MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

DEFINITE INTEGRATION

Examples

1. Evaluate the following definite integrals as limit of sum $\int_2^1 x^2 dx.$

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2. Evaluate: $\int_a^b e^x dx$ using limit of sum

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3. Evaluate: $\int_a^b \sin x dx$ using limit of sum

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4. Evaluate $\int_a^b \frac{dx}{\sqrt{x}}, where a, b > 0.$

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5. If $f(x) = \min\left(|x|, 1 - |x|, \frac{1}{4}\right) \forall x \in R,$ then find the value of $\int_{-1}^{1} f(x) dx.$

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6. Evaluate: $\int_{-\frac{\pi}{2}}^{2\pi} \sin^{-1}(\sin x) dx$

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7. Evaluate $\int_0^1 \frac{1}{\sqrt{1-x^2}\sin^{-1}\left(2x\sqrt{1-x^2}\right)dx}$.

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8. Evaluate: $\int_0^{2\pi}[\sin x]dx, where[.]$ denotes the greatest integer function.

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9. Prove that $\frac{1+\sqrt{2}}{2} < \int_0^{\pi/2}\frac{\sin x}{x}dx < \frac{\pi+2\sqrt{2}}{4}$

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10. Evaluate: $\int_{-1}^0 \frac{dx}{x^2+2x+2}$

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11. Let $P(x)$ be a polynomial of least degree whose graph has three points of inflection $(-1, -1), (1.1)$and a point with abscissa 0 at which the curve is inclined to the axis of abscissa at an angle of 60°. Then find the value of $\int_0^1 p(x)dx$.

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12. Let f be a continuous function on $[a, b]\cdot$ Prove that there exists a number $x \in [a, b]$ such that $\int_a^x f(t)dx = \int_x^b f(t)dt\cdot$

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13. $\int_0^1 \frac{dx}{e^x + e^{-x}}$

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14. Evaluate $\left(\frac{\int_0^\pi}{2}\right)\frac{\tan x dx}{1 + m^2 \tan^2 x}$

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15. Find the mistake of the following evaluation of the integral $I=\int_0^{\pi}\frac{dx}{1+2\sin^2 x}$ $I=\int_0^{\pi}\frac{dx}{\cos^2 x+3\sin^2 x}$ $=\int_0^{\pi}\frac{\sec^2 x dx}{1+3\tan^2 x}=\frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\sqrt{3}\tan x\right)\right]\pi 0=0$

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16. Let $\frac{d}{dx}(F(x))=\frac{e^{\sin x}}{x}, x>0$. If $\int_1^4 2\frac{e^{\sin\left(x^2\right)}}{x}dx=F(k)-F(1)$, then possible value of k is:

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17. If $\int_a^b (f(x)-3x)dx=a^2-b^2$ then the value of $f\left(\frac{\pi}{6}\right)$ is ___

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18. If $f(0) = 1,\ f(2) = 3,\ f(2) = 5, thenf \in dthevalueof \int_0^1 x f^{2x} dx$

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19. $F \in dthevalueof \int_0^1 \log x dx \cdot$

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20. Evaluate: $\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} dx$

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21. If $\lambda = \int_0^1 \frac{e^t}{1 + t}$, then $\int_0^1 e^t \log_e (1 + t) dt$ is equal to

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22. If $\int_0^1 e^{-x} \hat{} 2dx = a,$ then find the value of $\int_0^1 x^2 e^{-x} \hat{} 2dx$ in terms of a .

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23. If $f(x) = x + \sin x,$ then find the value of $\int_{\pi}^{2\pi} f^{-1}(x)dx.$

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24. Find the value of $\int_0^{\pi/2} \cos^5 x \sin^7 x dx$

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25. Evalute $\lim_{n \to \infty} \left[\frac{1}{(n+1)(n+2)} + \frac{1}{(n+2)(n+4)} + \ldots\ldots + \frac{1}{6n^2}\right]$

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26. Evaluate: $\left(\lim\right)_{n\overrightarrow{\infty}} n\left[\frac{1}{na}+\frac{1}{na+1}+\frac{1}{na+2}++\frac{1}{nb}\right]$

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27. Evaluate: $\left(\lim\right)_{n\overrightarrow{\infty}}\left(\frac{(n+1)(n+2)(n+n)^{\frac{1}{n}}}{n}\right.$

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28. Evaluate: $\lim_{n\to\infty}\frac{\left(1^2+2^2+3^3+\ldots+n^2\right)\left(1^3+2^3+3^3+\ldots+n^3\right)}{1^6+2^6+3^6+\ldots+n^6}$

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29. `P rov et h a t0

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30. $Provethat \frac{1}{2} \leq \int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^{2n}}} \leq \frac{\pi}{6} f \text{ or } n \geq 1.$

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31.

$$Let I_1 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{x} dx, \ I_2 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\sin \frac{\sin x}{\sin x} dx, \ I_3 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\frac{\sin(\tan x)}{\tan x} dx \right.\right.$$

Then arrange in the decreasing order in which values I_1, I_2, I_3 lie.

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32. `P rov et h a t1

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33. Estimate the absolute value of the integral $\int_{10}^{19} \frac{\sin x}{1+x^8} dx$

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34. Prove that $\int_0^1 \sqrt{(1+x)(1+x^3)}\,dx$ cannot exceed $\sqrt{\frac{15}{8}}$.

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35. Prove that $\int_a^b f(x)dx = (b-a)\int_0^1 f((b-a)x+a)dx$

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36. Evaluate $\int_{-1}^{2} |x^3 - x| dx$

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37. Evaluate: $\int_{-1}^{3/2} |x \sin \pi x| dx$

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38. Show that $\int_a^b \frac{|x|}{x} dx = |b| = |a|\cdot$

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39. If $f(n) = \int_0^{2015} \frac{e^x}{1+x^n} dx$, then find the value of $\lim_{n \to \infty} f(n)$

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40. Let: $f(x) = \int_0^x |2t - 3| dt\cdot$ Then discuss continuity and differentiability of $f(x) at x = \frac{3}{2}$

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41. A continuous real function f satisfies $f(2x) = 3\left(f(x) \forall x \in R\dot{I} f \int_0^1 f(x) dx = 1,\right.$ then find the value of $\int_1^2 f(x) dx$

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42. Let $g(x)=\int_0^x f(t)dt$, where f is such that $\frac{1}{2}\leq f(t)\leq 1$, for $t\varepsilon[0,1]$ and $0\leq f(t)\leq\frac{1}{2}$, for $t\varepsilon[1,2]$. Then prove that $\frac{1}{2}\leq g(2)\leq\frac{3}{2}$.

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43. If $[x]$ denotes the greatest integer less than or equal to x, then find the value of the integral $\int_0^2 x^2[x]dx\cdot$

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44. Evaluate: $\int_0^{\frac{5\pi}{12}}[\tan x]dx, where[.]$ deontoes the greatest integer function.

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45. Evaluate: $\int_0^{10\pi} \left[\tan^{-1} x\right] dx, where[x]$ represents greatest integer function.

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46. Evaluate: $\int_0^2 \left[x^2 - x + 1\right] dx, where[.]$ denotos the greatest integer function.

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47. Prove that $\int_0^{\infty} \left[ne^{-x}\right] dx = 1n\left(\frac{n^n}{n!}\right), wheren$ is a natural number greater than 1 and [.] denotes the greatest integer function..

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48. Evaluate: $\int_0^{\sqrt{3}} \frac{1}{1+x^2} \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

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49. Evalaute $\int_{\pi/6}^{\pi/3} \frac{\sqrt{(\sin x)}dx}{\sqrt{(\sin x)} + \sqrt{(\cos x)}}$

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50. Evaluate: $\int_0^a \frac{dx}{x + \sqrt{(a^2 - x^2)}}$ or $\int_0^{\frac{\pi}{2}} \frac{dth\eta}{1 + \tan\theta}$

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51. Evaluate $\int_0^{\pi} \frac{\sin 6x}{\sin x} dx.$

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52. Evaluate: $\int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$

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53. Evaluate: $\int_{-\pi}^{3\pi} \log(sec\theta - tan\theta)dth\eta$

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54. Prove that $\int_0^{2a} f(x)dx = \int_a^a [f(a-x) + f(a+x)]dx$

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55. Evaluate $\int_0^{\pi/4} In(1 + \tan x)dx$

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56. Evaluate: $\int_{-5}^{5} x^2 \left[x + \frac{1}{2}\right] dx (where[.]$ denotes the greatest integer function).

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57. Evaluate: $\int_{-\pi}^{\pi} \frac{x \sin x dx}{e^x + 1}$

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58. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$

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59. Prove that $\int_0^1 \tan^{-1}\left(\frac{1}{1 - x + x^2}\right) dx = 2\int_0^1 \tan^{-1} x dx$. Hence or otherwise, evaluate the integral $\int_0^1 \tan^{-1}(1 - x + x^2) dx$

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60. Show that $\int_0^{\frac{\pi}{2}} \sqrt{(\sin 2\theta)} \sin \theta d th \eta = \frac{\pi}{4}$

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61. For

$\theta \in \left(= , \frac{\pi}{2} \right), provethat \int_0^{\theta} \log(1 + \tan\theta \tan x) dx = \theta \log(\sec\theta)$

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62. Evaluate the definite integral: $\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \left(\frac{x^4}{1 - x^4} \right) \cos^{01} \left(\frac{2x}{1 + x^2} \right) dx.$

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63. Evaluate $\int_0^1 \frac{dx}{(5 + 2x - 2x^2)(1 + e^{2-4x})} dx$

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64. Evaluate $\int_0^{2\pi} \frac{dx}{1 + 3\cos^2 x}$

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65. Evaluate $\int_0^{2\pi} \frac{x\cos^{2n}x}{\cos^{2n}x+\sin^{2n}x}dx$

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66. Evaluate $\int_0^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right)+3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x dx.$

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67. $\int_0^{\pi} x\log\sin x dx$

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68. Evaluate: $\int_{\pi/4}^{\pi/4} \log(\sin x+\cos x)dx$

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69. Evaluate: $\int_0^{\frac{\pi}{2}} x\cot x dx$

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70. Evaluate: $\int_0^{\infty} \log\left(x+\frac{1}{x}\right)\frac{dx}{1+x^2}$

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71. Evaluate $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\,dx.$

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72. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{a - s\int h\eta}{a+\sin\theta}\right) dth\eta, a > 0$

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73. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left\{\frac{ax^2+bx+c}{ax^2-bx+c}(a+b)|\sin x|\right\}dx$

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74. Evaluate: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^9 - 3x^5 + 7x^3 - x + 1}{\cos^2 x} dx$

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75. If f is an odd function, then evaluate $I = \int_{-a}^{a} \left(\frac{f(\sin x)}{f(\cos x) + f(\sin^2 x) dx} \right.$

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76. Evaluate: $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left[\left(\frac{x+1}{x-1} \right)^2 + \left(\frac{x-1}{x+1} \right)^2 - 2 \right]^{\frac{1}{2}} dx$

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77. Find the value of $\int_{-2}^{2} \frac{\sin^{-1}(\sin x) + \cos^{-1}(\cos x)}{(1 + x^2)\left(1 + \left[\frac{x^2}{5}\right]\right)} dx$, where [.] represents the greatest integer function.

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78. Determine the value of $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx.$

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79. Evalaute $\int_0^{16\pi/3} |\sin x| dx.$

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80. Evaluate $\frac{\int_0^n [x] dx}{\int_0^n \{x\} dx}$ (where $[x]$ and $\{x\}$ are integral and fractional parts of x respectively and $n \varepsilon N$).

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81. Let $f(x)$ be a continuous and periodic function such that $f(x) = f(x+T)$ for all $x \varepsilon R, T > 0$.If $\int_{-2T}^{a+5T} f(x)dx = 19(ag0)$ and $\int_0^T f(x)dx = 2$, then find the value of $\int_0^a f(x)dx$.

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82. If $g(x) = \int_0^x \cos^4 t dt$, then prove that $g(x+\pi) = g(x) + g(\pi)$.

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83. Evaluate: $\int_{-\frac{\pi}{4}}^{n\pi - \frac{\pi}{4}} |\sin x + \cos x| dx$

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84. Evaluate: $\int_0^x [\cos t] dt where n \in \left(2n\pi, \left(4n + 1\frac{\pi}{2}\right), n \in N, and [.]$ denotes the greatest integer function.

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85. Let f be a real-valued function satisfying $f(x)+f(x+4)=f(x+2)+f(x+6)$ Prove that $\int_x^{x+8} f(t)dt$ is constant function.

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86. A periodic function with period 1 is integrable over any finite interval. Also, for two real numbers a, b and two unequal non-zero positive integers $mandn$ $\int_a^{a+n} f(x)dx = \int_b^{b+m} f(x)dx \dot{C}alc\underline{a}tethevalueof \int_m^n f(x)dx$

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87. If $y=\int_{x^2}^{x^3} \frac{1}{\log t}dt (x>0)$, then find $\frac{dy}{dx}$

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88. If $\int_0^y \cos t^2\, dt = \int_0^{x^2} \frac{\sin t}{t}\, dt$, then $\frac{dy}{dx}$ is equal to

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89. $Ifx = \int_0^y \frac{dt}{\sqrt{1+9t^2}} and \frac{d^2y}{dx^2} = ay, thenf \in da$

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90. If $\int_{\sin x}^1 t^2 f(t) dt = 1 - \sin x, x\varepsilon\left(0, \frac{\pi}{2}\right)$ then find the value of $f\left(\frac{1}{\sqrt{3}}\right)$

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91. Let $f: \overrightarrow{RR}$ be a differentiable function having $f(2) = 6, f'(2) = \frac{1}{48}$.

Then evaluate $(\lim)_{x \overrightarrow{2}} \int_6^{f(x)} \frac{4t^3}{x-2} dt$

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92. Evaluate: $(\lim)_{x\overrightarrow{\infty}}\frac{\left(\int 0xe^{x}\ \hat{}\ 2dx\right)^{2}}{\int 0xe^{2x}\ \hat{}\ 2dx}$

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93. Prove that: $y=\int_{\frac{1}{8}}^{\sin^{2}x}\sin^{-1}\sqrt{t}dt+\int_{\frac{1}{8}}^{\cos^{2}x}\cos^{-1}, where0\leq x\leq\frac{\pi}{2},$ is the equation of a straight line parallel to the x-axis. Find the equation.

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94. If $f(x)=\int_{x^{2}}^{x^{2}+1}e^{-t^{2}}dt$, then find the interval in which $f(x)$ increases.

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95. Let $f:(0,\infty)\overrightarrow{0,\infty}$ be a differentiable function satisfying, $\xi nt_0^x(1-t)f(t)dt=\int_0^x tf(t)dtx\in R^+ andf(1)=1$. Determine $f(x)\cdot$

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96. Let $f:R-(0,\infty)$ be a real valued function satisfying $\int_0^1 tf(x-t)dt=e^{2x}-1$ then which of the following is/are correct?

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97. Let $f:R\to R$ be a differntiable function satisfying $f(x)=x^2+3\int_0^x e^{-t^3}.\,f(x-t^3)dt$. Then find $f(x)$.

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98. $Ify=\int_0^x f(t)\sin\left\{k(x-t)dt, thenprovethat\frac{dt^2y}{dx^2}+k^2y=kf(x)\cdot\right.$

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99. Prove that $\int_0^x e^{xt} e^{-t^2} dt = e^{x^2/4} \int_0^x e^{-t^2/4} dt.$

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100. show that the sum of the two integrals $\int_{-4}^{-5} e^{(x+5)^2} dx + 3\int_{\frac{1}{3}}^{\frac{2}{3}} e^{9\left(x-\frac{2}{3}\right)^2} dx$ is zero

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101. Compute the integrals: $\int_0^\infty f\left(x^n + x^{-n}\right) \log x \frac{dx}{x}$

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102. Compute the integrals: $\int_0^\infty f\left(x^n + x^{-n}\right) \log x \frac{dx}{1+x^2}$

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103. Compute the integrals: $\int_{\frac{1}{e}}^{e} \frac{1}{x}\sin\left(x - \frac{1}{x}\right)dx$

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104. $LetA = \int_0^\infty \frac{\log x}{1+x^3} dx \dot{T}henf \in dthevalueof \int_0^\infty \frac{x \log x}{1+x^3} dx$ in terms of $A \cdot$

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105. If $\int_0^1 \frac{\sin t}{1+t} dt = \alpha$, then find the value of $\int_{4\pi-2}^{4\pi} \frac{\sin\frac{t}{2}}{4\pi|2-t|} dt$

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106. Prove that $\frac{\int_0^{\tan^{-1} x}}{x} dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{x}{\sin x} dx.$

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107. For $x > 0, let f(x) = \int_1^x \frac{\log t}{1+t} dt$. Find the function $f(x) + f\left(\frac{1}{x}\right)$ and find the value of $f(e) + f\left(\frac{1}{e}\right)$.

