MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

DETERMINANT

Single Correct Answer

1. $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\beta \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$

if $\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$

A. 1

B. 2

C. 3/2

D. 1/2

Answer: A

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2. If α, β, γ are roots of the equation $x^2(px+q)=r(x+1)$, then the value of determinant $\begin{vmatrix} 1+\alpha & 1 & 1 \\ 1 & 1+\beta & 1 \\ 1 & 1 & 1+\gamma \end{vmatrix}$ is

A. $\alpha\beta\gamma$

B. $1+\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

C. 0

D. none of these

Answer: C

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3. if $\omega \neq 1$ is cube root of unity and $x+y+z \neq 0$ then

$$\begin{vmatrix} \frac{x}{1+\omega} & \frac{y}{\omega+\omega^2} & \frac{z}{\omega^2+1} \\ \frac{y}{\omega+\omega^2} & \frac{z}{\omega^2+1} & \frac{x}{1+\omega} \\ \frac{z}{\omega^2+1} & \frac{x}{1+\omega} & \frac{y}{\omega+\omega^2} \end{vmatrix} = 0 \text{ if}$$

A. $x^2 + y^2 + z^2 = 0$

B. $x + y\omega + z\omega^2 = 0$ or $x = y = z$

C. $x \neq y \neq z \neq 0$

D. $x = 2y = 3z$

Answer: B

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4. If $a = \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)$ then $\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix}$

A. purely real

B. purely imaginary

C. 0

D. none of these

Answer: B

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5. If α is a root of $x^4 = 1$ with negative principal argument then the principal argument of $\Delta(\alpha) = \begin{vmatrix} 1 & 1 & 1 \\ \alpha^n & \alpha^{n+1} & \alpha^{n+3} \\ \frac{1}{\alpha^{n+1}} & \frac{1}{\alpha^n} & 0 \end{vmatrix}$ is

A. $\frac{5\pi}{14}$

B. - $\frac{3\pi}{4}$

C. $\frac{\pi}{4}$

D. - $\frac{\pi}{4}$

Answer: B

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6. $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinations then

A. $\Delta_1 = 3(\Delta_2)^2$

B. $\frac{d}{dx}(\Delta_1) = 3\Delta_2$

C. $\frac{d}{dx}(\Delta_1) = 3(\Delta_2)^2$

D. $\Delta_1 = 3\Delta_2^{3/2}$

Answer: B

7. If $a^2+b^2+c^2+ab+bc+ca\le 0$ for all, $a,b,c\in R$, then the value of the determinant

$$\begin{vmatrix}(a+b+2)^2 & a^2+b^2 & 1\\ 1 & (b+c+2)^2 & b^2+c^2\\ c^2+a^2 & 1 & (c+a+2)^2\end{vmatrix}$$, is equal to

A. 65

B. $a^2+b^2+c^2+31$

C. $4\left(a^2+b^2+c^2\right)$

D. 0

Answer: A

8. Product of roots of equation $\begin{vmatrix} 1+2x & 1 & 1-x \\ 2-x & 2+x & 3+x \\ x & 1+x & 1-x^2 \end{vmatrix} = 0$ is

A. $1/2$

B. $3/4$

C. $4/3$

D. $1/4$

Answer: A

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9. If $x \neq 0, y \neq 0, z \neq 0 and |1 + x111 + y1 + 2y11 + z1 + z1 + 3z| = 0$, then $x^{-1} + y^{-1} + z^{-1}$ is equal to 1 b. -1 c. -3 d. none of these

A. 0

B. 1

C. 3

D. 6

Answer: C

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10. If $Y = SX$, $Z = tX$ all the variables being differentiable functions of x and lower suffices denote the derivative with respect to x and

$$\begin{vmatrix} X & Y & X \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} + \begin{vmatrix} S_1 & t_1 \\ S_2 & t_2 \end{vmatrix} = X^n, \text{ then } n =$$

A. 1

B. 2

C. 3

D. 4

Answer: C

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11. If $w \neq 1$ is a cube root of unity and $\Delta = \begin{vmatrix} x+w^2 & w & 1 \\ w & w^2 & 1+x \\ 1 & x+w & w^2 \end{vmatrix} = 0,$

then value of x is

A. 0

B. 2

C. -1

D. None of these

Answer: A

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12. Let $|A| = \left|a_{ij}\right|_{3\times 3} \neq 0$ Each element a_{ij} is multiplied by by k^{i-j} Let $|B|$ the resulting determinant, where $k_1|A| + k_2|B| = a$ then $k_1 + k_2 =$

A. 1

B. -1

C. 0

D. 2

Answer: C

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13. If $\alpha, \beta. \gamma$ are the roots of $x^3 + px^2 + q = 0$, where $q = 0$, then

$$\Delta = \begin{vmatrix} \frac{1}{\alpha} & \frac{1}{\beta} & \frac{1}{\gamma} \\ \frac{1}{\beta} & \frac{1}{\gamma} & \frac{1}{\alpha} \\ \frac{1}{\gamma} & \frac{1}{\alpha} & \frac{1}{\beta} \end{vmatrix}$$ equals

A. $\alpha\beta\gamma$

B. $\alpha + \beta + \gamma$

C. 0

D. None of these

Answer: C

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14. If $a - 2b + c = 1$, then the value of $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ is

A. x

B. $-x$

C. -1

D. 1

Answer: C

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15. Let $x > 0, y > 0, z > 0$ are respectively the $2^{nd}, 3^{rd}, 4^{th}$ terms of a $G.P.$

and $\Delta = \begin{vmatrix} x^k & x^{k+1} & x^{k+2} \\ y^k & y^{k+1} & y^{k+2} \\ z^k & z^{k+1} & z^{k+2} \end{vmatrix} = (r - 1)^2\left(1 - \frac{1}{r^2}\right)$ (where r is the common ratio), then

A. $k = -1$

B. $k = 1$

C. $k = 0$

D. None of these

Answer: A

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16. If $a, b, c, d > 0$, $x \in R$ and $\left(a^2 + b^2 + c^2\right)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \leq 0,$ then

$$\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} =$$

A. 1

B. -1

C. 0

D. none of these

Answer: C

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17. Show that

$$\left| \wedge xC^x_r C^x_{r+1} C^y_{r+2} C^y_r C^y_{r+1} C^z_{r+2} C^z_r C^z_{r+1} C_{r+1} \right| = \left| \wedge xC^{x+1}_r C^{x+2}_{r+1} C^y_{r+2} C^{y+1}_r C^{y+2}_{r+1} C \right.$$

.

A. 0

B. 2^n

C. ${}^{x+y+z}C_r$

D. ${}^{x+y+z}C_{r+2}$

Answer: A

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18. If $\begin{vmatrix} .{}^{9}C_4 & .{}^{9}C_5 & .{}^{10}C_r \\ .{}^{10}C_6 & .{}^{10}C_7 & .{}^{11}C_{r+2} \\ .{}^{11}C_8 & .{}^{11}C_9 & .{}^{12}C_{r+4} \end{vmatrix} = 0$, then the value of r is equal to

A. 3

B. 4

C. 5

D. 6

Answer: C

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19. If either of the two P, Q and R are equal and $P+Q+R=180^{\circ}$, then the value of $\begin{vmatrix} 1 & 1+\sin P & \sin P(1+\sin P) \\ 1 & 1+\sin Q & \sin Q(1+\sin Q) \\ 1 & 1+\sin R & \sin R(1+\sin R) \end{vmatrix}$ is

A. 0

B. 1

C. $\sin(P+Q+R)$

D. $\sin\Psi nQ\sin R$

Answer: A

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20. In a triangle ABC, if a, b, c are the sides opposite to angles A, B, C respectively, then the value of $\begin{vmatrix} b\cos C & a & c\cos B \\ c\cos A & b & a\cos C \\ a\cos B & c & b\cos A \end{vmatrix}$ is

A. 1

B. -1

C. 0

D. $a\cos A + b\cos B + c\cos C$

Answer: C

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21. If $\begin{array}{l} a = 1 + 2 + 4 + \text{...to n terms} \\ b = 1 + 3 + 9 + \text{...to n terms} \\ c = 1 + 5 + 25 + \text{...to n terms} \end{array}$

then $\begin{vmatrix} a & 2b & 4c \\ 2 & 2 & 2 \\ 2^n & 3^n & 5^n \end{vmatrix} =$

A. $(30)^n$

B. $(10)^n$

C. 0

D. $2^n + 3^n + 5^n$

Answer: C

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22. If $a_1, a_2, a_3, 54, a_6, a_7, a_8, a_9$ are in H.P., and $D = \left|a_1 a_2 a_3 54 a_6 a_7 a_8 a_9\right|$, then the value of $[D] is where [.]$ represents the greatest integer function

A. 4

B. 5

C. 6

D. 7

Answer: B

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23. $$\begin{vmatrix} \frac{1}{c} & \frac{1}{c} & -\frac{a+b}{c^2} \\ -\frac{b+c}{c^2} & \frac{1}{a} & \frac{1}{a} \\ \frac{-b(b+c)}{a^2c} & \frac{a+2b+c}{ac} & \frac{-b(a+b)}{ac^2} \end{vmatrix}$$ is

A. dependent on a, b, c

B. dependent on a

C. dependent on b

D. independent on a, b and c

Answer: D

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24. The equation $\begin{vmatrix} (1+x)^2 & (1-x)^2 & -\left(2+x^2\right) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$ has has (a) no real solution (b) 4 real solutions (c) two real and two non-real solutions (d) infinite number of solutions, real or non-real

A. has no real solution

B. has 4 real solutions

C. has two real and two non-real solutions

D. has infinite number of solutions, real or non-real

Answer: D

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25. Let $\Delta_1 = \begin{bmatrix} ap^2 & 2ap & 1 \\ aq^2 & 2aq & 1 \\ ar^2 & 2ar & 1 \end{bmatrix}$ and $\Delta_2 = \begin{bmatrix} apq & a(p+q) & 1 \\ aqr & a(q+r) & 1 \\ arp & a(r+p) & 1 \end{bmatrix}$ then

A. $\Delta_1 = \Delta_2$

B. $\Delta_2 = 2\Delta_1$

C. $\Delta_1 = 2\Delta_2$

D. $\Delta_1 + 2\Delta_2 = 0$

Answer: D

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26. Area of triangle whose vertices are $\left(a, a^2\right), \left(b, b^2\right), \left(c, c^2\right)$ is $\frac{1}{2}$. and area of another triangle whose vertices are $\left(p, p^2\right), \left(q, q^2\right)$ and $\left(r, r^2\right)$ is

4, then the value of $\begin{vmatrix} (1+ap)^2 & (1+bp)^2 & (1+cp)^2 \\ (1+aq)^2 & (1+bp)^2 & (1+cq)^2 \\ (1+ar)^2 & (1+br)^2 & (1+cr)^2 \end{vmatrix}$ is (A) 2 (B) 4 (C) 8 (D) 16

A. 2

B. 4

C. 8

D. 16

Answer: D

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27. The value of $\begin{vmatrix} \beta\gamma & \beta\gamma' + \beta'\gamma & \beta'\gamma' \\ \gamma\alpha & \gamma\alpha' + \gamma'\alpha & \gamma'\alpha' \\ \alpha\beta & \alpha\beta' + \alpha'\beta & \alpha'\beta' \end{vmatrix}$ is

A. $(\alpha\beta' - \alpha'\beta)(\beta\gamma' - \beta'\gamma)(\gamma\alpha' - \gamma'\alpha)$

B. $(\alpha\alpha' - \beta\beta')(\beta\beta' - \gamma\gamma')(\gamma\gamma' - \alpha\alpha')$

C. $(\alpha\beta' + \alpha'\beta)(\beta\gamma' + \beta'\gamma)(\gamma\alpha' + \gamma'\alpha)$

D. None of these

Answer: A

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28. If $\begin{vmatrix} a & b & a \\ b & c & 1 \\ c & a & 1 \end{vmatrix} = 2010$ and if $\begin{vmatrix} c-a & c-b & ab \\ a-b & a-c & bc \\ b-c & b-a & ca \end{vmatrix} - \begin{vmatrix} c-a & c-b & c^2 \\ a-b & a-c & a^2 \\ b-c & b-a & b^2 \end{vmatrix} = p$,

then the number of positive divisors of p is

A. 36

B. 49

C. 64

D. 81

Answer: D

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29. Let $\begin{vmatrix} a & l & m \\ l & b & n \\ m & n & c \end{vmatrix} \begin{vmatrix} bc - n^2 & mn - lc & \ln - bm \\ mn - lc & ac - m^2 & ml - an \\ \ln - bm & lm - an & ab - l^2 \end{vmatrix} = 64.$ If the value of

$\begin{vmatrix} 2a + 3l & 3l + 5m & 5m + 4a \\ 2l + 3b & 3b + 5n & 5n + 4l \\ 2m + 3n & 3n + 5c & 5c + 4m \end{vmatrix} = \lambda$ then $\left[\frac{\lambda}{2}\right]$ equals

A. 120

B. 240

C. 360

D. 480

Answer: C

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30. The value of $\begin{vmatrix} x^2+y^2 & ax+by & x+y \\ ax+by & a^2+b^2 & a+b \\ x+y & a+b & 2 \end{vmatrix}$ depends on

A. a

B. b

C. x

D. none of these

Answer: D

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31. If $u = ax + by + cz$, $v = ay + bz + cx$, $w = ax + bx + cy$, then the value of

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix}$ is

A. $u^2 + v^2 + w^2 - 2uvw$

B. $u^3 + v^3 + w^3 - 3uvw$

C. 0

D. none of these

Answer: B

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32. If the number of positive integral solutions of $u + v + w = n$ be denoted by P_n then the absolute value of $\begin{vmatrix} P_n & P_{n+1} & P_{n+2} \\ P_{n+1} & P_{n+2} & P_{n+3} \\ P_{n+2} & P_{n+3} & P_{n+4} \end{vmatrix}$ is

A. -1

B. 2

C. 3

D. 4

Answer: A

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33. If $f(x), h(x)$ are polynomials of degree 4 and $\begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ p & q & r \end{vmatrix}$ $= mx^4 + nx^3 + rx^2 + sx + r$ be an identity in x, then $\begin{vmatrix} f''(0) - f''(0) & g''(0) - g''(0) & h''(0) - h''(0) \\ a & b & c \\ p & q & r \end{vmatrix}$ is

A. $2(3n - r)$

B. $2(2n - 3r)$

C. $3(n - 2r)$

D. none of these

Answer: A

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34. If $f(x) = \begin{vmatrix} x-2 & (x-1)^2 & x^3 \\ (x-1) & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+2)^3 \end{vmatrix}$ then coefficient of x in $f(x)$ is

A. -4

B. -2

C. -6

D. 0

Answer: B

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35. If $f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & x \end{vmatrix}$, then $\lim_{x \to 0} \frac{f(x)}{x^2} =$

A. 0

B. 3

C. 2

D. 1

Answer: D

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36. If the system of linear equations $x+2ay+az=0, x+3by+bz=0$ and $x+4cy+cz=0$ has a non-zero solution, then a, b, c are in

A. $A. P.$

B. $G. P.$

C. $H. P.$

D. satisfies $a+2b+3c=0$

Answer: C

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37. Find all values of λ for which the $(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 02x + (3\lambda + 1)y + 3(\lambda$

possess non-trivial solution and find the ratios $x:y:z,$ where λ has the smallest of these value.

A. $3:2:1$

B. $3:3:2$

C. $1:3:1$

D. $1:1:1$

Answer: D

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38. The system of homogenous equations

$tx + (t + 1)y + (t - 1)z = 0,$ $(t + 1)x + ty + (t + 2)z = 0,$

$(t - 1)x + (t + 2)y + tz = 0$ has a non trivial solution for

A. exactly three real values of t

B. exactly two real values of t

C. exactly one real values of t

D. infinite number of values of t

Answer: C

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39. If a, b, c are non-zero, then the system of equations $(\alpha+a)x+\alpha y+\alpha z=0, \alpha x+(\alpha+b)y+\alpha z=0, \alpha x+\alpha y+(\alpha+c)z=0$ has a non-trivial solution if $\alpha^{-1}=-\left(a^{-1}+b^{-1}+c^{-1}\right)$ b. $\alpha^{-1}=a+b+c$ c. $\alpha+a+b+c=1$ d. none of these

A. $2\alpha=a+b+c$

B. $\alpha^{-1}=a+b+c$

C. $\alpha+a+b+c=1$

D. $\alpha^{-1}=-\left(a^{-1}+b^{-1}+c^{-1}\right)$

Answer: D

40. The values of θ, λ for which the following equations $\sin\theta x - \cos\theta y + (\lambda + 1)z = 0$, $\cos\theta x + \sin\theta y - \lambda z = 0$, $\lambda x + (\lambda + 1)y + \cos\theta z = 0$ have non trivial solution, is

A. $\theta = n\pi, \lambda \in R - \{0\}$

B. $\theta = 2n\pi, \lambda$ is any rational number

C. $\theta = (2n + 1)\pi, \lambda \in R^+, n \in I$

D. $\theta = (2n + 1)\frac{\pi}{2}, \lambda \in R, n \in I$

Answer: D

41. If the system of equation

$(, x - 2y + z = a), (2x + y - 2z = b)$, and $, (x + 3y - 3z = c)$

have at least one solution, then

A. $a + b + c = 0$

B. $a - b + c = 0$

C. $-a + b + c = 0$

D. $a + b - c = 0$

Answer: B

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42. If A, B, C are the angles of a triangle, the system of equations

$(\sin A)x + y + z = \cos A, x + (\sin B)y + z = \cos B, x + y + (\sin C)z = 1 - \cos C$ has

A. No solution

B. Unique solution

C. Infinitely many solutions

D. Finitely many solutions

Answer: B

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Comprehension

1. A 3×3 determinant has entries either 1 or -1.

Let $S_3 =$ set of all determinants which contain determinants such that product of elements of any row or any column is -1 For example

$\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix}$ is an element of the set S_3.

Number of elements of the set $S_3 =$

A. 10

B. 16

C. 12

D. 18

Answer: B

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2. A 3×3 determinant has entries either 1 or -1.

Let $S_3 =$ set of all determinants which contain determinants such that product of elements of any row or any column is -1 For example

$$\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix}$$ is an element of the set S_3.

Number of elements of the set $S_n =$

A. 2^n

B. 2^{n-1}

C. 2^{2n}

D. $2^{(n-1)^2}$

Answer: D

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Multiple Correct Answer

1. If $x \in R, a_i, b_i, c_i \in R$ for $i = 1, 2, 3$ and $\begin{vmatrix} a_1 + b_1 x & a_1 x + b_1 & c_1 \\ a_2 + b_2 x & a_2 x + b_2 & c_2 \\ a_3 + b_3 x & a_3 x + b_3 & c_3 \end{vmatrix} = 0,$

then which of the following may be true ?

A. $x = 1$

B. $x = -1$

C. $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

D. none of these

Answer: A::B::C

2. If $a_i,\ i = 1, 2, \ldots\ldots, 9$ are perfect odd squares, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is always a multiple of

A. 4

B. 7

C. 16

D. 64

Answer: A::C::D

3. The value of the determinant $\begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta & \cos\theta & \sin\alpha \\ -\cos\theta & \sin\theta & \lambda\cos\alpha \end{vmatrix}$ is

A. independent of θ for all $\lambda \in R$

B. independent of θ and α when $\lambda = 1$

C. independent of θ and α when $\lambda = -1$

D. independent of λ for all θ

Answer: A::C

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4. A solution set of the equations $x + 2y + z = 1$, $x + 3y + 4z = k$, $x + 5y + 10z = k^2$ is

A. $(1 + 5\lambda, -3\lambda, \lambda)$

B. $(5\lambda - 1, 1 - 3\lambda, \lambda)$

C. $(1 + 6\lambda, \ - 2\lambda, \lambda)$

D. $(1 - 6\lambda, \lambda, \lambda)$

Answer: A::B

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5. Consider the system of equations : $x\sin\theta - 2y\cos\theta - az = 0$, $x + 2y + z = 0$, $-x + y + z = 0, \theta \in R$

A. The given system will have infinite solutions for $a = 2$

B. The number of integer values of a is 3 for the system to have nontrivial solutions.

C. For $a = 1$ there exists θ for which the system will have infinite solutions

D. For $a = 3$ there exists θ for which the system will have unique solutions

Answer: B::C::D

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Examples

1. find the value of $\begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$

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2. Prove that the determinant $\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

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3. The parameter on which the value of the determinant $\left|1aa^2\cos(p-d)x\cos px\cos(p+d)x\sin(p-d)x\sin px\sin(p+d)x\right|$ does not depend is a b. p c. d d. x

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4. Let a, b, c be positive and not all equal. Show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative

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5. If $a, b, c \in R,$ then find the number of real roots of the equation

$= |xc - b - cxab - ax| = 0$

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6. If $x + y + z = 0$, prove that $\begin{vmatrix} ax & by & cz \\ cy & az & bx \\ bz & cx & ay \end{vmatrix} = xyz \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

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7. If $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2 + 3\lambda & \lambda - 1 & \lambda + 3 \\ \lambda^2 + 1 & 2 - \lambda & \lambda - 3 \\ \lambda^2 - 3 & \lambda + 4 & 3\lambda \end{vmatrix}$ then $t =$

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8. The largest value of a third order determinant whose elements are equal to 1 or 0 is

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9. Prove that the value of the determinant

$$\begin{vmatrix} -7 & 5+3i & \frac{2}{3}-4i \\ 5-3i & 8 & 4+5i \\ \frac{2}{3}+4i & 4-5i & 9 \end{vmatrix} \text{ is real}$$

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10. Without expanding the determinants Prove that

$$\begin{vmatrix} 103 & 115 & 114 \\ 111 & 108 & 106 \\ 104 & 113 & 116 \end{vmatrix} + \begin{vmatrix} 113 & 116 & 104 \\ 108 & 106 & 111 \\ 115 & 114 & 103 \end{vmatrix} = 0$$

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11. Prove that $\begin{vmatrix} ax & by & cz \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} a & c & c \\ x & y & z \\ yz & xz & xy \end{vmatrix}$

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12. for $x, x, z > 0$ Prove that $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} = 0$

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13. witout expanding at any stage Prove that

$$\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$$

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14. consider the fourth -degree polynomial equation

$$\begin{vmatrix} a_1 + b_1 x & a_1 x^2 + b_1 & c_1 \\ a_2 + b_2 x^2 & a_2 x^2 + b_2 & c_2 \\ a_3 + b_3 x^2 & a_3 x^2 + b_3 & c_3 \end{vmatrix} = 0$$

Without expanding the determinant find all the roots of the equation.

