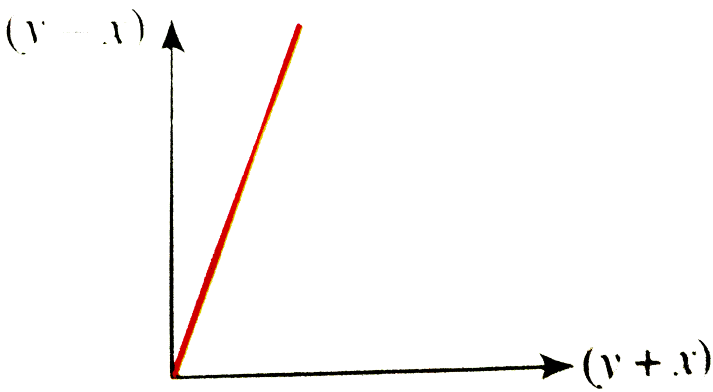
# MATHS

## BOOKS - CENGAGE MATHS (HINGLISH)

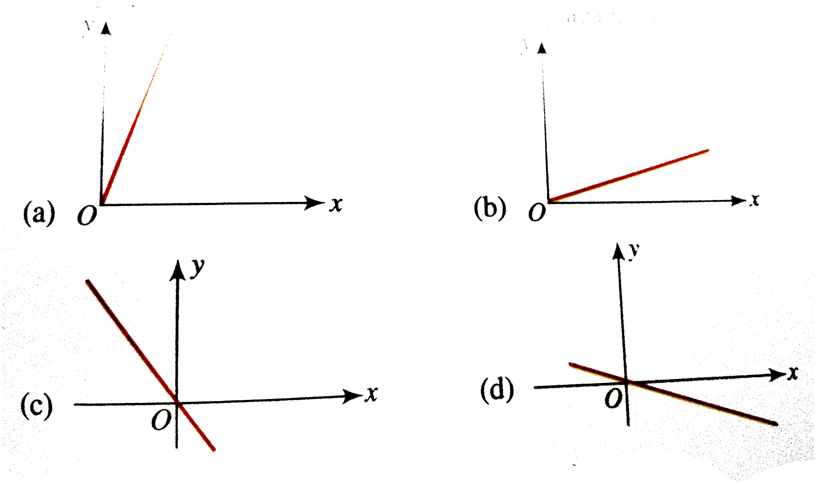
## GRAPHS OF ELEMENTARY FUNCTIONS

### Illustrations

**1.** The graph of $(y - x)$ against $(y + x)$ is shown below.



Which one of the following shows the graph of $y$ against $x$ ?

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2. Draw the graph of $f(x) = \dfrac{x^3 - x}{x^2 - 1}$.

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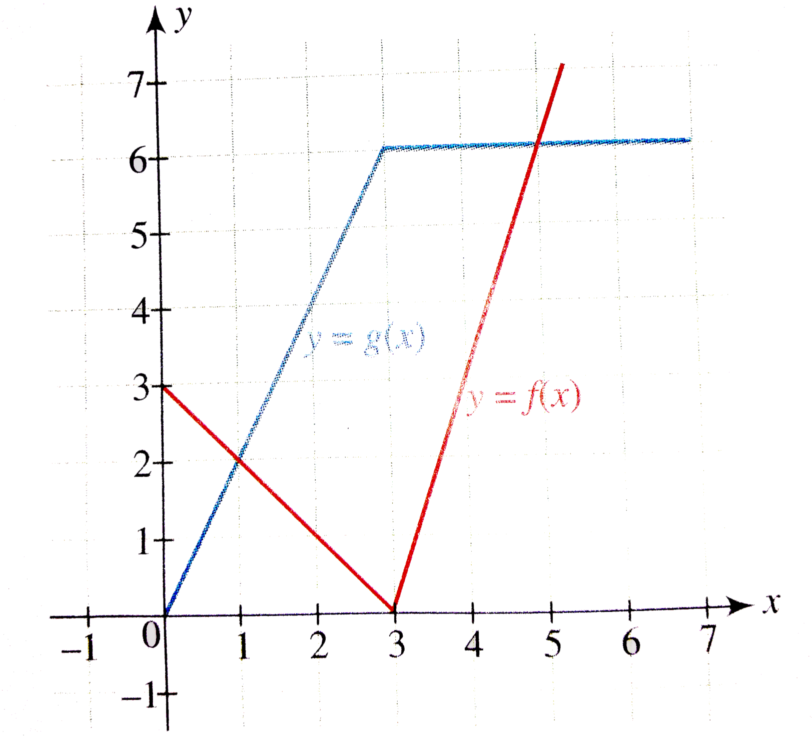
3. Graph of $y = f(x)$ and $y = g(x)$ is given in the following figure. If $h(x) = f(g(x))$, then find the value of $h'(2)$.

