MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

LIMITS

Examples

1. Find the value of $\lim_{x \to 3^-} \frac{x-2}{x-3}$.

Watch Video Solution

2. Prove that $\lim_{x \to 2} [x]$ does not exists, where [.] represents the greatest integer function.

Watch Video Solution

3. $Let f(x) = \begin{cases} x+1 & , \quad \text{if} \quad x \geq 0 \\ x-1 & , \quad \text{if} \quad x < 0 \end{cases}$. Then prove that $\lim_{x \to 0} f(x)$ does not exist.

Watch Video Solution

4. IF $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 1-x & \text{if } x \text{ is irrational} \end{cases}$, then find $\lim_{x \to 1/2} f(x)$ if exists.

Watch Video Solution

5. Evalate $\lim_{x \to 1^+} 2^{-2^{\frac{1}{1-x}}}$.

Watch Video Solution

6. Evaluate $\lim_{x \to 0^-} \frac{x^2 - 3x + 2}{x^3 - 2x^2}$.

Watch Video Solution

7. Evaluate $\lim_{x \to 0} \frac{\sin x - 2}{\cos x - 1}$.

Watch Video Solution

8. If a and b are positive and [x] denotes greatest integer less than or equal to x, then find $\lim_{x \to 0^+} \frac{x}{a}\left[\frac{b}{x}\right]$.

Watch Video Solution

9. $f(x) = \begin{cases} \frac{|x-4|}{2(x-4)} & \text{if } x \neq 4 \\ 0 & \text{if } x = 4 \end{cases}$ at $x = 4$ is.

Watch Video Solution

10. Evaluate the left- and right-hand limits of the function defined $f(x) = \begin{cases} 1 + x^2, \text{if } 0 \leq x < 1 \\ 2 - x, \text{if} x > 1 \end{cases}$ at $x = 1$. *Also, showt*ˆlim_(xto1) f(x)` does not exist.

Watch Video Solution

Watch Video Solution

11. Let $f(x) = \begin{cases} \cos[x], & x \leq 0 \\ |x| + a, & x < 0 \end{cases}$. Then find the value of a, so that $\lim_{x \to 0} f(x)$ exists, where [x] denotes the greatest integer function less than or equal to x.

Watch Video Solution

12. Evaluate: $\lim_{x \to \frac{5\pi}{4}} [\sin x + \cos x]$, [.] denotes the greatest integer function.

Watch Video Solution

13. Let

$$f(x) = \begin{cases} x + 1, & xl > 0 \\ 2 - x, & x \leq 0 \end{cases} \text{and} g(x) = \begin{cases} x + 3, & x < 1 \\ x^2 - 2x - 2, & 1 \leq x < 2 \\ x - 5, & x \geq 2 \end{cases}$$

Find the LHL and RHL of g(f(x)) at x=0 and, hence, find $\lim_{x \to 0} g(f(x))$.

Watch Video Solution

14. If $\lim_{x \to a} [f(x) + g(x)] = 2$ and $\lim_{x \to a} [f(x) - g(x)] = 1$, then find the value of $\lim_{x \to a} f(x)g(x)$.

Watch Video Solution

15. Find the following limits.

(i) $\lim_{x \to 2} \frac{4x}{x^3 - 3}$ (ii) $\lim_{x \to 1} \frac{\log_{10} x - 3}{3x - 2}$ (iii) $\lim_{x \to \pi} \frac{3 + \cos}{2 - \sin}$

Watch Video Solution

16. Evaluate $\lim_{x \to 0} \left(\frac{x + 4}{2 - x} \right)^{\frac{x^2 + 2x - 3}{x - 1}}$

Watch Video Solution

17. If $a_1 = 1$ and $a_n + 1 = \frac{4 + 3a_n}{3 + 2a_n}, n \geq 1$and if $\lim_{n \to \infty} a_n = a$, then find the value of a.

Watch Video Solution

18. Evaluate $\lim_{x \to \infty} \frac{\sin x}{x}$.

Watch Video Solution

19. Find $\lim_{x \to 0} [x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$, (where $[.]$ represents the greatest integer funciton).

Watch Video Solution

20. In the neighbourhood of $x = 0$ it is known that $1 + |x| < \frac{e^x - 1}{x} < 1 - |x|$ then find $\lim_{x \to 0} \frac{e^x - 1}{x}$.

Watch Video Solution

21. If $0 < \log_e x < \sqrt{x}$ for all $x < 1$, then find the value of $\lim_{x \to \infty} \frac{\log_e x}{x}$.

Watch Video Solution

22. If $3 - \left(\frac{x^2}{12}\right) \leq f(x) \leq 3 + \left(\frac{x^3}{9}\right)$ in the neighborhood of x=0, then find the value of $\lim_{x \to 0} f(x)$.

Watch Video Solution

23. Evaluate $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 4}$.

Watch Video Solution

24. Evaluate $\lim_{x \to 1} \left(\frac{2}{1 - x^2} - \frac{1}{1 - x}\right)$.

Watch Video Solution

25. Evaluate $\lim_{x \to 1} \frac{x^2 + x \log_e x - \log_e x - 1}{(x^2) - 1}$

Watch Video Solution

26. The value of $\lim_{x \to \frac{3\pi}{4}} \frac{1 + \sqrt[3]{\tan x}}{1 - 2\cos^2 x}$ is

Watch Video Solution

27. Evaluate $\lim_{x \to \infty} \frac{\log_e x}{x}$

Watch Video Solution

28. Evaluate $\lim_{n \to \infty} (4^n + 5^n)^{1/n}$

Watch Video Solution

29. Evaluate $\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$.

Watch Video Solution

30. Evaluate $\lim_{x \to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}, (a \neq 0)$.

Watch Video Solution

31. Evaluate

$$\lim_{x \to \pi/2} \tan^2 x \sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2}.$$

Watch Video Solution

32. Evaluate $\lim_{n \to \infty} \frac{1^3 + 2^3 + 3^3 + \ldots + n^3}{\sqrt{4n^8 + 1}}$.

Watch Video Solution

33. If $f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 7x + 12}$, then which of the following limits exists?

(i) $\lim\limits_{x \to \infty} \sin^{-1} f(x)$ $\quad$ (ii) $\lim\limits_{x \to \infty} \cos^{-1} f(x)$

Watch Video Solution

34. Evaluate $\lim\limits_{x \to \infty} \dfrac{\sqrt{3x^2 - 1} - \sqrt{2x^2 - 1}}{4x + 3}$.

Watch Video Solution

35. Evaluate $\lim\limits_{x \to \infty} \sqrt{x}\left(\sqrt{x + c} - \sqrt{x}\right.$.

Watch Video Solution

36. Find the value of $\lim\limits_{x \to 0^+} \dfrac{3(\log_e x)^2 + 5\log_e x + 6}{1 + (\log_e x)^2}$.

Watch Video Solution

37. Evaluate $\lim_{x \to \infty} \frac{3^{\sin x} + 2x + 1}{\sin x - \sqrt{x^2} + 1}$.

Watch Video Solution

38. Evaluate $\lim_{x \to \infty} \frac{x + 7x \sin x}{-2x + 13}$ using sandwich theorem.

Watch Video Solution

39. Evaluate $\lim_{x \to \infty} \frac{\sqrt{x^2 + 1} - \sqrt[3]{x^3 + 1}}{\sqrt[4]{x^4 + 1} - \sqrt[5]{x^4 + 1}}$

Watch Video Solution

40. Evaluate $\lim_{x \to \infty} \left(\sqrt{25x^2 - 3x} + 5x\right)$.

Watch Video Solution

41. Evaluate $\lim_{x \to \infty} \left(\frac{x^2 + x - 1}{3x^2 + 2x + 4} \right)^{\frac{3x^2 + x}{x - 2}}$

Watch Video Solution

42. Evaluate $\lim_{n \to \infty} \sin^n \left(\frac{2\pi n}{3n + 1} \right), n \in N.$

Watch Video Solution

43. Evaluate $\lim_{x \to \infty} \left(\sqrt[3]{(x + 1)(x + 2)(x + 3)} - x \right).$

Watch Video Solution

44. Evaluate $\lim_{n \to \infty} \frac{1}{1 + n^2} + \frac{2}{2 + n^2} + \ldots + \frac{n}{n + n^2}.$

Watch Video Solution

45. If [x] denotes the greatest integer less than or equal to x, then evaluate $\lim_{n\to\infty} \frac{1}{n^3}\left([1^1x]+[2^2x]+[3^2x]+\ldots+[n^2x]\right)$.

Watch Video Solution

46. If $\lim_{x\to\infty}\left\{\frac{x^2+1}{x+1}-(ax+b)\right\}=0$, then find the values of a and b.

Watch Video Solution

47. Evaluate $\lim_{n\to\infty}\frac{n^p\sin^2(n!)}{n+1}$, wher $0<p<1$.

Watch Video Solution

48. Evaluate $\lim_{n\to\infty}(-1)^{n-1}\sin\left(\pi\sqrt{n^2+0.5n+1}\right)$, where $n\in N$

Watch Video Solution

49. Evaluate $\lim_{x \to 2} \frac{x^{10} - 1024}{x^5 - 32}$.

Watch Video Solution

50. If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$ and $n \in N$, then find the value of n.

Watch Video Solution

51. Evaluate $\lim_{x \to 1} \frac{\sqrt{x} + \sqrt{\sqrt{x}} + \sqrt{\sqrt{\sqrt{x}}} + \sqrt{\sqrt{\sqrt{\sqrt{x}}}} - 4}{x - 1}$.

Watch Video Solution

52. Evaluate $\lim_{x \to a} \frac{(x+2)^{5/3} - (a+2)^{5/3}}{x - a}$.

Watch Video Solution

53. Evaluate $\lim_{x \to 2} \dfrac{\sqrt{(x+7)} - 3\sqrt{(2x-3)}}{\sqrt[3]{(x+6)} - 2\sqrt[3]{(3x-5)}}$.

Watch Video Solution

54. If $\lim_{x \to 0} \dfrac{(4x-1)^{\frac{1}{3}} + a + bx}{x} = \dfrac{1}{3}$ then find the values of a and b.

Watch Video Solution

55. Evaluate $\lim_{x \to 0} \dfrac{\sin x - x}{x^3}$.

Watch Video Solution

56. Evaluate $\lim_{x \to 0} \dfrac{5\sin x - 7\sin 2x + 3\sin 3x}{x^2 \sin x}$.

Watch Video Solution

57. Evaluate $\lim_{x \to 0} \frac{\sin x + \log(1 - x)}{x^2}$.

Watch Video Solution

58. Evaluate $\lim_{x \to 0} \frac{e^{\sin x} - (1 + \sin x)}{(\tan(\sin x))^2}$

Watch Video Solution

59. Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$.

Watch Video Solution

60. If $\lim_{x \to 0} \frac{\cos 4x + a \cos 2x + b}{x^4}$ is finite, find a and b using expansion formula. Also, find the limit.

Watch Video Solution

61. Find integral value of n for which $\lim_{x \to 0} \frac{\cos^2 x - \cos x - e^x \cos x + e^x - \frac{x^3}{2}}{x^n}$ is a finite nonzero number.

Watch Video Solution

62. Find the limits of the following:

(i) $\lim_{x \to 0} \frac{\sin 3x}{x}$ (ii) $\lim_{x \to 0} \frac{\sin 7x}{\sin 4x}$ (iii) $\lim_{x \to 0} \frac{1 - \cos^2 x}{x^2}$

Watch Video Solution

63. Find the following limits:

(i) $\lim_{x \to 0} \frac{1}{x} \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$ (ii) $\lim_{x \to 0} \frac{1}{x} \sin^{-1}\left(3x - 4x^3\right)$

Watch Video Solution

64. Evaluate $\lim_{x \to \infty} 2^{x-1} \tan\left(\frac{a}{2^x}\right)$.

Watch Video Solution

65. Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2}$.

Watch Video Solution

66. Evaluate $\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 2x - \sin(x - 2)}$.

Watch Video Solution

67. Find $\lim_{x \to 0} \frac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x}$ if exists.

Watch Video Solution

68. Evaluate $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$.

Watch Video Solution

69. Evaluate $\lim_{x \to \frac{\pi}{6}} \frac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2}$.

Watch Video Solution

70. Evaluate $\lim_{x \to \pi} \frac{\sin^{-1}(1 + \cos x).\sec\left(\frac{x}{2}\right)}{(x - \pi)}$.

Watch Video Solution

71. Evaluate $\lim_{x \to \infty} \left[\frac{x^4 \sin\left(\frac{1}{x}\right) + x^2}{\left(1 + |x|^3\right)}\right]$.

Watch Video Solution

72. Evaluate $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$.

Watch Video Solution

73. Evaluate $\lim_{x \to \infty} x\left(\tan^{-1}\frac{x+1}{x+4} - \frac{\pi}{4}\right)$.

Watch Video Solution

74. Evaluate $\lim_{n \to \infty} n\sin\left(2\pi\sqrt{1+n^2}\right), (n \in N)$.

Watch Video Solution

75. Evaluate $\lim_{x \to -1^+} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{1+x}}$.

Watch Video Solution

76. Evaluate $\lim_{x \to 0} x\left[\tan^{-1}\left(\frac{x+1}{x+2}\right) - \tan^{-1}\left(\frac{x}{x+2}\right)\right]$.

Watch Video Solution

77. Evaluate $\lim_{x \to 0} \frac{1 - \cos(1 - \cos x)}{x^4}$.

Watch Video Solution

78. Using $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$, prove that area of circle of radius R is πR^2.

Watch Video Solution

79. Prove that $\left[\lim_{x \to 0} \frac{\sin x}{x} \right] = 0$, where $[.]$ represents the greatest integer function.

Watch Video Solution

80. Prove that $\left[\lim_{x \to 0} \frac{\tan^{-1} x}{x} \right] = 0$, where $[.]$ represents the greatest integer function.

Watch Video Solution

81. If $L = \lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$ is finite, then find the value of a and L.

Watch Video Solution

82. If $m, n \in I_0$ and $\lim_{x \to 0} \frac{\tan 2x - n \sin x}{x^3}$ = some integer, then find the value of n and also the value of limit.