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15. Let $\Delta_r = \begin{vmatrix} r-1 & n & 6 \\ (r-1)^2 & 2n^2 & 4n-2 \\ (r-1)^2 & 3n^3 & 3n^2-3n \end{vmatrix}$. Show that $\Sigma_{r=1}^{n} \Delta_r$ is constant.

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16. Find the value of $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$

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17. Find the value of determinant

$$\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$$

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18. Find the value of the determinant $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix}$

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19. Prove that $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix} = 0$

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20. Using properties of determinants, solve the following for x:

$|x - 22x - 33x - 4x - 42x - 93x - 16x - 82x - 273x - 64| = 0$

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21. By using properties of determinants, prove the following:

$|x + 42x2x2 \times \ + 42x2x2 \times \ + 4| = (5x + 4)(4 - x)^2$

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22. prove that $\begin{bmatrix} a - b - c & 2a & 2a \\ 2b & b - c - a & 2b \\ 2c & 2c & c - a - b \end{bmatrix} = (a + b + c)^3$

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23. if $x_i = a_ib_iC_i, i = 1, 2, 3$ are three- digit positive integer such that each x_i is a mulptiple of 19 then prove that det $\begin{Bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{Bmatrix}$ is divisible by 19.

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24. If a, b and c are real numbers, and $\Delta = |b + \ + aa + bc + aa + \ + ca + \ + \ + a| = 0$ .Show that either $a + b + c = 0$ or $a = b = c$.

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25. Find the value of the determinant $|baabpqr111|, wherea, b, andc$ are respectively, the pth, qth, and rth terms of a harmonic progression.

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26.

if $a_a, a_2, a_3 \ldots .$ are in A.P, then find the value of the following determinant:

$$\begin{vmatrix} a_p + a_{p+m} + a_{p+2m} & 2a_p + 3a_{p+m} + 4a_{p+2m} & 4a_p + 9a_{p+m} + 16a_{p+2m} \\ a_p + a_{q+m} + a_{q+2m} & 2a_q + 3a_{q+m} + 4a_{q+2m} & 4a_q + 9a_{q+m} + 16a_{q+2m} \\ a_r + a_{r+m} + a_{r+2m} & 2a_r + 3a_{r+m} + 4a_{r+2m} & 4a_r + 9a_{r+m} + 16a_{r+2m} \end{vmatrix}$$

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27. Prove that

$$\left|\left(1, , \beta\gamma + \alpha\delta, , \beta^2\gamma^2 + \alpha^2, \delta^2\right), \left(1, , \gamma\alpha + \beta\delta, , \gamma^2\alpha^2 + \beta^2\delta^2\right), \left(1, , \alpha\beta + \gamma\delta, , \alpha^2\beta^2 +\right.\right.$$

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28. Prove that

$$\left|ab + ca^2bc + ab^2ca + bc^2\right| = -(a + b + c) \times (a - b)(b - c)(c - a)$$

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29. Prove that $\begin{vmatrix} x^2 & x^2-(y-z)^2 & yz \\ y^2 & y^2-(z-x)^2 & zx \\ z^2 & z^2-(x-y)^2 & xy \end{vmatrix}$

$= (x-y)(y-z)(z-x)(x+y+z)\left(x^2+y^2+z^2\right)$

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30. If a,b,c are all distinct and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix}$ =0, show that abc(ab+bc+ac) = a+b+c

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31. Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

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32. prove that $\begin{vmatrix} (b+c)^2 & bc & ac \\ ba & (c+a)^2 & cb \\ ca & cb & (a+b)^2 \end{vmatrix}$

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

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33. Let a,b,c be real numbers with $a^2 + b^2 + c^2 = 1$. Show that the equation $\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix} = 0$ represents a straight line.

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34. If $a^2 + b^2 + c^2 = 1,$ then prove that

$|a^2 + \left(b^2 + c^2\right)\cos\varphi ab(1 - \cos\varphi)ac(1 - \cos\varphi)ba(1 - \cos\varphi)b^2 + \left(c^2 + a^2\right)\cos\varphi bc(1 -$

is independent of $a, b, \cdot$

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35. Find the area of a triangle whose vertices are

$A(3, 2), B(11, 8)andC(8, 12)^{\cdot}$

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36. If the lines $a_1x + b_1y + 1 = 0,\ a_2x + b_2y + 1 = 0\ and\ a_3x + b_3y + 1 = 0$ are concurrent, show that the points $\left(a_1, b_1\right),\ \left(a_2, b_2\right) and\ \left(a_3, b_3\right)$ are collinear.

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37. The number of values of a for which the lines $2x + y - 1 = 0$, $ax + 3y - 3 = 0,$ and $3x + 2y - 2 = 0$ are concurrent is 0 (b) 1 (c) 2 (d) infinite

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38. If the lines $ax + y + 1 = 0, x + by + 1 = 0andx + y + c = 0(a, b, c$ being distinct and different from $1)$ are concurrent, then prove that $\frac{1}{1 - a} + \frac{1}{1 - b} + \frac{1}{1 - c} = 1.$

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39. Find the value of λ if $2x^2 + 6xy + 3y^2 + 8x + 14y + \lambda = 0$ represent a pair of straight lines.

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40. show that the determinant

$$\begin{vmatrix} a^2 + b^2 + c^2 & bc + ca + ab & bc + ca + ab \\ bc + ca + ab & a^2 + b^2 + c^2 & bc + ca + ab \\ bc + ca + ab & bc + ca + ab & a^2 + b^2 + c^2 \end{vmatrix}$$

is always non- negative.

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41. Factorize the following

$$\left|3a + b + ca^3 + b^3 + c^3a + b + ca^2 + b^2 + c^2a^4 + b^4 + c^4a^2 + b^2 + c^2a^3 + b^3 + c^3\right.$$

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42. prove that $\begin{vmatrix} (a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-z)^2 \end{vmatrix}$

$$\begin{vmatrix} (1+ax)^2 & (1+bx)^2 & (1+cx)^2 \\ (1+ay)^2 & (1+by)^2 & (1+cy)^2 \\ (1+az)^2 & (1+bx)^2 & (1+cz)^2 \end{vmatrix}$$

$= 2(b-c)(c-a)(a-b) \times (y-z)(z-x)(x-y)$

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43. If α, β, γ are real numbers, then without expanding at any stage, show that $|1\cos(\beta-\alpha)\cos(\gamma-\alpha)\cos(\alpha-\beta)1\cos(\gamma-\beta)\cos(\alpha-\gamma)\cos(\beta-\gamma)1| = 0$

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44. If $\begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix} = 3$ then find the value of $\begin{vmatrix} x^3-1 & 0 & x-x^4 \\ 0 & x-x^4 & x^3-1 \\ x-x^4 & x^3-1 & 0 \end{vmatrix}$

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45. Show that $\begin{vmatrix} bc - a^2 & ca - b^2 & ab - c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ca - b^2 \end{vmatrix}$

$$\begin{vmatrix} a^2 & c^2 & 2ca - b^2 \\ 2ab - c^2 & b^2 & a^2 \\ b^2 & 2ac - a^2 & c^2 \end{vmatrix}.$$

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46. Let $= \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$ then find the values of $f(0)$ and f' $(\pi/2)$.

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47. if $= \begin{vmatrix} x^n & n! & 2 \\ \cos x & \cos\frac{n\pi}{2} & 4 \\ \sin x & \sin\frac{n\pi}{2} & 8 \end{vmatrix}$, then find the value of

$\frac{d^n}{dx^n}[f(x)]_{x=0}(n \in z)$

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48. if f, g, and h are differentiable function of x and

$\Delta(x) = \begin{vmatrix} f & g & h \\ (xf)' & (xg)' & (xh)' \\ \left(x^2f\right)'' & \left(x^2g\right)'' & \left(x^2h\right)'' \end{vmatrix}$ prove that

$\Delta(x) = \begin{vmatrix} f & g & h \\ f' & g & h \\ \left(x^3f''\right)' & \left(x^3g''\right)' & \left(x^3h''\right)' \end{vmatrix}$ where prime(′)

denotes the derivatives .

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49. Let α be a repeated root of a quadratic equation $f(x) = 0andA(x), B(x), C(x)$ be polynomials of degrees 3, 4, and 5, respectively, then show that $|A(x)B(x)C(x)A(\alpha)B(\alpha)C(\alpha)A'(\alpha)B'(\alpha)C'(\alpha)|$ is divisible by $f(x)$, where prime $(')$ denotes the derivatives.

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50. if $\Delta(x) = \begin{vmatrix} a_1 + x & b_1 + x & c_1 + x \\ a_2 + x & b_2 + x & c_2 + x \\ a_3 + x & b_3 + x & c_3 + x \end{vmatrix}$ then show that $\Delta(x) = 0$

and that $\Delta(x) = \Delta(0) + sx.$ where s denotes the sum of all the cofactors of all the elements in $\Delta(0)$

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51. If $\Delta(x) = \left| 1 \quad x^2 \quad x^26 \quad 4x \quad 39 \quad x \quad -7 \right|$ then find the value of $\int_0^1 \Delta(x)dx$

without expanding $\Delta(x)$.

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52. Find the value of $aandb$ if the system of equation $a^2x - by = a^2 - bandbx = b^2y = 2 + 4b$ (i) posses unique solution (ii) infinite solutions

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53. If a system of three linear equations

$x + 4ay + a = 0, x + 3by + b = 0, \text{and } x + 2cy + c = 0$

is consistent, then prove that a, b, c are in H.P.

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54. Solve by Cramers rule $x + y + z = 6\ x - y + z = 2\ 3x + 2y - 4z = -5$

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55. For what values of p and q the system of equations $2x + py + 6z = 8, x + 2y + qz = 5, x + y + 3z = 4$ has i no solution ii a unique solution iii in finitely many solutions.

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56. If $2ax - 2y + 3z = 0, x + ay + 2z = 0, and 2 + az = 0$ have a nontrivial solution, find the value of $\dot{a}$

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57. For what values of k, the following system of equations possesses a nontrival solution over the set of rationals: $c + ky + 3z = 0, 3c + ky - 2z = 02c + 3y - 4x = 0$. Also find the solution for this value of $\dot{k}$

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58. Prove that:

$|-2aa+ba+cb+a-2+\ +ac+b-2c|=4(a+b)(b+c)(c+a)$

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59. If a,b and c are non- zero real number then prove that

$$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}=0$$

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60. Prove that

$|ax-by-czay+bxcx+azay+bxby-cz-axbz+cycx+azbz+cycz-ax-by|=\Big($

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61. If $f(x)$ is a polynomial of degree < 3, prove that

$$|1af(a)/(x - a)1bf(b)/(x - b)1cf(c)/(x - c)| \div \left|1aa^2 1 \wedge 21 \wedge 2\right| = \frac{f(x)}{(x - a)(x - b)(x - c}$$

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62. $\begin{vmatrix} 2a_1b_1 & a_1b_2 + a_2b_1 & a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 & 2a_2b_2 & a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 & a_3b_2 + a_2b_3 & 2a_3b_3 \end{vmatrix} =$

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63. Find the value of $\begin{vmatrix} \cos\left(\frac{2\pi}{63}\right) & \cos\left(\frac{3\pi}{70}\right) & \cos\left(\frac{4\pi}{77}\right) \\ \cos\left(\frac{\pi}{72}\right) & \cos\left(\frac{\pi}{40}\right) & \cos\left(\frac{3\pi}{88}\right) \\ 1 & \cos\left(\frac{\pi}{90}\right) & \cos\left(\frac{2\pi}{99}\right) \end{vmatrix}$

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64. $Let f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$ .Then find $\lim_{t \to 0} \frac{f(t)}{t^2}$

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65. it

$x_1^2 + 2y_1^2 + 3z_1^2 = x_2^2 + 2y_2^2 + 3z_2^2 = x_3^2 + 2y_3^2 + 3z_3^2 = 2$ and $x_2x_3 + 2y_2y_3 + 3z_2z_3$

Then find the value of $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$

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66. Let α_1, α_2 and β_1, β_2 be the roots of the equation $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y_\beta \_ 2z = 0$ has a non trivial solution then prove that $\frac{b^2}{q^2} = \frac{ac}{pr}$

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67. If $bc + qr = ca + rp = ab + pq = -1$ and $(abc, pqr \neq 0)$ then $\begin{vmatrix} ap & a & p \\ bq & b & q \\ cr & c & r \end{vmatrix}$

is (A) 1 (B) 2 (C) 0 (D) 3

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Exercise 12 1

1. Evaluate $|\cos\alpha\cos\beta\cos\alpha\sin\beta - \sin\alpha - \sin\beta\cos\beta 0\sin\alpha\cos\beta\sin\alpha\sin\beta\cos\alpha|$

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2. If A,B,C are the angles of a non right angled triangle ABC. Then find the

value of: $\begin{vmatrix} \tan A & 1 & 1 \\ 1 & \tan B & 1 \\ 1 & 1 & \tan C \end{vmatrix}$

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3. If $e^{i\theta} = \cos\theta + is\int h\eta,$ find the value of

$\left|1e^{i\pi/3}e^{i\pi/4}e^{-i\pi/3}1e^{i2\pi/3}e^{-i\pi/4}e^{-i2\pi/3}1\right|$

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4. Find the number of real root of the equation

$|0x - ax - bx + a0x - cx + bx + c0| = 0, a \neq b \neq candb(a + c) > ac$

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5. If α, β, γ are the roots of $ax^3 + bx^2 + cx + d = 0and|\alpha\beta\gamma\beta\gamma\alpha\gamma\alpha\beta| = 0, \alpha \neq \beta \neq \gamma$ then find the equation whose roots are $\alpha + \beta - \gamma, \beta + \gamma - \alpha, and\gamma + \alpha - \beta$

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6. A triangle has vertices $A_i\left(x_i, y_i\right)$ for i= 1,2,3,. If the orthocenter of triangle is (0,0) then prove that

$$\begin{vmatrix} x_2 - x_3 & y_2 - y_3 & y_1\left(y_2 - y_3\right) + x_1\left(x_2 - x_3\right) \\ x_3 - x_1 & y_3 - y_1 & y_2\left(y_3 - y_1\right) + x_2\left(x_3 - x_1\right) \\ x_1 - x_2 & y_1 - y_2 & y_3\left(y_1 - y_2\right) + x_3\left(x_1 - x_2\right) \end{vmatrix} = 0$$

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7. if $\omega \ne 1$ is cube root of unity and x+y+z $\ne$ 0 then

$$\begin{vmatrix} \frac{x}{1+\omega} & \frac{y}{\omega+\omega^2} & \frac{z}{\omega^2+1} \\ \frac{y}{\omega+\omega^2} & \frac{z}{\omega^2+1} & \frac{x}{1+\omega} \\ \frac{z}{\omega^2+1} & \frac{x}{1+\omega} & \frac{y}{\omega+\omega^2} \end{vmatrix}$$ =0 if

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Exercise 12 2

1. Prove that the value of determinant $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$

where ω is complex cube root of unity .

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2. Prove that $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ac \\ c & c^2 & ab \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

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3. if $\Delta = \begin{vmatrix} abc & a_2 & c^2b \\ abc & c^2a & ca^2 \\ abc & a^2b & b^2a \end{vmatrix} = 0, (a, b, c \in R$ and are all

different and non- zero) the prove that $a + b + c = 0$

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4. if $a_r = (\cos 2r\pi + I\sin 2r\pi)^{1/9}$ then prove that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a^4 & a^5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = 0$$

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5. Given $A = |ab2cde2flm2n|, B = |f2de2n4l2mc2ab|$, then the value of B/A is __________.

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Exercise 12 3

1. Prove that the value of each the following determinants zero:

$$(a)\begin{vmatrix} a_1 & la_1 + mb_1 & b_1 \\ a_2 & la_2 + mb_2 & b_2 \\ a_3 & la_3 + mb_3 & b_3 \end{vmatrix}$$

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2. using properties of determinants evaluate

$$\begin{vmatrix} 18 & 40 & 89 \\ 40 & 89 & 198 \\ 89 & 198 & 440 \end{vmatrix}$$

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3. Prove that $\begin{vmatrix} 18 & 40 & 89 \\ 40 & 89 & 198 \\ 89 & 198 & 440 \end{vmatrix} = 3abc - a^3 - b^3 - c^3$

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4. Show that $|11 + p1 + p + q23 + 2p1 + 3p + 2q36 + 3p106p + 3q| = 1.$

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5. Show that $\begin{vmatrix} a - x & c & b \\ c & b - x & a \\ b & a & c - x \end{vmatrix} = 0$

where $a + b + c \neq 0$

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6. Show that:

$|3a - a + b - a + c - b + a3b - b + c - c + a - c + b3c| = 3(a + b + c)(ab + bc + ca)$.

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7. Using properties of determinants Prove that

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

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8. Solve $\begin{vmatrix} x^2-1 & x^2+2x+1 & 2x^2+3x+1 \\ 2x^2+x-1 & 2x^2+5x-3 & 4x^2+4x-3 \\ 6x^2-x-2 & 6x^2-7x+2 & 12x^2-5x-2 \end{vmatrix} = 0$

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9. Show that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

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10. Show that if $x_1, x_2, x_3 \neq 0$

$$\begin{vmatrix} x_1 + a_1b_1 & a_1b_2 & a_1b_3 \\ a_2b_1 & x_2 + a_2b_2 & a_2b_3 \\ a_3b_1 & a_3b_2 & x_3 + a_3b_3 \end{vmatrix}$$

$$= x_1x_2x_3\left(1 + \frac{a_1b_1}{x_1} + \frac{a_2b_2}{x_2} + \frac{a_3b_3}{x_3}\right)$$

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11. If $A, BandC$ are the angels of a triangle, show that

$|-1 + \cos B \cos C + \cos B \cos B \cos C + \cos A - 1 + \cos A \cos A - 1 + \cos B - 1 + \cos A - 1|$

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12. If $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+b)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = k(a-b)(b-c)(c-a)$ then the value of

k is a. 4 b. -2 c.-4 d. 2

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13. Prove that $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc + 1^2 & ac + b^2 & ab + c^2 \end{vmatrix}$

$= 2(a - b)(b - c)(c - 1)$

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14. Evaluate $\begin{vmatrix} .^{x}C_1 & .^{x}C_2 & .^{x}C_3 \\ .^{y}C_1 & .^{y}C_2 & .^{y}C_3 \\ .^{x}C_1 & .^{z}C_2 & .^{z}C_3 \end{vmatrix}$

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15. if $\Delta_r = \begin{vmatrix} 2^{r-1} & 2 \times 3^{r-1} & 4 \times 5^{r-1} \\ \alpha & \beta & \gamma \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$

then find the value of $\Sigma_{r=1}^{n} \Delta r$.

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16. Prove that $|1 + a11111 + b11111 + c11111 + d| = abcd\left(a + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right).$ Hence find the value of the determinant if a, b, c, d are the roots of the equation $px^4 + qx^3 + rx^2 + sx + t = 0$.

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17. Prove that $\begin{vmatrix} 1 & ab & \frac{1}{a} + \frac{1}{b} \\ 1 & bc & \frac{1}{b} + \frac{1}{c} \\ 1 & ca & \frac{1}{c} + \frac{1}{a} \end{vmatrix} = 0$

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18. Prove the identities: $\left|b^2 + c^2 abacbac^2 + a^2 bacba^2 + b^2\right| = 4a^2b^2c^2$

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19. Show that $\begin{vmatrix} a & b - c & c + b \\ a + c & b & c - a \\ a - b & b + a & c \end{vmatrix} = (a + b + c)\left(a^2 + b^2 + c^2\right)$

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Exercise 12 4

1. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with same common ratio, then prove that the points $\left(x_1, y_1\right), \left(x_2, y_2\right), and\left(x_3, y_3\right)$ are collinear.

