MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

INTRODUCTION TO VECTORS

Examples

1. The vector $\vec{a} + \vec{b}$ bisects the angle between the vectors $\hat{a}$ and $\hat{b}$ if

(A) $\left|\vec{a}\right| + \left|\vec{b}\right| = 0$ (B) angle between $\vec{a}$ and $\vec{b}$ is zero (C) $\left|\vec{a}\right| = \left|\vec{b}\right| = 0$ (D) none of these

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2. if $\vec{A}o + \vec{O}B = \vec{B}O + \vec{O}C$,than prove that B is the midpoint of AC.

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3. $ABCDE$ is pentagon, prove that $\vec{A}B + \vec{B}C + \vec{C}D + \vec{D}E + \vec{E}A = \vec{0}$

$\vec{A}B + \vec{A}E + \vec{B}C + \vec{D}C + \vec{E}D + \vec{A}C = 3\vec{A}C$

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4. Prove that the resultant of two forces acting at point O and represented by $\vec{O}B$ and $\vec{O}C$ is given by $2\vec{O}D$,where D is the midpoint of BC.

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5. Prove that the sum of three vectors determined by the medians of a triangle directed from the vertices is zero.

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6. ABC is a triangle and P any point on BC. if $\overrightarrow{P}Q$ is the sum of $\overrightarrow{A}P$ + $\overrightarrow{P}B$ +$\overrightarrow{P}C$, show that ABPQ is a parallelogram and Q , therefore , is a fixed point.

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7. Two forces $\overrightarrow{A}B$ and $\overrightarrow{A}D$ are acting at vertex A of a quadrilateral ABCD and two forces $\overrightarrow{C}B$ and $\overrightarrow{C}D$ at C prove that their resultant is given by 4 $\overrightarrow{E}F$, where E and F are the midpoints of AC and BD, respectively.

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8. If O($\overrightarrow{0}$) is the circumcentre and O' the orthocentre of a triangle ABC, then prove that

i. $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \overrightarrow{OO'}$

ii. $\overrightarrow{O'A} + \overrightarrow{O'B} + \overrightarrow{O'C} = 2\overrightarrow{O'O}$

iii. $\overrightarrow{AO'} + \overrightarrow{O'B} + \overrightarrow{O'C} = 2\overrightarrow{AO} = \overrightarrow{AP}$

where AP is the diameter through A of the circumcircle.

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9. A unit vector of modulus 2 is equally inclined to x - and y -axes angle at an angle $\pi/3$. Find the length of projection of the vector on the z -axis.

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10. If the projections of vector $\vec{a}$ on x -, y - and z -axes are 2, 1 and 2 units ,respectively, find the angle at which vector $\vec{a}$ is inclined to the z -axis.

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11. Find a vector of magnitude 8 units in the direction of the vector $\left(5\hat{i}-\hat{j}+2\hat{k}\right)$.

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12. सदिश $\overline{PQ}$, के अनुदिश मात्रक सदिश ज्ञात कीजिए जहाँ बिंदु P और Q क्रमश: (1,2,3) और (4,5,6) है!

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13. If $\overrightarrow{a} = \left(-\hat{i} + \hat{j} - \hat{k}\right)$ and $\overrightarrow{b} = \left(2\hat{i} - 2\hat{j} + 2\hat{k}\right)$ then find the unit vector in the direction of $\left(\overrightarrow{a} + \overrightarrow{b}\right)$.

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14. Show that the points A, B and C having position vectors $\left(3\hat{i} - 4\hat{j} - 4\hat{k}\right), \left(2\hat{i} - \hat{j} + \hat{k}\right)$and $\left(\hat{i} - 3\hat{j} - 5\hat{k}\right)$ respectively, from the vertices of a right-angled triangle.

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15. If $2\overrightarrow{A}C = 3\overrightarrow{C}B$, then prove that $2\overrightarrow{O}A = 3\overrightarrow{C}B$ then prove that $2\overrightarrow{O}A + 3\overrightarrow{O}B = 5\overrightarrow{O}C$ where O is the origin.

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16. Prove that points $\hat{i} + 2\hat{j} - 3\hat{k}, 2\hat{i} - \hat{j} + \hat{k}$ and $2\hat{i} + 5\hat{j} - \hat{k}$ form a triangle in space.

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17. Find the position vector of a point R which divides the line joining the point $P\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $Q\left(-\hat{i} + \hat{j} + \hat{k}\right)$ in the ratio $2:1$, (i) internally and (ii) externally.

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18. If $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}, \overrightarrow{d}$ are the position vectors of points A, B, C and D, respectively referred to the same origin O such that no three of these points are collinear and $\overrightarrow{a}+\overrightarrow{c}=\overrightarrow{b}+\overrightarrow{d}$, then prove that quadrilateral $ABCD$ is a parallelogram.

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19. Find the point of intersection of AB and $A($ 6,-7,0),B(16,-19,-4,) , C(0,3,-6) and D(2,-5,10).

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20. Find the angle of vector $\overrightarrow{a}=6\hat{i}+2\hat{j}-3\hat{k}$ with x-axis.

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21. i. Show that the lines joining the vertices of a tetrahedron to the centroids of opposite faces are concurrent.

ii. Show that the joins of the midpoints of the opposite edges of a tetrahedron intersect and bisect each other.

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22. The midpoint of two opposite sides of a quadrilateral and the midpoint of the diagonals are the vertices of a parallelogram. Prove that using vectors.

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23. Check whether the three vectors $2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-3\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{c}=3\hat{i}+4\hat{k}$ form a triangle or not.

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24. Find the resultant of vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$. Find the unit vector in the direction of the resultant vector.

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25. If in parallelogram ABCD, diagonal vectors are $\overrightarrow{AC} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\overrightarrow{BD} = -6\hat{i} + 7\hat{j} - 2\hat{k}$, then find the adjacent side vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$.

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26. If two sides of a triangle are $\hat{i} + 2\hat{j}$ and $\hat{i} + \hat{k}$, then find the length of the third side.

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27. Three coinitial vectors of magnitudes a, 2a and 3a meet at a point and their directions are along the diagonals if three adjacent faces if a cube. Determined their resultant R. Also prove that the sum of the three vectors determinate by the diagonals of three adjacent faces of a cube passing through the same corner, the vectors being directed from the corner, is twice the vector determined by the diagonal of the cube.

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28. The axes of coordinates are rotated about the z-axis though an angle of $\pi/4$ in the anticlockwise direction and the components of a vector are $2\sqrt{2}$, $3\sqrt{2}, 4.$ Prove that the components of the same vector in the original system are -1,5,4.

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29. If the resultant of two forces is equal in magnitude to one of the components and perpendicular to it direction, find the other components using the vector method.

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30. A man travelling towards east at 8km/h finds that the wind seems to blow directly from the north On doubling the speed, he finds that it appears to come from the north-east. Find the velocity of the wind.

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31. OABCDE is a regular hexagon of side 2 units in the XY-plane in the first quadrant. O being the origin and OA taken along the x-axis. A point P is taken on a line parallel to the z-axis through the centre of the hexagon at a distance of 3 unit from O in the positive Z direction. Then find vector AP.

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32. If $\overrightarrow{a} = 7\hat{i} - 4\hat{j} - 4\hat{k}$ and $\overrightarrow{b} = -2\hat{i} - \hat{j} + 2\hat{k}$, determine vector $\overrightarrow{c}$ along the internal bisector of the angle between vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ such that $\left|\overrightarrow{c}\right| = 5\sqrt{6}$.

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33. Find a unit vector $\overrightarrow{c}$ if $-\hat{i} + \hat{j} - \hat{k}$ bisects the angle between vectors $\overrightarrow{c}$ and $3\hat{i} + 4\hat{j}$.

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34. The vectors $2\hat{i} + 3\hat{j}, 5\hat{i} + 6\hat{j}$ and $8\hat{j} + \lambda\hat{j}$ have their initial points at (1, 1). The value of λ so that the vectors terminate on one straight line, is

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35. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero vectors, no two of which are collinear, $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$, then find the value of $\left|\vec{a} + 2\vec{b} + 6\vec{c}\right|$.