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108. Determine a positive integer n such that $\int_0^{\frac{\pi}{2}} x^n \sin x dx = \frac{3}{4}\left(\pi^2 - 8\right)$

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109. Determine a positive integer $n \leq 5$ such that $\int_0^1 e^x (x-1)^n = 16 - 6e$

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110. Prove that: $I_n = \int_0^\infty x^{2n+1} e^{-x} \hat{} 2dx = \frac{n!}{2}, n \in N$.

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111. If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}; n \in N,$ then prove that $2nI_{n+1} = 2^{-n} + (2n-1)I_n$

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112. If $I_n = \int_0^{\frac{\pi}{2}} \sin^x x dx$, then show that $I_n = ((n-1)n)I_{n-2}$.

Hence prove that

$$I_n = \begin{cases} \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\ldots\ldots\ldots\left(\frac{1}{2}\right)\frac{\pi}{2} & \text{if } n \text{is even} \\ \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\ldots\ldots\ldots\left(\frac{2}{3}\right)1 & \text{if } n \text{is odd} \end{cases}$$

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113. $f, g, h,$ are continuous in $[0, a], f(a-x) = f(x), g(a-x) = -g(x), 3h(x) - 4h(a-x) = 5.$

Then prove that $\int_0^a f(x)g(x)h(x)dx = 0$

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114. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin 3x}{\sin x + \cos x} dx.$

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115. Given a function $f : [0, 4] \to R$ is differentiable ,then prove that for some $\alpha, \beta \varepsilon (0, 2)$, $\int_0^4 f(t)dt = 2\alpha f(\alpha^2) + 2\beta f(\beta^2)$.

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116. Prove that $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx = \int_0^{\infty} (\sin x) x dx$

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117. If $\int_0^{\frac{\pi}{2}} \log \sin \theta d th\eta = k$, then find the value of $\int_{\pi}^{\frac{\pi}{2}} \left(\frac{\theta}{s \int h\eta}\right)^2 d th\eta$ in terms of k

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118. Evaluate : $\int_0^{\pi} \frac{x^2 \sin 2x \cdot \sin\left(\frac{\pi}{2} \cdot \cos x\right)}{2x - \pi} dx$

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119. Find the value of $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\pi + 4x^3}{2 - \cos\left(|x| \frac{\pi}{3}\right)} dx$

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120. It is known that $f(x)$ is an odd function and has a period p. Prove that $\int_a^x f(1) dt$ is also periodic function with the same period.

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121. Evaluate: $\int_0^{\frac{\pi}{4}} \left(\tan^{-1}\left(\frac{2\cos^2\theta}{2 - \sin 2\theta}\right)\right) \sec^2 \theta dth\eta\cdot$

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122. If $f(x)=\frac{\sin x}{x}\forall x\in(0,\pi]$, prove that $\frac{\pi}{2}\int_0^{\frac{\pi}{2}} f(x)f\left(\frac{\pi}{2}-x\right)dx=\int_0^{\pi} f(x)dx$

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123. Let $f(x)$ be a continuous function $\forall x\in R$, except at $x=0$, such that $\int_x^a \frac{f(t)}{t}dt, provethat \int_0^a f(x)dx=\int_0^a g(x)dx$

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124. If $\xi nt_0^x \sin(f(t))dt=(x+2)\int_0^x t\sin(f(t))dt, wherex>0,$ then show that $f'(x)\cot f(x)+\frac{3}{1+x}0.$

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125. Show that: $\int_0^{\pi/2} f(\sin 2x)\sin x dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x dx.$

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126. Let $a + b = 4, wherea < 2, andletg(x)$ be a differentiable function. If $\frac{dg}{dx} > 0$ for all x, prove that $\int_0^a g(x)dx + \int_0^b g(x)dx \in crerasesas(b-a) \in crerases.$

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127. If" f, is a continuous function with $\int_0^x f(t)dt \to \infty$ as $|x \mapsto \infty$then show that every line $y = mx$ intersects the curve $y^2 + \int_0^x f(t)dt = 2$

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128. If $f(x + f(y)) = f(x) + y \forall x, y \in R and f(0) = 1,$ then prove that $\int_0^2 f(2-x)dx = 2\int_0^1 f(x)dx.$

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129. Suppose f is a real-valued differentiable function defined on $[1, \infty]$ with $f(x1) = 1.$ Moreover, suppose that f satisfies $f'(x) = \frac{1}{x^2 + f^2(x)} Showthat f(x) < 1 + \frac{\pi}{4} \forall x \geq 1.$

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130. Let f be a continuous function on $[a, b]\cdot$ If $F(x) = \left(\int_a^x f(t)dt - \int_x^b f(t)dt\right)(2x - (a + b)),$ then prove that there exist some $c \in (a, b)$ such that $\int_a^c f(t)dt - \int_c^b f(t)dt = f(c)(a + b - 2c)\cdot$

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131. $f(x)$ is a continuous and bijective function on $R\cdot$ If $\forall t \in R,$ then the area bounded by $y = f(x), x = a - t, x = a,$ and the x-axis is equal to

the area bounded by $y = f(x), x = a + t, x = a,$ and the x-axis. Then prove that $\int_{-\lambda}^{\lambda} f^{-1}(x)dx = 2a\lambda(giventhatf(a) = 0)\cdot$

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132. $Iff(x) = x + \int_0^1 t(x+t)f(t)dt,$ $thenf \in dthevalueofthedef \in ite \in tegral \int_0^1 f(x)dx\cdot$

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Exercise 8.1

1. Evaluate the following integrals using limit of sum.

$$\int_a^b \cos x dx$$

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2. Evaluate the following integrals using limit of sum.

$$\int_a^b x^3 dx$$

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3. Find the value of $\int_0^4 [x]dx$, where $[.]$ represents the gretest integer function.

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4. If

$f(x) = \{1 - |x|, |x| \leq 10, |x| > 1' and g(x) = f(x-1) + f(x+1),$

find the value of $\int_{-3}^{5} g(x)dx.$

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Exercise 8.2

1. Consider the integral $I = \int_0^{2\pi} \frac{dx}{5 - 2\cos x}$

Making the substitution $\tan \frac{1}{2} x = t$, we have

$$I = \int_0^{2\pi} \frac{dx}{5 - 2\cos x} = \int_0^0 \frac{2dt}{(1 + t^2)[5 - 2(1 - t^2)/(1 + t^2)]} = 0$$

The result is obviously wrong, since the integrand is positive and consequently the integral of this function cannot be equal to zero. Find the mistake.

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2. Evaluate the following : $\int_0^{\pi} \frac{dx}{1 + \sin x}$

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3. Evaluate: $\int_1^{\infty} \left(e^{x+1} + e^{3-1}\right)^{-1} dx$

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4. Evaluate: $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)\sqrt{1-x^2}} dx$

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5. Evaluate: $\int_0^1 \frac{2-x^2}{(1+x)\sqrt{1-x^2}} dx$

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6. Evaluate the following : $\int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

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7. Evaluate: $\int_{\pi/6}^{\pi/4} \frac{1+\cot x}{e^x \sin x} dx$

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8. Evaluate $\int_0^1 \frac{e^{-x} dx}{1+e^x}$

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9. Prove that $\int_0^{102} (x-1)(x-2)(x-100) x\left(\frac{1}{(x-1)+\frac{1}{(x-2)}+\frac{1}{(x-100)}} dx = 101! - 100!\right.$

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10. Show that : $\int_0^1 \frac{\log x}{(1+x)} dx = -\int_0^1 \frac{\log(1+x)}{x} dx$

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11. If $\int_0^1 \frac{e^t}{1+t} dt = a$, then find the value of $\int_0^1 \frac{e^t}{(1+t)^2} dt$ in terms of a.

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12. Let f be a one to one continuous function such that $f(2) = 3$ and $f(5) = 6$. Given $\int_2^5 f(x)dx = 17$, then find the value of $\int_3^7 f^{-1}(x)dx$.

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13. Evaluate: $(\lim)_{n \to \infty}\left(\frac{1}{\sqrt{4n^2 - 1}} + \frac{1}{\sqrt{4n^2 - 2^2}} + + \frac{1}{\sqrt{3n^2}}\right)$

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14.

$$\lim_{n \to \infty}\left[\frac{1}{n^2} \cdot \sec^2\left(\frac{1}{n^2}\right) + \frac{2}{n^2} \cdot \sec^2\left(\frac{4}{n^2}\right) + \ldots\ldots\ldots\ldots. + \frac{1}{n} \cdot \sec^2 1\right]$$

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15. Evaluate $(\lim)_{n \to \infty} \sum_{k=1}^{n} \frac{k}{n^2 + k^2}$

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16. Evaluate the following limit:

$$\lim_{n \to \infty} \frac{\sum_{r=1}^{n} \sqrt{r} \sum_{r=1}^{h} \frac{1}{\sqrt{r}}}{\sum_{r=1}^{n} r}$$

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17. Evaluate the following limit:

$$\lim_{n \to \infty} \left[\frac{n!}{n^n}\right]^{1/n}$$

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Exercise 8.3

1. $Provethat4 \leq \int_1^3 \sqrt{3+x^2} \leq 4\sqrt{3}$

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2.

$$If I_1 = \int_0^1 2x,\ I_2 = \int_0^1 2^x \text{ ^ } 3dx,\ I_3 = \int_1^{22} \text{ ^ } x^2dx,\ I_4 = \int_1^2 2^x \text{ ^ } 3dx,$$

then which of the following is/are ture? $I_1 > I_2$ (b) $I_2 > I_2$ $I_3 > I_4$ (d)

`I_3

A. $I_1 > I_2$

B. $I_2 > I_1$

C. $I_3 > I_4$

D. $I_3 < I_4$

Answer: A::D

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3.

$$If I_I = \int_0^{\pi/2} \cos(\sin x)dx,\ I_2 = \int_0^{\frac{\pi}{2}} \sin(\cos x)d,\ and I_3 = \int_0^{\frac{\pi}{2}} \cos x dx,$$

then find the order in which the values $I_1, I_2, I_3,$ exist.

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4. Prove that `pi/6

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Exercise 8.4

1. Evaluate $\int_0^{\pi/2} |\sin x - \cos x| dx.$

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2. Evaluate: $\int_{-1}^{4} f(x)dx = 4 and \int_2^4 (3 - f(x))dx = 7,$ then find the value of $\int_2^{-1} f(x)dx \cdot$

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3. Evaluate $\int_1^5 \sqrt{x-2\sqrt{x-1}}dx.$

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4. Evaluate: $\int_{-1}^{3}\left(\frac{\tan^{-1}d}{x^2+1}+\frac{x^2+1}{x}\right)dx$

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5. Evaluate $\int_1^a x.\,a^{-[\log_e x]}dx, (a>1)$.Here $[.]$ represents the greatest integer function.

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6. Evaluate $\int_1^{e^6}\left[\frac{\log x}{3}\right]dx,$ where [.] denotes the greatest integer function.

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7. Find the value of $\int_{-1}^{1}\left[x^2+\{x\}\right]dx, where[.]and\{.\}$ denote the greatest function and fractional parts of x, respectively.

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8. Prove that $\int_0^x\left[\cot^{-1}x\right]dx, where[.]$ denotes the greatest integer function.

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9. Prove that $\int_0^x [t]dt = \frac{[x]([x]-1)}{2} + [x](x-[x]),$ where [.] denotes the greatest integer function.

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10. Evaluate: $\int_0^\infty\left[2e^{-e}\right]dx, where[x]$ represents greatest integer function.

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Exercise 8.5

1. If $f(a+b-x)=f(x)$, then prove that $\int_a^b x f(x)dx \frac{a+b}{2}\int_a^b f(x)dx \cdot$

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2. The value of the integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is

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3. Find the value of $\int_0^1 \sqrt[3]{2x(3)-3x^2-x+1dx}.$

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4. Show that $\int_0^{\pi} fx(\sin x)dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx\cdot$

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5. Find the value of $\int_0^1 x(1-x)^n dx$

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6. If a continuous function f on $[0, a]$ satisfies $f(x)f(a-x) = 1, a > 0,$ then find the value of $\int_0^a \frac{dx}{1+f(x)}$.

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7. If $fandg$ are continuous function on $[0, a]$ satisfying $f(x) = f(a-x)andg(x)(a-x) = 2,$ then show that $\int_0^a f(x)g(x)dx = \int_0^a f(x)dx\cdot$

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8. Find the value of $\int_0^{\pi/2} \sin 2x \log \tan x dx.$

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9. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}. dx$,a>0 is

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10. Find the value of $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

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11. Evaluate $\int_0^{\pi} \frac{x dx}{1+\cos\alpha \sin x}$,where $0 < \alpha < \pi.$

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Exercise 8.6

1. Find the value of $\int_0^{2\pi} \frac{1}{1+\tan^4 x} dx$

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2. $\int_0^{2\pi} \sin^{100} x \cos^{99} x dx$

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3. For $U_n = \int_0^1 x^n (2-x)^n dx; V_n = \int_0^1 x^n (1-x)^n dx n \in N,$ which of the following statement(s) is/are true? $U_n = 2^n V_n$ (b) $U_n = 2^{-n} V_n$ $U_n = 2^{2n} V_n$ (d) $V_n = 2^{-2n} U_n$

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4. Evaluate: $\int_0^{\pi} \log(1+\cos x) dx$

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5. Find the value of $\int_0^1 \{(\sin^{-1} x)/x\}dx$

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6. Evaluate $\int_{-\infty}^{0} \frac{te^t}{\sqrt{1-e^{2t}}}dt$

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7. If $I_1 = \int_0^{\pi} x f(\sin^3 x + \cos^2 x)dx and$
$I_2 = \int_0^{\frac{\pi}{2}} f(\sin^3 x + \cos^2 x)dx, then relate I_1 and I_2$

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Exercise 8.7

1. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos^2 x(\sin x + \cos x)dx$

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2. Evaluate: $\int_{-1}^{1} \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx$

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3. Evaluate the following: $\int_{-\pi}^{\pi} (1 - x^2) x \sin x \cos^2 x dx$

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4. Evaluate the following: $\int_{-1}^{1} \frac{\sin x - x^2}{3 - |x|} dx$

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5. Evaluate the following: $\int_{-1/2}^{1/2} \cos x \log \frac{1 - x}{1 + x} dx$

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6. $\int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}}\left\{(\pi+x)^3+\cos^2(x+3\pi)\right\}dx$ is equal to (A) $\frac{\pi}{4}-1$ (B) $\frac{\pi^4}{32}$ (C) $\frac{\pi^4}{32}+\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

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Exercise 8.8

1. Evaluate: $\int_0^{100}(x-[x]dx(where[.]$ represents the greatest integer function).

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2. Evaluate: $\int_0^{100\pi}\sqrt{(1-\cos 2x)}dx.$

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3. $If\int_0^{n\pi} f\left(\cos^2 x\right)dx = k\int_0^{\pi} f\left(c0s^2 x\right)dx, thenf \in dthevaluek\cdot$

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4. Evaluate $\int_0^{n\pi + t} (|\cos x| + |\sin x|)dx,$ where $n\varepsilon N$ and $t\varepsilon[0, \pi/2]$.

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5. Find the value of : $\int_0^{10} e^{2x - [2x]} d(x - [x]) where [.]$ denotes the greatest integer function).