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4. Let $f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}$ for all real x and y. If f'(0) exists and equals-1 and f(0)=1, find f(2)

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**5.** Sketch the regions satisfying the following inequalities:

(a) $x > 2$

(b) $|y| \geq 1$

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**6.** Shade the regions where points satisfy $|x - y| < 1.$

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**7.** Plot the region satisfying $|x| + |y| \leq 2 \text{ and } |x| + |y| > 2.$

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**8.** If $x < 2$, then find the values of $x^2$ graphically.

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9. If $x < -1$, then find the vallues of $x^2$ graphically.

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10. Draw the graph of $f(x) = \begin{cases} x^3, x^2 < 1 \\ x, x^2 \leq 1 \end{cases}$

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11. If $x > 2$, then find the values of $1/x$ graphically.

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12. If $x < -1$, then find the values of $1/x$ graphically.

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13. When $x > -2$, find the values of $1/x$.

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**14.** When $x < 3$, find the values of $1/x$.

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**15.** Draw the graph of $\frac{1}{x} + \frac{1}{y} = 1$.

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**16.** Draw the graph of $y = \frac{1}{x^2}$.

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**17.** Draw the graphs of following quadratic functions.

(i) $y = x^2 + x + 1$ (ii) $y = x^2 - 2x - 3$

(iii) $y = 2 + x - x^2$ (iv) $y = x - 1 - x^2$

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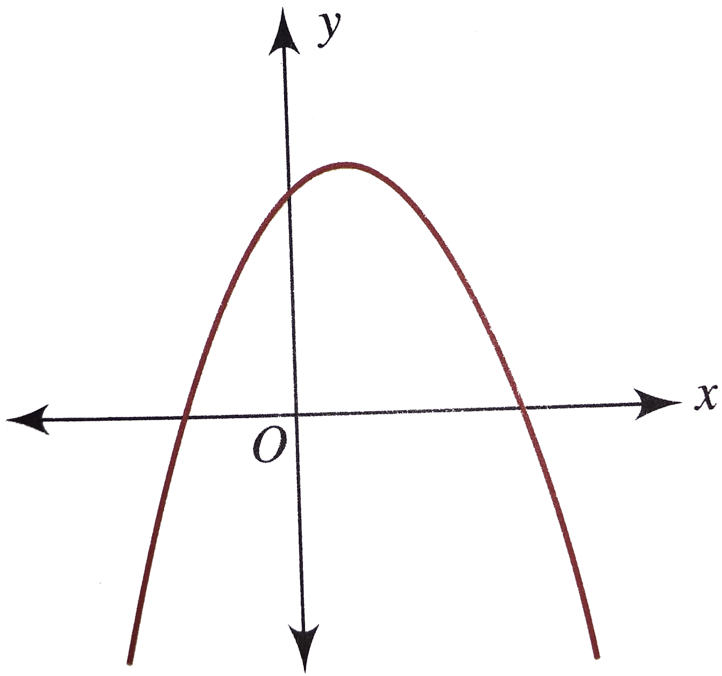
**18.** The following figure shows the graph of $f(x) = ax^2 + bx + c$, find the sign of $a, b$ and $c$.