Watch Video Solution

83. Evaluate $\lim_{x \to 0} \frac{3^{2x} - 2^{3x}}{x}$.

Watch Video Solution

84. Evaluate $\lim_{x \to 0} \frac{10^x - 2^x - 5^x + 1}{x \tan x}$.

Watch Video Solution

85. Evaluate $\lim_{x \to 0} \frac{2^x - 1}{\sqrt{1+x} - 1}$.

Watch Video Solution

86. Evaluate $\lim_{x \to 1} \frac{a^{x-1} - 1}{\sin \pi x}$.

Watch Video Solution

87. Evaluate $\lim_{x \to 0} \frac{e^x - e^{x \cos x}}{x + \sin x}$.

Watch Video Solution

88. Evaluate x $\lim_{x \to 2} \frac{x - 2}{\log_a (x - 1)}$.

Watch Video Solution

89. Evaluate $\lim_{x \to a} \frac{\log x - \log a}{x - a}$.

Watch Video Solution

90. Evaluate $\lim_{x \to 0} \frac{\log(5+x) - \log(5-x)}{x}$.

Watch Video Solution

91. Evaluate $\lim_{h \to 0} \frac{\log_e(1+2h) - 2\log_e(1+h)}{h^2}$.

Watch Video Solution

92. Let $P_n = a^{P_{n-1}} - 1, \forall n = 2, 3, \ldots,$ and let $P_1 = a^x - 1,$ where $a \in R^+$. Then evaluate $\lim_{x \to 0} \frac{P_n}{x}$.

Watch Video Solution

93. If $\lim_{x \to 0} \frac{ae^x - b}{x}$=2, then find the values of a and b.

Watch Video Solution

94. Find the following limits:

$(i)\ \lim_{x \to 0} (1-x)^{\frac{1}{x}}$ $(ii)\ \lim_{x \to 1} (1+\log_e x)^{\frac{1}{\log_e x}}$

$(iii)\ \lim_{x \to 0} (1+\sin x)^{\frac{1}{x}}$

Watch Video Solution

95. Evaluate $\lim_{x \to 0} (\cos x)^{\cot x}$.

Watch Video Solution

96. The population of a country increases by 2% every year. If it increases k times in a century, then prove that $[k] = 7, where[.]$ represents the greatest integer function.

Watch Video Solution

97. If $\lim_{x \to 0} \left(1 + ax + bx^2\right)^{2/x} = e^3$, then find the values of a and b.

Watch Video Solution

98. Evaluate $\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{\left(\frac{\sin x}{x - \sin x}\right)}$.

Watch Video Solution

99. Evaluate $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{2/x}, (a, b, c > 0)$

Watch Video Solution

100. If $f(n) = \lim_{x \to 0} \left\{\left(1 + \sin\frac{x}{2}\right)\left(1 + \sin\frac{x}{2^2}\right)\ldots\left(1 + \sin\frac{x}{2^n}\right)\right\}^{\frac{1}{x}}$ then find $\lim_{n \to \infty} f(n)$.

Watch Video Solution

101. Find the following using L'Hospital's rule

$(i)\ \lim_{x \to 0} \frac{(16+5x)^{1/4}-2}{(32+3x)^{1/5}-2}$ $(ii)\ \lim_{x \to \pi/2} [x \tan x - (\pi/2)\sec x]$

Watch Video Solution

102. Let $f(x)$ be a twice-differentiable function and $f''(0) = 2$. Then evaluate $\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$.

Watch Video Solution

103. Let $f(a) = g(a) = k$ and their nth derivatives exist and be not equal for some n.

If $\lim_{x \to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4$ then find the value of k.

Watch Video Solution

104. Evaluate $\lim_{x \to 0} \left(\log_{\tan^2 x}\left(\tan^2 2x\right)\right.$.

Watch Video Solution

105. Evaluate $\lim_{x \to 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$.

Watch Video Solution

106. The graph of the function $y = f(x)$ has a unique tangent at the point (a,0) through which the graph passes. Then evaluate

$\lim_{x \to a} \frac{\log_e\{1 + 6f(x)\}}{3f(x)}$.

Watch Video Solution

107. Evaluate $\lim_{x \to \infty} x \log_e \left\{\frac{\sin\left(a + \frac{1}{x}\right)}{\sin a}\right\}, 0 < a < \frac{\pi}{2}$.

Watch Video Solution

108. Find the value of a so that $\lim_{x \to 0} (e^{ax} - e^x - x) = \frac{3}{2}$.

Watch Video Solution

109. Find the value of $\lim_{x \to 0} \frac{\sin x + \log_e\left(\sqrt{1 + \sin^2 x} - \sin x\right)}{\sin^3 x}$.

Watch Video Solution

110. Evaluate $\lim_{x \to \infty} x^{\frac{1}{x}}$.

Watch Video Solution

111. Evaluate $\lim_{x \to \frac{\pi}{2}^-} (\cos x)^{\cos x}$.

Watch Video Solution

112. Evaluate $\lim_{x \to 0^+} (x)^{\frac{1}{\log_e \sin x}}$.

Watch Video Solution

113. Evaluate

$$\lim_{n \to \infty} n^2 \left\{ \sqrt{\left(1 - \cos\frac{1}{n}\right)\sqrt{\left(1 - \cos\frac{1}{n}\right)\sqrt{\left(1 - \cos\frac{1}{n}\right)\ldots}}} \right\}.$$

Watch Video Solution

114. Evaluate $\lim_{n \to \infty} \left\{ \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\ldots\cos\left(\frac{x}{2^n}\right) \right\}$.

Watch Video Solution

115. Evaluate $\lim_{x \to 1} \sec\frac{\pi}{2^x} . \log_e x$.

Watch Video Solution

116. Evaluate $\lim_{x \to 0^+} \frac{1}{x} \cos^{-1}\left(\frac{\sin x}{x}\right)$.

Watch Video Solution

117. If $f(x) = \frac{\tan x}{x}$, then find $\lim_{x \to 0} \left([f(x)] + x^2\right)^{\frac{1}{[f(x)]}}$, where $[.]$ and $\{.\}$ denotes greatest integer and fractional part function respectively.

Watch Video Solution

118. Evaluate $\lim_{n \to \infty} \frac{1}{n^{2(\log_e n - \log_e (n-1))} + n}$.

Watch Video Solution

119. Evaluate $\lim_{x \to 0} \frac{(1+x)^{1/x} - e + \frac{1}{2}es}{x^2}$.

Watch Video Solution

120. Evaluate $\lim_{x \to 0^+} x^m (\log x)^m, m, n, \in N.$

Watch Video Solution

121. If $\alpha_1, , \alpha_2, , \alpha_n$ are the roots of equation $x^n + nax - b = 0,$ show that $(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_n) = n(\alpha 1n - 1 + a)$

Watch Video Solution

122. Evaluate $\lim_{x \to 0} \left\{1^{1/\sin^2 x} + 2^{1/\sin^2 x} + \ldots + n^{1/\sin^2 x}\right\}^{\sin^2 x}.$

Watch Video Solution

123. Evaluate $\lim_{x \to \pi/2} \frac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \log_e \sin x}.$

Watch Video Solution

124. Evaluate $\lim_{x \to 0} \frac{e - (1+x)^{1/x}}{x}$.

Watch Video Solution

125. Evaluate

$\lim_{n \to \infty} n^{-n^2}\left[\left(n+2^0\right)\left(n+2^{-1}\right)\left(n+2^{-2}\right)\ldots\left(n+2^{-n+1}\right)\right]^n$.

Watch Video Solution

126. ABC is an isosceles triangle inscribed in a circle of radius r. If $AB = AC$ and h is the altitude form A to BC. If P is perimeter and A is the area of the triangle then find the value of $\lim_{h \to 0} \frac{A}{P^3}$.

View Text Solution

127. At the endpoint and midpoint of a circular are AB, tangent lines are drawn, and the points, A and B are jointed with a chord. Prove that the

ratio of the areas of the triangles thus formed tends to 4 as the arc AB decreases infinitely.

Watch Video Solution

Exercise 2.1

1. Evaluate $\lim_{x \to -2^+} \frac{x^2 - 1}{2x + 4}$.

Watch Video Solution

2. Evaluate $\lim_{x \to 2^+} \frac{[x - 2]}{\log(x - 2)}$, where $[.]$ represents the greatest integer function.

Watch Video Solution

3. Evaluate $\lim_{x \to 0} \frac{\sin[\cos x]}{1 + [\cos x]}$ ($[.]$ denotes the greatest integer function).

Watch Video Solution

Watch Video Solution

4. If $f(x)=\begin{cases}\frac{x-|x|}{x}, x\neq 0\\ 2, \quad x=0\end{cases}$, show that $\lim_{x\to 0} f(x)$ does not exist.

Watch Video Solution

5. Show that $\lim_{x\to 0}\frac{e^{1/x}-1}{e^{1/x}+1}$ does not exist.

Watch Video Solution

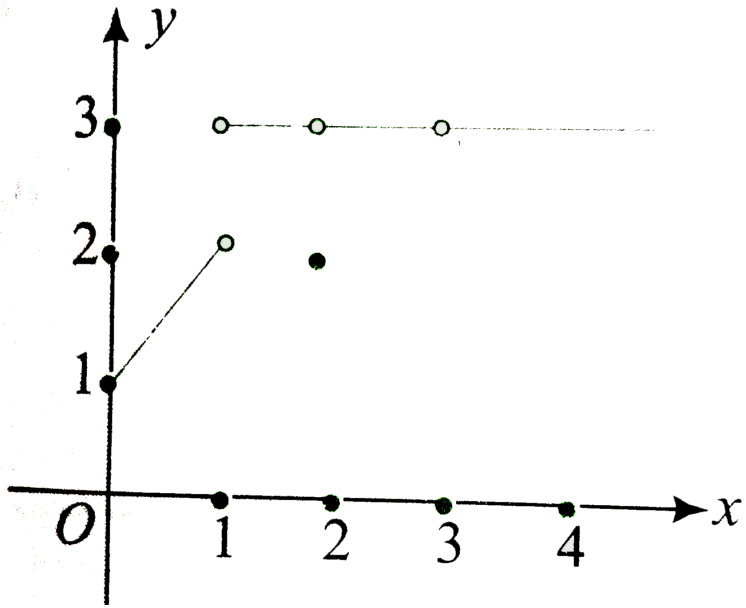
6. Evaluate $\lim_{x\to 0}\frac{3x+|x|}{7x-5|x|}$.

Watch Video Solution

7. If $f(x)=\begin{cases}x, & x<0\\ 1, & x=0\\ x^2, & x>0\end{cases}$, then find $\lim_{x\to 0} f(x)$ if exists.

Watch Video Solution

8. Consider the following graph of the function y=f(x). Which of the following is//are correct?



(a) $\lim_{x \to 1} f(x)$ does not exist.

(b) $\lim_{x \to 2} f(x)$ does not exist.

(c) $\lim_{x \to 3} f(x) = 3$.

(d) $\lim_{x \to 1.99} f(x)$exists.

Watch Video Solution

9. Evaluate $\lim_{x \to 0} \frac{\tan(sgn(x))}{sgn(x)}$ if exists.

Watch Video Solution

10. If $f(x) = \begin{cases} \sin x, & x \neq n\pi, \ n \in I \\ 2, & \text{otherwise} \end{cases}$ and $g(x) = \begin{cases} x^2 + 1, & x \neq 0, \ 2 \\ 4, & x = 0 \\ 5, & x = 2 \end{cases}$ then find $\lim_{x \to 0} g\{f(x)\}$.

Watch Video Solution

Exercise 2.2

1. If $|f(x)| \leq x^2$, then prove that $\lim_{x \to 0} \frac{f(x)}{x} = 0$.

Watch Video Solution

2. If $f(x) = sgn(x)$ and $g(x) = x^3$,then prove that $\lim_{x \to 0} f(x).\, g(x)$ exists though $\lim_{x \to 0} f(x)$ does not exist.

Watch Video Solution

3. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & \text{for } [x] \neq 0 \\ 0, & \text{for } [x] = 0 \end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x. Then find $\lim_{x \to 0} f(x)$.

Watch Video Solution

4. Find the value of $\lim_{x \to 0^+} (\sin x)^{\frac{1}{x}}$.

Watch Video Solution

5. Let the sequence $< b_n >$ of real numbers satisfy the recurrence relation $b_{n+1} = \frac{1}{3}\left(2b_n + \frac{125}{b_n^2}\right), b_n \neq 0$. Then find $\lim_{n \to o0} b_n$.

Watch Video Solution

Watch Video Solution

6. Let $f:(1,2) \to R$ satisfy the inequality

$$\frac{\cos(2x-4)-33}{2} < f(x) < \frac{x^2|4x-8|}{x-2} \forall x \in (1,2).$$

Then find $\lim_{x \to 2^-} f(x)$.

Watch Video Solution

7. If $\frac{x^2+x-2}{x+3} \leq \frac{f(x)}{x^2} \leq \frac{x^2+2x-1}{x+3}$ holds for a certain interval containing the value of $\lim_{x \to -1} f(x)$.

Watch Video Solution

Exercise 2.3

1. Evaluate $\lim_{x \to 1} \frac{(2x-3)\left(\sqrt{x}-1\right)}{2x^2+x-3}$.

Watch Video Solution

2. Evauate $\lim_{x \to 1} \frac{x^4 - 3x^4 + 2}{x^3 - 5x^2 + 3x + 1}$.

Watch Video Solution

3. Evaluate $\lim_{x \to \frac{\pi}{4}} \frac{1 - \sin 2x}{1 + \cos 4x}$.

Watch Video Solution

4. Evaluate $\lim_{x \to \pi/4} \frac{1 - \cot^3 x}{2 - \cot x - \cot^3 x}$.

Watch Video Solution

5. Evaluate $\lim_{x \to a} \frac{\sqrt{3x - a} - \sqrt{x + a}}{x - a}$.

Watch Video Solution

6. Evaluate $\lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$.

Watch Video Solution

7. Evaluate $\lim_{x \to \sqrt{10}} \frac{\sqrt{7 + 2x} - \left(\sqrt{5} + \sqrt{2}\right)}{x^2 - 10}$.

Watch Video Solution

8. Evaluate $\lim_{n \to \infty} \frac{\left(1^2 - 2^2 + 3^2 - 4^2 + 5^2 + \ldots n \text{ terms}\right)}{n^2}$.

Watch Video Solution

9. Evaluate $\lim_{x \to \infty} \left[\sqrt{a^2x^2 + ax + 1} - \sqrt{a^2x^2 + 1}\right]$.

Watch Video Solution

10. If $[x]$ denotes the greatest integer less than or equal to x, then evaluate $\lim_{n \to \infty} \frac{1}{n^2}([1.x] + [2.x] + [3.x] + \ldots + [n.x])$.