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6. If $f(x)$ is a function satisfying $f(x+a) + f(x) = 0$ for all $x \in R$ and positive constant a such that $\int_b^{c+b} f(x)dx$ is independent of $b,$ then find the least positive value of $\cdot$

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7. Show that $\int_0^{n\pi+v}|\sin x|dx = 2n+1-\cos v$, where n is a positive integer and `,lt=v

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Exercise 8.9

1. $If\int_{\frac{\pi}{3}}^{x}\sqrt{(3-\sin^2 t)}dt+\int_0^y \cos tdt = 0, the \neq valuate\frac{dy}{dx}$

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2. $Iff(x) = e^{g(x)}andg(x) = \int_2^x \frac{tdt}{1+t^4}$, then find the value of $f'(2)$

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3. Evaluate $\left(\lim\right)_{x \xrightarrow{} 4} \int_4^x \frac{(4t - f(t))}{(x - 4)} dt$

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4. Evaluate: $(\lim)_{x \xrightarrow{} 2} \frac{\int 0\text{xcos}\, t^2 dt}{x}$

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5. Find the points of minima for $f(x) = \int_0^x t(t-1)(t-2)dt$

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6. Find the equation of tangent to $y = \int_{x^2}^{x^3} \frac{dt}{\sqrt{1+t^2}} atx = 1.$

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7. $If f(x) = \int_{\frac{x^2}{16}}^{x^2} \frac{\sin x \sin \sqrt{\theta}}{1 + \cos^2 \sqrt{\theta}} dth\eta,$ then find the value of $f'\left(\frac{\pi}{2}\right)$.

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8. Let $f(x)$ be a continuous and differentiable function such that $f(x) = \int_0^x \sin\left(t^2 - t + x\right) dt \dot{T} henprovethat f^x + f(x) = \cos x^2 + 2x \sin x$

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9. Let $f(x)$ be a differentiable function satisfying $f(x) = \int_0^x e^{\left(2tx - t^2\right)} \cos(x - t) dt,$ then find the value of $f''(0)$.

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Exercise 8.10

1. $If \int_0^1 \frac{e^t dt}{t + 1} = a, the \neq valuate \int_{b-1}^{b} \frac{e^{-t} dt}{t - b - 1}$

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2. If $f(x)=\int_1^x (\log t)(1+t+t^2)\,dt\,\forall x\geq 1,$ then prove that $f(x)=f\left(\frac{1}{x}\right)$.

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3. $f(x)=\int_1^x \frac{\tan^{-1}(t)}{t}dt\,\forall x\in R^+, thenf\in dthevalueof$ $f(e^2)-f\left(\frac{1}{e^2}\right)$

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4. Evaluate: $\int_{\sqrt{2}}^{\sqrt{2}+1}\frac{(x^2-1)}{(x^2+1)^2}dx$

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5. Evaluate: $\int_0^{e-1} \frac{\frac{x^2+2x-1}{2}}{x+1} dx + \int_1^e x \log x e^{\frac{x^2-2}{2}} dx$

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6. Find the value of $\int_{\frac{1}{2}}^{2} e^{\left|x - \frac{1}{x}\right|} dx$.

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7. If $I_1 = \int_0^1 \frac{dx}{e^x(1+x)}$ and $I_2 = \int_0^{\pi/4} \frac{e^{\tan^7\theta}\sin\theta}{(2-\tan^2\theta)\cos^3\theta d\theta}$,then find the value of $\frac{l_1}{l_2}$.

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Exercise 8.11

1. If $I_K = \int_1^e (1nx)^k dx \left(k \in I^+\right) dx \left(k \in I^+\right)$, then find the value of I_4.

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2. Given $I_m = \int_1^e (\log x)^m dx, then prove that \frac{I_m}{1-m} + mI_{m-2} = e$

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3. If $I_n = \int_0^\pi x^n \sin x dx$, then find the value of $I_5 + 20I_3$.

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4. If $L(m, n) = \int_0^1 t^m (1+t)^n, dt$, then prove that

$$L(m, n) = \frac{2^n}{m+1} - \frac{n}{m+1} L(m+1, n-1)$$

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5. $If I_n = \int_0^1 x^n (\tan^{-1} x) dx, then prove that$

$$(n+1)I_n + (n-1)I_{n-2} = -\frac{1}{n} + \frac{\pi}{2}$$

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6. $IfI_{m,n}=\int_0^{\frac{\pi}{2}}\sin^m x\cos^n x dx,$ Then show that $I_{m,n}=\frac{m-1}{m+n}I_m-2n(m,n\in N)$ Hence, prove that $I_{m,n}=f(x)=\{\frac{(n-1)(n-3)(m-5)(n-1)(n-3)(n-5)}{(m+n)(m+n-2)(m+n-4)}\frac{\pi}{4}when \perp$

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Exercise (Single)

1. Let $f(x)=\lim_{n\to\infty}\frac{1}{n}\left(\left(x+\frac{1}{n}\right)^2+\left(x+\frac{2}{n}\right)^2+\ldots\ldots\ldots+\left(x+\frac{n-1}{n}\right)\right)$

Then the minimum value of $f(x)$ is

A. $1/4$

B. $1/6$

C. $1/9$

D. $1/12$

Answer: D

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2. $IfS_n = \left[\frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + + \frac{1}{n+\sqrt{n^2}}\right], then(\lim)_{n \overrightarrow{\infty}} S_n$ is equal to log 2 (b) log 4 log 8 (d) none of these

A. $\log 2$

B. $\log 4$

C. $\log 8$

D. none of these

Answer: B

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3. The value of $(\lim)_{n \overrightarrow{\infty}} \sum_{r=1}^{4n} \frac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r}+\sqrt{n}\right)^2}$ is equal to

A. $\frac{1}{35}$

B. $\frac{1}{14}$

C. $\frac{1}{10}$

D. $\frac{1}{5}$

Answer: C

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4. The value of

$$\lim_{n \to \infty} \left(\frac{1^2 + 2^2 + \dots\dots\dots + n^2 1^3 + 2^3 + \dots\dots\dots + n^3 \left(1^4 + 2^4 + \dots\right.}{\left(1^5 + 2^5 + \dots\dots\dots\dots + n^5\right)^2} \right.$$

is equal to

A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{2}{5}$

D. $\frac{1}{5}$

Answer: A

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5. $The value of (\lim)_{n \overrightarrow{\infty}} \left[tan\frac{\pi}{2n}\frac{\tan(2\pi)}{2n}\frac{\tan(\dot{n}\pi)}{2n}\right]^{1/n} is\ e$ (b) e^2 (c) 1 (d) e^3

A. e

B. e^2

C. 1

D. e^3

Answer: C

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6. $\int_{2-a}^{2+a} f(x)dxisequa < o[wheref(2-\alpha) = f(2+\alpha)\,\forall\alpha \in R]$ 2 $\int_{2}^{2+a} f(x)dx$ (b) $2\int_{0}^{a} f(x)dx$ $2\int_{2}^{2} f(x)dx$ (d) none of these

A. $2\int_{2}^{2+a} f(x)dx$

B. $2\int_{0}^{a} f(x)dx$

C. $2\int_{2}^{2} f(x)dx$

D. none of these

Answer: A

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7. If $f(x) = \min(\{x\}, \{-x\})x \in R$, where {x} denotes the fractional part of x, then $\int_{-100}^{100} f(x)dx$ is

A. 50

B. 100

C. 200

D. none of these

Answer: A

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8. Which of the following is incorrect ?

A. $\int_{a+c}^{b+c} f(x)dx = \int_{a}^{b} f(x+c)dx$

B. $\int_{ac}^{bc} f(x)dx = c\int_{a}^{b} f(cx)dx$

C. $\int_{-a}^{a} f(x)dx = \frac{1}{2}\int_{-a}^{a} f(x) + f(-x)dx$

D. none of these

Answer: D

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9. $\int_{-1}^{\frac{1}{2}} \frac{e^x (2 - x^2) dx}{(1 - x)\sqrt{1 - x^2}}$ is equal to

A. $\frac{\sqrt{e}}{2}(\sqrt{3} + 1)$

B. $\frac{\sqrt{3e}}{2}$

C. $\sqrt{3e}$

D. $\sqrt{\frac{e}{3}}$

Answer: C

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10. $If \int_{\log 2}^{x} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}, thenxisequal \rightarrow$

A. 4

B. $In8$

C. $In4$

D. none of these

Answer: C

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11. $\int_{\frac{5}{2}}^{5} \frac{\sqrt{(25-x^2)^3}}{x^4} dxisequa < o \frac{\pi}{6}$ (b) $\frac{2\pi}{3}$ $\frac{5\pi}{6}$ (d) $\frac{\pi}{3}$

A. $\frac{\pi}{6}$

B. $\frac{2\pi}{3}$

C. $\frac{5\pi}{6}$

D. $\frac{\pi}{3}$

Answer: D

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12. If $f(x)$ satisfies the condition of Rolle's theorem in $[1, 2]$, then $\int_1^2 f'(x)dx$ is equal to (a) 1 (b) 3 (c) 0 (d) none of these

A. 1

B. 3

C. 0

D. none of these

Answer: C

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13. The value of the integral $\int_0^{\log 5} \frac{e^x\sqrt{e^x - 1}}{e^x + 3} dx$

A. $3 + 2\pi$

B. $4 - \pi$

C. $2 + \pi$

D. none of these

Answer: B

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14. The value of the integral $\int_0^1 \frac{dx}{x^2 + 2x\cos\alpha + 1}, 0 < \alpha < \pi$ is

A. $\sin\alpha$

B. $\alpha\sin\alpha$

C. $\frac{\alpha}{\sin\alpha}$

D. $\frac{\alpha}{2}\sin\alpha$

Answer: C

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15. $\int_0^\infty \frac{dx}{\left[x + \sqrt{x^2+1}\right]^3} isequa < o \frac{3}{8}$ (b) $\frac{1}{8}$ (c) $-\frac{3}{8}$ (d) none of these

A. $\frac{3}{8}$

B. $\frac{1}{8}$

C. $-\frac{3}{8}$

D. none of these

Answer: A

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16. If $f(y) = e^y, g(y) = y, y > 0, \text{ and } F(t) = \int_0^t f(t-y)g(y)dy$, then

A. $F(t) = e^t - (1+t)$

B. $F(t) = te^t$

C. $F(t) = te^{-t}$

D. $F(t) = 1 - e^t(1+t)$

Answer: A

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17. If $P(x)$ is a polynomial of the least degree that has a maximum equal to 6 at $x = 1$, and a minimum equalto 2 at $x = 3$,then $\int_0^1 P(x)dx$ equals:

A. $\frac{17}{4}$

B. $\frac{13}{4}$

C. $\frac{19}{4}$

D. $\frac{5}{4}$

Answer: C

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18. The numbers of possible continuous $f(x)$ defined in $[0, 1]$ for which $I_1 = \int_0^1 f(x)dx = 1, I_2 = \int_0^1 xf(x)dx - a, I_3 = \int_0^1 x^2 f(x)dx = a^2 is / a$
1 (b) ∞ (c) 2 (d) 0

A. 1

B. ∞

C. 2

D. 0

Answer: D

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19. Suppose that F (x) is an antiderivative of $f(x) = \frac{\sin x}{x}, x > 0$, then $\int_1^3 \frac{\sin 2x}{x} dx$ can be expressed as

A. $F(6) - F(2)$

B. $\frac{1}{2}(F(6) - F(2))$

C. $\frac{1}{2}(F(3) - F(1))$

D. $2(F(6) - F(2))$

Answer: A

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20. $\int_{-\frac{\pi}{3}}^{0}\left[\cot^{-1}\left(\frac{2}{2\cos x-1}\right)+\cot^{-1}\left(\cos x-\frac{1}{2}\right)\right]dx isequa < o$

$\frac{\pi^2}{6}$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{3\pi^2}{8}$

A. $\frac{\pi^2}{6}$

B. $\frac{\pi^2}{3}$

C. $\frac{\pi^2}{8}$

D. $\frac{3\pi^2}{8}$

Answer: A

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21. Evaluate the definite integrals $\int 0\frac{\pi}{4}\frac{\sin x+\cos x}{9+16\sin 2x}dx$

A. $\frac{1}{20}\log 3$

B. $\frac{1}{40}\log 3$

C. $\frac{1}{20}\log 6$

D. $10 \log 3$

Answer: A

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22. $\int_{-1}^{1} \frac{e^{-\frac{1}{x}}}{x^2\left(1+e^{-\frac{2}{x}}\right)} dx$ is equal to :

A. $\frac{\pi}{2} = 2\tan^{-1} e$

B. $\frac{\pi}{2} - 2\cot^{-1} e$

C. $2\tan^{-1} e$

D. $\pi - 2\tan^{-1} e$

Answer: D

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23. If $\int_{0}^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$, then $\int_{0}^{\infty} \frac{\sin^3 x}{x} dx$ is equal to

A. $\pi/2$

B. $\pi/4$

C. $\pi/6$

D. $3\pi/2$

Answer: B

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24. The range of the function $f(x)=\int_{-1}^{1}\frac{\sin x dt}{1+2t\cos x+t^2}$ is

A. $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

B. $[0,\pi]$

C. $\{0,\pi\}$

D. $\left\{-\frac{\pi}{2},\frac{\pi}{2}\right\}$

Answer: D

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25. If the function $f:[0,8] \to R$ is differentiable, then for $0 < \beta < 1$ and $0 < \alpha < 2$, $\int_0^8 f(t)dt$ is equal to

A. $3\left[\alpha^3 f(\alpha^2) + \beta^2 f(\beta^2)\right]$

B. $3\left[\alpha^3 f(\alpha) + \beta^3 f(\beta)\right]$

C. $3\left[\alpha^2 f(\alpha^3) + \beta^2 f(\beta^3)\right]$

D. $3\left[\alpha^2 f(\alpha^2) + \beta^2 f(\beta^2)\right]$

Answer: C

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26. If $f(x) = x^5 + 5x - 1$ then $\int_5^{41} \frac{dx}{(f^{-1}(x))^5 + 5f^{-1}(x)}$ equals

A. 0

B. $\log_e 3$

C. $\log_e 4$

D. $\log_e 7$

Answer: D

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27. Let $f(0) = 0 and \int_0^2 f'(2t)e^{f(2t)}dt = 5. thenvalueoff f(4) is$ log 2 (b) log 7 (c) log 11 (d) log 13

A. $\log 2$

B. $\log 7$

C. $\log 11$

D. $\log 13$

Answer: C

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28. If $f(x) - 3\cos\left(\tan^{-1} x\right)$, then the value of the integral $\int_0^1 x f''(x) dx$ is

A. $\frac{3-\sqrt{2}}{2}$

B. $\frac{3+\sqrt{2}}{2}$

C. 1

D. $1 - \frac{3}{2\sqrt{2}}$

Answer: D

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29. The equation of the curve is $y = f(x)$. The tangents at $[1, f(1), [2, f(2)], and[3, f(3)]$ make angles $\frac{\pi}{6}, \frac{\pi}{3}, and \frac{\pi}{4}$, respectively, with the positive direction of x-axis. Then the value of $\int_2^3 f'(x) f^x dx + \int_1^3 f^x dx$ is equal to $-\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$ (e) 0 (d) none of these

A. $-1/\sqrt{3}$

B. $1/\sqrt{3}$

C. 0

D. none of these

Answer: A

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30. The value of $\int_1^e \left(\frac{\tan^{-1}x}{x} + \frac{\log x}{1+x^2}\right) dx is$ $\tan e$ (b) $\tan^{-1} e$ $\tan^{-1}\left(\frac{1}{e}\right)$ (d) none of these

A. $\tan e$

B. $\tan^{-1} e$

C. $\tan^{-1}(1/e)$

D. none of these

Answer: B

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31. If $f(\pi) = 2$ and $\int_0^\pi (f(x) + f''(x))\sin x dx = 5$, then $f(0)$ is equal to (it is given that $f(x)$ is continuous in $[0, \pi]$)

A. 7

B. 3

C. 5

D. 1

Answer: B

Watch Video Solution

32. If $\int_1^2 e^x \hat{} 2dx = a, then \int_e^{e^4} \sqrt{1nx} dxisequa < o\ 2e^4 - 2e - a$ (b) $2e^4 - e - a\ 2e^4 - e - 2a$ (d) $e^4 - e - a$

A. $2e^4 - 2e - a$

B. $2e^4 - e - a$

C. $2e^4 - e - 2a$

D. $e^4 - e - a$

Answer: B

Watch Video Solution

33. If $f(x)$ is continuous for all real values of x, then $\sum_{r=1}^{n} f(r - 1 + x)dxisequa < o \int_0^n f(x)dx$ (b) $\int_0^1 f(x)dx \; n\int_0^1 f(x)dx$ (d) $(n-1)\int_0^1 f(x)dx$