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2. If lines $px + qy + r = 0, qx + ry + p = 0andrx + py + q = 0$ are concurrent, then prove that $p + q + r = 0(wherep, q, r$ are distinct$\Big).$

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3. If

$$\left(x_1, x_2\right)^2 + \left(y_1 - y_2\right)^2 = a^2, \left(x_2 - x_3\right)^2 + \left(y_2 - y_3\right)^2 = b^2\left(x_3 - x_1\right)^2 + \left(y_3 - y_1\right)^2$$

where a,b,c are positive then prove that

$$4\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = (a + b + c)(b + c - a)(c + a - b)(a + b - c)$$

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4. it is known that the equation of hyperbola and that of its pair of asymptotes differ by constant . If equation of hyperbola is $x^2 + 4xy + 3y^2 - 4x + 2y + 1 = 0$ then find the equation of its pair of asymptoes.

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Exercise 12 5

1. Prove that

$|(b + x)(c + x)(v + x)(a + x)(a + x)(b + x)(b + y)(c + y)(c + x)(a + t)(a + y)(b + y)(b$

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2. $\Delta = \begin{vmatrix} 1+a^2+a^4 & 1+ab+a^2b^2 & 1+ac+a^2c^2 \\ 1+ab+a^2b^2 & 1+b^2+b^4 & 1+bc+b^2c^2 \\ 1+ac+a^2c^2 & 1+bc+b^2c^2 & 1+c^2c^4 \end{vmatrix}$ is equal to

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3. Prove that

$|2\alpha + \beta + \gamma + \delta\alpha\beta + \gamma\delta\alpha + \beta + \gamma + \delta2(\alpha + \beta)(\gamma + \delta)\alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta)\alpha\beta + \gamma\delta\alpha\beta(\gamma$

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4. For all values of $A, B, CandP, Q, R$ show that

$|\cos(A - P)\cos(A - Q)\cos(A - R)\cos(B - P)\cos(B - Q)\cos(B - R)\cos(C - P)\cos(C - Q)$

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5. Show that: $\left|b^2 + c^2abacbac^2 + a^2bacba^2 + b^2\right| = 4a^2b^2c^2$

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6. Express $= \left|2bc - a^2c^2b^2c^22ca - b^2a^2b62a^22ab - c^2\right|$ as square of a determinant of hence evaluate if.

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Exercise 12 6

1. Let

$f(x) = \left|\cos\left(x + x^2\right)\sin\left(x + x^2\right) - \cos\left(x + x^2\right)\sin\left(x - x^2\right)\cos\left(x - x^2\right)\sin\left(x - x^2\right)\sin2\right.$

. Find the value of $f'(0)$

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2. If $f(x), g(x) and h(x)$ are three polynomial of degree 2, then prove that $\varphi(x) = \mid f(x)g(x)h(x)f'(x)g'(xh'(x)f''(x)g''(xh''(x)\mid$ is a constant polynomial.

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3. If $g(x) - \frac{f(x)}{(x-a)(x-b)(x-c)}, where f(x)$ is a polynomial of degree < 3, then prove that $\frac{dg(x)}{dx} = \left|1af(a)(x-a)^{-2}1bf(b)(x-b)^{-2}1cf(c)(x-c)^{-2}\right| + \left|a^2a1b^2b1c^2c1\right|$

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4. If $f(x) = \begin{vmatrix} \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$ and $f(0) = 2$ theb find the value of $\Sigma_{r=1}^{30}|f(r)|$.

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5. $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$ then find the value of

$\lim_{x \to 0} \frac{f(x)}{x}$

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Exercise 12 7

1. Find the following system of equations is consistent,

$(a+1)^3 x + (a+2)^3 y = (a+3)^3, \quad (a+1)x + (a+2)y = a+3, x+y=1,$ then find the value of a

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2. Solve the system of the equations: $ax + by + cz = d\ a^2x + b^2y + c^2z = d^2$ $a^3x + b^3y + c^3z = d^3$ Will the solution always exist and be unique?

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3. consider the system of equations : ltbr. $3x - y + 4z = 3$

$x + 2y - 3z = - 2$

$6x + 5y + \lambda z = - 3$

Prove that system of equation has at least one solution for all real values of λ.also prove that infinite solutions of the system of equations satisfy $\frac{7x - 4}{-5} = \frac{7y + 9}{13} = z$

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4. If the equation $2x + 3y + 1 = 0, 3x + y - 2 = 0, andax + 2y - b = 0$ are consistent, then prove that $a - b = 2$.

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5. if x,y and z are not all zero and connected by the equations $a_1x + b_1y + c_1z = 0, a_zx + b_2y + c_2z = 0$ and

$\left(p_1 + \lambda q_1\right)x + \left(p_2 + \lambda q_2\right)y + \left(p_3 + \lambda q_3\right)z = 0$ show that

$$\lambda = - \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ p_1 & p_2 & p_3 \end{vmatrix} \div \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ q_1 & q_2 & q_3 \end{vmatrix}$$

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Exercise Single

1. If $\theta \in R$ then maximum value of $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 + \sin\theta & 1 \\ 1 & 1 & 1 + \cos\theta \end{vmatrix}$ is

A. $\sqrt{3}/2$

B. $1/2$

C. $1/\sqrt{2}$

D. None of these

Answer: B

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2. If $p+q+r=a+b+c=0$, then the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ equals

A. 0

B. $pa+qb+rc$

C. 1

D. none of these

Answer: A

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3. If α, β, γ are the roots of $px^2+qx^2+r=0,$ then the value of the determinant $|\alpha\beta\beta\gamma\gamma\alpha\beta\gamma\gamma\alpha\alpha\beta\gamma\alpha\alpha\beta\beta\gamma|$ is p b. q c. 0 d. r

A. p

B. q

C. 0

D. r

Answer: C

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4. If $f(x) = a = bx + cx^2 and\alpha, \beta, \gamma$ are the roots of the equation $x^3 = 1, then|abcbcacab|$ is equal to $f(\alpha) + f(\beta) + f(\gamma)$ $f(\alpha)f(\beta) + f(\beta)f(\gamma) + f(\gamma)f(\alpha)$ $f(\alpha)f(\beta)f(\gamma)$ - $f(\alpha)f(\beta)f(\gamma)$

A. $f(\alpha) + f(\beta) + f(\gamma)$

B. $f(\alpha)f(\beta) + f(\beta)f(\gamma) + f(\gamma)f(\alpha)$

C. $f(\alpha)f(\beta)f(\gamma)$

D. - $f(\alpha)f(\beta)f(\gamma)$

Answer: D

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5. If [] denotes the greatest integer less than or equal to the ral number under consideration, and -$1 \leq x < 0, 0 \leq y < 1, 1 \leq a < 2$, then the value of the determinant $|[x] + 1[y][z][x][y] + 1[z][x][y][z] + 1|$ is $[x]$ b. $[y]$ c. $[z]$ d. none of these

A. $[x]$

B. $[y]$

C. $[z]$

D. none of these

Answer: C

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6. If $a = \cos\theta + i\sin\theta, b = \cos 2\theta - i\sin 2\theta, c = \cos 3\theta + i\sin 3\theta$ and if

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \text{ then}$$

A. $\theta = 2k\pi, k \in Z$

B. $\theta = (2k+1)\pi, k \in Z$

C. $\theta = (4k+1)\pi, k \in Z$

D. none of these

Answer: A

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7. If $\begin{vmatrix} x^n & x^{n+2} & x^{n+3} \\ y^n & y^{n+2} & y^{n+3} \\ z^n & z^{n+2} & z^{n+3} \end{vmatrix} = (x - y)(y - z)(z - x)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$, then n equals

A. 1

B. -1

C. 2

D. -2

Answer: B

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8. If the determinant $\begin{vmatrix} \cos 2x & \sin^2 x & \cos 4x \\ \sin^2 x & \cos 2x & \cos^2 x \\ \cos 4x & \cos^2 x & \cos 2x \end{vmatrix}$ is expanded in powers of $\sin x$, then the constant term is

A. 1

B. 0

C. -1

D. 2

Answer: C

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9. If a determinant of order 3×3 is formed by using the numbers 1 or -1 then minimum value of determinant is :

A. -2

B. -4

C. 0

D. -8

Answer: B

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10. If $A+B+C=\pi$ and $e^{i\theta}=\cos\theta+i\sin\theta$ and $z=\begin{vmatrix} e^{2iA} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2iB} & e^{-iA} \\ e^{-iB} & e^{-iA} & e^{2iC} \end{vmatrix}$, then

A. 1

B. -1

C. -2

D. -4

Answer: D

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11. If a, b, c are different, then the value of x for which

$$\begin{vmatrix} 0 & x^2-a & x^3-b \\ x^2+a & 0 & x^2+c \\ x^4+b & x-c & 0 \end{vmatrix}=0 \text{ is}$$

A. a

B. c

C. b

D. 0

Answer: D

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12. if the value of the determinant $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ is positivie then

$(a, b, c < 0)$

A. $abc > 1$

B. $abc > - 8$

C. $abc > - 9$

D. $abc > - 2$

Answer: B

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13. if $A_1, B_1, C_1\ldots\ldots$ are respectively the cofactors of the elements $a_1, b_1, c_1\ldots\ldots$ of the determinant

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta \neq 0 \text{ then the value of } \begin{vmatrix} B_2 & C_2 \\ B_3 & C_3 \end{vmatrix} \text{ is equal to}$$

A. $a_1^2\Delta$

B. $a_1\Delta$

C. $a_1\Delta^2$

D. $a_1^2\Delta^2$

Answer: B

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14. If $a, b, c, d, e, andf$ are in G.P. then the value of $\left|a^2d^2xb^2e^2yc^2f^2z\right|$ depends on $xandy$ b. $xandz$ c. $yandz$ d. independent of $x, y, andz$

A. x and y

B. x and z

C. y and z

D. independent of x,y and z

Answer: D

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15. Let $x < 1,$ then value of $\left|x^2 + 22x + 112x + 1x + 21331\right|$ is a. none-negative b. none-positive c. negative d. positive

A. non-negative

B. non- positive

C. begative

D. positive

Answer: C

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16. The value of $\begin{vmatrix} -1 & 2 & 1 \\ 3+2\sqrt{2} & 2+2\sqrt{2} & 1 \\ 3-2\sqrt{2} & 2-2\sqrt{2} & 1 \end{vmatrix}$ is equal to

A. 0

B. $-16\sqrt{2}$

C. $-8\sqrt{2}$

D. none of these

Answer: B

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17. Let $\left\{D_1, D_2, D_3, D_n\right\}$ be the set of third order determinant that can be made with the distinct non-zero real numbers $a_1, a_2, a_q^{\cdot}$ Then $\sum_{i=1}^{n} D_i = 1$ b. $\sum_{i=1}^{n} D_i = 0$ c. D_i - D_j, $\forall i, j$ d. none of these

A. $\Sigma i \overset{n}{=} 1 D_i = 1$

B. $\Sigma i \overset{n}{=} 1 D_i = 0$

C. $D_i D_j, \ \forall I, j$

D. None of these

Answer: B

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18. if w is a complex cube root to unity then value of

$$\Delta = \begin{vmatrix} a_1 + b_1 w & a_1 w^2 + b_1 & c_1 + b_1 \bar{w} \\ a_2 + b_2 w & a_2 w^2 + b_2 & c_2 + b_2 \bar{w} \\ a_3 + b_3 w & a_3 w^2 + b_3 & c_3 + b_3 \bar{w} \end{vmatrix}$$ is

A. 0

B. -1

C. 2

D. none of these

Answer: A

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19. If $a + b + c = 0$, one root of $|a - xcbcb - xabac - x| = 0$ is $x = 1$ b. $x = 2$ c. $x = a^2 + b^2 + c^2$ d. $x = 0$

A. $x = 1$

B. $x = 2$

C. $x = a^2 + b^2 + c^2$

D. $x = 0$

Answer: D

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20. If x,y, z are in A.P., then the values of the determinant

$$\begin{vmatrix} a+2 & a+3 & a+2y \\ a+3 & a+4 & a+2y \\ a+4 & a+5 & a+2z \end{vmatrix}, \text{ is}$$

A. 1

B. 0

C. 2a

D. a

Answer: B

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21. If $a_1, a_2, a_3, ...$ are in G.P. then the value of determinant

$$\begin{vmatrix} \log(a_n) & \log(a_{n+1}) & \log(a_{n+2}) \\ \log(a_{n+3}) & \log(a_{n+4}) & \log(a_{n+5}) \\ \log(a_{n+6}) & \log(a_{n+7}) & \log(a_{n+8}) \end{vmatrix}$$ equals

A. 1

B. 0

C. 2a

D. a

Answer: B

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22. Value of $|x + yzzxy + zxyyz + x|, wherex, y, z$ are nonzero real number, is equal to xyz b. $2xyz$ c. $3xyz$ d. $4xyz$

A. xyz

B. 2xyz

C. 3xyz

D. 4xyz

Answer: D

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23. Which of the following is not the root of the equation $|x - 6 - 12 - 3 \times\ - 3 - 32 \times\ + 2| = 0 ?\ 2$ b. 0 c. 1 d. -3

A. 2

B. 0

C. 1

D. -3

Answer: B

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24. The value of the determinant $\left|kak^2+a^21kbk^2+b^21kck^2+c^21\right|$ is

$k(a+b)(b+c)(c+a)$ $kabc\left(a^2+b^{f2}+c^2\right)$ $k(a-b)(b-c)(c-a)$

$k(a+b-c)(b+c-a)(c+a-b)$

A. $k(a+b)(b+c)(c+a)$

B. $kabc\left(a^2+b^2+c^2\right)$

C. $k(a-b)(b-c)(c-a)$

D. $k(a+b-c)(b+c-a)(c+a-b)$

Answer: C

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25. If $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

where a, b, c are all different, then the determinant

$\begin{vmatrix} 1 & 1 & 1 \\ (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (x-b)(x-c) & (x-c)(x-a) & (x-a)(x-b) \end{vmatrix}$ vanishes when

A. $a+b+c=0$

B. $x=\frac{1}{3}(a+b+c)$

C. $x=\frac{1}{2}(a+b+c)$

D. $x=a+b+c$

Answer: B

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26. if f(x) = $\begin{vmatrix} mx & mx-p & mx-p \\ n & n+p & n-p \\ mx+2n & mx+2n+p & mx+2n-p \end{vmatrix}$ then y=f(x) represents

A. a straight line parallet to x-axis

B. a striaght line parallel to y-axis

C. parabola

D. a striaght line with negative slope

Answer: B

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27. if $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$ then x is equal to

A. 0

B. -9

C. 3

D. none of these

Answer: B

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28. If $\left|x^n x^{n+2} x^{2n} 1 x^a a x^{n+5} x^{a+6} x^{2n+5}\right| = 0, \ \forall x \in R, where n \in N,$ then value of a is n b. n - 1 c. $n + 1$ d. none of these

A. n

B. n-1

C. n+1

D. none of these

Answer: C

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29. for the equation $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 6 \end{vmatrix} = 0$

A. There are exactly two distinct roots

B. there is one pair of equation real roots

C. There are three pairs of equal roots

D. Modulus of each root is 2

Answer: C

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30. If $a^2 + b^2 + c^2 = -2andf(x) =$

$\left|a + a^2x\left(1 + b^2\right)x\left(1 + c^2\right)x\left(1 + a^2\right)x1 + b^2x\left(1 + c^2\right)x\left(1 + a^2\right)x\left(1 + b^2\right)x1 + c^2x\right.$

, then $f(x)$ is a polynomial of degree 0 b. 1 c. 2 d. 3

A. 0

B. 1

C. 2

D. 3

Answer: C

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31. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ .^{m}C_1 & .^{m+1}C_1 & .^{m+2}C_1 \\ .^{m}C_2 & .^{m+1}C_2 & .^{m+2}C_2 \end{vmatrix}$ is equal to

A. 1

B. -1

C. 0

D. none of these

Answer: A

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32. the value of the determinant

$$\begin{vmatrix} .^{n}C_{r-1} & .^{n}C_{r} & (r+1)^{n+2}C_{r+1} \\ .^{n}C_{r} & .^{n}C_{r+1} & (r+2)^{n+2}C_{r+2} \\ .^{n}C_{r+1} & .^{n}C_{r+2} & (r+3)^{n+2}C_{r+3} \end{vmatrix} \text{ is}$$

A. $n^2 + n - 1\big)$

B. 0

C. $.^{n+3}C_{r+3}$

D. $.^{n}C_{r-1} +^{n}C_{r} +^{n}C_{r+1}$

Answer: B

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33. if $f(x) = \begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = 0$ then

A. $f(x) = 0$ and $f(x) = 0$ has one common root

B. $f(x) = 0$ and $f(x) = 0$ has one common root

C. sum of roots of f(x) =0 is -3a

D. none of these

Answer: B

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34. If $x \ne y \ne z$ and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0,$ then xyz =

A. 1

B. 2

C. -1

D. -2

Answer: C

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35. if $x \neq 0, y \neq 0, z \neq 0$ and $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0$ then $x^{-1} + y^{-1} + z^{-1}$ is equal to

A. -1

B. -2

C. -3

D. none of these

Answer: C

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36. if $a_1b_1c_1, a_2b_2c_2$ and $a_3b_3c_3$ are three-digit even natural numbers and

$$\Delta = \begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix} \text{ then } \Delta \text{ is}$$

A. divisible by 2 but not necessarily by 4

B. divisible by 4 but not necessarily by 8

C. divisible by 8

D. none of these

Answer: A

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37. if $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$ then the value of k is

A. 1

B. 2

C. 3

D. 4

Answer: B

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38. suppose $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} a_1 + pb_1 & b_1 + qc_1 & c_1 + ra_1 \\ a_2 + pb_2 & b_2 + qc_2 & c_2 + ra_2 \\ a_3 + pb_3 & b_3 + qc_3 & c_3 + ra_3 \end{vmatrix}$. Then

A. $D' = D$

B. $D' = D(1 - pqr)$

C. $D = D(1 + p + q + r)$

D. $D' = D(1 + pqr)$

Answer: D

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39. The value of the determinant $\begin{vmatrix} \log_a\left(\frac{x}{y}\right) & \log_a\left(\frac{y}{z}\right) & \log_a\left(\frac{z}{x}\right) \\ \log_b\left(\frac{y}{z}\right) & \log_b\left(\frac{z}{x}\right) & \log_b\left(\frac{x}{y}\right) \\ \log_c\left(\frac{z}{x}\right) & \log_c\left(\frac{x}{y}\right) & \log_c\left(\frac{y}{z}\right) \end{vmatrix}$

A. 1

B. -1

C. 0

D. $\frac{1}{6}\log_a xyz$

Answer: C

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40. If $a > 0, b > 0, c > 0$ are respectively the pth, qth, rth terms of a G.P , then the value of the determinant

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}, \text{ is}$$

A. 0

B. $\log(abc)$

C. $-(p+q+r)$

D. none of these

Answer: A

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41. If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is negative, then

$= abax + \mathbf{c}bx + cax + \mathbf{x} + c0$ is $+ve$ b. $(ac - b)^2\left(ax^2 + 2bx + c\right)$ c. $-ve$ d. 0

A. $+ve$

B. $(ac - b)^2\big)\big(ax^2 + 2bx + c\big)$

C. $-ve$

D. 0

Answer: C

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42. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

A. 0

B. 2

C. 1

D. 3

Answer: C

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43. if $D_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n+1 & n^2+n \\ 2k-1 & n^2 & n^2+n+1 \end{vmatrix}$ and $\Sigma_{k=1}^{n} D_k = 56$ then n equals

A. 4

B. 6

C. 8

D. 7

Answer: D

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44. the value of $\Sigma_{r=2}^{n}(-2)^{r}\begin{vmatrix} {}^{n-2}C_{r-2} & {}^{n-2}C_{r-1} & {}^{n-2}C_{r} \\ -3 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix}(n > 2)$

A. $2n - 1 + (-1)^{n}$

B. $2n + 1 + (-1)^{n-1}$

C. $2n - 3 + (-1)^{n}$

D. none of these

Answer: A

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45. if $\Delta = \begin{vmatrix} 3 & 4 & 5 & x \\ 4 & 5 & 6 & y \\ 5 & 6 & 7 & z \\ x & y & z & 0 \end{vmatrix} = 0$ then

A. x, y, z are in A.P.

B. x, y, z are in G.P

C. x, y, z are in H.P

D. none of these

Answer: A

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46. Roots of the equations $\begin{vmatrix} x & m & n & 1 \\ a & x & n & 1 \\ a & b & x & 1 \\ a & b & c & 1 \end{vmatrix} = 0$ are

A. independent of m and n

B. independent of a, b and c

C. depend on m, n and a, b, c

D. inedependent of m, n and a, b, c

Answer: A

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47. If x, y, z are different from zero and $\text{Delta} = ab - yc - za - xbc - za - xb - yc = 0,$ then the value of the expression $\frac{a}{x} + \frac{b}{y} + \frac{c}{z}$ is 0 b. -1 c. 1 d. 2

A. 0

B. -1

C. 1

D. 2

Answer: D

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48. The number of positive integral solutions of the equation

$$\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & z^2y & z^3+1 \end{vmatrix} = 11 \text{ is}$$

A. 0

B. 3

C. 6

D. 12

Answer: B

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49. In triangle ABC, if $\begin{vmatrix} 1 & 1 & 1 \\ \cot\left(\frac{A}{2}\right) & \cot\left(\frac{B}{2}\right) & \cot\left(\frac{C}{2}\right) \\ \tan\left(\frac{B}{2}\right)+\tan\left(\frac{C}{2}\right) & \tan\left(\frac{C}{2}\right)+\tan\left(\frac{A}{2}\right) & \tan\left(\frac{A}{2}\right)+\tan\left(\frac{B}{2}\right) \end{vmatrix}$ then the triangle must be (A) Equilateral (B) Isoceless (C) Right Angle (D) none of these