Watch Video Solution

11. Evaluate $\lim_{x \to \infty} x^3 \left\{ \sqrt{x^2 + \sqrt{1 + x^4}} - x\sqrt{2} \right\}$.

Watch Video Solution

12. Evaluate $\lim_{x \to \infty} \left(\frac{7x^2 + 1}{5x^2 - 1} \right)^{\frac{x^5}{1 - x^3}}$.

Watch Video Solution

13. Evaluate $\lim_{n \to \infty} \cos\left(\pi\sqrt{n^2 + n}\right)$ when n is an integer.

Watch Video Solution

14. Evaluate $\lim_{x \to 1} \sum_{k=1}^{100} x^k - 100 \frac{)}{x-1}$.

Watch Video Solution

15. Evaluate $\lim_{h \to 0} \left[\frac{1}{h^3 \sqrt{8+h}} - \frac{1}{2h} \right]$.

Watch Video Solution

Exercise 2.4

1. Evaluate $\lim_{x \to 0} \left\{ \frac{\sin x - x + \frac{x^3}{6}}{x^5} \right\}$.

Watch Video Solution

2. Evaluate $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$.

Watch Video Solution

3. Evaluate $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$.

Watch Video Solution

4. If $\lim_{x \to 0} \frac{1 - \cos x}{e^{ax} - bx - 1} = 2$ then find the values of a and b.

Watch Video Solution

5. Find the values of a and b in order that

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1.$$

Watch Video Solution

Exercise 2.5

1. Evaluate $\lim_{x \to \infty} \frac{\sin x^0}{x}$.

Watch Video Solution

2. Evaluate $\lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx}$.

Watch Video Solution

3. Evaluate $\lim_{x \to 0} \frac{\cot 2x - \operatorname{cosec} 2x}{x}$.

Watch Video Solution

4. Evaluate $\lim_{x \to 0} \frac{\tan 2x - x}{3x - \sin x}$.

Watch Video Solution

5. Evaluate $\lim_{n \to \infty} n \cos\left(\frac{\pi}{4n}\right) \sin\left(\frac{\pi}{4n}\right)$.

Watch Video Solution

6. Evaluate $\lim_{x \to 0} \frac{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)}{\sin^{-1} x}$.

Watch Video Solution

7. Evaluate $\lim_{h \to 0} \frac{2\left[\sqrt{3}\sin\left(\frac{\pi}{6}+h\right) - \cos\left(\frac{\pi}{6}+h\right)\right]}{\sqrt{3}h\left(\sqrt{3}\cosh - \sinh\right)}$.

Watch Video Solution

8. Evaluate $\lim_{x \to 0} \frac{8}{x^8}\left\{1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4}\right\}$.

Watch Video Solution

9. Evaluate $\lim_{y \to 0} \frac{y^2 + \sin x}{x^2 + \sin y^2}$, where $(x, y) \to (0, 0)$ along the curve $x = y^2$.

Watch Video Solution

10. Evaluate $\lim_{x \to 1} (1-x)\tan\frac{\pi x}{2}$.

Watch Video Solution

11. Evaluate $\lim_{x \to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$.

Watch Video Solution

12. If $\lim_{x \to 2} \frac{\tan(x-2).\left(x^2+(k-2)x-2k\right)}{(x^2-4x+4)} = 5$, then find the value of k.

Watch Video Solution

Exercise 2.6

1. Evaluate $\lim_{x \to \infty} \left[x\left(a^{1/x}-1\right)\right], a > 1$

Watch Video Solution

2. Evalaute $\lim_{x \to 0} \frac{x2^x - x}{1 - \cos x}$

Watch Video Solution

3. Evaluate $\lim_{x \to 2} \frac{\sin\left(e^{x-2} - 1\right)}{\log(x - 1)}$

Watch Video Solution

4. Evaluate $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$

Watch Video Solution

5. Evaluate $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2}$

Watch Video Solution

6. $\lim_{x \to a} \frac{\log(x-a)}{\log(e^x - e^a)}$

Watch Video Solution

7. Evaluate $\lim_{x \to 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}, a > 0$

Watch Video Solution

8. $\lim_{x \to 0} \frac{(1 - 3^x - 4^x + 12^x)}{\sqrt{(2\cos x + 7)} - 3}$

Watch Video Solution

9. $\lim_{x \to 0} \frac{(729)^x - (243)^x - (81)^x + 9^x + 3^x - 1}{x^3}$

Watch Video Solution

Exercise 2.7

1. Evaluate $\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x$.

A. 0

B. 1

C. -1

D. does not exist

Answer: e^2

Watch Video Solution

2. Evaluate $\lim_{x \to \infty} (\log_3 3x)^{\log_x 3}$.

A. e^(-1)

B. e

C. -1

D. 1

Answer: B

Watch Video Solution

3. Evaluate $\lim_{x \to \infty} \left(\frac{x+2}{x+1} \right)^{x+3}$.

A. 0

B. e

C. ∞

D. does not exist

Answer: B

Watch Video Solution

4. Evaluate $\lim_{x \to \infty} \left(1 + \frac{1}{a+bx} \right)^{c+dx}$, where a, b, c, and d are positive.

A. 4

B. 2

C. -1

D. 0

Answer: $e^{d/b}$

Watch Video Solution

5. Evaluate $\lim_{x \to 7/2} \left(2x^2 - 9x + 8\right)^{\cot(2x-7)}$.

A. both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ must exist

B. $\lim_{x \to a} f(x)$ need not exist but $\lim_{x \to a} g(x)$ exists

C. neither $\lim_{x \to a} f(x)$ nor $\lim_{x \to a} g(x)$ may exist

D. $\lim_{x \to a} f(x)$ exists but $\lim_{x \to a} g(x)$ need not exist

Answer: $e^{5/2}$

Watch Video Solution

6. If x_1 and x_2 are the real and distinct roots of $ax^2 + bx + c = 0$ then prove that $\lim_{x \to x1} \left(1 + \sin\left(ax^2 + bx + c\right)\right)^{\frac{1}{x - x_1}} = e^{a(x_1 - x_2)}$.

A. does not exist

B. 1

C. ∞

D. $\frac{1}{2}$

Watch Video Solution

7. Evaluate $\lim_{x \to 0} \left\{\sin^2\left(\frac{\pi}{2 - px}\right)\right\}^{\sec^2\left(\frac{\pi}{2 - qx}\right)}$.

A. -1

B. 2

C. $\sqrt{5}$

D. e^{-p^2/q^2}

Answer: D

Watch Video Solution

Exercise 2.8

1. Evaluate $\lim_{x \to 1} \frac{\cos \frac{\pi}{2} x}{1 - \sqrt{x}}$

A. $-\pi$

B. π

C. 0

D. does not exist

Answer: B

Watch Video Solution

2. Evalaute $\lim_{x \to 1} \frac{1 + \log x - x}{1 - 2x + x^2}$

A. 0

B. 1

C. $\frac{1}{3}$

D. $\frac{1}{2}$

Answer: $-1/2$

Watch Video Solution

3. Evaluate $\lim_{x \to \pi/2} \tan x \log \sin x$

A. -2

B. -1

C. $-\frac{2}{7}$

D. 0

Answer: D

Watch Video Solution

4. Evaluate $\lim_{x \to 0} \frac{\log \cos x}{x}$

A. 1

B. 0

C. $-1/4$

D. $3/2$

Answer: B

Watch Video Solution

5. Evaluate $\lim_{x \to 0} \frac{2^x - 1}{(1+x)^{1/2} - 1}$

A. $\frac{1}{8\sqrt{3}}$

B. $\frac{1}{4\sqrt{3}}$

C. 0

D. none of these

Answer: $\log 4$

Watch Video Solution

6. Evaluate $\lim_{x \to \pi/4} (2 - \tan x)^{1/1n(\tan x)}$

A. 16

B. 8

C. 4

D. 2

Answer: e^{-1}

Watch Video Solution

7. Evaluate $\lim_{x \to 0^+} x^x$ and $\lim_{x \to 0^+} x^{x^x}$

A. $1/2$

B. 2

C. 1

D. None of these

Answer: $1 \quad \text{and} \quad 0$

Watch Video Solution

8. If $\lim_{x \to a} \frac{a^x - x^a}{x^x - a^a} = -1 \quad \text{and} \quad a > 0, \quad \text{then find the value of a.}$

A. 43529

B. 2

C. $\sin\frac{2}{5}$

D. $\sin\frac{1}{5}$

Answer: $a = 1$

Watch Video Solution

Exercise (Single)

1. $\lim_{x \to 0} \left[\frac{\sin(sgn(x))}{(sgn(x))} \right]$, where $[.]$ denotes the greatest integer function, is equal to

A. $\hat{} \ (2n)p_n$

B. $\hat{} \ (2n)C_n$

C. $(2n)!$

D. none of these

Answer: A

Watch Video Solution

2. Let $\lim_{x \to 0} \frac{[x]^2}{x^2} = m$, where $[.]$ denotes greatest integer. Then,

A. $-\frac{1}{\sqrt{2}}$

B. $\frac{1}{\sqrt{2}}$

C. $\sqrt{2}$

D. $-\sqrt{2}$

Answer: B

View Text Solution

3. $\lim_{x \to 1} \left[\cos ec \frac{\pi x}{2}\right]^{1/(1-x)}$ (where $[.]$ represents the greatest integer function) is equal to

A. 0

B. 1

C. ∞

D. Does not exists

Answer: B

Watch Video Solution

4. The value of the limit $\lim_{x \to 0} \frac{a^{\sqrt{x}} - a^{1/\sqrt{x}}}{a^{\sqrt{x}} + a^{1/\sqrt{x}}}, a > 1$, is

A. does not exist

B. $1/3$

C. 0

D. $2/9$

Answer: C

Watch Video Solution

5. If $\lim_{x \to a} \left\{ \frac{f(x)}{g(x)} \right\}$ exists, then

A. 0

B. 2

C. 4

D. ∞

Answer: C

View Text Solution

6. $(\lim)_{x \to 1} \frac{1}{\sqrt{|x| - \{-x\}}} (where \{x\}$ denotes the fractional part of (x) is equal to does not exist (b) 1 ∞ (d) $\frac{1}{2}$

A. 16

B. 24

C. 32

D. 8

Answer: A

Watch Video Solution

7. If $x_1 = 3$ and $x_{n+1} = \sqrt{2 + x_n} \quad , n \geq 1,$ then $\lim_{n \to \infty} x_n$ is

A. 0

B. $\frac{1}{2}$

C. log2

D. e^4

Answer: B

Watch Video Solution

8. $\lim_{x \to 0^+} \frac{\sum_{r=1}^{2n+1} [x^r] + (n+1)}{1 + [x] + |x| + 2x}$, where $n \in N$ and $[.]$ denotes the greatest integer function, equals

A. 0

B. 1

C. -1

D. does not exist

Answer: C

View Text Solution

9. $\lim_{x \to \infty} \frac{\sin^4 x - \sin^2 x + 1}{\cos^4 x - \cos^2 x + 1}$ is equal to

A. 0

B. 1

C. 10

D. 100

Answer: B

Watch Video Solution

10. If $f(x) = \frac{2}{x-3}, g(x) = \frac{x-3}{x+4}$, and $h(x) = -\frac{2(2x+1)}{x^2+x-12}$, then $\lim_{x}$

A. 1

B. ∞

C. $\sqrt{2}$

D. none of these

Answer: C

Watch Video Solution

11. The value of $\lim_{x \to \pi} \frac{1+\cos^3 x}{\sin^2 x}$ is

A. [2, 5)

B. (1, 5)

C. (-1, 5)

D. $(-\infty, \infty)$

Answer: D

Watch Video Solution

12. The value of $\lim_{x \to 2} \frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2}$ is

A. 0

B. e^x

C. $\log_e x$

D. none of these

Answer: A

Watch Video Solution

13. The value of $\lim_{x \to 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$ is

A. $|2x| > \sqrt{3}$

B. $|2x| < \sqrt{3}$

C. $|2x| \geq \sqrt{3}$

D. $|2x| \leq \sqrt{3}$

Answer: B

Watch Video Solution

14. The value of $\lim_{x \to 2} \left(\left(\frac{x^3 - 4x}{x^3 - 8} \right)^{-1} - \left(\frac{x + \sqrt{2x}}{x - 2} - \frac{\sqrt{2}}{\sqrt{x} - \sqrt{2}} \right)^{-1} \right)$ is

A. 1

B. $1/2$

C. 2

D. none of these

Answer: A

Watch Video Solution

15. If $\lim_{x \to -2^-} \frac{ae^{1/|x+2|} - 1}{2 - e^{1/|x+2|}} = \lim_{x \to -2^+} \sin\left(\frac{x^4 - 16}{x^5 + 32}\right)$, then a is

A. 1

B. -1

C. 0

D. none of these

Answer: C

Watch Video Solution

16. $\lim_{x \to 1} \frac{(1-x)(1-x^2)...(1-x^{2n})}{\{(1-x)(1-x^2)...(1-x^n)\}^2}$, $n \in N$, equals

A. ${}^{2n}P_n$

B. ${}^{2n}C_n$

C. $2n!$

D. none of these

Answer: B

Watch Video Solution

17. The value of $\lim_{x \to \frac{1}{\sqrt{2}}} \frac{x - \cos\left(\sin^{-1} x\right)}{1 - \tan\left(\sin^{-1} x\right)}$ is

A. $-\frac{1}{\sqrt{2}}$

B. -1

C. non-existent

D. none of these

Answer: A

Watch Video Solution

18. Among (i) $\lim_{x \to \infty} \sec^{-1}\left(\frac{x}{\sin x}\right)$ and (ii) $\lim_{x \to \infty} \sec^{-1}\left(\frac{\sin x}{x}\right)$.