A. $\int_0^n f(x)dx$

B. $\int_0^1 f(x)dx$

C. $n\int_0^1 f(x)dx$

D. $(n-1)\int_0^1 f(x)dx$

Answer: A

Watch Video Solution

34. The value of $\int_0^{\frac{\pi}{2}} \sin|2x - \alpha| dx$, where $\alpha \in [0, \pi]$, is

A. $1 - \cos\alpha$

B. $1 + \cos\alpha$

C. 1

D. $\cos\alpha$

Answer: C

Watch Video Solution

35. $f(x)$ is a continuous function for all real values of x and satisfies $\int_n^{n+1} f(x)dx = \frac{n^2}{2} \forall n \in I$. Then $\int_{-3}^{5} f(|x|)dx$ is equal to $\frac{19}{2}$ (b) $\frac{35}{2}$ (c) $\frac{17}{2}$ (d) none of these

A. $19/2$

B. $35/2$

C. $17/2$

D. none of these

Answer: B

Watch Video Solution

36. If $f(x) = \int_{-1}^{x} |t| dt$, then for any $x \geq 0$, $f(x)$ equals

A. $\frac{1}{2}(1 - x^2)$

B. $\frac{1}{2}x^2$

C. $\frac{1}{2}(1 + x^2)$

D. none of these

Answer: C

Watch Video Solution

37. If $a > 0$ and $A = \int_0^a \cos^{-1} x dx$, and $\int_{-a}^{a} \left(\cos^{-1} x - \sin^{-1} \sqrt{1 - x^2}\right) dx = \pi a - \lambda A$. Then λ is

A. 0

B. 2

C. 3

D. none of these

Answer: B

Watch Video Solution

38. The value of $\int_1^a [x] f'(x) dx f'(x) dx, where a > 1, and [x]$ denotes the greatest integer not exceeding x, is $af(a) - \{f(1) f(2) + \; + f([a])\}$ $[a] f(a) - \{f(1) + f(2) + \; + f([a])\}$ $[a] f(a) - \{f(1) + f(2) + \; + fA\}$ $af([a]) - \{f(1) + f(2) + \; + fA\}$

A. $af(a) - (f(1) + f(2) + \ldots\ldots\ldots\ldots + f([a]))$

B. $[a]f(a)-(f(1)+f(2))+\ldots\ldots\ldots+f([a]))$

C. $[a]f([a])-(f(1)+f(2)+\ldots\ldots\ldots+f(a))$

D. $af([a])-(f(1)+f(2)+\ldots\ldots\ldots+f(a))$

Answer: B

Watch Video Solution

39. $\int_3^{10}[\log[x]]dx$ is equal to (where [.] represents the greatest integer function)

A. 9

B. $16-e$

C. 10

D. $10+e$

Answer: A

Watch Video Solution

40. $\int_{-1}^{2}\left[\frac{[x]}{1+x^2}\right]dx$, where [.] denotes the greatest integer function, is equal to

A. -2

B. -1

C. zero

D. none of these

Answer: B

Watch Video Solution

41. The value of $\int_{-g}^{1}\left[x\left[1+\cos\left(\frac{\pi x}{2}\right)\right]+1\right]dx$, where [.] denotes greatest integer function is

A. 1

B. $1/2$

C. 2

D. none of these

Answer: C

Watch Video Solution

42. The value of $\int_0^{2\pi} [2\sin x] dx$, where $[.]$ represents the greatest integral functions, is

A. $\frac{-5\pi}{3}$

B. $-\pi$

C. $\frac{5\pi}{3}$

D. -2π

Answer: B

Watch Video Solution

43. $I_1 = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx,\ I_2 = \int_0^{2\pi} \cos^6 dx,$
$I_3 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x dx,\ I_4 = \int_0^1 In\left(\frac{1}{x} - 1\right) dx$. Then

A. $I_2 = I_3 = I_4 = 0,\ I_1 \neq 0$

B. $I_1 = I_2 = I_3 = 0,\ I_4 \neq 0$

C. $I_1 = I_3 = I_4 = 0,\ I_2 \neq 0$

D. $I_1 = I_2 = I_3 = 0,\ I_4 \neq 0$

Answer: C

Watch Video Solution

44. Given $\int_0^{\pi/2} \frac{dx}{1 + \sin x + \cos x} = A$. Then the value of the definite integral $\int_0^{\pi/2} \frac{\sin x}{1 + \sin x + \cos x} dx$ is equal to

A. $\frac{1}{2}A$

B. $\frac{\pi}{2} - A$

C. $\frac{\pi}{4} - \frac{1}{2}A$

D. $\frac{\pi}{2} + A$

Answer: C

Watch Video Solution

45. $\text{I f} I_1 = \int_{-100}^{101} \frac{dx}{(5 + 2x - 2x^2)(1 + e^{2-4x})}$ $\text{and} I_2 = \int_{-100}^{101} \frac{dx}{5 + 2x - 2x^2}, then \frac{I_1}{I_2}$ i s 2 (b) $\frac{1}{2}$ (c) 1 (d) $-\frac{1}{2}$

A. 2

B. $\frac{1}{2}$

C. 1

D. $-\frac{1}{2}$

Answer: B

Watch Video Solution

46. The value of $\int_0^{\infty} \frac{x dx}{(1+x)(1+x^2)}$ is equal to

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. none of these

Answer: A

Watch Video Solution

47. For any integer n, the integral $\int_0^{\pi} e^{\cos x} \cos^3(2n+1)x dx$ has the value

A. π

B. 1

C. 0

D. none of these

Answer: C

Watch Video Solution

48. Let f be a positive function. If $I_1 = \int_{1-k}^{k} x f[x(1-x)]\,dx$ and $I_2 = \int_{1-k}^{k} f[x(1-x)]\,dx$, where $2k-1>0$. Then $\frac{I_1}{I_2}$ is

A. 2

B. k

C. $\frac{1}{2}$

D. 1

Answer: C

Watch Video Solution

49. $Iff(x) = \frac{e^x}{1+e^x}, I_1 = \int_{f(-a)}^{f(a)} xg(x(1-x)dx, and$

$I_2 = \int_{f(-a)}^{f(a)} g(x(1-x))dx, thenthevalueo\frac{f(I_2)}{I_1}is -1$ (b) -2 (c) 2 (d)

1

A. -1

B. -2

C. 2

D. 1

Answer: C

Watch Video Solution

50. The value of $\int_1^2 \frac{x^2+1}{x^4-x^2+1}\log\left(1+x-\frac{1}{x}\right)dx$ is

A. $\frac{\pi}{8}\log_e 2$

B. $\frac{\pi}{2}\log_e 2$

C. $-\frac{\pi}{2}\log_e 2$

D. none of these

Answer: A

Watch Video Solution

51. The value of the definite integral $\int_0^{\frac{\pi}{2}} \sqrt{\tan x} dx$ is $\sqrt{2}\pi$ (b) $\frac{\pi}{\sqrt{2}}$ $2\sqrt{2}\pi$

(d) $\frac{\pi}{2\sqrt{2}}$

A. $\sqrt{2}\pi$

B. $\frac{\pi}{\sqrt{2}}$

C. $2\sqrt{2}\pi$

D. $\frac{\pi}{2\sqrt{2}}$

Answer: B

Watch Video Solution

52. $f(x) > 0 \forall x \in Randisbounde\ddot{I}f$

$$(\lim)_{n\overrightarrow{\infty}}\left[\int_0^a \frac{f(x)dx}{f(x)+f(a-x)} + a^2 + a\int_a^{2a} \frac{f(x)dx}{f(x)+f(3a-x)} + \int_{2a}^{3a}\right.$$

(where $a < 1$), then a is equal to $\frac{2}{7}$ (b) $\frac{1}{7}$ (c) $\frac{14}{19}$ (d) $\frac{9}{14}$

A. $\frac{2}{7}$

B. $\frac{1}{7}$

C. $\frac{14}{19}$

D. $\frac{9}{14}$

Answer: C

Watch Video Solution

53. $If\int_0^1 \cot^{-1}(1-x+x^2)dx = \lambda\int_0^1 \tan^{-1} xdx, then\lambda isequa < o$ 1

(b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

Answer: B

Watch Video Solution

54. The value of the definite integral $\int_{-1}^{1}(1+x)^{1/2}(1-x)^{3/2}dx$ equals

A. π

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

Answer: D

Watch Video Solution

55. The value of the integral $\int_{-3\pi/4}^{5\pi/4} \frac{(\sin x + \cos x)}{e^{x-\pi/4}+1} dx$ is

A. 0

B. 1

C. 2

D. none of these

Answer: A

Watch Video Solution

56. $I_1 = \int_0^{\frac{\pi}{2}} In(\sin x) dx$, $I_2 = \int_{-\pi/4}^{\pi/4} In(\sin x + \cos x) dx$. Then

A. $I_1 = 2I_2$

B. $I_2 = 2I_1$

C. $I_1 = 4I_2$

D. $I_2 = 4I_1$

Answer: A

Watch Video Solution

57. $If I_1 = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{1+\cos^2 x} dx, I_2 = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\sin^2 x} dx$
$I_3 = \int_0^{\frac{\pi}{2}} \frac{1+2\cos^2 x \sin^2 x}{4+2\cos^2 x \sin^2 x} dx, then \quad I_1 = I_2 > I_3$ (b) $I_3 > I_1 = I_2$
$I_1 = I_2 = I_3$ (d) none of these

A. $I_1 = I_2 > I_3$

B. $I_3 > I_1 = I_2$

C. $I_1 = I_2 = I_3$

D. none of these

Answer: C

Watch Video Solution

58. $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\cos^4 x + \sin^4 x} dx =$

A. $\frac{\pi^2}{2}$

B. $\frac{\pi^2}{4}$

C. $\frac{\pi^2}{8}$

D. $\frac{\pi^2}{16}$

Answer: D

Watch Video Solution

59. For $x \varepsilon R$, and a continuous function f let $I_1 = \int_{\sin^2 t}^{1+\cos^2 t} x f\{x(2-x)\} dx$ and $I_2 = \int_{\sin^2 t}^{1+\cos^2 t} f\{x(2-x)\} dx$. Then $\frac{I_1}{I_2}$ is

A. -1

B. 1

C. 2

D. 3

Answer: B

Watch Video Solution

60.

$If\int_{-\pi}^{\frac{3\pi}{4}}\frac{e^{\frac{\pi}{4}}dx}{\left(e^x+e^{\frac{\pi}{4}}\right)(\sin x+\cos x)=k\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sec x dx, thenthevalueofkis}$

$\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $-\frac{1}{\sqrt{2}}$

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{2\sqrt{2}}$

D. $-\frac{1}{\sqrt{2}}$

Answer: C

Watch Video Solution

61. The value of the definite integral $\int_2^4 (x(3-x)(4+x)(6-x)(10-x)+\sin x)dx$ equals

A. $\cos 2 + \cos 4$

B. $\cos 2 - \cos 4$

C. $\sin 2 + \sin 4$

D. $\sin 2 - \sin 4$

Answer: B

Watch Video Solution

62. If $I = \int_{-20\pi}^{20\pi} |\sin x| [\sin x] dx$ (where [.] denotes the greatest integer function) then the value of I is

A. -40

B. 40

C. 20

D. -20

Answer: A

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63. The function f and g are positive and continuous. If f is increasing and g is decreasing, then $\int_0^1 f(x)[g(x)-g(1-x)]dx$ is always non-positive is always non-negative can take positive and negative values none of these

A. is always non-positive

B. is always non-negative

C. can take positive and negative values

D. none of these

Answer: A

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64. $\int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx$ is

A. $\frac{\pi^2}{4}$

B. $\frac{\pi^2}{2}$

C. $\frac{3\pi^2}{2}$

D. $\frac{\pi^2}{3}$

Answer: A

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65. If $f(x) = \int_0^{\pi} \frac{t \sin t dt}{\sqrt{1 + \tan^2 x \sin^2 t}}$ for $0 < x < \frac{\pi}{2}$ then

A. $f(0^+) = -\pi$

B. $f\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8}$

C. f is continuous and differntiable in $\left(0, \frac{\pi}{2}\right)$

D. f is continuous but not differentiable in $\left(0, \frac{\pi}{2}\right)$

Answer: C

Watch Video Solution

66. $\int_{-3}^{3} x^8\{x^{11}\}dx$ is equal to (where {.} is the fractional part of x)

A. 3^8

B. 3^7

C. 3^9

D. none of these

Answer: B

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67. The value of $\int_0^{4\pi} \log_e\left|3\sin x + 3\sqrt{3}\cos x\right| dx$ then the value of I is equal to

A. $\pi \log_e 3$

B. $2\pi \log_e 3$

C. $4\pi \log_e 3$

D. $8\pi \log_e 3$

Answer: C

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68. The value of $\int_0^{\pi} \frac{|x|\sin^2 x}{1 + 2 + \cos x \mid \sin x} dx$ is equal to

A. $\pi / 4$

B. $\pi / 2$

C. π

D. 2π

Answer: B

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69. The value f the integral $\int_{-\pi}^{\pi} \sin mx \sin nx dx,$ for $m \neq n(m, n \in I), is$ 0 (b) π (c) $\frac{\pi}{2}$ (d) 2π

A. 0

B. π

C. $\pi / 2$

D. 2π

Answer: A

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70. If $f(x)$ and $g(x)$ are continuous functions, then $\int_{In\lambda}^{In(1/\lambda)}\frac{f(x^2/4)[f(x)-f(-x)]}{g(x^2/4)[g(x)+g(-x)]}dx$ is

A. depenent on λ

B. a non zero constant

C. zero

D. none of these

Answer: C

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71. The value of $\int_0^1\frac{\tan^{-1}\left(\frac{x}{x+1}\right)}{0\tan^{-1}\left(\frac{1+2x-2x^2}{2}\right)}dx$ is

A. $1/4$

B. $1/2$

C. 1

D. 2

Answer: B

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72. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^{|\sin x|}\cos x}{(1+e^{\tan x})dxisequa < o}$ $e+1$ (b) $1-e$ $e-1$ (d) none of these

A. $e+1$

B. $2e$

C. $e-1$

D. $e-2$

Answer: C

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73. The value of $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is

A. π

B. π^2

C. $2\pi^2$

D. $\pi^2/2$

Answer: B

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74. [The value of int_(-pi)^(pi)sum_(r=0)^(999)cos rx(1+sum_(r=1)^(999)sin rx)dx, is [(1) 2 pi, (2) 999 pi, (3) 0]]

A. 2π

B. 999π

C. 0

D. π

Answer: A

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75. Let $T > 0$ be a fixed real number. Suppose f is continuous function such that for all $x \in R$, $f(x+T) = f(x)\cdot$ If $I = \int_0^T f(x)dx,$ then the value of $\int_3^{3+3T} f(2x)dx$ is $\frac{3}{2}I$ (b) $2I$ (c) $3I$ (d) $6I$

A. $\frac{3}{2}I$

B. $2I$

C. $3I$

D. $6I$

Answer: C

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76. $\int_1^4 (x - 0.4) dx equals (where \{x\} is a actional par \rightarrow f(x)$ 13 (b) 6.3 (c) 1.5 (d) 7.5

A. 13

B. 6.3

C. 1.5

D. 7.5

Answer: C

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77. The value of $\int_0^x [\cos t] dt, x \in \left[(4n+1)\frac{\pi}{2}, (4n+3)\frac{\pi}{2}\right] and n \in N,$ is equal to where [.] represents greatest integer function. $\frac{\pi}{2}(2n-1) - 2x$ $\frac{\pi}{2}(2n-1) + x$ $\frac{\pi}{2}(2n+1) - x$ (d) $\frac{\pi}{2}(2n+1) + x$

A. $\frac{\pi}{2}(2n-1) - 2x$

B. $\frac{\pi}{2}(2n-1) + x$

C. $\frac{\pi}{2}(2n+1)-x$

D. $\frac{\pi}{2}(2n+1)+x$

Answer: C

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78. $\int_0^x [\sin t]dt, where x \in (2n\pi, (2n+1)\pi), n \in N, and [.]$ denotes the greatest integer function is equal to $-n\pi$ (b) $-(n+1)\pi$ $2n\pi$ (d) $-(2n+1)\pi$

A. $4n-\cos x$

B. $4n-\sin x$

C. $4n+1-\cos x$

D. $4n-1-\cos x$

Answer: C

Watch Video Solution

79. $\int_0^x \frac{2^t}{2^{[t]}} dt$, where [.] denotes the greatest integer function, and $x \varepsilon R^+$ is equal to

