MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

DIFFERENTIAL EQUATIONS

Examples

1. Find the order and degree of the following differential equations. i) $\frac{dy}{dx}+y=\frac{1}{\frac{dy}{dx}}$, ii) $\frac{e^{e^3 y}}{dx^3}-x\frac{d^2y}{dx^2}+y=0$, iii) $\frac{\sin^{-1}(dy)}{dx}=x+y$, iv) $\frac{\log_e(dy)}{dx}=ax+by$

v) $y\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-4y\frac{dy}{dx}=0$

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2. Form the differential equation of family of lines concurrent at the origin.

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3. Form the differential equation of all circle touching the x-axis at the origin and centre on the y-axis.

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4. Form the differential equation of family of lines situated at a constant distance p from the origin.

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5. From the differential equation of the family of parabolas with focus at the origin and axis of symmetry along the x-axis. Find the order and degree of the differential equation.

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6. The differential equation of all parabolas whose axis are parallel to the y-axis is (a) $(b)(c)(d)\frac{(e)(f)d^{(g)3(h)}(i)y}{j}\left((k)d(l)x^{(m)3(n)}(o)\right)(p)(q)=0(r)$ (s) (b) $(t)(u)(v)\frac{(w)(x)d^{(y)2(z)}(aa)x}{bb}\left((cc)d(dd)y^{(ee)2(ff)}(gg)\right)(hh)(ii)=C(jj$ (kk) (c) *[Math Processing Error]* (ii) (d) *[Math Processing Error]* (ggg)

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7. From the differential equation of the family curves having equation $y=\left(\sin^{-1}x\right)^2+A\cos^{-1}x+B.$

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8. What is the order of the differential equation whose general solution is

$y = c_1 \cos 2x + c_2 \sin^2 x + c_3 \cos^2 x + c_4 e^{2x} + c_5 e^{2x + c6}$?

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9. Find the particular solution of the differential equation

$(1 + e^{2x})dy + (1 + y^2)e^x dx = 0,$ given that $y = 1$ when $x = 0.$

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10. Solve $\frac{\log(dy)}{dx} = 4x - 2y - 2$, given that $y = 1$ when $x = 1.$

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11. Solve the differential equation $xy\frac{dx}{ydx} = \frac{1 + y^2}{1 + x^2}(1 + x + x^2)$

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12. Solve the following differential equation: $\sqrt{1+x^2+y^2+x^2 y^2}+xy\frac{dy}{dx}=0$

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13. Solve $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$

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14. Solve $\frac{dy}{dx}=(x+y)^2$

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15. Solve $\frac{dy}{dx}\sqrt{1+x+y}=x+y-1$

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16. Show that the differential equation $(x^2 + xy)dy = (x^2 + y^2)dx$ is homogenous and solve it.

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17. Show that the given differential equation $xdy - ydx = \sqrt{x^2 + y^2}$ dx is homogenous and solve it.

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18. Solve $\left(x\frac{\sin y}{x}\right)dy = \left(y\frac{\sin y}{x} - x\right)dx.$

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19. Solve $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$

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20. Solve $xdy = \left(y + x\frac{f\left(\frac{y}{x}\right)}{f'\left(\frac{y}{x}\right)}\right)dx$

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21. Find the real value of m for which the substitution $y = u^m$ will transform the differential equation $2x^4 y\frac{dy}{dx} + y^4 = 4x^6$ in to a homogeneous equation.

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22. Solve $\frac{dy}{dx} = \frac{x + 2y + 3}{2x + 3y + 4}$

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23. Solve:

$xdy + ydx = \frac{xdy - ydx}{x^2 + y^2}$

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24. Solve: $\left[(x+1)\frac{y}{x}+\sin y\right]dx+[x+\log_e x+x\cos y]dy=0$

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25. Solve: $(x\cos x-\sin x)dx=\frac{x}{y}\sin x\,dy$

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26. Solve: $y^4dx+2xy^3dy=\frac{ydx-xdy}{x^3y^3}$

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27. Solve:

$$\frac{dy}{dx}=\frac{yf'(x)-y^2}{f(x)}$$

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28. Solve the differential equation $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y^2\right)dy (y \neq 0)$

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29. Solve $\frac{dy}{dx} = \frac{x - 2y + 5}{2x + y - 1}$

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30. Solve $\frac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \frac{y^4}{x^2}$

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31. Solve:

$$1\left(\frac{1}{y}\frac{\sin x}{y} - \frac{y}{x^2}\frac{\cos y}{x} + 1\right)dx + \left(\frac{1}{x}\frac{\cos y}{x} - \frac{x}{y^2}\frac{\sin y}{x} + \frac{1}{y^2}\right)dy = 0$$

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32. Solve $x^2\left(\frac{dy}{dx}\right)+y=1$

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33. Solve:

$ydx - xdy + \log x dx = 0$

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34. Solve $(1+y+x^2y)dx+(x+x^3)dy=0$

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35. Solve : $(x+2y^3)\frac{dy}{dx}=y$

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36. Solve the differential equation: (i) $\left(1+y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$

(ii) $x\frac{dy}{dx}+\cos^2 y=\tan y\frac{dy}{dx}$

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37. Let $u(x)$ and $v(x)$ satisfy the differential equation $\frac{du}{dx}+p(x)u=f(x)$ and $\frac{dv}{dx}+p(x)v=g(x)$ are continuous functions. If $u(x_1)$ for some x_1 and $f(x)>g(x)$ for all $x>x_1,$ prove that any point $(x,y),$ where $x>x_1,$ does not satisfy the equations $y=u(x)$ and $y=v(x)$.

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38. Solve $\left(\frac{dy}{dx}\right)+\left(\frac{y}{x}\right)=y^3$

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39. Solve $\left(\frac{dy}{dx}\right)=e^{x-y}(e^{x}-e^{y}).$

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40. Solve $(x-1)dy+ydx=x(x-1)y^{\frac{1}{3}}dx.$

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41. The solution of the differential equation, $e^{x}(x+1)dx+(ye^{y}-xe^{x})dy=0$ with initial condition f(0) = 0, is

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42. The slope of a curve, passing through (3,4) at any point is the reciprocal of twice the ordinate of that point. Show that it is parabola.

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43. Find the equation of a curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

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44. Find the equation of the curve passing through (2,1) which has constant sub-tangent.

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45. If length of tangent at any point on th curve $y = f(x)$ intercepted between the point and the x-axis is of length 1. Find the equation of the curve.

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46. Find the equation of a curve passing through the point (1.1) if the perpendicular distance of the origin from the normal at any point P(x, y) of the curve is equal to the distance of P from the x-axis.

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47. Find the equation of the curve such that the portion of the x-axis cut off between the origin and the tangent art a point is twice the abscissa and which passes through the point (1,2).

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48. Find the equation of the curve passing through the point, (5,4) if the sum of reciprocal of the intercepts of the normal drawn at any point P(x,y) on it is 1.

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49. Find the equation of the curve passing through the origin if the middle point of the segment of its normal from any point of the curve to the x-axis lies on the parabola $2y^2 = x$

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50. Find the equation of family of curves which intersect the family of curves xy=c at an angle 45°.

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51. Find the orthogonal trajectory of $y^2 = 4ax$ (a being the parameter).

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52. The population of a certain is known to increase at a rate proportional to the number of people presently living in the country. If after two years the population has doubled, and after three years the population is 20,000 estimates the number of people initially living in the country.

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53. What constant interest rate is required if an initial deposit placed into an account accrues interest compounded continuously is to double its value in six years? $(\ln|x| = 0.6930)$

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54. Suppose that a mothball loses volume by evaporation at a rate proportional to its instantaneous area. If the diameter of the ball decreases from 2cm to 1cm in 3 months, how long will it take until the ball has practically gone?

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55. A body at a temperature of $50^0 F$ is placed outdoors where the temperature is $100^0 F$. If the rate of change of the temperature of a body is proportional to the temperature difference between the body and its surrounding medium. If after 5 min the temperature of the body is $60^0 F$, find (a) how long it will take the body to reach a temperature of 75 $\hat{}$ 0 F and (b) the temperature of the body after 20 min.

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56. Find the time required for a cylindrical tank of radius r and height H to empty through a round hole of area a at the bottom. The flow through the hole is according to the law $v(t) = k\sqrt{2gh(t)}$, where $v(t)$ and $h(t)$, are respectively, the velocity of flow through the hole and the height of the water level above the hole at time $t,$ and g is the acceleration due to gravity.

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57. A country has a food deficit of 10%. Its population grows continuously at a rate of 3% per year. Its annual food production every year is 4% more than that of the last year. Assuming that the average food requirement per person remains constant, prove that the country will become self-sufficient in food after n years, where n is the smallest integer bigger than or equal to $\frac{\ln 10 - \ln 9}{\ln(1.04) - 0.03}$

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58. A hemi-spherical tank of radius 2 m is initially full of water and has an outlet of $12cm^2$ cross-sectional area at the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law $v(t) = \sqrt{0.62gh(t)}$, where $v(t)$ and $h(t)$ are, respectively, the velocity of the flow through the outlet and the height of water level above the outlet and the height of water level above the outlet at time t, and g is the acceleration due to gravity. Find the time it takes to empty the tank.

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59. Solve $\frac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \frac{y^4}{x^2}$

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60. Solve :

$$\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}ydx = \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}xdy$$

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61. Solve $\frac{dy}{dx} = \frac{(x+y)^2}{(x+2)(y-2)}$

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62. Solve $y\left(\frac{dy}{dx}\right)^2$

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63. If $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$, find $y(x)$.

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64. If $\int_a^x ty(t)dt = x^2 + y(x)$, then find $y(x)$

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65. Given a function ' g' which has a derivative $g'(x)$ for every real x and satisfies $g'(0) = 2$ and $g(x+y) = e^y g(x) + e^y g(y)$ for all x and y then:

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66. Solve $\frac{dy}{dx} = \frac{s \in}{\sin 2y - x \cos y}$

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67. Solve:

$$\frac{dy}{dx}+\frac{3y}{x}=g(x)$$, where g(x) = $\begin{cases} 1 & \text{if } 0 \le x \le 1 \\ \frac{1}{x} & \text{if } x > 1 \end{cases}$

$y\left(\frac{1}{2}\right)=\frac{1}{8}$ and y(x) is continous on $[0, \infty]$.

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68. Solve: $(x\cos y - y\sin y)dy + (x\sin y + y\cos y)dx = 0$

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69. If y_1 and y_2 are the solution of the differential equation $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x alone and $y_2 = y_1 z$, then prove that $z = 1 + \cdot e^{-\int \frac{Q}{y_1} dx}$, where c is an arbitrary constant.

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70. If y_1 and y_2 are two solutions to the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$. Then prove that $y = y_1 + c(y_1 - y_2)$ is the general solution to the equation where c is any constant.

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71. Let $f(x), x \geq 0,$ be a non-negative continuous function, and let $f(x) = \int_0^x f(t)dt, x \geq 0,$ if for some $c > 0, f(x) \leq cF(x)$ for all $x \geq 0,$ then show that $f(x) = 0$ for all $x \geq 0.$

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72. Find a pair of curves such that (a) the tangents drawn at points with equal abscissas intersect on the y-axis. (b) the normal drawn at points with equal abscissas intersect on the x-axis. (c) one curve passes through (1,1) and other passes through (2, 3).

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73. Given two curves: $y = f(x)$ passing through the point $(0, 1)$ and $g(x) = \int_{-\infty}^{x} f(t)dt$ passing through the point $\left(0, \frac{1}{n}\right)$. The tangents drawn to both the curves at the points with equal abscissas intersect on the x-axis. Find the curve $y = f(x)$.

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74. A cyclist moving on a level road at 4 m/s stops pedalling and lets the wheels come to rest. The retardation of the cycle has two components: a constant 0.08 m/s^2 due to friction in the working parts and a resistance of $0.02v^2/m$, where v is speed in meters per second. What distance is traversed by the cycle before it comes to rest? (consider 1n 5=1.61).

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75. The force of resistance encountered by water on a motor boat of mass m going in still water with velocity v is proportional to the velocity v. At $t = 0$ when its velocity is v_0, then engine shuts off. Find an expression for the position of motor boat at time t and also the distance travelled by the boat before it comes to rest. Take the proportionality constant as $k > 0$.

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76. A and B are two separate reservoirs of water. Capacity of reservoir A is double the capacity of reservoir B. Both the reservoirs are filled completely with water, their inlets are closed and then the water is released simultaneously from both the reservoirs. The rate of flow of water out of each reservoir at any instant of time is proportional to the quantity of water in the reservoir at the time. One hour after the water is released, the quantity of water is reservoir A is $1\frac{1}{2}$ times the quantity of water in reservoir B. After how many hours do both the reservoirs have the same quantity of water?

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Exercise 10.1

1. Find the order and degree (if defined) of the following differential equations:

$$\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^4\right\}^{5/3}$$

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2. Find the order and degree (if defined) of the equation:

$$\frac{d^3y}{dx^3} = x \ln\left(\frac{dy}{dx}\right)$$

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3. $\left(\frac{d^4y}{dx^4}\right)^3 + 3\left(\frac{d^2y}{dx^2}\right)^6 + \sin x = 2\cos x$

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4. Find the order and degree (if defined) of the equation:

$$\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} + 4 - 3\frac{d^2}{dx^2} + 5\frac{dy}{dx} = 0$$

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5. Find the order and degree (if defined) of the equation:

$$a = \frac{1\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}},$$ where a is constant

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6. Find the order and degree (if defined) of the equation:

$$\frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3}\right) = 0$$

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Exercise 10.2

1. Find the differential equation of All-horizontal lines in a plane. All non-vertical lines in a plane.

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2. Form the differential equation of family of circles having center at origin.

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3. Find the differential equation of all parabolas whose axes are parallel to the x-axis an having latus rectum a.

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4. Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitrary constants.

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5. The degree of the differential equation satisfying the relation $\sqrt{1+x^2}+\sqrt{1+y^2}=\lambda\left(x\sqrt{1+y^2}-y\sqrt{1+x^2}\right)$ is

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6. Find the differential equation whose general solution is given by $y=(c_1+c_2)\cos(x+c_3)-c_4e^{x+c}$, where c_1, c_2, c_3, c_4, c_5 are arbitary constants.

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Exercise 10.3

1. Solve $e^{\frac{dy}{dx}}=x+1,$ given that when $x=0, y=3.$

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2. Solve $(x - y^2 x)\,dx = (y - x^2 y)\,dy.$

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3. Solve $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

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4. Solve the following differential equation:

$e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$

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5. Solve the following differential equations: $\frac{dy}{dx} = 1 + x + y + xy$ (ii)

$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$

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6. Solve $(x-y)(dx+dy)=dx-dy$, given that $y=-1$, where $x=0$.

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7. Solve $\frac{dy}{dx}=yf'(x)=f(x)f'(x)$, where $f(x)$ is a given integrable function of x.

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8. Solve $\frac{dy}{dx}=\cos(x+y)-\sin(x+y)$.

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9. If a function 'f' satisfies the relation $f(x)f''(x)-f(x)f'(x)-f'(x)^2=0 \forall x \in R$ and $f(0)=1=f'(0)$. Then find $f(x)$.

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Exercise 10.4

1. Solve the following differential equation: $x\frac{dy}{dx} - y = 2\sqrt{y^2 - x^2}$

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2. Solve $\left[2\sqrt{xy} - x\right]dy + ydx = 0$

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3. $x\frac{dy}{dx} = y(\log y - \log x + 1)$

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4. Solve $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0; y = \frac{\pi}{4}$ when $x = 1$.

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5. Show that the differential equation $y^3 dy - (x + y^2) dx = 0$ can be reduced to a homogenous equation.

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6. Solve $\frac{dy}{dx} = \frac{2x - y + 1}{x + 2y - 3}$

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Exercise 10.5

1. The solution of $ydx - xdy + 3x^2 y^2 e^{x^3}$ dx = 0 is

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2. $\frac{dy}{dx} = \frac{2xy}{x^2 - 1 - 2y}$

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3. The solution of the differential equation $ydx + (x + x^2y)dy = 0$ is

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4. The solution of the differential equation $(xy^4 + y)dx - xdy = 0,$ is

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5. $y(x^2y + e^x)dx - e^x dy = 0$

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6. $\frac{dy}{dx} = -\frac{y + \sin x}{x}$ satisfying condition $y(0) = 1$

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7. $y(xy+1)dx + x\left(1+xy+x^2y^2\right)dy = 0$

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Exercise 10.6

1. What is the integrating factor of the differential equation $\left(1-y^2\right)\frac{dx}{dy} + yx = ay\,(-1 < y < 1)$?

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2. Solve $\frac{dy}{dx} + y\cot x = \sin x$

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3. Solve `(x+y+1)(dy//dx)=1

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4. Solve the equation $(1-x^2)\left(\frac{dy}{dx}\right)+2xy=x\sqrt{1-x^2}$

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5. Solve the equation $\frac{dy}{dx}=\frac{y}{2y1ny+y-x}$

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6. Solve the equation $ydx+(x-y^2)dy=0$

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7. Find the equation of a curve passing through $(0, 1)$ and having gradient $\frac{1(y+y^3)}{1+x+xy}at(x, y)$

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Exercise 10.7

1. Solve the equation $\frac{dy}{dx} = \frac{1}{x} = \frac{e^y}{x^2}$

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2. $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$

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3. $\frac{dy}{dx} + \frac{xy}{1-x^2} = x\sqrt{y}$

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4. $\frac{dy}{dx} = \left(x^3 - 2x\tan^{-1}y\right)\left(1+y^2\right)$

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5. $\frac{dy}{dx} - \frac{\tan x}{1+x} e^x \sec y$

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Exercise 10.8

1. Find the equation of the curve in which the subnormal varies as the square of the ordinate.

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2. Find the curve for which the length of normal is equal to the radius vector.

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3. Find the curve for which the perpendicular from the foot of the ordinate to the tangent is of constant length.

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4. A curve $y=f(x)$ passes through the origin. Through any point (x, y) on the curve, lines are drawn parallel to the co-ordinate axes. If the curve divides the area formed by these lines and co-ordinates axes in the ratio $m:n,$ find the curve.

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5. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis at $Q\cdot$ If PQ has constant length $k,$ then show that the differential equation describing such curves is $y\frac{dy}{dx}=\pm\sqrt{k^2-y^2}$. Find the equation of such a curve passing through $(0, k)\cdot$

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6. Find the orthogonal trajectories of family of curves $x^2 + y^2 = cx$

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7. A curve C has the property that if the tangent drawn at any point P on C meets the co-ordinate axis at A and B , then P is the mid-point of AB. The curve passes through the point (1,1). Determine the equation of the curve.

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Exercise 10.9

1. A person places Rs 500 in an account that interest compounded continuously. Assuming no additional deposits or withdrawals, how much will be in the account after seven years if the interest rate isa constant 8.5 percent for the first four year and a constant 9.25 percent for the last three years

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2. Find the time required for a cylindrical tank of radius 2.5 m and height 3 m to empty through a round hole of 2.5 cm with a velocity $2.5\sqrt{h}$ m/s, h being the depth of the water in the tank.

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3. If the population of country double in 50 years, in how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants.

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4. The rate at which a substance cools in moving air is proportional to the difference between the temperatures of the substance and that of the air. If the temperature of the air is 290 K and the substance cools from 370 K to 330 K in 10 min, when will the temperature be 295 K?