A. $-\frac{1}{2\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. does not exist

Answer: A

Watch Video Solution

19. $\lim_{x \to \infty} \left(\frac{x^3}{3x^2 - 4} - \frac{x^2}{3x + 2}\right)$ is equal to

A. 2

B. -1

C. 0

D. $\frac{2}{9}$

Answer: D

Watch Video Solution

20. $\lim_{n \to \infty} \frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}$ is equal to

A. 1

B. 0

C. 2

D. none of these

Answer: C

Watch Video Solution

21. $\lim_{x \to \infty} \frac{(2x+1)^{40}(4x+1)^5}{(2x+3)^{45}}$ is equal to

A. $\frac{1}{2\pi}$

B. $\frac{-1}{\pi}$

C. $\frac{-2}{\pi}$

D. none of these

Answer: C

Watch Video Solution

22. $\lim_{x \to \infty} \left[\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}\right]$ is equal to

A. 1

B. 0

C. 2

D. none of these

Answer: B

Watch Video Solution

23. $\lim_{x \to \infty} \frac{2 + 2x + \sin 2x}{(2x + \sin 2x)e^{\sin x}}$ is equal to

A. $\sec x(x \tan x + 1)$

B. $x \tan x + \sec x$

C. $x \sec x + \tan x$

D. none of these

Answer: D

Watch Video Solution

24. $\lim_{x \to \infty} \frac{(x+1)^{10} + (x+2)^{10} + \ldots + (x+100)^{10}}{x^{10} + 10^{10}}$ is equal to

A. 0

B. 1

C. 2

D. 4

Answer: D

Watch Video Solution

25. $\lim_{x \to \infty} \frac{2\sqrt{x} + 3\sqrt[3]{x} + 4\sqrt[4]{x} + \ldots + n\sqrt[n]{x}}{\sqrt{(2x-3)} + \sqrt[3]{(2x-3)} + \ldots + \sqrt[n]{(2x-3)}}$ is equal to

A. 0

B. 1

C. $\sqrt{2}$.

D. $2\sqrt{2}$.

Answer: C

Watch Video Solution

26. If $\lim_{n \to \infty} \frac{n.3^n}{n(x-2)^n + n.3^{n+1} - 3^n} = \frac{1}{3}$, then the range of x is (where $n \in N$)

A. 1

B. $(2/3)^{1/2}$

C. $(3/2)^{1/2}$

D. $e^{1/2}$

Answer: A

Watch Video Solution

27. $\lim_{n \to \infty} n^2 \left(x^{1/n} - x^{1/(n+1)} \right), x > 0,$ is equal to

A. π

B. 2π

C. $\pi/2$

D. none of these

Answer: C

Watch Video Solution

28. $Let f(x) = \lim_{n \to \infty} \frac{1}{\left(\frac{3}{\pi}\tan^{-1} 2x\right)^{2n} + 5}$. Then the set of values of x for which $f(x) = 0$ is

A. 199

B. 198

C. 0

D. none of these

Answer: A

Watch Video Solution

29. $f(x) = \frac{\ln(x^2 + e^x)}{\ln(x^4 + e^{2x})}$. Then $\lim_{x \to \infty}$ f(x) is equal to

A. $\frac{2a}{\pi}$

B. $-\frac{2a}{\pi}$

C. $\frac{4a}{\pi}$

D. $-\frac{4a}{\pi}$

Answer: B

Watch Video Solution

30. The value of $\lim_{n\to\infty}\left[\frac{2n}{2n^2-1}\cos\frac{n+1}{2n+1}-\frac{n}{1-2n}.\frac{n}{n^2+1}\right]$ is

A. 5

B. 6

C. 7

D. none of these

Answer: C

Watch Video Solution

31. If $f(x) = 0$ is a quadratic equation such that $f(-\pi) = f(\pi) = 0$ and $f\left(\frac{\pi}{2}\right) = -\frac{3\pi^2}{4}$, then $\lim_{x \to -\pi^+} \frac{f(x)}{\sin(\sin x)}$ is equal to

A. $-\frac{3}{2}\pi$

B. $-\frac{1}{2}$

C. (2π)

D. $\frac{3}{2}\pi$

Answer: C

Watch Video Solution

32. $\lim_{x \to 1} \frac{x \sin(x - [x])}{x - 1}$, where $[.]$ denotes the greatest integer function, is equal to

A. $\frac{2}{\pi - 1}$

B. $\frac{\pi + 1}{2}$

C. $\frac{2}{\pi + 1}$

D. $\frac{2(\pi+1)}{\pi-1}$

Answer: C

Watch Video Solution

33. $\lim_{x \to \infty} \frac{x^2 \tan\frac{1}{x}}{\sqrt{8x^2+7x+1}}$ is equal to

A. 4

B. $(1/2$

C. 2

D. $1/4$

Answer: A

Watch Video Solution

34. $\lim_{x \to 0} \frac{x^a \sin^b x}{\sin(x^c)}$, where $a, b, c \in R\text{~}\{0\}$, exists and has non-zero value. Then,

A. 0

B. $\pi / 2$

C. π

D. 2π

Answer: C

Watch Video Solution

35. $\lim_{x \to 0} \frac{x^4\left(\cot^4 x - \cot^2 x + 1\right)}{\left(\tan^4 x - \tan^2 x + 1\right)}$ is equal to

A. -1

B. 1

C. 0

D. none of these

Answer: A

Watch Video Solution

36. $\lim_{x \to 1} \frac{1 - x^2}{\sin 2\pi x}$ is equal to

A. 0

B. ∞

C. -2

D. 2

Answer: B

Watch Video Solution

37. $\lim_{x \to 0} \frac{1}{x} \cos^1 \left(\frac{1 - x^2}{1 + x^2} \right)$ is equal to

A. $f(x)f(y)$

B. $f(x)+f(y)$

C. $f(x)-f(y)$

D. none of these

Answer: D

Watch Video Solution

38. $\lim_{y\to 0}\frac{(x+y)\sec(x+y)-x\sec x}{y}$ is equal to

Watch Video Solution

39. $\lim_{x\to 1}\frac{1+\sin\pi\left(\frac{3x}{1+x^2}\right)}{1+\cos\pi x}$ is equal to

A. $\log n\left(\frac{2}{3}\right)$

B. 0

C. $n \log n\left(\frac{2}{3}\right)$

D. not defined

Answer: A

Watch Video Solution

40. $\lim_{n \to \infty} \sum_{x=1}^{20} \cos^{2n}(x - 10)$ is equal to

A. 2

B. -2

C. 1

D. -1

Answer: B

Watch Video Solution

41. $\lim_{x \to -1} \left(\frac{x^4 + x^2 + x + 1}{x^2 - x + 1} \right)^{\frac{1 - \cos(x+1)}{(x+1)^2}}$ is equal to

A. 1

B. -1

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

Answer: B

Watch Video Solution

42. $\lim_{x \to \infty} \{(x+5)\tan^{-1}(x+5) - (x+1)\tan^{-1}(x+1)\}$ is equal to

A. $\frac{1}{m2^m} - \frac{1}{n2^n}$

B. $\frac{1}{m2^m} + \frac{1}{n2^n}$

C. $\frac{1}{m2^{m-1}} - \frac{1}{n2^{n-1}}$

D. $\frac{1}{m2^{m-1}} + \frac{1}{n2^{n-1}}$

Answer: B

Watch Video Solution

43. The value of $\lim_{x \to 0} \left(\left[\frac{100x}{\sin x} \right] + \left[\frac{99 \sin x}{x} \right] \right)$ (where $[.]$ represents the greatest integral function) is

A. 1

B. 0

C. $e - 1$

D. $e + 1$

Answer: B

Watch Video Solution

44. The value of $\lim_{x \to a} \sqrt{a^2 - x^2} \cot \frac{\pi}{2} \sqrt{\frac{a - x}{a + x}}$ is

A. $\frac{5050}{\pi e}$

B. $100\frac{)}{\pi e}$

C. $-\frac{5050}{\pi e}$

D. $-\frac{4950}{\pi e}$

Answer: C

Watch Video Solution

45. $\lim_{x \to 0} \left[\min \left(y^2 - 4y + 11\right) \frac{\sin x}{x} \right]$ (where $[.]$ denotes the greatest integer function) is

A. -1

B. 1

C. 0

D. 2

Answer: B

Watch Video Solution

46. The value of $\lim_{x \to 0} \frac{1 - (\cos x)\sqrt{\cos 2x}}{x^2}$ is

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{1}{6}$

D. $\frac{5}{12}$

Answer: D

Watch Video Solution

47. $\lim_{x \to \infty} \frac{1}{x+1} \tan\left(\frac{\pi x + 1}{2x + 2}\right)$ is equal to

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 0

D. none of these

Answer: A

Watch Video Solution

48. The value of $\lim_{x \to 1^-} \frac{1 - \sqrt{x}}{(\cos^{-1} x)^2}$ is

A. $1/6$

B. $-1/3$

C. $1/2$

D. 1

Answer: D

Watch Video Solution

49. $\lim_{x \to \pi/2} \frac{\sin(x \cos x)}{\cos(x \sin x)}$ is equal to

A. $a = 3$ and $b = 9/2$

B. $a = 3$ and $b = 9/2$

C. $a = -3$ and $b = -9/2$

D. $a = 3$ and $b = -9/2$

Answer: B

Watch Video Solution

50. $\lim_{x \to 0} \left[(1 - e^x) \frac{\sin x}{|x|} \right]$ is (where $[.]$ represents the greatest integer function)

A. 1

B. 2

C. 3

D. none of these

Answer: A

Watch Video Solution

51. Evaluate $\lim_{x \to 0} \frac{x(e^x - 1)}{1 - \cos x}$ is equal to

A. e

B. $\frac{1}{e}$

C. 1

D. none of these

Answer: D

Watch Video Solution

52. If $f(x) = \lim_{n \to \infty} n\left(x^{1/n} - 1\right)$, then for $x > 0, y > 0, f(xy)$ is equal to

A. $f(2x) + f(2y)$

B. $f(x) + f(y)$

C. `f(x+y)

D. $f(x) - f(y)$

Answer: B

Watch Video Solution

53. $\lim_{x \to 0} \left\{(1+x)^{\frac{2}{x}}\right\}$ (where $\{.\}$ denotes the fractional part of x) is equal to

A. 1

B. e

C. e^{-1}

D. none of these

Answer: A

Watch Video Solution

54. The value of $\lim_{x \to \infty} \frac{\left(2^{x^n}\right) e^{\frac{1}{x}} - \left(3^{x^n}\right) e^{\frac{1}{x}}}{x^n}$ (where $n \in N$) is

A. e

B. e^2

C. e^{-1}

D. 1

Answer: B

Watch Video Solution

55. $\lim_{x \to 0} \frac{\sin\left(x^2\right)}{\ln(\cos(2x^2 - x))}$ is equal to

A. e^a

B. $-a$

C. e^{1-a}

D. e^{1+a}

Answer: B

Watch Video Solution

56. $\lim_{x \to \infty} \frac{e^{1/x^2} - 1}{2\tan^{-1}(x^2) - \pi}$ is equal to

A. $e^{(1-e)}$

B. $e^{\left(\frac{1-e}{e}\right)}$

C. $e^{\left(\frac{e}{1-e}\right)}$

D. $e^{\frac{1+e}{e}}$

Answer: D

Watch Video Solution

57. $\lim_{x \to 0} \frac{(2^m + x)^{1/m} - (2^n + x)^{1/n}}{x}$ is equal to

A. $(n!)^n$

B. $(n!)^{1/n}$

C. $n!$

D. $\ln(n!)$

Answer: C

Watch Video Solution

58. The value of $\lim_{n \to \infty} \left[\frac{1}{n} + \frac{e^{1/n}}{n} + \frac{e^{2/n}}{n} + \ldots + \frac{e^{(n-1)/n}}{n} \right]$ is

A. $\frac{p+q}{2}$

B. $\frac{pq}{2}$

C. $\frac{p-q}{2}$

D. $\sqrt{\frac{p}{q}}$

Answer: C

Watch Video Solution

59. $\lim_{x \to 1} \frac{nx^{n-1} - (n+1)x^n + 1}{(e^x - e)\sin \pi x}$, where $n = 100$,is equal to

A. 0

B. -1

C. 1

D. does not exist

Answer: C

Watch Video Solution

60. $\lim_{x \to 0} \frac{\log\left(1 + x + x^2\right) + \log\left(1 - x + x^2\right)}{\sec x - \cos x} =$

A. 2

B. 1

C. $\log_a 2$

D. 0

Answer: B

Watch Video Solution

61. The value of $\lim_{x \to 0} \left(\sqrt[3]{x^3 + 2x^2} - \sqrt{x^2 + x} \right)$ is

A. e

B. e^2

C. $\sqrt{€}$

D. e^{-1}

Answer: C

Watch Video Solution

62. The value of $\lim_{x \to 0} \frac{1 + \sin x - \cos x + \log(1 - x)}{x^3}$ is

A. 1

B. -1

C. 2

D. -2

Answer: B

Watch Video Solution

63. $\lim_{x \to 0} \frac{\cos(\tan x) - \cos x}{x^4}$ is equal to

A. $f(1 + 0) = -1, f(1 - 0) = 0$

B. $f(1 + 0) = 0 = f(1 - 0)$

C. $\lim_{x \to 1} f(x)$ exists

D. $\lim_{x \to 1} f(x)$ does not exist

Answer: B

Watch Video Solution

64. If $\lim_{x \to 0} \left(x^{-3} \sin 3x + ax^{-2} + b\right)$ exists and is equal to 0, then

A. $a = 1$

B. $a = 0$

C. $b = 1$

D. $b = -1$

Answer: A

Watch Video Solution

65. If $\lim_{x \to 0} \frac{x^n \sin^n x}{x^n - \sin^n x}$ is non-zero finite, then n is equal to

A. $1, \quad \text{if} \quad n = m$

B. $0, \quad \text{if} \quad n > m$

C. $\infty, \quad \text{if} \quad n < m$

D. $n/m, \quad \text{if} \quad n < m$

Answer: B

Watch Video Solution

66. $\lim_{x \to 0} \left(\frac{1 + \tan x}{1 + \sin x} \right)^{\cos ecx}$ is equal to

A. $\lim_{x \to 0} f(x)$ exists for $n > 0$

B. $\lim_{x \to 0} f(x)$ does not exists for $n < 0$

C. $\lim_{x \to 0} f(x)$ does not exists for any value of n

D. $\lim_{x \to 0} f(x)$ exists for any value of n

Answer: C

Watch Video Solution

67. The value of $\lim_{x \to 1} (2-x)^{\tan\left(\frac{\pi x}{2}\right)}$ is

A. $a = 1/4$

B. $b = 3/4$

C. $L = -1/32$

D. $L = 1/32$

Answer: C

Watch Video Solution

68. The value of $\lim_{m \to \infty} \left(\cos\frac{x}{m}\right)^m$ is

A. $\lim_{x \to \infty} \frac{\log_e x}{\{x\}} = \infty$

B. $\lim_{x \to 2^+} \frac{x}{x^2 - x - 2} = \infty$

C. $\lim_{x \to -1^-} \frac{x}{x^2 - x - 2} = \infty$

D. $\lim_{x \to \infty} \frac{\log_{0.5} x}{\{x\}} = \infty$

Answer: A

Watch Video Solution

69. $\lim_{n \to \infty} \left(\frac{n^2 - n + 1}{n^2 - n - 1} \right)^{n(n-1)}$ is equal to

A. $\lim_{x \to 0} \frac{[x + |x|]}{x} = 0$, where $[x]$ denotes the greatest integer functions.

B. $\lim_{x \to 0} \frac{x e^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}} = 0$

C. $\lim_{x \to 3} (x - 3)^{\frac{1}{5}} sgn(x - 3) = 0$, where sgn stands for signum function.

D. $\lim_{x \to 0} \frac{\tan^{-1}|x|}{x} = 0$

Answer: B

Watch Video Solution

70. $\lim_{n\to\infty}\left\{\left(\frac{n}{n+1}\right)^{a}+\sin\frac{1}{n}\right\}^{n}$ (where $\alpha\in Q$) is equal to

A. $a=1/3, b=1$

B. $a=1, b=-1$

C. $a=9, b=-9$

D. $a=2, b=2/3$

Answer: C

Watch Video Solution

71. $\lim_{x\to\infty}\left[\left(\frac{e}{1-e}\right)\left(\frac{1}{e}-\frac{x}{1+x}\right)\right]^{x}$ is

A. limit does not exist when $a=\pi/6$

B. $L=-1$ when $a=\pi$

C. $L=1$ when $a=\pi/2$

D. $L=1$ when $a=0$

Answer: C

Watch Video Solution

72. $\lim_{x \to 0} \left(\frac{1^x + 2^x + 3^x + ... + n^x}{n} \right)^{\frac{1}{x}}$