A. $\frac{1}{1n2}\left([x]+2^{\{x\}}-1\right)$

B. $\frac{1}{1n2}\left([x]+2^{\{x\}}\right)$

C. $\frac{1}{1n2}\left([x]-2^{\{x\}}\right)$

D. $\frac{1}{1n2}\left([x]+2^{\{x\}}+1\right)$

Answer: A

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80. f is an odd function, It is also known that $f(x)$ is continuous for all values of x and is periodic with period 2. If $g(x)=\int_0^x f(t)dt$, then $g(x) is odd$ (b) $g(n)=0, n \in N$ $g(2n)=0, n \in N$ (d) $g(x)$ is non-periodic

A. $g(x)$ is odd

B. $2(n) = 0, n\varepsilon N$

C. $g(2n) = 0, n\varepsilon N$

D. $g(x)$ is non-periodic

Answer: C

Watch Video Solution

81. $Ifg(x) = \int_0^x (|\sin t| + |\cos t|)dt, theng\left(x + \frac{\pi n}{2}\right)$ is equal to, where $n \in N$, $g(x) + g(\pi)$ (b) $g(x) + g\left(\frac{n\pi}{n2}\right)$ $g(x) + g\left(\frac{\pi}{2}\right)$ (d) none of these

A. $g(x) + g(\pi)$

B. $g(x) + ng\left(\frac{\pi}{2}\right)$

C. $g(x) + g\left(\frac{\pi}{2}\right)$

D. none of these

Answer: B

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82. $Ifx = \int_c^{\sin t} \sin^{-1} zdz, y = \int_k^{\sqrt{t}} \frac{\sin z^2}{z} dz, then \frac{dy}{dx} isequa < o$

$\frac{\tan t}{2t}$ (b) $\frac{\tan t}{t^2}$ $\frac{t}{2t^2}$ (d) $\frac{ta\ \hat{}\ 2}{2t^2}$

A. $\frac{\tan t}{2t}$

B. $\frac{\tan t}{t^2}$

C. $\frac{\tan t}{2t^2}$

D. $\frac{\tan t^2}{2t^2}$

Answer: C

Watch Video Solution

83. Let $f(x) = \int_2^x \frac{dt}{\sqrt{1+t^4}}$ and g be the inverse of f then the value of

$g'(0)$ is

A. 1

B. 17

C. $\sqrt{17}$

D. none of these

Answer: C

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84. If $f(x)$ is differentiable and $\int_0^{t^2} x f(x) dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals $\frac{2}{5}$ (b) $-\frac{5}{2}$ 1 (d) $\frac{5}{2}$

A. $2/5$

B. $-5/2$

C. 1

D. $5/2$

Answer: A

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85. If $f(x) = \cos x - \int_0^x (x-t)f(t)dt, then f'(x) + f(x)$ is equal to $-\cos x$ (b) $-\sin x \int_0^x (x-t)f(t)dt$ (d) 0

A. $-\cos x$

B. $-\sin x$

C. $\int_0^x (x-t)f(t)dt$

D. 0

Answer: A

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86. A function f is continuous for all x (and not everywhere zero) such that $f^2(x) = \int_0^x f(t)\frac{\cos t}{2+\sin t}dt$. Then $f(x)$ is

A. $\frac{1}{2}In\left(\frac{x+\cos x}{2}\right)$

B. $\frac{1}{2}In\left(\frac{3}{2+\cos x}\right)$

C. $\frac{1}{2}In\left(\frac{2+\sin x}{2}\right)$

D. $\frac{\cos x+\sin x}{2+\sin x}$

Answer: C

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87. $\lim_{x\to 0}\frac{1}{x}\left[\int_{y\to a} e^{\sin^2 t}dt-\int_{x+y\to a} e^{\sin^2 t}dt\right]$ is equal to

A. $e^{\sin^2 y}$

B. $\sin 2ye^{\sin^2 y}$

C. 0

D. none of these

Answer: A

Watch Video Solution

88. Let $f(x) = \int_1^x \frac{e^t}{t} dt, x \in R^+$. Then complete set of valuesof x for which $f(x) \leq In$ x is

A. $(0, 1]$

B. $[1, \infty)$

C. $(0, \infty)$

D. none of these

Answer: A

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89. If $\int_0^x f(t)dt = x + \int_x^1 f(t)dt$,then the value of $f(1)$ is

A. $1/2$

B. 0

C. 1

D. $-1/2$

Answer: A

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90. $Iff(x) = 1 + \frac{1}{\xi} nt_1^x f(t)dt, thenthevalueof\left(e^{-1}\right)is$ 1 (b) 0 (c) -1

(d) none of these

A. 1

B. 0

C. -1

D. none of these

Answer: B

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91. If $\left[f\left(\frac{\sqrt{3}}{2}\right)\right]$ is [.] denotes the greatest integer function) 4 (b) 5 (c) 6 (d) -7

A. 4

B. 5

C. 6

D. -7

Answer: B

Watch Video Solution

92. $f(x)$ is continuous function for all real values of x and satisfies $\int_0^x f(t)dt = \int_x^1 t^2 f(t)dt + \frac{x^{16}}{8} + \frac{x^6}{3} + a\cdot$ Then the value of a is equal to: $-\frac{1}{24}$ (b) $\frac{17}{168}$ (c) $\frac{1}{7}$ (d) $-\frac{167}{840}$

A. $-\frac{1}{24}$

B. $\frac{17}{168}$

C. $\frac{1}{7}$

D. $-\frac{167}{840}$

Answer: D

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93. the value of $\int_{\frac{1}{e} \to \tan x} \frac{tdt}{1+t^2} + \int_{\frac{1}{e} \to \cot x} \frac{dt}{t \cdot (1+t^2)} =$

A. 0

B. 2

C. 1

D. none of these

Answer: C

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94. $\lim_{x \to \infty} \frac{\int_0^x \tan^{-1} dt}{\sqrt{x^2+1}}$ is equal to

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. 1

D. π

Answer: A

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95. A function f is defined by $f(x) = \int_0^\pi \cos t \cos(x-t)dt, 0 \leq x \leq 2\pi$ then which of the following.hold(s) good?

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{-\pi}{2}$

D. $\frac{-\pi}{4}$

Answer: C

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96. If f' is a differentiable function satisfying $f(x)=\int_0^x \sqrt{1-f^2(t)}dt+\frac{1}{2}$ then the value of $f(\pi)$ is equal to

A. $-\frac{\sqrt{3}}{2}$

B. $-\frac{1}{2}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1}{2}$

Answer: B

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97. If $\int_0^1 e^{x^2}(x-\alpha)dx = 0$, then

A. $1 < \alpha < 2$

B. $\alpha < 0$

C. $0 < \alpha < 1$

D. $\alpha = 0$

Answer: C

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98. The value of the integral $\int_0^1 e^x \hat{} 2dx$ lies in the interval $(0, 1)$ (b) $(-1, 0)$ $(1, e)$ (d) none of these

A. $(0, 1)$

B. $(-1, 0)$

C. $(1, e)$

D. none of these

Answer: C

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99. Given that f satisfies $|f(u) - f(v)| \le |u - v| f \text{ or } u and v$ in $[a, b]$. Then $\left|\int_a^b f(x)dx - b(b-a)f(a)\right| \le \frac{(b-a)}{2}$ (b) $\frac{(b-a)^2}{2}$ $(b-a)^2$ (d) none of these

A. $\frac{(b-a)}{2}$

B. $\frac{(b-a)^2}{2}$

C. $(b-a)^2$

D. none of these

Answer: B

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100. The value of the integral $\int_0^{\infty} \frac{x \log x}{(1+x^2)^2} dxis$ 0 (b) log 7 (c) 5 log 13

(d) none of these

A. 0

B. $\log 7$

C. $5 \log 13$

D. none of these

Answer: A

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101. $\int_0^{\infty} \left(\frac{\pi}{1+\pi^2 x^2} - \frac{1}{1+x^2}\right) \log x dxisequa < o \quad -\frac{\pi}{21} n\pi$ (b) 0 $\frac{\pi}{21} n2$ (d) none of these

A. $-\frac{\pi}{2} In\pi$

B. 0

C. $\frac{\pi}{2} In2$

D. none of these

Answer: A

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102. If $A = \int_0^{\pi} \frac{\cos x}{(x+2)^2} dx$, then $\int_0^{\pi/2} \frac{\sin 2x}{x+1} dx$ is equal to

A. $\frac{1}{2} + \frac{1}{\pi+2} - A$

B. $\frac{1}{\pi+2} - A$

C. $1 + \frac{1}{\pi+2} - A$

D. $A - \frac{1}{2} - \frac{1}{\pi+2}$

Answer: A

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103. $\int_0^4 \frac{(y^2 - 4y + 5)\sin(y-2)dy}{[2y^2 - 8y + 1]}$ is equal to

A. 0

B. 2

C. -2

D. none of these

Answer: A

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104. $\int_{\sin\theta}^{\cos\theta} f(x\tan\theta)dx$ (where $\theta \neq \frac{n\pi}{2}, n\varepsilon I$) is equal to

A. $-\cos\theta\int_{1}^{\tan\theta} f(x\sin\theta)dx$

B. $-\tan\theta\int_{\cos\theta}^{\sin\theta} f(x)dx$

C. $\sin\theta\int_{1}^{\tan\theta} f(x\cos\theta)dx$

D. $\frac{1}{\tan\theta}\int_{\sin\theta}^{\sin\theta\tan\theta} f(x)dx$

Answer: A

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105. If $I_1 = \int_0^1 \frac{e^x}{1+x} dx$and $I_2 = \int_0^1 \frac{x^2}{e^{x^3}(2-x^3)} dx$ then $\frac{I_1}{I_2}$ is

A. $3/e$

B. $e/3$

C. $3e$

D. $1/3e$

Answer: C

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106. $Let I_1 = \int_{-2}^{2} \frac{x^6+3x^5+7x^4}{x^4+2} dx and$ $I_2 = \int_{-3}^{1} \frac{2(x+1)^2+11(x+1)+14}{(x+1)^4+2} dt$. Then the value of $I_1 + I_2 is$ 8 (b) $\frac{200}{3}$ (c) $\frac{100}{3}$ (d) $none of these$

A. 8

B. $200/3$

C. $100/3$

D. noe

Answer: C

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107. Let f beintegrable over $[0, a]$ for any real value of a.

If $I_1 = \int_0^{\pi/2} \cos\theta f(\sin\theta + \cos^2\theta)\,d\theta$ and $I_2 = \int_0^{\pi/2} \sin 2\theta f(\sin\theta + \cos^2\theta)\,d\theta$, then

A. $I_1 = -2I_2$

B. $I_1 = I_2$

C. $2I_1 = I_2$

D. $I_1 = -I_2$

Answer: B

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108. The value of $\int_a^b (x-a)^3(b-x)^4 dx is$ $\frac{(b-a)^4}{6^4}$ (b) $\frac{(b-a)^8}{280}$ $\frac{(b-a)^7}{7^3}$ (d) none of these

A. $\frac{(b-a)^4}{6^4}$

B. $\frac{(b-a)^8}{280}$

C. $\frac{(b-a)^7}{7^3}$

D. none of these

Answer: B

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109. If $I(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx$, then

A. $I(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m-n}}dx$

B. $I(m,n) = \int_0^{\infty} \frac{x^m}{(1+x)^{m+n}} dx$

C. $I(m,n) = \int_0^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}} dx$

D. $I(m,n) = \int_0^{\infty} \frac{x^n}{(1+x)^{m+n}} dx$

Answer: C

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110. $The value of the def \in ite \in tegral \int_0^{\frac{\pi}{2}} \frac{\sin 5x}{\sin x} dx is$ 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

A. 0

B. $\frac{\pi}{2}$

C. π

D. 2π

Answer: A

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111. If $I_n = \int_0^{\pi} e^x (\sin x)^n dx$, then $\frac{I_3}{I_1}$ is equal to

A. $3/5$

B. $1/5$

C. 1

D. $2/5$

Answer: A

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112. If $f'(x) = f(x) + \int_0^1 f(x)dx$,given $f(0) = 1$, then the value of $f(\log_e 2)$ is

A. $\frac{1}{3+e}$

B. $\frac{5-e}{3-e}$

C. $\frac{2+e}{e-2}$

D. none of these

Answer: B

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113. Let $f(x)$ be positive, continuous, and differentiable on the interval $(a, b) and (\lim)_{x \vec{a}^+} f(x) = 1, (\lim)_{x \vec{b}^-} f(x) = 3^{\frac{1}{4}} \dot{I} f f f'(x) \geq f^3(x) + \frac{1}{f(x)}$ then the greatest value of $b - a$ is $\frac{\pi}{48}$ (b) $\frac{\pi}{36}$ $\frac{\pi}{24}$ (d) $\frac{\pi}{12}$

A. $\frac{\pi}{48}$

B. $\frac{\pi}{36}$

C. $\frac{\pi}{24}$

D. $\frac{\pi}{12}$

Answer: C

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Exercise (Multiple)

1. If $f(x)$ is integrable over $[1, 2]$ then $\int_1^2 f(x)dx$ is equal to

A. $\lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right)$

B. $\lim_{n \to \infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$

C. $\lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r+n}{n}\right)$

D. $\lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

Answer: B::C

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2. If $L = \lim_{n \to \infty} \frac{n^3\left(e^{1/n} + e^{2/n} + \dots\dots\dots + e\right)}{(n+1)^m\left(1^m + 4^m + \dots. + n^{2m}\right)}$ is non zer finite real, then

A. $L = 3(e-1)$

B. $L = 2(e-1)$

C. $m = 1/3$

D. $m = 1/3$

Answer: A::C

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3. Let $p = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots. + \frac{1}{\sqrt{120}}$ and $q = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots. + \frac{1}{\sqrt{121}}$ then

A. $p > 20$

B. $q < 20$

C. $p + q < 40$

D. $p + q > 40$

Answer: A::B::D

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4. Let $S_n = \sum_{k=0}^{n} \frac{n}{n^2 + kn + k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$,for $n = 1, 2, 3, \ldots\ldots,$ then

A. $S_n < \frac{\pi}{3\sqrt{3}}$

B. $S_n > \frac{\pi}{3\sqrt{3}}$

C. $T_n < \frac{\pi}{3\sqrt{3}}$

D. $T_n > \frac{\pi}{3\sqrt{3}}$

Answer: A::D

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5. $Thevalueof \int_0^1 \frac{2x^2 + 3x + 3}{(x+1)(x^2 + 2x + 2)} dxis \quad \frac{\pi}{4} + 2\log 2 - \tan^{-1} 2$
$\frac{\pi}{4} + 2\log 2 - \frac{\tan^{-1} 1}{3} \; 2\log 2 - \cot^{-1} 3 \text{ (d) } - \frac{\pi}{4} + \log 4 + \cot^{-1} 2$

A. $\frac{\pi}{4} + 2\log 2 - \tan^{-1} 2$

B. $\frac{\pi}{4} + 2\log 2 - \tan^{-1} \frac{1}{3}$

C. $2\log 2 - \cot^{-1} 3$

D. $-\frac{\pi}{4}+\log 4+\cot^{-1} 2$

Answer: A::C::D

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6. Let $f(x)=\int_1^x \frac{3^t}{1+t^2} dt$, where $x>0$, Then

A. for $0<\alpha<\beta, f(\alpha)<f(\beta)$

B. for $0<\alpha<\beta, f(\alpha)>f(\beta)$

C. $f(x)+\pi/4<\tan^{-1} x \, \forall x \geq 1$

D. $f(x)+\pi/4>\tan^{-1} x \, \forall x \geq 1$

Answer: A::D

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7. If $\int_a^b |\sin x| dx = 8$ and $\int_0^{a+b} |\cos x| dx = 9$, then find the value of $\int_a^b x \sin x dx.$

A. $a + b = \frac{9\pi}{2}$

B. $|a = b| = 4\pi$

C. $\frac{a}{b} = 15$

D. $\int_a^b \sec^2 x dx = 0$

Answer: A::B

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8. $Iff(x) = \int_0^x 2|t| dt, then \;\; g(x) = x|x| \;\; g(x)$ is monotonic $g(x)$ is differentiable at $x = 0$ $g'(x)$ is differentiable at $x = 0$