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5. A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant k is positive). Suppose that r(t) is the radius of the liquid cone at time t. The time after which the cone is empty is

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Exercise (Single)

1. The degree of the differential equation satisfying

$\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, is

A. 1

B. 2

C. 3

D. None of these

Answer: A

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2. Number of values of $m \in N$ for which $y = e^{mx}$ is a solution of the differential equation $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 12y = 0$ (a) 0 (b) 1 (c) 2 (d) More than 2

A. 0

B. 1

C. 2

D. More than 2

Answer: C

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3. The order of differential equation of family of circles passing through intersection of $3x+4y-7=0$ and $S=-x^2+y^2-2x+1=0$ is

A. 1

B. 2

C. 3

D. 4

Answer: A

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4. The differential equation of all non-horizontal lines in a plane is (a) $(b)(c)(d)\frac{(e)(f)d^{(g)2(h)}(i)y}{j}\left((k)d(l)x^{(m)2(n)}(o)\right)(p)(q)(r)$ (s) (b) $(t)(u)(v)\frac{(w)(x)d^{(y)2(z)}(aa)x}{bb}\left((cc)d(dd)y^{(ee)2(ff)}(gg)\right)(hh)(ii)=0(jj$ (kk) (c) $(d)(e)(f)\frac{(g)dy}{h}((i)dx)(j)(k)=0(l)$ (m) (d) $(n)(o)(p)\frac{(q)dx}{r}((s)dy)(t)(u)=0(v)$ (w)

A. $\frac{d^2y}{dx^2}$

B. $\frac{d^2x}{dy^2}=0$

C. $\frac{dy}{dx}=0$

D. $\frac{dx}{dy}=0$

Answer: B

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5. The differential equation of the family of curves $y=e^x(A\cos x+B\sin x)$, where A and B are arbitrary constants is (a) $(b)(c)(d)\frac{(e)(f)d^{(g)2(h)}(i)y}{j}\left((k)d(l)x^{(m)2(n)}(o)\right)(p)(q)-2(r)\frac{(s)dy}{t}((u$ (y) (z) *[Math Processing Error]* (xx) (yy) *[Math Processing Error]* (eeee) (ffff) *[Math Processing Error]* (ddddd)

A. $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

B. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-2y=0$

C. $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+y=0$

D. $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 2y = 0$

Answer: A

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6. Differential equation of the family of circles touching the line $y = 2$ at $(0, 2)$ is (a)

$(b)(c)(d)x^{(e)2(f)}(g) + (h)(i)((j)(k)y - 2(l))^{(m)2(n)}(o) + (p)\frac{(q)dy}{r}((s)$

(z) (aa) *[Math Processing Error]* (uu) (vv)

$(ww)(\times)(yy)x^{(zz)2(aaa)}(bbb) + (ccc)(ddd)((eee)(fff)y - 2(ggg))^{(hhh)2}$

$- 2(vvv)\Big) = 0(www)$

(xxx) (yyy) None of these

A. $x^2 + (y-2)^2 + \frac{dy}{dx}(y-2) = 0$

B. $x^2 + (y-2)\left(2 - 2x\frac{dx}{dy} - y\right) = 0$

C. $x^2 + (y-2)^2 + \left(\frac{dx}{dy} + y - 2\right)(y-2) = 0$

D. None of these

Answer: D

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7. The order and degree of the differential equation of all tangent lines to the parabola $y = x^2$ is (a) 1,2 (b) 2,3 (c) 2,1 (d) 1,1

A. 1,2

B. 2,3

C. 2,1

D. 1,1

Answer: A

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8. The differential equation for the family of curve $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant, is (a)

$(b)(c)2\left((d)(e)(f)x^{(g)2(h)}(i)-(j)y^{(k)2(l)}(m)(n)\right)(o)y^{(p)'(q)}(r)=xy($

(t) (u)

$(v)(w)2\left((x)(y)(z)x^{(aa)2(bb)}(cc)+(dd)y^{(ee)2(ff)}(gg)(hh)\right)(ii)y^{(jj)'(kk)}$

(nn) (oo) [Math Processing Error] (hhh) (iii)

$(jjj)(kkk)\left((lll)(mmm)\left(\bigcap\right)x^{(ooo)2(ppp)}(qqq)+(rrr)y^{(sss)2(ttt)}(uuu)\right($

(bbbb)

A. $2(x^2-y^2)y'=xy$

B. $2(x^2+y^2)y'=xy$

C. $(x^2-y^2)y'=2xy$

D. $(x^2+y^2)y'=2xy$

Answer: C

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9. The differential equation whose general solution is given by $y=\left(c_1\cos(x+c_2)-\left(c_3e^{(-x+c4)}+(c_5\sin x),\right.\right.$ where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is (a)

$$(b)(c)(d)\frac{(e)(f)d^{(g)4(h)}(i)y}{j}\Big((k)d(l)x^{(m)4(n)}(o)\Big)(p)(q)-(r)\frac{(s)(t)d^{(u)}}{x}$$

(gg) (hh)

$$(ii)(jj)(kk)\frac{(ll)(mm)d^{(nn)3(oo)}(pp)y}{qq}\Big((rr)d(ss)x^{(tt)3(uu)}(vv)\Big)(ww)(\times$$

$$+(mmm)\frac{(nnn)dy}{ooo}((ppp)dx)(qqq)(rrr)+y=0(sss)$$

(ttt) (uuu)

$$(vvv)(www)(\times x)\frac{(yyy)(zzz)d^{(aaaa)5(bbbb)}(cccc)}{dddd}\Big((eeee)d(ffff)x^{(gggg)5(}$$

(mmmm) (nnnn)

$$(oooo)(pppp)(qqqq)\frac{(rrrr)(ssss)d^{(tttt)3(uuuu)}(vvvv)y}{wwww}\Big((xxxx)d(yyyy)x^{(zzz}$$

$$-(eeeee)\frac{(fffff)(ggggg)d^{(hhhhh)2(iiiii)}(jjjjj)y}{kkkkk}\Big((lllll)d(mmmmm)x^{(nn}$$

$$=0(yyyyy)$$

(zzzzz)

A. $\frac{d^4y}{dx^4}-\frac{d^2y}{dx^2}+y=0$

B. $\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0$

C. $\frac{d^5y}{dx^5}+y=0$

D. $\frac{d^3y}{dx^3}-\frac{d^2y}{dx^2}+\frac{dy}{dx}-y=0$

Answer: B

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10. If $y = \frac{x}{In|cx|}$ (where c is an arbitrary constant) is the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$ then function $\phi\left(\frac{x}{y}\right)$ is:

A. x^2 / y^2

B. $-x^2 / y^2$

C. y^2 / x^2

D. $-y^2 / x^2$

Answer: D

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11. The differential equation of the curve $\frac{x}{c-1} + \frac{y}{c+1} = 1$ is (a) *[Math Processing Error]* (cc) (dd)

$(ee)(ff)\left((gg)(hh)(ii)\frac{(jj)dy}{kk}((ll)dx)(mm)(\cap) + 1(oo)\right)\left((pp)(qq)y - x\right.$

(fff) (ggg)

$(hhh)(iii)\left((jjj)(kkk)(lll)\frac{(mmm)dy}{nnn}((ooo)dx)(ppp)(qqq) + 1(rrr)\right)\left((ss\right.$

$= 2(bbbb)\frac{(cccc)dy}{dddd}((eeee)dx)(ffff)(gggg)(hhhh)$

(iiii)

A. $\frac{dy}{dx} - 1\left(y + x\frac{dy}{dx}\right) = 2\frac{dy}{dx}$

B. $\left(\frac{dy}{dx} + 1\right)\left(y - x\frac{dy}{dx}\right) = \frac{dy}{dx}$

C. $\left(\frac{dy}{dx} + 1\right)\left(y - x\frac{dy}{dx}\right) = 2\frac{dy}{dx}$

D. y^2 / x^2

Answer: C

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12. If $y = y(x)$ and $\left(\frac{2 + \sin x}{y + 1}\right)\frac{dy}{dx} = -\cos x,\ y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ equals

A. $1/3$

B. $2/3$

C. $-1/3$

D. 1

Answer: A

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13. The equation of the curves through the point (1, 0) and whose slope is $\frac{y-1}{x^2+x}$ is (a) $(b)(c)((d)(e)y-1(f))((g)(h)x+1(i))+2x=0(j)$ (k) (l) $(m)(n)2x((o)(p)y-1(q))+x+1=0(r)$ (s) (t) $(u)(v)x((w)(x)y-1(y))((z)(aa)x+1(bb))+2=0(cc)$ (dd) (ee)None of these

A. $(y-1)(x+1)+2x=0$

B. $2x(y-1)+x+1=0$

C. $x(y-1)(x+1)+2=0$

D. None of these

Answer: A

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14. Solve the differential equation $\frac{dy}{dx} = \frac{x(2\log x + 1)}{(\sin y + y\cos y)}$.

A. $y\sin y = x^2\log x + \frac{x^2}{2} + c$

B. $y\cos y = x^2(\log x + 1) + c$

C. $y\cos y = x^2\log x + \frac{x^2}{2} + c$

D. $y\sin y = x^2\log x + c$

Answer: D

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15. The solution of the equation $(x^2y + x^2)dx + y^2(x-1)dy = 0$ is given by (a)

$(b)(c)(d)x^{(e)2(f)}(g) + (h)y^{(i)2(j)}(k) + 2((l)(m)x - y(n)) + 21n(o)\frac{(p)(}{}$

(aa) (bb)

$(cc)(dd)(ee)x^{(ff)2(gg)}(hh) + (ii)y^{(jj)2(kk)}(ll) + 2((mm)(\cap)x - y(\infty)$

(bbb) (ccc)

$(ddd)(eee)(fff)x^{(ggg)2(hhh)}(iii) + (jjj)y^{(kkk)2(lll)}(mmm) + 2((nnn)($

$= 0(bbbb)$

(cccc) (dddd) None of these

A. $x^2 + y^2 + 2(x - y) + 2\ln\frac{(x-1)(y+1)}{c} = 0$

B. $x^2 + y^2 + 2(x - y) + \ln\frac{(x-1)(y+1)}{c} = 0$

C. $x^2 + y^2 + 2(x - y) - 2\ln\frac{(x-1)(y+1)}{c} = 0$

D. None of these

Answer: A

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16. Solve the following differential equations

$$\frac{dy}{dx} = \sin x \cdot \sin y$$

A. $e^{\cos x}\frac{\tan y}{2}=c$

B. $e^{\cos x}\tan y=c$

C. $\cos x\tan y=c$

D. $\cos x\sin y=c$

Answer: A

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17. The solution of $\frac{dv}{dt}+\frac{k}{m}v=-g$ is (a) $(b)(c)v=c(d)e^{(e)(f)(g)\frac{k}{h}g(i)(j)t(k)}(l)-(m)\frac{(n)mg}{o}k(p)(q)(r)$ (s) (b) *[Math Processing Error]* (kk) (c) $(d)(e)v(f)e^{(g)(h)(i)\frac{k}{j}m(k)(l)t(m)}(n)=c-(o)\frac{(p)mg}{q}k(r)(s)(t)$ (u) (d) *[Math Processing Error]* (mm)

A. $v=ce^{-\frac{k}{m}t}-\frac{mg}{k}$

B. $v=c-\frac{mg}{k}e^{-\frac{k}{m}t}$

C. $ve^{-\frac{k}{m}t}=c-\frac{mg}{k}$

D. $ve^{\frac{k}{m}t} = c - \frac{mg}{k}$

Answer: A

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18. The general solution of the differential equation $\frac{dy}{dx} + \frac{\sin(x+y)}{2} = \frac{\sin(x-y)}{2}$ is (a) $(b)(c)\log\tan\left((d)(e)(f)\frac{y}{g}2(h)(i)(j)\right) = c - 2\sin x(k)$ (l) (m) *[Math Processing Error]* (ee) (ff) *[Math Processing Error]* (uu) (vv) $(ww)(\times)\log\tan\left((yy)(zz)(aaa)\frac{y}{bbb}4(ccc)(ddd) + (eee)\frac{\pi}{fff}4(ggg)(hhh)\right.$ (rrr)

A. $\log\tan\left(\frac{y}{2}\right) = c - 2\sin x$

B. $\log\tan\left(\frac{y}{2}\right) = c - 2\sin\left(\frac{x}{2}\right)$

C. $\log\tan\left(\frac{y}{2} + \frac{\pi}{4}\right) = c - 2\sin x$

D. $\log\tan\left(\frac{y}{4} + \frac{\pi}{4}\right) = c - 2\sin\left(\frac{x}{2}\right)$

Answer: B

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19. if $y+x\frac{dy}{dx}=x\frac{\phi(xy)}{\phi'(xy)}$ then $\phi(xy)$ is equation to

A. $ke^{x^2/2}$

B. $ke^{y^2/2}$

C. $ke^{xy/2}$

D. ke^{xy}

Answer: A

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20. The solution of differential equation

$x^2=1+\left(\frac{x}{y}\right)^{-1}\frac{dy}{dx}+\frac{\left(\frac{x}{y}\right)^{-2}\left(\frac{dy}{dx}\right)^2}{2}+\ldots.$

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21. The solution of the differential equation $\frac{x + \frac{x^3}{3!} + \frac{x^5}{5!} +}{1 + \frac{x^2}{2!} + \frac{x^4}{4!} +} = \frac{dx - dy}{dx + dy} is$ (a)

$(b)(c)2y(d)e^{(e)\,(f)\,2x\,(g)}(h) = C(i)e^{(j)\,(k)\,2x\,(l)}(m) + 1(n)$ (o) (p)

$(q)(r)2y(s)e^{(t)\,(u)\,2x\,(v)}(w) = C(x)e^{(y)\,(z)\,2x\,(aa)}(bb) - 1(cc)$ (dd) (ee)

$(ff)(gg)y(hh)e^{(ii)\,(jj)\,2x\,(kk)}(ll) = C(mm)e^{(nn)\,(\infty)\,2x\,(pp)}(qq) + 2(rr)$

(ss) (d) None of these

A. $2ye^{2x} = Ce^{2x} + 1$

B. $2ye^{2x} = Ce^{2x} - 1$

C. $ye^{2x} = Ce^{2x} + 2$

D. None of these

Answer: B

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22. The solution of the differential equation $x = 1 + xy\frac{dy}{dx} + \frac{x^2y^2}{2!}\left(\frac{dy}{d}\right)^2 + \frac{x^3y^3}{3!}\left(\frac{dy}{dx}\right)^3 + is$ (a)

$(b)(c)y = 1n((d)x(e)) + c(f)$ (g) (b)

$(h)(i)(j)y^{(k)2(l)}(m) = (n)(o)((p)(q)1nx(r))^{(s)2(t)}(u) + c(v)$ (w) (c)

$(d)(e)y = \log x + xy(f)$ (g) (d) $(h)(i)xy = (j)x^{(k)y(l)}(m) + c(n)$ (o)

A. $y = \ln(x) + C$

B. $y^2 = (\ln\ x)^2 + c$

C. $y = \log x + xy$

D. $xy = x^y + c$

Answer: B

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23. A curve passing through $(2, 3)$ and satisfying the differential equation $\int_0^x ty(t)dt = x^2y(x), (x > 0)$ is (a)

$(b)(c)(d)x^{(e)2(f)}(g) + (h)y^{(i)2(j)}(k) = 13(l)$ (m) (b)

$(n)(o)(p)y^{(q)2(r)}(s) = (t)\frac{9}{u}2(v)(w)x(x)$ (y) (c)

$(d)(e)(f)\frac{(g)(h)x^{(i)2(j)}(k)}{l}8(m)(n) + (o)\frac{(p)(q)y^{(r)2(s)}(t)}{u}((v)18)(w)(x)$

(z) (d) *[Math Processing Error]* (dd)

A. $x^2 + y^2 = 13$

B. $y^2 = \frac{9}{2}x$

C. $\frac{x^2}{8} + \frac{y^2}{18} = 1$

D. $xy = 6$

Answer: D

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24. The solution of the differential equation $\frac{d^2y}{dx^2} = \sin 3x + e^x + x^2$ when $y_1(0) = 1$ and $y(0)$ is (a)

$(b)(c)(d)\frac{(e) - \sin 3x}{f}9(g)(h) + (i)e^{(j)x(k)}(l) + (m)\frac{(n)(o)x^{(p)4(q)}(r)}{s}((($

(bb) (cc) *[Math Processing Error]* (ddd) (eee)

$(fff)(ggg)(hhh)\frac{(iii) - \cos 3x}{jjj} 3(kkk)(lll) + (mmm)e^{(nnn)x(\infty o)}(ppp) +$

(ffff) (d) None of these

A. $y = \frac{-\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1$

B. $y = \frac{-\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x$

C. $y = \frac{-\cos 3x}{3} + e^x + \frac{x^4}{12} + \frac{1}{3}x + 1$

D. None of these

Answer: A

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25. The solution of the differential equation $y'y''' = 3(y'')^2$ is

A. $x = A_1y^2 + A_2y + A_3$

B. $x = A_1y + A_2$

C. $x = A_1y^2 + A_2y$

D. None of these

Answer: A

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26. The solution of the differential equation $y^{-8}y^{=}0,$ where $y(0)=\frac{1}{8}, y'(0)=0, y^{0}=1$, is (a) $(b)(c)y=(d)\frac{1}{e}8(f)(g)\left((h)(i)(j)\frac{(k)(l)e^{(m)\ (n)\ 8x\ (o)}\ (p)}{q}8(r)(s)+x-(t)\right.$

(z) (aa) *[Math Processing Error]* (zz) (aaa) $(bbb)(c)y=(ddd)\frac{1}{eee}8(fff)(ggg)\left((hhh)(iii)(jjj)\frac{(kkk)(lll)e^{(mmm)\ (\cap)\ 8}}{qqq}\right.$

(zzz) (d) None of these

A. $y=\frac{1}{8}\left(\frac{e^{8x}}{8}+x-\frac{7}{9}\right)$

B. $y=\frac{1}{8}\left(\frac{e^{8x}}{8}+x+\frac{7}{8}\right)$

C. $y=\frac{1}{8}\left(\frac{e^{8x}}{8}-x+\frac{7}{8}\right)$

D. None of these

Answer: C

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27. The slope of the tangent at (x, y) to a curve passing through $\left(1, \frac{\pi}{4}\right)$ is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$, then the equation of the curve is (a) $(b)(c)y = (d)(e)\tan^{(f)\ (g)\ -1(h)}(i)\left((j)(k)\log\left((l)(m)(n)\frac{e}{o}x(p)(q)(r)\right)(s)$

(u) (v) *[Math Processing Error]* (pp) (qq) *[Math Processing Error]* (kkk) (d)

none of these

A. $y = \tan^{-1}\log\left(\frac{e}{x}\right)$

B. $y = x\tan^{-1}\log\left(\frac{x}{e}\right)$

C. $y = x\tan^{-1}\log\left(\frac{e}{x}\right)$

D. None of these

Answer: C

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28. $x\frac{dy}{dx} = y(\log y - \log x + 1)$

A. $\dfrac{\log x}{y} = cy$

B. $\dfrac{\log y}{x} = cy$

C. $\dfrac{\log y}{x} = cx$

D. None of these

Answer: C

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29. The solution of differential equation $xy' = x\left(\dfrac{y^2}{x^2} + \dfrac{f\left(\frac{y^2}{x^2}\right)}{f'\left(\frac{y^2}{x^2}\right)}\right)$ is

(a)

$$(b)(c)f\left((d)(e)(f)\frac{(g)(h)y^{(i)2(j)}(k)}{l}\left((m)(n)x^{(o)2(p)}(q)\right)(r)(s)(t)\right) = c($$

(z) (b) *[Math Processing Error]* (ggg) (c)

$$(d)(e)(f)x^{(g)2(h)}(i)f\left((j)(k)(l)\frac{(m)(n)y^{(o)2(p)}(q)}{r}\left((s)(t)x^{(u)2(v)}(w)\right)\right.($$

(bb) (d) *[Math Processing Error]* (bbb)

A. $f\left(y^2 / x^2\right) = cx^2$

B. $x^2 f(y^2/x^2) = c^2 y^2$

C. $x^2 f(y^2/x^2) = c$

D. $f(y^2/x^2) = cy/x$

Answer: A

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30. The solution of $(x^2 + xy)dy = (x^2 + y^2)dx$ is (a) $(b)(c)\log x = \log((d)(e)x - y(f)) + (g)\frac{y}{h}x(i)(j) + c(k)$ (l) (m) $(n)(o)\log x = 2\log((p)(q)x - y(r)) + (s)\frac{y}{t}x(u)(v) + c(w)$ (x) (y) *[Math Processing Error]* (jj) (kk) None of these

A. $\log x = \log(x - y) + \frac{y}{x} + c$

B. $\log x = 2\log(x - y) + \frac{y}{x} + c$

C. $\log x = \log(x - y) + \frac{x}{y} + c$

D. None of these

Answer: B

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31. The solution of $(y+x+5)dy=(y-x+1)dx$ is (a) *[Math Processing Error]* (jj) (kk)

$(ll)(mm)\log\left((nn)(\infty)(pp)(qq)((rr)(ss)y+3(tt))^{(uu)2(vv)}(ww)+(xx)(y$

$+(ggg)(hhh)\tan^{(iii)(jjj)-1(kkk)}(lll)(mmm)\frac{(nnn)y-3}{ooo}((ppp)y-2)(qq$

(ttt) (uuu) *[Math Processing Error]* (ddddd) (eeeee) *[Math Processing Error]* (nnnnnn)

A. $\log\left((y+3)^2+(x+2)^2\right)+\frac{\tan^{-1}(y+3)}{y+2}+C$

B. $\log\left((y+3)^2+(x-2)^2\right)+\frac{\tan^{-1}(y-3)}{x-2}=C$

C. $\log\left((y+3)^2+(x+2)^2\right)+2\frac{\tan^{-1}(y+3)}{x+2}=C$

D. $\log\left((y+3)^2+(x+2)^2\right)-2\frac{\tan^{-1}(y+3)}{x+2}=C$

Answer: A

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32. The slope of the tangent at (x, y) to a curve passing through a point $(2, 1)$ is $\frac{x^2 + y^2}{2xy}$, then the equation of the curve is (a) $(b)(c)2\left((d)(e)(f)x^{(g)2(h)}(i) - (j)y^{(k)2(l)}(m)(n)\right) = 3x(o)$ (p) (b) *[Math Processing Error]* (ee) (c) $(d)(e)x\left((f)(g)(h)x^{(i)2(j)}(k) - (l)y^{(m)2(n)}(o)(p)\right) = 6(q)$ (r) (d) $(s)(t)x\left((u)(v)(w)x^{(x)2(y)}(z) + (aa)y^{(bb)2(cc)}(dd)(ee)\right) = 10(ff)$

(gg)

A. $2(x^2 - y^2) = 3x$

B. $2(x^2 - y^2) = 6y$

C. $x(x^2 - y^2) = 6$

D. $x(x^2 + y^2) = 10$

Answer: A

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33. Solutionof the differential equation $ydx - xdy + x\sqrt{xy}dy = 0$ is