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36. i. Prove that the points $\vec{a} - 2\vec{b} + 3\vec{c}, 2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-7\vec{b} + 10\vec{c}$ are collinear, where $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar.

ii. Prove that the points $A(1, 2, 3), B(3, 4, 7)$ and $C(-3, -2, -5)$ are collinear. Find the ratio in which point C divides AB.

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37. Check whether the given three vectors are coplnar or non- coplanar :

$-2\hat{i} - 2\hat{j} + 4\hat{k}, -2\hat{i} + 4\hat{j} - 2\hat{k}, 4\hat{i} - 2\hat{j} - 2\hat{k}.$

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38. Prove that the four points $6\hat{i}-7\hat{j}, 16\hat{i}-19\hat{j}-4\hat{k}, 3\hat{j}-6\hat{k}$ and $2\hat{i}+5\hat{j}+10\hat{k}$ form a tetrahedron in spacel.

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39. If $\overrightarrow{a} and \overrightarrow{b}$ are two non-collinear vectors, show that points $l_1\overrightarrow{a}+m_1\overrightarrow{b}, l_2\overrightarrow{a}+m_2\overrightarrow{b}$ and $l_3\overrightarrow{a}+m_3\overrightarrow{b}$ are collinear if $|l_1l_2l_3m_1m_2m_3111|=0.$

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40. The vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ are non collinear. Find for what value of x the vectors $\overrightarrow{c}=(x-2)\overrightarrow{a}+\overrightarrow{b}$ and $\overrightarrow{d}=(2x+1)\overrightarrow{a}-\overrightarrow{b}$ are collinear.?

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41. The median AD of the triangle ABC is bisected at E and BE meets AC at F. Find AF:FC.

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42. Prove that the necessary and sufficient condition for any four points in three-dimensional space to be coplanar is that there exists a liner relation connecting their position vectors such that the algebraic sum of the coefficients (not all zero) in it is zero.

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43. i. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar vectors, prove that vectors $3\vec{a} - 7\vec{b} - 4\vec{c}, 3\vec{a} - 2\vec{b} + \vec{c}$ and $\vec{a} + \vec{b} + 2\vec{c}$ are coplanar.

ii. If the vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} + 2\hat{h} - 3\hat{k}$ and $3\hat{i} + a\hat{j} + 5\hat{k}$ are coplanar, the prove that $a = 4$.

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44. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-coplanar vectors, prove that the four points $2\vec{a}+3\vec{b}-\vec{c}$, $\vec{a}-2\vec{b}+3\vec{c}$, $3\vec{a}+4\vec{b}-2\vec{c}$ and $\vec{a}-6\vec{b}+6\vec{c}$ are coplanar.

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45. Let P be an interior point of a triangle ABC and AP, BP, CP meet the sides BC, CA, AB in D, E, F, respectively. Show that $\frac{AP}{PD}=\frac{AF}{FB}+\frac{AE}{EC}$.

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46. Points $A\left(\vec{a}\right), B\left(\vec{b}\right), C\left(\vec{c}\right)$ and $D\left(\vec{d}\right)$ are related as $x\vec{a}+y\vec{b}+z\vec{c}+w\vec{d}=0$ and $x+y+z+w=0$, where x, y, z and w are scalars (sum of any two of x, y, z and w is not zero). Prove that if A, B, C and D are concyclic, then $|xy|\left|\vec{a}-\vec{b}\right|^2=|wz|\left|\vec{c}-\vec{d}\right|^2$.

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Exercise 1 1

1. Find the unit vector in the direction of the vector $\overrightarrow{a} = \hat{i} + \hat{j} + 2\hat{k}$.

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2. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

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3. Find the direction cosines of the vector joining the points $A(1, 2, -3)$ and $B(-1, -2, 1)$ directed from A to B.

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4. The position vectors of P and Q are $5\hat{i}+4\hat{j}+a\hat{k}$ and $-\hat{i}+2\hat{j}-2\hat{k}$, respectively. If the distance between them is 7, then find the value of a.

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5. Given three points are $A(-3,-2,0), B(3,-3,1) and C(5,0,2)$. Then find a vector having the same direction as that of $\overrightarrow{A}B$ and magnitude equal to $\left|\overrightarrow{A}C\right|$.

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6. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors

$$\overrightarrow{a}=2\hat{i}+3\hat{j}-\hat{k} \text{ and } \overrightarrow{b}=\hat{i}-2\hat{j}+\hat{k}.$$

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7. Show that the points $A(1, -2, -8), B(5, 0, -2)$ and $C(11, 3, 7)$ are collinear, and find the ratio in which B divides AC.

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8. If $ABCD$ is a rhombus whose diagonals cut at the origin O, then proved that $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D + \vec{O}\cdot$

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9. Let $D, EandF$ be the middle points of the sides $BC, CAandAB$, respectively of a triangle $ABC\cdot$ Then prove that $\vec{A}D + \vec{B}E + \vec{C}F = \vec{0}$.

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10. Let $ABCD$ be a p[arallelogram whose diagonals intersect at P and let O be the origin. Then prove that $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D = 4\vec{O}P\cdot$

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11. If $ABCD$ is quadrilateral and $EandF$ are the mid-points of $ACandBD$ respectively, prove that $\vec{A}B + \vec{A}D$ +$\vec{C}B$ +$\vec{C}D$ =4 $\vec{E}F$.

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12. If $\vec{A}O + \vec{O}B = \vec{B}O + \vec{O}C$, then $A, BnadC$ are (where O is the origin) a. coplanar b. collinear c. non-collinear d. none of these

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13. If the sides of an angle are given by vectors $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$, then find the internal bisector of the angle.

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14. $ABCD$ is a parallelogram. If $LandM$ are the mid-points of $BCandDC$ respectively, then express $\overrightarrow{A}LandA\overrightarrow{M}$ in terms of $\overrightarrow{A}BandA\overrightarrow{D}$. Also, prove that $\overrightarrow{A}L+\overrightarrow{A}M=\frac{3}{2}\overrightarrow{A}C\cdot$

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15. $ABCD$ is a quadrilateral and E and the point intersection of the lines joining the middle points of opposite side. Show that the resultant of $\overrightarrow{O}A, \overrightarrow{O}B, \overrightarrow{O}CandO\overrightarrow{D}$ is equal to 4 $\overrightarrow{O}E,$ where O is any point.

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16. What is the unit vector parallel to $\overrightarrow{a}=3\hat{i}+4\hat{j}-2\hat{k}$? What vector should be added to $\overrightarrow{a}$ so that the resultant is the unit vector $\hat{i}$?

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17. The position vectors of points A and B w.r.t. the origin are $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$, respectively. Determine vector $\overrightarrow{OP}$ which bisects angle AOB, where P is a point on AB.

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18. If $\vec{r}_1, \vec{r}_2, \vec{r}_3$ are the position vectors off thee collinear points and scalar $pandq$ exist such that $\vec{r}_3 = p\vec{r}_1 + q\vec{r}_2$, then show that $p + q = 1$.

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19. If $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 1 inclined at 120°, then find the angle between $\vec{b}$ and $\vec{b} - \vec{a}$.

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20. Find the vector of magnitude 3, bisecting the angle between the vectors $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

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Exercise 1 2

1. If $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are four vectors in three-dimensional space with the same initial point and such that $3\vec{a} + 2\vec{b} + \vec{c} - 2\vec{d} = 0$, show that terminals $A, B, CandD$ of these vectors are coplanar. Find the point at which $ACandBD$ meet. Find the ratio in which P divides $ACandBD$.

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2. Show that the vectors $2\vec{a} - \vec{b} + 3\vec{c}, \vec{a} + \vec{b} - 2\vec{c}$ and $\vec{a} + \vec{b} - 3\vec{c}$ are non-coplanar vectors (where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors).

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3. Examine the following vectors for linear independence :

i. $\vec{i}+\vec{j}+\vec{k}, 2\vec{i}+\vec{j}-\vec{k}, -\vec{i}-2\vec{j}+2\vec{k}$

ii. $3\vec{i}+\vec{j}-\vec{k}, 2\vec{i}-\vec{j}+7\vec{k}, 7\vec{i}-\vec{j}+13\vec{k}$

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4. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors and $\vec{A}=(p+4q)\vec{a}+(2p+q+1)\vec{b}$ and $\vec{B}=(-2p+q+2)\vec{a}+(2p-3$, and if $3\vec{A}=2\vec{B}$, then determine p and q.