A. $f(1^+) + f(1^-) = 0$

B. $f(1^+) + f(1^-) + f(1) = 3/2$

C. $f(-1^+) + f(-1^-) = -1$

D. $f(1^+) + f(-1^-) = 0$

Answer: B

Watch Video Solution

73. The value of $\lim_{x \to 1} \left(\frac{p}{1 - x^p} - \frac{q}{1 - xq} \right), p, q, \in N,$ equals

A. $-\frac{3}{4}$

B. 0 if n is even

C. $-\frac{3}{4}$ if n is odd

D. none of these

Answer: C

Watch Video Solution

74. $\lim_{x \to \infty} \frac{x(\log x)^3}{1 + x + x^2}$ equals

A. $\lim_{x \to 0^+} f(x) = 1$

B. $\lim_{x \to 0^-} f(x) = \cot 1$

C. $\cot^{-1}\left(\lim_{x \to 0^-} f(X)\right)^2 = 1$

D. $\tan^{-1}\left(\lim_{x \to 0^+} f(x)\right) = \frac{\pi}{4}$

Answer: A

Watch Video Solution

75. $\lim_{x \to \infty} \frac{\cot^{-1}(x^{-a} \log_a x)}{\sec^{-1}(a^x \log_x a)}, (a > 1),$ is equal to

A. $\lim_{x \to 1} f(x)$ exists $\Rightarrow a = -2$

B. $f(x)$ exists $\Rightarrow a = 13$
$x \to -2$

C. $f(x) = 4/3$ if it exists
$x \to 1$

D. $\lim_{x \to -2} f(x) = -1/3$ if it exists

Answer: B

Watch Video Solution

76. The value of $\lim_{n \to \infty} \frac{e^n}{\left(1 + \frac{1}{n}\right)^{n^2}}$ is

A. -1

B. 0

C. 1

D. ∞

Answer: C

Watch Video Solution

Exercise (Multiple)

1. Let $f(x) = \begin{cases} 1 + \frac{2x}{a}, & 0 \le x < 1 \\ ax, & 1 \le x < 2 \end{cases}$. If $\lim_{x \to 1} f(x)$ exists, then a is

A. $\lim_{x \to 5^-} f(x) = 0$

B. $\lim_{x \to 5^+} f(x) = 1$

C. $\lim_{x \to 5} f(x)$ does not exist

D. none of these

Answer: B::C

Watch Video Solution

2. If $f(x) = |x-1| - [x]$, where $[x]$ is the greatest integer less than or equal to x, then

A. $\lim_{x \to 0} [f(x)] = 0$

B. $\lim_{x \to 0} [f(x)] = 1$

C. $\lim_{x \to 0} \left[\frac{f(x)}{x}\right]$ does not exist

D. $\lim_{x \to 0} \left[\frac{f(x)}{x}\right]$ exists

Answer: A::D

Watch Video Solution

3. If $\lim_{n \to \infty} \left(an - \frac{1+n^2}{1+n}\right) = b$, where a is a finite number, then

A. $f(0) = 1$

B. $f\left(\frac{\pi}{2}\right) = 1$

C. $f(a) = (\cos a)^{\cos^2 a} . (\sin a)^{\sin^2 a}$ if $a \in \left(0, \frac{\pi}{2}\right)$

D. $f(a) = \dfrac{(\sin a)^{\sin^2 a}}{(\cos a)^{\cos^2 a}}$ if $a \in \left(0, \dfrac{\pi}{2}\right)$

Answer: A::C

Watch Video Solution

4. If $m, n \in N$, $\lim\limits_{x \to 0} \dfrac{\sin x^n}{(\sin x)^m}$ is

A. $\dfrac{p}{2}$

B. $\dfrac{\pi}{2\sqrt{2}}$

C. $\dfrac{\pi}{\sqrt{2}}$

D. $\sqrt{2}\pi$

Answer: A::B::C

Watch Video Solution

5. Let $f(x) = \begin{cases} x^n \sin\left(1/x^2\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, $(n \in I)$. Then

A. $\frac{\pi}{2}$

B. $\frac{\pi}{2\sqrt{2}}$

C. $\frac{\pi}{\sqrt{2}}$

D. $\sqrt{2}\pi$

Answer: A::B

View Text Solution

6. If $L = \lim_{x \to 0} \frac{1}{x^3}\left(\frac{1}{\sqrt{1+x}} - \frac{1+ax}{1+bx}\right)$ exists,then

A. $\cos L < \cos R$

B. $\tan(2L) < an < 2R$

C. $\sin L > \sin R$

D. `None of these

Answer: A::B::C

Watch Video Solution

Watch Video Solution

7. Which of the following is true ($\{.\}$ denotes the fractional part of the function)?

A. always 1

B. always -1

C. $(-1)^{n-m+1}$

D. $(-1)^{n-m}$

Answer: A::B::C

View Text Solution

8. Which of the following is/are correct?

A. always 1

B. always -1

C. $(-1)^{m+1}$

D. $(-1)^{n-m}$

Answer: A::B::C

View Text Solution

9. If $\lim_{x \to 1} (2 - x + a[x-1] + b[1+x])$ exists, then a and b can take the values (where $[.]$ denotes the greatest integer function)

A. is always equal to -1

B. is always equal to +1

C. does not exist None of these

D.

Answer: B::C

Watch Video Solution

10. Let $L = \lim_{x \to a} \frac{|2\sin x - 1|}{2\sin x - 1}$. Then

A. $1/2$

B. $-1/3$

C. $-1/6$

D. 3

Answer: A::B::C

Watch Video Solution

11. Let $f(x) = \lim_{n \to \infty} \frac{x}{x^{2n} + 1}$. Then

A. real and equal roots

B. complex roots

C. unequal positive real roots

D. unequal roots

Answer: B::C::D

Watch Video Solution

12. $\lim_{n \to \infty} \frac{-3n + (-1)^n}{4n - (-1)^n}$ is equal to $(n \in N)$

A. $[-2, 2]$

B. $[0, 2]$

C. [-1,1]`

D. $[-2, 1]$

Answer: A::C

Watch Video Solution

13. Given a real valued function f such that

$f(x) = \begin{cases} \frac{\tan^2[x]}{x^2 - [x]^2}, x < 0 \text{ and } 1, x = 0 \text{ and } \sqrt{\{x\}\cot\{x\}}, x < 0 \end{cases}$

where [.] represents greatest integer function then

A. $p_1 \ \ln \ a_1 + p_2 \ \ln \ a_2 + \ldots + p_n \ \ln \ a_n$

B. $a_1^{p_1} + a_2^{p_2} + \ldots + a_n^{p_n}$

C. $a_1^{p_1} . a_2^{p_2} \ldots a_n^{p_n}$

D. $\sum_{r=1}^{n} a_r p_r$

Answer: A::B::C::D

Watch Video Solution

14. If $f(x) = \frac{3x^2 + ax + a + 1}{x^2 + x - 2}$, then which of the following can be correct $(\lim)_{x \to 1} f(x) e\xi stsa = -2$ $(\lim)_{x \to -2} f(x) e\xi stsa = 13$ $(\lim)_{x \to 1} f(x) = \frac{4}{3}$ $(\lim)_{x \to -2} f(x) = -\frac{1}{3}$

A. $\ln \ a_1$

B. e^{a_n}

C. a_(1)`

D. a_n

Answer: A::B::C::D

Watch Video Solution

15. The value of $\lim_{n \to \infty} \frac{1}{1 + n \sin^2 nx}$ can be $(n \in N)$

A. $\ln \ a_n$

B. e^{a_1}

C. a_1

D. a_n

Answer: B::C

Watch Video Solution

16. Let $f(x) = \dfrac{x^2 - 9x + 20}{x - [x]}$ where $[x]$ denotes greatest integer less than or equal to x), then

A. -1

B. $1/2$

C. 1

D. $3/2$

Answer: A::B::C

Watch Video Solution

17. Given $\lim_{x \to 0} \dfrac{f(x)}{x^2} = 2$, where $[.]$ denotes the greatest integer function, then

A. $\dfrac{1}{3}$

B. $\dfrac{1}{4}$

C. $\dfrac{1}{2}$

D. $\frac{2}{3}$

Answer: A::C

Watch Video Solution

18. If $f(a) = \lim_{x \to 2} (\sin^x a + \cos^x a)^{\frac{1}{(x-2)}}$ for $a \in \left[0, \frac{\pi}{2}\right]$, then

A. $f(0) = 1$

B. $f\left(\frac{\pi}{2}\right) = 1$

C. $f(\alpha) = (\cos \alpha)^{\cos^2 \alpha} \cdot (\sin \alpha)^{\sin^2 \alpha}$ if $\alpha \in \left(0, \frac{\pi}{2}\right)$

D. $f(\alpha) = \frac{(\sin \alpha)^{\sin^2 \alpha}}{(\cos \alpha)^{\cos^2 \alpha}}$ if $\alpha \in \left(0, \frac{\pi}{2}\right)$

Answer: A::B::C

Watch Video Solution

Exercise (Comprehension)

1. $Let f(x) = \frac{\sin^{-1}(1-\{x\}) \times \cos^{-1}(1-\{x\})}{\sqrt{2\{x\}} \times (1-\{x\})}$, where $\{x\}$ denotes the fractional part of x.

$R = \lim_{x \to 0+} f(x)$ is equal to

A. $np(1-n)$

B. $-np(1+n)$

C. $n^2 p$

D. $np(1+n)$

Answer: A

Watch Video Solution

2. $Let f(x) = \frac{\sin^{-1}(1-\{x\}) \times \cos^{-1}(1-\{x\})}{\sqrt{2\{x\}} \times (1-\{x\})}$, where $\{x\}$ denotes the fractional part of x.

$L = \lim_{x \to 0-} f(x)$ is equal to

A. $\frac{\pi}{2}$

B. $\frac{\pi}{2\sqrt{2}}$

C. (π)

D. $\frac{1}{2\sqrt{2}}$

Answer: B

Watch Video Solution

3. let $f(x) = \frac{\cos^{-1}(1-\{x\})\sin^{-1}(1-\{x\})}{\sqrt{2\{x\}(1-\{x\})}}$ where $\{x\}$ denotes the fractional part of x then

A. $\frac{1}{96}$

B. $\frac{1}{48}$

C. $\frac{1}{24}$

D. 1

Answer: D

Watch Video Solution

4. $A_i = \frac{x - a_i}{|x - a_i|}, i = 1, 2, ..., n, \quad \text{and} \quad a_1 < a_2 < a_3 < ... < a_n.$

If $1 \leq m \leq n, \ \min N,$ then the value of $L = \lim_{x \to a_m} (A_1 A_2 ... A_n)$is

A. 2

B. -1

C. not exist

D. 1

Answer: C

Watch Video Solution

5. $A_i = \frac{x - a_i}{|x - a_i|}, i = 1, 2, ..., n, \quad \text{and} \quad a_1 < a_2 < a_3 < ... < a_n.$

If $1 \leq m \leq n, \ \min N,$ then the value of $R = \lim_{x \to a_m +} (A_1 A_2 ... A_n)$is

A. $e^{-\frac{1}{4}}$

B. $e^{-\frac{1}{2}}$

C. e^{-2}

D. e^{-4}

Answer: D

Watch Video Solution

6. $A_i = \frac{x - a_i}{|x - a_i|}, i = 1, 2, ..., n,$ and $a_1 < a_2 < a_3 < ... < a_n.$

If $1 \leq m \leq n,$ min $N,$ then $\lim_{x \to a_m} (A_1 A_2 ... A_n)$

Watch Video Solution

7. If $L = \lim_{x \to 0} \frac{\sin x + ae^x + be^{-x} + c \log_e (1 + x)}{x^3}$ exists finitely, then

The value of L is

Watch Video Solution

8. If $L = \lim_{x \to 0} \frac{\sin x + ae^x + be^{-x} + c\log_e(1+x)}{x^3}$ exists finitely, then

Equation $ax^2 + bx + c = 0$ has

Watch Video Solution

9. If $L = \lim_{x \to 0} \frac{\sin x + ae^x + be^{-x} + c\log_e(1+x)}{x^3}$ exists finitely, then

The solutions set of $||x + c| - 2a| < 4b$ is

A. $\begin{matrix} a & b & c & d \\ s & r & q & p \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ q & s & r & p \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ s & r & p & q \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ s & p & q & r \end{matrix}$

Answer: C

Watch Video Solution

10. Let $a_1 > a_2 > a_3 > \ldots > a_n > 1$.

$p_1 > p_2 > p_3 > \ldots > p_n > 0$ such that $p_1 + p_2 + p_3 + \ldots + p_n = 1$.

Also, $F(x) = \left(p_1 a_1^x + p_n a_n^x\right)^{1/x}$.

$\lim_{x \to 0^+} F(x)$ equals

A. $a \quad b \quad c \quad d$
$s \quad r \quad q \quad p$

B. $a \quad b \quad c \quad d$
$q \quad p \quad s \quad p$

C. $a \quad b \quad c \quad d$
$s \quad r \quad p \quad q$

D. $a \quad b \quad c \quad d$
$p \quad p \quad q \quad r$

Answer: C

Watch Video Solution

11. Let $a_1 > a_2 > a_3 > \ldots > a_n > 1$.

$p_1 > p_2 > p_3 > \ldots > p_n > 0$ such that $p_1 + p_2 + p_3 + \ldots + p_n = 1$.

Also, $F(x) = \left(p_1 a_1^x + p_n a_n^x\right)^{1/x}$.

$\lim_{x \to \infty} F(x)$ equals

Watch Video Solution

12. Let $a_1 < a_2 < a_3 ... a_< 1 \quad p_1 > p_2 > ... p_n > 0$; such that $p_1 + p_2 + p_3 + p_n = 1$. Also $F(x) = \left(p_1 a_1^x + p_2 a_2^x + ... + p_n an^x\right)^{\frac{1}{x}}$

Compute

Watch Video Solution

13. If $L = \lim_{x \to \infty} \left(x + 1 - \sqrt{ax^2 + x + 3}\right)$ exists infinetely then

The value of a is

Watch Video Solution

14. If $L = \lim_{x \to \infty} \left(x + 1 - \sqrt{ax^2 + x + 3}\right)$ exists finitely then

The value of L is

Watch Video Solution

15.

$Let f: R \to R$ be a real function. The function f is double differentiable.

$\lim_{x \to \infty} x^{n+1} f(x)$, then

$\lim_{x \to \infty} x^{n+1} f'(x)$ is equal to

Watch Video Solution

16.

$Let f: R \to R$ be a real function. The function f is double differentiable.