A. equilateral

B. isosceles

C. obtuse angled

D. none of these

Answer: B

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50. If $\begin{vmatrix} a & b - c & b + c \\ a + c & b & c - a \\ a - b & a + b & c \end{vmatrix} = 0$ then the line $ax + by + c = 0$ passes through the fixed point which is

A. $(1, 2)$

B. $(1, 1)$

C. $(- 2, 1)$

D. $(1, 0)$

Answer: B

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51. The determinant $\begin{vmatrix} y^2 & -xy & x^2 \\ a & b & c \\ a' & b' & c' \end{vmatrix}$ is equal to

A. $\begin{vmatrix} bx + ay & cx + by \\ b' x + a' y & c' x + b' y \end{vmatrix}$

B. $\begin{vmatrix} ax+by & bx+cy \\ a'x+b'y & b'x+c'y \end{vmatrix}$

C. $\begin{vmatrix} bx+cy & ax+by \\ b'x+c'y & a'x+b'y \end{vmatrix}$

D. $\begin{vmatrix} ax+by & bc+cy \\ a'x+b'y & b'x+c'y \end{vmatrix}$

Answer: D

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52. Let $\vec{a_r} = x_r\hat{i} + y_r\hat{j} + z_r\hat{k}, r = 1, 2, 3$ three mutually prependicular unit vectors then the value of $\begin{vmatrix} x_1 & -x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$ is equal to

A. zero

B. ± 1

C. ± 2

D. none of these

Answer: B

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53. Let

$$\begin{vmatrix} y^5z^6\left(z^3 - y^3\right) & x^4z^6\left(x^3 - z^3\right) & x^4y^5\left(y^3 - x^3\right) \\ y^2z^3\left(y^6 - z^6\right) & xz^3\left(z^6 - x^6\right) & xy^2\left(x^6 - y^6\right) \\ y^2 \wedge (3)\left(z^3 - y^3\right) & xz^3\left(x^3 - z^3\right) & xy^2\left(y^3 - x^3\right) \end{vmatrix} \text{ and } \Delta_2 = \begin{vmatrix} x & y^2 & z^3 \\ x^4 & y^5 & z^6 \\ x^7 & y^8 & z^9 \end{vmatrix}$$

.Then $\Delta_1\Delta_2$ is equal to

A. Δ_2^6

B. Δ_2^4

C. Δ_2^3

D. Δ_2^2

Answer: C

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54. the value of the determinant

$$\begin{vmatrix} (a_1 - b_1)^2 & (a_1 - b_2)^2 & (a_1 - b_3)^2 & (a_1 - b_4)^2 \\ (a_2 - b_1)^2 & (a_2 - b_2)^2 & (a_2 - b_3)^2 & (a_3 - b_4)^2 \\ (a_3 - b_1)^2 & (a_3 - b_2)^2 & (a_3 - b_3)^2 & (a_3 - b_4)^2 \\ (a_4 - b_1)^2 & (a_4 - b_2)^2 & (a_4 - b_3)^2 & (a_4 - b_4)^2 \end{vmatrix} \text{is}$$

A. dependant on $a_i, i = 1, 2, 3, 4$

B. dependant on $b_i, i = 1, 2, 3, 4$

C. dependant on $a_{ij}, b_i i = 1, 2, 3, 4$

D. 0

Answer: D

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55. if $\Delta(x) = \begin{vmatrix} \tan x & \tan(x+h) & \tan(x+2h) \\ \tan(x+2h) & \tan x & \tan(x+h) \\ \tan(x+h) & \tan(x+2h) & \tan x \end{vmatrix}$, then

The value of $\lim_{h \to 0}. \dfrac{\Delta(\pi/3)}{\sqrt{3}h^2}$ is

A. 144

B. 81

C. 64

D. 36

Answer: A

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56. Value of $\begin{vmatrix} 1+x_1 & 1+x_1x & 1+x_1x^2 \\ 1+x_2 & 1+x_2x & 1+x_2x^2 \\ 1+x_3 & 1+x_3x & 1+x_3x^2 \end{vmatrix}$ depends upon

A. x only

B. $x_1$only

C. $x_2$only

D. none of these

Answer: D

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57. If

$\left|a^2 + \lambda^2 ab + c\lambda ca - b\lambda ab - c\lambda b^2 + \lambda^2 bc + aca + b\lambda bc - a\lambda c^2 + \lambda^2\right| |\lambda c - b - c\lambda ab - a\lambda|$

, then he value of λ is 8 b. 27 c. 1 d. -1

A. 8

B. 27

C. 1

D. -1

Answer: C

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58. Let $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$ Then, the value of $5a + 4b + 3c + 2d + e$ is equal to

A. zero

B. -16

C. 16

D. -11

Answer: D

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59. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinants then

A. $\Delta_1 = 3(\Delta_2)^2$

B. $\frac{d}{dx}(\Delta_1) = 3\Delta_2$

C. $\frac{d}{dx}(\Delta_1) = 3(\Delta_2)^2$

D. $\Delta_1 = 3\Delta_2^{3/2}$

Answer: B

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60. if y= sin mx, them the value of the determinant

$\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$ Where $y_n = \frac{d^n y}{dx^n}$ is

A. m^9

B. m^2

C. m^3

D. 0

Answer: D

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61. Let $f(x) = \begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$, then the value of $\int_0^{\pi/2} \{f(x) + f'(x)\}dx$ is

A. π

B. $\pi/2$

C. 2π

D. $3\pi/2$

Answer: A

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62. a, b, c are distinct real numbers not equal to one. If $ax + y + z = 0, x + by + z = 0, andx + y + cz = 0$ have nontrivial solution, then the value of $\frac{1}{1 - a} + \frac{1}{1 - b} + \frac{}{1 - c}$ is equal to 1 b. -1 c.$zero$ d. none of these

A. -1

B. 1

C. zero

D. none of these

Answer: B

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63. If the system of linear equation $x + y + z = 6, x + 2y + 3c = 14, and 2x + 5y + \lambda z = \mu(\lambda, \mu R)$ has a unique solution, then $\lambda = 8$ b. $\lambda = 8, \mu = 36$ c.$\lambda = 8, \mu \neq 36$ d. none of these

A. $\lambda \neq 8$

B. $\lambda = 8, \mu \neq 36$

C. $\lambda = 8, \mu = 36$

D. none of these

Answer: A

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64. If α, β, γ are the angles of a triangle and system of equations

$\cos(\alpha - \beta)x + \cos(\beta - \gamma)y + \cos(\gamma - \alpha)z = 0$

$\cos(\alpha + \beta)x + \cos(\beta + \gamma)y + \cos(\gamma + \alpha)z = 0$

$\sin(\alpha + \beta)x + \sin(\beta + \gamma)y + \sin(\gamma + \alpha)z = 0$ has non-trivial solutions, then triangle is necessarily a. equilateral b. isosceles c. right angled d. acute angled

A. equiliateral

B. isosoceles

C. right angled

D. acute angled

Answer: B

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65. Given $a = x/(y - z), b = y/(z - x), andc = z/(x - y), wherex, y, zandz$ are not all zero, then the value of $ab + bc + ca$ is 0 b. 1 c.-1 d. none of these

A. 0

B. 1

C. -1

D. none of these

Answer: C

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66. If $pqr \neq 0$ and the system of equation $(p + a)x + by - cz = 0$ $ax + (q + b)y + cz = 0$ $ac + by + (r + c)z = 0$ has nontrivial solution, then value of $\frac{1}{p} + \frac{b}{q} + \frac{c}{r}$ is -1 b. 0 c.0 d. $\neg - 2$

A. -1

B. 0

C. 1

D. 2

Answer: A

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67. The value of $|\alpha|$ for which the system of equation

$\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1$

$x + y + \alpha z = \alpha - 1$

has no solution , is _____

A. either -2 or 1

B. -2

C. 1

D. not-2

Answer: B

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68. the set of equations $\lambda x - y + (\cos\theta)z = 0, 3x + y + 2z = 0$, $(\cos\theta)$ x+y+2z=0 ,$\theta \leq 0 < 2\pi$ has non-trivial solution (s)

A. for no value of λ and 0

B. for all values of λ and 0

C. for all values of λ and only tow values of 0

D. for only one value of λ and all values of 0

Answer: A

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69. If $c < 1$ and the system of equations $x + y - 1 = 0, 2x - y - c = 0, \text{and } bx + 3by - c = 0$ is consistent, then the possible real values of b are

A. $b \in \left(- 3\frac{3}{4} \right)$

B. $b \in \left(- \frac{3}{2}, 4 \right)$

C. $b \in \left(- \frac{3}{4}, 3 \right)$

D. none of these

Answer: C

70. If a, b, c are in G.P. with common ratio $r_1 and \alpha, \beta, \gamma$ are in G.P. with common ratio r_2 and equations $ax + \alpha y + z = 0, bx + \beta y + z = 0, cx + \gamma y + z = 0$ have only zero solution, then which of the following is not true? $a + b + c$ b. abc c.1 d. none of these

A. $r_1 \neq 1$

B. $r_2 \neq 1$

C. $r_1 \neq r_2$

D. none of these

Answer: D

71. if the system of equations

$(a - t)x + by + cz = 0$

$bx + (c - t)y + az = 0$

$cx + ay + (b - t)z = 0$

has non-trivial solutions then product of all possible values of t is

A. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

B. $a + b + c$

C. $a^2 + b^2 + c^2$

D. 1

Answer: A

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72. Let λ and α be real. Then the numbers of intergral values λ for which the system of linear equations

$\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$

$-x + (\sin\alpha)y - (\cos\alpha)z = 0$ has non-trivial solutions is

A. 0

B. 1

C. 2

D. 3

Answer: D

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Exercise Multiple

1. Which of the following has/have value equal to zero ?

A. $\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$

B. $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ac \\ 1/c & c^2 & ab \end{vmatrix}$

C. $\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix}$

D. $\begin{vmatrix} 2 & 43 & 6 \\ 7 & 35 & 4 \\ 3 & 17 & 2 \end{vmatrix}$

Answer: A::B::C

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2. If $f(\alpha, \beta) = \begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1 \end{vmatrix}$, then

A. f(300,200=f(400,200)

B. f(200,400)=f(200,600)

C. f(100,200)=f(200,200)

D. none of these

Answer: A::C

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3. if $f(0) = \begin{vmatrix} \sin 0 & \cos 0 & \sin 0 \\ \cos 0 & \sin 0 & \cos 0 \\ \cos 0 & \sin 0 & \sin 0 \end{vmatrix}$ then

A. f(0)= 0 has exactly 2 real solutions in $[0, \pi]$

B. f(0) =0 has exactly 3 real solutions in $[0, \pi]$

C. range of function $\frac{f(0)}{1 - \sin 20}$ is $\left[-\sqrt{2}, \sqrt{2}\right]$

D. range of fucntion .$\frac{f(0)}{\sin 20 - 1}$ is $[-3, 3]$ is $[-3, 3]$

Answer: A::C

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4. If $f(x) = \left|a - 10axa - 1ax^2axa\right|, thenf(2x) - f(x)$ is divisible by a b. b c.c, d, e

d. none of these

A. x

B. a

C. $2a + 3x$

D. x^2

Answer: A::B::C

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5. $\Delta = \begin{vmatrix} 1 & 1 + ac & 1 + bc \\ 1 & 1 + ad & 1 + bd \\ 1 & 1 + ae & 1 + be \end{vmatrix}$ is independent of

A. a

B. b

C. c,d,e

D. none of these

Answer: A::B::C

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6. if $\Delta = \begin{vmatrix} -x & a & b \\ b & -x & a \\ a & b & -x \end{vmatrix}$ then a factor of Δ is

A. $a + b + x$

B. $x^2 - (a - b)x + a^2 + b^2 + ab$

C. $x^2 + (a + b)x + a^2 + b^2 - ab$

D. $a + b - x$

Answer: C::D

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7. the determinant $\Delta = \begin{vmatrix} a^2+x & ab & ac \\ ab & b^2+x & bc \\ ac & bc & c^2+x \end{vmatrix}$ is divisible by

A. x

B. x^2

C. x^3

D. none of these

Answer: A::B

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8. $= \left| aa^2 012a+b(a+b)012a+3b \right|$ is divisible by $a+b$ b. $a+2b$ c. $2a+3b$ d. a^2

A. $a+b$

B. $a+2b$

C. $2a + 3b$

D. a^2

Answer: A::B

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9. the roots of the equations $\begin{vmatrix} .^{x}C_r & .^{n-1}C_r & .^{n-1}C_{r-1} \\ .^{x+1}C_r & .^{n}C_r & .^{n}C_{r-1} \\ .^{x+2}C_r & .^{n+1}C_r & .^{n+1}C_{r-1} \end{vmatrix} = 0$

A. $x = n$

B. $x = n + 1$

C. $x = n - 1$

D. $x = n - 2$

Answer: A::C

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10. If $f(x) = \begin{vmatrix} 3 & 3x & 3x^2 + 2a^2 \\ 3x & 3x^2 + 2a^2 & 3x^3 + 6a^2x \\ 3x^2 + 2a^2 & 3x^3 + \ + 6a^2x & 3x^4 + 12a^2x^2 + 2a^4 \end{vmatrix}$ then

A. f'(x)=0

B. y=f(x) is a straight line parallel to x-axis

C. $\int_0^2 f(x)dx = 32a^4$

D. none of these

Answer: A::B

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11. Let $f(n) = \begin{vmatrix} n & n+1 & n+1 \\ .^{n}P_{n} & .^{n+1}P_{n+1} & .^{n+2}P_{n+2} \\ .^{n}C_{n} & .^{n+1}C_{n+1} & .^{n+2}C_{n+2} \end{vmatrix}$ where the sysmbols have

their usual neanings .then f(n) is divisible by

A. $n^2 + n + 1$

B. $(n + 1)!$

C. $n!$

D. none of these

Answer: A::C

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12. the determinant $\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix} = 0$ is equal to zero if

A. a,b,c are in A.P

B. a,b,c are in G.P.

C. α is a root of the equation $ax^2 + bx + c = 0$

D. $(x - \alpha)$ is a factor fo $ax^2 + 2bx + c$

Answer: B::D

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13. if $\begin{vmatrix} \sin x & \sin y & \sin z \\ \cos x & \cos y & \cos z \\ \cos^3 x & \cos^3 y & \cos^3 z \end{vmatrix} = 0$ then which of the following is/are possible ?

A. $x = y$

B. $y = z$

C. $x = z$

D. $x + y + z = \pi/2$

Answer: A::B::C::D

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14. If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = xA + B$ then

A. $\begin{vmatrix} 1 & 1 & 1 \\ -1 & -3 & 3 \\ 4 & 0 & 0 \end{vmatrix}$

B. $\begin{vmatrix} 0 & 1 & 2 \\ 1 & -2 & 3 \\ -4 & 0 & 0 \end{vmatrix}$

C. $\begin{vmatrix} 1 & 1 & -2 \\ -3 & -2 & 3 \\ 4 & 0 & 1 \end{vmatrix}$

D. $\begin{vmatrix} 0 & 1 & -2 \\ -1 & -3 & 3 \\ 4 & 0 & 0 \end{vmatrix}$

Answer: A::D

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15. if $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$ where a,b,c are distinct positive reals then the possible values of abc is/are

A. $\frac{1}{18}$

B. $\frac{1}{63}$

C. $\frac{1}{27}$

D. $\frac{1}{9}$

Answer: A::B

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16. $\begin{vmatrix} .^{x}C_{r} & .^{x}C_{r+1} & .^{x}C_{r+2} \\ .^{y}C_{r} & .^{y}C_{r+1} & .^{y}C_{r+2} \\ .^{z}C_{r} & .^{z}C_{r+1} & .^{z}C_{r+2} \end{vmatrix}$ is equal to

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17. If $\begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & \theta \end{vmatrix}$ then

A. Δ is independent of theta

B. Δ is independent of ϕ

C. $\Delta$is a constant

D. $\left[\frac{d\Delta}{d}(\theta)\right]_{\theta=\pi/2} = 0$

Answer: B::D

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18. If $f(\theta) = \left|\sin^2 A \cot A 1 \sin^2 B \cos B 1 \sin^2 C \cos C 1\right|$, then $tanA + tanB + c$

$\cot A \cot B \cot C$ $\sin^2 A + \sin^2 B + \sin^2 C$ 0

A. $\tan A + \tan B + C$

B. $\cot A \cot B \cot C$

C. $\sin^2 A + \sin^2 B + \sin^2 C$

D. 0

Answer: D

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19. if determinant $\begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix}$ is

A. non-negative

B. independent of theta

C. independent of ϕ

D. none of these

Answer: A::B

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20. If $g(x) = \begin{vmatrix} a^{-x} & e^{x\log_e a} & x^2 \\ a^{-3x} & e^{3x\log_e a} & x^4 \\ a^{-5x} & e^{5x\log_e a} & 1 \end{vmatrix}$ then

A. graphs of g(x) is symmetrical about the origin

B. gpaphs of g(x) is symmertical about the y-axis

C. $\frac{d^4 g(x)}{dx^4} \mid_{x=0} = 0$

D. $f(x) = g(x) \times \log.\left(\frac{a-x}{a+x}\right)$ is an odd function

Answer: A::C

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21. If

$(x) = \left|x^2 + 4x - 32x + 4132x^2 + 5x - 94x + 5268x^2 - 6x + 116x - 6104\right| = ax^3 + bx$

then $a = 3$ b. $b = 0$ c.$c = 0$ d. none of these

A. $a = 3$

B. $b = 0$

C. $c = 0$

D. None of these

Answer: B::C

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22. if $\begin{vmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ xz - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix} = \begin{vmatrix} r^2 & u^2 & u^2 \\ u^2 & r^2 & u^2 \\ u^2 & u^2 & r^2 \end{vmatrix}$ then

A. $r^2 = x + y + z$

B. $r^2 = x^2 = y^2 + z^2$

C. $u^2 = yz + zx + xy$

D. $u^2 = xyz$

Answer: B::C

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23. which of the following is/are true for

$$\Delta = \begin{vmatrix} a^2 & 1 & a+c \\ 0 & b^2+1 & b+c \\ 0 & b+c & c^2+1 \end{vmatrix} ?$$

A. $\Delta \geq 0$ for real values of a,b,c

B. $\Delta \leq 0$ for real values of a,b,c

C. $\Delta = \begin{vmatrix} bc-1 & 0 & 0 \\ 1 & ac & -a \\ -b & -a & ab \end{vmatrix}$

D. $\Delta = 0$ if bc =1 where a,b,c are non-zero

Answer: A::C::D

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24. The values of $k \in R$ for which the system of equations $x + ky + 3z = 0, kx + 2y + 2z = 0, 2x + 3y + 4z = 0$ admits of nontrivial solution is 2 b. $5/2$ c. 3 d. $5/4$

A. 2

B. $5/2$

C. 3

D. $5/4$

Answer: A::B

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25. The system of equations $-2x + y + z = a\ x - 2y + z = b\ x + y - 2z = c$ has

A. no solution if $a + b + c \neq 0$

B. unique solution if $a + b + c = 0$

C. infinite number of solutions if $a + b + c = 0$

D. None of these

Answer: A::C

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26. Let α, β and γ be the roots of the equations $x^3 + ax^2 + bx + c = 0, (a \neq 0)$. If the system of equations $\alpha x + \beta y + \gamma z = 0$ $\beta x + \gamma y + \alpha z = 0$ and $\gamma x = \alpha y + \beta z = 0$ has non-trivial solution then

A. $a^2 = 3b$

B. $a^3 = 27c$

C. $b^3 = 27c^2$

D. $\alpha + \beta + \gamma = 0$

Answer: A::B::C

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Exercise Comprehension

1. Consider the function f(x) = $\begin{vmatrix} a^2+x & ab & ac \\ ab & b^2+x & bc \\ ac & bc & c^2+x \end{vmatrix}$

Which of the following is true ?

A. f(x) =0 and f(x) =0 have one positive common root

B. f(x)=0 and f(x)=0 have one negative common root

C. f(x) =0 and f(x) =0 have no common root

D. None of these

Answer: D

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2. Consider the function f(x) = $\begin{vmatrix} a^2+x & ab & ac \\ ab & b^2+x & bc \\ ac & bc & c^2+x \end{vmatrix}$

which of the following is true ?

