that $\angle RPQ - \angle RQP$ is a positive constant 2α. Find the locus of the point R.

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33. If the coordinates of a variable point P are $(a\cos\theta, b\sin\theta)$, where θ is a variable quantity, then find the locus of P.

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34. Find the locus of a point whose distance from (a, 0) is equal to its distance from the y-axis.

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35. The coordinates of the point $A and B$ are (a,0) and $(-a, 0)$, respectively. If a point P moves so that $PA^2 - PB^2 = 2k^2$, when k is constant, then find the equation to the locus of the point P.

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36. Find the locus of the foot of perpendicular from the point (2, 1) on the variable line passing through the point (0, 0).

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37. A variable line through the point $P(2, 1)$ meets the axes at $A and B$. Find the locus of the centroid of triangle OAB (where O is the origin).

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38. If $A(\cos\alpha, \sin\alpha), B(\sin\alpha, -\cos\alpha), C(1, 2)$ are the vertices of ABC, then as α varies, find the locus of its centroid.

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39. Let $A(2, -3) and B(-2, 1)$ be the vertices of ABC. If the centroid of the triangle moves on the line $2x + 3y = 1$, then find the locus of the vertex C.

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40. A straight line is drawn through $P(3, 4)$ to meet the axis of x and y at $A and B$, respectively. If the rectangle $OACB$ is completed, then find the locus of C.

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41. A variable line through point $P(2, 1)$ meets the axes at $AandB$. Find the locus of the circumcenter of triangle OAB (where O is the origin).

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42. A point moves such that the area of the triangle formed by it with the points (1, 5) and $(3, -7)is21sq\dot{u}nits\cdot$ Then, find the locus of the point.

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43. Find the locus of the point of intersection of lines $x\cos\alpha + y\sin\alpha = a$ and $x\sin\alpha - y\cos\alpha = b(\alpha$ is a variable).

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44. Find the locus of the middle point of the portion of the line $x\cos\alpha + y\sin\alpha = p$ which is intercepted between the axes, given that p

remains constant.

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45. Q is a variable point whose locus is $2x + 3y + 4 = 0$; corresponding to a particular position of Q, P is the point of section of OQ, O being the origin, such that $OP : PQ = 3 : 1$. Find the locus of P.

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46. Convert $y = 10$ into a polar equation.

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47. Find the minimum distance of any point on the line $3x + 4y - 10 = 0$ from the origin using polar coordinates.

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48. Express the polar equation $r = 2\cos\theta$ in rectangular coordinates.

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49. Convert $x^2 - y^2 = 4$ into a polar equation.

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50. Convert $r\sin\theta = r\cos\theta + 4$ into its equivalent Cartesian equation.

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51. Convert $r = \cos ec\theta e^{r\cos\theta}$ into its equivalent Cartesian equation.

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52. Find the maximum distance of any point on the curve $x^2 + 2y^2 + 2xy = 1$ from the origin.

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53. Convert $r = 4 \tan\theta \sec\theta$ into its equivalent Cartesian equation.

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54. Given the equation $4x^2 + 2\sqrt{3}xy + 2y^2 = 1$. Through what angle should the axes be rotated so that the term xy is removed from the transformed equation.

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55. The equation of a curve referred to a given system of axes is $3x^2 + 2xy + 3y^2 = 10.$ Find its equation if the axes are rotated through an angle 45^0 , the origin remaining unchanged.

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56. Determine x so that the line passing through $(3, 4) and (x, 5)$ makes an angle of 135^0 with the positive direction of the x-axis.

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57. What does the equation $2x^2 + 4xy - 5y^2 + 20x - 22y - 14 = 0$ become when referred to the rectangular axes through the point $(-2, -3)$, the new axes being inclined at an angle at 45^0 with the old axes?

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58. Shift the origin to a suitable point so that the equation $y^2 + 4y + 8x - 2 = 0$ will not contain a term in y and the constant term.

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59. At what point should the origin be shifted if the coordinates of a point $(4, 5)$ become $(-3, 9)$?

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60. Find the equation to which the equation $x^2 + 7xy - 2y^2 + 17x - 26y - 60 = 0$ is transformed if the origin is shifted to the point $(2, -3)$, the axes remaining parallel to the original axies.

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61. The equation of curve referred to the new axes, axes retaining their directions, and origin $(4, 5)$ is $X^2 + Y^2 = 36$. Find the equation referred to the original axes.

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62. If the point $(2, 3), (1, 1), and(x, 3x)$ are collinear, then find the value of x, using slope method.