A. $\dfrac{x^2 + y^2}{2} + \tan^{-1}\sqrt{\dfrac{y}{x}} = c$

B. $\dfrac{x^2 + y^2}{2} + 2\tan^{-1}\sqrt{\dfrac{x}{y}} = C$

C. $\dfrac{x^2 + y^2}{2} + 2\cot^{-1}\sqrt{\dfrac{x}{y}} = c$

D. None of these

Answer: D

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34. The solution of $x^2\dfrac{dy}{dx} - xy = 1 + \dfrac{\cos y}{x}$ is

A. $\tan\left(\dfrac{y}{2x}\right) = c - \dfrac{1}{2x^2}$

B. $\dfrac{\tan y}{x} = c + \dfrac{1}{x}$

C. $\cos\left(\dfrac{y}{x}\right) = 1 + \dfrac{c}{x}$

D. $x^2 = (c + x^2)\dfrac{\tan y}{x}$

Answer: A

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35. The solution of the differential equation

$2x^2y\frac{dy}{dx} = \tan(x^2y^2) - 2xy^2$, given $y(1) = \sqrt{\frac{\pi}{2}}$, is

A. $\sin^2 y^2 = e^{x+1}$

B. $\sin(x^2y^2) = x$

C. $\cos x^2y^2 + x = 0$

D. $\sin(x^2y^2) = e^{x-1}$

Answer: D

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36. The solution of the differential equation

$\left\{\frac{1}{x} - \frac{y^2}{(x-y)^2}\right\}dx + \left\{\frac{x^2}{(x-y)^2} - \frac{1}{y}\right\}dy = 0$ is

A. $\ln\left|\frac{x}{y}\right| + \frac{xy}{x-y} = c$

B. $\frac{xy}{x-y} = ce^{x/y}$

C. $\ln|xy| + \frac{x^4y^4}{4} = C$

D. None of these

Answer: A

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37. The solution of differential equation $(2y + xy^3)dx + (x + x^2y^2)dy = 0$ is (a)

$(b)(c)(d)x^{(e)2(f)}(g)y + (h)\frac{(i)(j)x^{(k)3(l)}(m)(n)y^{(o)3(p)}(q)}{r}3(s)(t) = c($

(v) (b)

$(w)(x)x(y)y^{(z)2(aa)}(bb) + (cc)\frac{(dd)(ee)x^{(ff)3(gg)}(hh)(ii)y^{(jj)3(kk)}(ll)}{mm}3($

(qq) (c)

$(d)(e)(f)x^{(g)2(h)}(i)y + (j)\frac{(k)(l)x^{(m)4(n)}(o)(p)y^{(q)4(r)}(s)}{t}4(u)(v) = c($

(x) (d) None of these

A. $x^2y + \frac{x^3y^3}{3} = c$

B. $xy^2 + \frac{x^3y^3}{3} = c$

C. $x^2y + \frac{x^4y^4}{4} = c$

D. None of these

Answer: A

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38. The solution of $ye^{-\frac{x}{y}}dx - \left(xe^{\left(-\frac{x}{y}\right)} + y^3\right)dy = 0$ is (a) $(b)(c)(d)e^{(e)(f)-(g)\frac{x}{h}y(i)(j)(k)}(l) + (m)y^{(n)2(o)}(p) = C(q)$ (r) (b) *[Math Processing Error]* (ee) (c) $(d)(e)2(f)e^{(g)(h)-(i)\frac{x}{j}y(k)(l)(m)}(n) + (o)y^{(p)2(q)}(r) = C(s)$ (t) (d) *[Math Processing Error]* (kk)

A. $e^{-x/y} + y^2 = C$

B. $xe^{-x/y} + y = C$

C. $2e^{-x/y} + y^2 = C$

D. $e^{-x/y} + 2y^2 = C$

Answer: C

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39. The curve satisfying the equation $\frac{dy}{dx} = \frac{y(x + y^3)}{x(y^3 - x)}$ and passing through the point $(4, -2)$ is (a) $(b)(c)(d)y^{(e)2(f)}(g) = -2x(h)$ (i) (b) $(j)(k)y = -2x(l)$ (m) (c) $(d)(e)(f)y^{(g)3(h)}(i) = -2x(j)$ (k) (d) None of these

A. $y^2 = -2x$

B. $y = -2x$

C. $y^3 = -2x$

D. None of these

Answer: C

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40. The solution of differential equation

$$\frac{x+y\frac{dy}{dx}}{y-x\frac{dy}{dx}}=x\left(\frac{\cos^2(x^2+y^2)}{y^3}\right)$$ is

A. $\tan(x^2+y^2)=\frac{x^2}{y^2}+c$

B. $\cot(x^2+y^2)=\frac{x^2}{y^2}+c$

C. $\tan(x^2+y^2)=\frac{y^2}{x^2}+c$

D. $\cot(x^2+y^2)=\frac{y^2}{x^2}+c$

Answer: A

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41. The solution of the differential equation $\frac{dy}{dx}=\frac{3x^2y^4+2xy}{x^2-2x^3y^3}$ is (a)

$(b)(c)(d)\frac{(e)(f)y^{(g)2(h)}(i)}{j}x(k)(l)-(m)x^{(n)3(o)}(p)(q)y^{(r)2(s)}(t)=c(u$

(v) (w) *[Math Processing Error]* (ww) (xx)

$(yy)(zz)(aaa)\frac{(bbb)(c)x^{(ddd)2(eee)}(fff)}{ggg}y(hhh)(iii)+(jjj)x^{(kkk)3(lll)}(m$

(sss) (d)

$(ttt)(\text{U})(\text{V})\frac{(www)(\times x)x^{(yyy)2(zzz)}(aaaa)}{bbbb}((cccc)3y)(dddd)(eeee)$ –

(oooo)

A. $\frac{y^2}{x} - x^3y^2 = c$

B. $\frac{x^2}{y^2} + x^3y^3 = c$

C. $\frac{x^2}{y} + x^3y^2 = c$

D. $\frac{x^2}{3y} - 2x^3y^2 = c$

Answer: C

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42. The solution of the differential equation $\left\{1 + x\sqrt{(x^2 + y^2)}\right\}dx + \left\{\sqrt{(x^2 + y^2)} - 1\right\}ydy = 0$ is equal to (a) $(b)(c)(d)x^{(e)2(f)}(g) + (h)\frac{(i)(j)y^{(k)2(l)}(m)}{n}2(o)(p) + (q)\frac{1}{r}3(s)(t)(u)(v$

(qq) (rr) *[Math Processing Error]* (dddd) (eeee) *[Math Processing Error]*

(qqqqq)

A. $x^2 + \frac{y^2}{2} + \frac{1}{3}(x^2 + y^2)^{3/2} = c$

B. $x - \frac{y^3}{3} + \frac{1}{2}(x^2 + y^2)^{1/2} = c$

C. $x - \frac{y^2}{2} + \frac{1}{3}(x^2 + y^2)^{3/2} = c$

D. None of these

Answer: C

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43. The solution of the differential equation

$y(2x^4 + y)\frac{dy}{dx} = (1 - 4xy^2)x^2$ is given by

A. $3(x^2y^2)^2 + y^3 - x^3 = c$

B. $xy^2 + \frac{y^3}{3} - \frac{x^3}{3} + c = 0$

C. $\frac{2}{5}yx^5 + \frac{y^3}{3} = \frac{x^3}{3} - \frac{4xy^3}{3} + c$

D. None of these

Answer: A

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44. The solution of the differential equation $(x \cot y + \log \cos x)dy + (\log \sin y - y \tan x)dx = 0$ is

A. $(\sin x)^{y}(\cos y)^{x} = c$

B. $(\sin y)^{x}(\cos x)^{y} = c$

C. $(\sin x)^{x}(\cos y)^{y} = c$

D. None of these

Answer: B

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45. If $= (e^{y} - x)^{-1}$, where $y(0) = 0$, then y is expressed explicitly as

(a) $(b)(c)(d)\frac{1}{e}2(f)(g)1n\Big((h)(i)1 + (j)x^{(k)2(l)}(m)(n)\Big)(o)$ (p) (b) $(q)(r)1n\Big((s)(t)1 + (u)x^{(v)2(w)}(x)(y)\Big)(z)$ (aa) (c)

$(d)(e)1n\left((f)(g)x+\sqrt{(h)(i)1+(j)x^{(k)2(l)}(m)(n)(o)(p)}\right)(q)$ (r) (d)

[Math Processing Error] (gg)

A. $\frac{1}{2}\log_e(1+x^2)$

B. $\log_e(1+x^2)$

C. $\log_e\left(x+\sqrt{1+x^2}\right)=c$

D. None of these

Answer: C

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46. The general solution of the differential equation, $y'+y\varphi'(x)=0$, where $\varphi(x)$ is a known function, is (a)

$(b)(c)y=c(d)e^{(e)(f)-\varphi((g)x(h))(i)}(j)+\varphi((k)x(l))-1(m)$ (n) (o)

$(p)(q)y=c(r)e^{(s)(t)+\varphi((u)x(v))(w)}(x)+\varphi((y)x(z))-1(aa)$ (bb)

(cc)

$(dd)(ee)y=c(ff)e^{(gg)(hh)-\varphi((ii)x(jj))(kk)}(ll)-\varphi((mm)x(\cap))+1(o$

(pp) (d)

$(qq)(rr)y = c(ss)e^{(tt)(\cup)-\varphi((vv)x(ww))(xx)}(yy) + \varphi((zz)x(aaa)) + 1(b$

(ccc)

A. $y = ce^{-\phi(x)} + \phi(x) - 1$

B. $y = ce^{+\phi(x)} + \phi(x) - 1$

C. $y = ce^{-\phi(x)} - \phi(x) + 1$

D. $y = ce^{-\phi(x)} + \phi(x) + 1$

Answer: A

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47. The integrating factor of the differential equation $\frac{dy}{dx}\left(x(\log)_e x\right) + y = 2(\log)_e x$ is given by (a) $(b)x(c)$ (d) (b) $(e)(f)(g)e^{(h)x(i)}(j)(k)$ (l) (c) $(m)(n)(o)((p)\log)_q e(r)(s)x(t)$ (u) (d)

[Math Processing Error] (ii)

A. x

B. e^x

C. $\log_e x$

D. $\log_e(\log_e x)$

Answer: C

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48. The solution of the differential equation $x(x^2+1)(dy/dx) = y(1-x^2) + x^3 \log x$ is

A. $y(x^2+1)/x = \frac{1}{4}x^2 \log x + \frac{1}{2}x^2 + c$

B. $y^2(x^2-1)/x = \frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + c$

C. $y(x^2+1)x = \frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + c$

D. None of these

Answer: C

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49. Integrating factor of differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$ is

(a) $(b)(c)\cos x(d)$ (e) (b) $(f)(g)\tan x(h)$ (i) (c) $(d)(e)\sec x(f)$ (g) (d) $(h)(i)\sin x(j)$ (k)

A. $\cos x$

B. $\tan x$

C. $\sec x$

D. $\sin x$

Answer: C

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50. Solution of the equation `cos^2x(dy)/(dx)-(tan2x)y=cos^4x ,|x|

A. $y = \tan 2x \cos^2 x$

B. $y = \cot 2x \cos^2 x$

C. $y = \frac{1}{2} \tan 2x \cos^2 x$

D. $y=\frac{1}{2}\cot 2x\cos^2 x$

Answer: C

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51. If integrating factor of $x(1-x^2)dy+(2x^2y-y-ax^3)dx=0$ is $e^{\int pdx}$, then P is equal to (a)

$(b)(c)(d)\frac{(e)2(f)x^{(g)2(h)}(i)-a(j)x^{(k)3(l)}(m)}{n}\Big((o)x\Big((p)(q)1-(r)x^{(s)2(}$

(z) (b) [Math Processing Error] (hh) (c)

$(d)(e)(f)\frac{(g)2(h)x^{(i)2(j)}(k)-a}{l}\Big((m)a(n)x^{(o)3(p)}(q)\Big)(r)(s)(t)$ (u)

(d)

$(v)(w)(x)\frac{(y)2(z)x^{(aa)2(bb)}(cc)-1}{dd}\Big((ee)x\Big((ff)(gg)1-(hh)x^{(ii)2(jj)}(k$

(pp)

A. $\frac{2x^2-ax^2}{x(1-x^2)}$

B. $2x^3-1$

C. $\frac{2x^2-a}{ax^3}$

D. $\dfrac{2x^2-1}{x(1-x^2)}$

Answer: D

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52. A function $y=f(x)$ satisfies $(x+1)f'(x)-2(x^2+x)f(x)=\dfrac{e^x\ \hat{}\ 2}{(x+1)}, \forall x \succ 1.$ If $f(0)=5,$ then $f(x)$ is (a) $(b)(c)\left((d)(e)(f)\dfrac{(g)3x+5}{h}((i)x+1)(j)(k)(l)\right)(m)e^n(o)(p)x^{((q)2(r))(s}$ (w) (b) *[Math Processing Error]* (ss) (c) $(d)(e)\left((f)(g)(h)\dfrac{(i)6x+5}{j}((k)x+1)(l)(m)(n)\right)(o)e^p(q)(r)x^{((s)2(t))(u}$ (y) (d) *[Math Processing Error]* (uu)

A. $\left(\dfrac{3x+5}{x+1}\right)e^{x^2}$

B. $\left(\dfrac{6x+5}{x+1}\right)e^{x^2}$

C. $\left(\dfrac{6x+5}{(x+1)^2}\right)e^{x^2}$

D. $\left(\dfrac{5-6x}{x+1}\right)e^{x^2}$

Answer: B

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53. The general solution of the equation $\frac{dy}{dx}=1+xy$ is (a) $(b)(c)y=c(d)e^{e}(f)-(g)\left((h)(i)\frac{x^{((j)2(k))(l)}}{m}2(n)(o)(p)\right)(q)(r)$ (s)

(b)

$(t)(u)y=c(v)e^{w}(x)(y)\left((z)(aa)\frac{x^{((bb)2(cc))(dd)}}{ee}2(ff)(gg)(hh)\right)(ii)(jj)$

(kk) (c)

$(d)(e)y=((f)(g)x+c(h)),(i)e^{j}(k)-(l)\left((m)(n)\frac{x^{((o)2(p))(q)}}{r}2(s)(t)($

(x) (d) None of these

A. $y=ce^{-x^2/2}$

B. $y=ce^{x^2/2}$

C. $y=(x+c), e^{-x^2/2}$

D. None of these

Answer: D

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54. The solution of the differential equation $\frac{(x+2y^3)dy}{dx}=y$ is (a) $(b)(c)(d)\frac{x}{e}\left((f)(g)y^{(h)2(i)}(j)\right)(k)(l)=y+c(m)$ (n) (b) $(o)(p)(q)\frac{x}{r}y(s)(t)=(u)y^{(v)2(w)}(x)+c(y)$ (z) (c) $(d)(e)(f)\frac{(g)(h)x^{(i)2(j)}(k)}{l}y(m)(n)=(o)y^{(p)2(q)}(r)+c(s)$ (t) (d) $(u)(v)(w)\frac{y}{x}x(y)(z)=(aa)x^{(bb)2(cc)}(dd)+c(ee)$ (ff)

A. $\frac{x}{y^2}=y+c$

B. $\frac{x}{y}=y^2+c$

C. $\frac{x^2}{y}=y^2+c$

D. $\frac{y}{x}=x^2+c$

Answer: B

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55. The solution of the differential equation $x^2 \frac{dy}{dx} \frac{\cos 1}{x} - y \frac{\sin 1}{x} = -1$, where $\overrightarrow{-} 1$ as$x \overrightarrow{\infty}$ is (a) $(b)(c)y = \sin(d)\frac{1}{e}x(f)(g) - \cos(h)\frac{1}{i}x(j)(k)(l)$ (m) (b) $(n)(o)y = (p)\frac{(q)x+1}{r}\left((s)x\sin(t)\frac{1}{u}x(v)(w)\right)(x)(y)(z)$ (aa) (c) $(d)(e)y = \cos(f)\frac{1}{g}x(h)(i) + \sin(j)\frac{1}{k}x(l)(m)(n)$ (o) (d) *[Math Processing Error]* (cc)

A. $y = \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$

B. $y = \frac{x+1}{x\sin\left(\frac{1}{x}\right)}$

C. $y = \cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)$

D. $y = \frac{x+1}{x\cos(1/x)}$

Answer: A

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56. The solution of $\frac{dy}{dx} = \frac{x^2 + y^2 + 1}{2xy}$ satisfying $y(1) = 1$ is given by (a) a system of parabolas (b) a system of circles (c) $(d)(e)(f)y^{(g)2(h)}(i) = x((j)(k)1 + x(l)) - 1(m)$ (n) (d) *[Math Processing Error]* (hh)

A. a system of parabolas

B. a system of circles

C. $y^2 = x(1 + x) - 1$

D. $(x - 2)^2 + (y - 3)^2 = 5$

Answer: C

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57. The solution of the differential equation $\frac{dy}{dx} = \frac{1}{xy\left[x^2 \sin y^2 + 1\right]}$ is

A. $x^2\left(\cos y^2 - \sin y^2 - 2Ce^{-y^2}\right) = 2$

B. $y^2\left(\cos x^2 - \sin y^2 - 2Ce^{-y^2}\right) = 4C$

C. None of these

D. a system of circles

Answer: A

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58. The equation of a curve passing through $\left(2, \frac{7}{2}\right)$ and having gradient $1 - \frac{1}{x^2}$ at (x, y) is (a) $(b)(c)y = (d)x^{(e)2(f)}(g) + x + 1(h)$ (i) (b) $(j)(k)xy = (l)x^{(m)2(n)}(o) + x + 1(p)$ (q) (c) $(d)(e)xy = x + 1(f)$ (g)

(d) None of these

A. $y = x^2 + x + 1$

B. $xy = x^2 + x + 1$

C. $xy = x + 1$

D. None of these

Answer: B

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59. Which of the following is not the differential equation of family of curves whose tangent from an angle of $\frac{\pi}{4}$ with the hyperbola $xy = c^2$?