A. f(x) has one +ve point of maxima.

B. f(x) has one -ve point of minima

C. f(x)=0 has three distinct roots

D. Local minimum value of f(x) is zero

Answer: D

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3. Consider the function f(x) = $\begin{vmatrix} a^2+x & ab & ac \\ ab & b^2+x & bc \\ ac & bc & c^2+x \end{vmatrix}$

In which of the following interval f(x) is strictly increasing

A. $(-\infty, \infty)$

B. $(-\infty, 0)$

C. $(0, \infty)$

D. None of these

Answer: C

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4. Let $\Delta = \begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & -ab \end{vmatrix}$ and the equation

$px^3 + qx^2 + rx + s = 0$ has roots a,b,c where $a, b, c \in R^+$

the value of Δ is

A. r^2/p^2

B. r^3/p^3

C. $-s/p$

D. none of these

Answer: B

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5. Let $\Delta = \begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$ and the equation

$px^3 + qx^2 + rx + s = 0$ has roots a,b,c where $a, b, c \in R^+$

The value of Δ is

A. $\leq 9r^2/p^2$

B. $\geq 27s^2/p^2$

C. $\leq 27s^3/p^3$

D. none of these

Answer: B

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6. Let $\Delta = \begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$ and the equation

$px^3 + qx^2 + rx + s = 0$ has roots a,b,c where $a, b, c \in R^+$

$\text{if } \Delta = 27 \text{ and } a^2 + b^2 + c^2 = 3$then

A. $3p + 2q = 0$

B. $4p + 3q = 0$

C. $3p + q = 0$

D. none of these

Answer: C

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7. if $x > m, y > n, z > r(x, y, z > 0)$ such that $\begin{vmatrix} x & n & r \\ m & y & r \\ m & n & z \end{vmatrix} = 0$

The value of $\frac{x}{x - m} + \frac{y}{y - n} + \frac{z}{z - r}$ is

A. 1

B. -1

C. 2

D. -2

Answer: C

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8. if $x > m, y > n, z > r(x, y, z > 0)$ such that $\begin{vmatrix} x & n & r \\ m & y & r \\ m & n & z \end{vmatrix} = 0$

the value of $\frac{m}{x - m} + \frac{n}{y - n} + \frac{r}{z - r}$ is

A. -2

B. -4

C. 0

D. -1

Answer: D

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9. if $x > m, y > n, z > r(x, y, z > 0)$ such that $\begin{vmatrix} x & n & r \\ m & y & r \\ m & n & z \end{vmatrix} = 0$

the value $\dfrac{xyz}{(x - m)(y - n)(z - r)}$ is

A. 27

B. $\dfrac{8}{27}$

C. $\dfrac{64}{27}$

D. None of these

Answer: B

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10. $f(x) = \begin{vmatrix} x+c_1 & x+a & x+a \\ x+b & x+c_2 & x+a \\ x+b & x+b & x+c_3 \end{vmatrix}$ and $g(x) = \left(C_1 - x\right)\left(c_3 - x\right)$

Coefficient of x in f(x) is

A. $\dfrac{g(a) - f(b)}{b - a}$

B. $\dfrac{g(-a) - g(-b)}{b - a}$

C. $\dfrac{g(a) - g(b)}{b - a}$

D. none of these

Answer: C

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11. $f(x) = \begin{vmatrix} x + c_1 & x + a & x + a \\ x + b & x + c_2 & x + a \\ x + b & x + b & x + c_3 \end{vmatrix}$ and $g(x) = \left(C_1 - x\right)\left(c_3 - x\right)$

Which of the following is not a constant term in f(x) ?

A. $\dfrac{bg(a) - ag(b)}{(b - a)}$

B. $\dfrac{bf(a) - af(- b)}{(b - a)}$

C. $\dfrac{bf(- a) - ag(b)}{(b - a)}$

D. none of these

Answer: D

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12. $f(x) = \begin{vmatrix} x + c_1 & x + a & x + a \\ x + b & x + c_2 & x + a \\ x + b & x + b & x + c_3 \end{vmatrix}$ and $g(x) = \left(C_1 - x\right)\left(c_3 - x\right)$

`Which of the following is not true ?

A. $f(-a) = g(a)$

B. $f(-a) = g(-a)$

C. $f(-b) = g(b)$

D. none of these

Answer: B

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13. Suppose f(x) is a function satisfying the following conditions :

(i) f(0)=2,f(1)=1

(ii) f has a minimum value at $x = 5/2$

(iii) for all x, $f(x) = \begin{vmatrix} 2ax & 2ax - 1 & 2ax + b + 1 \\ b & b + 1 & -1 \\ 2(ax + b) & 2ax + 2b + 1 & 2ax + b \end{vmatrix}$

The value of f(2) is

A. $1/4$

B. $1/2$

C. -1

D. 3

Answer: B

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14. Suppose f(x) is a function satisfying the following conditions :

(i) f(0)=2,f(1)=1

(ii) f has a minimum value at $x = 5/2$

(iii) for all x, $f(x) = \begin{vmatrix} 2ax & 2ax - 1 & 2ax + b + 1 \\ b & b + 1 & -1 \\ 2(ax + b) & 2ax + 2b + 1 & 2ax + b \end{vmatrix}$

f(x)=0 has

A. both roots positive

B. both roots negative

C. roots of opposite sign

D. imaginary roots

Answer: D

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15. Suppose f(x) is a function satisfying the following conditions :

(i) f(0)=2,f(1)=1

(ii) f has a minimum value at $x = 5/2$

(iii) for all x, $f(x) = \begin{vmatrix} 2ax & 2ax - 1 & 2ax + b + 1 \\ b & b + 1 & -1 \\ 2(ax + b) & 2ax + 2b + 1 & 2ax + b \end{vmatrix}$

Range of f(x) is

A. $[7/16, \infty)$

B. $(-\infty, 15/16]$

C. $[3/4, \infty)$

D. none of these

Answer: A

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16. Consider the polynomial fucntion

$$f(x) = \begin{vmatrix} (1+x)^a & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix}$$ a,b being positive integers. The

constant term in f(x) is

A. 2

B. 1

C. -1

D. 0

Answer: D

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17. Consider the polynomial fucntion

$$f(x) = \begin{vmatrix} (1+x)^2 & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix}$$

a,b being positive integers.

the coefficient of x in f(x) is

A. 2^a

B. $2^a - 3 \times 2^b + 1$

C. 0

D. none of these

Answer: C

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18. Consider the polynomial fucntion

$$f(x) = \begin{vmatrix} (1+x)^2 & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix}$$

a,b being positive integers.

Which of the following is true ?

A. All the roots of the equation f(x)=0 are positive

B. All the roots of the equation f(x)=0 are negative

C. At least one of the equation f(x)=0 is repeating one .

D. None of these

Answer: C

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19. Given that the system of equations $x = cy + bz, y = az + cx, z = bx + ay$ has nonzero solutions and and at least one of the a,b,c is a proper fraction.

$a^2 + b^2 + c^2$ is

A. > 2

B. > 3

C. < 3

D. < 2

Answer: C

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20. Given that the system of equations $x = cy + bz, y = az + cx, z = bx + ay$ has nonzero solutions and and at least one of the a,b,c is a proper fraction.

abc is

A. $> \,$ - 1

B. > 1

C. < 2

D. < 3

Answer: A

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21. Given that the system of equations $x = cy + bz, y = az + cx, z = bx + ay$ has nonzero solutions and and at least one of the a,b,c is a proper fraction.

System has solution such that

A. $x, y, z \equiv \left(1 - 2a^2\right) : \left(1 - 2b^2\right) : \left(1 - 2c^2\right)$

B. $x. y. z \equiv \frac{1}{1 - 2a^2} : \frac{1}{1 - 2b^2} : \frac{1}{1 - 2c^2}$

C. $x. y. z \equiv \frac{a}{1 - a^2} : \frac{b}{1 - b^2} : \frac{c}{1 - c^2}$

D. $x. y. z \equiv \sqrt{1 - a^2} : \sqrt{1 - b^2} : \sqrt{1 - c^2}$

Answer: D

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22. Consider the system of equations

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$

the system has unique solution if

A. $\lambda \neq 3$

B. $\lambda = 3, \mu = 10$

C. $\lambda = 3, \mu \neq 10$

D. none of these

Answer: A

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23. Consider the system of equations

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$

the system has infinite solutions if

A. $\lambda \neq 3$

B. $\lambda = 3, \mu = 10$

C. $\lambda = 3, \mu \neq 10$

D. $\lambda = 3, \mu \neq 10$

Answer: B

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24. Consider the system of equations

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$

The system has no solution if

A. $\lambda \neq 3$

B. $\lambda = 3, \mu = 10$

C. $\lambda = 3, \mu \neq 10$

D. none of these

Answer: C

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Matrix Match Type

1. Match the following lists :

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2. Match the following lists:

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3. If α, β, γ are the roots of $x^3 - 3x^2 + 3x - 1 = \begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$

then match the list I with list II

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4. consider the system of equations

$\lambda x + y + z = 1$

$x + \lambda y + z = \lambda :$

$x + y + \lambda z = \lambda^2$

Now match the following lists:

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5. consider determinant $\Delta = \left|a_{ij}\right|$ of order 3. If $\Delta = 2$ the match the following lists.

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Exercise Numerical

1. If $a_1, a_2, a_3, 54, a_6, a_7, a_8, a_9$ are in H.P., and $D = \left|a_1a_2a_354a_6a_7a_8a_9\right|$, then the value of $[D]iswhere[.]$ represents the greatest integer function

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2. the sum of values of p for which the equations x+y+z=1x+2y +4z=p and x+4y +10z =p^2 have a solution is ____

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3. The sum of roots of the equations

$$\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0 \text{ is } \_\_\_\_$$

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4. Prove that

$$\begin{vmatrix} (\beta+\gamma-\alpha-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 & 1 \\ (\gamma+\alpha-\beta-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 & 1 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1 \end{vmatrix} = -64(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\beta-\delta)(\gamma-\delta)$$

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5. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then the value of

f(500) ______

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6. If

$|(x, x+y, x+y+z), (2x+3x+2y, 4x+3y+2z), (3x+6x+3y, 10x+6y+3z)| = 6$

then the real value of x is

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7. Let $D_1 = |aba+bcdc+daba-b| andD_2 = |aca+cbdb+daca+b+c|$ then

the value of $\left|\frac{D_1}{D_2}\right|$, $whereb \neq 0andad \neq bc,$ is _____.

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8. if $a_1, a_2, a +_3 \ldots\ldots, a_{12}$ are in A.P and

$$\Delta_1 = \begin{vmatrix} a_1a_5 & a_1 & a_2 \\ a_2a_6 & a_2 & a_3 \\ a_3a_7 & a_3 & a_4 \end{vmatrix} \Delta_3 = \begin{vmatrix} a_2b_{10} & a_2 & a_3 \\ a_3a_{11} & a_3 & a_4 \\ a_3a_{12} & a_4 & a_5 \end{vmatrix}$$

then $\Delta_2 : \Delta_2 =$ ______

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9. if $\left(1 + ax + bx^2\right)^4 = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_8x^8,$ where $a, b, a_0, a_1 \ldots\ldots, a_8 \in R$ such that $a_0 + a_1 + a_2 \neq 0$ and

$$\begin{vmatrix} a_0 & a_1 & a_2 \\ a_1 & a_2 & a_0 \\ a_2 & a_0 & a_1 \end{vmatrix} = 0$$ then the value of $5.\frac{a}{b}$ is _____

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10. $\begin{vmatrix} 5\sqrt{\log_e 3} & 5\sqrt{\log_e 3} & 3\sqrt{\log_e 3} \\ 3^{-\log_{1/3}4} & (0.1)^{\log_{0.01}4} & 7^{\log_7 3} \\ 7 & 3 & 5 \end{vmatrix}$ is equal to _____

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11. Let a+b+c =s and $\begin{vmatrix} s+c & a & b \\ c & s+a & b \\ c & a & s+b \end{vmatrix} = 532$ then the value of s is

____

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12. Let $a, b, c, \in R$ not all are equal and $\Delta_1 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$\Delta_2 = \begin{vmatrix} a+2b & b+3c & c+4a \\ b+2c & c+3a & a+4b \\ c+2a & a+3b & b+4c \end{vmatrix}$ then $\dfrac{\Delta_2}{\Delta_1} =$ ____

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13. Three distinct points $P\left(3u^2, 2u^3\right); Q\left(3v^2, 2v^3\right)$ and $R\left(3w^2, 2w^3\right)$ are collinear then

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14. if $\Delta_r = \begin{vmatrix} r & 612 & 915 \\ 101r^2 & 2r & 3r \\ r & \frac{1}{r} & \frac{1}{r^2} \end{vmatrix}$ then the value of

$\lim n \to \infty . \frac{1}{n^3}\left(\Sigma_{r=1}^{n}\Delta_r\right.$ is ____

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15. if x=31,y=32 and z=33 then the value of

$\begin{vmatrix} \left(x^2+1\right)^2 & (xy+1)^2 & (xz+1)^2 \\ (xy+1)^2 & \left(y^2+1\right)^2 & (yz+1)^2 \\ (xz+1)^2 & (yz+1)^2 & \left(z^2+1\right)^2 \end{vmatrix}$ is ____

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16. Let α, β, γ are the real roots of the equation $x^3 + ax^2 + bx + c = 0(a, b, c \in Randa \neq 0)^{\cdot}$ If the system of equations $(\in u, v, andw)$ given by $\alpha u + \beta v + \gamma w = 0$ $\beta u + \gamma v + \alpha w = 0$ $\gamma u + \alpha v + \beta w = 0$ has non-trivial solutions then the value of a^2/b is _______.

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17. The value of $|\alpha|$ for which the system of equation

$\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1$

$x + y + \alpha z = \alpha - 1$

has no solution , is ____

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18. Number of values of θ lying I $[0,100\pi]$ for which the system of equations (sin 3θ) x-y+z=0, (cos 2θ) x+4y +3z=0, 2x+ 7y+7z =0 has non-trivial solution is ____

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Jee Main Previous Year

1. Let a,b,c be such that b(a+c) $\neq 0$. If

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0,$$ Then the value of 'n' is:

A. zero

B. any even integer

C. any odd integer

D. any integer

Answer: 3

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2. Consider the system of linear equations:

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

The system has

A. no solution

B. infinite number of solutions

C. exactly three solutions.

D. a unique solution

Answer: 1

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3. The number of values of k for which the linear equations $4x + ky + 2z = 0$ $kx + 4y + z = 0$ $2x + 2y + z = 0$ posses a non-zero solution is : (1) 3 (2) 2 (3) 1 (4) zero

A. zero

B. 3

C. 2

D. 1

Answer: 3

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4. The number of values of k for which the system of equations:

$kx + (3k + 2)y = 4k$

$(3k - 1)x + (9k + 1)y = 4(k + 1)$ has no solution, are

A. infinite

B. 1

C. 2

D. 3

Answer: 2

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5. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and $|31 + f(1)1 + f(2)1 + f(1)1 + f(2)1 + f(3)1 + f(2)1 + f(3)1 + f(4)| = K(1 - \alpha)^2(1 - \beta)^2$, then K is equal to (1) $\alpha\beta$ (2) $\frac{1}{\alpha\beta}$ (3) 1 (4) -1

A. $\alpha\beta$

B. $\frac{1}{\alpha\beta}$

C. 1

D. -1

Answer: 3

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6. The set of the all values of λ for which the system of linear equations

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$ has a non-trivial solution,

A. is an empty set

B. is a singleton set

C. contains two elements

D. contains more than two elements

Answer: 3

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7. The system of linear equations $x + \lambda y - z = 0$ $\lambda x - y - z = 0$ $x + y - \lambda z = 0$ has a non-trivial solution for : (1) infinitely many values of λ . (2) exactly one value of λ . (3) exactly two values of λ . (4) exactly three values of λ .

A. Exactly one value of λ

B. Exactly two values of λ

C. Exactly three values of λ

D. Infinitely many values of λ

Answer: 3

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8. If S is the set of distinct values of $'b$ for which the following system of linear equations $x+y+z=1$ $x+ay+z=1$ $ax+by+z=0$ has no solution, then S is : a finite set containing two or more elements (2) a singleton an empty set (4) an infinite set

A. a singleton set

B. an empty set

C. an infinite set

D. a finite set containing two or more elements

Answer: 1

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9. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}.$

$If\left|1111 - \omega^2 - 1\omega^2 1\omega^2\omega^7\right| = 3k,$ then$k$ is equal to : -1 (2) 1 (3) $-z$ (4) z

A. 1

B. $-z$

C. z

D. -1

Answer: 2

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10. If the system of linear equations x+ky+3z=0 3x+ky-2z=0 2x+4y-3z=0 has a non-zero solution (x,y,z) then $\frac{xz}{y^2}$ is equal to

A. 30

B. -10

C. 10

D. -30

Answer: 3

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11. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$ then the ordered pair (A,B) is equal to

A. $(4, 5)$

B. $(-4, -5)$

C. $(-4, 3)$

D. $(-4, 5)$

Answer: D

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Jee Advanced Previous Year

1. Which of the following values of α satisfying the equation

$$\left|(1+\alpha)^2(1+2\alpha)^2(1+3\alpha)^2(2+\alpha)^2(2+2\alpha)^2(2+3\alpha)^2(3+\alpha)^2(3+2\alpha)^2(3+3\alpha)^2\right|$$

-4 b. 9 c. -9 d. 4

A. -4

B. 9

C. -9

D. 4

Answer: 2,3

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2. Let $a, \lambda, \mu \in R,$ Consider the system of linear equations $ax + 2y = \lambda 3x - 2y = \mu$ Which of the flollowing statement (s) is (are) correct?

A. If $\alpha = -3$ then the system has infinitely many solutions for all values of λ and μ

B. If $\alpha \neq -3$ then the system has a unique solution for all values of λ and μ

C. If $\lambda + \mu = 0$ then the system has infiniely many solutions for $\alpha = -3$

D. if $\lambda + \mu \neq 0$ then the system has no solution for $\alpha = -3$

Answer: 2,3,4

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3. Let ω be the complex number $\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$. Then the number of distinct complex cos numbers z satisfying $\Delta = \begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is

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4. The total number of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is (A) 0 (B) 1 (C) 2 (D) 3

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5. For a real number α, if the system $\left[1\alpha\alpha^2\alpha1\alpha\alpha^2\alpha1\right][xyz] = [1 - 11]$ of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

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6. Let P be a matrix of order 3×3 such that all the entries in P are from the set $\{-1, 0, 1\}$. Then, the maximum possible value of the determinant of P is _____.

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Solved Examples And Exercises

1. If $f(\theta) = \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cos B & 1 \\ \sin^2 C & \cos C & 1 \end{vmatrix}$, then (a) $tanA + tanB + c$ (b)$\cot A \cot B \cot C$ (c) $\sin^2 A + \sin^2 B + \sin^2 C$ (d)0

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2. Let $\Delta(x) = \begin{vmatrix} 3 & 3x & 3x^2+2a^2 \\ 3x & 3x^2+2a^2 & 3x^3+6a^2x \\ 3x^2+2a^2 & 3x^3+6a^2x & 3x^4+12a^2x^2+2a^4 \end{vmatrix}$ then

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3. The determinant $\text{Delta} = \left|a^2 + xabaca \wedge 2 + xbcacb \wedge 2 + x\right|$ is divisible by x b. x^2 c.x^3 d. none of these

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4. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, $then f(2x) - f(x)$ is divisible by (a) a (b) b (c) c, d, e

(d) none of these

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5. If

$(x) = \left|x^2 + 4x - 32x + 4132x^2 + 5x - 94x + 5268x^2 - 6x + 116x - 6104\right| = ax^3 + bx^2$

then $a = 3$ b. $b = 0$ c.$c = 0$ d. none of these

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6. If $\left|x^n x^{n+2} x^{2n} 1 x^a a x^{n+5} x^{a+6} x^{2n+5}\right| = 0, \ \forall x \in R, where n \in N,$ then value of a is n b. $n - 1$ c. $n + 1$ d. none of these

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7. Let $x < 1,$ then value of $\begin{bmatrix} x^2 + 2 & 2x + 1 & 1 \\ 2x + 1 & x + 2 & 1 \\ 3 & 3 & 1 \end{bmatrix}$ is a. none-negative b. none-positive c. negative d. positive

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8. Find the number of real root of the equation

$$\begin{vmatrix} 0 & x - a & x - b \\ x + a & 0 & x - c \\ x + b & x + c & 0 \end{vmatrix} = 0, a \neq b \neq c \text{ and} b(a + c) > ac$$

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9. Value of $\begin{bmatrix} x + y & z & z \\ x & y + z & x \\ y & y & z + x \end{bmatrix}$, where x ,y ,z are nonzero real number, is equal to xyz b. $2xyz$ c. $3xyz$ d. $4xyz$

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10. If $e^{i\theta} = \cos\theta + i\sin\theta$, find the value of

$$\left|1e^{i\pi/3}e^{i\pi/4}e^{-i\pi/3}1e^{i2\pi/3}e^{-i\pi/4}e^{-i2\pi/3}1\right|$$

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11. Which of the following is not the root of the equation

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0?$$ a.2 b. 0 c. 1 d. -3

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12. If A,B,C are the angles of a non right angled triangle ABC. Then find the

value of: $\begin{vmatrix} \tan A & 1 & 1 \\ 1 & \tan B & 1 \\ 1 & 1 & \tan C \end{vmatrix}$

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13. If $f(x) = |xaaaxaaax| = 0,$ then $f'(x) = 0 and f^x = 0$ has common root $f^x = 0 and f'(x) = 0$ has common root sum of roots of $f(x) = 0$ is $-3a$ none of these

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14. $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

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15. Roots of the equation $|xmn1axn1abx1abc1| = 0$ are independent of $m, andn$ independent of $a, b, andc$ depend on $m, n, anda, b, c$ independent of $m, nanda, b, c$

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16. Prove that the value of each the following determinants is zero: $|a = - - ax - yy - zz - xp - qq - rr - p|$

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17. If a, b, c are different, then the value of $\left|0x^2 - ax^3 - bx^2 + a0x^2 + cx^4 + bx - c0\right| = 0$ is c b. c c. b d. 0

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18. Prove that the value of each the following determinants is zero:

$$\begin{vmatrix} a_1 & la_1 + mb_1 & b_1 \\ a_2 & la_2 + mb_2 & b_2 \\ a_3 & la_3 + mb_3 & b_3 \end{vmatrix}$$

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19. If $|b + \ + aa + ba + \ + \ + ac + aa + \ + c| = k|abaca|$, then value of k is 1 b. 2 c. 3 d. 4

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20. A triangle has vertices $A_i\left(x_i, y_i\right)f \text{ or } i = 1, 2, 3$ If the orthocentre of triangle is $(0, 0)$, then prove that

$\left|x_2 - x_3 y_2 - y_3 y_1\left(y_2 - y_3\right) + x_1\left(x_2 - x_3\right) x_3 - x_1 y_2 - y_3 y_2\left(y_3 - y_1\right) + x_1\left(x_3 - x_1\right) x_1 -\right.$

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21. If α, β, γ are the roots of $ax^3 + bx^2 + cx + d = 0 and |\alpha\beta\gamma\beta\gamma\alpha\gamma\alpha\beta| = 0, \alpha \neq \beta \neq \gamma$ then find the equation whose roots are $\alpha + \beta - \gamma, \beta + \gamma - \alpha, and \gamma + \alpha - \beta$.