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**19.** Let $f(x)=2x(2-x), 0 \leq x \leq 2$. Then find the number of solutions of $f(f(f(x)))=\frac{x}{2}$.

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**20.** $f: R \to R$ is defined as $f(x)=\begin{cases} x^2+kx+3, & \text{for } x \geq 0 \\ 2kx+3, & \text{for } x<0 \end{cases}$. If $f(x)$ is injective, then find the values of $k$.

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**21.** If $f(x)=x^3+4x^2+\lambda x+1$ is a monotonically decreasing function of $x$ in the largest possible interval $\left(-2, -\frac{2}{3}\right)$. Then $\lambda=4$ (b) $\lambda=2$ $\lambda=-1$ (d) $\lambda$ has no real value

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22. For what real values of $a$ do the roots of the equation $x^2 - 2x - (a^2 - 1) = 0$ lie between the roots of the equation $x^2 - 2(a+1)x + a(a-1) = 0$.

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23. Find the value of $a$ for which $ax^2 + (a-3)x + 1 < 0$ for at least one positive real $x$ .

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24. Consider the inequality, $9^x - a.3^x - a + 3 \leq 0$, where $'a'$ is a real parameter.

(a) Find the value of $'a'$ for which the inequality has at least one negative solution.

(b) Find the values of $'a'$ for which the inequality has at least one positive solution.

(c) Find the vlaues of $'a'$ for which the inequality has at least one real solution.

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**25.** Let $a, b, c$ be real. If $ax^2 + bx + c = 0$ has two real roots $\alpha and \beta, where \alpha \langle -1 and \beta \rangle 1$, then show that $1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0$

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**26.** If $b > a$, then the equation $(x-a)(x-b) - 1 = 0$ has

(a) Both roots in $(a, b)$

(b) Both roots in $(-\infty, a)$

(c) Both roots in $(b, +\infty)$

(d) One root in $(-\infty, a)$ and the other in $(b, +\infty)$

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**27.** When $x > -2$, find the values of $|x|$ graphically.

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**28.** When $x < 3$, find the values of $|x|$ graphically.

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**29.** If $2 \leq |x| \leq 5$, then find the values of x from the graph of $y = |x|$.

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**30.** Draw the graph of $f(x) = \frac{|x-1|}{x-1}$.

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**31.** Draw the graph of $x + |y| = 2y$ and check the differentiability.

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**32.** Draw the graph of $f(x) = (x+2)|x-1|$.

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**33.** Draw the graph of the function $f(x) = x - \left|x - x^2\right|, \ -1 \leq x \leq 1$ and find the points of non-differentiability.

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**34.** Solve : $x^2 - |x+2| + x > 0$

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**35.** Draw the graph of $f(x) = |2x-1| + |2x-3|$. Find the range of the function.

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**36.** Draw the graph of $f(x) = |x| - |2x - 3|$. Find the range of the function.

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**37.** Let $f(x) = x + 2|x + 1| + 2|x - 1|$. Find the values of $k$ if $f(x) = k$

(i) has exactly one real solution,

(ii) has two negative solutions,

(iii) has two solutions of opposite sign.

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**38.** $f(x) = |ax - b| + c|x| \forall x \in (-\infty, \infty)$, where $a > 0, b > 0, c > 0$. Find the condition if $f(x)$ attains the minimum value only at one point.

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**39.** The tangent to the curve $y = e^x$ drawn at the point $(c, e^c)$ intersects the line joining $(c-1, e^{c-1})$ and $(c+1, e^{c+1})$ (a) on the left of $n = c$ (b) on the right of $n = c$ (c) at no points (d) at all points

A. On the left of $x = c$

B. On the right of $x = c$

C. At no point

D. At all points

**Answer:**

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**40.** If a continuous function $f$ defined on the real line R assume positive and negative values in R, then the equation $f(x) = 0$ has a root in R. For example, if it is known that a continuous function $f$ on R is positive at some point and its minimum value is negative, then the equation $f(x) = 0$ has a root in R. Consider $f(x) = ke^x - x$, for all real x where k is a real constant.