A. $g(x) = x|x|$

B. g(x) is monotonic

C. g(x) is differentiable at x = 0

D. g'(x) is differentiable at x = 0

Answer: A::B::C

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9. $If A_n = \int_0^{\frac{\pi}{2}} \frac{\sin(2n-1)x}{\sin x} dx, b_n = \int_0^{\frac{\pi}{2}} \left(\frac{\sin nx}{\sin x}\right)^2 dx f \text{ or } n \in N,$

Then $A_{n+1} = A_n$ (b) $B_{n+1} = B_n$ $A_{n+1} - A_n = B_{n+1}$ (d) $B_{n+1} - B_n = A_{n+1}$

A. $A_{n+1} = A_n$

B. $B_{n+1} = B_n$

C. $A_{n+1} - A_n = B_{n+1}$

D. $B_{n+1} - B_n = A_{n+1}$

Answer: A::D

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10. $The value of \int_0^\infty \frac{dx}{1+x^4} is \quad same as tha \to f\int_0^\infty \frac{x^2+1dx}{1+x} \quad \frac{\pi}{2\sqrt{2}}$
$same as tha \to f\int_0^\infty \frac{x^2+1dx}{1+x^4}$ (d) $\frac{\pi}{\sqrt{2}}$

A. same as that of $\int_0^\infty \frac{x^2+1dx}{1+x^4}$

B. $\frac{\pi}{2\sqrt{2}}$

C. same as that of $\int_0^\infty \frac{x^2dx}{1+x^4}$

D. $\frac{\pi}{\sqrt{2}}$

Answer: B::C

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11. The value of $\int_0^1 e^x \hat{} (2-x)dx$ is < 1 (b) > 1 (c) $> e^{-\frac{1}{4}}$ (d) `

A. < 1

B. > 1

C. $> e^{-\frac{1}{4}}$

D. $< e^{-\frac{1}{4}}$

Answer: A::C

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12. If $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx = 10$, then

A. $b = 22, a = 2$

B. $b = 15, a = -5$

C. $b = 10, a = -10$

D. $b = 10, a = -2$

Answer: A::B::C

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13. The values of a for which the integral $\int_0^2 |x-a|dx \geq 1$ is satisfied are $(2, \infty)$ (b) $(-\infty, 0)$ $(0, 2)$ (d) none of these

A. $[2, \infty)$

B. $(-\infty, 0]$

C. $(0, 2)$

D. none of these

Answer: A::B::C

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14. If $f(x) = \int_0^x |t-1|dt$, where $0 \leq x \leq 2$, then

A. range of $f(x)$ is $[0, 1]$

B. $f(x)$ is differentiable at $x = 1$

C. $f(x) = \cos^{-1} x$ has two real roots

D. $f'(1/2) = 1/2$

Answer: B

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15. If $f(2-x) = f(2+x)$ and $f(4x) = f(4+x)$ for all x and $f(x)$ is a function for which $\int_0^2 f(x)dx = 5, then \int_0^{50} f(x)dx$ is equal to

A. 125

B. $\int_{-4}^{46} f(x)dt$

C. $\int_1^{51} f(x)dx$

D. $\int_2^{52} f(x)dx$

Answer: A::B::D

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16. $Off(x)=\int_0^x (\cos(\sin t)+\cos(\cos t)dt, then f(x+\pi) is$

$f(x)+f(\pi)$ (b) $f(x)+2(\pi)$ $f(x)+f\left(\frac{\pi}{2}\right)$ (d) $f(x)+2f\left(\frac{\pi}{2}\right)$

A. $f(x)+f(\pi)$

B. $f(x)+2f(\pi)$

C. $f(x)+f\left(\frac{\pi}{2}\right)$

D. $f(x)+2f\left(\frac{\pi}{2}\right)$

Answer: A::D

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17. If $I_n=\int_0^{\pi/4}\tan^n x dx$, $(n>1$ is an integer), then

A. $I_n+I_{n-2}=\frac{1}{n+1}$

B. $I_n+I_{n-2}=\frac{1}{n-1}$

C. $I_2+I_4, I_6, \ldots\ldots$ are in H.P.

D. $\frac{1}{2(n+1)} < I_n < \frac{1}{2(n-1)}$

Answer: B::C::D

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18. $If I_n = \int_0^1 \frac{dx}{(1+x^2)^n}, where n \in N,$ which of the following statements hold good? $2nI_{n+1} = 2^{-n} + (2n-1)I_n$ $I_2 = \frac{\pi}{8} + \frac{1}{4}$ (c) $I_2 = \frac{\pi}{8} - \frac{1}{4}$ $I_3 = \frac{3\pi}{32} + \frac{1}{4}$

A. $2nI_{n+1} = 2^{-n} + (2n-1)I_n$

B. $I_2 = \frac{\pi}{8} + \frac{1}{4}$

C. $I_2 = \frac{\pi}{8} - \frac{1}{4}$

D. $I_3 = \frac{3\pi}{32} + \frac{1}{4}$

Answer: A::B::D

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19. $Let f:[1,\infty)\vec{R} and f(x)=\int_1^x \frac{e^t}{t}dt-e^x\dot{T}hen\ f(x)$ is an increasing function $(\lim)_{x\vec{\infty}} f(x)\vec{\infty}$ $f'(x)$ has a maxima at $x=e$ $f(x)$ is a decreasing function

A. $f(x)$ is an increasing function

B. $\lim_{x\to\infty} f(x)\to\infty$

C. $f'(x)$ has a maxima at $x=e$

D. $f(x)$ is a decreasing function

Answer: A::B

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20. $If f(x)=\int_a^x [f(x)]^{-1}dx and \int_a^1 [f(x)]^{-1}dx=\sqrt{2}, then\ f(2)=2$ (b) $f'(2)=\frac{1}{2}$ $f'(2)=2$ (d) $\int_0^1 f(x)dx=\sqrt{2}$

A. $f(2)=2$

B. $f'(2)=1/2$

C. $f^{-1}(2) = 2$

D. $\int_0^1 f(x)dx = \sqrt{2}$

Answer: A::B::C

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21. A continuous function $f(x)$ satisfies the relation $f(x) = e^x + \int_0^1 e^x f(t)dt$ then $f(1) =$

A. $f(0) < 0$

B. $f(x)$ is a decreasing function

C. $f(x)$ is increasing function

D. $\int_0^1 f(x)dx > 0$

Answer: A::B

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22. $\int_0^x \left\{\int_0^u f(t)dx\right\} duisequa < o$ $\int_0^x (x-u)f(u)du$ $\int_0^x uf(x-u)du \xi nt_0^x f(u)du$ (d) $\xi nt_0^x uf(u-x)du$

A. $\int_0^x (x-u)f(u)du$

B. $\int_0^x uf(x-u)du$

C. $x\int_0^x f(u)du$

D. $x\int_0^x uf(u-x)du$

Answer: A::B

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23. Which of the following statement(s) is/are TRUE?

A. If function $y = f(x)$ is continuous at $x = c$ such that $f(c) \neq 0$, then $f(x)f(c) > 0 \forall x \varepsilon (c-h, c+h)$, where h is sufficiently small positive quantity.

B. $\lim_{n\to\infty} \frac{1}{n}\left(n\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right).....\left(1+\frac{n}{n}\right)\right) = 1 + 2In2.$

C. Let f be a continuous and non-negative function defined on $[a, b]$ If

$$\int_a^b f(x)dx = 0, \text{ then } f(x) = \forall x \varepsilon [a, b]$$

D. Let f be continuous function defined on $[a, b]$ such that

$\int_a^b f(x)dx = 0$.Then there exists at least one $c\varepsilon(a, b)$ for which

$f(c) = 0$

Answer: A::C::D

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24. If $\int_0^x [x]dx = \int_0^{|x|} xdx$, (where [.] and {.} denotes the greatest integer and fractional part respectively), then

A. $x\varepsilon[0, 1)$

B. $\{x\} = 1/2$

C. $\{x\} = 1/3$

D. $x > 0$

Answer: A::B

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25. Consider the function $f(\theta)=\int_0^1 \frac{\left|\sqrt{1-x^2}-\sin\theta\right|}{\sqrt{1-x^2}}dx,$ where $0\leq\theta\leq\frac{\pi}{2}$, then

A. $f_{\min}=\sqrt{2}-1$

B. $f_{\min}=\sqrt{2}+1$

C. $f_{\max}=1$

D. $f_{\max}=\frac{\pi}{2}-1$

Answer: A::B::C::D

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26. $f:[0,1)\to R$ be a non increasing function the for $\alpha\varepsilon(0,1)$

A. $\alpha \int_0^1 f(x)dx \le \int_0^{\alpha} f(x)dx$

B. $\alpha \int_0^1 f(x)dx \ge \int_0^{\alpha} f(x)dx$

C. $\alpha^2 \int_0^1 f(x)dx \le \int_0^{\alpha} f(x)dx$

D. $\sqrt{\alpha} \int_0^1 f(x)dx \ge \int_0^{\alpha} f(x)dx$

Answer: A::C

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27. Let f(x) be a non-constant twice differentiable function defined on (∞, ∞) such that $f(x) = f(1-x)$ and f"(1/4) = 0`. Then

A. $f'(x)$ vanishes at least twice on $[0, 1]$

B. $f'\left(\frac{1}{2}\right) = 0$

C. $\int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right) \sin x dx = 0$

D. $\int_0^{1/2} f(t) e^{\sin \pi t} dt = \int_{t/2}^{1} f(1-t) e^{\sin \pi t} dt$

Answer: A::B::C::D

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Exercise (Comprehension)

1. $y = f(x)$ satisfies the relation $\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$

The range of $y = f(x)$ is

A. $[0, \infty)$

B. R

C. $(-\infty, 0]$

D. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Answer: D

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2. $y = f(x)$ satisfies the relation $\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$

The value of $\int_{-2}^{2} f(x)dx$ is

A. 0

B. -2

C. $2\log_e 2$

D. none of these

Answer: A

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3. $y = f(x)$ satisfies the relation $\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$

The value of x for which $f(x)$ is increasing is

A. $(-\infty, 1]$

B. $[-1, \infty)$

C. $[-1, 1]$

D. none of these

Answer: C

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4. Let $f: R \to R$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x-t) dt.$

$f(x)$ increases for

A. $x > 1$

B. $x < -2$

C. $x > 2$

D. none of these

Answer: B

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5. Let $f: R \to R$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x-t) dt$.

$y = f(x)$ is

A. injective but not surjective

B. surjective but not injective

C. bijective

D. neither injective nor surjective

Answer: B

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6. Let $f(x)$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x-t) dt$ then $\int_0^1 f(x) dx =$

A. $\frac{1}{4}$

B. $-\frac{1}{12}$

C. $\frac{5}{12}$

D. $\frac{12}{7}$

Answer: C

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7. $f(x)$ satisfies the relation $f(x) - \lambda\int_0^{\pi/2} \sin x \cdot \cos t f(t) dt = \sin x$ If $\lambda > 2$ then $f(x)$ decreases in

A. $(0, \pi)$

B. $\left(\frac{\pi}{2}, 3\pi/2\right)$

C. $(-\pi/2, \pi/2)$

D. none of these

Answer: C

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8. $f(x)$ satisfies the relation $f(x) - \lambda \int_0^{\pi/2} \sin x \cos t f(t) dt = \sin x$

If $f(x) = 2$ has the least one real root, then

A. $\lambda \varepsilon [1, 4]$

B. $\lambda \varepsilon [-1, 2]$

C. $\lambda \varepsilon [0, 1]$

D. $\lambda \varepsilon [1, 3]$

Answer: D

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9. $f(x)$ satisfies the relation $f(x) - \lambda \int_0^{\pi/2} \sin x \cdot \cos t f(t) dt = \sin x$ If

$\lambda > 2$ then $f(x)$ decreases in

A. 1

B. $3/2$

C. $4/3$

D. none of these

Answer: C

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10. Let $f(x)$ and $\phi(x)$ are two continuous function on R satisfying $\phi(x)=\int_a^x f(t)dt, a\neq 0$ and another continuous function $g(x)$ satisfying $g(x+\alpha)+g(x)=0 \forall x \varepsilon R, \alpha>0,$ and $\int_b^{2k} g(t)dt$ is independent of b

If $f(x)$ is an odd function, then

A. $\phi(x)$ is also an odd function

B. $\phi(x)$ is an even function

C. $\phi(x)$ is neither an even nor an odd function

D. for $\phi(x)$ to be an even function, it must satisfy $\int_0^a f(x)dx=0$

Answer: B

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11. Let $f(x)$ and $\phi(x)$ are two continuous function on R satisfying $\phi(x) = \int_a^x f(t)dt, a \neq 0$ and another continuous function $g(x)$ satisfying $g(x + \alpha) + g(x) = 0 \forall x \varepsilon R, \alpha > 0,$ and $\int_b^{2k} g(t)dt$ is independent of b

If $f(x)$ is an even function, then

A. $\phi(x)$ is also an even function

B. $\phi(x)$ is an odd function

C. if $f(a - x) = -f(x)$, then $\phi(x)$ is an even function

D. if $f(a - x) = -f(x)$ then $\phi(x)$ is an odd function

Answer: D

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12. Let $f(x)$ and $\phi(x)$ are two continuous function on R satisfying $\phi(x) = \int_a^x f(t)dt, a \neq 0$ and another continuous function $g(x)$ satisfying $g(x+\alpha) + g(x) = 0 \forall x \varepsilon R, \alpha > 0,$ and $\int_b^{2k} g(t)dt$ is independent of b

Least positive value fo c if c, k, b are n A.P. is

A. 0

B. 1

C. α

D. 2α

Answer: D

Watch Video Solution

13. Let $f(x)$ and $\phi(x)$ are two continuous function on R satisfying $\phi(x) = \int_a^x f(t)dt, a \neq 0$ and another continuous function $g(x)$ satisfying $g(x+\alpha) + g(x) = 0 \forall x \varepsilon R, \alpha > 0,$ and $\int_b^{2k} g(t)dt$ is independent of b

If m, n are even integers and $p, q \varepsilon R$, then $\int_{p+n\alpha}^{q+n\alpha} g(t)dt$ is equal to

A. $\int_{p}^{q} g(x)dx$

B. $(n-m)\int_{0}^{\alpha} g(x)dx$

C. $\int_{p}^{\alpha} g(x)dx + (n-m)\int_{0}^{\alpha} g(2x)dx$

D. $\int_{p}^{q} g(x)dx + \frac{(n-m)}{2}\int_{0}^{2\alpha} g(x)dx$

Answer: D

View Text Solution

14. Evaluating integrals dependent on a parameter:

Differentiate I with respect to the parameter with in the sign an integrals taking variable of the integrand as constant. Now evaluate the integral so obtained as a function of the parameter then integrate then result of get I. Constant of integration can be computed by giving some arbitrary

values to the parameter and the corresponding value of I.

The value of $\int_0^1 \frac{x^a - 1}{\log x} dx$ is

A. $\log(a-1)$

B. $\log(a+1)$

C. $a\log(a+1)$

D. none of these

Answer: B

Watch Video Solution

15. Evaluating integrals dependent on a parameter:

Differentiate I with respect to the parameter within the sign an integrals taking variable of the integrand as constant. Now evaluate the integral so obtained as a function of the parameter then integrate then result of get I. Constant of integration can be computed by giving some arbitrary

values to the parameter and the corresponding value of I.

The value $\int_0^{\pi/2} \log\left(\sin^2\theta + k^2\cos^2\theta\right)d\theta$, where $k \geq 0$, is

A. $\pi\log(1+k)+\pi\log 2$

B. $p\log(1+k)$

C. $\pi\log(1+k)-\pi\log 2$

D. $\log(1+k)-\log 2$

Answer: C

Watch Video Solution

16. Evaluating integrals dependent on a parameter:

Differentiate I with respect to the parameter within the sign an integrals taking variable of the integrand as constant. Now evaluate the integral so obtained as a function of the parameter then integrate then result of get I. Constant of integration can be computed by giving some arbitrary values to the parameter and the corresponding value of I.

The value of $\frac{dI}{da}$ when $I = \int_0^{\pi/2} \log\left(\frac{1 + a\sin x}{1 - a\sin x}\right)\frac{dx}{\sin x}$ (where $|a| < 1$) is

A. $\frac{\pi}{\sqrt{1 - a^2}}$

B. $-\pi\sqrt{1 - a^2}$

C. $\sqrt{1 - a^2}$

D. $\frac{\sqrt{1 - a^2}}{\pi}$

Answer: A

Watch Video Solution

17. Evaluating integrals dependent on a parameter:

Differentiate I with respect to the parameter within the sign an integrals taking variable of the integrand as constant. Now evaluate the integral so obtained as a function of the parameter then integrate then result of get I. Constant of integration can be computed by giving some arbitrary

values to the parameter and the corresponding value of I.