A. $\frac{dy}{dx} = \frac{x-y}{x+y}$

B. $\frac{dy}{dx} = \frac{x}{x-y}$

C. $\frac{dy}{dx} = \frac{x+y}{y-x}$

D. None of these

Answer: B

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60. Tangent to a curve intercepts the y-axis at a point P. A line perpendicular to this tangent through P passes through another point

(1,0). The differential equation of the curve is (a)

$$(b)(c)y(d)\frac{(e)dy}{f}((g)dx)(h)(i) - x(j)(k)\left((l)(m)(n)\frac{(o)dy}{p}((q)dx)(r)(s)(t$$

(y) (b) [Math Processing Error] (eee) (c)

$(d)(e)y(f)\frac{(g)dx}{h}((i)dy)(j)(k) + x = 1(l)$ (m) (d) None of these

A. $y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 1$

B. $\frac{xd^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

C. $y\frac{dy}{dx} + x = 1$

D. None of these

Answer: A

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61. Orthogonal trajectories of family of the curve $x^{\frac{2}{3}} + \frac{y^2}{3} = a^{\left(\frac{2}{3}\right)}$, where a is any arbitrary constant, is (a)

$$(b)(c)(d)x^{(e)(f)(g)\frac{2}{h}3(i)(j)(k)}(l) - (m)y^{(n)(o)(p)\frac{2}{q}3(r)(s)(t)}(u) = c(v$$

(w) (b) [Math Processing Error] (ss) (c)

$(d)(e)(f)x^{(g)\,(h)\,(i)\,\frac{4}{j}3\,(k)\,(l)\,(m)}(n) + (o)y^{(p)\,(q)\,(r)\,\frac{4}{s}3\,(t)\,(u)\,(v)}(w) = c(x$

(y) (d) *[Math Processing Error]* (uu)

A. $x^{2/3} - y^{2/3} = c$

B. $x^{4/3} - y^{4/3} = c$

C. $x^{4/3} + y^{4/3} = c$

D. $x^{\frac{1}{3}} - y^{1/3} = c$

Answer: B

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62. The curve in the first quadrant for which the normal at any point (x, y) and the line joining the origin to that point form an isosceles triangle with the x-axis as base is (a) an ellipse (b) a rectangular hyperbola (c) a circle (d) None of these

A. an ellipse

B. a rectangular hyperbola

C. a circle

D. None of these

Answer: B

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63. The equation of the curve which is such that the portion of the axis of x cut-off between the origin and tangent at any point is proportional to the ordinate of that point is

A. $x = y(a - b\log y)$

B. $\log_x = by^2 + a$

C. $x^2 = y(a - b\log y)$

D. None of these

Answer: A

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64. The family of curves represented by $\frac{dy}{dx} = \frac{x^2+x+1}{y^2+y+1}$ and the family represented by $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$

A. Touch each other

B. Are orthogonal

C. Are one and the same

D. None of these

Answer: B

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65. A normal at $P(x, y)$ on a curve meets the x-axis at Q and N is the foot of the ordinate at P. If $NQ = \frac{x(1+y^2)}{1+x^2}$, then the equation of curve given that it passes through the point $(3, 1)$ is (a) $(b)(c)(d)x^{(e)2(f)}(g) - (h)y^{(i)2(j)}(k) = 8(l)$ (m) (b)

$(n)(o)(p)x^{(q)2(r)}(s)+2(t)y^{(u)2(v)}(w)=11(x)$ (y) (c)

$(d)(e)(f)x^{(g)2(h)}(i)-5(j)y^{(k)2(l)}(m)=4(n)$ (o) (d) None of these

A. $x^2-y^2=8$

B. $x^2+2y^2=11$

C. $x^2-5y^2=4$

D. None of these

Answer: C

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66. A curve is such that the mid-point of the portion of the tangent intercepted between the point where the tangent is drawn and the point where the tangent meets the y-axis lies on the line $y=x$. If the curve passes through $(1,0)$, then the curve is (a)

$(b)(c)2y=(d)x^{(e)2(f)}(g)-x(h)$ (i) (b)

$(j)(k)y=(l)x^{(m)2(n)}(o)-x(p)$ (q) (c)

$(d)(e)y = x - (f)x^{(g)2(h)}(i)(j)$ (k) (d)

$(l)(m)y = 2\left((n)(o)x - (p)x^{(q)2(r)}(s)(t)\right)(u)$ (v)

A. $2y = x^2 - x$

B. $y = x^2 - x$

C. $y = x - x^2$

D. $y = 2(x - x^2)$

Answer: C

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67. The normal to a curve at P(x, y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a (1) ellipse (2) parabola (3) circle (4) hyperbola

A. parabola

B. circle

C. hyperbola

D. ellipse

Answer: C

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68. The x-intercept of the tangent to a curve is equal to the ordinate of the point of contact. The equation of the curve through the point (1,1) is

A. $ye^{x/y} = e$

B. $xe^{x/y} = e$

C. $xe^{y/x} = e$

D. $ye^{y/x} = e$

Answer: A

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69. The equation of a curve passing through (1,0) for which the product of the abscissa of a point P and the intercept made by a normal at P on the x-axis equal twice the square of the radius vector of the point P is (a) $(b)(c)(d)x^{(e)2(f)}(g)+(h)y^{(i)2(j)}(k)=(l)x^{(m)4(n)}(o)(p)$ (q) (b) $(r)(s)(t)x^{(u)2(v)}(w)+(x)y^{(y)2(z)}(aa)=2(bb)x^{(cc)4(dd)}(ee)(ff)$ (gg) (c) $(d)(e)(f)x^{(g)2(h)}(i)+(j)y^{(k)2(l)}(m)=4(n)x^{(o)4(p)}(q)(r)$ (s) (d) None of these

A. $x^2+y^2=x^4$

B. $x^2+y^2=2x^4$

C. $x^2+y^2=4x^4$

D. None of these

Answer: A

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70. The curve with the property that the projection of the ordinate on the normal is constant and has a length equal to a is (a)

$(b)(c)a1n\left((d)(e)\sqrt{(f)(g)(h)y^{(i)2(j)}(k)-(l)a^{(m)2(n)}(o)(p)}(q)+y(r)\right)$

(t) (u)

$(v)(w)x+\sqrt{(x)(y)(z)a^{(aa)2(bb)}(cc)-(dd)y^{(ee)2(ff)}(gg)(hh)}(ii)=c(j$

(kk) (ll) $(mm)(\cap)(\infty)(pp)((qq)(rr)y-a(ss))^{(tt)2(uu)}(vv)=cx(ww)$

(xx) (yy)

$(zz)(aaa)ay=(bbb)(c)\tan^{(ddd)(eee)-1(fff)}(ggg)((hhh)(iii)x+c(jjj))($

(lll)

A. $a \ \ln\left(\sqrt{y^2-a^2}\right)=x+c$

B. $x+\sqrt{a^2-y^2}=c$

C. $(y-a)^2=cx$

D. $ay=\tan^{-1}(x+c)$

Answer: A

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71. Spherical rain drop evaporates at a rate proportional to its surface area. The differential equation corresponding to the rate of change of the radius of the rain drop if the constant of proportionality is $K > 0$ is (a)

$(b)(c)(d)\frac{(e)dy}{f}((g)dt)(h)(i) + K = 0(j)$ (k) (b)

$(l)(m)(n)\frac{(o)dr}{p}((q)dt)(r)(s) - K = 0(t)$ (u) (c)

$(d)(e)(f)\frac{(g)dr}{h}((i)dt)(j)(k) = Kr(l)$ (m) (d) None of these

A. $\frac{dr}{dt} + K = 0$

B. $\frac{dr}{dt} - K = 0$

C. $\frac{dr}{dt} = Kr$

D. None of these

Answer: A

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72. Water is drained from a vertical cylindrical tank by opening a valve at the base of the tank. It is known that the rate at which the water level drops is proportional to the square root of water depth y, where the constant of proportionality $k > 0$ depends on the acceleration due to gravity and the geometry of the hole. If t is measured in minutes and $k = \frac{1}{15}$, then the time to drain the tank if the water is 4 m deep to start with is (a) 30 min (b) 45 min (c) 60 min (d) 80 min

A. 30 min

B. 45 min

C. 60 min

D. 80 min

Answer: C

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73. The population of a country increases at a rate proportional to the number of inhabitants. f is the population which doubles in 30 years, then the population will triple in approximately. (a) 30 years (b) 45 years (c) 48 years (d) 54 years

A. 30 years

B. 45 years

C. 48 years

D. 54 years

Answer: C

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74. An object falling from rest in air is subject not only to the gravitational force but also to air resistance. Assume that the air resistance is proportional to the velocity with constant of proportionality as $k > 0$, and acts in a direction opposite to motion $\left(g = 9.8\frac{m}{s^2}\right)$. Then velocity cannot exceed. (c) $(d)(e)9.8/km/s(f)$ (g) (b) $(h)(i)98/km/s(j)$ (k) (c) $(d)(e)(f)\frac{k}{g}((h)9.8)(i)(j)m/s(k)$ (l) (d) None of these

A. 9.8/km/s

B. 98/k m/s

C. $\frac{k}{9.8}m/x$

D. None of these

Answer: A

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Exercise (Multiple)

1. Which one of the following function(s) is/are homogeneous?

A. $f(x, y) = \frac{x - y}{x^2 + y^2}$

B. $f(x, y) = x^{\frac{1}{3}}y^{-\frac{2}{3}}\frac{\tan^{-1}x}{y}$

C. $f(x, y) = x\left(\ln\sqrt{x^2 + y^2}\right) - \ln y + ye^{x/y}$

D. $f(x, y) = x\left[\ln(2x^2 + y^2)x - \ln(x + y)\right] + y^2\frac{\tan(x + 2y)}{3x - y}$

Answer: A::B::C

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2. The differential equation whose solution is $(x-h)^2+(y-k)^2=a^2$ is (a is a constant)

A. order is 2

B. order is 3

C. degree is 2

D. degree is 3

Answer: A::C

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3. The equation of the curve satisfying the differential equation $y\left(\frac{dy}{dx}\right)^2+(x-y)\frac{dy}{dx}-x=0$ can be a (a) circle (b) Straight line (c) Parabola (d) Ellipse

A. Circle

B. Straight line

C. Parabola

D. Ellipse

Answer: A::B

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4. Which of the following equation(s) is/are linear?

A. $\frac{dy}{dx} + \frac{y}{x} = \log x$

B. $y\frac{dy}{dx} + 4x = 0$

C. $(2x + y^3)\frac{dy}{dx} = 3y$

D. None of these

Answer: A::C

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5. The solution of $\frac{dy}{dx}=\frac{ax+h}{by+k}$ represents a parabola when

A. $a=0, b\neq 0$

B. $a\neq 0, b\neq 0$

C. $b=0, a\neq 0$

D. $a=0, b\in R$

Answer: A::C

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6. The equation of the curve satisfying the differential equation $y_2(x^2+1)=2xy_1$ passing through the point (0,1) and having slope of tangent at $x=0$ as 3 (where y_2 and y_1 represent 2nd and 1st order derivative), then

A. $y=f(x)$ is a strictly increasing function

B. $y = f(x)$ is a non-monoatomic function

C. $y = f(x)$ has three distinct real root

D. $y = f(x)$ has only one negative root

Answer: A::D

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7. Identify the statement(s) which is/are true.

A. $f(x, y) = e^{y/x} = e^{\frac{y}{x}} + \frac{\tan y}{x}$ is a homogeneous of degree zero.

B. $x \ \ln \ \frac{y}{x} dx + \frac{y^2}{x} \frac{\sin^{-1} y}{x} dx = 0$ is a homogenous differential equation.

C. $f(x, y) = x^2 + \sin x \cos y$ is a non-homogenous

D. $(x^2 + y^2) dx - (xy^2 - y^3) dy = 0$ is a homogenous differential equationn.

Answer: A::B::C

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8. The graph of the function y = f(x) passing through the point (0, 1) and satisfying the differential equation $\frac{dy}{dx} + y \cos x = \cos x$ is such that

A. It is constant function

B. It is periodic

C. it is neither an even nor an odd function.

D. it is continuous and differentiable for all x

Answer: A::B::D

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9. If f(x), g(x) be twice differentiable functions on [0,2] satisfying $f''(x) = g''(x)$,$f'(1) = 2g'(1) = 4$ and $f(2) = 3g(2) = 9,$ then $f(x) - g(x)$ at x = 4 equals (A) 0 (B) 10 (C) 8 (D) 2

A. $f(4) - g(4) = 10$

B. $|f(x) - g(x)| < 2 \Rightarrow -2 < x < 0$

C. $f(2) = g(2) \Rightarrow x = -1$

D. $f(x) - g(x) = 2x$ has real root

Answer: A::B::C

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10. The solution of the differential equation $(x^2y^2 - 1)dy + 2xy^3dx = 0$ is (a) $(b)(c)1 + (d)x^{(e)2(f)}(g)(h)y^{(i)2(j)}(k) = cx(l)$ (m) (b) $(n)(o)1 + (p)x^{(q)2(r)}(s)(t)y^{(u)2(v)}(w) = cy(x)$ (y) (c) $(d)(e)y = 0(f)$ (g) (d) $(h)(i)y = -(j)\frac{1}{k}\left((l)(m)x^{(n)2(o)}(p)\right)(q)(r)(s)$ (t)

A. $1 + x^2y^2 = cx$

B. $1 + x^2y^2 = cy$

C. $y = 0$

D. $y = -\frac{1}{x^2}$

Answer: B

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11. $y = ae^{-\frac{1}{x}} + b$ is a solution of $\frac{dy}{dx} = \frac{y}{x^2}$, then (a) $(b)(c)a \in R(d)$ (e) (b) 0 (c) $(d)(e)b = 1(f)$ (g) (d) $(h)a(i)$ (j) takes finite number of values

A. $x \in R - \{0\}$

B. $b = 0$

C. $b = 1$

D. a takes finite number of values

Answer: A::B

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12. The equation of the curve whose subnormal is constant is

A. its eccentricity is 1

B. its eccentricity is $\sqrt{2}$

C. its axis is the x-axis

D. its axis is the y-axis

Answer: B

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13. The solution of $\frac{xdd+ydy}{xdy-ydx}=\sqrt{\frac{1-x^2-y^2}{x^2+y^2}}$ is (a) *[Math Processing Error]* (dd) (ee) *[Math Processing Error]* (hhh) (iii)

$$(jjj)(kkk)\sqrt{(lll)(mmm)\left(\bigcap\right)x^{(ooo)2(ppp)}(qqq)+(rrr)y^{(sss)2(ttt)}(uuu)}$$

$$=\left(\tan\left\{si(\times x)n^{(yyy)(zzz)-1(aaaa)}(bbbb)\left((cccc)(dddd)(eeee)\frac{y}{ffff}x(g\right.\right.\right.$$

(kkkk) (llll)

$$(mmmm)\left(\bigcap n\right)y$$

$$+x\tan\left((oooo)(pppp)c+(qqqq)(rrrr)\sin^{(ssss)(tt)-1(uuuu)}(vvvv)\sqrt{(wwv}\right.$$

$$(jjjjj)$$

(kkkkk)

A. $\sqrt{x^2+y^2}=\sin\{(\tan^{-1}y/x)+C\}$

B. $\sqrt{x^2+y^2}=\cos\{(\tan^{-1}y/x)+C\}$

C. $\sqrt{x^2+y^2}=(\tan(\sin^{-1}y/x)+C)$

D. $y=x\tan\left(c+\sin^{-1}\sqrt{x^2+y}\right)$

Answer: A::D

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14. Find the curves for which the length of normal is equal to the radius vector.

A. circles

B. rectangular hyperbola

C. ellipses

D. straight lines

Answer: A::B

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15. In which of the following differential equation degree is not defined?

(a) [Math Processing Error] (uu) (vv) [Math Processing Error] (pppp) (qqqq) [Math Processing Error] (ddddd) (eeeee) $(fffff)(ggggg)x - 2y = \log\left((hhhhh)(iiiii)(jjjjj)\frac{(kkkkk)dy}{lllll}((mmmm$

(rrrrr)

A. $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x\frac{\log(d^2y)}{dx^2}$

B. $\frac{d^2y}{\left(dx^2\right)^2} + \left(\frac{dy}{dx}\right)^2 = x\frac{\sin(d^2y)}{dx^2}$

C. $x = \frac{\sin(dy)}{dx} - 2y, |x| < 1$

D. $x - 2y = \frac{\log(dy)}{dx}$

Answer: A::B

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16. If $y=f(x)$ is the solution of equation $ydx+dy=-e^{x}y^{2}$dy, f(0)=1 and area bounded by the curve $y=f(x), y=e^{x}$ and x=1 is A, then

A. curve y=f(x) is passing through $(-2, e)$.

B. Curve $y=f(x)$ is passing through $(1, 1/e)$

C. curve $y=f(x)$ is passing through $(1, 1/3)$

D. $A=e+\dfrac{2}{\sqrt{e}}-3$

Answer: A::D

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17. A particle falls in a medium whose resistance is propotional to the square of the velocity of the particles. If the differential equation of the free fall is $\dfrac{dv}{dt}=g-kv^{2}$ (k is constant) then

A. $v=2\sqrt{\dfrac{g}{k}}\dfrac{e^{2t\sqrt{g/t}}+1}{e^{2r\sqrt{g/k}}-1}$

B. $v=\sqrt{\dfrac{g}{k}}\dfrac{e^{2t\sqrt{gk}}-1}{e^{2t\sqrt{gk}}+1}$

C. $v \to 0$ as $t \to \infty$

D. $v \to \sqrt{\frac{g}{k}}$ as $t \to \infty$

Answer: B::D

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Exercise (Comprehension)

1. For certain curves y= f(x) satisfying $\frac{d^2y}{dx^2} = 6x - 4$, f(x) has local minimum value 5 when x=1. 9. Number of critical point for y=f(x) for x â, [0,2] (a) 0 (b)1. c).2 d) 3 10. Global minimum value y = f(x) for x â, [0,2] is (a)5 (b)7 (c)8 d) 9 11 Global maximum value of y = f(x) for x â, [0,2] is (a) 5 (b) 7 (c) 8 (d) 9

A. 0

B. 1

C. 2

D. 3

Answer: C

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2. For certain curves y= f(x) satisfying $\frac{d^2y}{dx^2} = 6x - 4$, f(x) has local minimum value 5 when x=1. 9. Number of critical point for y=f(x) for x â, [0,2] (a) 0 (b)1. c).2 d) 3 10. Global minimum value y = f(x) for x â, [0,2] is (a)5 (b)7 (c)8 d) 9 11 Global maximum value of y = f(x) for x â, [0,2] is (a) 5 (b) 7 (c) 8 (d) 9

A. 5

B. 7

C. 8

D. 9

Answer: A

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3. For certain curve $y = f(x)$ satisfying $\frac{d^2y}{dx^2} = 6x - 4$, $f(x)$ has local minimum value 5 when $x = 1$

Global maximum value of $y = f(x)$ for $x \in [0, 2]$ is

A. 5

B. 7

C. 8

D. 9

Answer: B

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4. The differential equation $y = px + f(p)$, ...(i) where $p = \frac{dy}{dx}$,is known as Clairout's equation. To solve equation i) differentiate it with respect to x, which gives either $\frac{dp}{dx} = 0 \Rightarrow p = c$..............(ii) or

$x + f^i(p) = 0$............(iii) Which of the following is true about solutions of differential equation $y = xy' + \sqrt{1 + y^{'2}}$?

A. the general solution of equation is family of parabolas

B. the general solution of equation is family of circles

C. the singular solution of equation is circle

D. the singular solution of equation is ellipse

Answer: C

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5. The differential equation $y = px + f(p)$, ...(i)

where $p = \dfrac{dy}{dx}$,is known as Clairout's equation. To solve equation i) differentiate it with respect to x, which gives either

$\dfrac{dp}{dx} = 0 \Rightarrow p = c$.............(ii)

or $x + f^i(p) = 0$............(iii)

The number of solution of the equation $f(x) = -1$ and the singular solution of the equation $y = x\dfrac{dy}{dx} + \left(\dfrac{dy}{dx}\right)^2$ is

A. 1

B. 2

C. 4

D. 0

Answer: B

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6. The differential equation $y = px + f(p)$, ...(i)

where $p = \frac{dy}{dx}$,is known as Clairout's equation. To solve equation i) differentiate it with respect to x, which gives either

$\frac{dp}{dx} = 0 \Rightarrow p = c$..............(ii)

or $x + f^{i}(p) = 0$............(iii)

The singular solution of the differential equation $y = mx + m - m^3$, where $m = \frac{dy}{dx}$, passes through the point.

A. (0,0)

B. (0,1)

C. (1,0)

D. (-1,0)

Answer: D

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7. Let $f(x)$ be a non-positive continuous function and $F(x) = \int_0^x f(t)dt \, \forall x \geq 0$ and $f(x) \geq cF(x)$ where $c < 0$ and let $g: [0, \infty) \to R$ be a function such that $\frac{dg(x)}{dx} < g(x) \, \forall x > 0$ and $g(0) = 0$

The total number of root(s) of the equation $f(x) = g(x)$ is/ are

A. ∞

B. 1

C. 2

D. 0

Answer: B

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8. Let $f(x)$ be a non-positive continuous function and $F(x) = \int_0^x f(t)dt \, \forall x \geq 0$ and $f(x) \geq cF(x)$ where $c < 0$ and let $g: [0, \infty) \to R$ be a function such that $\frac{dg(x)}{dx} < g(x) \, \forall x > 0$ and $g(0) = 0$

The number of solution(s) of the equation

$|x^2 + x - 6| = f(x) + g(x)$ is/are

A. 2

B. 1

C. 0

D. 3

Answer: C

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9. Let $f(x)$ be a non-positive continuous function and $F(x) = \int_0^x f(t)dt \, \forall x \geq 0$ and $f(x) \geq cF(x)$ where $c < 0$ and let $g: [0, \infty) \to R$ be a function such that $\frac{dg(x)}{dx} < g(x) \, \forall x > 0$ and $g(0) = 0$

The solution set of inequation $g(x)\left(\cos^{-1} x - \sin^{-1}\right) \leq 0$

A. $\left[-1, \frac{1}{\sqrt{2}}\right]$

B. $\left[\frac{1}{\sqrt{2}}, 1\right]$

C. $\left[0, \frac{1}{\sqrt{2}}\right]$

D. $\left[0, \frac{1}{\sqrt{2}}\right]$

Answer: A

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10. A curve 'C' with negative slope through the point(0,1) lies in the I Quadrant. The tangent at any point 'P' on it meets the x-axis at 'Q'. Such that $PQ = 1$. Then

The curve in parametric form is

A. $x = \cos\theta + \log_e \tan(\theta/2), y = \sin\theta$

B. $x = -\cos\theta + \log_e \tan(\theta/2), y = \sin\theta$

C. $x = -\cos\theta - \log_e \tan\theta/2, y = \sin\theta$

D. None of these

Answer: C

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11. A curve 'C' with negative slope through the point(0,1) lies in the I Quadrant. The tangent at any point 'P' on it meets the x-axis at 'Q'. Such that $PQ = 1$. Then

The area bounded by 'C' and the co-ordinate axes is

A. 1

B. $\log_e 2$

C. $\pi/4$

D. $\pi/2$

Answer: C

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12. A curve 'C' with negative slope through the point(0,1) lies in the I Quadrant. The tangent at any point 'P' on it meets the x-axis at 'Q'. Such that $PQ = 1$. Then

The orthogonal trajectories of 'C' are

A. Circles

B. Parabolas

C. Ellipses

D. Hyperbolas

Answer: A

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13. Let $y = f(x)$ satisfies the equation $f(x) = (e^{-x} + e^{x})\cos x - 2x - \int_0^x (x-t)f'(t)dt$ y satisfies the differential equation

A. $\frac{dy}{dx} + y = e^{x}(\cos x - \sin x) - e^{-x}(\cos x + \sin x)$

B. $\frac{dy}{dx} - y = e^{x}(\cos x - \sin x) + e^{-x}(\cos x + \sin x)$

C. $\frac{dy}{dx} + y = e^{x}(\cos x + \sin x) - e^{-x}(\cos x - \sin x)$

D. $\frac{dy}{dx} - y = e^{x}(\cos x - \sin x) + e^{-x}(\cos x - \sin x)$

Answer: A

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14. Let $y = f(x)$ satisfies the equation

$f(x) = (e^{-x} + e^{x})\cos x - 2x + \int_0^x (x-t)f'(t)dt$

The value of $f(0) + f'(0)$ equal

A. -1

B. 0

C. 1

D. 1

Answer: B

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15. Let $y = f(x)$ satisfies the equation $f(x) = \left(e^{-x} + e^{x}\right)\cos x - 2x - \int_0^x (x-t) f'(t) dt$

A. $e^{-x}(\cos x - \sin x) + \frac{e^x}{5}(3\cos x + \sin x) + \frac{2}{5}e^{-x}$

B. $e^{-x}(\cos x + \sin x) + \frac{e^x}{5}(3\cos x - \sin x) - \frac{2}{5}e^{-x}$

C. $e^{-x}(\cos x - \sin x) + \frac{e^x}{5}(3\cos x - \sin x) + \frac{2}{5}e^{-x}$

D. $e^{-x}(\cos x + \sin x) + \frac{e^x}{5}(3\cos x - \sin x) - \frac{2}{5}e^{-x}$

Answer: C

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16. A certain radioactive material is known to decay at a rate proportional to the amount present. If initially there is 50 kg of the material present and after two hours it is observed that the material has lost 10% of its original mass, find () and expression for the mass of the material remaining at any time t, (ii) the mass of the material after four hours and (iii) the time at which the material has decayed to one half of its initial mass.