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5. If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three non-coplanar vectors, then prove that points

$l_1\vec{a}+m_1\vec{b}+n_1\vec{c}, l_2\vec{a}+m_2\vec{b}+n_2\vec{c}, l_3\vec{a}+m_3\vec{b}+n_3\vec{c}, l_4\vec{a}+m_4$

are coplanar if $\begin{vmatrix} l_1 & l_2 & l_3 & l_4 \\ m_1 & m_2 & m_3 & m_4 \\ n_1 & n_2 & n_3 & n_4 \\ 1 & 1 & 1 & 1 \end{vmatrix}=0$

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6. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero, non-coplanar vectors,then find the linear relation between the following four vectors : $\vec{a}-2\vec{b}+3\vec{c}, 2\vec{a}-3\vec{b}+4\vec{c}, 3\vec{a}-4\vec{b}+5\vec{c}, 7\vec{a}-11\vec{b}+15\vec{c}$

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7. Let a, b, c be distinct non-negative numbers and the vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}, c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, and then prove that the quadratic equation $ax^2+2cx+b=0$ has equal roots.

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Exercise Subjective

1. The position vectors of the vertices A, B and C of triangle are $\hat{i}+\hat{j}, \hat{j}+\hat{k}$ and $\hat{i}+\hat{k}$, respectively. Find the unit vectors $\hat{r}$ lying in the plane of ABC and perpendicular to IA, where I is the incentre of the triangle.

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2. A ship is sailing towards the north at a speed of 1.25 m/s. The current is taking it towards the east at the rate of 1 m/s and a sailor is climbing a vertical pole on the ship at the rate of 0.5 m/s. Find the velocity of the sailor in space.

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3. Given four points $P_1, P_2, P_3 and P_4$ on the coordinate plane with origin O which satisfy the condition $\left(\vec{O}P\right)_{n-1}\left(\overrightarrow{+} OP\right)_{n-1}=\frac{3}{2}\vec{O}P_n$. i. If

$ii. iii. iv. P_v.1vi. vii. andviii. P_ix.2x. \xi. \xi i.$ xiii. lie on the curve $xiv. xv. xy = 1, xvi.$ xvii. then prove that $xviii. \xi x. \times . P_x xi.3xxii. \times iii. \times iv.$ xxv. does not lie on the curve.

xxvi. If

$xxvii. \times viii. \times ix. P_x xx.1xxxi. \times \xi i. , xxxiii. P_x xxiv.2xxxv. \times xvi. a$

xlii. lie on the circle $xliii. xliv. xlv. x^{xlvi.2xlvii.} xlviii. + xlix. y^{l.2li.} lii. = 1, liii.$ liv. then prove that $lv. lvi. lvii. P_l viii.4lix. lx. l\xi.$ lxii. also lies on this circle.

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4. $ABCD$ is a tetrahedron and O is any point. If the lines joining O to the vrticfes meet the opposite faces at $P, Q, RandS,$ prove that $\frac{OP}{AP} + \frac{OQ}{BQ} + \frac{OR}{CR} + \frac{OS}{DS} = 1.$

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5. A pyramid with vertex at point P has a regular hexagonal base ABCDEF. Position vectors of points A and B are $\hat{i}$ and $\hat{i}+2\hat{j}$, respectively. The centre of the base has the position vector $\hat{i}+\hat{j}+\sqrt{3}\hat{k}$.

Altitude drawn from P on the base meets the diagonal AD at point G. Find all possible vectors of G. It is given that the volume of the pyramid is $6\sqrt{3}$ cubic units and AP is 5 units.

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6. A straight line L cuts the lines $AB, ACandAD$ of a parallelogram $ABCD$ at points $B_1, C_1andD_1,$ respectively. If $\left(\vec{A}B\right)_1, \lambda_1\vec{A}B, \left(\vec{A}D\right)_1 = \lambda_2\vec{A}Dand\left(\vec{A}C\right)_1 = \lambda_3\vec{A}C,$ then prove that $\frac{1}{\lambda_3}=\frac{1}{\lambda_1}+\frac{1}{\lambda_2}$.

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7. The position vectors of the points P and Q are $5\hat{i}+7\hat{j}-2\hat{k}$ and $-3\hat{i}+3\hat{j}+6\hat{k}$, respectively. Vector

$\overrightarrow{A} = 3\hat{i} - \hat{j} + \hat{k}$ passes through point P and vector $\overrightarrow{B} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ passes through point Q. A third vector $2\hat{i} + 7\hat{j} - 5\hat{k}$ intersects vectors A and B. Find the position vectors of points of intersection.

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8. Show that $x_1\hat{i} + y_1\hat{j} + z_1\hat{k}, x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ and $x_3\hat{i} + y_3\hat{j} + z_3\hat{k}$ are non-coplnar if $|x_1| > |y_1| + |z_1|$, $|y_2| > |x_2| + |z_2|$ and $|z_3| > |x_3| + |y_3|$.

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9. If $\overrightarrow{A}$ and $\overrightarrow{B}$ are two vectors and k any scalar quantity greater than zero, then prove that $\left|\overrightarrow{A} + \overrightarrow{B}\right|^2 \leq (1+k)\left|\overrightarrow{A}\right|^2 + \left(1 + \frac{1}{k}\right)\left|\overrightarrow{B}\right|^2$

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10. Consider the vectors $\hat{i} + \cos(\beta - \alpha)\hat{j} + \cos(\gamma - \alpha)\hat{k}, \cos(\alpha - \beta)\hat{i} + \hat{j} + \cos(\gamma - \beta)\hat{k} and \cos(\alpha$

are different angles. If these vectors are coplanar, show that a is independent of $\alpha, \beta, and \gamma.$

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11. In a triangle $PQR, S and T$ are points on $QR and PR,$ respectively, such that $QS = 3SR and PT = 4TR.$ Let M be the point of intersection of $PS and QT.$ Determine the ratio $QM : MT$ using the vector method .

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12. A boat moves in still water with a velocity which is k times less than the river flow velocity. Find the angle to the stream direction at which the boat should be rowed to minimize drifting.

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13. If $D, EandF$ are three points on the sides $BC, CAandAB$, respectively, of a triangle ABC such that the $\frac{BD}{CD}, \frac{CE}{AE}, \frac{AF}{BF} = -1$

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14. In a quadrilateral $PQRS, \overrightarrow{P}Q = \overrightarrow{a}, \overrightarrow{Q}R, \overrightarrow{b}, \overrightarrow{S}P = \overrightarrow{a} - \overrightarrow{b}, M$ is the midpoint of $\overrightarrow{Q}RandX$ is a point on SM such that $SX = \frac{4}{5}SM$. Prove that $P, XandR$ are collinear.

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Exercise Single

1. Four non zero vectors will always be a. linearly dependent b. linearly independent c. either a or b d. none of these

A. linearly dependent

B. linearly independent

C. either a or b

D. none of these

Answer: A

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2. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $3\vec{a} + 4\vec{b} + 5\vec{c} = \vec{0}$. Then which of the following statements is true? (A) $\vec{a}$ is parrallel to vecb (B)vec$a is perpendic\underline{a}r \rightarrow \vec{b}$ (C) $\vec{a}$ is neither parralel nor perpendicular to $\vec{b}$ (D) $\vec{a}, \vec{b}, \vec{c}$ are copalanar

A. $\vec{a}$ is parallel to $\vec{b}$

B. $\vec{a}$ is perpendicular to $\vec{b}$

C. $\vec{a}$ is neither parallel nor perpendicular to $\vec{b}$

D. none of these

Answer: D

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3. Let ABC be a triangle the position vectors of whose vertices are respectively $\hat{i}+2\hat{j}+4\hat{k}, -2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}-3\hat{k}$. Then the $\triangle ABC$ is (A) isosceles (B) equilateral (C) righat angled (D) none of these

A. isosceles

B. equilateral

C. right angled

D. none of these

Answer: C

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4. If $\left|\vec{a}+\vec{b}\right| < \left|\vec{a}-\vec{b}\right|$, then the angle between $\vec{a}$ and $\vec{b}$ can lie in the interval

A. $(-\pi/2, \pi/2)$

B. $(0, \pi)$

C. $(\pi/2, 3\pi/2)$

D. $(0, 2\pi)$

Answer: C

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5. A point O is the centre of a circle circumscribed about a triangle ABC. Then , $\vec{O}A \sin 2A + b\vec{O}B \sin 2B + \vec{O}C \sin 2C$ is equal to

A. $\left(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}\right) \sin 2A$

B. $3\overrightarrow{OG}$, where G is the centroid of triangle ABC

C. $\vec{0}$

D. none of these

Answer: C

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6. If G is the centroid of a triangle ABC, prove that $\vec{G}A + \vec{G}B + \vec{G}C = \vec{0}$.