If $\int_0^{\pi} \frac{dx}{(a - \cos x)} = \frac{\pi}{\sqrt{a^2 - 1}}$, then the value of $\frac{dx}{\left(\sqrt{10} - \cos x\right)}$ is

A. $\frac{\pi}{81}$

B. $\frac{7\pi}{162}$

C. $\frac{7\pi}{81}$

D. none of these

Answer: C

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18. $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t \cos x) f(t) dt$

The range of $f(x)$ is

A. $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$

B. $\left[-\frac{\sqrt{5}}{3}, \frac{\sqrt{5}}{3}\right]$

C. $\left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$

D. none of these

Answer: B

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19. $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t\cos x) f(t) dt$

$f(x)$ is not invertible for

A. $x\varepsilon\left[-\frac{\pi}{2} - \tan^{-1} 2, \frac{\pi}{2} - \tan^{-1} 2\right]$

B. $x\varepsilon\left[\frac{\tan^{-1} 1}{2}, \pi + \tan^{-1}\frac{1}{2}\right]$

C. $x\varepsilon\left[\pi + \cot^{-1} 2, 2\pi + \cot^{-1} 2\right]$

D. none of these

Answer: D

Watch Video Solution

20. $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t\cos x) f(t) dt$

The value of $\int_0^{\pi/2} f(x) dx$ is

A. 1

B. -2

C. -1

D. 2

Answer: C

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21. Let $u = \int_0^{\infty} \frac{dx}{x^4 + 7x^2 + 1}$ and $v = \int_0^{x} \frac{x^2 dx}{x^4 + 7x^2 + 1}$ then

A. $\pi/3$

B. $\pi/6$

C. $\pi/12$

D. $\pi/9$

Answer: B

Watch Video Solution

22. Let $u=\int_0^\infty \frac{dx}{x^4+7x^2+1}$ and $v=\int_0^x \frac{x^2dx}{x^4+7x^2+1}$ then

A. $\pi/3$

B. $\pi/6$

C. $\pi/12$

D. $\pi/9$

Answer: B

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23. If $f(x)=\int_0^1 \frac{dt}{1+|x-t|}, x\varepsilon R$

the value of $f'(1/2)$ is equal to

A. $1/2$

B. 0

C. 1

D. 2

Answer: B

View Text Solution

24. If $f(x)=\int_0^1 \frac{dt}{1+|x-t|}, x\varepsilon R$

Which of the following is not true about $f(x)$?

A. $f(x)$ is decreasing for $x>1$

B. $f(x)$ is increasing for $x<1$

C. $f(1) = \log_e 2$

D. $f(1/2) = \log_e(3/2)$

Answer: D

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25. Let f be a differentiable function satisfying $\int_0^{f(x)} f^{-1}(t)dt - \int_0^x (\cos t - f(t)dt = 0$ and $f\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$

The value of $\int_0^{\pi/2} f(x)dx$ lies in the interval

A. $\left(\frac{2}{\pi}, 1\right)$

B. $\left(1, \frac{\pi}{2}\right)$

C. $\left(\frac{3}{2}, \frac{\pi}{2}\right)$

D. $\left(0, \frac{2}{\pi}\right)$

Answer: B

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26. Let f be a differentiable function satisfying $\int_0^{f(x)} f^{-1}(t)dt - \int_0^x (\cos t - f(t)dt = 0$ and $f\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$

The value of $\lim_{x \to 0} \frac{\cos x}{f(x)}$ is equal to where [.] denotes greatest integer function

A. 0

B. 1

C. $1/2$

D. 2

Answer: B

Watch Video Solution

27. If $U_n = \int_0^\pi \frac{1 - \cos nx}{1 - \cos x} dx$ where n is positive integer of zero, then

The value of U_n is

A. $\pi / 2$

B. π

C. $n\pi / 2$

D. $n\pi$

Answer: D

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28. If $U_n = \int_0^{\pi} \frac{1 - \cos nx}{1 - \cos x} dx$, where n is positive integer or zero, then show that $U_{n+2} + U_n = 2U_{n+1}$. Hence, deduce that $\int_0^{\frac{\pi}{2}} \frac{\sin^2 n\theta}{\sin^2 \theta} = \frac{1}{2} n\pi$.

A. $\pi / 2$

B. π

C. $n\pi / 2$

D. $n\pi$

Answer: C

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29. Data could not be retrieved.

A. $\frac{\pi}{8}(1+\sqrt{2})$

B. $\frac{\pi}{4}(1+\sqrt{2})$

C. $\frac{\pi}{8\sqrt{2}}$

D. $\frac{\pi}{4\sqrt{2}}$

Answer: A

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30. Data could not be retrieved.

A. 4

B. 3

C. 2

D. 1

Answer: D

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31. Let the definite integral be defined by the formula $\int_a^b f(x)dx = \frac{b-a}{2}(f(a)+f(b))$. For more accurate result, for $c\varepsilon(a,b)$, we can use $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx = F(c)$ so that for $c = \frac{a+b}{2}$ we get $\int_a^b f(x)dx = \frac{b-a}{4}(f(a)+f(b)+2f(c))$.

If $f''(x) < 0 \forall x\varepsilon(a,b)$ and c is a point such that $a < c < b$, and $(c, f(c))$ is the point lying on the curve for which $F(c)$ is maximum then $f'(c)$ is equal to

A. $\frac{f(b)-f(a)}{b-a}$

B. $\frac{2(f(b)-f(a))}{b-a}$

C. $\dfrac{2f(b)-f(a)}{2b-a}$

D. 0

Answer: B

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Exercise (Matrix)

1. If [.] denotes the greatest integer function, then match the following lists:

	List I	List II
a.	$\int_{-1}^{1}[x+[x+[x]]]\,dx$	p. 3
b.	$\int_{2}^{5}([x]+[-x])\,dx$	q. 5
c.	$\int_{-1}^{3}\operatorname{sgn}(x-[x])dx$	r. 4
d.	$25\int_{0}^{\pi/4}\left(\tan^{6}(x-[x])+\tan^{4}(x-[x])\right)dx$	s. −3

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2. Match the following lists:

View Text Solution

3. Match the following lists:

List I	List II
a. If $f(x)$ is an integrable function for $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and $I_1 = \int_{\pi/6}^{\pi/3} \sec^2\theta f(2\sin 2\theta)\, d\theta$, and $I_2 = \int_{\pi/6}^{\pi/3} \operatorname{cosec}^2\theta f(2\sin 2\theta) d\theta$, then $I_1/I_2 =$	**p.** 3
b. If $f(x+1) = f(3+x)\ \forall\ x$, and the value of $\int_a^{a+b} f(x)dx$ is independent of a, then the value of b can be	**q.** 1
c. The value of $2\int_1^4 \frac{\tan^{-1}[x^2]}{\tan^{-1}[x^2] + \tan^{-1}[25 + x^2 - 10x]} dx$ (where [.]denotes the greatest integer function) is	**r.** 2
d. If $I = \int_0^2 \sqrt{x + \sqrt{x + \sqrt{x + \cdots \infty}}}\ dx$ (where $x > 0$), then $[\,I\,]$ is equal to (where [.] denotes the greatest integer function)	**s.** 4

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4. Match the following lists:

View Text Solution

5. Let $\int_0^\infty \frac{\sin x}{x} dx = \alpha$ Then match the following lists and choose the correct code. :

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6. If $I(m) = \int_0^\pi \log_e \left(1 - 2m \cos x + m^2\right) dx$, Then match the following lists and choose the correct code. :

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Exercise (Numerical)

1. If the value of $\lim_{n \to \infty} \left(n^{-3/2}\right) \cdot \sum_{j=1}^{6n} \sqrt{j}$ is equal to $\sqrt{N}$, then the value of N is________.

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2. $\left(\lim\right)_{n \overrightarrow{\infty}} \frac{n}{2^n} \int_0^2 x^n dxequals\_\_$

Watch Video Solution

3. A continuous real function f satisfies $f(2x) = 3\left(f(x) \forall x \in R \dot{I} f \int_0^1 f(x) dx = 1,\right.$ then find the value of $\int_1^2 f(x) dx$

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4. Consider the polynomial $f(x) = ax^2 + bx + \cdot$ If $f(0), f(2) = 2$, then the minimum value of $\int_0^2 |f'(x)dxis\_\_$

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5. If $I = \int_0^{3\pi/4} ((1 + x)\sin x(1 - x)\cos x)dx$, then the value of $(\sqrt{2} - 1)I$ is______

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6. If $\int_0^{100} f(x)dx = 7$, then $\sum_{r=1}^{100} \int_0^1 (r - 1 + x)dx =$ ________.

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7. The value of $\int_0^{\frac{3\pi}{2}} \frac{|\tan^{-1}\tan x| - |\sin^{-1}\sin x|}{|\tan^{-1}\tan x| + |\sin^{-1}\sin x|} dx$ is equal to

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8. $Let f(x) = x^3 = \frac{3x^2}{2} + x + \frac{1}{4}$ Then the value of $\left(\int_{\frac{1}{4}}^{\frac{3}{4}} f(f(x))dx\right)^{-1}$ os____

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9. The value of $\int_0^1 \frac{\tan^{-1} x}{\cot^{-1}(1 - x + x^2)dx}$ is____.

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10. Let $f(x)$ be differentiate function symmetric about $x = 2$, then the value of $\int_0^4 \cos(\pi x) f'(x) dx$ is equal to_______.

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11. Let $f: [0, \infty)\overrightarrow{R}$ be a continuous strictly increasing function, such that $f^3(x) = \int_0^x t\dot{f}^2(t)dt$ for every $x \geq 0$. Then value of $f(6)$ is______

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12. If f is continuous function and $F(x) = \int_0^x \left((2t+3)\int_t^2 f(u)du \right) dt,$ then $\left| \frac{F^2}{f(2)} \right|$ is equal to____

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13. If the value of the definite integral $\int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2 - x + 1} dx$ is $\frac{\pi^2}{\sqrt{n}}$ (where $n \in N),$ then the value of $\frac{n}{27}$ is

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14. Let $f(x) = \int_0^x \frac{dt}{\sqrt{1+t^3}}$ and $g(x)$ be the inverse of $f(x)$. Then the value of $4\frac{g''(x)}{g(x)^2}$ is_______.

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15. Let $g(x)$ be differentiable on R and $\int_{\sin t}^{1} x^2 g(x)dx = (1 - \sin t)$, where $t \in \left(0, \frac{\pi}{2}\right)$. Then the value of $g\left(\frac{1}{\sqrt{2}}\right)$ is____

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16. If $\int_0^{\infty} x^{2n+1} \dot{e}^{-xdx} = 360,$ then the value of n is___

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17. Let $f(x)$ be a derivable function satisfying $f(x) = \int_0^x e^t \sin(x - t) dt and g(x) = f^x - f(x)$ Then the possible integers in the range of $g(x)$ is______

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18. Let $f(x) = \frac{1}{x^2} \int_0^x \left(4t^2 - 2f'(t)\right) dt$ then find $9f'(4)$

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19. If the value of the definite integral $\int_0^1 \hat{} (2007)C_7 x^{2000} 1 \dot{-} x^7 dx$ is equal to $\frac{1}{k}$, $where k \in N$, then the value of $\frac{k}{26}$ is____

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20. $If I_n = \int_0^1 (1 - x^5)^n dx, then \frac{55}{7} \frac{I_{10}}{I_{11}} isequa < o\_\_$

Watch Video Solution

21. Evaluate: $5050 \frac{\int 01 (1 - x^{50})^{100} dx}{\int 01 (1 - x^{50})^{101} dx}$

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22. $Let J = \int_{-5}^{-4} (3 - x^2) \tan(3 - x^2) dx and K = \int_{-2}^{-1} (6 - 6x + x^2)$ $\tan(6x - x^2 - 6) dx \dot{T}hen(J + K)$ equals _____

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23. The value of the definite integral $\int_{2-1}^{\sqrt{2}+1} \frac{x^4+x^2+2}{(x^2+1)^2} dx equals$ _ _

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24. Consider a real valued continuous function f such that $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t(f(t))dt$. If M and m are maximum and minimum values of function f, then the value of M/m is__________.

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25. $If f(x) = x + \int_0^1 t(x+t)f(t)dt, then the value o \frac{f(23)}{2} f(0)$ is equal to ________

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26. Let $y = f(x) = 4x^3 + 2x - 6$, then the value of $\int_0^2 f(x)dx + \int_0^{30} f^{-1}(y)dy$ is equal to ________.

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27. The value of $\int_1^3 \left(\sqrt{1 + (x-1)^3} + (x^2 - 1)^{\frac{1}{3}} + 1\right)dx$ is ________.

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28. The value of $\int_0^1 \cos^{-1}(x - x^2) - \sqrt{(1 - x^2)(2x - x^2)}dx$ is equal to ________.

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JEE Main Previous Year

1. $\int_0^{\pi} [\cot x] dx,$ where [.] denotes the greatest integer function, is equal to

A. $\frac{\pi}{2}$

B. 1

C. -1

D. $-\frac{\pi}{2}$

Answer: D

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2. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1-x)$ for all $x \varepsilon [0, 1], p(0) = 1,$ and $p(1) = 41.$

Then $\int_0^1 p(x) dx$ is equal to

A. 42

B. $\sqrt{41}$

C. 21

D. 41

Answer: C

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3. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is

A. $\log 2$

B. $\pi \log 2$

C. $\frac{\pi}{8}\log 2$

D. $\frac{\pi}{2}\log 2$

Answer: B

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4. For $x\varepsilon\left(0, \frac{5\pi}{2}\right)$, definite $f(x) = \int_0^x \sqrt{t}\sin t dt$. Then f has

A. local maximum at π and local minima at 2π

B. local maximum at π and 2π

C. local minimum at π and 2π

D. local minimum at π and local maximum at `2pi

Answer: A

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5. If $g(x) = \int_0^x \cos^4 t dt,$ then $g(x+\pi)$ equals

A. $\frac{g(x)}{g(\pi)}$

B. $g(x) + g(\pi)$

C. $g(x) - g(\pi)$

D. $g(x).\ g(\pi)$

Answer: B

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6. Statement I: The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.