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22. If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} a_1 + pb_1 & b_1 + qc_1 & c_1 + ra_1 \\ a_2 + pb_2 & b_2 + qc^2 & c^2 + ra^2 \\ a_3 + pb_3 & b_3 + qc_3 & c_3 + ra_3 \end{vmatrix}$ then

$\Delta_1 =$

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23. Prove that the value of each the following determinants is zero:

$$\begin{vmatrix} \left(a^x + a^{-x}\right)^2 & \left(a^x - a^{-x}\right)^2 & 1 \\ \left(b^y + b^{-y}\right)^2 & \left(b^y - + b^{-y}\right)^2 & 1 \\ \left(c^z + c^{-z}\right)^2 & \left(c^z - c^{-z}\right)^2 & 1 \end{vmatrix}$$

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24. The value of the determinant $\begin{bmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{bmatrix}$ is equal to 1 b. 0 c. 2 d. 3

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25. If $f(x) = |1 \times + 12 \times (x - 1)(x + 1)x3x(x - 1)x(x - 1)(x - 2)(x + 1)x(x - 1)|$ then $f(5000)$ is equal to 0 b. 1 c. 500 d. -500

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26. Prove that the value of each the following determinants is zero: $|\log x\log y\log z\log 2x\log 2y\log 2z\log 3x\log 3y\log 3z|$

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27. If the system of equations $x - ky - z = 0, kx - y - z = 0, x + y - z = 0$ has a nonzero solution, then the possible value of k are $-1, 2$ b. $1, 2$ c. $0, 1$ d. $-1, 1$

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28. Show that $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} =$

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29. The determinant $|aba\alpha + \mathbf{c}b\alpha + ca\alpha + \boldsymbol{\alpha} + c0| = 0,$ if a, b, c are in A.P. a, b, c are in G.P. a, b, c are in H.P. α is a root of the equation $ax^2 + bc + c = 0$ $(x - \alpha)$ is a factor of $ax^2 + 2bx + c$

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30. By using properties of determinants. Show that:

$$\left|1 + a^2 - b^2 2ab - 2b 2ab 1 - a^2 + b^2 2a 2b - 2a 1 - a^2 - b^2\right| = \left(1 + a^2 + b^2\right)^3$$

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31. Which of the following values of α satisfying the equation

$$\left|(1 + \alpha)^2(1 + 2\alpha)^2(1 + 3\alpha)^2(2 + \alpha)^2(2 + 2\alpha)^2(2 + 3\alpha)^2(3 + \alpha)^2(3 + 2\alpha)^2(3 + 3\alpha)^2\right|$$

-4 b. 9 c. -9 d. 4

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32. Show that:

$|3a - a + b - a + c - b + a3b - b + c - c + a - c + b3c| = 3(a + b + c)(ab + bc + ca)$

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33. If the system of equations $x + ay = 0, az + y = 0, andax + z = 0$ has infinite solutions, then the value of equation has no solution is -3 b. 1 c. 0 d. 3

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34. If $|6i - 3i143i - 1203i| = x + iy,$ then a.$x = 3, y = 1$ b. $x = 1, y = 3$ c. $x = 0, y = 3$ d. $x = 0, y = 0$

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35. The value of $\begin{vmatrix} yz & zx & xy \\ p & 2p & 3r \\ 1 & 1 & 1 \end{vmatrix}$, $wherex, y, z$ are respectively, pth, $(2q)$ th, and $(3r)$ th terms of an H.P. is a.-1 b. 0 c. 1 d. none of these

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36. Prove that

$$\begin{vmatrix} (\beta+\gamma-\alpha-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 & 1 \\ (\gamma+\alpha-\beta-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 & 1 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1 \end{vmatrix} = -64(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\beta-\delta)(\gamma-\delta)$$

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37. Prove that the value of each the following determinants is zero:

$$|\sin^2\left(x+\frac{3\pi}{2}\right)\sin^2\left(x+\frac{5\pi}{2}\right)\sin^2\left(x+\frac{7\pi}{2}\right)\sin^{x+\frac{3\pi}{2}}\sin^{x+\frac{5\pi}{2}}\sin^{x+\frac{7\pi}{2}}\sin^{x-\frac{3\pi}{2}}\sin^{x-\frac{5\pi}{2}}\mathrm{si}$$

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38. The number of values of k for which the system of the equations $(k+1)x+8y=4kandkx+(k+3)y=3k-1$ has infinitely many solutions is 0 b. 1 c. 2 d. infinite

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39. Prove that $|b+ca-bac+ab-cba+bc-ac|=3ab-a^3-b^3-c^3$

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40. If $\left|a^2b^2c^2(a+1)^2(b+1)^2(c+1)^2(a-1)^2(b-1)^2(c-1)^2\right|=k(a-b)(b-c)(c-a),$ then find the value of k

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41. Statement 1: $\text{Delta} = |my + nzmq + nrmb + nckz - mxkr - mpkc - ma - nx - ky - np - kq - na - kb|$ is equal to 0. Statement 2: The value of skew symmetric matrix of order 3 is zero.

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42. Prove that $= \left|111abcbc + a^2ac + b^2ab + c^2\right| = 2(a - b)(b - c)(c - a)$

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43. If $= \left|s\int h\eta\cos\varphi\sin\theta s \in \varphi\cos\theta\cos\theta\cos\varphi\cos\theta s \in \varphi - s\int h\eta - s\int h\eta s \in \varphi s\int h\eta\cos\varphi 0\right|$ then is independent of θ is independent of φ Delta is a constant $\left(\frac{d\text{Delta}}{dth\eta}\right])_{\theta = \pi/2} = 0$

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44. The system of equations $-2x + y + z = a$, $x - 2y + z = b$, $x + y - 2z = c$, has: (a)no solution if $a + b + c \neq 0$ (b)unique solution if $a + b + c = 0$ (c)infinite number of solutions if $a + b + c = 0$ (d)none of these

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45. Show that if $x_1, x_2, x_3 \neq 0$

$$\left|x_1 + a_1 b1 a_1 b_2 a_1 b_3 a_2 b_1 x_2 + a_2 b_2 a_2 b_3 a_3 b_1 a_3 b_2 x_3 + a_3 b_3\right| = x_1 x_2 x_3 \left(1 + \frac{a_1 b_1}{x_1} + \right.$$

.

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46. If determinant $\left|\cos(\theta + \varphi) - \sin(\theta + \varphi)\cos 2\varphi s\int h\eta\cos\theta s \in \varphi - \cos\theta s\int h\eta\cos\varphi\right|$ is a. positive b. independent of q c. independent of φ d. none of these

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47. If $\Delta_r = \begin{vmatrix} 2^{r-1} & 2.3^{r-1} & 4.5^{r-1} \\ \alpha & \beta & \gamma \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$, then find the value of Δ

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48. The values of $k \in R$ for which the system of equations $x + ky + 3z = 0, kx + 2y + 2z = 0, 2x + 3y + 4z = 0$ admits of nontrivial solution is 2 b. $5/2$ c. 3 d. $5/4$

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49. A determinant of second order is made with the elements 0 and 1. Find the number of determinants with non-negative values.

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50. If a, b, c are nonzero real numbers such that $|baabcaacaa| = 0, then$ $\frac{1}{a} + \frac{1}{b\omega} + \frac{1}{c\omega^2} = 0$ b. $\frac{1}{a} + \frac{1}{b\omega^2} + \frac{1}{c\omega^{\square}} = 0$ c. $\frac{1}{a\omega} + \frac{1}{b\omega^2} + \frac{1}{c} = 0$ d. none of these

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51. Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ

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52. Consider the determinant $f(x) = \left|0x^2 - ax^3 - bx^2 + a0x^2 + cx^4 + bx - c0\right|.$

Statement 1: $f(x) = 0$ has one root $x = 0.$ Statement 2: The value of skew symmetric determinant of odd order is always zero.

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53. Find the value of $|124 - 130410|$

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54. Consider the system of the equation $kx + y + z = 1, x + ky + z = k, andx + y + kz = k^2$. Statement 1: System equations has infinite solutions when $k = 1$. Statement 2: If the determinant $\left|111kk1k^2 1k\right| = 0$, t hen $k = -1$.

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55. If $x + y + z = 0$ prove that $|axbyczcyazbxbzcxay| = xyz|abaca|$

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56. If $a, b, c \in R,$ then find the number of real roots of the equation

$$\Delta = \begin{vmatrix} x & c & -b \\ -c & x & a \\ b & -a & x \end{vmatrix} = 0$$

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57. $\Delta = \begin{vmatrix} a & a^2 & 0 \\ 1 & 2a+b & (a+b) \\ 0 & 1 & 2a+3b \end{vmatrix}$ is divisible by $a+b$ b. $a+2b$ c. $2a+3b$ d. a^2

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58. Without expanding at any stage, prove that the value of each of the following determinants is zero. (1) $\begin{vmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{vmatrix}$ (2) $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$ (3)

$\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$, where w is cube root of unity

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59. If a,b,c are positive and are the $pth, qthrth$ terms respectively of a GP then $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} =$

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60. If the entries in a 3×3 determinant are either 0 or 1, then the greatest value of their determinants is

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61. The value of the determinant of n^{th} order, being given by $|x111x111x|$ is $(x-1)^{n-1}(x+n-1)$ b. $(x-1)^{n}(x+n-1)$ c. $(1-x)^{-1}(x+n-1)$ d. none of these

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62. Prove that $a \neq 0$, $\begin{bmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a^2 \end{bmatrix} = 0$ represents a straight line parallel to the y-axis.

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63. If $a_1b_1, c_1, a_2b_2c_2anda_3b_3c_3$ are three digit even natural numbers and $= \left|c_1a_1b_1c_2a_2b_2c_3a_3b_3\right|, thenis$ divisible by 2 but not necessarily by 4 divisible by 4 but not necessarily by 8 divisible by 8 none of these

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64. If $= \begin{vmatrix} abc & b^2c & c^2b \\ abc & c^2a & ca^2 \\ abc & a^2b & b^2a \end{vmatrix} = 0, (a, b, c \in R)$ and are all different and nonzero), then prove that $a + b + c = 0.$

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65. The value of $\begin{vmatrix} yz & zx & xy \\ p & 2p & 3r \\ 1 & 1 & 1 \end{vmatrix}$$, where x, y, z$ are respectively, pth, $(2q)$ th, and $(3r)$ th terms of an H.P. is a.-1 b. 0 c. 1 d. none of these

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66. Show that the determinant

$\left|a^2+b^2+c^2bc+ca+a\mathbf{c}+ca+a\mathbf{c}+ca+aba^2+b^c+c^2bc+ca+a\mathbf{c}+ca+a\mathbf{c}+\right.$

is always non-negative. When is the determinant zero?

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67. If $x \neq 0, y \neq 0, z \neq 0$ and $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0,$ then $x^{-1}+y^{-1}+z^{-1}$ is equal to a.1 b.-1 c.-3 d. none of these

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68. If $x \neq y \neq z and \begin{vmatrix} x & x^2 & 1 + x^3 \\ y & y^2 & 1 + y^3 \\ z & z^2 & 1 + z^3 \end{vmatrix} = 0,$ then the value of xyz is a.1 b. 2 c. -1

d. 2

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69. Prove that

$|2\alpha + \beta + \gamma + \delta\alpha\beta + \gamma\delta\alpha + \beta + \gamma + \delta2(\alpha + \beta)(\gamma + \delta)\alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta)\alpha\beta + \gamma\delta\alpha\beta(\gamma$

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70. Solve for x, $|x - 6 - 12 - 3 \times \ - 3 - 32 \times \ + 2| = 0.$

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71. The value of determinant $\left|111^{m}C_{1}^{m+1}C_{1}^{m+2}C_{1}^{m}C_{2}^{m+1}C_{2}^{m+2}C_{2}\right|$ is equal to 1 b. -1 c. 0 d. none of these

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72. Solve the equation $|a-xcbcb-xabac-x|=0wherea+b+c\neq 0.$

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73. If $a^2+b^2+c^2=-2andf(x)=$ $\left|a+a^2x\left(1+b^2\right)x\left(1+c^2\right)x\left(1+a^2\right)x1+b^2x\left(1+c^2\right)x\left(1+a^2\right)x\left(1+b^2\right)x1+c^2\right.$, then $f(x)$ is a polynomial of degree 0 b. 1 c. 2 d. 3

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74. Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of the set of all determinants with value -1. Then

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75. Solve:

$|x^2 - 1x^2 + 2x + 12x^2 + 3x + 12x^2 + x - 12x^2 + 5x - 32x^2 + 4x - 36x^2 - x - 26x^2 - 7x$

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76. If $|x3636x6x3| = |2x7x7272x| = |45x5x4x45| = 0,$ then x is equal to 0 b. -9

c. 3 d. none of these

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77. If $A_1B_1C_1, A_2B_2C_2$ and $A_3B_3C_3$ are three digit numbers, each of which

is divisible by k, then $\Delta = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$ is

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78. If $f'(x) = |mxmx - pmx + p \cap \ + pn - pmx + 2nmx + 2n + pmx + 2n - p|$, then $y = f(x)$ represents a straight line parallel to x-axis a straight line parallel to y-axis parabola a straight line with negative slope

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79. If $\sum_{n=1}^{n} \alpha_n = an^2 + bn, wherea, b$ are constants and $\alpha_1, \alpha_2, \alpha_3 \in \{1, 2, 3, \ldots\ldots, 9\}and25\alpha_1, 37\alpha_2, 49\alpha_3$ be three digit number, then prove that $\left|\alpha_1\alpha_2\alpha_3 57925\alpha_1 37\alpha_2 49\alpha_3\right| = 0$

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80. The determinant $|xp + yxyyp + zyz0xp + yyp + z| = 0$ if x, y, z

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81. Using properties of determinant prove that

$|a + b + c - c - b - ca + b + c - a - b - aa + b + c| = 2(a + b)(b + c)(c + a)$

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82. Show that

$$\left| \wedge xC_r^x C_{r+1}^x C_{r+2}^y C_r^y C_{r+1}^y C_{r+2}^z C_r^z C_{r+1}^z C_{r+1} \right| = \left| \wedge xC_r^{x+1} C_{r+1}^{x+2} C_{r+2}^y C_r^{y+1} C_{r+1}^{y+2} C$$

.

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83. If $\omega \neq 1$ is a cube root of unity and $x + y + z \neq 0$, then prove that

$$\left| \frac{x}{1 + \omega} \frac{y}{\omega + \omega^2} \frac{z}{\omega^2 + 1} \frac{y}{\omega + \omega^2} \frac{z}{\omega^2 + 1} \frac{x}{1 + \omega} \frac{zz}{\omega^2 + 1} \frac{x}{1 + \omega} \frac{y}{\omega + \omega^2} \right| = 0$$ if

$x = y = z$

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84. Statement 1: If the system of equation $\lambda x + (b - a)y + (c - a)z = 0, (a - b)x + \lambda y + (c - b)z = 0, and(a - c)x + (b - c)y + \lambda z =$

has a non trivial solution, then the value of λ is $0.$ Statement 2: the value of skew symmetric matrix of order 3 is order.

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85. If $A, BandC$ are the angels of a triangle, show that

$| - 1 + \cos B\cos C + \cos B\cos B\cos C + \cos A - 1 + \cos A\cos A - 1 + \cos B - 1 + \cos A - 1|$

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86. If α, β, γ are the angles of a triangle and system of equations

$\cos(\alpha - \beta)x + \cos(\beta - \gamma)y + \cos(\gamma - \alpha)z = 0$

$\cos(\alpha + \beta)x + \cos(\beta + \gamma)y + \cos(\gamma + \alpha)z = 0$

$\sin(\alpha + \beta)x + \sin(\beta + \gamma)y + \sin(\gamma + \alpha)z = 0$ has non-trivial solutions, then triangle is necessarily a. equilateral b. isosceles c. right angled d. acute angled

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87. Without expanding the determinants, prove that $|103115114111108106104113116| + |113116104108106111115114103| = 0$

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88. Given $a = x/(y - z), b = y/(z - x), andc = z/(x - y), wherex, y, zandz$ are not all zero, then the value of $ab + bc + ca$ is 0 b. 1 c.-1 d. none of these

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89. Find the value of determinant $\left|\sqrt{(13)} + \sqrt{3}2\sqrt{5}\sqrt{5}\sqrt{(15)} + \sqrt{(26)}5\sqrt{(10)}3 + \sqrt{(65)}\sqrt{(15)}5\right|$

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90. a, b, c are distinct real numbers not equal to one. If $ax + y + z = 0, x + by + z = 0, andx + y + cz = 0$ have nontrivial solution, then the value of $\frac{1}{1 - a} + \frac{1}{1 - b} + \frac{}{1 - c}$ is equal to a.1 b. -1 c.$zero$ d. none of these

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91. Prove that the value of the determinant $\left| -75 + 3i\frac{2}{3} - 4i5 - 3i84 + 5i\frac{2}{3} + 4i4 - fi9 \right|$ is real.

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92. If the system of linear equation $x + y + z = 6, x + 2y + 3c = 14, and2x + 5y + \lambda z = \mu(\lambda, \mu R)$ has a unique solution, then $\lambda = 8$ b. $\lambda = 8, \mu = 36$ c.$\lambda = 8, \mu \neq 36$ d. none of these

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93. If $a_r = (\cos 2r\pi + is \in 2r\pi)^{1/9}$, then prove that $\left|a_1a_2a_3a_4a_5a_6a_7a_8a_9\right| = 0.$

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94. Let $f(x) = \left|2\cos^2 x\sin 2x - \sin x\sin 2x 2\sin^2 x\cos x\sin x - \cos x 0\right|$. Then the value of $\int_0^{\pi/2}\left[f(x) + f'(x)\right]dx$ is a.π b. $\pi/2$ c.2π d. $3\pi/2$

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95. Using properties of determinants, prove that

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

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96. The number of positive integral solutions of the equation

$$\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & z^2y & z^3+1 \end{vmatrix} = 11 \text{ is}$$

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97. If the value of the determinant $|a111b111c|$ is positive then $(a, b, c > 0)$

$abc > 1$ b. $abc < -8$ c.$abc < -8$ d. $abc \succ 2$

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98. By using properties of determinants. Show that: (i)

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x-4)(4-x)^2 \text{ (ii) } \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$$

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99. Using properties of determinants, evaluate $|184089408919889198440|$

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100. If $A_1, B_1, C_1, ,$ are respectively, the cofactors of the elements $a_1, b_1c_1, ,$ of the determinant $\text{Delta} = \left|a_1b_1c_1a_1b_2c_2a_3b_3c_3\right|, \text{Delta} \neq 0$, then the value of $\left|B_2C_2B_3C_3\right|$ is equal to $a12$ b. a_1 c.a_1^2 d. $a12^2$

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101. Solve for x $|x - 22x - 33 \times - 4x - 42x - 93x - 16x - 82x - 273x - 64| = 0.$

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102. If $\text{Delta}_1 = |x\mathbf{a}xbaax| and\text{Delta}_2 = |xbax|$ are the given determinants, then $\text{Delta}_1 = 3\left(\text{Delta}_2\right)^2$ b.$\frac{d}{dx}\left(\text{Delta}_1\right) = 3\text{Delta}_2$ c. $\frac{d}{dx}\left(\text{Delta}_1\right) = 3\left(\text{Delta}_2\right)^2$ d. $\text{Delta}_1 = 3\text{Delta}23/2$

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103. Without expanding evaluate the determinant

$$\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$$

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104. If $y = \sin mx,$ then the value of the determinant $\left|yy_1y_2y_3y_4y_5y_6y_7y_8\right|, where y_n = \dfrac{d^n y}{dx^n}$ is m^9 b. m^2 c.m^3 d. none of these

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105. Find the value of the determinant $|111123413610141020|$

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106. The parameter on which the value of the determinant $\left|1aa^2\cos(p-d)x\cos px\cos(p+d)x\sin(p-d)x\sin px\sin(p+d)x\right|$ does not depend is a b. p c. d d. x

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107. If $x = cy + bz, y = az + cx, z = x + ay, where x, y, z$ are not all zeros, then find the value of $a^2 + b^2 + c^2 + 2abc$.