The line $y = x$ meets $y = ke^x$ for $k \leq 0$ at

A. No point

B. One point

C. Two points

D. More than two points

**Answer:**

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**41.** If a continuous function $f$ defined on the real line R assume positive and negative values in R, then the equation $f(x) = 0$ has a root in R. For example, if it is known that a continuous function $f$ on R is positive at some point and its minimum value is negative, then the equation $f(x) = 0$ has a root in R. Consider $f(x) = ke^x - x$, for all real x where k is a real constant.

The line $y = x$ meets $y = ke^x$ for $k \leq 0$ at

A. $\frac{1}{e}$

B. 1

C. $e$

D. $\log_e 2$

**Answer:**

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**42.** If a continuous function $f$ defined on the real line R assume positive and negative values in R, then the equation $f(x) = 0$ has a root in R. For example, if it is known that a continuous function $f$ on R is positive at some point and its minimum value is negative, then the equation $f(x) = 0$ has a root in R. Consider $f(x) = ke^x - x$, for all real x where k is a real constant.

The line $y = x$ meets $y = ke^x$ for $k \leq 0$ at

A. $\left(0, \frac{1}{e}\right)$

B. $\left(\frac{1}{e}, 1\right)$

C. $\left(\frac{1}{e}, \infty\right)$

D. $(0, 1)$

**Answer:**

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**43.** Find the number of solution of $2x^2 + 3^x + 4^x - 5^x = 0$

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**44.** Draw the graph of $y = \log_x \sqrt{x}$

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**45.** Find the number of roots of the equation $x \log_e x = 1.$

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**46.** If the graphs of the functions $y = \log_e x$ and $y = ax$ intersect at exactly two points,then find the value of $a$.

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**47.** draw the graph of f(x)=x+[x], [.] denotes greatest integer function.

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**48.** Draw the graph of the function $f(x) = \left|\left(x + \frac{1}{2}\right)[x]\right|, \ -2 \leq x \leq 2$, where $[\cdot]$ denotes the greatest integer function. Find the points of discontinuity and non-differentiability.

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**49.** Draw the graph of $f(x) = [x^2], x \in [0, 2)$, where $[\cdot]$ denotes the greatest integer function.

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**50.** Draw the graph of $f(x) = [\sqrt{x}], x \in [0, 16)$, where $[\cdot]$ denotes the greatest ineger function.

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**51.** Draw the graph of $y = [x] + \sqrt{x - [x]}$, where$[\cdot]$ denotes the greatest ineger function.

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**52.** Draw the graph of $f(x) = [\log_e x], e^{-2} < x < 10$, where $[\cdot]$ represents the greatest integer function.

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**53.** Solve $x^2 - 4 - [x] = 0$ (where [] denotes the greatest integer function).

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**54.** Sketch the region of relation $[x] + [y] = 5, x, y \geq 0$, where $[\cdot]$ denots the greatest integer function.

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**55.** Draw the graph of $f(x) = \{2x\}$, where $\{\cdot\}$ represents the fractional part function.

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**56.** Find the domain of $f(x) = \sqrt{|x| - \{x\}}$ (where $\{\cdot\}$ denots the fractional part of x).

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**57.** Solve : $x^2 = \{x\}$, where $\{x\}$ represents the fractional part function.

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**58.** Draw the graph of $y^2 = \{x\}$, where $\{\cdot\}$ represents the fractional part function.

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**59.** Draw the graph of $y = \frac{1}{\{x\}}$, where $\{\cdot\}$ denotes the fractional part function.

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**60.** Solve : $4\{x\} = x + [x]$(where $[\cdot]$ denotes the greatest integer function and $\{\cdot\}$ denotes the fractional part function.