A. $N = 50e^{(1/2)(\ln 9)t}$

B. $50e^{(1/4)(\ln 9)t}$

C. $N = 50e^{-(\ln 0.9)t}$

D. None of these

Answer: A

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17. A certain radioactive material is known to decay at a rate proportional to the amount present. If initially there is 50 kg of the material present and after two hours it is observed that the material has lost 10% of its original mass, find () and expression for the mass of the material remaining at any time t, (ii) the mass of the material after four hours and (iii) the time at which the material has decayed to one half of its initial mass.

A. $50^{-0.5 \ln 9}$

B. $50e^{-2 \ln 9}$

C. $50e^{-2 \ln 0.9}$

D. None of these

Answer: C

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18. A certain radioactive material is known to decay at a rate proportional to the amount present. If initially there is 50 kg of the material present and after two hours it is observed that the material has lost 10% of its original mass, find () and expression for the mass of the material remaining at any time t, (ii) the mass of the material after four hours and (iii) the time at which the material has decayed to one half of its initial mass.

A. $\frac{\ln\ 0.25}{\ln\ 0.9}h$

B. $\frac{\ln\ 0.5}{\ln\ 0.81}h$

C. $\frac{\ln\ 0.25}{\ln\ 0.81}h$

D. None of these

Answer: C

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19. Consider a tank which initially holds V_0 liter of brine that contains a lb of salt. Another brine solution, containing b lb of salt per liter is poured into the tank at the rate of eL/min. The problem is to find the amount of salt in the tank at any time t.

Let Q denote the amount of salt in the tank at any time. The time rate of change of Q, $\frac{dQ}{dt}$, equals the rate at which salt enters the tank at the rate at which salt leaves the tank. Salt enters the tank at the rate of be lb/min.

To determine the rate at which salt leaves the tank, we first calculate the volume of brine in the tank at any time t, which is the initial volume V_0 plus the volume of brine added et minus the volume of brine removed ft. Thus, the volume of brine at any time is

$V_0 + et - ft$

The concentration of salt in the tank at any time is $Q/(V_0 + et - ft)$, from which it follows that salt leaves the tank at the rate of $f\left(\frac{Q}{V_0 + et - ft}\right)$lb/min. Thus,

$$\frac{dQ}{dt} = be - f\left(\frac{Q}{V_0 + et - ft}\right)Q = be$$

A tank initially holds 100 L of a brine solution containing 20 lb of salt. At t=0, fresh water is poured into the tank at the rate of 5 L/min, while the well-stirred mixture leaves the tank at the same rate. Then the amount of salt in the tank after 20 min.

A. 20/e

B. 10/e

C. $40 / e^2$

D. 5/e L

Answer: A

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20. A 50 L tank initailly contains 10 L of fresh water, At t=0, a brine solution containing 1 lb of salt per gallon is poured into the tank at the rate of 4 L/min, while the well-stirred mixture leaves the tank at the rate of 2 L/min. Then the amount of time required for overflow to occur in

A. 30 min

B. 20 min

C. 10 min

D. 40 min

Answer: B

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21. In the above question, the amount of salt in the tank at the moment of overflow is

A. 20 lb

B. 50 lb

C. 30 lb

D. None of these

Answer: D

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Matrix Match Type

1. Match the following lists:

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2. Match the following lists:

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3. Match the differential equation in List I with its solution in List II and then choose the correct code.

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Exercise (Numerical)

1. If $y = y(x)$ and it follows the relation $4xe^{xy} = y + 5\sin^2 x$, then $y'(0)$ is equal to_____

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2. If $x\frac{dy}{dx} = x^2 + y - 2, y(1) = 1$, then $y(2)$ equals ....

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3. If the dependent variable y is changed to z by the substitution method y=tanz then the differential equation $d^2\frac{y}{dx^2} = 1 + 2\frac{1+y}{1+y^2}\left(\frac{dy}{dx}\right)^2$ is changed to $d^2\frac{z}{dx^2} = \cos^2 z + k\left(\frac{dz}{dx}\right)^2$ then find the value off k

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4. Let y=y(t) be a solution to the differential equation $y' + 2ty = t^2$, then $16 \lim_{t \to \infty t} \frac{y}{t}$ is..................

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5. If the solution of the differential equation $\frac{dy}{dx} = \frac{1}{x \cos y + \sin 2y}$ is $x = ce^{\sin y} - k(1 + \sin y)$, then the value of k is______

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6. If the independent variable x is changed to y, then the differential equation $x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} = 0$ is changed to $x\frac{d^2x}{dy^2} + \left(\frac{dx}{dy}\right)^2 = k$ where k equals___

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7. Let y_1 and y_2 be two different solutions of the equation $\frac{dy}{dx} + P(x).\,y = Q(x)$. Then $\alpha y_1 + \beta y_2$ will be solution of the given equation if $\alpha + \beta = \ldots\ldots\ldots\ldots\ldots\ldots$

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8. Tangent is drawn at the point (x_i, y_i) on the curve $y = f(x)$, which intersects the x-axis at $(x_{i+1}, 0)$. Now, again a tangent is drawn at (x_{i+1}, y_{i+1}) on the curve which intersect the x-axis at $(x_{i+2}, 0)$ and the process is repeated n times, i.e. $i = 1, 2, 3; n\cdot$ If $x_1, x_2, x_3, ;\, x_n$ from an arithmetic progression with common difference equal to $(\log)_2 e$ and curve passes through $(0, 2)\cdot$ Now if curve passes through the point $(-2, k)$, then the value of k is____

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9. The perpendicular from the origin to the tangent at any point on a curve is equal to the abscissa of the point of contact. Also curve passes through the point (1,1). Then the length of intercept of the curve on the x-axis is__________

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10. If the eccentricity of the curve for which tangent at point P intersects the y-axis at M such that the point of tangency is equidistant from M and the origin is e, then the value of $5e^2$ is___

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11. If the solution of the differential equation $\frac{dy}{dx} - y = 1 - e^{-x}$ and $y(0) = y_0$ has a finite value, when $x \vec{\infty}$, then the value of $\left|\frac{2}{y_0}\right|$ is__

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12. Let f be a function defined on the interval $[0, 2\pi]$ such that $\int_0^x (f'(t) - \sin 2t)dt = \int_x^0 f(t)\tan t dt$ and $f(0) = 1$. Then the maximum value of $f(x)$is......................

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13. Let $y(x)$ be a function satisfying $\frac{d^2y}{dx^2} - \frac{dy}{dx} + e^{2x} = 0$, y(0)= and $y'(0) = 1$. If maximum value of $y(x)$ is $y(\alpha)$, then integral part of 2α is.................

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14. if the differential equation of a curve, passing through $\left(0, -\frac{\pi}{4}\right)$ and $(t, 0)$ is $\cos y\left(\frac{dy}{dx} + e^{-x}\right) + \sin y\left(e^{-x} - \frac{dy}{dx}\right) = e^{e^{-x}}$ then find the value of $t.\ e^{e^{-1}}$

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15. Let f be a continuous function satisfying the equation $\int_0^x f(t)dt + \int_0^x tf(x-t)dt = e^{-x} - 1$, then find the value of $e^9 f(9)$ is equal to......................

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JEE Main Previous Year

1. The differential equaiotn which represents the family of curves $y = C_1 e^{C_2 x}$, where C_1 and C_2 are arbitrary constants, is

A. $y' = y^2$

B. $y'' = y' y$

C. $yy'' = y'$

D. $yy'' = (y')^2$

Answer: D

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2. Solution of the differential equation

$\cos x dy = y(\sin x - y)dx, 0 < x < \frac{\pi}{2}$ is

A. $\tan x = (\sec x + c)y$

B. $\sec x = (\tan x + c)y$

C. $y \sec x = \tan x + c$

D. $y \tan x = \sec x + c$

Answer: B

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3. Let I be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\left(dV\frac{t}{dt} = -k(T-t)\right.$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is : (1) $T^2 - \frac{1}{k}$ (2) $I - \frac{kT^2}{2}$ (3) $I - \frac{k(T-t)^2}{2}$ (4) e^{-kT}

A. e^{-kT}

B. $T^2 - \frac{I}{k}$

C. $I - \frac{kT^2}{2}$

D. $I - \frac{k(T-t)^2}{2}$

Answer: C

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4. If $\frac{dy}{dx} = y + 3 \text{ and } y(0) = 2$, then y(ln 2) is equal to

A. -2

B. 7

C. 5

D. 13

Answer: B

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5. A spherical balloon is filled with 4500p cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is (1) $\frac{9}{7}$ (2) $\frac{7}{9}$ (3) $\frac{2}{9}$ (4) $\frac{9}{2}$

A. $6/7$

B. $4/9$

C. $2/9$

D. None of these

Answer: C

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6. The population p(t) at time t of a certain mouse species satisfies the differential equation $\left(dp\frac{t}{dt} = 0.5p(t) - 450\right.$ If $p(0) = 850$, then the time at which the population becomes zero is (1) 2 ln 18 (2) ln 9 (3) $\frac{1}{2}$ ln 18 (4) ln 18

A. 2 ln 18

B. ln 9

C. $\frac{1}{2}$ ln 18

D. ln 18

Answer: A

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7. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}.$ If the firm employs 25 more workers, then the new level of production of items is (1) 3000 (2) 3500 (3) 4500 (4) 2500

A. 2500

B. 3000

C. 3500

D. 4500

Answer: C

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8. Let the population of rabbits surviving at a time t be governed by the differential equation $\left(dp\frac{t}{dt} = \frac{1}{2}p(t) - 200.\right.$ If $p(0) = 100$, then p(t) equals (1) $400 - 300e^{t/2}$ (2) $300 - 200e^{-t/2}$ (3) $600 - 500e^{t/2}$ (4) $400 - 300e^{-t/2}$

A. $40 - 300e^{t/2}$

B. $200 - 200e^{-t/2}$

C. $600 - 500e^{t/2}$

D. $400 - 300e^{-t/2}$

Answer: A

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9. Let y(x) be the solution of the differential equation $(x\log x)\frac{dy}{dx}+y=2x\log x, (x\geq 1)$, Then y(e) is equal to

A. e

B. 0

C. 2

D. 2e

Answer: C

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10. If a curve $y=f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation , $y(1+xy)dx=xdy$, then $f\left(-\frac{1}{2}\right)$ is equal to: (1) $-\frac{2}{5}$ (2) $-\frac{4}{5}$ (3) $\frac{2}{5}$ (4) $\frac{4}{5}$

A. $-\frac{4}{5}$

B. $\frac{2}{5}$

C. $\frac{4}{5}$

D. $-\frac{2}{5}$

Answer: C

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11. If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and $y(0)=1$, then $y\left(\frac{\pi}{2}\right)$ is equal to

A. $\frac{4}{3}$

B. $\frac{1}{3}$

C. $-\frac{2}{3}$

D. $-\frac{1}{3}$

Answer: B

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12. Let $y = g(x)$ be the solution of the differential equation $\frac{\sin(dy)}{dx} + y\cos x = 4x, x \in (0, \pi)$ If y(pi/2)=0, $then$y(pi/6)` is equal to

A. $-\frac{4}{9}\pi^2$

B. $\frac{4}{9\sqrt{3}}\pi^2$

C. $-\frac{8}{9\sqrt{3}}\pi^2$

D. $-\frac{8}{9}\pi^2$

Answer: D

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JEE Advanced Previous Year

1. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0$. Then the equation of the

curve is (a) $(b)(c)\sin\left((d)(e)(f)\frac{y}{g}x(h)(i)(j)\right) = \log x + (k)\frac{1}{l}2(m)(n)(o)$ (p) (q) $(r)(s)\mathrm{cosec}\left((t)(u)(v)\frac{y}{w}x(x)(y)(z)\right) = \log x + 2(aa)$ (bb) (cc) $(dd)(ee)\sec\left((ff)(gg)(hh)\frac{(ii)2y}{jj}x(kk)(ll)(mm)\right) = \log x + 2(nn)$

(oo) (pp) *[Math Processing Error]* (fff)

A. $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$

B. $\mathrm{cosec}\left(\frac{y}{x}\right) = \log x + 2$

C. $\frac{\sec(2y)}{x} = \log x + 2$

D. $\frac{\cos(2y)}{x} = \log x + \frac{1}{2}$

Answer: A

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2. The function $y = f(x)$ is the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$ in $(-1, 1)$ satisfying $f(0) = 0$. Then $\int_{\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is (a) $(b)(c)(d)\frac{\pi}{e}3(f)(g) - (h)\frac{(i)\sqrt{(j)3(k)(l)}}{m}2(n)(o)(p)$

(q) (b) $(r)(s)(t)\frac{\pi}{u}3(v)(w) - (x)\frac{(y)\sqrt{(z)3(aa)}(bb)}{cc}4(dd)(ee)(ff)$ (gg)

(c) $(d)(e)(f)\frac{\pi}{g}6(h)(i) - (j)\frac{(k)\sqrt{(l)3(m)}(n)}{o}4(p)(q)(r)$ (s) (d)

$(t)(u)(v)\frac{\pi}{w}6(x)(y) - (z)\frac{(aa)\sqrt{(bb)3(cc)}(dd)}{ee}2(ff)(gg)(hh)$ (ii)

A. $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$

B. $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

C. $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$

D. $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

Answer: B

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3. If $y = y(x)$ satisfies the differential equation $8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx, x > 0$ and $y(0) = \sqrt{7},$ then $y(256) =$

A. 3

B. 9

C. 16

D. 80

Answer: A

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4. If $y(x)$ satisfies the differential equation $y' - y\tan x = 2x secx$ and $y(0) = 0$, then (a)

$(b)(c)y\left((d)(e)(f)\frac{\pi}{g}4(h)(i)(j)\right) = (k)\frac{(l)(m)\pi^{(n)2(o)}(p)}{q}\left((r)8\sqrt{(s)2(t)(}\right.$

(y) (b) [Math Processing Error] (xx) (c)

$(d)(e)y\left((f)(g)(h)\frac{\pi}{i}3(j)(k)(l)\right) = (m)\frac{(n)(o)\pi^{(p)2(q)}(r)}{s}9(t)(u)(v)$

(w) (d) [Math Processing Error] (ddd)

A. $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$

B. $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$

C. $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$

D. $y'\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$

Answer: A::D

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5. Consider the family of all circles whose centers lie on the straight line $y=x$. If this family of circles is represented by the differential equation $Py^{+}Qy'+1=0$, where P, Q are functions of x, y and $y'\left(herey'=\frac{dy}{dx}, y^{=}\frac{d^2y}{dx^2}\right)$, then which of the following statements is (are) true? (a) $(b)(c)P=y+x(d)$ (e) (b) $(f)(g)P=y-x(h)$ (i) (c) $(d)(e)P+Q=1-x+y+y+(f)(g)\left((h)(i)y^{(j)\ '(k)}(l)(m)\right)^{(n)2(o)}$ (p)

(r) (s)

$(t)(u)P-Q=x+y-y-(v)(w)\left((x)(y)y^{(z)\ '(aa)}(bb)(cc)\right)^{(dd)2(ee)}$ $(f$

(hh)

A. $P=y+x$

B. $P=y-x$

C. $P+Q=1-x+y+y'+(y')^2$

D. $P - Q = x + y - y' - (y')^2$

Answer: B::C

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6. Let $y(x)$ be a solution of the differential equation $(1 + e^x)y' + ye^x = 1$. If $y(0) = 2$, then which of the following statements is (are) true? (a) $(b)(c)y((d)(e) - 4(f)) = 0(g)$ (h) (b) $(i)(j)y((k)(l) - 2(m)) = 0(n)$ (o) (c) $(d)(e)y((f)x(g))(h)$ (i) has a critical point in the interval $(j)(k)((l)(m) - 1, 0(n))(o)$ (p) (q) $(r)(s)y((t)x(u))(v)$ (w) has no critical point in the interval $(x)(y)((z)(aa) - 1, 0(bb))(cc)$ (dd)

A. $y(-4) = 0$

B. $y(-2) = 0$

C. $y(x)$ has a critical point in the interval $(-1, 0)$

D. $y(x)$ has no critical point in the interval $(-1, 0)$

Answer: A::C

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7. A solution curve of the differential equation $\left(x^2 + xy + 4x + 2y + 4\right)\left(\frac{dy}{dx}\right) - y^2 = 0$ passes through the point $(1, 3)$ Then the solution curve is

A. intersects $y = x + 2$ exactly at one point

B. intersects $y = x + 2$ exactly at two points

C. intersects $y = (x + 2)^2$

D. does NOT intersect $y = (x + 3)^2$

Answer: A::D

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8. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be two non-constant differentiable functions. If $f'(x) = \left(e^{(f(x) - g(x))}\right) g'(x)$ for all $x \in R$, and $f(1) = g(2) = 1$, then which of the following statement(s) is (are) TRUE? $f(2) < 1 - (\log)_e 2$ (b) $f(2) > 1 - (\log)_e 2$ (c) $g(1) > 1 - (\log)_e 2$ (d) $g(1) < 1 - (\log)_e 2$

A. $f(2) < 1 - \log_e 2$

B. $f(2) > 1 - \log_e(2)$

C. $g(1) < 1 - \log_e 2$

D. $g(1) < 1 - \log_e 2$

Answer: B::C

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9. Let f be a real-valued differentiable function on R (the set of all real numbers) such that $f(1) = 1$. If the $y - \in tercept$ of the tangent at any point $P(x, y)$ on the curve $y = f(x)$ is equal to the cube of the abscissa of $P,$ then the value of $f(-3)$ is equal to_______

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10. Let $y'(x) + y(x)g'(x) = g(x)g'(x), y(0), x \in R,$ where $f'(x)$ denotes $\dfrac{dy(x)}{dx}$, and $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0.$ Then the value of $y(2)$ is_____

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11. Let $f: [1, \infty]$ be a differentiable function such that $f(1) = 2.$ If $\int_1^x f(t)dt = 3xf(x) - x^3$ for all $x \geq 1,$ then the value of $f(2)$ is

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12. Let $f: R \rightarrow R$ be a differentiable function with $f(0) = 0$. If $y = f(x)$ satisfies the differential equation

$\frac{dy}{dx} = (2+5y)(5y-2)$, then the value of $\lim_{x \to \infty} f(x)$ is..................