A. $\vec{0}$

B. $3\overrightarrow{GA}$

C. $3\overrightarrow{GB}$

D. $3\overrightarrow{GC}$

Answer: A

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7. If $\vec{a}$ is a non zero vecrtor iof modulus $\vec{a}$ $and\, m$ is a non zero scalar such that ma is a unit vector, write the value of m.

A. $m = \pm 1$

B. $a = |m|$

C. $a = 1/|m|$

D. $a = \frac{1}{m}$

Answer: C

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8. ABCD a parallelogram, and A_1 and B_1 are the midpoints of sides BC and CD, respectively. If $\overrightarrow{aA_1} + \overrightarrow{AB_1} = \lambda\overrightarrow{AC}$, then λ is equal to `

A. $\frac{1}{2}$

B. 1

C. $\frac{3}{2}$

D. 2

Answer: C

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9. The position vectors of the points P and Q with respect to the origin O are $\overrightarrow{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\overrightarrow{b} = 3\hat{i} - \hat{j} - 2\hat{k}$, respectively. If M is a point on PQ, such that OM is the bisector of POQ, then $\overrightarrow{OM}$ is

A. $2\left(\hat{i} - \hat{j} + \hat{k}\right)$

B. $2\hat{i} + \hat{j} - 2\hat{k}$

C. $2\left(-\hat{i} + \hat{j} - \hat{k}\right)$

D. $2\left(\hat{i} + \hat{j} + \hat{k}\right)$

Answer: B

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10. $ABCD$ is a quadrilateral. E is the point of intersection of the line joining the midpoints of the opposite sides. If O is any point and $\overrightarrow{O}A + \overrightarrow{O}B + \overrightarrow{O}C + \overrightarrow{O}D = x\overrightarrow{O}E, thenx$ is equal to a. 3 b. 9 c. 7 d. 4

A. 3

B. 9

C. 7

D. 4

Answer: D

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11. The vector $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are sides of a triangle ABC. The length of the median through A is (A) $\sqrt{18}$ (B) $\sqrt{72}$ (C) $\sqrt{33}$ (D) $\sqrt{288}$

A. $\sqrt{14}$

B. $\sqrt{18}$

C. $\sqrt{29}$

D. 5

Answer: B

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12. A, B, C and D have position vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$, repectively, such that $\vec{a} - \vec{b} = 2\left(\vec{d} - \vec{c}\right)$. Then

A. AB and CD bisect each other

B. BD and AC bisect each other

C. AB and CD trisect each other

D. BD and AC trisect each other

Answer: D

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13. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then the unit vector along the angular bisector of $\vec{a}$ and $\vec{b}$ will be given by

A. $\dfrac{\vec{a} - \vec{b}}{2\cos(\theta/2)}$

B. $\dfrac{\vec{a} + \vec{b}}{2\cos(\theta/2)}$

C. $\dfrac{\vec{a} - \vec{b}}{\cos(\theta/2)}$

D. none of these

Answer: B

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14. let us define , the length of a vector as $|a| + |b| + |c|$. this definition coincides with the usual definition of the length of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ if

A. $a = b = c = 0$

B. any two of a, b and c are zero

C. any one of a, b and c is zero

D. $a + b + c = 0$

Answer: B

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15. Given three vectors $\vec{a} = \hat{i} - 3\hat{j}, \vec{b} = 2\hat{i} - t\hat{j}$ and $\vec{c} = -2\hat{i} + 21\hat{j}$ such that $\vec{\alpha} = \vec{a} + \vec{b} + \vec{c}$. Then the resolution of te vector $\vec{\alpha}$ into components with respect to $\vec{a}$ and $\vec{b}$ is given by (A) $3\vec{a} - 2\vec{b}$ (B) $2\vec{a} - 3\vec{b}$ (C) $3\vec{b} - 2\vec{a}$ (D) none of these

A. $3\vec{a} - 2\vec{b}$

B. $3\vec{b} - 2\vec{a}$

C. $2\vec{a} - 3\vec{b}$

D. $\vec{a} - 2\vec{b}$

Answer: C

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16. If $\vec{\alpha}+\vec{\beta}+\vec{\gamma}=a\vec{\delta}\, and \vec{\beta}+\vec{\gamma}+\vec{\delta}=b\vec{\alpha}, \vec{\alpha}\, and\, \vec{\delta}$ are non-colliner, then $\vec{\alpha}+\vec{\beta}+\vec{\gamma}+\vec{\delta}$ equals a. $a\vec{\alpha}$ b. $b\vec{\delta}$ c. 0 d. $(a+b)\vec{\gamma}$

A. $a\vec{\alpha}$

B. $b\vec{\delta}$

C. 0

D. $(a+b)\vec{\gamma}$

Answer: C

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17. In triangle $ABC, \angle A=30^0, H$ is the orthocenter and D is the midpoint of $BC\cdot$ Segment HD is produced to T such that $HD=DT\cdot$ The length AT is equal to a. $2BC$ b. $3BC$ c. $\frac{4}{2}BC$ d. none of these

A. 2 BC

B. 3 BC

C. $\frac{4}{3}BC$

D. none of these

Answer: A

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18. Let vecr_1, vecr_2,......vecr_n$bethepositionofp\int s$P_1,P_2,.........,P_n

$respectivelyrelative \rightarrow an$ or $ig \in O. Show t \widehat{\text{ if }} the \overrightarrow{\longrightarrow} requation$

a_1vecr_1+a_2vecr_2+..+a_nvecr_n=vec0

$holds, thenasimilarequationwillalsoholdg\infty dwi < hrespect \rightarrow anyothe$

a_1+a_2+......+a_n=0`

A. $a_1 + a_2 + \ldots. + a_n = n$

B. $a_1 + a_2 + \ldots. + a_n = 1$

C. $a_1 + a_2 + \ldots + a_n = 0$

D. $a_1 = a_2 = a_3 = \ldots. = a_n = 0$

Answer: C

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19. Given three non-zero, non-coplanar vectors $\vec{a}, \vec{b}$ and $\vec{c}$. $\vec{r}_1 = p\vec{a} + q\vec{b} + \vec{c}$ and $\vec{r}_2 = \vec{a} + p\vec{b} + q\vec{c}$. If the vectors $\vec{r}_1 + 2\vec{r}_2$ and $2\vec{r}_1 + \vec{r}_2$ are collinear, then (p, q) is

A. $(0, 0)$

B. $(1, -1)$

C. $(-1, 1)$

D. $(1, 1)$

Answer: D

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20. If the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ are linearly independent and satisfying $\left(\sqrt{3}\tan\theta - 1\right)\overrightarrow{a} + \left(\sqrt{3}\sec\theta - 2\right)\overrightarrow{b} = \overrightarrow{0}$,then the most general values of θ are:

A. $n\pi - \frac{\pi}{6}, n \in Z$

B. $2n\pi \pm \frac{11\pi}{6}, n \in Z$

C. $n\pi \pm \frac{\pi}{6}, n \in Z$

D. $2n\pi + \frac{11\pi}{6}, n \in Z$

Answer: D

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21. In a trapezium ABCD the vector $\overrightarrow{BC} = \lambda\overrightarrow{AD}$. If $\overrightarrow{p} = \overrightarrow{AC} + \overrightarrow{BD}$ is coillinear with $\overrightarrow{AD}$ such that $\overrightarrow{p} = \mu\overrightarrow{AD}$, then

A. $\mu = \alpha + 2$

B. $\mu + \alpha = 1$

C. $\alpha = \mu + 1$

D. $\mu = \alpha + 1$

Answer: D

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22. Vectors $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 4\hat{k}$ are so placed that the end point of one vector is the starting point of the next vector. Then the vectors are