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108. If $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = ka^2b^2c^2,$ then the value of k is abc b. $a^2b^2c^2$ c. $bc + ca + ab$ d. none of these

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109. The value of $\left|-1213+2\sqrt{22}+2\sqrt{213}-2\sqrt{22}-2\sqrt{21}\right|$ is equal to a. zero b. $-16\sqrt{2}$ c. $-8\sqrt{2}$ d. none of these

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110. Find the following system of equations is consistent, $(a+1)^3x+(a+2)^3y=(a+3)^3$ $(a+1)x+(a+2)y=a+3$ $x+y=1,$ then find the value of a

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111. Prove the identities: $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}=4a^2b^2c^2$

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112. If $\left|111abca^3b^2c^3\right| = (a - b)(b - c)(c - a)(a + b + c), wherea, b, c$ are different, then the determinant $\left|111(x - a)^2(x - b)^2(x - c)^2(x - b)(x - c)(x - c)(x - a)(x - a)(x - b)\right|$ vanishes when a.$a + b + c = 0$ b. $x = \frac{1}{3}(a + b + c)$ c. $x = \frac{1}{2}(a + b + c)$ d. $x = a + b + c$

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113. Prove that

$$\begin{bmatrix} 1 + a^2 + a^4 & 1 + ab + a^2b^2 & 1 + ac + a^2c^2 \\ 1 + ab + a^2b^2 & 1 + b^2 + b^4 & 1 + bc + b^2c^2 \\ 1 + ac + a^2c^2 & 1 + bc + b^2c^2 & 1 + c^2 + c^2 \end{bmatrix} = (a - b)^2(b - c)^2(c - a)^2$$

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114. The determinant $\left|y^2 - xyx^2abca'b'c'\right|$ is equal to a. $\left|bx + aycx + byb'x + a'yc'x + b'y\right|$ b. $\left|ax + bybx + cya'x + b'yb'x + c'y\right|$ c. $\left|bx + cyax + byb'x + c'ya'x + b'y\right|$ d. $\left|ax + bybx + cya'x + b'yb'x + c'y\right|$

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115. If x, y, z are in A.P. , then the value of the determinant are in A.P. , then the value of the determinant $|a+2a+3a+2xa+3a+4a+2ya+4a+5a+2z|$ is a. 1 b. 0 c. $2a$ d. a

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116. Prove that $|(b+x)(c+x)(v+x)(a+x)(a+x)(b+x)(b+y)(c+y)(c+x)(a+t)(a+y)(b+y)(b$

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117. If $a+b+c=0,$ one root of $\begin{bmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{bmatrix}=0$ is a. $x=1$ b. $x=2$ c. $x=a^2+b^2+c^2$ d. $x=0$

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118. Factorise the following $\begin{vmatrix} 3 & a+b+c & a^3+b^3+c^3 \\ a+b+c & a^2+b^2+c^2 & a^4+b^4+c^4 \\ a^2+b^2+c^2 & a^3+b^3+c^3 & a^5+b^5+c^5 \end{vmatrix}$

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119. Let $\{D_1, D_2, D_3, D_n\}$ be the set of third order determinant that can be made with the distinct non-zero real numbers a_1, a_2, a_q. Then $\sum_{i=1}^{n} D_i = 1$ b. $\sum_{i=1}^{n} D_i = 0$ c. $D_i - D_j, \forall i, j$ d. none of these

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120. If $g(x) = \dfrac{f(x)}{(x-a)(x-b)(x-c)}$,where f(x) is a polynomial of degree < 3 , then $\int g(x)dx = \begin{vmatrix} 1 & a & f(a)\log|x-a| \\ 1 & b & f(b)\log|x-b| \\ 1 & c & f(c)\log|x-c| \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + k$

$$\frac{dg(x)}{dx} = \begin{vmatrix} 1 & a & -f(a)(x-a)^{-2} \\ 1 & b & -f(b)(x-b)^{-2} \\ 1 & c & -f(c)(x-c)^{-2} \end{vmatrix} : - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

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121. If the equation $2x + 3y + 1 = 0, 3x + y - 2 = 0, and ax + 2y - b = 0$ are consistent, then prove that $a - b = 2$.

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122. If w is a complex cube root of unity, then value of

$$= \left| a_1 + b_1 w a_1 w^2 + b_1 c_1 + b_1 w a_2 + b_2 w a_2 w^2 + b_2 c_2 + b_2 w a_3 + b_3 w a_3 w^2 + b_3 c_3 \right.$$

is a. 0 b. -1 c. 2 d. none of these

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123. Let

$f(x) = \left|\cos\left(x + x^2\right)\sin\left(x + x^2\right) - \cos\left(x + x^2\right)\sin\left(x - x^2\right)\cos\left(x - x^2\right)\sin\left(x - x^2\right)\sin 2$

. Find the value of $f'(0)$

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124. If $f(x), g(x) and h(x)$ are three polynomial of degree 2, then prove that $\phi(x) = | f(x)g(x)h(x)f'(x)g'(xh'(x)f''(x)g''(x)h''(x)|$ is a constant polynomial.

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125. If a, b, c are in G.P. with common ratio $r_1 and \alpha, \beta, \gamma$ are in G.P. with common ratio r_2 and equations $ax + \alpha y + z = 0, bx + \beta y + z = 0, cx + \gamma y + z = 0$ have only zero solution, then which of the following is not true? a.$a + b + c$ b. abc c.1 d. none of these

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126. Show that: $|ab - \ + ba + cbc - aa - \ + ac| = (a + b + c)\left(a^2 + b^2 + c^2\right)^{\cdot}$

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127. In triangle ABC , if $\left|111\frac{\cot A}{2}\frac{\cot B}{2}\frac{\cot C}{2}\frac{\tan B}{2} + \frac{\tan C}{2}\frac{\tan B}{2} + \frac{\tan A}{2}\frac{\tan A}{2} + \frac{\tan B}{2}\right|$ then the triangle must be a. equilateral b. isosceles c. obtuse angled d. none of these

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128. If $a, b, c, d, e,$ and $f$are in G.P. then the value of $\begin{vmatrix} \left(a^2\right) & \left(d^2\right) & x \\ \left(b^2\right) & \left(e^2\right) & y \\ \left(c^2\right) & \left(f^2\right) & z \end{vmatrix}$ depends on (A) x and y (B) x and z (C) y and z (D) independent of x ,y ,and z

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129. Prove that: $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

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130. If $x_i = a_i b_i c + i, i = 1, , 23$ are three-digit positive integer such that each x_1 is a multiple of 19, then for some integers $n,$ prove that $\left|a_1 a_2 a_3 b_1 b_2 b_3 c_1 c_2 c_3\right|$ is divisible by 19.

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131. The system of linear equations $x + \lambda y - z = 0 \ \lambda x - y - z = 0 \ x + y - \lambda z = 0$ has a non-trivial solution for : (1) infinitely many values of λ . (2) exactly one value of λ . (3) exactly two values of λ . (4) exactly three values of λ .

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132. If $pqr \neq 0$ and the system of equation $(p + a)x + by + cz = 0$ $ax + (q + b)y + cz = 0$ $ax + by + (r + c)z = 0$ has nontrivial solution, then value of $\frac{a}{p} + \frac{b}{q} + \frac{c}{r}$ is -1 b. 0 c.0 d. $\neg - 2$

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133. If $c < 1$ and the system of equations $x + y - 1 = 0, 2x - y - c = 0, andbx + 3by - c = 0$ is consistent, then the possible real values of b are $b\left(-3\frac{3}{4}\right)$ b. $b\left(-\frac{3}{2}, 4\right)$ c.$b\left(-\frac{3}{4}, 3\right)$ d. none of these

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134. Prove that

$$\left|x^2x^2 - (y - z)^2yzy^2y^2 - (z - x)^2zxz^2z^2 - (x - y)^2xy\right| = (x - y)(y - z)(z - x)(x + y + z)\Big(x$$

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135. If a, b and c are real numbers, and $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0.$

Show that either $a + b + c = 0$ or $a = b = c$

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136. If a, b, c are non-zero, then the system of equations $(\alpha + a)x + \alpha y + \alpha z = 0, \alpha x + (\alpha + b)y + \alpha z = 0, \alpha x + \alpha y + (\alpha + c)z = 0$ has a non-trivial solution if $\alpha^{-1} =$ (A) $-\left(a^{-1} + b^{-1} + c^{-1}\right)$ (B) $a + b + c$ (C) $\alpha + a + b + c = 1$ (D) none of these

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137. Prove that

$$\left|ab + ca^2bc + ab^2ca + bc^2\right| = -(a+b+c) \times (a-b)(b-c)(c-a)$$

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138. If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is negative, then

$\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is $+ve$ b. $(ac-b)^2\left(ax^2+2bx+c\right)$ c. $-ve$ d. 0

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139. Using the properties of determinants, prove that following

$$\begin{vmatrix} a-b & -c^2 & a^2 \\ a^2 & -c & -a^2 \\ b^2 & c^2 & -a-b \end{vmatrix} = (a+b+c)^3$$

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140. If $a_1, a_2, a_3, \ldots . a_n \ldots$ are in G.P. then the determinant

$\Delta = \begin{vmatrix} \log a_n, \log a_{n+1}, \log a_{n+2} \\ \log a_{n+3}, \log a_{n+4}, \log a_{n+5} \\ \log a_{n+6}, \log a_{n+7}, \log a_{n+8} \end{vmatrix}$ is equal to- (A) -2 (B) 1 (C) -1 (D) 0

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141. Using properties of determinants, prove the following

$\begin{vmatrix} a^2 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2.$

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142. Let $\vec{a}_r = x_r\hat{i} + y_r\hat{j} + z_r\hat{k}, r = 1, 2, 3$ be three mutually perpendicular unit vectors, then the value of $\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$ is equal to 0 b. ± 1 c. ± 2 d.

none of these

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143. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $\pi/4 \leq x \leq \pi/4$ is 0 b. 2 c. 1 d. 3

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144. Let a, b, c be real numbers with $a^2 + b^2 + c^2 = 1.$ Show that the equation

$|ax - by - cbx - aycx + abx + ay - ax + by - y + bcx + acy + b - ax - by + c| = 0$

represents a straight line.

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145. If lines $px + qy + r = 0, qx + ry + p = 0andrx + py + q = 0$ are concurrent, then prove that $p + q + r = 0(wherep, q, r$ are distinct $\Big)$.

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146. If $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2 + 3\lambda & \lambda - 1 & \lambda + 3 \\ \lambda^2 + 1 & 2 - \lambda & \lambda - 3 \\ \lambda^2 - 3 & \lambda + 4 & 3\lambda \end{vmatrix}$ then $t =$

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147. Find the value of λ if $2x^2 + 7xy + 3y^2 + 8x + 14y + \lambda = 0$ represent a pair of straight lines.

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148. If x, y, z are different from zero and Delta $= \begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0,$

then the value of the expression $\frac{a}{x} + \frac{b}{y} + \frac{c}{z}$ is 0 b. -1 c. 1 d. 2

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149. If A, B, C are angles of a triangles, then the value of

$\begin{vmatrix} e^{2iA} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2iB} & e^{-iA} \\ e^{-iB} & e^{-iA} & e^{2iC} \end{vmatrix}$ is 1 b. -1 c. -2 d. -4

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150. If $a^2 + b^2 + c^2 = 1,$ then prove that

$\left|a^2 + \left(b^2 + c^2\right)\cos\varphi ab(1 - \cos\varphi)ac(1 - \cos\varphi)ba(1 - \cos\varphi)b^2 + \left(c^2 + a^2\right)\cos\varphi bc(1 -\right.$

is independent of $a, b,$ ·

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151. For the equation $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = 0,$ There are exactly two distinct roots There is one pair of equation real roots. There are three pairs of equal roots Modulus of each root is 2

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152. Let $\text{Delta}_r = \left| r - 1n6(r - 1)^2 2n^2 4n - 2(r - 1)^2 3n^3 3n^2 - 3n \right|^{\cdot}$ Show that $\sum_{r=1}^{n} \text{Delta}_r$ is contant.

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153. Let m be a positive integer and $\Delta r = \begin{vmatrix} 2r - 1 & .^{m}C_{r} & 1 \\ m^{2} - 1 & 2^{m} & m + 1 \\ \sin^{2}\left(m^{2}\right) & \sin^{2}m & \sin\left(m^{2}\right) \end{vmatrix}$. Then the value of $\sum_{r=0}^{m} \Delta r$

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154. Prove that $|1 + a11111 + b11111 + c11111 + d| = abcd\left(a + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right)^{\cdot}$ Hence find the value of the determinant if a, b, c, d are the roots of the equation $px^{4} + qx^{3} + rx^{2} + sx + t = 0.$

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155. Find the area of a triangle whose vertices are $A(3, 2), B(11, 8)andC(8, 12)^{\cdot}$

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156. If $D_k = 1 \cap 2kn^2 + n + 1n^2 + n2k - 1n^2n^2 + n + 1and\sum_{k=1}^{n} D_k = 56.$ then n equals 4 b. 6 c. 8 d. none of these

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157. If If lines $px + by = c, ax + qy = c$ and $ax + by = r(a \neq p, b \neq q, c \neq r)$ are concurrent then find the value and $\frac{p}{p - a} + \frac{q}{q - b} + \frac{r}{r - c}.$

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158. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with same common ratio, then prove that the points $\left(x_1, y_1\right), \left(x_2, y_2\right), and\left(x_3, y_3\right)$ are collinear.

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159. If the lines $a_1x + b_1y + 1 = 0, a_2x + b_2y + 1 = 0 and a_3x + b_3y + 1 = 0$ are concurrent, show that the point $\left(a_1, b_1\right), \left(a_1, b_2\right) and \left(a_3, b_3\right)$ are collinear.

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160. For a fixed positive integer n, if $= |n!(n+1)!(n+2)!(n+1)!(n+2)!(n+3)!(n+2)!(n+3)!(n+4)!|$, then show that $\left[/\left((n!)^3\right) - 4\right]$ is divisible by n

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161. Find the value of a for which the lines $2x + y - 1 = 0$ $2x + y - 1 = 0$ $ax + 3y - 3 = 0$ $3x + 2y - 2 = 0$ are concurrent.

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162. If the lines $ax + y + 1 = 0, x + by + 1 = 0 and x + y + c = 0(a, b, c$ being distinct and different from $1)$ are concurrent, then prove that $\frac{1}{1 - a} + \frac{1}{1 - b} + \frac{1}{1 - c} = 1.$

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163. If $\left(1 + ax + bx^2\right)^4 = a_0 + a_1x + a_2x^2 + \ldots + a_8x^8$ when $a, b, a_0, a_1, a_2..., a_8 \in R$ such that $a_0 + a_1 + a_2 \neq 0$ and $\begin{vmatrix} a_0 & a_1 & a_2 \\ a_1 & a_2 & a_3 \\ a_2 & a_0 & a_1 \end{vmatrix} = 0$ then the value of $5\frac{a}{b}$ (A) 6 (B) 8 (C) 10 (D) 12

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164. Find the value of λ for which the homogeneous system of equations: $2x + 3y - 2z = 0$ $2x - y + 3z = 0$ $7x + \lambda y - z = 0$ has non-trivial solutions. Find the solution.

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165. If $f, g, andh$ are differentiable functions of $xand(x) = \left|fgh(xf)'(xg)'(xh)'\left(x^{f2}f\right)''\left(x^2g\right)''\left(x^2h\right)''\right|$ prove that $\wedge(') = \left|fgff'g'h'\left(x^3f''\right)'\left(x^3g''\right)'\left(x^3h''\right)'\right|$

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166. Let a, b, c be positive and not all equal. Show that the value of the determinant $|abcbcacab|$ is negative.

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167. $\text{if} \quad = \begin{vmatrix} x^n & n! & 2 \\ \cos x & \cos.\frac{n\pi}{2} & 4 \\ \sin x & \sin.\frac{n\pi}{2} & 8 \end{vmatrix}$, then find the value of

$\frac{d^n}{dx^n}[f(x)]_{x=0}.(n \in z).$

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168. Without expanding a determinant at any stage, show that |

$\left|x^2 + x, x + 1, x + 2\right|, \left|, 2x^2 + 3x - 13x, 3x - 3\right|, \left|x^2 + 2x + 3, 2x - 1, 2x - 1\right| \; | \quad = xA$

.

are determinant of order 3 not involving x

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169. Show that $\begin{vmatrix} bc - a^2 & ca - b^2 & ab - c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ca - b^2 \end{vmatrix}$

$$\begin{vmatrix} a^2 & c^2 & 2ca - b^2 \\ 2ab - c^2 & b^2 & a^2 \\ b^2 & 2ac - a^2 & c^2 \end{vmatrix}.$$

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170. Show that the system of equations $3x - y + 4z = 3, x + 2y - 3z = -2$ and $6x + 5y + \lambda z = -3$ has at least one solution for any real number λ. Find the set of solutions of $\lambda = -5$

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171. Express $\Delta = \begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ca - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix}$ as square of a determinant of hence evaluate if.

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172. If $y = |\sin x \cos x \sin x \cos x - \sin x \cos \times 11| = f \in d\frac{dy}{dx}\cdot$

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173. Consider the system of equation $x + y + z = 6, x + 2y + 3z = 10, andx + 2y + \lambda z = \mu$ Statement 1: if the system has infinite number of solutions, then $\mu = 10$. Statement 2: The determinant $|116121012\mu| = 0f$ or $\mu = 10$.

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174. Consider the system linear equations in $x, y, andz$ given by $(\sin 3\theta)x - y + z = 0, (\cos 2\theta)x + 4y + 3z = 0, 2x + 7y + 7z = 0$. the value of θ for which the system has a non-trivial solution : (A) $\theta = n\frac{\pi}{2}$ (B) $\theta = (2n + 1)\frac{\pi}{6}$ (C) $\theta = n\pi$ or $n\pi + (- 1)^n\frac{\pi}{6}$ (D) none of these

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175. Prove without expansion that $|ah + bggab + chbf + bafhb + bcaf + bbg + fc| = a|ah + bgahbf + bahbaf + bcgf|$

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176. If $\alpha, \beta and \gamma$ are real number without expanding at any stage prove that

$$\begin{vmatrix} 1 & \cos(\beta - \alpha) & \cos(\gamma - \alpha) \\ \cos(\alpha - \beta) & 1 & \cos(\gamma - \beta) \\ \cos(\alpha - \gamma) & \cos(\beta - \gamma) & 1 \end{vmatrix} = 0.$$

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177. If 3 digit numbers $A28, 3B9$ and $62C$ are divisible by a fixed constant 'K' where A, B, C are integers lying between 0 and 9, then determinant

$\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is always divisible by

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178. If $a = \cos\theta + i\sin\theta, b = \cos 2\theta - i\sin 2\theta, c = \cos 3\theta + i\sin 3\theta$ and if

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then $\theta = \ ?$

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179. Prove that

$\left|a_1\alpha_1 + b_1\beta_1 a_1\alpha_2 + b_2\beta_2 a_1\alpha_3 + b_1\beta_3 a_2\alpha_1 + b_2\beta_1 a_2\alpha_2 + b_2\beta_2 a_2\alpha_3 + b_2\beta_3 a_3\alpha_1 + b\right.$

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180. If α, β, γ are the roots of $px^3 + qx^2 + r = 0,$ then the value of the determinant $\begin{vmatrix} \alpha\beta & \beta\gamma & \gamma\alpha \\ \beta\gamma & \gamma\alpha & \alpha\beta \\ \gamma\alpha & \alpha\beta & \beta\gamma \end{vmatrix}$ is p b. q c. 0 d. r

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181. Prove that

$\left|(a - x)^2(a - y)^2(a - z)^2(b - x)^2(b - y)^2(b - z)^2(c - x)^2(c - y)^2(c - z)^2\right| =$

$\left|(1 + ax)^2(1 + bx)^2(1 + cx)^2(1 + ay)^2(1 + by)^2(1 + cy)^2(1 + az)^2(1 + bz)^2(1 + cz)^2\right|$

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182. If $\left|1 \times^2 \times^2 1x^2 1x\right| = ,$ then find the value of $\left|x^3 - 10x - x^4 0x - x^4 x^3 - 1x - x^4 x^3 - 10\right|$

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183. If

$$\left|x^n x^{n+2} x^{n+3} y^n y^{n+2} y^{n+3} z^n z^{n+2} z^{n+3}\right| = (x - y)(y - z)(z - x)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right),$$

then n equals 1 b. -1 c. 2 d. -2

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184. Solve the system of the equations: $ax + by + cz = d$ $a^2x + b^2y + c^2z = d^2$ $a^3x + b^3y + c^3z = d^3$ Will the solution always exist and be unique?