Statement II: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

A. Statement I is true, statement II is true, statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is a not a correct explanation for statement I

C. Statement I is true, statement II is false

D. Statement I is false, statement II is true

Answer: D

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7. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t| dt, x \in R,$ which are parallel to the line $y = 2x$, are equal to

(1) ± 2 (2) ± 3 (3) ± 4 (4) ± 1

A. ± 1

B. ± 2

C. ± 3

D. ± 4

Answer: A

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8. The integral $\int_0^\pi \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}} dx$ equals

A. $\pi - 4$

B. $\frac{2\pi}{3} - 4 - \sqrt{3}$

C. $4\sqrt{3} - 4$

D. $4\sqrt{3} - 4 - \frac{\pi}{3}$

Answer: D

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9. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$ is equal to

A. 2

B. 4

C. 1

D. 6

Answer: C

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10. $\lim_{n \to \infty} \left(\frac{(n+1)(n+2)(n+3).......3n}{n^{2n}} \right)^{\frac{1}{n}}$ is equal to

A. $\dfrac{27}{e^{2}0}$

B. $\dfrac{9}{e^{2}}$

C. $3\log 3-2$

D. $\dfrac{18}{e^{4}}$

Answer: A

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11. The Integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{dx}{1+\cos x}$ is equal to: (2) (3) (4)

A. -1

B. -2

C. 2

D. 4

Answer: C

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12. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx$ is

A. $\pi/4$

B. $\pi/8$

C. $\pi/2$

D. 4π

Answer: A

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JEE Advanced Previous Year

1. Let f be a non-negative function defined on the interval .[0,1].If $\int_0^x \sqrt{1-(f'(t))^2}.\, dt = \int_0^x f(t).\, dt, 0 \leq x \leq 1$ and f(0)=0,then

A. $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

B. $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

C. $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

D. $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

Answer: C

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2. The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$ is

A. $\frac{22}{7} - \pi$

B. $\frac{2}{105}$

C. 0

D. $\frac{71}{15} - \frac{3\pi}{2}$

Answer: A

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3. Let f be a real-valued function defined on the inverval $(-1, 1)$ such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1} dt,$ for all, $x \in (-1, 1) and let f^{-1}$ be the inverse function of f. Then $(f^{-1})'(2)$ is equal to 1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{e}$

A. 1

B. $1/3$

C. $1/2$

D. $1/e$

Answer: B

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4. The valued of $\int_{\sqrt{In2}}^{\sqrt{In3}} \frac{x \sin x^2}{\sin x^2 + \sin(In6 - x^2)} dx$ is

A. $\frac{1}{4} In \frac{3}{2}$

B. $\frac{1}{2}In\frac{3}{2}$

C. $In\frac{3}{2}$

D. $\frac{1}{6}In\frac{3}{2}$

Answer: A

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5. Let $f:[-1,2]\overrightarrow{0,\infty}$ be a continuous function such that $f(x)=f(1-x)f$ or $allx\in[-1,2]$. Let $R_1=\int_{-1}^{2}xf(x)dx,$ and R_2 be the area of the region bounded by $y=f(x), x=-1, x=2,$ and the $x-a\xi s$. Then $R_1=2R_2$ (b) $R_1=3R_2$ $2R_1$ (d) $3R_1=R_2$

A. $R_1=2R_2$

B. $R_1=3R_2$

C. $2R_1=R_2$

D. $3R_1=R_2$

Answer: C

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6. Let $f:\left[\frac{1}{2},1\right]\vec{R}$ (the set of all real numbers) be a positive, non-constant, and differentiable function such that $f'(x) < 2f9x\Big)andf\left(\frac{1}{2}\right)=1$. Then the value of $\int_{\frac{1}{2}}^{1} f(x)dx$ lies in the interval $(2e-1, 2e)$ (b) $(3-1, 2e-1)$ $\left(\frac{e-1}{2}, e-1\right)$ (d) $\left(0, \frac{e-1}{2}\right)$

A. $(2e-1, 2e)$

B. $(e-1, 2e-1)$

C. $\left(\frac{e-1}{2}, e-1\right)$

D. $\left(0, \frac{e-1}{2}\right)$

Answer: D

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7. Let $f:[0,2]\overrightarrow{R}$ be a function which is continuous on [0,2] and is differentiable on (0,2) with $f(0)=1$

$Let: F(x) = \int_0^{x^2} f(\sqrt{t})\,dtf$ or $x \in [0,2]\dot{I}fF'(x) = f'(x)$. for all $x \in (0,2)$, then $F(2)$ equals $e^2 - 1$ (b) $e^4 - 1$ $e - 1$ (d) e^4

A. $e^2 - 1$

B. $e^4 - 1$

C. $e - 1$

D. e^4

Answer: B

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8. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2\cos ecx)^{17}$ dx

A. $\int_0^{\log\left(1+\sqrt{2}\right)} 2\left(e^u + e^{-u}\right)^{16} du$

B. $\int_0^{\log\left(1+\sqrt{2}\right)} 2\left(e^u + e^{-u}\right)^{17} du$

C. $\int_0^{\log\left(1+\sqrt{2}\right)} 2\left(e^u - e^{-u}\right)^{17} du$

D. $\int_0^{\log\left(1+\sqrt{2}\right)} 2\left(e^u - e^{-u}\right)^{16} du$

Answer: A

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9. Let $f'(x) = \frac{192x^3}{2 + \sin^4 \pi x}$ for all $x \in \mathbb{R}$ with $f\left(\frac{1}{2}\right) = 0$. If $m \leq \int_{\frac{1}{2}}^{1} f(x)dx \leq M$ then the possible values of m and M are (i) $m = 13, M = 24$ (ii)$m = \frac{1}{4}, M = \frac{1}{2}$ (iii) $m = -11, M = 0$ (iv) $m = 1, M = 12$

A. $m = 13, M = 24$

B. $m = \frac{1}{4}, M = \frac{1}{2}$

C. $m = -11, M = 0$

D. $m = 1, M = 12$

Answer: D

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10. Evaluate: $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$

A. $\frac{\pi^2}{4} - 2$

B. $\frac{\pi^2}{4} + 2$

C. $\pi^2 - e^{\frac{\pi}{2}}$

D. $\pi^2 + \frac{e^{\pi}}{2}$

Answer: A

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11. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx, n = 0, 1, 2, \ldots\ldots$ then which one of the following is not true ?

A. $I_n = I_{n+2}$

B. $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

C. $\sum_{m=1}^{10} I_{2m} = 0$

D. $I_n = I_{n+1}$

Answer: A::B::C

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12. Let f be a real-valued function defined on interval $(0, \infty)$,by $f(x) = \ln x + \int_0^x \sqrt{1+\sin t}.\, dt.$ Then which of the following statement(s) is (are) true? (A). f"(x) exists for all $\in (0, \infty)$. (B). f'(x) exists for all x $\in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$. (C). there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all x $\in (\alpha, \infty)$. (D). there exists $\beta > 1$ such that $|f(x)| + |f'(x)| \leq \beta$ for all x $\in (0, \infty)$.

A. $f''(x)$ exists for al $x\varepsilon(0, \infty)$

B. $f'(x)$ exists for all $x\varepsilon(0, \infty)$ and f' is continuous on $(0, \infty)$ but not differentiable on $(0, \infty)$.

C. There exists $\alpha > 1$ suchthat $|f'(x)| < \ | f(x) +$ for all $x\varepsilon(\alpha, \infty)$

D. There exists $\beta > 0$ such that $|f(x)| + \ | f'(x) + \ \leq \beta$ for all $x\varepsilon(0, \infty)$

Answer: B::C

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13. Let S be the area of the region enclosed by $y = e^{-x} \ \hat{}\ 2, y = 0, x = 0, andx = 1.$ Then $S \geq \frac{1}{e}$ (b) $S \geq 1 = \frac{1}{e}$ $S \leq \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$ (d) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

A. $S \geq \frac{1}{e}$

B. $S \geq 1 - \frac{1}{e}$

C. $S \leq \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$

D. $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

Answer: A::B::D

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14. Find a for which $\lim_{n \to \infty} \frac{1^a + 2^a + 3^a + \dots + n^a}{(n+1)^{a-1}[(na+1) + (na+2) + \dots + (na+n)]} = \frac{1}{60}$

A. 5

B. 7

C. $\frac{-15}{2}$

D. $\frac{-17}{2}$

Answer: B::D

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15. Let $f: [a, b] \to [1, \infty)$ be a continuous function and let $g: R \to R$ be defined as

$$g(x) = \begin{cases} 0 & \text{if} \quad x < a \\ \int_a^x f(t)dt & \text{if} \quad a \le x \le b \\ \int_a^b f(t)dt & \text{if} \quad x > b \end{cases}$$ Then

A. $g(x)$ is continuous but not differentiable at a

B. $g(x)$ is differentiable on R

C. $g(x)$ is continuous but not differentiable at b

D. $g(x)$ is continuous and differentiable at either a or b but not both

Answer: A::C

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16. Let $f: (0, \infty) \in R$ be given

$$f(x) = \int_{1/x}^{x} e^{-\left(t + \frac{1}{t}\right)} \frac{1}{t} dt,$$ then

A. $f(x)$ is monotonically increasing on $[1, \infty)$

B. $f(x)$ is monotonocally decreasing on$(0, 1)$

C. $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x\varepsilon(0, \infty)$

D. $f(2^x)$ is an odd function of x on R

Answer: A::C::D

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17. The option(s) with the values of $aandL$ that satisfy the following equation is (are) $\frac{\int 04\pi e^t \left(s \in^6 at + \cos^4 at\right) dt}{\int 0\pi e^t (s \in^6 at + \cos^4 at) dt} = L$ $a = 2, L = \frac{e^{4\pi} - 1}{e^\pi - 1}$ (b) $a = 2, L = \frac{e^{4\pi} + 1}{e^\pi + 1}$ $a = 4, L = \frac{e^{4\pi} - 1}{e^\pi - 1}$ (d) $a = 4, L = \frac{e^{4\pi} + 1}{e^\pi + 1}$

A. $a = 2, L = \frac{e^{4\pi} - 1}{e^\pi - 1}$

B. $a = 2, L = \frac{e^{4\pi + 1}}{e^\pi + 1}$

C. $a = 4, L = \frac{e^{4\pi} - 1}{e^\pi - 1}$

D. $a = 4, L = \frac{e^{4\pi} + 1}{e^\pi + 1}$

Answer: A::C

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18. Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the correct expression (s) is (are) $\int_0^{\frac{\pi}{4}} x f(x) dx = \frac{1}{12}$ $\int_0^{\frac{\pi}{4}} f(x) dx = 0$ $\int_0^{\frac{\pi}{4}} x f(x) = \frac{1}{6}$ (d) $\int_0^{\frac{\pi}{4}} f(x) dx = \frac{1}{12}$

A. $\int_0^{\pi/4} x f(x) dx = \frac{1}{12}$

B. $\int_0^{\pi/4} f(x) dx = 0$

C. $\int_0^{\pi/4} x f(x) = \frac{1}{6}$

D. $\int_0^{\pi/4} f(x) dx = 1$

Answer: A::B

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19. Let $f(x) = \lim_{n \to \infty} \left(\frac{n^n (x+n)\left(x+\frac{n}{2}\right)....\left(x+\frac{n}{n}\right)}{n!(x^2+n^2)\left(x^2+\frac{n^2}{4}\right).....\left(x^2+\frac{n^2}{n^2}\right)} \right)^{\frac{x}{n}}$ foe all $x > 0$. Then

A. $f\left(\frac{1}{2}\right) \geq f(1)$

B. $f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$

C. $f'(2) \leq 0$

D. $\frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f'(2)}$

Answer: B::C

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20. Let $f : \overrightarrow{R0, 1}$ be a continuous function. Then, which of the following function (s) has (have) the value zero at some point in the interval (0,1)?

$e^x - \int_0^x f(t)\sin t dt$ (b) $f(x) + \int_0^{\frac{\pi}{2}} f(t)\sin t dt$ $x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t dt$

(d) $x^9 - f(x)$

A. $e^{x} - \int_0^x f(t)\sin t dt$

B. $x^9 - f(x)$

C. $f(x) + \int_0^{\frac{\pi}{2}} f(t)\sin t dt$

D. $x - \int_0^{\frac{\pi}{2} - x} f(t)\cos t dt$

Answer: B::D

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21. If $I \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)} dx, then: \quad I < \frac{49}{50}$ (b) $I > (\log)_e 99$ $I > (\log)_e 99$ (d) $I < (\log)_e 99$

A. $I > \log_e 99$

B. $I < \log_e 99$

C. $I < \frac{49}{50}$

D. $I > \frac{49}{50}$

Answer: B::D

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22. If $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)dt, then: \quad g'\left(\frac{\pi}{2}\right) = -2\pi$ (b) $g'\left(-\frac{\pi}{2}\right) = -2\pi$ $g'\left(-\frac{\pi}{2}\right) = 2\pi$ (d) $g'\left(\frac{\pi}{2}\right) = 2\pi$

A. $g'\left(\frac{\pi}{2}\right) = -2\pi$

B. $g'\left(-\frac{\pi}{2}\right) = 2\pi$

C. $g'\left(\frac{\pi}{2}\right) = 2\pi$

D. $g'\left(-\frac{\pi}{2}\right) = -2\pi$

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23. Given that for each $a\varepsilon(0, 1)$, $\lim_{h \to 0^+} \int_h^{1-h} f^{-a}(1-t)^{a-1}dt$ exists. Let this limit be $g(a)$. In addition it is given the function $g(a)$ is differentiable on $(0, 1)$.

The value of $g\left(\frac{1}{2}\right)$ is

A. π

B. 2π

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: A

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24. Given that for each $a\varepsilon(0, 1)$, $\lim\left(h \to 0^{+}\right)\int_{h}^{1-h} f^{-a}(1-t)^{a-1}dt$ exists. Let this limit be $g(a)$. In addition it is given the function $g(a)$ is differentiable on $(0, 1)$.

The value of $g'\left(\frac{1}{2}\right)$ is

A. $\frac{\pi}{2}$

B. π

C. $-\frac{\pi}{2}$

D. 0

Answer: D

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25. Let $F: R \to R$ be a thrice differntiable function. Suppose that $F(1) = 0, F(3) = -4$ and $F'(x) < 0$ for all $x \varepsilon (1/2, 3)$. Let $f(x) = xF(x)$ for all $x \varepsilon R$.

The correct statement(s) is (are)

A. $f'(1) < 0$

B. $f(2) < 0$

C. $f'(x) \neq 0$ for an $x \varepsilon (1, 3)$

D. $f'(x) = 0$ for some $x \varepsilon (1, 3)$

Answer: A::B::C

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26. Let $F : R - R$ be a thrice differentiable function. Suppose that $F(1) = 0, F(3) = -4$ and $F(x) < 0$ for all $x \in \left(\frac{1}{2}, 3\right)$. Let $f(x) = xF(x)$ for all $x \in R$ The correct statement is

A. $9f'(3) + f'(1) - 32 = 0$

B. $\int_1^3 f(x)dx = 12$

C. $9f'(3) - f'(1) + 32 = 0$

D. $\int_1^3 f(x)dx = -12$

Answer: C::D

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27. Match the statements/expressions given in List (with the value given in List II.

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28. For any real number $x, let[x]$ denote the largest integer less than or equal to $x, Let f$ be a real-valued function defined on the interval $[-10, 10]$ be $f(x) = \{x - [x], \quad \text{if} \quad [x] is odd 1 + [x] - x, \quad \text{if} \quad [x] is even$ Then the value of $\frac{\pi^2}{10}\int_{-1}^{10} f(x)\cos \pi x dx is$ _ _

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29. Let $y'(x) + y(x)g'(x) = g(x)g'(x), y(0), x \in R,$ where $f'(x)$ denotes $\frac{dy(x)}{dx}$, and $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0.$ Then the value of $y(2)$ is_____

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30. The value of $\int_0^1 4x^3\left\{\frac{d^2}{dx^2}(1 - x^2)^5\right\} dx is$

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31. Let $f: R\overrightarrow{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^{x} f(t)dtf \text{ or } allx \in [-1, 2] andG(x) = \int_{-1}^{x} t|f(f(t))|dtf \text{ or } a$

Then the value of $f\left(\frac{1}{2}\right)$ is

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32. If $\alpha = \int_0^1 \left(e^9x + 3\tan^{(-1)x}\right)\left(\frac{12 + 9x^2}{1 + x^2}\right)dxwherm^{-1}$ takes only principal values, then the value of $\left((\log)_e|1 + \alpha| - \frac{3\pi}{4}\right)is$

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33. Let $F(x) = \int_x^{x^2 + \frac{\pi}{6}} \left[2\cos^2 t.\, dt\right]$ for all $x \in R$ and $f: \left[0, \frac{1}{2}\right] \to [0, \infty)$ be a continuous function.For $a \in \left[0, \frac{1}{2}\right]$, if F'(a)+2 is the area of the region bounded by x=0,y=0,y=f(x) and x=a, then f(0) is

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34. Let $f:R\overrightarrow{R}$ be a function defined by $f(x)=\{[x], x\leq 20, x>2$ where $[x]$ is the greatest integer less than or equal to $x\cdot$ If $I=\int_{-1}^{2}\frac{xf(x^2)}{2+f(x+1)}dx, thenthevalueof f(4I-1) is$

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35. The total number for distinct $x\in[0,1]$ for which $\int_0^x \frac{t^2}{1+t^4}dt=2x-1$ is ________.

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36. Let $f:R\overrightarrow{R}$ be a differentiable function such that $f(0), f\left(\frac{\pi}{2}\right)=3 and f'(0)=1.$ If $g(x)=\int_x^{\frac{\pi}{2}}\left[f'(t)\cos ect-\cot t\cos ect f(t)\right]dt f$ or $x\left(0,\frac{\pi}{2}\right]$, then $(\lim)_{x\overrightarrow{0}}g(x)=$

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37. For each positive integer n, let $y_n = \frac{1}{n}\left((n+1)(n+2)n \dot{+} n\right)^{\frac{1}{n}}$ For $x \in R$ let $[x]$ be the greatest integer less than or equal to x. If $(\lim)_{n \to \infty} y_n = L$, then the value of $[L]$ is ______.

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38. The value of the integral $\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{\left((x+1)^2(1-x)^6\right)^{\frac{1}{4}}} dx$ is ______.

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