A. not coplanar

B. coplanar but cannot form a triangle

C. coplanar and form a triangle

D. coplanar and can form a right-angled triangle

Answer: B

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23. Vectors $\overrightarrow{a} = -4\hat{i} + 3\hat{k}$; $\overrightarrow{b} = 14\hat{i} + 2\hat{j} - 5\hat{k}$ are laid off from one point. Vector $\hat{d}$, which is being laid of from the same point dividing the angle between vectors $\overrightarrow{a}\ and\ \overrightarrow{b}$ in equal halves and having the magnitude $\sqrt{6}$, is a. $\hat{i} + \hat{j} + 2\hat{k}$ b. $\hat{i} - \hat{j} + 2\hat{k}$ c. $\hat{i} + \hat{j} - 2\hat{k}$ d. $2\hat{i} - \hat{j} - 2\hat{k}$

A. $\hat{i} + \hat{j} + 2\hat{k}$

B. $\hat{i} - \hat{j} + 2\hat{k}$

C. $\hat{i} + \hat{j} - 2\hat{k}$

D. $2\hat{i} - \hat{j} - 2\hat{k}$

Answer: A

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24. If $\hat{i} - 3\hat{j} + 5\hat{k}$ bisects the angle between $\hat{a}$ and $-\hat{i} + 2\hat{j} + 2\hat{k}$, where $\hat{a}$ is a unit vector, then

A. $\hat{a} = \frac{1}{150}\left(41\hat{i} + 88\hat{j} - 40\hat{k}\right)$

B. $\hat{a} = \frac{1}{105}\left(41\hat{i} + 88\hat{j} + 40\hat{k}\right)$

C. $\hat{a} = \frac{1}{105}\left(-41\hat{i} + 88\hat{j} - 40\hat{k}\right)$

D. $\hat{a} = \frac{1}{105}\left(41\hat{i} - 88\hat{j} - 40\hat{k}\right)$

Answer: D

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25. If $4\hat{i} + 7\hat{j} + 8\hat{k}$, $2\hat{i} + 3\hat{j} + 4\hat{k}$ and $2\hat{i} + 5\hat{j} + 7\hat{k}$ are the position vectors of the vertices A, B and C, respectively, of triangle ABC, then the position vector of the point where the bisector of angle A meets BC is

A. $\frac{2}{3}\left(-6\hat{i} - 8\hat{j} - 6\hat{k}\right)$

B. $\frac{2}{3}\left(6\hat{i} + 8\hat{j} + 6\hat{k}\right)$

C. $\frac{1}{3}\left(6\hat{i} + 13\hat{j} + 18\hat{k}\right)$

D. $\frac{1}{3}\left(5\hat{j} + 12\hat{k}\right)$

Answer: C

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26. If $\overrightarrow{b}$ is a vector whose initial point divides thejoin of $5\hat{i}and5\hat{j}$ in the ratio $k:1$ and whose terminal point is the origin and $\left|\overrightarrow{b}\right| \leq \sqrt{37}, thenk$ lies in the interval a. $[-6, -1/6]$ b. $(-\infty, -6] \cup [-1/6, \infty)$ c. $[0, 6]$ d. none of these

A. $[-6, -1/16]$

B. $(-\infty, -6] \cup [-1/6, \infty)$

C. $[0, 6]$

D. none of these

Answer: B

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27. The value of theλ so that P, Q, R, S on the sides OA, OB, OC and AB of a regular tetrahedron are coplanar. When $\frac{OP}{OA}=\frac{1}{3}; \frac{OQ}{OB}=\frac{1}{2}$ and $\frac{OS}{AB}=\lambda$ is (A) $\lambda=\frac{1}{2}$ (B) $\lambda=-1$ (C) $\lambda=0$ (D) $\lambda=2$

A. $\lambda=\frac{1}{2}$

B. $\lambda=-1$

C. $\lambda=0$

D. for no value of λ

Answer: B

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28. $'I'$ is the incentre of triangle ABC whose corresponding sides are $a, b, c,$ rspectively. $a\overrightarrow{I}A+b\overrightarrow{I}B+c\overrightarrow{I}C$ is always equal to a. $\overrightarrow{0}$ b. $(a+b+c)\overrightarrow{B}C$ c. $\left(\overrightarrow{a}+\overrightarrow{b}+\overrightarrow{c}\right)\overrightarrow{A}C$ d. $(a+b+c)\overrightarrow{A}B$

A. $\overrightarrow{0}$

B. $(a+b+c)\overrightarrow{BC}$

C. $\left(\vec{a}+\vec{b}+\vec{c}\right)\overrightarrow{AC}$

D. $(a+b+c)\overrightarrow{AB}$

Answer: A

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29. Let $x^2+3y^2=3$ be the equation of an ellipse in the $x-y$ plane. $AandB$ are two points whose position vectors are $-\sqrt{3}\hat{i}and-\sqrt{3}\hat{i}+2\hat{k}$. Then the position vector of a point P on the ellipse such that $\angle APB=\pi/4$ is a. $\pm\hat{j}$ b. $\pm\left(\hat{i}+\hat{j}\right)$ c. $\pm\hat{i}$ d. none of these

A. $\pm\hat{j}$

B. $\pm\left(\hat{i}+\hat{j}\right)$

C. $\pm\hat{i}$

D. none of these

Answer: A

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30. Locus of the point P, for which $\overrightarrow{OP}$ represents a vector with direction cosine $\cos\alpha = \frac{1}{2}$ (where O is the origin) is

A. a circle parallel to the y-z plane with centre on the x-axis

B. a conic concentric with the positive x-axis having vertex at the origin and slant height equal to the magnitude of the vector

C. a ray emanating from the origin and making an angle of 60° with the x-axis

D. a dise parallel to the y-z plane with centre on the x-axis and radius equal to $\left|\overrightarrow{OP}\right|\sin 60^\circ$.

Answer: B

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31. If $\overrightarrow{x}$ and $\overrightarrow{y}$ are two non-collinear vectors and ABC is a triangle with side lengths a,b and c satisfying (20a-15b) $\overrightarrow{x}$ + (15b-12c) $\overrightarrow{y}$ + (12c-20a) $\overrightarrow{x} \times \overrightarrow{y}$ is:

A. an acute-angled triangle

B. an obtuse-angled triangle

C. a right-angled triangle

D. an isosceles triangle

Answer: C

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32. A uni-modular tangent vector on the curve $x = t^2 + 2, y = 4t - 5, z = 2t^2 - 6t = 2$ is a. $\frac{1}{3}\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$ b. $\frac{1}{3}\left(\hat{i} - \hat{j} - \hat{k}\right)$ c. $\frac{1}{6}\left(2\hat{i} + \hat{j} + \hat{k}\right)$ d. $\frac{2}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$

A. $\frac{1}{3}\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$

B. $\frac{1}{3}\left(\hat{i}-\hat{j}-\hat{k}\right)$

C. $\frac{1}{6}\left(2\hat{i}+\hat{j}+\hat{k}\right)$

D. $\frac{2}{3}\left(\hat{i}+\hat{j}+\hat{k}\right)$

Answer: A

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33. If $\overrightarrow{x}$ and $\overrightarrow{y}$ are two non-collinear vectors and a, b and c represent the sides of a ΔABC satisfying $(a-b)\overrightarrow{x}+(b-c)\overrightarrow{y}+(c-a)\left(\overrightarrow{\times}x\overrightarrow{y}\right)=0$, then ΔABC is (where $\overrightarrow{x}\times\overrightarrow{y}$ is perpendicular to the plane of $\overrightarrow{x}$ and $\overrightarrow{y}$)

A. an acute-angled triangle

B. an obtuse-angled triangle

C. a right-angled triangle

D. a scalene triangle

Answer: A

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34. $\vec{A}$ isa vector with direction cosines $\cos\alpha, \cos\beta and \cos\gamma$. Assuming the $y-z$ plane as a mirror, the directin cosines of the reflected image of $\vec{A}$ in the plane are a. $\cos\alpha, \cos\beta, \cos\gamma$ b. $\cos\alpha, -\cos\beta, \cos\gamma$ c. $-\cos\alpha, \cos\beta, \cos\gamma$ d. $-\cos\alpha, -\cos\beta, -\cos\gamma$