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185. If $|ab - \ + ba + cbc - aa - ba + bc| = 0$, then the line $ax + by + c = 0$ posses through the fixed point which is $(1, 2)$ b. $(1, 1)$ c. $(- 2, 1)$ d. $(1, 0)$

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186. If $2ax - 2y + 3z = 0, x + ay + 2z = 0, and2 + az = 0$ have a nontrivial solution, find the value of $a^{.}$

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187. If [] denotes the greatest integer less than or equal to the ral number under consideration, and $-1 \leq x < 0, 0 \leq y < 1, 1 \leq a < 2,$ then the value of the determinant $|[x] + 1[y][z][x][y] + 1[z][x][y][z] + 1|$ is $[x]$ b. $[y]$ c. $[z]$ d. none of these

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188. If $f(x) = a + bx + cx^2$ and α, β, γ are the roots of the equation $x^3 = 1, then|abcbcacab|$ is equal to a.$f(\alpha) + f(\beta) + f(\gamma)$ b $f(\alpha)f(\beta) + f(\beta)f(\gamma) + f(\gamma)f(\alpha)$ c. $f(\alpha)f(\beta)f(\gamma)$ d. $-f(\alpha)f(\beta)f(\gamma)$

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189. If $x, yandz$ are not all zero and connected by the equations $a_1x + b_1y + c_1z = 0, a_2x + b_2y + c_2z = 0, and\left(p_1 + \lambda q_1\right)x + \left(p_2 + \lambda q_2\right) + \left(p_3 + \lambda$

, show that $\lambda = - \left|a_1b_1c_1a_2b_2c_2p_1p_2p_3\right| \div \left|a_1b_1c_1a_2b_2c_2q_1q_2q_3\right|$

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190. Find λ for which the system of equations $x + y - 2z = 0, 2x - 3y + z = 0, x - 5y + 4z = \lambda$ is consistent and find the solutions for all such values of λ.

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191. If p, q, r are in A.P. then value of determinant

$\left|a^2 + 2^{n+1} + 2pb^2 + 2^{n+2} + 3qc^2 + p2^n + p2^{n+1}2qa^2 + 2^n + pb^2 + 2^{n+1}c^2 - r\right|$

is 0 (b) Independent from a, b, c $a^2b^2c^2 - 2^n$ (d) Independent from n

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192. For what values of $k,$ the following system of equations possesses a nontrival solution over the set of rationals: $c + ky + 3z = 0, 3c + ky - 2z = 02c + 3y - 4x = 0.$ Also find the solution for this value of k

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193. Let $a, b, c \in R$ such that no two of them are equal and satisfy

$\begin{vmatrix} 2a & b & c \\ b & c & 2a \\ c & 2a & b \end{vmatrix} = 0,$ then equation $24ax^2 + 8bx + 4c = 0$ has (a) at last one root in $[0, 1]$ (b) at last one root in $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (c) at last one root in $[-1, 0]$

(d) at last two roots in $[0, 2]$

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194. Solve by Cramers rule $x + y + z = 6$ $x - y + z = 2$ $3x + 2y - 4z = -5$

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195. The value of the determinant $\begin{vmatrix} ka & k^2 + a^2 & 1 \\ kb & k^2 + b^2 & 1 \\ kc & k^2 + c^2 & 1 \end{vmatrix}$ is (A) $k(a + b)(b + c)(c + a)$ (B) $kabc\left(a^2 + b^2 + c^2\right)$ (C) $k(a - b)(b - c)(c - a)$ (D) $k(a + b - c)(b + c - a)(c + a - b)$

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196. Find the value of $aandb$ if the system of equation $a^2x - by = a^2 - bandbx = b^2y = 2 + 4b$ (i) posses unique solution (ii) infinite solutions

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197. If $\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2 = a^2$, $\left(x_2 - x_3\right)^2 + \left(y_2 - y_3\right)^2 = b^2$, $\left(x_3 - x_1\right)^2 + \left(y_3 - y_1\right)^2 = c^2$, and $k\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = (a + b + c)(b + c - b)(c + a - b) \times (a + b - c)$, then the value of k is 1 b. 2 c. 4 d. none of these

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198. $f(x) = \left|\cos \times 12\sin \times {}^22x\tan \times 1\right|$. Then value of $(\lim)_{x\vec{0}}\frac{f(x)}{x}$ is equal to 1 b. -1 c. zero d. none of these

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199. If a system of three linear equations $x + 4ay + a = 0, x + 3by + b = 0, andx + 2cy + c = 0$ is consistent, then prove that a, b, c are in H.P.

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200. Let α be a repeated root of a quadratic equation $f(x) = 0$ and $A(x), B(x), C(x)$ be polynomials of degrees 3, 4, and 5, respectively, then show that $\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$ is divisible by $f(x)$, where prime $(')$ denotes the derivatives.

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201. If

$$\left|a^2 + \lambda^2 ab + c\lambda ca - b\lambda ab - c\lambda b^2 + \lambda^2 bc + aca + b\lambda bc - a\lambda c^2 + \lambda^2\right| |\lambda c - b - c\lambda ab - a\lambda|$$

, then he value of λ is 8 b. 27 c. 1 d. -1

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202. Value of $\left|1+x_1 1+x_1 x 1+x_1 x^2 1+x_2 1+x_2 x 1+x_2 x^2 1+x_3 1+x_3 x 1+x_3 x^2\right|$ depends upon $\mathrm{xo\ n\ l\ y}$ b. $\mathrm{x}_1\mathrm{o\ n\ l\ y}$ c. $\mathrm{x}_2 only$ d. none of these

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203. If $(x) = |\alpha + x\theta + x\lambda + x\beta + x\varphi + x\mu + x\gamma + x\psi + xv + x|$ show that $\mathrm{Delta}^{\wedge}(x) = 0 and \mathrm{Delta}(0) + Sx, where S$ denotes he sum of all the cofactors of all elements in $\mathrm{Delta}(0) and$ dash denotes the derivative with respect of $\dot{x}$

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204. The value of determinant $\left|bc - a^2ac - b^2ab - c^2ac - b^2ab - c^2bc - a^2ab - c^2bc - a^2ac - b^2\right|$ is a. always positive b. always negative c. always zero d. cannot say anything

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205. If $l_1^2 + m_1^2 + n_1^2 = 1$ etc., and $l_1l_2 + m_1m_2 + n_1n_2 = 0$, etc. and

$$\Delta = \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$$ then

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206. Let $\left|x2 \times^2 x6 \times 6\right| = Ax^4 + Bx^3 + Cx^2 + Dx + E^{\cdot}$ Then the value of $5A + 4B + 3C + 2D + E$ is equal to a. zero b. -16 c. 11 d. 11

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207. If the determinant

$\left|b - \ - aa - \ \wedge (') - c'c^{'} - a'a^{'} - b'b^{-c_{'}'}c^{-a_{'}'}a^{-b_{'}'}\right| = m|abca'b'c'a''b''c''|$,

then the value of m s 0 b. 2 c. 1 d. -1

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208. Let a, b, c be the real numbers. The following system of equations in $x, y, and z$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{a^2} = 1, \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{a^2} = 1, - \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{a^2} = 1$ has a. no solution b. unique solution c. infinitely many solutions d. finitely many solutions

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209. The value of the determinant

$|[\sin\theta, \cos\theta, \sin 2\theta], \left[\sin\left(\theta + \frac{2\pi}{3}\right), \cos\left(\theta + \frac{2\pi}{3}\right), \sin\left(2\theta + \frac{4\pi}{3}\right)\right], \left[\sin\left(\theta - \frac{2\pi}{3}\right),$

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210. Find the value of the determinant $|baabpqr111|, wherea, b, andc$ are respectively, the pth, qth, and rth terms of a harmonic progression.

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211. let $a > 0, d > 0$ find the value of the determinant

$$\begin{vmatrix} \frac{1}{a} & \frac{1}{a(a+d)} & \frac{1}{(a+d)(a+2d)} \\ \frac{1}{a+d} & \frac{1}{(a+d)(a+2d)} & \frac{1}{(a+2d)(a+3d)} \\ \frac{1}{a+2d} & \frac{1}{(a+2d)(a+3d)} & \frac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

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212. Prove that

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0.$$

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213. Let $\lambda and \alpha$ be real. Find the set of all values of λ for which the system of linear equations $\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0, x + (\cos\alpha)y + (\sin\alpha)z = 0, - x + (\sin\alpha)y - (\cos\alpha)z = 0.$ have trivial solution

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214. Prove that $|ax - by - czay + bxcx + azay + bxby - cz - axbz + cycx + azbz + cycz - ax - by| = \Big($

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215. If $(x) = \left|a_1 + xb_1 + xc_1 + xa_2 + xb_2 + xc_2 + xa_3 + xb_3 + xc_3 + x\right|$, show that $\wedge (x) = 0$ and that$(x) = (0) + Sx, whereS$ denotes the sum of all the cofactors of all the element in $\dot{(0)}$

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216. If α, β, γ are different from 1 and are the roots of $ax^3 + bx^2 + cx + d = 0and(\beta - \gamma)(\gamma - \alpha)(\alpha - \beta) = \frac{25}{2}$, then prove that $\left|\frac{\alpha}{1 - \alpha}\frac{\beta}{1 - \beta}\frac{\gamma}{1 - \gamma}\alpha\beta\gamma\alpha^2\beta^2\gamma^2\right| = \frac{25d}{2(a + b + c + d)}$

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217. If $p + q + r = 0 = a + b + c,$ then the value of the determnalnt $|paqbrcqcrapbrbpcqa|is\ 0$ b. $pa + qb + rc$ c. 1 d. none of these

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218. Let

$= \left|2a_1b_1a_1b_2 + a_2b_1a_1b_3 + a_3b_1a_1b_2 + a_2b_12a_2b_2a_2b_3 + a_3b_2a_1b_3 + a_3b_1a_3b_2\right.$

. Expressing as the product of two determinants, show that $= 0.$ Hence, show that if $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = (lx + my + n)\left(l'x + m'y + n\right), then|ahghbfgfc|$

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219. If $2s = a + b + c$ and $A = \begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix}$ then det A

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220. Evaluate $\left| \wedge\ xC_1^xC_2^xC_3^yC_1^yC_2^yC_3^zC_1^zC_2^zC_3 \right|$

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221. Prove that: $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(a+b)(b+c)(c+a)$

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222. If a determinant of order 3×3 is formed by using the numbers 1 or -1 then minimum value of determinant is :

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223. If $z = | - 53 + 4i5 - 7i3 - 4i68 + 7i5 + 7i8 - 7i9|, thenz$ is purely real purely imaginary $a + ib, wherea \neq 0, b \neq 0$ d. $a + ib, whereb = 4$

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224. Let ω be the complex number $\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$. Then the number of distinct complex cos numbers z satisfying

$$\Delta = \begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0 \text{ is}$$

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225. If ω is the complex cube root of unity then

$$\begin{vmatrix} 1 & 1 + i + \omega^2 & \omega^2 \\ 1 - i & -1 & \omega^2 - 1 \\ -i & -i + \omega - 1 & -1 \end{vmatrix} =$$

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226. If

$ax12 + by12 + cz12 = ax22 + by22 + cz22 = ax32 + by32 + cz32 = d, ax23 + by_2y$

then prove that $\left|x_1y_1z_1x_2y_2z_2x_3y_3z_3\right| = (d - f)\left\{\dfrac{(d + 2f)}{abc}\right\}^{1/2}$

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227. If $A, B, andC$ are the angles of triangle, show that the system of equations

$x\sin 2A + yisnC + z\sin B = 0, \sin C + y\sin 2B + z\sin A = 0, andx\sin B + y\sin A + z\sin 2C$

posses nontrivial solution. Hence, system has infinite solutions.

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228. Let α, β, γ are the real roots of the equation $x^3 + ax^2 + bx + c = 0(a, b, c \in Randa \neq 0)$ If the system of equations $(\in u, v, andw)$ given by $\alpha u + \beta v + \gamma w = 0$ $\beta u + \gamma v + \alpha w = 0$ $\gamma u + \alpha v + \beta w = 0$ has non-trivial solutions then the value of a^2/b is ______.

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229. If $a_1, a_2, a_3, 54, a_6, a_7, a_8, a_9$ are in H.P., and $D = \left|a_1a_2a_3 54a_6a_7a_8a_9\right|$, then the value of $[D]iswhere[.]$ represents the greatest integer function

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230. If $f(x)$ is a polynomial of degree $< 3,$ prove that $|1af(a)/(x - a)1bf(b)/(x - b)1cf(c)/(x - c)| \div \left|1aa^21 \wedge 21 \wedge 2\right| = \frac{f(x)}{(x - a)(x - b)(x - c}$

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231. Prove that $= |a - aa + \ - cb + ca - \ - c0a - cxyz1 + x + y| = 0$ implies that a, b, c are in A.P. or a, c, b are in G.P.

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232. Solve for $x \in R$: $\begin{vmatrix} (x+a)(x-a) & (x+b)(x-b) & (x+c)(x-c) \\ (x-a)^3 & (x-b)^3 & (x-c)^3 \\ (x+a)^3 & (x+b)^3 & (x+c)^3 \end{vmatrix}$ =0, a,b and c being distinct real numbers.

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233. Absolute value of sum of roots of the equation $|x + 22x + 33x + 42x + 33x + 44x + 53x + 55c + 810x + 17| = 0$ is ______.

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234. Let α_1, α_2 and β_1, β_2 be the roots of the equation $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y_{\beta} \_ 2z = 0$ has a non trivial solution then prove that $\frac{b^2}{q^2} = \frac{ac}{pr}$

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235. The product of all values of t, for which the system of equations $(a - t)x + by + cz = 0, bx + (c - t)y + az = 0, cx + ay + (b - t)z = 0$ has non-trivial solution, is $|a - c - b - cb - a - b - ac|$ (b) $|abcbcacab|$ $|aca$**a**$ba|$ (d) $|aa + \ + c + \ + a + aa + b|$

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236. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is (A) 3ω (B) $3\omega(\omega - 1)$ (C) $3\omega^2$ (D) $3\omega(1 - \omega)$

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237. If $= | - xa - xaab - x|$, then a factor is $a + b + x$ $x^2 - (a - b) + x + a^2 + b^2 + ab\ x^2 - (a + b) + x + a^2 + b^2 - ab\ a + b - x$

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238. If $g(x) = \left|a^{-x}e^{x\log_e a}x^2 a^{-3x}e^{3x\log_e a}x^4 a^{-5x}e^{5x\log_e a}1\right|$, then graphs of $g(x)$ is symmetrical about the origin graph of $g(x)$ is symmetrical about the y-axis $\left(\frac{d^4g(x)}{dx^4}\ |\ \right)_{x=0} = 0\ f(x) = g(x) \times \log\left(\frac{a - x}{a + x}\right)$ is an odd function

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239. the sum of values of p for which the equations x+y+z=1x+2y +4z=p and x+4y +10z =p^2 have a solution is ____

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240. The value of $|\alpha|$ for which the system of equation

$\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1$

$x + y + \alpha z = \alpha - 1$

has no solution , is _____

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241. Let $D_1 = |aba + bcdc + daba - b| and D_2 = |aca + cbdb + daca + b + c|$

then the value of $\left|\frac{D_1}{D_2}\right|$, $where b \neq 0 and ad \neq bc,$ is _____.

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242. Three distinct points $P\left(3u^2, 2u^3\right); Q\left(3v^2, 2v^3\right)$ and $R\left(3w^2, 2w^3\right)$ are collinear then

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243. If $|\times\ + yx + y + z2x3x + 2y4x + 3y + 2z3x6x + 3y10x + 6y + 3z| = 64,$ then the real value of x is ________.

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244. If $= \left|13\cos\theta 1s\int h\eta 13\cos\theta 1s\int h\eta 1\right|$, then the value of $(\ \_\ (max))/2$

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245. If

$$\left|x^n x^{n+2} x^{n+4} y^n y^{n+2} y^{n+4} z^n z^{n+2} z^{n+4}\right| = \left(\frac{1}{y^2} - \frac{1}{x^2}\right)\left(\frac{1}{z^2} - \frac{1}{y^2}\right)\left(\frac{1}{x^2} - \frac{1}{z^2}\right)$$

then n is ________.

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246. The value of

$\left|2x_1y_1x_1y_2 + x_2y_1x_1y_3 + x_3y_1x_1y_2 + x_2y_12x_2y_2x_2y_3 + x_3y_2x_1y_3 + x_3y_1x_2y_3 + x_3y_2\right.$

is.

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247. Given $A = |ab2cde2flm2n|, B = |f2de2n4l2mc2ab|$, then the value of B/A is __________.

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Question Bank

1. Number of real roots of the equation

$[[1, x, x], [x, 1, x], [x, x, 1]| + |[1 - x, 1, 1], [1, 1 - x, 1], [1, 1, 1 - x]\ | \ = 0$ is

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2. If $\begin{vmatrix} \sqrt{3}x - x^2 & x - 1 & x - \sqrt{3} \\ \sqrt{3}x & x & \sqrt{3} - 1 \\ x - 1 & \sqrt{3} - x & x - 1 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$ and $a + c = p + \sqrt{q}$, then $(p - q)$ is equal to

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3. If

$f(x) = \ | \cos(x + \alpha), \cos(x + \beta), \cos(x + \gamma)], [\sin(x + \alpha), \sin(x + \beta), \sin(x + \gamma)], [\sin($

and

$f(0) = -2$ then $\sum_{r=1}^{30} |f(r)|$equals

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4. If the equations $px + 2y - 3 = 0, x + 3y - 4 = 0$ and $px^2 + 3y^2 + 3xy + (q - 3)x - 3y - 1 = 0 (p \neq 23)$ has a unique solution then find the value of $(p + q)$.

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5. Let $f(x) = \begin{vmatrix} 1 + \sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1 + \cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1 + 4\sin 2x \end{vmatrix}$, the maximum value of $f(x)$ is

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6. Given $2x - y + 2z = 1, x - 2y + z = -4,$ and $x + y + \lambda z = 4$. Then the. value of λ such that the given system of equation has no solution is

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7. If M and m are maximum and minimum value, respectively of $\begin{vmatrix} 1 & \cos\theta & 1 \\ \cos\theta & 1 & \cos\theta \\ -1 & \cos\theta & 1 \end{vmatrix}$, then value of $(M + m)$ is

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8. The value of $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$ is

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9. If $x \in \left[0, \frac{\pi}{2}\right]$ such that $\left|(\sin x + \cos x) - \sqrt{2}\right| + \left|y + \frac{1}{y} - 2\right| + |z| \leq 0$, then. the valuc of

|[sin 4 x , cos ^2 x , 1],[y+2 , y^3 , (1)/(y)],[ln (z+1) , z+5 , z^(3)+2]|` is

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10. If α, β, γ are roots of the equation $x^3 + x^2 + 2x - 4 = 0$, then value of

$\begin{vmatrix} 1+\alpha & 1 & 1 \\ 1 & 4-\beta^3 & 1 \\ 1 & 1 & 4-\gamma^3 \end{vmatrix}$ (where α is real)

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11. If p and q are real so that sysiem of equations $px + 4y + z = 0, 2y + 3z = 1$ and $3x - qz = -2$ has infinite solution, then $\sqrt{q^2 - p^2}$ is equal to

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12. If |[1 , x, x^2],[x , x^2 , 1],[x^2 , 1 , x]|=3, *thenthevalueof*|[x^(3)-1, 0 , x-x^4],[0, x-x^(4) , x^(3)-1],[x-x^(4) , x^(3)-1 , 0]|`

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13. Let $f(x) = \begin{vmatrix} x & 1 & \frac{-3}{2} \\ 2 & 2 & 1 \\ \frac{1}{x-1} & 0 & \frac{1}{2} \end{vmatrix}$. The miniumu value of f(x) (given $x > 1$) is

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14. In the determinant, $\Delta = \begin{vmatrix} -2 & 5 & -1 \\ 4 & 7 & 0 \\ 5 & -3 & 1 \end{vmatrix}$, the absolutevalue of sum of the minors of elements of third row is

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15. If |[x^2, x^4 , (x-1)^2],[0, 4, 6 x^2],[4 ,1,7] |= overset(6) underset(I = 0) sum a_i x, then |a_0| + |a_6|` is equal to

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16. If a, b, c are the sides of a scalene triangle such that |[a , a^2 , a^(3)-1], [b , b^2 , b^(3)-1],[c, c^2 , c^(3)-1]|=0, $thenthe \geq ometricmeanof$a, b c` is

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17. Number of real roots of the equation $\begin{vmatrix} \sin x & x \\ 1 & 1 \end{vmatrix} \begin{vmatrix} \cos x & \\ x & 1 \end{vmatrix} = 0$

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18. Let $a+b+c=s$ and |[s+c, a, b],[c ,s+a, b],[c, a , s+b]| $iscqual \to 54thenthevalueofs$` is

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19. In triangle ABC, if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$ then the value of $\sin^2 A + \cos^2 B + \tan^2 C$ is equal to

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20. If system of linear equations $(a-1)x + z = \alpha_1 x + (b-1)y = \beta$ and $y + (c-1)z = \gamma$ where $a, b, c \in I$ does not have a unique solution, then maximum possible value of $|a + b \div c|$ is

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21. If $a_1, a_2, a_3, 5, 4, a_6, a_7, a_8, a_9$ are in $H.P,$ then the value of the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ can be expressed in the lowest form as $\underline{P}$, find $(p+q)$

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22. If $0 \leq [x] < 2,\ -1 \leq [y] < 1$ and $1 \leq [z] < 3$ where $[.]$ denotes greatest integral function then the maximum walue of the determinant.

$D = \begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is

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23. If $f(x) = \tan x$ and A, B, C are the angles of $\triangle ABC,$ then

$$\begin{vmatrix} f(A) & f\left(\frac{\pi}{4}\right) & f\left(\frac{\pi}{4}\right) \\ f\left(\frac{\pi}{4}\right) & f(B) & f\left(\frac{\pi}{4}\right) \\ f\left(\frac{\pi}{4}\right) & f\left(\frac{\pi}{4}\right) & f(C) \end{vmatrix} =$$

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