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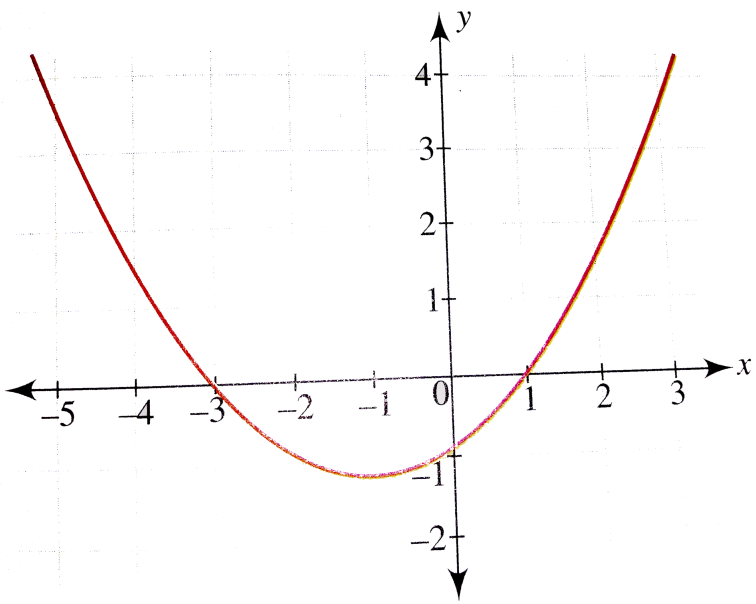
**61.** Given the graph of the function $y = f(x)$, draw the graph of $y = \text{sgn}(x)$.

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**62.** Draw the graph of $f(x) = \operatorname{sgn}(x^3 - x)$.

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**63.** Draw the graph of $f(x) = \operatorname{sgn}(\log_e x)$.

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**64.** Let a function $f(x)$ be defined in $[-2, 2]$ as

$$f(x) = \begin{cases} \{x\}, & -2 \le x < -1 \\ |\operatorname{sgn}\ x|, & -1 \le x \le 1 \\ \{-x\}, & 1 < x \le 2 \end{cases}$$

where $\{x\}$ and sgn $x$ denote fractional part and signum functions, respectively. Then find the area bounded by the graph of $f(x)$ an the x-axis.

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65. Let $f: R \rightarrow R$ be defined as $f(x) = e^{\operatorname{sgn} x} + e^{x^2}$. Then find the range of the function, and also indentify the type of the function : one-one or many-one.

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66. Draw the graph of the function $f(x) = \max . \{x, x^2\}$ and write its equivalent definition.

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67. Let $f: R \rightarrow R$ be a function defined by $f(x) = \max . \{x, x^3\}$. The set of all points where $f(x)$ is NOT differenctiable is

(a) $\{-1, 1\}$

(b) $\{-1, 0\}$

(c ) $\{0, 1\}$

(d) $\{-1, 0, 1\}$

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68. Find the equivalent definition of

$f(x) = max x^2, (-x)^{2^{\cdot}}, 2x(1-x) whre 0 \leq x \leq 1$

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69. Let $f: R \to R$ and $g: R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1.$ Define $h: R \to R$ by $h(x) = \{ \max\{f(x), g(x)\}, \text{ if } x \leq 0 \text{ and } \min\{f(x), g(x)\}, \text{ if } x >$

.The number of points at which $h(x)$ is not differentiable is

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70. Sketch the region of the points satisfying $\max . \{|x|, |y|\} \leq 4.$

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**71.** Consider the regions $A = \left\{(x, y) \mid x^2 + y^2 \leq 100\right\}$ and $B = \{(x, y) \mid \sin(x + y) > 0\}$ in the plane. Then find the area of the region $A \cup B$.

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**72.** Draw the graphs of the following parabolas :

(i) $x = y^2 - 2y - 3$

(ii) $x = 6 + y - y^2$

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**73.** Find the number of roots of the equation $e^x = \sqrt{-x}$

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**74.** Let $g(x) = \sqrt{x - 2k}, \forall 2k \leq x < 2(k+1),$ where $k \in$ integer. Check whether $g(x)$ is periodic or not.