$\lim_{x \to \infty} x^{n+1} f(x)$, then

$\lim_{x \to \infty} x^{n+2} f''(x)$ is equal to

Watch Video Solution

17. $Let f(x)$ be a polynomial satisfying $\lim_{x \to \infty} \frac{x^2 f(x)}{2x^5 + 3} = 6$ and $f(1) = 3, f(3) = 7$ and $f(5) = 11$. Then The value of $f(0)$ is

Watch Video Solution

18. $Let f(x)$ be a polynomial satisfying $\lim_{x \to \infty} \frac{x^2 f(x)}{2x^5 + 3} = 6$ and $f(1) = 3, f(3) = 7$ and $f(5) = 11$. Then $\lim_{x \to 1} \frac{x - 1}{\sin(f(x) - 2x - 1)}$ is equal to

Watch Video Solution

19. If $\lim_{x \to 0} \frac{f(x)}{\sin^2 x} = 8$, $\lim_{x \to 0} \frac{g(x)}{2\cos x - xe^x + x^3 + x - 2} = \lambda$ and $\lim_{x \to 0} (1 + 2f(x))^{\frac{1}{g(x)}} = \frac{1}{e}$, then

The value of λ is

Watch Video Solution

20. If $\lim_{x \to 0} \frac{f(x)}{\sin^2 x} = 8$, $\lim_{x \to 0} \frac{g(x)}{2\cos x - xe^x + x^3 + x - 2} = \lambda$ and $\lim_{x \to 0} (1 + 2f(x))^{\frac{1}{g(x)}} = \frac{1}{e}$, then

$\lim_{x \to 0} (1 + f(x))^{\frac{1}{2g(x)}}$ is equal to

Watch Video Solution

Exercise (Matrix)

1. Match the following lists:

List I	List II
a. If $L = \lim_{x \to -1} \frac{\sqrt[3]{(7-x)} - 2}{(x+1)}$, then $12L =$	**p.** -2
b. If $L = \lim_{x \to \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$, then $-L/4 =$	**q.** 2
c. If $L = \lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2 + x - 3}$, then $20L =$	**r.** 1
d. If $L = \lim_{x \to \infty} \frac{\log x^n - [x]}{[x]}$, where $n \in N$, ($[x]$ denotes greatest integer less than or equal to x), then $-2L =$	**s.** -1

View Text Solution

2. Match the following lists:

List I ([·] denotes the greatest integer function)	List II
a. $\lim_{x\to 0}\left(\left[100\frac{\sin x}{x}\right]+\left[100\frac{\tan x}{x}\right]\right)$	**p.** 198
b. $\lim_{x\to 0}\left(\left[100\frac{x}{\sin x}\right]+\left[100\frac{\tan x}{x}\right]\right)$	**q.** 199
c. $\lim_{x\to 0}\left(\left[100\frac{\sin^{-1}x}{x}\right]+\left[100\frac{\tan^{-1}x}{x}\right]\right)$	**r.** 200
d. $\lim_{x\to 0}\left(\left[100\frac{x}{\sin^{-1}x}\right]+\left[100\frac{\tan^{-1}x}{x}\right]\right)$	**s.** 201

View Text Solution

3. Match the following lists:

List I	List II
a. If $\lim_{x\to\infty}(\sqrt{(x^2-x-1)}-ax-b)=0$, where $a>0$, then there exists at least one a and b for which point $(a, 2b)$ lies on the line.	**p.** $y=-3$
b. If $\lim_{x\to\infty}\frac{(1+a^3)+8e^{1/x}}{1+(1-b^3)e^{1/x}}=2$, then there exists at least one a and b for which point (a, b^3) lies on the line.	**q.** $3x-2y-5=0$
c. If $\lim_{x\to\infty}(\sqrt{(x^4-x^2+1)}-ax^2-b)=0$, then there exists at least one a and b for which point $(a, -4b)$ lies on the line.	**r.** $15x-2y-11=0$
d. If $\lim_{x\to-a}\frac{x^7+a^7}{x+a}=7$, where $a<0$, then there exists at least one a for which point $(-a, 2)$ lies on the line.	**s.** $y=2$

View Text Solution

4. Consider $\lim_{x\to\infty}\left(\frac{x^3+x^2+x+\sin x}{x^2+2\cos x}-a\sin x-bx+c\right)=4.$

Now, match the following lists and then choose the correct code.

List I	List II
a. The value of a is	p. 1
b. The value of b is	q. 3
c. The value of c is	r. 2
d. Number of real roots of equation $cx^2 + bx + a = 0$ is	s. 0

Watch Video Solution

5. Match the following lists (where $[x]$ represents the greatest integer function) and then choose the correct code.

List I	List II
a. $\lim_{x\to 0} x(-1)^{[1/x]}$	p. Does not exist
b. $\lim_{x\to 2}(-1)^{[x]}$	q. is 0
c. $\lim_{x\to \frac{3}{2}}(x-[x])$	r. is 1
d. $\lim_{x\to 0}[x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$	s. is 2

Codes :

	a	b	c	d
(1)	s	r	q	p
(2)	q	p	s	p
(3)	s	r	p	q
(4)	p	p	q	r

Watch Video Solution

Exercise (Numerical)

1. The reciprocal of the value of

$$\lim_{n\to\infty}\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\left(1-\frac{1}{n^2}\right)\text{is}\_\_\_\_\_.$$

Watch Video Solution

2. $\lim_{x\to\infty} f(x)$, where $\frac{2x-3}{x} < f(x) < \frac{2x^2+5x}{x^2}\ \forall x > 0$, is _____.

Watch Video Solution

3. If $f(x) = \begin{cases} x-1, & x \geq 1 \\ 2x^2-2, & x < 1 \end{cases}, g(x) = \begin{cases} x+1, & x > 0 \\ -x^2+1, & x \leq 0 \end{cases}$, and $h(x) = |x|$, then $\lim_{x \to 0} f(g(h(x)))$ is_____.

Watch Video Solution

4. If $\lim_{x \to \infty} f(x)$ exists and is finite and nonzero and if $\lim_{x \to \infty} \left\{ f(x) + \frac{3f(x)-1}{f^2(x)} \right\} = 3$, then the value of $\lim_{x \to \infty} f(x)$ is _____.

Watch Video Solution

5. If $L = \lim_{x \to 2} \frac{(10-x)^{1/3} - 2}{x-2}$, then the value of $|1/(4L)|$ is _____.

Watch Video Solution

6. If $\lim_{x \to 0} \frac{p \sin 2x + (1 - \cos 2x)}{x + \tan x} = 1$, then the value of p is_____.

Watch Video Solution

7. The value of $\lim_{x \to \infty} \left(\frac{100}{1 - x^{100}} - \frac{50}{1 - x^{50}} \right)$ is _____.

Watch Video Solution

8. If $L = \lim_{x \to 2} \frac{\sqrt[3]{60 + x^2} - 4}{\sin(x - 2)}$, then the value of $1 / L$ is_____.

Watch Video Solution

9. The value of $\lim_{x \to \infty} \left(\frac{20^x - 1}{19(5^x)} \right)^{1/x}$ is _____.

A. 3

B. 1

C. $\frac{2}{3}$

D. $\frac{3}{2}$

Answer: (4)

Watch Video Solution

10. The value of $\lim_{n \to \infty} \left[\sqrt[3]{(n+1)^2} - \sqrt[3]{(n-1)^2}\right]$ is _____.

A. Equals $\frac{1}{\sqrt{2}}$

B. Does not exist

C. Equals $\sqrt{2}$

D. Equals -sqrt(2)`

Answer: (0)

Watch Video Solution

11. If $L = -\lim_{n \to \infty} \left(2 \times 3^2 \times 2^3 \times 3^4 \ldots \times 2^{n-1} \times 3^n\right)^{\frac{1}{(n^2+1)}}$, then the value of L^4 is _____.

A. $-\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. 2

Answer: (6)

Watch Video Solution

12. The value of $\lim_{x \to \infty} \frac{\log_e(\log_e x)}{e\sqrt{x}}$ is _____.

A. $\frac{\pi}{2}$

B. 1

C. $-\pi$

D. π

Answer: (0)

Watch Video Solution

13. If $\lim_{x \to 0} \frac{1 - \sqrt{\cos 2x} . \sqrt[3]{\cos 3x} . \sqrt[4]{\cos 4x} \ldots \sqrt[n]{\cos nx}}{x^2}$ has the value equal to 10, then the value of n equals_____.

A. 4

B. 3

C. 2

D. $1/2$

Answer: (6)

Watch Video Solution

14. The value of $\lim_{x \to \infty} \left(x - x^2 \log_e \left(1 + \frac{1}{x} \right) \right)$ is _____.

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. 2

Answer: (0.5)

Watch Video Solution

15. Let $S_n = 1 + 2 + 3 + \ldots + n$ and $P_n = \frac{S_2}{S_2 - 1} \cdot \frac{S_3}{S_3 - 1} \cdot \frac{S_4}{S_4 - 1} \ldots \frac{S_n}{S_n - 1}$, where $n \in N, (n \geq 2) Then \lim_{n \to \infty} P_n$=_____.

A. $\frac{1}{4}$

B. $\frac{1}{24}$

C. $\frac{1}{16}$

D. $\frac{1}{8}$

Answer: (3)

Watch Video Solution

16. If $\lim_{x \to 1} \frac{a \sin(x-1) + b \cos(x-1) + 4}{x^2 - 1} = -2$, then $|a+b|$ is_____.

A. does not exist (in R)

B. is equal to 0

C. is equal to 15

D. is equal to 120

Answer: (8)

Watch Video Solution

17. Let $\lim_{x \to 1} \frac{x^a - ax + a - 1}{(x-1)^2} = f(a)$. Then the value of $f(4)$ is_____.

A. $\pm \frac{\pi}{4}$

B. $\pm \frac{\pi}{3}$

C. $\pm \frac{\pi}{6}$

D. $\pm\frac{\pi}{2}$

Answer: (6)

Watch Video Solution

18. Number of integral values of k for which

$\lim_{x\to 1}\sin^{-1}\left(\frac{k}{\log_e x}-\frac{k}{x-1}\right)$ exists is _____.

A. $a=1, b=4$

B. $a=1b=-4$

C. $a=2, b=-3$

D. $a=2, b=3$

Answer: (5)

Watch Video Solution

19. If $\lim_{x \to 1} \left(1 + ax + bx^2\right)^{\frac{e}{(x-1)}} = e^3$, then the value of bc is _____.

A. $a = 2$

B. $a = 1$

C. $L = \frac{1}{64}$

D. L=(1)/(32)`

Answer: (3)

Watch Video Solution

20. Let $f''(x)$ be continuous at $x = 0$

If $\lim_{x \to 0} \frac{2f(x) - 3af(2x) + bf(8x)}{\sin^2 x}$ exists and $f(0) \neq 0, f'(0) \neq 0,$ then the value of $3a / b$ is_____.

A. $\lim_{x \to 1^+} f(x) = 0$

B. $\lim_{x \to 1^-} f(x)$ does not exist

C.

D.

Answer: (7)

Watch Video Solution

21. If $L = \lim_{x \to 0} \frac{e^{-x^2/2} - \cos x}{x^3 \sin x}$, then the value of $1/(3L)$ is _____.

Watch Video Solution

22. The integer n for which $\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite nonzero number is_____.

Watch Video Solution

23. If $\lim_{x \to 0} \left[1 + x + \frac{f(x)}{x}\right]^{1/x} = e^3$, then the value of $\ln\left(\lim_{x \to 0} \left[1 + \frac{f(x)}{x}\right]^{1/x}\right)$ is _____.

Watch Video Solution

JEE Main Previous Year

1. Let $f: R \to R$ be a positive, increasing function with $\lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x \to \infty} \frac{f(2x)}{f(x)}$ is equal to

Watch Video Solution

2. Show that the $\lim_{x \to 2} \left(\frac{\sqrt{1 - \cos\{2(x-2)\}}}{x - 2} \right)$ doesnot exist.

Watch Video Solution

3. $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to

Watch Video Solution

4. $\lim_{x \to 0} \frac{\sin\left(\pi \cos^2 x\right)}{x^2}$ is equal to

Watch Video Solution

5. $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to

Watch Video Solution

6. Let $p = \lim_{x \to 0^+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$. Then $\log_e p$ is equal to

Watch Video Solution

7. The $\lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals

Watch Video Solution

8. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then $\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right]\right)$

Watch Video Solution

JEE Advanced Previous Year

1. If

$$\lim_{x \to 0} \left[1 + x \ln\left(1 + b^2\right)\right]^{1/x} = 2b \sin^2 \theta, b > 0, \quad \text{and} \quad \theta \in (-\pi, \pi],$$

then the value of θ is

Watch Video Solution

2. If $\lim_{x \to \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b\right) = 4$, then

Watch Video Solution

3. Let $L = \lim_{x \to 0} \dfrac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then

Watch Video Solution

4. Let $f(x) = \left(\dfrac{1 - x(1 + |1 - x|)}{|1 - x|}\right)\cos\left(\dfrac{1}{1 - x}\right)$ for $x \neq 1$

Watch Video Solution

5. The largest value of the non-negative integer a for which

$$\lim_{x \to 1} \left\{\frac{-ax + \sin(x - 1) + a}{x + \sin(x - 1)a}\right\}^{\frac{1 - x}{1 - \sqrt{x}}} = \frac{1}{4} \text{ is } \_\_\_\_\_.$$

Watch Video Solution

6. Let m and n be two positive integers greater than 1. If

$$\lim_{a \to 0} \frac{e^{\cos(\alpha^n)} - e}{\alpha^m} = -\frac{e}{2}, \text{ then the value of } \frac{m}{n} \text{ is } \_\_\_\_\_.$$

Watch Video Solution

7. Let $\alpha, \beta \in R$ be such that $\lim_{x \to 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals_____.

Watch Video Solution

Single Correct Answer Type

1. If $f: R \to R$ is defined by $f(x) = [x - 3] + |x - 4|$ for $x \in R$, then $\lim_{x \to 3^-} f(x)$ is equal to (where $[.]$ represents the greatest integer function)

A. -2

B. -1

C. 0

D. 1

Answer: C

Watch Video Solution

2. $\lim_{x-\frac{\pi}{2}}\left[\frac{[\sin x]-[\cos x]+1}{3}\right]=$ (where $[.]$ denotes the greatest integer integer function)

A. 0

B. 1

C. -1

D. does not exist

Answer: A

Watch Video Solution

3. $\lim_{x\to\frac{-1}{3}}\frac{1}{x}\left[\frac{-1}{x}\right]=$ (where [.] denotes the greatest integer function)

A. -9

B. -12

C. -6

D. 0

Answer: C

Watch Video Solution

4. If

$$f(x) = \begin{cases} x + \frac{1}{2}, & x < 0 \\ 2x + \frac{3}{4}, & x \geq 0 \end{cases}, \text{then} \left[\lim_{x \to 0} f(x) \right] = (\text{where}[.] \text{ denotes the grea}$$

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. does not exist

D. none of these

Answer: C

Watch Video Solution

5. $\lim_{x \to -7} \frac{[x]^2 + 15[x] + 56}{\sin(x+7)\sin(x+8)} =$ (where [.] denotes the greatest integer function)

A. is 0

B. is 1

C. is -1

D. does not exist

Answer: A

Watch Video Solution

6. Which of the following limits exists finitely?

A. $\lim_{x \to 0^+} (x)^{\log_e x}$

B. $\lim_{x \to 1^+} \frac{x^2 - 9 - \sqrt{x^2 - 6x + 6}}{|x-1| - 2}$

C. $\lim_{x \to 1^+} ([x])^{\frac{1}{x-1}} =$ (where [.] denotes the greatest integer function)

D. none of these

Answer: C

View Text Solution

7. Let $L_1 = \lim_{x \to 4} (x-6)^x$ and $L_2 = \lim_{x \to 4} (x-6)^4$.

Which of the following is true?