A. $\cos\alpha, \cos\beta, \cos\gamma$

B. $\cos\alpha, -\cos\beta, \cos\gamma$

C. $-\cos\alpha, \cos\beta, \cos\gamma$

D. $-\cos\alpha, -\cos\beta, -\cos\gamma$

Answer: C

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Exercise Multiple

1. The vectors $x\hat{i} + (x+1)\hat{j} + (x+2)\hat{k}, (x+3)\hat{i} + (x+4)\hat{j} + (x+5)\hat{k}$ and $(x+6)\hat{i}$

are coplanar if x is equal to

A. 1

B. -3

C. 4

D. 0

Answer: A::B::C::D

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2. The sides of a parallelogram are $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$. The unit vector parallel to one of the diagonals is

A. $\frac{1}{7}\left(3\hat{i} + 6\hat{j} - 2\hat{k}\right)$

B. $\frac{1}{7}\left(3\hat{i} - 6\hat{j} - 2\hat{k}\right)$

C. $\frac{1}{\sqrt{69}}\left(\hat{i} + 2\hat{j} + 8\hat{k}\right)$

D. $\frac{1}{\sqrt{69}}\left(-\hat{i} - 2\hat{j} + 8\hat{k}\right)$

Answer: A::D

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3. The vector $\overrightarrow{a}$ has the components $2p$ and 1 w.r.t. a rectangular Cartesian system. This system is rotated through a certain angel about the origin in the counterclockwise sense. If, with respect to a new system, $\overrightarrow{a}$ has components $(p+1) and 1$, then p is equal to a. -4 b. $-1/3$ c. 1 d. 2

A. -1

B. $-1/3$

C. 1

D. 2

Answer: B::C

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4. If points $\hat{i}+\hat{j}, \hat{i}-\hat{j}$ and $p\hat{i}+q\hat{j}+r\hat{k}$ are collinear, then

A. $p=1$

B. $r=0$

C. $q \in R$

D. $q \neq 1$

Answer: A::B::D

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5. If $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ are non coplanar vectors and λ is a real number, then the vectors $\overrightarrow{a}+2\overrightarrow{b}+3\overrightarrow{c}, \lambda\overrightarrow{b}+4\overrightarrow{}$ and $(2\lambda-1)\overrightarrow{c}$ are non coplanar for

A. $\mu \in R$

B. $\lambda = \frac{1}{2}$

C. $\lambda = 0$

D. no value of λ

Answer: A::B::C

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6. If the resultant of three forces $\vec{F}_1 = p\hat{i} + 3\hat{j} - \hat{k}$, $\vec{F}_2 = 6\hat{i} - \hat{k}$ and $\vec{F}_3 = -5\hat{i} + \hat{j} + 2\hat{k}$ acting on a particle has a magnitude equal to 5 units, then the value of p is

A. -6

B. -4

C. 2

D. 4

Answer: B::C

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7. If the vectors $\hat{i}-\hat{j}, \hat{j}+\hat{k}$ and $\overrightarrow{a}$ form a triangle then $\overrightarrow{a}$ may be (A) $-\hat{i}-\hat{k}$ (B) $\hat{i}-2\hat{j}-\hat{k}$ (C) $2\hat{i}+\hat{j}+\hat{j}k$ (D) hati+hatk`

A. $-\hat{i}-\hat{k}$

B. $\hat{i}-2\hat{j}-\hat{k}$

C. $2\hat{i}+\hat{j}+\hat{k}$

D. $\hat{i}+\hat{k}$

Answer: A::B::D

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8. The vector $\hat{i}+x\hat{j}+3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i}+(4x-2)\hat{j}+2\hat{k}$. Then values of x are

(A) $-\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 2

A. 1

B. $-2/3$

C. 2

D. $4/3$

Answer: B::C

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9. $\vec{a}, \vec{b}$ and $\vec{c}$ are three coplanar unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$. If three vectors $\vec{p}, \vec{q}$ and $\vec{r}$ are parallel to $\vec{a}, \vec{b}$ and $\vec{c}$, respectively, and have integral but different magnitudes, then among the following options, $\left|\vec{p}+\vec{q}+\vec{r}\right|$ can take a value equal to

A. 1

B. 0

C. $\sqrt{3}$

D. 2

Answer: C::D

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10. If non-zero vectors $\vec{a}$ and $\vec{b}$ are equally inclined to coplanar vector $\vec{c}$, then $\vec{c}$ can be

A. $\dfrac{|\vec{a}|}{|\vec{a}|+2|\vec{b}|}\vec{a}+\dfrac{|\vec{b}|}{|\vec{a}|+|\vec{b}|}\vec{b}$

B. $\dfrac{|\vec{b}|}{|\vec{a}|+|\vec{b}|}\vec{a}+\dfrac{|\vec{a}|}{|\vec{a}|+|\vec{b}|}\vec{b}$

C. $\dfrac{|\vec{a}|}{|\vec{a}|+2|\vec{b}|}\vec{a}+\dfrac{|\vec{b}|}{|\vec{a}|+2|\vec{b}|}\vec{b}$

D. $\dfrac{|\vec{b}|}{2|\vec{a}|+|\vec{b}|}\vec{a}+\dfrac{|\vec{a}|}{2|\vec{a}|+|\vec{b}|}\vec{b}$

Answer: B::D

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11. If $A(-4, 0, 3) and B(14, 2, -5)$, then which one of the following points lie on the bisector of the angle between $\vec{O}A and \vec{O}B (O$ is the origin of reference)? a. $(2, 2, 4)$ b. $(2, 11, 5)$ c. $(-3, -3, -6)$ d. $(1, 1, 2)$

A. $(2, 2, 4)$

B. $(2, 11, 5)$

C. $(-3, -3, -6)$

D. $(1, 1, 2)$

Answer: A::C::D

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12. In a four-dimensional space where unit vectors along the axes are $\hat{i}, \hat{j}, \hat{k} and \hat{l}, and \vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are four non-zero vectors such that no vector can be expressed as a linear combination of others and $(\lambda - 1)\left(\vec{a}_1 - \vec{a}_2\right) + \mu\left(\vec{a}_2 + \vec{a}_3\right) + \gamma\left(\vec{a}_3 + \vec{a}_4 - 2\vec{a}_2\right) + \vec{a}_3 + \delta \vec{a}$

then a. $\lambda = 1$ b. $\mu = -2/3$ c. $\gamma = 2/3$ d. $\delta = 1/3$

A. $\lambda = 1$

B. $\mu = -2/3$

C. $\gamma = 2/3$

D. $\delta = 1/3$

Answer: A::B::D

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13. Let ABC be a triangle, the position vectors of whose vertices are respectively

$7\hat{j} + 10\hat{k}, -\hat{i} + 6\hat{j} + 6\hat{k}$ and $-4\hat{i} + 9\hat{j} + 6\hat{k}$. Then, ΔABC is

A. isosceles

B. equilateral

C. right angled

D. none of these

Answer: A::C

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Exercise Reasoning Questions

1. Each question has four choices a, b, c, and d, out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. a. Both the statements are TRUE and statement 2 is the correct explanation for Statement 1. b. Both the statements are TRUE but Statement 2 is NOT the correct explanation for Statement 1. c. Statement 1 is TRUE and Statement 2 is FALSE. d. Statement 1 is FALSE and Statement 2 is TRUE. A vector has components p and 1 with respect to a rectangular Cartesian system. The axes are rotted through an angel α about the origin the anticlockwise sense. Statement 1: IF the vector has component $p+2$ and 1 with respect to the new system, then $p=-1$. Statement 2: Magnitude of the origin vector and the new vector remains the same.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

Answer: A

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2. Statement 1: if three points $P, QandR$ have position vectors $\vec{a}, \vec{b}, and\,\vec{c}$, respectively, and $2\vec{a}+3\vec{b}-5\vec{c}=0$, then the points

$P, Q, and R$ must be collinear. Statement 2: If for three points $A, B, and C, \vec{A}B = \lambda \vec{A}C$, then points $A, B, and C$ must be collinear.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