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63. Find the orthocentre of ΔABC with vertices $A(1, 0), B(-2, 1)$, and $C(5, 2)$

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64. The angle between the line joining the points (1,-2) , (3,2) and the line x+2y -7 =0 is

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65. The line joining the points $A(2, 1), and B(3, 2)$ is perpendicular to the line $(a^2)x + (a + 2)y + 2 = 0$. Find the values of a.

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66. For what value of k are the points $(k, 2 - 2k), (-k + 1, 2k) and (-4 - k, 6 - 2k)$ collinear?

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67. Find the area of the quadrilateral $ABCD$ having vertices $A(1, 1), B(7, -3), C(12, 2),$ and $D(7, 21)$.

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68. Given that $P(3, 1), Q(6. 5),$ and $R(x, y)$ are three points such that the angle PQR is a right angle and the area of RQP is 7, find the number of such points R.

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69. If O is the origin and if the coordinates of any two points $Q_1 and Q_2$ are $(x_1, y_1) and (x_2, y_2)$, respectively, prove that $OQ_1 \dot{O} Q_2 \cos \angle Q_1 O Q_2 = x_1 x_2 + y_1 y_2$.

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70. Prove that the area of the triangle whose vertices are $(t, t-2), (t+2, t+2),$ and $(t+3, t)$ is independent of t.

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71. Find the area of a triangle having vertices $A(3, 2), B(11, 8),$ and $C(8, 12)$.

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72. In ABC Prove that $AB^2 + AC^2 = 2(AO^2 + BO^2)$, where O is the middle point of BC

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73. Two points $O(0, 0)$ and $A(3, \sqrt{3})$ with another point P form an equilateral triangle. Find the coordinates of P.

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74. Find the coordinates of the circumcenter of the triangle whose vertices are $(A(5, -1), B(-1, 5),$ and $C(6, 6)$. Find its radius also.

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75. Find the orthocentre of ΔABC with vertices $A(1, 0), B(-2, 1)$, and $C(5, 2)$

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76. If $(b_2 - b_1)(b_3 - b_1) + (a_2 - a_1)(a_3 - a_1) = 0$, then prove that the circumcenter of the triangle having vertices $(a_1, b_1), (a_2, b_2)$ and (a_3, b_3) is $\left(\frac{a_{2+a_3}}{2}, \frac{b_{2+b_3}}{2}\right)$

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77. If line $3x - ay - 1 = 0$ is parallel to the line $(a+2)x - y + 3 = 0$ then find the value of $a\cdot$

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78. If $A(2, -1) and B(6, 5)$ are two points, then find the ratio in which the food of the perpendicular from $(4, 1)$ to AB divides it.

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79. Angle of a line with the positive direction of the x-axis is θ . The line is rotated about some point on it in anticlockwise direction by angle 45^0 and its slope becomes 3. Find the angle θ.

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80. Let $A(6, 4) and B(2, 12)$ be two given point. Find the slope of a line perpendicular to AB.

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81. If the points $(a, 0), (b, 0), (0, c)$, and $(0, d)$ are concyclic $(a, b, c, d > 0)$, then prove that $ab = cd$.

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82. If three points are $A(-2, 1)B(2, 3), andC(-2, -4)$, then find the angle between $ABandBC$.

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83. The line joining the points $(x, 2x)and(3, 5)$ makes an obtuse angle with the positive direction of the x-axis. Then find the values of x.

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84. If the line passing through $(4, 3)and(2, k)$ is parallel to the line $y = 2x + 3,$ then find the value of k.

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85. Find the area of the pentagon whose vertices are $A(1, 1), B(7, 21), C(7, -3), D(12, 2),$ and $E(0, -3)$

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86. Let $A = (3, 4)$ and B is a variable point on the lines $|x|$ =6. IF $AB \leq 4$, then find the number of position of B with integral coordinates.

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87. The three points $(-2, 2)(9, -2), and(-4, -3)$ are the vertices of (a) an isosceles triangle (b) an equilateral triangle (c) a right-angled triangle (d) none of these

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88. The points $(-a, -b), (a, b), (a^2, ab)$ are (a) vertices of an equilateral triangle (b) vertices of a right angled triangle (c) vertices of an isosceles triangle (d) collinear

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89. Find the length of altitude through A of the triangle ABC, where $A \equiv (-3, 0) B \equiv (4, -1), C \equiv (5, 2)$

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90. If the coordinates of two points A and B are (3, 4) and $(5, -2)$, respectively, find the coordinates of any point P if $PA = PB$. Area of PAB is 10 sq. units.

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91. If the vertices of a triangle have rational coordinates, then prove that the triangle cannot be equilateral.

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