A. Both L_1 and L_2 exists

B. Neither L_1 nor L_2 exists

C. L_1 exists but L_2 does not exist

D. L_2 exists but L_1 does not exist

Answer: D

Watch Video Solution

8. Set of all values of x such that $\lim_{n \to \infty} \frac{1}{1 + \left(\frac{4 \tan^{-1}(2\pi x)}{\pi} \right)^{4n}}$ is non-zero and finite number when $n \in N$ is

A. $\left(0, \frac{1}{2\pi}\right)$

B. $\left(-\frac{1}{\pi}, \frac{1}{\pi}\right)$

C. $\left[-\frac{1}{2\pi}, \frac{1}{2\pi}\right]$

D. $\left(-\frac{1}{2\pi}, 0\right)$

Answer: C

Watch Video Solution

9. $\lim_{x \to \infty} \left[x - \log_e \left(\frac{e^x + e^{-x}}{2} \right) \right] =$

A. $\log_e 4$

B. 0

C. $\log_e 2$

D. ∞

Answer: C

Watch Video Solution

10. $\lim_{x \to \infty} \left\{ (e^x + \pi^x)^{\frac{1}{x}} \right\}$= (where {.} denotes the fractional part of x) is equal to

A. $\pi - e$

B. $\pi - 3$

C. $e - 2$

D. $3 - e$

Answer: B

Watch Video Solution

11. If $\frac{\cos x}{\sin ax}$ is periodic function, then $\lim_{m \to \infty} \left(1 + \cos^{2m} n!\pi a\right)$ is equal to

A. 0

B. 1

C. 2

D. -1

Answer: C

Watch Video Solution

12. The value of $\lim_{x \to 0} \frac{\sqrt{1 - \cos x^2}}{1 - \cos x}$ is

A. $\frac{1}{2}$

B. 2

C. $\sqrt{2}$

D. none of these

Answer: C

Watch Video Solution

13. $\lim_{x \to \frac{\pi}{2}} (1 - \sin x)\tan x =$

A. $\frac{\pi}{2}$

B. 1

C. 0

D. ∞

Answer: C

Watch Video Solution

14. The value of $\lim_{x \to \infty} x^2 \left(1 - \cos. \frac{1}{x}\right)$ is

A. 0

B. $1/4$

C. $1/2$

D. 1

Answer: C

Watch Video Solution

15. $\lim_{x \to \infty} \sqrt[3]{x}\left(\sqrt[3]{(x+1)^2} - \sqrt[3]{(x-1)^2}\right) =$

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. 1

D. $\frac{4}{3}$

Answer: D

View Text Solution

16. $\lim_{n \to \infty} \frac{3.2^{n+1} - 4.5^{n+1}}{5.2^n + 7.5^n} =$

A. 0

B. 43529

C. $-4/7$

D. $-20/7$

Answer: D

Watch Video Solution

17. $\lim\limits_{\quad} (X \to 2^+)\{x\}\frac{\sin(x-2)}{(x-2)^2} =$ (where $\{.\}$ denotes the fractional part function)

A. 0

B. 2

C. 1

D. does not exist

Answer: C

Watch Video Solution

18. $\lim_{x \to \infty} \frac{\cot^{-1}\left(\sqrt{x+1}+\sqrt{x}\right)}{\sec^{-1}\left\{\left(\frac{2x+1}{x-1}\right)^{x}\right\}} =$

A. 1

B. 0

C. $\pi/2$

D. non existent

Answer: A

Watch Video Solution

19. $\lim_{x \to 0} \frac{3\tan 3x - 4\tan 2x - \tan x}{4x^2 \tan x}$

A. 0

B. 1

C. 3

D. 4

Answer: D

Watch Video Solution

20. $\lim_{x \to 0} \left[\frac{\sin^{-1} x}{\tan^{-1} x} \right] =$ (where $[.]$ denotes the greatest integer function)

A. 0

B. 1

C. -1

D. none of these

Answer: B

Watch Video Solution

21. The value of $\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{1-\sqrt{\sin 2x}}}{\pi - 4x}$ is

A. $\frac{1}{4}$

B. $-\frac{1}{4}$

C. 1

D. does not exist

Answer: D

Watch Video Solution

22. The value of $\lim_{x \to \infty} \left(e^{\sqrt{x^4+}} - e^{(x^2+1)}\right)$ is

A. 0

B. e

C. 1/e

D. $-\infty$

Answer: D

Watch Video Solution

23. The value of $\lim_{x \to \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ is

A. 8

B. 4

C. -8

D. -2

Answer: C

Watch Video Solution

24. $\lim_{x \to \frac{\pi}{2}} \frac{(1-\sin x)\left(8x^3-\pi^3\right)}{(\pi-2x)^4}$

A. $-\frac{\pi^2}{16}$

B. $\frac{3\pi^2}{16}$

C. $\frac{\pi^2}{16}$

D. $-\frac{3\pi^2}{16}$

Answer: D

Watch Video Solution

25. $\lim_{x \to \infty} \frac{\sum_{r=1}^{10}(x+r)^{2010}}{(x^{1006}+1)(2x^{1004}+1)} =$

A. 5

B. 2010

C. $\frac{502}{1005}$

D. 0

Answer: A

Watch Video Solution

26. If $\lim_{x \to 0} \frac{f(x)}{x^2} = a$ and $\lim_{x \to 0} \frac{f(1-\cos x)}{g(x)\sin^2 x} = b$ (where $b \neq 0$), then $\lim_{x \to 0} \frac{g(1-\cos 2x)}{x^4}$ is

A. $\frac{4a}{b}$

B. $\frac{a}{4b}$

C. $\frac{a}{b}$

D. none of these

Answer: C

View Text Solution

27. If

$$f(x)=\begin{cases}\frac{x}{\sin x}, & x>0\\ 2-x, & x\le 0\end{cases} \text{ and } g(x)=\begin{cases}x+3, & x<1\\ x^2-2x-2, & 1\le x<2\\ x-5, & x\ge 2\end{cases}$$

Then the value of $\lim_{x\to 0} g(f(x))$

A. is -2

B. is -3

C. is 1

D. does not exist

Answer: B

Watch Video Solution

28. If $k\in I$ such that $\lim_{n\to\infty}\left(\cos.\frac{k\pi}{4}\right)^{2n}-\left(\cos.\frac{k\pi}{6}\right)^{2n}=0,$ then

A. k must bot be divisible by 24

B. k is divisible by 24 or k is divisible neither by 4 nor by 6

C. k must be divisible by 12 but not necessarity by 24

D. none of these

Answer: B

Watch Video Solution

29. If a_n and b_n are positive integers and $a_n + \sqrt{2}b_n = \left(2 + \sqrt{2}\right)^n$, then $\lim_{n \to \infty} \left(\frac{a_n}{b_n}\right)$=

A. $\sqrt{2}$

B. 2

C. $e^{\sqrt{2}}$

D. e^2

Answer: A

Watch Video Solution

30. The value of $\lim_{x \to 0} \frac{\tan x^{\frac{1}{5}}}{\left(\tan^{-1}\sqrt{x}\right)^2} \frac{\log(1+5x)}{e^{3\sqrt[5]{x}}-1}$ is

A. $\frac{3}{5}$

B. $\frac{5}{3}$

C. 1

D. none of these

Answer: B

Watch Video Solution

31. The value of $\lim_{x \to 3} \frac{\left(x^3+27\right)\log_e(x-2)}{x^2-9}$ is

A. 9

B. 18

C. 27

D. $5/3$

Answer: A

Watch Video Solution

32. The value of $\lim_{x \to 0^+} \left(\frac{1 - \cos\left(\sin^2 x\right)}{x^2} \right)^{\frac{\log_e\left(1 - 2x^2\right)}{\sin^2 x}}$ is

A. 0

B. e

C. -1

D. ∞

Answer: D

Watch Video Solution

33. $\lim_{x \to 0} \frac{1}{x^2} \begin{vmatrix} 1 - \cos 3x & \log_e(1 + 4x) \\ \sin^{-1}(x^x -) & \tan^{-1}(2x) \end{vmatrix}$ is equal to

A. 2

B. -4

C. 6

D. 4

Answer: B

Watch Video Solution

34. If graph of the function $y = f(x)$ is continuous and passes through point (3, 1) then $\lim_{x \to 3} \frac{\log_e(3f(x) - 2)}{2(1 - f(x))}$ is equal

A. $\frac{3}{2}$

B. $\frac{1}{2}$

C. $-\frac{3}{2}$

D. $-\frac{1}{2}$

Answer: C

Watch Video Solution

35. Let f(x) be defined for all $x \in R$ such that $\lim_{x \to 0}\left[f(x) + \log\left(1 - \frac{1}{e^{f(x)}}\right) - \log(f(x))\right] = 0$. Then f(0) is

A. 0

B. 1

C. 2

D. 3

Answer: A

Watch Video Solution

36. $\lim_{x \to \infty} x^2 \sin\left(\log_e \sqrt{\frac{\cos(\pi)}{x}}\right)$

A. 0

B. $-\frac{\pi^2}{2}$

C. $-\frac{\pi^2}{4}$

D. $-\frac{\pi^2}{8}$

Answer: C

Watch Video Solution

37. If $\lim_{x \to \infty} \left(\frac{x+c}{x-c}\right)^x = 4$ then the value of e^c is

A. $1/4$

B. $1/2$

C. 1

D. 2

Answer: D

Watch Video Solution

38. If $\lim_{x \to 0} \left[1 + x + \frac{f(x)}{x}\right]^{1/x} = e^3$, then $\lim_{x \to 0} \left[1 + \frac{f(x)}{x}\right]^{1/x} =$

A. e

B. e^2

C. e^3

D. none of these

Answer: B

Watch Video Solution

39. $\lim_{x \to \frac{\pi}{2}^-} \left[1 + (\cos x)^{\cos x}\right]^2 =$

A. Does not exist

B. 1

C. e

D. 4

Answer: D

Watch Video Solution

40. If $a > 0, b > 0$ than $\lim_{n \to \infty} \left(\frac{a - 1 + b^{\frac{1}{n}}}{a} \right)^n =$

A. $b^{\frac{1}{a}}$

B. $a^{\frac{1}{b}}$

C. a^b

D. b^a

Answer: A

Watch Video Solution

41. If $f(x) = \lim_{n \to \infty} \left(\frac{\cos(x)}{\sqrt{n}} \right)^n$, then the value of $\lim_{x \to 0} \frac{f(x) - 1}{x}$ is

A. 0

B. 1

C. 2

D. $3/2$

Answer: A

Watch Video Solution

42. $\lim_{x \to 0} \frac{\log\left(x^{x^2} + 2\sqrt{x}\right)}{\tan\sqrt{x}}$ is equal to

A. 0

B. 1

C. e^2

D. 2

Answer: D

Watch Video Solution

43. Let $f: R \to R$ be such that $f(a) = 1, f(a) = 2$. Then $\lim_{x \to 0} \left(\frac{f^2(a+x)}{f(a)} \right)^{1/x}$ is

A. e^2

B. e^4

C. e^{-4}

D. $1/e$

Answer: B

Watch Video Solution

44. The value of $\lim_{n \to \infty} \left(\frac{\sqrt{n^2} + n - 1}{n} \right)^{2\sqrt{n^2+n-1}}$ is

A. e

B. $1/e$

C. e^2

D. e^{-2}

Answer: B

Watch Video Solution

45. If

$$f(n) = \lim_{x \to 0} \left(\frac{1+\sin.(x)}{2}\right)\left(1+\sin.\frac{x}{2^2}\right)\ldots\left(1+\sin.\frac{x}{2^n}\right)^{\frac{1}{x}} \text{ then } \lim_{n \to}$$

A. 1

B. e

C. 0

D. ∞

Answer: B

Watch Video Solution

46. $\lim_{n \to \infty} \left(1 - x + x.\sqrt[n]{e}\right)^n$ is equal to

A. e^x

B. e^{-x}

C. e^{2x}

D. none of these

Answer: A

Watch Video Solution

47. The value of $\lim_{x \to 1} \frac{\sqrt[13]{x} - \sqrt[7]{x}}{\sqrt[5]{x} - \sqrt[3]{x}}$ is

A. $\frac{44}{91}$

B. $\frac{45}{89}$

C. $\frac{45}{89}$

D. $\frac{40}{93}$

Answer: B

Watch Video Solution

48. $\lim_{x \to 1} \frac{\sqrt[13]{x} - \sqrt[7]{x}}{\sqrt[5]{x} - \sqrt[3]{x}}$ is

A. 1

B. -1

C. 0

D. $-1/2$

Answer: D

Watch Video Solution

49. The value of $\lim_{x \to 0} \frac{1 - \cos 2x}{e^{x^2} - e^x + x}$ is

A. 0

B. 2

C. 4

D. 8

Answer: C

Watch Video Solution

50. If $f(a) = \frac{1}{4}$, then $\lim_{h \to 0} \frac{f(a+2h^2) - f(a-2h^2)}{f(a+h^3-h^2) - f(a-h^3+h^2)}$=

A. 0

B. 1

C. -2

D. none of these

Answer: C

Watch Video Solution

51. $\lim_{x \to 0^+} \frac{1}{x\sqrt{x}} \left(a \tan^{-1}. \frac{\sqrt{x}}{a} - b \tan^{-1}. \frac{\sqrt{x}}{b} \right)$ has the value equal to

A. $\frac{a-b}{3}$

B. 0

C. $\frac{(a^2-b^2)}{6a^2b^2}$

D. $\frac{a^2-b^2}{3a^2b^2}$

Answer: D

Watch Video Solution

52. The value of $\lim_{x \to 0} \left(\frac{1+2x}{1+3x} \right)^{\frac{1}{x^2}} . e^{\frac{1}{e^x}}$ is

A. $e\left(\frac{5}{2}\right)$

B. e^2

C. e^{-2}

D. 1

Answer: A

Watch Video Solution

53. If $F: R \to R$ be a differentiable function at x = 0 satisfying $f(0) = 0$ and $f'(0) = 1$, then the value of

$$\lim_{x \to 0} \frac{1}{x} \sum_{n=1}^{\infty} (-1)^n f\left(\frac{x}{n}\right) =$$