Answer: A

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3. Statement 1: If $\vec{u} and \vec{v}$ are unit vectors inclined at an angle $\alpha and \vec{x}$ is a unit vector bisecting the angle between them, then $\vec{x} = \left(\vec{u} + \vec{v}\right) / (2\sin(\alpha / 2)$. Statement 2: If $\mathrm{Delta}ABC$ is an isosceles triangle with $AB = AC = 1$, then the vector representing the bisector of angel A is given by $\vec{A}D = \left(\vec{A}B + \vec{A}C\right)/2$.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

Answer: D

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4. Statement 1: If $\cos\alpha, \cos\beta, and \cos\gamma$ are the direction cosines of any line segment, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Statement 2: If $\cos\alpha, \cos\beta, and \cos\gamma$ are the direction cosines of any line segment, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

Answer: B

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5. Statement 1: The direction cosines of one of the angular bisectors of two intersecting line having direction cosines as $l_1, m_1, n_1 and l_2, m_2, n_2$ are proportional to $l_1 + l_2, m_1 + m_2, n_1 + n_2.$ Statement 2: The angle between the two intersection lines having direction cosines as $l_1, m_1, n_1 and l_2, m_2, n_2$ is given by $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2.$

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

Answer: B

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6. Statement 1: In $\text{Delta}ABC, \vec{A}B + \vec{A}B + \vec{C}A = 0$ Statement 2: If $\vec{O}A = \vec{a}, \vec{O}B = \vec{b}, then\vec{A}B = \vec{a} + \vec{b}$

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

Answer: C

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7. Statement 1 : $\overrightarrow{a} = 3\overrightarrow{i} + p\overrightarrow{j} + 3\overrightarrow{k}$ and $\overrightarrow{b} = 2\overrightarrow{i} + 3\overrightarrow{j} + q\overrightarrow{k}$ are parallel vectors if $p = 9/2$ and $q = 2$.

Statement 2 : If $\overrightarrow{a} = a_1\overrightarrow{i} + a_2\overrightarrow{j} + a_3\overrightarrow{k}$ and $\overrightarrow{b} = b_1\overrightarrow{i} + b_2\overrightarrow{j} + b_3\overrightarrow{k}$ are parallel, then $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

Answer: A

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8. Statement 1 : If $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|$, then $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

Statement 2 : If the diagonals of a parallelogram are equal in magnitude, then the parallelogram is a rectangle.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

Answer: A

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9. Statement 1 : Let $A\left(\vec{a}\right), B\left(\vec{b}\right)$ and $C\left(\vec{c}\right)$ be three points such that $\vec{a} = 2\hat{i} + \hat{k}, veb = 3\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = -\hat{i} + 7\hat{j} - 5\hat{k}$. Then OABC is tetrahedron.

Statement 2 : Let $A\left(\vec{a}\right), B\left(\vec{b}\right)$ and $C\left(\vec{c}\right)$ be three points such that vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar. Then OABC is a tetrahedron, where O is the origin.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

Answer: A

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10. Statement 1: Let $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ be the position vectors of four points $A, B, CandD$ and $3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = 0.$ Then points $A, B, C, andD$ are coplanar. Statement 2: Three non-zero, linearly dependent coinitial vector $\left(\vec{P}Q, \vec{P}RandP\vec{S}\right)$ are coplanar. Then $\vec{P}Q = \lambda\vec{P}R + \mu\vec{P}S, where\lambda and\mu$ are scalars.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

Answer: A

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11. Statement 1 : If $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4$ and $\left|\vec{a} + \vec{b}\right| = 5,$ then $\left|\vec{a} - \vec{b}\right| = 5.$

Statement 2 : The length of the diagonals of a rectangle is the same.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

Answer: A

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Exercise Comprehension

1. ABCD is a parallelogram. L is a point on BC which divides BC in the ratio $1:2$. AL intersects BD at P.M is a point on DC which divides DC in the ratio $1:2$ and AM intersects BD in Q.

Point P divides AL in the ratio

A. $1:2$

B. $1:3$

C. $3:1$

D. $2:1$

Answer: C

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2. ABCD is a parallelogram. L is a point on BC which divides BC in the ratio $1:2$. AL intersects BD at P.M is a point on DC which divides DC in the ratio $1:2$ and AM intersects BD in Q.

Point Q divides DB in the ratio

A. $1:2$

B. $1:3$

C. $3:1$

D. $2:1$

Answer: B

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3. ABCD is a parallelogram. L is a point on BC which divides BC in the ratio $1:2$. AL intersects BD at P.M is a point on DC which divides DC in the ratio $1:2$ and AM intersects BD in Q.

$PQ:DB$ is equal to

A. $2/3$

B. $1/3$

C. $1/2$

D. $3/4$

Answer: C

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4. If ABCDEF is a regular hexagon then $\overrightarrow{AD}+\overrightarrow{EB}+\overrightarrow{FC}$ equals :

A. 2 $\overrightarrow{AB}$

B. 3 $\overrightarrow{AB}$

C. 4$A\overrightarrow{AB}$

D. none of these

Answer: C

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5. Consider the ragular hexagon ABCDEF with centre at O (origin).

Five forces $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}, \overrightarrow{AE}, \overrightarrow{AF}$ act at the vertex A of a regular hexagon ABCDEF. Then their resultant is

A. $3\overrightarrow{AO}$

B. $2\overrightarrow{AO}$

C. $4\overrightarrow{AO}$

D. $6\overrightarrow{AO}$

Answer: D

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6. Let A, B, C, D, E represent vertices of a regular pentagon ABCDE. Given the position vector of these vertices be $\vec{a}, \vec{a}+\vec{b}, \vec{b}, \lambda\vec{a}$ and $\lambda\vec{b}$, respectively.

The ratio $\dfrac{AD}{BC}$ is equal to

A. $1-\cos\frac{3\pi}{5}:\cos\frac{3\pi}{5}$

B. $1+2\cos\frac{2\pi}{5}:\cos\frac{\pi}{5}$

C. $1+2\cos\frac{\pi}{5}:2\cos\frac{\pi}{5}$

D. None of these

Answer: C

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7. Let A, B, C, D, E represent vertices of a regular pentagon ABCDE. Given the position vector of these vertices be $\vec{a}$, $\vec{a}+\vec{b}$, $\vec{b}$, $\lambda\vec{a}$ and $\lambda\vec{b}$, respectively.

AD divides EC in the ratio

A. $\cos\frac{2\pi}{5}:1$

B. $\cos\frac{3\pi}{5}:1$

C. $1:2\cos\frac{\pi}{5}$

D. $1:2$

Answer: C

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8. In a parallelogram OABC, vectors $\vec{a}, \vec{b}, \vec{c}$ are, respectively, tehe position vectors of vertices A, B, C with reference to O as origin. A point E is taken on the side BC which divides it in the ratio $2:1$. Also, the line segment AE intersects the line bisecting the angle $\angle AOC$ internally at point P. If CP when extended meets AB in point F, then

The position vector of point P is

A. $\dfrac{|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left(\dfrac{\vec{a}}{|\vec{a}|}+\dfrac{\vec{c}}{|\vec{c}|}\right)$

B. $\dfrac{3|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left(\dfrac{\vec{a}}{|\vec{a}|}+\dfrac{\vec{c}}{|\vec{c}|}\right)$

C. $\dfrac{2|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left(\dfrac{\vec{a}}{|\vec{a}|}+\dfrac{\vec{c}}{|\vec{c}|}\right)$

D. None of these

Answer: B

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9. In a parallelogram OABC, vectors $\vec{a}, \vec{b}, \vec{c}$ are, respectively, tehe position vectors of vertices A, B, C with reference to O as origin. A point E is taken on the side BC which divides it in the ratio $2:1$. Also, the line segment AE intersects the line bisecting the angle $\angle AOC$ internally at point P. If CP when extended meets AB in point F, then

The ratio in which F divides AB is

A. $\dfrac{2\left|\vec{a}\right|}{\left|\left|\vec{a} - 3\left|\vec{c}\right|\right|\right.}$

B. $\dfrac{\left|\vec{a}\right|}{\left|\left|\vec{a}\right| - 3\left|\vec{c}\right|\right|}$

C. $\dfrac{3\left|\vec{a}\right|}{\left|\left|\vec{a}\right| - 3\left|\vec{c}\right|\right|}$

D. $\dfrac{3\left|\vec{c}\right|}{3\left|\vec{c}\right| - \left|\vec{a}\right|}$

Answer: D

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Linked Comprehension Type

1. Let OABCD be a pentagon in which the sides OA and CB are parallel and the sides OD and AB are parallel. Also $OA : CB = 2 : 1$ and $OD : AB = 1 : 3$.