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**75.** Plot the region in the first quadrant in which points are nearer to the origin than to the line $x = 3$.

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**76.** Draw the graph of $y = \sqrt{x^2 - 1}$

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**77.** Draw the graph of $y = -\sqrt{6 - 3x^2}$

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**78.** The eccentricity of the ellipse $9x^2 + 25y^2 - 18x - 100y - 116 = 0$ is

$25/16$ b. $4/5$ c. $16/25$ d. $5/4$

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**79.** Find the area enclosed by the curves $y = \sqrt{x}$ and $x = -\sqrt{y}$ and the circle $x^2 + y^2 = 2$ above the x-axis.

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**80.** Consider a square with vertices at $(1, 1), (-1, 1), (-1, -1), and (1, -1)$. Set $S$ be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region $S$ and find its area.

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Exercise

**1.** Draw the graph of $y = \frac{1}{(1/x)}$.

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**2.** (a) Draw the graph of

$$f(x) = = \begin{cases} 1, & |x| \geq 1 \\ \frac{1}{n^2}, & \frac{1}{n} < |x| < \frac{1}{n-1}, n = 2,3,\ldots \\ 0, & x = 0 \end{cases}$$

(b) Sketch the region $y \leq -1$.

(c) Sketch the region $|x| < 3$.

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**3.** Sketch the regions which points satisfy $|x + y| \geq 2$.

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**4.** Sketch the region satisfying $|x| < |y|$.

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**5.** For a point $P$ in the plane, let $d_1(P) and d_2(P)$ be the distances of the point $P$ from the lines $x - y = 0 and x + y = 0$ respectively. The area of the region $R$ consisting of all points $P$ lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4,$ is

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**6.** Draw the graph of $y = \dfrac{x-1}{x-2}$.

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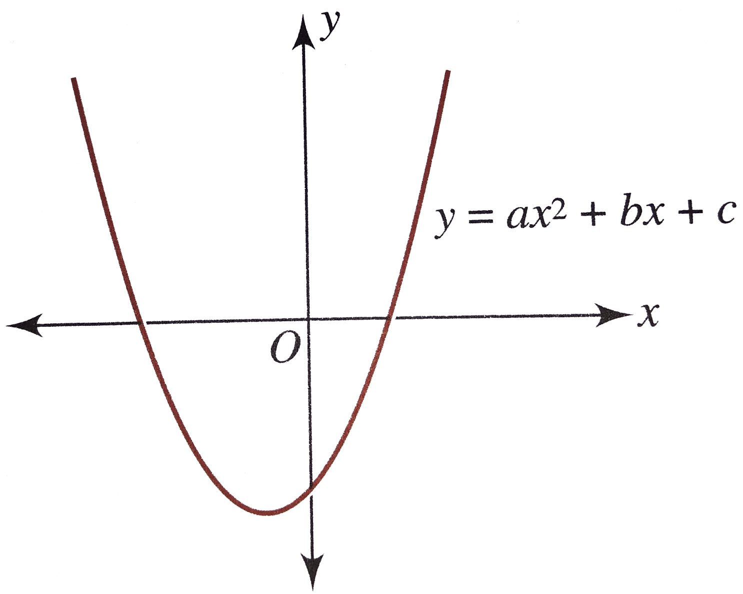
**7.** The following figure shows the graph of $f(x) = ax^2 + bx + c$, then find the sign of values of $a$, $b$ and $c$.

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**8.** The entire graph of the equation $y = x^2 + kx - x + 9$ in strictly above the $x - a\xi s$ if and only if $k < 7$ (b) `-5-5` (d) none of these

A. $k < 7$

B. $-5 < k < 7$

C. $k > -5$

D. None of these

**Answer:**

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**9.** If $x^2 + 2ax + a < 0 \forall x \in [1, 2]$, the find the values of $a\cdot$

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**10.** Draw the graph of $f(x) = x|x|$.