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Single Correct Answer Type

1. The order of the differential equation whose general solution is $y = c_1 \cos 2x + c_2 \cos^2 x + c_3 \sin^2 x + c_4$ is

A. 2

B. 4

C. 3

D. None of these

Answer: A

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2. Order of the differential equation whose general solution is $y = \frac{ax}{bx + c}$, where a, b, c are arbitrary constants is

A. 1

B. 2

C. 3

D. None of these

Answer: B

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3. If p and q are order and degree of differential equation $y^2\left(\frac{d^2y}{dx^2}\right)^2 + 3x\left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^2y^2 = \sin x, then$

A. $p > q$

B. $\frac{p}{q} = \frac{1}{2}$

C. $p = q$

D. $p < q$

Answer: D

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4. If m and n are the order and degree of the differential equation $(y'')^5 + 4.\frac{(y'')^3}{y'''} + y''' = \sin x, then$

A. m = 3, n = 5

B. m = 3, n = 1

C. m = 3, n = 3

D. m = 3, n = 2

Answer: D

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5. If the differential equation $\frac{dx}{3y+f}+\frac{dy}{px+g}=0$ represents a family of circle, then p=

A. g

B. f

C. 4

D. 3

Answer: D

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6. The general solution of $\frac{dy}{dx}=1-x^2-y^2+x^2y^2$ is

A. $2\sin^{-1}y=x\sqrt{1-y^2}+c$

B. $\sin^{-1}y=\frac{1}{2}\sin^{-1}x+c$

C. $\cos^{-1}y=x\cos^{-1}x+c$

D. $\frac{1}{2}\log\left(\frac{1+y}{1-y}\right) = x - \frac{x^3}{3} + c$

Answer: D

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7. The solution of the differential equation $e^{-x}(y+1)dy + \left(\cos^2 x - \sin 2x\right)y(dx) = 0$ subjected to the condition that y = 1 when x = 0 is

A. $(y+1) + e^x \cos^2 x = 2$

B. $y + \log y = e^x \cos^2 x$

C. $\log(y+1) + e^x \cos^2 x = 1$

D. $y = \log y + e^x \cos^2 x = 2$

Answer: D

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8. The solutio of the differential equation $\left(x^2 - xy^2\right)\dfrac{dy}{dx} + y^2 + xy^2 = 0$ is

A. $\log\left(\dfrac{x}{y}\right) = \dfrac{1}{x} + \dfrac{1}{y} + c$

B. $\log\left(\dfrac{y}{x}\right) = \dfrac{1}{x} + \dfrac{1}{y} + c$

C. $\log(xy) = \dfrac{1}{x} + \dfrac{1}{y} + c$

D. $\log(xy) + \dfrac{1}{x} + \dfrac{1}{y} = c$

Answer: A

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9. The family of curves passing through $(0, 0)$ and satisfying the differential equation $\dfrac{y_2}{y_1} = 1$ $\left(\text{where, } y_n = \dfrac{d^n y}{dx^n}\right)$ is (A) $y = k$ (B) $y = kx$ (C) $y = k(e^x + 1)$ (C) $y = k(e^x - 1)$

A. y = k

B. y = kx

C. $y = k(e^x + 1)$

D. $y = k(e^x - 1)$

Answer: D

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10. The solution of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$ is

A. $\tan^{-1}\left(\frac{x}{y}\right) + In \quad y + C = 0$

B. $2\tan^{-1}\left(\frac{x}{y}\right) + In \quad x + C = 0$

C. $In\left(y + \sqrt{x^2 + y^2}\right) + In \quad y + C = 0$

D. $In\left(x + \sqrt{x^2 + y^2} + C = 0\right.$

Answer: A

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11. The solution of differential equation $\left(1 - xy + x^2y^2\right)dx = x^2dy$ is

A. tan xy = log |cx|

B. tan (y/x) = tan log |cx|

C. xy = tan log |cx|

D. None of these

Answer: C

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12. If y(t) satisfies the differential equation $y'(t) + 2y(t) = 2e^{-2t}$, $y(0) = 2$ then y(1) equals

A. $\frac{3}{e}$

B. $\frac{3}{e^2}$

C. $\frac{4}{e}$

D. $\frac{4}{e^2}$

Answer: D

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13. If $\frac{dy}{dx} - y\log_e 2 = 2^{\sin x}(\cos x - 1)\log_e 2$, then y =

A. $2^{\sin x} + c2^x$

B. $2^{\cos x} + c2^x$

C. $2^{\sin x} + c2^{-x}$

D. $2^{\cos x} + c2^{-x}$

Answer: A

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14. If $ye^y dx = (y^3 + 2xe^y)dy, y(0) = 1$, then the value of x when y = 0 is

A. -1

B. 0

C. 1

D. 2

Answer: B

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15. If $y_1(x)$ is a solution of the differential equation $\frac{dy}{dx} - f(x)y = 0$, then a solution of the differential equation $\frac{dy}{dx} + f(x)y = r(x)$ is

A. $y = \frac{1}{y_1(x)} \int r(x)y_1(x)dx + \frac{c}{y_1(x)}$

B. $y = y_1(x) \int \frac{r(x)}{y_1(x)} dx + c$

C. $y = \int r(x)y_1(x)dx + c$

D. None of these

Answer: A

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16. The general solution of $x\left(\frac{dy}{dx}\right)+(\log x)y=x^{-\frac{1}{2}\log x}$ is

A. $y=x^{1-\frac{1}{2}\log x}+cx^{-\frac{1}{2}\log x}$

B. $y.\,x^{\frac{1}{2}\log x}=x^{\frac{1}{2}\log x}+c$

C. $y=e^{\frac{(\log x)^2}{2}}(x+c)$

D. $y=e^{\frac{1}{2}(\log x)^2}\left(x^{1-\frac{1}{2}(\log x)}-x^{-\frac{1}{2}\log x}\right)+c$

Answer: A

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17. Find the general solution of the differential equation $(1+\tan y)(dx-dy)+2xdy=0$

A. $x(\sin y+\cos y)=\sin y+ce^{y}$

B. $x(\sin y+\cos y)=\sin y+ce^{-y}$

C. $y(\sin x+\cos x)=\sin x+ce^{x}$

D. None of these

Answer: B

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18. Solution of differential equation $x^2y - x^3\frac{dy}{dx} = y^4 \cos x$ is

A. $x^2y^{-3} = 2\sin x + c$

B. $x^2y^{-3} = 3\cos x + c$

C. $x^3y^{-3} = 3\sin x + c$

D. $x^2y^3 = 3\sin x + cx^2y$

Answer: C

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19. Suppose a solutions of the differential equation $(xy^3 + x^2y^7)\frac{dy}{dx} = 1$ satisfies the initial conditions $y\left(\frac{1}{4}\right) = 1$. Then the value of $\frac{dy}{dx}$ when $y = -1$ is

A. $-\frac{3}{20}$

B. $-\frac{20}{3}$

C. $-\frac{5}{16}$

D. $-\frac{16}{5}$

Answer: D

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20. The general solution of the differential equation $\frac{dy}{dx} = y\tan x - y^2 \sec x$ is

A. $\tan x = (c + \sec x)y$

B. $\sec y = (c + \tan y)x$

C. $\sec x = (c + \tan x)y$

D. None of these

Answer: C

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21. The solution of differential equation $x^2(xdy + ydx) = (xy - 1)^2 dx$ is (where c is an arbitrary constant)

A. $xy - 1 = cx$

B. $xy - 1 = cx^2$

C. $\frac{1}{xy - 1} = \frac{1}{x} + c$

D. None of these

Answer: C

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22. Solution of the differential $(x + 2y^3) = \frac{dx}{dy} y$ is

A. $x = y^2(c + y^2)$

B. $x = y(c - y^2)$

C. $x = 2y(c - y^2)$

D. $x = y(c + y^2)$

Answer: D

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23. General solution of differential equation $x^2\left(x + y\frac{dy}{dx}\right) + \left(x\frac{dy}{dx} - y\right)\sqrt{x^2 + y^2} = 0$ is

A. $\frac{1}{\sqrt{x^2 + y^2}} + \frac{y}{x} = c$

B. $\sqrt{x^2 + y^2} - \frac{y}{x} = c$

C. $\sqrt{x^2 + y^2} + \frac{y}{x} = c$

D. $2\sqrt{x^2 + y^2} + \frac{y}{x} = c$

Answer: C

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24. Solution of the differential y' = $\frac{3yx^2}{x^3 + 2y^4}$ is

A. $x^3y^{-1} = \frac{2}{3}y^3 + c$

B. $x^2y^{-1} = \frac{2}{3}y^3 + c$

C. $xy^{-1} = \frac{2}{3}y^3 + c$

D. None of these

Answer: A

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25. For $y > 0 \text{ and } x \in R, ydx + y^2dy = xdy$ where y = f(x). If f(1)=1, then the value of f(-3) is

A. 1

B. 2

C. 3

D. 4

Answer: C

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26. An equation of the curve satisfying $xdy - ydx = \sqrt{x^2 - y^2}dx$ and y(1) = 0 is

A. $y = x^2 \log|\sin x|$

B. $y = x \sin(\log|x|)$

C. $y^2 = x(x - 1)^2$

D. $y = 2x^2(x - 1)$

Answer: B

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27. The solution of $\left(y\left(1+x^{-1}\right)+\sin y\right)dx+(x+\log x+x\cos y)dy=0$ is

A. $\left(1+y^{-1}\sin y\right)+x^{-1}\log x=c$

B. $(y+\sin y)+xy\log x=C$

C. $xy+y\log x+x\sin y=C$

D. None of these

Answer: C

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28. The solution of $(1+x)\dfrac{dy}{dx}+1=e^{x-y}$ is

A. $e^{y}(x+1)=c$

B. $e^{y}(x+1)=e^{x}+c$

C. $e^{y}(x+1) = ce^{x}$

D. $(x+1) = e^{x} + c$

Answer: B

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29. The solution of differential equation $x \sec\left(\frac{y}{x}\right)(ydx + xdy) = \text{y cosec}\left(\frac{y}{x}\right)(xdy - ydx)$ is

A. $xy = \text{c cosec}\left(\frac{y}{x}\right)$

B. $xy^{2}\sin\frac{y}{x} = c$

C. $xy\text{cosec}\frac{y}{x} = c$

D. $xy = c\sin\left(\frac{x}{y}\right)$

Answer: C

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30. The general solution of the differential equation $\sqrt{1-x^2y^2}dx = ydx + xdy$ is

A. $\sin(xy) = x + c$

B. $\sin^{-1}(xy) + x = c$

C. $\sin(x + c) = xy$

D. $\sin(xy) + x = c$

Answer: C

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31. $(1 + xy)ydx + (1 - xy)xdy = 0$

A. $\frac{x}{y} + \frac{1}{xy} = k$

B. $\log\left(\frac{x}{y}\right) = \frac{1}{xy} + k$

C. $\frac{x}{y} + \frac{1}{xy} = k$

D. $\log\left(\frac{x}{y}\right) = xy + k$

Answer: C

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32. Solution of the differential equation $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$, is

A. $e^{-2x}y^2 + 2In|y| = c$

B. $e^{2x}y^2 = 2In|y| = c$

C. $e^x + In|y| = c$

D. None of these

Answer: A

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33. A popular grows at the rate of 10% of the population per year. How long does it take for the population to doble ?

A. 2 log 10 years

B. 20 log 2 years

C. 10 log 2 years

D. 5 log 2 years

Answer: C

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34. A tangent drawn to the curve y = f(x) at P(x, y) cuts the x and y axes at A and B, respectively, such that AP : PB = 1 : 3. If f(1) = 1 then the curve passes through $\left(k, \frac{1}{8}\right)$ where k is

A. 1

B. 2

C. 3

D. 4

Answer: B

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35. A curve 'C' with negative slope through the point(0,1) lies in the I Quadrant. The tangent at any point 'P' on it meets the x-axis at 'Q'. Such that $PQ = 1$. Then

The curve in parametric form is

A. $x = \cos\theta + In \tan(\theta/2), y = \sin\theta$

B. $x = -\cos\theta + In \tan(\theta/2), y = \sin\theta$

C. $x = -\cos\theta - In \tan(\theta/2), y = \sin\theta$

D. None of these

Answer: C

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36. Tangent to a curve intercepts the y-axis at a point P. A line perpendicular to this tangent through P passes through another point (1,0). The differential equation of the curve is (a) $(b)(c)y(d)\frac{(e)dy}{f}((g)dx)(h)(i) - x(j)(k)\Big((l)(m)(n)\frac{(o)dy}{p}((q)dx)(r)(s)(t$ (y) (b) *[Math Processing Error]* (eee) (c) $(d)(e)y(f)\frac{(g)dx}{h}((i)dy)(j)(k) + x = 1(l)$ (m) (d) None of these

A. $y.\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 1$

B. $x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 1$

C. $y.\frac{dx}{dy} + x = 1$

D. None of these

Answer: A

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37. The orthogonal trajectories of the family of curves an $a^{n-1}y = x^n$ are given by (A) $x^n + n^2y = cons\tan t$ (B) $ny^2 + x^2 = cons\tan t$ (C)

$n^2x + y^n = cons\tan t$ (D) $y = x$

A. $x^n + n^2y$ = const

B. $ny^2 + x^2$ = const

C. $n^2x + y^n$ = const

D. $n^2x - y^n$ = const

Answer: B

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Multiple Correct Answer Type

1. The differential equation for the family of curves $y = c\sin x$ can be given by

A. $\left(\frac{dy}{dx}\right)^2 = y^2\cot^2 x$

B. $\left(\frac{dy}{dx}\right)^2 - \left(\sec x\frac{dy}{dx}\right)^2 + y^2 = 0$

C. $\left(\frac{dy}{dx}\right)^2 = \tan^2 x$

D. $\frac{dy}{dx} = y \cot x$

Answer: A::B::D

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2. If the solution of the equation $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = 0$ given that for $t = 0, x = 0$ and $\frac{dx}{dt} = 12$ is in the form $x = Ae^{-3t} + Be^{-t}$, then

A. $A + B = 0$

B. $A + B = 12$

C. $|AB| = 36$

D. $|AB| = 49$

Answer: A::C

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3. The solution of $\left(\frac{dy}{dx}\right)^2 - 2\left(x + \frac{1}{4x}\right)\frac{dy}{dx} + 1 = 0$

A. $y = x^2 + c$

B. $y = \frac{1}{2}In(x) + c, x > 0$

C. $y = \frac{x}{2} + c$

D. $y = \frac{x^2}{2} + c$

Answer: A::B

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4. $\left(\frac{dy}{dx}\right)^2 + 2y\cot x\frac{dy}{dx} = y^2$ has the solution

A. $y + \frac{c}{1 + \cos x} = 0$

B. $y = \frac{c}{1 - \cos x}$

C. $x = 2\sin^{-1}\sqrt{\frac{c}{2y}}$

D. $x = 2\cos^{-1}\left(\frac{c}{2y}\right)$

Answer: A::B

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5. Let $\frac{dy}{dx} + y = f(x)$ where y is a continuous function of x with y(0) = 1 and $f(x) = \begin{cases} e^{-x} & \text{if } o \leq x \leq 2 \\ e^{-2} & \text{if } x > 2 \end{cases}$ Which of the following hold(s) good ?

A. $y(1) = 2e^{-1}$

B. $y'(1) = -e^{-1}$

C. $y(3) = -2e^{-3}$

D. $y'(3) = -2e^{-3}$

Answer: A::B::D

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6. A differentiable function satisfies $f(x) = \int_0^x (f(t)\cot t - \cos(t - x))dt.$

Which of the following hold(s) good?

A. f(x) has a minimum value 1 - e

B. f(x) has a maximum value 1 - e^{-1}

C. $f''\left(\frac{\pi}{2}\right) = e$

D. $f'(0) = 1$

Answer: A::B::C

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7. y = f(x) which has differential equation $y(2xy + e^x)dx - e^x dy = 0$ passing through the point (0,n 1). Then which of the following is/are true about the function?

A. $x = 1 + \sqrt{2}$ is point of local maxima

B. $x = 1 - \sqrt{2}$ is point of local minima

C. $\lim_{x \to \infty} f(x) = -\infty$

D. $\lim_{x \to -\infty} f(x) = 0$

Answer: A::B::C::D

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8. Suppose a curve whose sub tangent is n times the abscissa of the point of contact and passes through the point (2, 3). Then

A. for n = 1, equation of the curve is $2y = 3x$

B. for n = 1, equation of the curve is $2y^2 = 9x$

C. for n = 2, equation of the curve is $2y = 3x$

D. for n = 2, equation of the curve is $2y^2 = 9x$

Answer: A::D

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9. Let C be a curve such that the normal at any point P on it meets x-axis and y-axis at A and B respectively. If BP : PA = 1 : 2 (internally) and the curve passes through the point (0, 4), then which of the following alternative(s) is/are correct?

A. The curves passes through $(\sqrt{10}, -6)$.

B. The equation of tangent at $(4, 4\sqrt{3})$ is $2x + \sqrt{3}y = 20$.

C. The differential equation for the curve is yy' + 2x = 0.

D. The curve represents a hyperbola.

Answer: A::D

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10. The normal at a general point (a, b) on curve makes an angle θ with x-axis which satisfies $b(a^2 \tan\theta - \cot\theta) = a(b^2 + 1)$. The equation of curve can be

A. $y = e^{x^2/2} + c$

B. $\log ky^2 = x^2$

C. $y = ke^{x^2/2}$

D. $x^2 - y^2 = k$

Answer: B::C::D

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Comprehension Type

1. Tangent is drawn at the point (x_i, y_i) on the curve $y = f(x)$, which intersects the x-axis at $(x_{i+1}, 0)$. Now, again a tangent is drawn at (x_{i+1}, y_{i+1}) on the curve which intersect the x-axis at $(x_{i+2}, 0)$ and the process is repeated n times, i.e. $i = 1, 2, 3; n\cdot$ If $x_1, x_2, x_3, ;\cdot x_n$ from an arithmetic progression with common difference equal to $(\log)_2 e$ and curve passes through $(0, 2)\cdot$ Now if curve passes through the point $(-2, k)$, then the value of k is____

A. (1, 4)

B. (5, 1/16)

C. (2, 1/2)

D. None of these

Answer: B

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2. Tangent is drawn at the point (x_i, y_i) on the curve y = f(x), which intersects the x-axis at $(x_{i+1}, 0)$. Now again tangent is drawn at (x_{i+1}, y_{i+1}) on the curve which intersects the x-axis at $(x_{i+2}, 0)$ and the process is repeated n times i.e. i = 1, 2, 3,.......,n.

If $x_1, x_2, x_3, \ldots, x_n$ form a geometric progression with common ratio equal to 2 and the curve passes through (1, 2), then the curve is

A. circle

B. hyperbola

C. ellipse

D. parabola

Answer: B

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Solved Examples And Exercises

1. If $y=\frac{1}{\sqrt{a^2-b^2}}\cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$, then $\frac{d^2y}{dx^2}=$ (i) $\frac{b\sin x}{(a+b\cos x)^2}$ (ii) $-\frac{b\sin x}{(a+b\cos x)^2}$ (iii) $\frac{b\cos x}{(a+b\cos x)^2}$ (iii) $-\frac{b\cos x}{(a+b\cos x)^2}$

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2. Find the differential equation of all non-vertical lines in a plane.

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3. Find the differential equation of all the ellipses whose center is at origin and axis are co-ordinate axis.

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4. Consider the equation $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1,$ where a and b are specified constants and λ is an arbitrary parameter. Find a differential equation satisfied by it.

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5. Form the differential equation of all circle touching the x-axis at the origin and centre on the y-axis.

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6. Form the differential equation of the family of parabolas with focus at the origin and the axis of symmetry along the axis.

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7. Form the differential equation of family of lines situated at a constant distance p from the origin.

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8. Find the differential equation of all parabolas whose axis are parallel to the x-axis.

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9. A body at a temperature of $50^0 F$ is placed outdoors where the temperature is $100^0 F$. If the rate of change of the temperature of a body is proportional to the temperature difference between the body and its surrounding medium. If after 5 min the temperature of the body is $60^0 F$, find (a) how long it will take the body to reach a temperature of 75 ^ 0 F and (b) the temperature of the body after 20 min.

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10. Solve $\frac{dy}{dx} = \frac{2x - y + 1}{x + 2y - 3}$

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11. Solve $x^2\left(\frac{dy}{dx}\right) + y = 1$

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12. Find the degree of the differential equation satisfying the relation $\sqrt{1 + x^2} + \sqrt{1 + y^2} = \lambda\left(x\sqrt{1 + y^2} - y\sqrt{1 + x^2}\right)$

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13. Find the particular solution of the differential equation $(1+e^{2x})dy+(1+y^2)e^x dx=0,$ given that $y=1$ when $x=0.$

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14. Find the time required for a cylindrical tank of radius 2.5 m and height 3 m to empty through a round hole of 2.5 cm with a velocity $2.5\sqrt{h}$ m/s, h being the depth of the water in the tank.

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15. Solve $\frac{dy}{dx}=(x+y)^2$

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16. Solve $\frac{dy}{dx}=\frac{x+2y+3}{2x+3y+4}$

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17. Solve $\left(\frac{dy}{dx}\right) + \left(\frac{y}{x}\right) = y^3$

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18. Solve $\left(\frac{dy}{dx}\right) = e^{x-y}(e^x - e^y)$.

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19. Solve the differential equation $xy\frac{dy}{dx} = \frac{1+y^2}{1+x^2}(1 + x + x^2)$

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20. Solve the equation $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$

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21. Solve the equation $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$

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22. Solve the equation $\frac{dy}{dx} + \frac{xy}{(1-x^2)} = x\sqrt{y}$

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23. Solve the equation $\frac{dy}{dx} = (x^3 - 2x\tan^{-1}y)(1+y^2)$

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24. Find the orthogonal trajectory of $y^2 = 4ax$ (a being the parameter).

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25. Solve $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0;\ y = \frac{\pi}{4}$ when $x = 1$.

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26. Find the orthogonal trajectories of family of curves $x^2+y^2=cx$

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27. Show that the differential equation $y^3dy+(x+y^2)dx=0$ can be reduced to a homogeneous equation.

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28. Solve $\left[2\sqrt{xy}-x\right]dy+ydx=0$

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29. Suppose that a mothball loses volume by evaporation at a rate proportional to its instantaneous area. If the diameter of the ball decreases from 2cm to 1cm in 3 months, how long will it take until the ball has practically gone?

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30. Find the equation of the curve which is such that the area of the rectangle constructed on the abscissa of any point and the intercept of the tangent at this point on the y-axis is equal to 4.