The ratio $\frac{OX}{XC}$ is

A. $3/4$

B. $1/3$

C. $2/5$

D. $1/2$

Answer: C

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2. Let OABCD be a pentagon in which the sides OA and CB are parallel and the sides OD and AB are parallel. Also $OA : CB = 2 : 1$ and $OD : AB = 1 : 3$.

The ratio $\frac{AX}{XD}$ is

A. $5/2$

B. 6

C. $7/3$

D. 4

Answer: B

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Matrix Match Type

1. Refer to the following diagram :

	Column I		Column II
$a.$	Collinear vectors	$p.$	$\vec{a}$
$b.$	Coinitial vectors	$q.$	$\vec{b}$
$c.$	Equal vectors	$r.$	$\vec{c}$
$d.$	Unlike vectors (same initial point)	$s.$	$\vec{d}$

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2. $\vec{a}$ and $\vec{b}$ form the consecutive sides of a regular hexagon ABCDEF.

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3.

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Exercise Numerical

1. Let ABC be a triangle whose centroid is G, orhtocentre is H and circumcentre is the origin 'O'. If D is any point in the plane of the triangle such that no three of O, A, C and D are collinear satisfying the relation $\overrightarrow{AD}+\overrightarrow{BD}+\overrightarrow{CH}+3\overrightarrow{HG}=\lambda\overrightarrow{HD}$, then what is the value of the scalar $'\lambda'$?

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2. If the resultant of three forces $\overrightarrow{F}_1=p\hat{i}+3\hat{j}-\hat{k}, \overrightarrow{F}_2=-5\hat{i}+\hat{j}+2\hat{k}$ and $\overrightarrow{F}_3=6\hat{i}-\hat{k}$ acting on a particle has a magnitude equal to 5 units, then what is difference in the values of p ?

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3. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be unit vector such that $\vec{a} + \vec{b} - \vec{c} = 0$. If the area of triangle formed by vectors $\vec{a}$ and $\vec{b}$ is A, then what is the value of $4A^2$?

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4. Find the least positive integral value of x form which the angle between vectors $\vec{a} = x\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = 2x\hat{i} + x\hat{j} - \hat{k}$ is acute.

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5. Vectors along the adjacent sides of parallelogram are $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$. Find the length of the longer diagonal of the parallelogram.

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6. If vectors $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + 2\hat{k}$ are coplanar, then find the value of $(\lambda - 4)$.

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Jee Previous Year

1. Find the all the values of lamda such that (x,y,z)!=(0,0,0) and x(hati+hatj+3hatk)+y(3hati-3hatj+hatk)+z(-4hati+5hatj)=lamda(xhati+yhatj+zhatk)`

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2. A vector a has components a_1, a_2, a_3 in a right handed rectangular cartesian coordinate system $OXYZ$ the coordinate axis is rotated about z axis through an angle $\frac{\pi}{2}$. The components of a in the new system

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3. The position vectors of the point A, B, C and D are $3\hat{i} - 2\hat{j} - \hat{k}, 2\hat{i} + 3\hat{j} - 4\hat{k}, -\hat{i} + \hat{j} + 2\hat{k}$ and $4\hat{i} + 5\hat{j} + \lambda\hat{k}$, respectively. If the points A, B, C and D lie on a plane, find the value of λ.

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4. Let $OACB$ be a parallelogram with O at the origin and OC a diagonal. Let D be the midpoint of OA. using vector methods prove that $BDandCO$ intersect in the same ratio. Determine this ratio.

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5. In a triangle $ABC, DandE$ are points on $BCandAC$, respectivley, such that $BD = 2DCandAE = 3EC$. Let P be the point of intersection of $ADandBE$. Find BP/PE using the vector method.

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6. Prove, by vector method or otherwise, that the point of intersection of the diagonals of a trapezium lies on the line passing through the midpoint of the parallel sides (you may assume that the trapezium is not a parallelogram).

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7. Show, by vector methods, that the angularbisectors of a triangle are concurrent and find an expression for the position vector of the point of concurrency in terms of the position vectors of the vertices.

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8. Let $A(t) = f_1(t)\vec{i} + f_2(t)\vec{j}$ and $\vec{B}(t) = g_1(t)\vec{i} + g_2(t)\vec{j}, t\varepsilon[0, 1] where f_1,$ are continuous functions. If $\vec{A}(t)$ and $\vec{B}(t)$ are non zero for all $t\varepsilon[0, 1]$ and $\vec{A}(0) = 2\vec{i} + 3\vec{j}, \vec{A}(1) = 6\vec{i} = 2\vec{j}, \vec{B}(0) = 3\vec{i} + 2\vec{j}$ a prove that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some $t\varepsilon(0, 1)$

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9. In a $\triangle OAB$,E is the mid point of OB and D is the point on AB such that $AD : DB = 2 : 1$ If OD and AE intersect at P then determine the ratio of $OP : PD$ using vector methods

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10. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and the vectors

$\vec{A} = (1, a, a^2), \vec{B} = (1, b, b^2), \vec{C}(1, c, c^2)$

are non-coplanar then the product abc = ...

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11. If the vectors $a\hat{i} + \hat{j} + \hat{k}, \hat{i} + b\hat{j} + \hat{k}, \hat{i} + \hat{j} + c\hat{k}(a \neq 1, b \neq 1, c \neq 1)$ are coplanat then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is (A) 0 (B) 1 (C) -1 (D) 2

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12. The points with position vectors $\vec{a}+\vec{b}, \vec{a}-\vec{b}$ and $\vec{a}+k\vec{b}$ are collinear for all real values of k.

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13. The points with position vectors $60\hat{i}+3\hat{j}, 40\hat{i}-8\hat{j}, 40\hat{i}-8\hat{j}, a\hat{i}-52\hat{j}$ are collinear iff (A) $a=-40$ (B) $a=40$ (C) $a=20$ (D) none of these

A. $a=-40$

B. $a=40$

C. $a=20$

D. none of these

Answer: A

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14. Let a, b and c be distinct non-negative numbers. If vectos $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar, then c is

A. the arithmetic mean of a and b

B. the geometric mean of a and b

C. the harmonic mean of a and b

D. equal to zero

Answer: B

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15. Let $\overrightarrow{a} = \overrightarrow{i} - \overrightarrow{k}, \overrightarrow{b} = x\overrightarrow{i} + \overrightarrow{j} + (1-x)\overrightarrow{k}$ and $\overrightarrow{c} = y\overrightarrow{i} + x\overrightarrow{j} + (1+x$

. Then $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are non-coplanar for

A. some values of x

B. some values of y

C. no values of x and y

D. for all values of x and y

Answer: D

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16. Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}, \beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}, \gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$

A. are collinear

B. form an equilateral triangle

C. form a scalene triangle

D. form a right-angled triangle

Answer: B

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17. The number of distinct values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is

A. zero

B. one

C. two

D. three

Answer: C

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18. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $\left|\vec{c}\right| = \sqrt{3}$ then

A. $\alpha = 1, \beta = -1$

B. $\alpha = 1, \beta = \pm 1$

C. $\alpha = -1, \beta = \pm 1$

D. $\alpha = \pm 1, \beta = 1$

Answer: D

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19. Consider the set of eight vector $V = \left\{a\hat{i} + b\hat{j} + c\hat{k}; a, bc \in \{-1, 1\}\right\}$. Three non-coplanar vectors can be chosen from V is 2^p ways. Then p is______.

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20. Suppose that $\vec{p}$, $\vec{q}$ and $\vec{r}$ are three non-coplanar vectors in R^3. Let the components of a vectors $\vec{s}$ along $\vec{p}$, $\vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $\left(-\vec{p} + \vec{q} + \vec{r}\right)$, $\left(\vec{p} - \vec{q} + \vec{r}\right)$ and $\left(-\vec{p} - \vec{q} + \vec{r}\right)$ are x, y and z, respectively, then the value of $2x + y + z$ is

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