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**11.** Draw the graph of the function: Solve $\left|\frac{x^2}{x-1}\right| \leq 1$ using the graphical method.

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**12.** Draw the graph of $y = |x^2 - 2x| - x$.

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**13.** Draw the graph of $y = \begin{cases} 2^x, & x^2 - 2x \leq 0 \\ 1 + 3.5x - x^2 & x^2 - 2x > 0 \end{cases}$

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**14.** Draw the graph of $f(x) = |x - 1| + |2x - 3|$. Find the range of the function.

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**15.** Draw the graph of $y = |x - 1| + 3|x - 2| - 5|x - 4|$ and find the values of $\lambda$ for which the equation $f(x) = \lambda$ has roots of opposite sign.

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16. Find the set of real value(s) of $a$ for which the equation $|2x+3|+2x-3| = ax+6$ has more than two solutions.

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17. Draw the graph of $y = 2^{\frac{(|x|+x)}{x}}$.

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18. Draw the graph of $y = x^{\frac{1}{\log_e x}}$.

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19. Find the number of solutions to the question $x + \log_e x = 0$.

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**20.** draw the graph of f(x)=x+[x], [.] denotes greatest integer function.

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**21.** Given $f(x)$ is a periodic function with period 2 and it is defined as

$$f(x) = \begin{cases} \left[\cos\frac{\pi x}{2}\right] + 1, & 0 < x < 1 \\ 2 - x, & 1 \leq x < 2 \end{cases}$$

Here $[\cdot]$ represents the greatest integer $\leq x$. If $f(0) = 1$, then draw the graph of the function for $x \in [-2, 2]$.

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**22.** Draw the region of relation $[x][y] = 6, x, y \geq 0$. Here $[\cdot]$ denotes the greatest integer function.

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**23.** Draw the graph of $y = \{x\}^2$, where $[\cdot]$ represents the fractional part function.

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**24.** Let $f(x) = \dfrac{[x]+1}{\{x\}+1}$ for $f : \left[0, \dfrac{5}{2}\right) \to \left(\dfrac{1}{2}, 3\right]$, where $[\cdot]$ represents the greatest integer function and $\{\cdot\}$ represents the fractional part of x. Draw the graph of $y = f(x)$. Prove that $y = f(x)$ is bijective. Also find the range of the function.

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**25.** Draw the graph of $y = 2^{\{x\}}$, where $\{\cdot\}$ represents the fractional part function.

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**26.** Find tha area of the region containing the points $(x, y)$ satisfying $4 \leq x^2 + y^2 \leq 2(|x| + |y|)$.

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**27.** Draw the graph of $y = -\sqrt{x^2 + 2}$

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**28.** Draw the graph of $y = |x|^{1/2}$ from $-1 \leq x \leq 1$.

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**29.** Draw the graph of $f(x) = sgn(\log_{0.5} x)$.

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**30.** Graph of $y = f(x)$ is given as shown in the following figure. Draw the graph of $y = \text{sgn}(f(x))$.

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**31.** Draw the graph of the function $y = f(x) = \lim_{n \to \infty} \frac{x^{2n} - 1}{x^{2n} + 1}$. Is this function same as the function $g(x) = \text{sgn}(x^2 - 1)$.

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**32.** An even periodic function $f: R \to R$ with period 4 is such that $f(x) = \begin{cases} \max .(|x|, x^2), & 0 \leq x < 1 \\ x, & 1 \leq x \leq 2 \end{cases}$. Then draw the graph of $y = f(x)$ for $x \in R$

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**33.** The function $f(x) = \max . \{(1 - x), (1 + x), 2\}, x \in (-\infty, \infty)$ is

A. Continuous at all points

B. Differentiable at all points

C. Differentiable at all points except at $x = 1$ and $x = -1$

D. Continuous at all points except at $x = 1$ and $x = -1$, where it is discontinuous

**Answer:**

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**34.** Check the differentiability if $f(x) = \min . \{1, x^2, x^3\}$.

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