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31. Solve $x\left(\frac{dy}{dx}\right) = y(\log y - \log x + 1)$

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32. If the population of country double in 50 years, in how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants.

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33. Find the equation of the curve passing through the origin if the middle point of the segment of its normal from any point of the curve to the x-axis lies on the parabola $2y^2 = x$

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34. Find the real value of m for which the substitution $y = u^m$ will transform the differential equation $2x^4 y \frac{dy}{dx} + y^4 = 4x^6$ in to a homogeneous equation.

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35. A curve $y = f(x)$ passes through the origin. Through any point (x, y) on the curve, lines are drawn parallel to the co-ordinate axes. If the curve divides the area formed by these lines and co-ordinates axes in the ratio $m : n,$ find the curve.

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36. Find the equation of a curve passing through $(0, 1)$ and having gradient $\frac{-(y+y^3)}{1+x+xy^2} at (x, y)$

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37. Solve $\frac{dy}{dx} = \frac{x-2y+5}{2x+y-1}$

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38. Solve $xdy = \left(y + x\frac{f\left(\frac{y}{x}\right)}{f'\left(\frac{y}{x}\right)}\right)dx$

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39. Solve the equation $ydx + (x-y^2)dy = 0$

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40. Find the orthogonal trajectory of $y^2 = 4ax$ (a being the parameter).

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41. Solve $e^{\frac{dy}{dx}} = x + 1,$ given that when $x = 0, y = 3.$

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42. Solve $\frac{\log(dy)}{dx} = 4x - 2y - 2$, given that $y = 1$ when $x = 1.$

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43. Form the differential equation of all concentric circles at the origin.

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44. Form the differential equation of family of lines concurrent at the origin.

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45. Solve $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

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46. Solve $\frac{dy}{dx}\sqrt{1+x+y} = x+y-1$

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47. Solve the equation: $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

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48. Solve $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y^2\right)dy(y \neq 0)$.

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49. Find the order and degree (if defined) of the equation: $\frac{d^3y}{dx^3} = x\ln\left(\frac{dy}{dx}\right)$

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50. Find the order and degree (if defined) of the equation: $\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^4\right\}^{\frac{5}{3}}$

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51. Find the time required for a cylindrical tank of radius r and height H to empty through a round hole of area a at the bottom. The flow through the hole is according to the law $v(t) = k\sqrt{2gh(t)}$, where $v(t)$ and $h(t)$,

are respectively, the velocity of flow through the hole and the height of the water level above the hole at time t, and g is the acceleration due to gravity.

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52. What constant interest rate is required if an initial deposit placed into an account accrues interest compounded continuously is to double its value in six years? $(\ln|x| = 0.6930)$

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53. Find the order and degree (if defined) of the equation: $\frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3}\right) = 0$

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54. Find the order and degree (if defined) of the equation:

$$a = \frac{1\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}},$$ where a is constant

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55. Find the order and degree (if defined) of the equation:

$$\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$$

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56. Find the order and degree (if defined) of the equation:

$$\frac{d^4y}{dx^4} + 3\left(\frac{d^2y}{dx^2}\right)^6 + \sin x = 2\cos x$$

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57. Solve the equation $\frac{dy}{dx} = \frac{y}{2y1ny + y - x}$

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58. Solve the equation $(1-x^2)\left(\frac{dy}{dx}\right)+2xy=x\sqrt{1-x^2}$

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59. Solve the equation $(x+y+1)\left(\frac{dy}{dx}\right)=1$

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60. Solve the equation $\frac{dy}{dx}+y\cot x=\sin x$

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61. Find the sum $\sum\sum_{0\le i<j\le n-1} j^{n}C_i$

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62. Solve the differential equation: (i) $\left(1+y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$

(ii) $x\frac{dy}{dx}+\cos^2 y=\tan y\frac{dy}{dx}$

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63. Solve $ydx - xdy + \log x dx = 0$

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64. Solve $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right]\frac{dx}{dy}=1(x\neq 0)$

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65. Find the order and degree of the following differential equation: $\sin^{-1}\left(\frac{dy}{dx}\right)=x+y$

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66. Find the order and degree of the following differential equation:

$$e^{\frac{d^3y}{dx^3}} - x\frac{d^2y}{dx^2} + y = 0$$

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67. Find the order and degree of the following differential equation:

$$\frac{dy}{dx} + y = \frac{1}{\frac{dy}{dx}}$$

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68. Find the curve for which the perpendicular from the foot of the ordinate to the tangent is of constant length.

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69. Find the orthogonal trajectories of $xy = \quad \cdot$

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70. Find the order and degree of the following differential equation:

$$\frac{d^2y}{dx^2} = \left[y + \left(\frac{dy}{dx}\right)^6\right]^{\frac{1}{4}}$$

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71. Find the order and degree of the following differential equation:

$$\ln\left(\frac{dy}{dx}\right) = ax + by$$

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72. Solve $\left(xy^2 - \frac{e^1}{x^3}\right) dx - x^2 y dy = 0$

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73. The rate at which a substance cools in moving air is proportional to the difference between the temperatures of the substance and that of the air. If the temperature of the air is 290 K and the substance cools from 370 K to 330 K in 10 min, when will the temperature be 295 K?

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74. Find the equation of the curve in which the subnormal varies as the square of the ordinate.

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75. Solve $\frac{dy}{dx} = \frac{\sin y + x}{\sin 2y - x \cos y}$

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76. Solve $(x-1)dy + ydx = x(x-1)y^{\frac{1}{3}}dx \cdot$

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77. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis and the y-axis in point A AND B, respectively, such that $\frac{1}{OA}+\frac{1}{OB}$ =1, where O is the origin. Find the equation of such a curve passing through $(5, 4)$

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78. Find the curve for which the length of normal is equal to the radius vector.

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79. What is the integrating factor of the differential equation $(1-y^2)\frac{dx}{dy}+\frac{y}{x}=ay(-1<y<1)$

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80. The solution of the differential equation $\left\{1 + x\sqrt{(x^2 + y^2)}\right\}dx + \left\{\sqrt{(x^2 + y^2)} - 1\right\}ydy = 0$ is equal to

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81. The solution of the differential equation $\frac{dy}{dx} = \frac{3x^2y^4 + 2xy}{x^2 - 2x^3y^3}$ is

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82. Solution of the differential equation $\left\{\frac{1}{x} - \frac{y^2}{(x-y)^2}\right\}dx + \left\{\frac{x^2}{(x-y)^2} - \frac{1}{y}\right\}dy = 0$ is

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83. The solution of the differential equation $2x^2y\frac{dy}{dx} = \tan(x^2y^2) - 2xy^2$, given $y(1) = \frac{\pi}{2}$, is

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84. The solution of the differential equation $x^2\frac{dy}{dx}\cos\left(\frac{1}{x}\right) - y\sin\left(\frac{1}{x}\right) = -1$, where $y \to -1$ as $x \to \infty$ is

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85. The solution of the differential equation $\frac{(x+2y^3)dy}{dx} = y$ is

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86. The curve satisfying the equation $\frac{dy}{dx} = \frac{y(x+y^3)}{x(y^3-x)}$ and passing through the point $(4, -2)$ is

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87. The solution of $ye^{-\frac{x}{y}}dx - \left(xe^{\left(-\frac{x}{y}\right)} + y^3\right)dy = 0$ is

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88. The solution of differential equation $\left(2y + xy^3\right)dx + \left(x + x^2y^2\right)dy = 0$ is

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89. If $y + x\dfrac{dy}{dx} = x\dfrac{\varphi(xy)}{\varphi'(xy)}$ then $\varphi(xy)$ is equal to

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90. Let $y'(x) + y(x)g'(x) = g(x)g'(x), y(0) = 0, x \in R,$ where $f'(x)$ denotes $\dfrac{dy(x)}{dx}$, and $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0.$ Then the value of $y(2)$ is_____

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91. Let $f:[1,\infty]$ be a differentiable function such that $f(1)=2$. If $6\int_1^x f(t)dt = 3xf(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is

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92. If $y(x)$ satisfies the differential equation $y' - y\tan x = 2x secx$ and $y(0)=0$, then

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93. Consider the family of all circles whose centers lie on the straight line $y=x$. If this family of circles is represented by the differential equation $Py'' + Qy' + 1 = 0$, where P, Q are functions of x, y and $y'\left(herey' = \frac{dy}{dx}, y = \frac{d^2y}{dx^2}\right)$, then which of the following statements is (are) true? (a)$P=y+x$ (b)$P=y-x$ (c) $P+Q=1-x+y+y'+(y')^2$ (d)$P-Q=x+y-y'-(y')^2$

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94. Let $y(x)$ be a solution of the differential equation $(1+e^x)y' + ye^x = 1$. If $y(0) = 2$, then which of the following statements is (are) true? (a)$y(-4) = 0$ (b)$y(-2) = 0$ (c)$y(x)$ has a critical point in the interval $(-1, 0)$ (d)$y(x)$ has no critical point in the interval$(-1, 0)$

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95. Let f be a real-valued differentiable function on R (the set of all real numbers) such that $f(1) = 1$. If the $y - \in tercept$ of the tangent at any point $P(x, y)$ on the curve $y = f(x)$ is equal to the cube of the abscissa of P, then the value of $f(-3)$ is equal to______

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96. The function $y = f(x)$ is the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^4+2x}{\sqrt{1-x^2}}$ in $(-1, 1)$ satisfying $f(0) = 0$. Then

$\int_{\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is

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97. The order of the differential equation whose general solution is given by $y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x + C_5}$, where C_1, C_2, C_3, C_4, C_5 , are arbitrary constants, is (a) 5 (b) 4 (c) 3 (d) 2

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98. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter, is of (A) order 1 (B) order 2 (C) degree 3 (D) degree 4

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99. A curve $y = f(x)$ passes through $(1, 1)$ and tangent at $P(x, y)$ cuts the x-axis and y-axis at A and B , respectively, such that $BP : AP = 3$, then (a) equation of curve is $xy' - 3y = 0$ (b) normal at $(1, 1)$ is $x + 3y = 4$ (c) curve passes through $2, 8$ (d) equation of curve is $xy' + 3y = 0$

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100. A curve is such that the mid-point of the portion of the tangent intercepted between the point where the tangent is drawn and the point where the tangent meets the y-axis lies on the line $y = x$. If the curve passes through $(1, 0)$, then the curve is

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101. The equation of a curve passing through $\left(2, \frac{7}{2}\right)$ and having gradient $1 - \frac{1}{x^2}$ at (x, y) is

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102. The equation of the curve which is such that the portion of the axis of x cut off between the origin and tangent at any point is proportional to the ordinate of that point is

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103. A normal at $P(x, y)$ on a curve meets the x-axis at Q and N is the foot of the ordinate at P. If $NQ = \frac{x(1+y^2)}{1+x^2}$, then the equation of curve given that it passes through the point $(3, 1)$ is

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104. The differential equation of all non-horizontal lines in a plane is

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105. The curve in the first quadrant for which the normal at any point (x, y) and the line joining the origin to that point form an isosceles triangle with the x-axis as base is (a) an ellipse (b) a rectangular hyperbola (c) a circle (d) None of these

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106. The differential equation of the curve for which the initial ordinate of any tangent is equal to the corresponding subnormal (a) is linear (b) is homogeneous of second degree (c) has separable variables (d) is of second order

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107. Orthogonal trajectories of family of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\left(\frac{2}{3}\right)}$, where a is any arbitrary constant, is

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108. Which of the following is not the differential equation of family of curves whose tangent form an angle of $\frac{\pi}{4}$ with the hyperbola $xy = c^2$?

$(a)\frac{dy}{dx} = \frac{x-y}{x+y}$ $(b)\frac{dy}{dx} = \frac{x}{x-y}$ $(c)\frac{dy}{dx} = \frac{x+y}{x-y}$ (d)N.O.T.

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109. Tangent to a curve intercepts the y-axis at a point P A line perpendicular to this tangent through P passes through another point $(1, 0)$. The differential equation of the curve is

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110. The solution of the differential equation $(x\cot y + \log\cos x)dy + (\log\sin y - y\tan x)dx = 0$ is

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111. The curve with the property that the projection of the ordinate on the normal is constant and has a length equal to a is

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112. The differential equation of all parabolas each of which has a latus rectum $4a$ and whose axes are parallel to the x-axis is (a) of order 1 and degree 2 (b) of order 2 and degree 3 (c) of order 2 and degree 1 (d) of order 2 and degree 2

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113. The solution of differential equation $\frac{y(2x^4 + y)dy}{dx} = (1 - 4xy^2)x^2$ is given by

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114. The normal to a curve at $P(x, y)$ meet the x-axis at G. If the distance of G from the origin is twice the abscissa of P , then the curve is a (a) parabola (b) circle (c) hyperbola (d) ellipse

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115. A normal at any point (x, y) to the curve $y = f(x)$ cuts a triangle of unit area with the axis, the differential equation of the curve is

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116. The equation of a curve passing through (1,0) for which the product of the abscissa of a point P and the intercept made by a normal at P on the x-axis equal twice the square of the radius vector of the point P is

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117. if a, b are two positive numbers such that $f(a+x) = b + \left[b^3 + 1 - 3b^2 f(x) + 3b\{f(x)\}^2 - \{f(x)\}^3\right]^{\frac{1}{3}}$ for all real x, then prove that $f(x)$ is peroidic and find its peroid?

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118. The differential equation whose general solution is given by $y = c_1 \cos(x + c_2) - c_3 e^{(-x + c4)} + (c_5 \sin x)$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is

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119. The solution to the differential equation $y \log y + xy' = 0$, where $y(1) = e$, is

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120. The form of the differential equation of the central conics $ax^2 + by^2 = 1$ is

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121. The differential equation for the family of curve $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant, is

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122. If $\frac{dy}{dx} = (e^y - x)^{-1}$, where $y(0) = 0$, then y is expressed explicitly as

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123. If $y = \frac{x}{\log|cx|}$ (where c is an arbitrary constant) is the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \varphi\left(\frac{x}{y}\right)$, then the function $\varphi\left(\frac{x}{y}\right)$ is

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124. Differential equation of the family of circles touching the line $y = 2$ at $(0, 2)$ is

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125. The differential equation of all parabolas whose axis are parallel to the y-axis is

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126. A differential equation associated to the primitive $y = a + be^{5x} + ce^{-7x}$ is (where y_n is nth derivative w.r.t. x) $(a) y_3 + 2y_2 - y_1 = 0 (b) 4y_3 + 5y_2 - 20y_1 = 0 (c) y_3 + 2y_2 - 35y_1 = 0$

(d) none of these

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127. The order and degree of the differential equation of all tangent lines to the parabola $y = x^2$ is (a) 1,2 (b) 2,3 (c) 2,1 (d) 1,1

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128. A solution of the differential equation, $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$

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129. A curve $y = f(x)$ passes through point $P(1, 1)$. The normal to the curve at P is a $(y - 1) + (x - 1) = 0$. If the slope of the tangent at any point on the curve is proportional to the ordinate of the point, then the equation of the curve is

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130. A spherical rain drop evaporates at a rate proportional to its surface area at any instant t. The differential equation giving the rate of change of the radius of the rain drop is ____

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131. If length of tangent at any point on the curve $y = f(x)$ intercepted between the point and the x-axis is of length l. Find the equation of the curve.

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132. For the primitive integral equation $ydx + y^2dy = xdy; x \in R, y > 0, y(1) = 1,$ then $y(-3)$ is (a) 3 (b) 2 (c) 1 (d) 5

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133. The solution of the primitive integral equation $(x^2 + y^2)dy = xydx$ is $y = y(x)$. If $y(1) = 1$ and $y(x_0) = e$, then x_0 is

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134. if $y = y(x)$ and $\frac{2 + \sin x}{y + 1}\left(\frac{dy}{dx}\right) = -\cos x, y(0) = 1,$ then $y\left(\frac{\pi}{2}\right) =$

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135. If $y(t)$ is a solution of $(1 + t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$ then $y(1)$ is

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136. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0.$ Then the equation of the

curve is

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137. The differential equation $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{y}$ determines a family of circle with (a) variable radii and a fixed centre at$(0, 1)$ (b) variable radii and a fixed centre at $(0, -1)$(c) Fixed radius 1 and variable centres along the x-axis. (d) Fixed radius 1 and variable centres along the y-axis.

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138. spherical rain drop evaporates at a rate proportional to its surface area. The differential equation corresponding to the rate of change of the radius of the rain drop if the constant of proportionality is $K > 0$ is

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139. Water is drained from a vertical cylindrical tank by opening a valve at the base of the tank. It is known that the rate at which the water level drops is proportional to the square root of water depth y, where the constant of proportionality $k > 0$ depends on the acceleration due to gravity and the geometry of the hole. If t is measured in minutes and $k = \frac{1}{15}$, then the time to drain the tank if the water is 4 m deep to start with is (a) 30 min (b) 45 min (c) 60 min (d) 80 min

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140. The population of a country increases at a rate proportional to the number of inhabitants. f is the population which doubles in 30 years, then the population will triple in approximately. (a) 30 years (b) 45 years (c) 48 years (d) 54 years

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141. An object falling from rest in air is subject not only to the gravitational force but also to air resistance. Assume that the air resistance is proportional to the velocity with constant of proportionality as $k > 0$, and acts in a direction opposite to motion $\left(g = 9.8\frac{m}{s^2}\right)$. Then velocity cannot exceed.

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142. The solution of differential equation $x^2 = 1 + \left(\frac{x}{y}\right)^{-1}\frac{dy}{dx} + \frac{\left(\frac{x}{y}\right)^{-2}\left(\frac{dy}{dx}\right)^2}{2!} + \frac{\left(\frac{x}{y}\right)^{-3}\left(\frac{dy}{dx}\right)^3}{3!} + \ldots is$

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143. The solution of the differential equation $y'y''' = 3(y'')^2$ is

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144. Number of values of $m \in N$ for which $y = e^{mx}$ is a solution of the differential equation $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 12y = 0$ (a) 0 (b) 1 (c) 2 (d) More than 2

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145. A curve passing through $(2, 3)$ and satisfying the differential equation $\int_0^x ty(t)dt = x^2y(x), (x > 0)$ is

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146. The solution of the differential equation $\frac{d^2y}{dx^2} = \sin 3x + e^x + x^2$ when $y_1(0) = 1$ and $y(0)$ is

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147. The solution of the differential equation $\frac{x + \frac{x^3}{3!} + \frac{x^5}{5!} + }{1 + \frac{x^2}{2!} + \frac{x^4}{4!} + } = \frac{dx - dy}{dx + dy} is$

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148. A and B are two separate reservoirs of water. Capacity of reservoir A is double the capacity of reservoir B. Both the reservoirs are filled completely with water, their inlets are closed and then the water is released simultaneously from both the reservoirs. The rate of flow of water out of each reservoir at any instant of time is proportional to the quantity of water in the reservoir at the time. One hour after the water is released, the quantity of water is reservoir A is $1\frac{1}{2}$ times the quantity of water in reservoir B. After how many hours do both the reservoirs have the same quantity of water?

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149. If $\int_a^x ty(t)dt = x^2 + y(x)$, then find $y(x)$

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150. Given a function ' g' which has a derivative $g'(x)$ for every real x and satisfies $g'(0) = 2$ and $g(x+y) = e^y g(x) + e^y g(y)$ for all x and y then:

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151. Solve $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$ given that $y(0) = \sqrt{5}$

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152. If $y + \frac{d}{dx}(xy) = x(\sin x + \log x), f \in dy(x)$.

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153. Solve $\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}ydx=\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}xdy$

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154. Solve $\frac{dy}{dx}=\frac{(x+y)^2}{(x+2)(y-2)}$

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155. Solve $\frac{x+y\frac{dy}{dx}}{y-x\frac{dy}{dx}}=x^2+2y^2+\frac{y^4}{x^2}$

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156. For the differential equation whose solution is $(x-h)^2+(y-k)^2=a^2$ (a is a constant), its (a) order is 2 (b) order is 3 (c) degree is 2 (d) degree is 3

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157. Which one of the following function(s) is/are homogeneous?

$(a) f(x, y) = \frac{x - y}{x^2 + y^2} (b) f(x, y) = x^{\frac{1}{3}} y^{-\frac{2}{3}} \tan^{-1}\left(\frac{x}{y}\right) (c) f(x, y) = x\left(\ln$

(d) none of these

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158. The solution of the differential equation $\left(e^{x^2} + e^{y^2}\right) y \frac{dy}{dx} + e^{x^2}\left(xy^2 - x\right) = 0 is$

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159. The solution of the differential equation $y^{'''} - 8y^{''} = 0,$ where $y(0) = \frac{1}{8}, y'(0) = 0, y^0 = 1$, is

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160. Differential equation of the family of curves $v = \frac{A}{r} + B$, where A and B are arbitrary constants, is

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161. The function $f(\theta) = \frac{d}{d(\theta)} \int_0^{\theta} \frac{dx}{1 - \cos\theta \cos x}$ satisfies the differential equation

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162. The differential equation of the curve $\frac{x}{c-1} + \frac{y}{c+1} = 1$ is

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163. The solution of the differential equation $x = 1 + xy\frac{dy}{dx} + \frac{x^2y^2}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{x^3y^3}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots is$

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164. Which of the following equation(s) is/are linear?

$(a)\frac{dy}{dx}+\frac{y}{x}=\ln x(b)y\left(\frac{dy}{dx}\right)+4x=0(c)\left(2x+y^3\right)\left(\frac{dy}{dx}\right)=3y(d)N.O$

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165. The equation of the curve satisfying the differential equation $y\left(\frac{dy}{dx}\right)^2+(x-y)\frac{dy}{dx}-x=0$ can be a (a) circle (b) Straight line (c) Parabola (d) Ellipse

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166. Find a pair of curves such that (a) the tangents drawn at points with equal abscissas intersect on the y-axis. (b) the normal drawn at points with equal abscissas intersect on the x-axis. (c) one curve passes through (1,1) and other passes through (2, 3).

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167. If y_1 and y_2 are two solutions to the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$. Then prove that $y = y_1 + c(y_1 - y_2)$ is the general solution to the equation where c is any constant.

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168. If y_1 and y_2 are the solution of the differential equation $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x alone and $y_2 = y_1 z$, then prove that $z = 1 + \cdot e^{-f \frac{Q}{y_1} dx}$, where c is an arbitrary constant.

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169. Let the function $\ln f(x)$ is defined where $f(x)$ exists for $x \geq 2$ and k is fixed positive real numbers prove that if $\frac{d}{dx}(x.\ f(x)) \geq -kf(x)$ then $f(x) \geq Ax^{-1-k}$ where A is independent of x.

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170. The degree of the differential equation satisfying $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ is (a) 1 (b) 2 (c) 3 (d) none of these

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171. The force of resistance encountered by water on a motor boat of mass m going in still water with velocity v is proportional to the velocity v. At $t=0$ when its velocity is v_0, then engine shuts off. Find an expression for the position of motor boat at time t and also the distance travelled by the boat before it comes to rest. Take the proportionality constant as $k>0$.

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172. A cyclist moving on a level road at 4 m/s stops pedalling and lets the wheels come to rest. The retardation of the cycle has two components: a

constant 0.08 m/s^2 due to friction in the working parts and a resistance of $0.02v^2/m$, where v is speed in meters per second. What distance is traversed by the cycle before it comes to rest? (consider 1n 5=1.61).

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173. Given two curves: $y = f(x)$ passing through the point $(0, 1)$ and $g(x) = \int_{-\infty}^{x} f(t)dt$ passing through the point $\left(0, \frac{1}{n}\right)$. The tangents drawn to both the curves at the points with equal abscissas intersect on the x-axis. Find the curve $y = f(x)$.

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174. The differential equation of the family of curves $y = e^x(A\cos x + B\sin x)$, where A and B are arbitrary constants is

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175. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants, is of (a) second order and second degree (b) first order and second degree (c) first order and first degree (d) second order and first degree

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176. Let $y = f(x)$ be a curve passing through $(1, 1)$ such that the triangle formed by the coordinate axes and the tangent at any point of the curve lies in the first quadrant and has area 2. Form the differential equation and determine all such possible curves.

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177. The solution of the equation $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$ is

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178. The solution of the equation $\log\left(\frac{dy}{dx}\right) = ax + by$ is

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179. If $y = y(x)$ and $\frac{2 + \sin x}{y + 1}\left(\frac{dy}{dx}\right) = -\cos x,\ y(0) = 1,$ then $y\left(\frac{\pi}{2}\right)$ equals

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180. The equation of the curves through the point (1, 0) and whose slope is $\frac{y - 1}{x^2 + x}$ is

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181. The solution of $\frac{dv}{dt} + \frac{k}{m}v = -g$ is

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182. The solution of the equation $\frac{dy}{dx} = \cos(x - y)$ is

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183. The solution of the equation $(x^2y + x^2)dx + y^2(x - 1)dy = 0$ is given by

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184. Solution of differential equation $dy - \sin x \sin y dx = 0$ is

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185. Solution of $\frac{dy}{dx} + 2xy = y$ is

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186. The general solution of the differential equation $\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$ is

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187. The solution of $\frac{xdx + ydy}{xdy - ydx} = \sqrt{\frac{1 - x^2 - y^2}{x^2 + y^2}}$ is

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188. The curve for which the length of the normal is equal to the length of the radius vector is/are (a) circles (b) rectangular hyperbola (c) ellipses (d) straight lines

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189. Identify the statement(s) which is/are true. (a) $f(x, y) = e^{\frac{y}{x}} + \frac{\tan y}{x}$ is a homogeneous of degree zero. (b) $x\frac{\ln y}{x}dx + \frac{y^2}{x} \cdot \sin^{-1}\left(\frac{y}{x}\right)dy = 0$

is a homogeneous differential equation. (c) $f(x, y) = x^2 + \sin x \cos y$ is a non homogeneous. (d) $(x^2 + y^2)dx - (xy^2 - y^3)dy = 0$ is a homogeneous differential equation.

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190. The graph of the function $y = f(x)$ passing through the point (0,1) and satisfying the differential equation $\frac{dy}{dx} + y\cos x = \cos x$ is such that (a) it is a constant function. (b) it is periodic (c) it is neither an even nor an odd function. (d) it is continuous and differentiable for all $f(x)$

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191. The solution of $\frac{dy}{dx} = \frac{ax + h}{by + k}$ represent a parabola when (a) (a) $a = 0, b \neq 0$ (b)$a \neq 0, b \neq 0$ (c) $b = 0, a \neq 0$ (d) $a = 0, b \in R$

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192. The equation of the curve satisfying the differential equation $y_2(x^2+1) = 2xy_1$ passing through the point (0,1) and having slope of tangent at $x = 0$ as 3 (where y_2 and y_1 represent 2nd and 1st order derivative), then (a)$y = f(x)$ is a strictly increasing function (b)$y = f(x)$ is a non-monotonic function (c)$y = f(x))$ has a three distinct real roots (d)$y = f(x)$ has only one negative root.

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193. $y = ae^{-\frac{1}{x}} + b$ is a solution of $\frac{dy}{dx} = \frac{y}{x^2}$,then (a)$a \in R$ (b)$b = 0$ (c) $b = 1$ (d)a takes finite number of values

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194. For equation of the curve whose subnormal is constant, then (a) its eccentricity is 1 (b) its eccentricity is $\sqrt{2}$ (c) its axis is the x-axis (d)its axis is the y-axis.

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195. If f(x), g(x) be twice differentiable functions on [0,2] satisfying $f''(x) = g''(x)$,$f'(1) = 2g'(1) = 4$ and $f(2) = 3g(2) = 9,$ then $f(x) - g(x)$ at x = 4 equals (A) 0 (B) 10 (C) 8 (D) 2

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196. The solution of the differential equation $(x^2y^2 - 1)dy + 2xy^3dx = 0$ is

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197. The solution of $\frac{x^2dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right)$ is

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198. The solution of $(x + y + 1)dy = dx$ is

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199. Solve the differential equation $x\frac{dy}{dx}=y(\log y-\log x+1)$.

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200. The slope of the tangent at (x, y) to a curve passing through $\left(1, \frac{\pi}{4}\right)$ is given by $\frac{y}{x}-\cos^2\left(\frac{y}{x}\right)$, then the equation of the curve is

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201. The solution of $\left(x^2+xy\right)dy=\left(x^2+y^2\right)dx$ is

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202. The solution of differential equation $yy'=x\left(\frac{y^2}{x^2}+\frac{f\left(\frac{y^2}{x^2}\right)}{f'\left(\frac{y^2}{x^2}\right)}\right)$ is

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203. The solution of $(y+x+5)dy=(y-x+1)dx$ is

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204. Solution of the differential equation $\left(y+x\sqrt{xy}(x+y)\right)dx+\left(y\sqrt{xy}(x+y)-x\right)dy=0$ is

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205. The slope of the tangent at (x, y) to a curve passing through a point $(2, 1)$ is $\dfrac{x^2+y^2}{2xy}$, then the equation of the curve is

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206. If $x\dfrac{dy}{dx}=x^2+y-2,\ y(1)=1$, then $y(2)$ equals ....

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207. If $y = y(x)$ and it follows the relation $4xe^{xy} = y + 5\sin^2 x$, then $y'(0)$ is equal to_____

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208. A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant k is positive). Suppose that r(t) is the radius of the liquid cone at time t. The time after which the cone is empty is

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209. A curve C passes through (2,0) and the slope at (x, y) as $\frac{(x+1)^2 + (y-3)}{x+1}$. Find the equation of the curve. Find the area bounded by curve and x-axis in the fourth quadrant.

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210. Let $u(x)$ and $v(x)$ satisfy the differential equation $\frac{du}{dx} + p(x)u = f(x)$ and $\frac{dv}{dx} + p(x)v = g(x)$ are continuous functions. If $u(x_1)$ for some x_1 and $f(x) > g(x)$ for all $x > x_1$, prove that any point (x, y), where $x > x_1$, does not satisfy the equations $y = u(x)$ and $y = v(x)$.

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211. Statement 1 : The differential equation of all circles in a plane must be of order 3. Statement 2 : There is only one circle passing through three non-collinear points.

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212. In which of the following differential equation degree is not defined?

(a) $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x \log\left(\frac{d^2y}{dx^2}\right)$ (b) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{d^2y}{dx^2}\right)$ (c) $x = \sin\left(\left(\frac{dy}{dx}\right) - 2y\right), |x| < 1$

(d) $x - 2y = \log\left(\frac{dy}{dx}\right)$

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213. Statement 1 : Degree of the differential equation $2x - 3y + 2 = \log\left(\frac{dy}{dx}\right)$ is not defined. Statement 2 : In the given differential equation, the power of highest order derivative when expressed as the polynomials of derivatives is called degree.

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214. Statement 1 : The differentia equation of the family of curves represented by $y = Ae^x$ is given by $\frac{dy}{dx} = y$ Statement 2 : $\frac{dy}{dx} = y$ is valid for every member of the given family.

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215. Statement 1 : The order of the differential equation whose general solution is $y = \; = c_1 \cos 2x + c_2 \sin^2 x + c_3 \cos^2 x + c_4 e^{2x} + c_5 e^{2x + c_6}$ is 3. Statement 2 : Total number of arbitrary parameters in the given general solution in the statement (1) is 3.

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216. Statement 1 : Order of a differential equation represents the number of arbitrary constants in the general solution. Statement 2 : Degree of a differential equation represents the number of family of curves.

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217. Determine the equation of the curve passing through the origin, in the form $y = f(x)$, which satisfies the differential equation $\frac{dy}{dx} = \sin(10x + 6y)$.

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218. Let $f(x), x \geq 0,$ be a non-negative continuous function, and let $F(x) = \int_0^x f(t)dt, x \geq 0,$ if for some $c > 0, f(x) \leq cF(x)$ for all $x \geq 0,$ then show that $f(x) = 0$ for all $x \geq 0.$

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219. A hemi-spherical tank of radius 2 m is initially full of water and has an outlet of $12cm^2$ cross-sectional area at the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law $v(t) = \sqrt{0.62gh(t)},$ where $v(t)$ and $h(t)$ are, respectively, the velocity of the flow through the outlet and the height of water level above the outlet and the height of water level above the outlet at time $t,$ and g is the acceleration due to gravity. Find the time it takes to empty the tank.

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220. A curve C has the property that if the tangent drawn at any point P on C meets the co-ordinate axis at A and B, then P is the mid-point of AB. The curve passes through the point (1,1). Determine the equation of the curve.

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221. A country has a food deficit of 10%. Its population ear. Its annual food production every year is 4% more than that of the last year Assuming that the average food requirement per person remains constant, prove that the country will become self-sufficient in food after n years, where n is the smallest integer bigger than or equal to $\dfrac{\log_e 10 - \log_e 9}{(\log_e 1.04) - 0.03}$

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222. Integrating factor of differential equation $\cos x \dfrac{dy}{dx} + y \sin x = 1$ is

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223. Solution of the equation $\cos^2 x \frac{dy}{dx} - (\tan 2x)y = \cos^4 x, |x| < \frac{\pi}{4}$, when $y\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{8}$ is

A. $y = \frac{\cos^2 x \tan 2x}{2}$

B. $y = \frac{\cos^2 x \tan 2x}{4}$

C. $y = \frac{\cos x \tan 2x}{4}$

D. $y = \frac{\cos x \tan 2x}{2}$

Answer: A

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224. If integrating factor of $x(1 - x^2)dy + (2x^2y - y - ax^3)dx = 0$ is $e^{\int pdx}$, then P is equal to

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225. A function $y = f(x)$ satisfies $(x+1)f'(x) - 2(x^2+x)f(x) = \frac{(e^x)^2}{(x+1)}, \forall x \succ 1$. If $f(0) = 5$, then $f(x)$ is

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226. The general solution of the differential equation, $y' + y\phi'(x) - \phi(x)\phi'(x) = 0$, where $\phi(x)$ is a known function, is

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227. The solution of $\frac{dy}{dx} = \frac{x^2+y^2+1}{2xy}$ satisfying $y(1) = 1$ is given by

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228. The integrating factor of the differential equation $\frac{dy}{dx}\left(x(\log)_e x\right) + y = 2(\log)_e x$ is given by

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229. The solution of the differential equation $x(x^2+1)\left(\frac{dy}{dx}\right)=y(1-x^2)+x^3\log x$ is

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230. The solution of the differential equation $\frac{dy}{dx}=\frac{1}{xy\left[x^2\sin y^2+1\right]}$ is

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231. The general solution of the equation $\frac{dy}{dx}=1+xy$ is

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232. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis at Q. If PQ has constant length k, then show that the differential equation describing such curves is $y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$. Find the equation of such a curve passing through $(0, k)$.

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233. If the solution of the differential equation $\frac{dy}{dx} - y = 1 - e^{-x}$ and $y(0) = y_0$ has a finite value, when $x \to \infty$, then the value of $\left|\frac{2}{y_0}\right|$ is__

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234. Let $y = y(t)$ be a solution to the differential equation $y' + 2ty = t^2$, then $16(\lim)_{t \to \infty} \frac{y}{t}$ is______

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235. If the dependent variable y is changed to z by the substitution $y = \tan z$ and the differential equation $\frac{d^2y}{dx^2} = 1 + \frac{2(1+y)}{1+y^2}\left(\frac{dy}{dx}\right)^2$ is changed to $\frac{d^2z}{dx^2} = \cos^2 z + k\left(\frac{dz}{dx}\right)^2$, then the value of k equal to______

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236. If the independent variable x is changed to y, then the differential equation $x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} = 0$ is changed to $x\frac{d^2x}{dy^2} + \left(\frac{dx}{dy}\right)^2 = k$ where k equals___

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237. If the solution of the differential equation $\frac{dy}{dx} = \frac{1}{x\cos y + \sin 2y}$ is $x = ce^{\sin y} - k(1 + \sin y)$, then the value of k is______

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238. Tangent is drawn at the point (x_i, y_i) on the curve $y = f(x)$, which intersects the x-axis at $(x_{i+1}, 0)$. Now, again a tangent is drawn at (x_{i+1}, y_{i+1}) on the curve which intersect the x-axis at $(x_{i+2}, 0)$ and the process is repeated n times, i.e. $i = 1, 2, 3; n$. If $x_1, x_2, x_3, ;\, x_n$ from an arithmetic progression with common difference equal to $(\log)_2 e$ and curve passes through $(0, 2)$. Now if curve passes through the point $(-2, k)$, then the value of k is___

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239. The curve passing through the point $(1, 1)$ satisfies the differential equation $\frac{dy}{dx} + \frac{\sqrt{(x^2 - 1)(y^2 - 1)}}{xy} = 0$. If the curve passes through the point $(\sqrt{2}, k)$, then the value of $[k]$ is (where [.] represents greatest integer function)____

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240. If the eccentricity of the curve for which tangent at point P intersects the y-axis at M such that the point of tangency is equidistant from M and the origin is e, then the value of $5e^2$ is___

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241. The perpendicular from the origin to the tangent at any point on a curve is equal to the abscissa of the point of contact. Also curve passes through the point (1,1). Then the length of intercept of the curve on the x-axis is__________

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Question Bank

1. A function is continuous and differentiable on R_0 satisfying $xf'(x) + f(x) = 1 \forall x$ in its domain. If $f(1) = 2$, then range of function does not contain

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2. A curve $y = f(x)$ is passing through $(0, 0)$. If the slope-of the curve at any point (x, y) is equal to $(x + xy)$, then the number of solution of the equation $f(x) = 1$, is

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3. If $y = f(x)$ satisfies the differential equation $\left(1 + x^2\right) f'(x)$=x(1-f(x)), f(0)=4/3, $thenf \in dthevalueof$f(sqrt8)+8/9`

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4. If $y(x)$ is solution of $(x + 1)\frac{dy}{dx} - xy = 1, y(0) = -1,$ then $y\left(-\frac{6}{5}\right)$ is equal to

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5. Let perpendicular distance of any variable tangent on the curve C from origin be equal to polar radius of point of tangency. If curve passes through $P(2\sqrt{3}, 2)$, then length of normal to the curve at point P 'is

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6. The number of straight lines which satisfies the differential equation $\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 - y = 0$ is

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7. The real valuc of m for which the 'substitution, $y = u^m$ will transform the differential equation, $2x^4 y \frac{dy}{dx} + y^4 = 4x^6$ into a homogeneous equation is

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8. A function $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} + x^2 y = -2x$, $f(1) = 1$. The value of $\left|f^{''}(1)\right|$ is

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9. If the differential equation representing the family of curves $y = C_1 \cos 2x + C_2 \cos^2 x + C_3 \sin^2 x + C_4$ is $\lambda y' = y^{''} \tan 2x$, then λ is

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10. The family of integral curves of the differential equation $\frac{dy}{dx} + x^3 y = x$ is cut by the line $x = 2$, the tangents at the points of intersection are concurrent at (λ, μ). Then find the value of $\left[\frac{\lambda}{\mu}\right]$, where $[.]$ denotes greatest integer function.

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11. If ' e' denotes tho cccentricity of the hyperbola, satisfying the differential equation $2xy\frac{dx}{dy} = x^2 + y^2$ and passing through the point $(2, 1)$, then $(e^2 - 1)$ is equal to

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12. A carve $y = f(x)$ passes through $O(0, 0)$ and slope of tangent line at any point $P(x, y)$ of the curve is $\frac{x^4 + 2xy - 1}{1 + x^2}$, then the value of least integer which is greater than or equal to $f(-1)$ is

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13. If $y = x\sin(\ln x)$ is the solution of

$x^2\left(\frac{dy}{dx}\right)^2 - (\lambda - 2)xy\left(\frac{dy}{dx}\right) + 2y^2 - x^2 - x^2 = 0$ then the value of (λ) is

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14. Let $y = f(x)$ be drawn with $f(0) = 2$ and for each real number ' a ' the line tangent to $y = f(x)$ at $(a, f(a))$, has x -intercept $(a - 2)$. If $f(x)$ is of the form of $ke^{\mu}x$, then $\left(\frac{k}{p}\right)$ has the value equal to

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15. If tangent to the curve $xy = x^2 + 1$ at (α, β) is normal to the curve $x^2 + y^2 + 2fy + c = 0$, then $|f. \alpha|$ is

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16. If $y = y(x)$ and it follows the relation $4xe^{v} = y + 5\sin^2 x$ then $y'(0)$ is equal to

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17. Given $y(0) = 2000$ and $\frac{dy}{dx} = 32000 - 20y^2$, then find the value of $\lim_{x \to \infty} y(x)$.

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18. If the differential equation corresponding to the family of curves, $y = (A + Bx)e^3x$ is given by $\frac{d^2y}{dx^2} = a\frac{dy}{dx} + by$ then $(a - b)$ equals

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19. A curve in the first quadrant is such that the area of the triangle formed in the first quadrant by the x -axis, a fangent to the curve at any of its point P and radius vector of the point P is $2sq$. units. If the curve passes through $(2, 1)$, and the equation of the curve $xy = k$ then k is equal to

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20. Number of values of m in N for which $y = e^{mx}$ is a solution of the differential equation

$D^3y - 3D^2y - 4Dy + 12y = 0$, is

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21. If $(x^2 + y^2)dy = xydx$ and $y(1) = 1, y(\alpha) = e$, then $\alpha^4 = ke^4$, where k equals, is

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22. If solution of the differetnial equation $\frac{dy}{dx} = \frac{2x - y - 1}{x + 5y - 6}$ is $ax(x - y) - 5y^2 - 2x + 12y = 0$, then ' a ' is equal to (where $y(0) = 0$)

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23. If $y(t)$ is the solution of $(1 + t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$ then absolute value of $y(1)$ is equal to

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24. Let $y = f(x)$ be a function satisfying the differcntial equation $\frac{xdy}{dx} + 2y = 4x^2$ and $f(1) = 1$. Then $f(-3)$ is ccual to

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25. Let $y = f(x)$ be an invertibie function such that x -intercept of the tangent at any point $P(x, y)$ on $y = f(x)$ is equal to 'the square of abscissa of the point of tangency. If $f(2) = 1$. Then $f^{-1}\left(\frac{5}{8}\right)$ equals

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