A. 0

B. $-\log 2$

C. 1

D. e

Answer: B

Watch Video Solution

54. The value of $\lim_{x \to \frac{3\pi}{4}} \frac{1 + \sqrt[3]{\tan x}}{1 - 2\cos^2 x}$ is

A. $-1/2$

B. $-2/3$

C. $-3/2$

D. $-1/3$

Answer: D

Watch Video Solution

55. Let $g(x) = \frac{(x-1)^n}{\log\cos^m(x-1)}, 0 < x < 2$ m and n integers, $m \neq 0, n > 0$ and. If $\lim_{x \to 1+} g(x) = -1$, then

A. n= 1, m = 1

B. n = 1, m = -1

C. n = 2, m = 2

D. n gt 2, m = n

Answer: C

Watch Video Solution

56. Number of integral values of λ for which $\lim_{x \to 1} \sec^{-1}\left(\frac{\lambda^2}{\log_e x} - \frac{\lambda^2}{x-1}\right)$ does not exist is

A. 1

B. 2

C. 3

D. 4

Answer: C

Watch Video Solution

57. If $\lim_{x \to 0} \frac{e^{ax} - e^x - x}{x^2} = b$ (finite), then

A. a = 2, b = 0

B. a = 0, $b = \frac{3}{2}$

C. $a = 2, b = \frac{3}{2}$

D. a = 0 , b = 2

Answer: C

Watch Video Solution

58. If $\lim_{x \to 0} \frac{x^3}{\sqrt{a+x}(bx - \sin x)} = 1, a > 0$, then $a + b$ is equal to

A. 36

B. 37

C. 38

D. 40

Answer: B

Watch Video Solution

59. If $\lim_{x \to \infty} x \log_e \left(\begin{vmatrix} \alpha/x & 1 & \gamma \\ 0 & 1/x & \beta \\ 1 & 0 & 1/x \end{vmatrix} \right) = -5.$ where α, β, γ are finite real numbers, then

A. $\alpha = 2, \beta = 1, \gamma \in R$

B. $\alpha = 2, \beta = 2, \gamma = 5$

C. $\alpha \in R, \beta = 1, \gamma \in R$

D. $\alpha \in R, \beta = 1, \gamma = 5$

Answer: D

View Text Solution

Multiple Correct Answers Type

1. If $A = \lim_{x \to 0} \frac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)}$ and $B = \lim_{x \to 0} \frac{[|x|]}{x}$, then

A. A = 1

B. A does not exist

C. B = 0

D. B = 1

Answer: B::C

Watch Video Solution

2. If $f(x) = x\left(\frac{e^{|x|+[x]} - 2}{|x| + [x]}\right)$ then (where [.] represent the greatest integer function)

A. $\lim_{x \to 0^+} f(x) = -1$

B. $\lim_{x \to 0^-} f(x) = 0$

C. $\lim_{x \to 0} f(0) = -1$

D. $\lim_{x \to 0} f(x) = 0$

Answer: A::B

Watch Video Solution

3. Assume that $\lim_{\theta \to -1} f(\theta)$ exists and $\frac{\theta^2+\theta-2}{\theta+3} \leq \frac{f(\theta)}{\theta^2} \leq \frac{\theta^2+2\theta-1}{\theta+3}$ holds for certain interval containing the point $\theta = -1$ then $\lim_{\theta \to -1} f(\theta)$

A. is equal to $f(-1)$

B. is equal to 1

C. is non-existent

D. is equal to -1

Answer: A::D

Watch Video Solution

4. Let $f(x) = \lim_{n \to \infty} \frac{\tan^{-1}(\tan x)}{1+(\log_x x)^n}, x \neq (2n+1)\frac{\pi}{2}$ then

A. $\forall 1 < x < \frac{\pi}{2}$, $f(x)$ is an identity function

B. $\forall \frac{\pi}{2} < x < \pi$, the graph of f(x) is a straight line having y intercept of $-\pi$

C. $\forall \frac{\pi}{2} < x < e$, the graph of f(x) is a straight line having y intercept of $-\pi$

D. $\forall x > e$, $f(x)$ is a constant function

Answer: A::C::D

View Text Solution

ComprehensionType

1. Let f(x) be the fourth degree polynomial such that $f'(0) = -6, f(0) = 2$ and $\lim_{x \to 1} \frac{f(x)}{(x-1)^2} = 1$

The value of f(2) is

A. 1

B. 0

C. 2

D. 3

Answer: C

Watch Video Solution

2. Let f(x) be the fourth degree polynomial such that $f'(0) = -6, f(0) = 2$ and $\lim_{x \to 1} \frac{f(x)}{(x-1)^2} = 1$

The value of f'(2) is

A. 4

B. 5

C. 6

D. 7

Answer: C

Watch Video Solution

Multiple Correct Answer Type

1. If $\lim_{x \to \infty} x \log_e \left(\begin{vmatrix} \alpha/x & 1 & \gamma \\ 0 & 1/x & \beta \\ 1 & 0 & 1/x \end{vmatrix} \right) = -5.$ then

A. a = 2

B. b = -4

C. c = 2

D. a + b + c = 8

Answer: A::B::C

Watch Video Solution

2. If $a \in I$, then value of a for which $\lim_{x \to a} \frac{\tan\left([x^2] - [x]^3 \right)}{(x-a)^3}$ exists finitely, is /are

A. 0

B. 1

C. -1

D. 2

Answer: A::B

Watch Video Solution

Comprehension Type

1. $L = \lim_{x \to 0} \frac{\sin(\sin x) - \sin x}{ax^5 + bx^3 + c} = -\frac{1}{12}$

The value/values of a is

A. $\in$ R

B. 2

C. 0

D. 1

Answer: A

Watch Video Solution

2. $L = \lim_{x \to 0} \frac{\sin(\sin x) - \sin x}{ax^5 + bx^3 + c} = -\frac{1}{12}$

The value/values of b is

A. $\in$ R

B. 2

C. 0

D. 1

Answer: B

Watch Video Solution

3. If

$f(x) = \lim_{n \to \infty} \frac{(x^2 + ax + 1) + x^{2n}(2x^2 + x + b)}{1 + x^{2n}}$ and $\lim_{x \to \pm 1} f(x)$

exists, then

The value of a is

A. -1

B. 1

C. 0

D. 2

Answer: B

View Text Solution

4. If $f(x) = \lim_{n \to \infty} \frac{(x^2 + ax + 1) + x^{2n}(2x^2 + x + b)}{1 + x^{2n}}$ and $\lim_{x \to \pm 1} f(x)$ exists, then

The value of b is

A. -1

B. 1

C. 0

D. 2

Answer: C

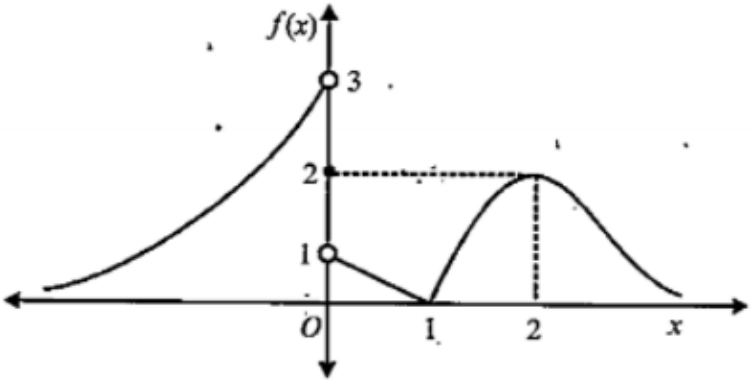
View Text Solution

Question Bank

1. If $\lim_{x \to 0} \frac{1 - \cos x}{e^{ax} - bx - 1}$ exist and is equal to 1, then $\left(a^2 + b^2\right)$ equals

View Text Solution

2. If graph of a function f(x) is shown in the adjacent figure, then $\lim_{x \to 0^-} \left[\frac{4f(x) - 6[f(x)]}{\tan(2f(x) - 6)}\right]$ is equal to



View Text Solution

3. If $\beta_1 + \beta_2 + \beta_3 + ... + \beta_n = p$ and $\lim_{x \to p} \left(\frac{3 - 3\tan\left(\frac{\pi}{4} - x\right)}{\sin 2x} \right) m^n$(where m and n are co-prime) then (m+n) equals (where $\beta_1, \beta_2, ... \beta_n are the values of \beta$)

View Text Solution

4. Let f(x) be a function continuous for all x except at x =0 such that $f'(x) > 0$ for $x > 0$ and $f'(x) < 0$ for $x < 0$. If $\lim_{x \to 0^+} f(x) = 10$, $\lim \left(x \to 0^-\right) f(x) = 15$ and f(0) = 12 then the

value of `(Lim_(x rarr 0)f(x^(3)-x^(2)))/(Lim_(x rarr0)f(2x^(4)-x^(5)))+(Lim_(x rarr0)[f(x^(3)-x^(2))])/(Lim_(x rarr 0) [f(2x^(4)-x^(5))]), is equal to [Note : [k] denote greatest integer less than or equal to k]

View Text Solution

5. Let f(x) $=(\sec x)^{\cos ecx}+(\cot x)^{\sin x}$. Then the value of $\lim_{x \to 0^+} f(x)$ is equal to

View Text Solution

6.

$$\lim_{z \to 0} \left\{ \left\{ \max \left(\left(\sin^{-1} x + \cos^{-1} x \right)^2, \min \left(x^2 + 4x + 7 \right) \right) \right\} . \frac{\sin^{-1} z}{z} \right]$$

is equal to (where [.] denotes greatest integer function)

View Text Solution

7. $\left(\frac{8e^a}{e}\right) \lim_{n \to \infty} \left(\left(\frac{n}{n+1}\right)^a + \sin\left(\frac{1}{n}\right)\right)^n$ where $a \in Q$ (rational number) is equal to

View Text Solution

8. If the value of $\lim_{x \to \infty} \left(\sqrt[3]{x^3 + x^2} - \sqrt[3]{x^3 - x^2}\right.$ can be expressed as (a/b), where $a, b \in N$ and a & b are relatively prime numbers), then (a+b) is

View Text Solution

9. Let f(x) be a polynomial satisfying $\lim_{x \to \infty} \frac{x^2 f(x)}{2x^5 + 3} = 6$ also f(1) = 3, f(3) =7 and f(5) =11, then find the value of ((f(6)+5f(4))/29)

View Text Solution

10. Let $L_1 = \lim_{x \to 0} \left(\cos(\pi x) \frac{e^{\lambda x} - 1}{\pi \sin x}$ and L_(2) = Lim_(x rarr 0) (In (1-x)+sin 2x)/x$If L_1 = L_2$ then the value of $[\lambda]$ is [Note: $[\lambda]$ denotes the largest integer less than or equal to lambda].

View Text Solution

11. If $\lim_{c \to 0} \frac{a \sin c + b \tan c}{c^3} = \frac{3}{2}$, find the value of |a+2b|.

View Text Solution

12. $\lim_{n \to \infty} \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right) \ldots \left(1 - \frac{1}{n^2}\right)$ equals

View Text Solution

13. The value of $\lim_{x \to 0} \frac{\sum_{k=1}^{2016} \left\{ \frac{\sin x}{x} + 2015k \right\}}{2016}$ is [where {x} represents fractional part of x]

View Text Solution

View Text Solution

14. $\lim_{x \to \infty} \frac{30 + 4\sqrt{x} + 7\sqrt[3]{x}}{2 + \sqrt{4x - 7} + \sqrt[3]{6x - 2}}$ equals

View Text Solution

15. If $\lim_{x \to 0} \left(\frac{a \sin x + b \tan x}{x^3} \right) = \frac{3}{2}$ then |a+2b| is equal to

View Text Solution

16. $\lim_{x \to 0} \left(\frac{1}{1 + n^2} + \frac{2}{1 + n^2} + \frac{3}{1 + n^2} + ... + \frac{n}{1 + n^2} \right)$ is equal to

View Text Solution

17. Find the value of $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{k^2 + k - 1}{(k + 1)!}$.

View Text Solution

18. If `Lim_(x rarr 0)(1-cos(1-cos(1-cosx)))/x^(a) is finite then teh maximum value of a is

View Text Solution

19. The value of $\lim_{x \to 0} \left((\tan x)^{\frac{1}{x}} + (1 + \sin x)^{x} \right)$ where $x > 0$ is equal to

View Text Solution

20. Let f(x) be a differentiable function satisfying $f(x + y) = f(x) + f(y) \, \forall x, y \in R$ and $f'(0) = 1$.Then `Lim_(x rarr0) (2^(f(tan^(2)x))-2^(f(sin^(2)x)))/(x^(3) f(sin x)) equals

View Text Solution

21. Let $\lim_{x \to 0} \frac{(8^x - 2^x - 4^x + 1)(128^x - 16^x - 8^x + 1)}{x^4}$ is equal to $(\ln b)^4$, then a/(2b) is

View Text Solution

22. For $n \in N, \leq ta_n = \sum_{k=1}^{n} 2k$and b_(n) = sum_(k=1)^(n)(2k-1). $Then$ Lim_(n rarr oo)(sqrt(a_(n))-sqrt(b_(n)))` is equal to

View Text Solution

23. If `Lim_(x rarr 0) (a sinx + bxe^(x)+3x^(2))/(sin x-2x+tanx) exists and has value equal to L, then the value of (b-L)/a is equal to

View Text Solution

24. If Lim_(x rarr a) (sqrt(x-b)-sqrt(a-b))/(x^(2)-a^(2))(a gt b) = 1/64 and Lim_(x rarr oo) (sqrt(x^(2)+ax)-sqrt(x^(2)+bx)) = 2` then the value of a/b, is

View Text Solution

25. Value of $\lim_{x \to 0} \frac{\sin(2\pi \sec x)}{x^2}$ is

View Text Solution

26. Iflim_(x rarr 0) (k+cos lx)/x^(2) exists and has the value equal to -4 then find l^2

View Text Solution

27. `Lim_(x rarr 0)((3sin x- sin 3x)^(4))/((sec x - cosx)^(6)) is equal to

View Text Solution

28. $\underset{x \to 0}{\text{Lim}} \frac{\tan x\sqrt{\tan x} - \sin x\sqrt{\sin x}}{x^3.\sqrt{x}}$ equals

View Text Solution

29. $\lim\limits_{x \to 1} \dfrac{\sin^2\left(x^3 + x^2 + x - 3\right)}{1 - \cos\left(x^2 - 4x + 3\right)}$ has the value equal to

View Text Solution

30. Let $f(x) = \dfrac{In\left(x^2 + e^x\right)}{In\left(x^4 + e^{2x}\right)}$, then value o f $\lim\limits_{x \to \infty} \left(\dfrac{1}{f(x)}\right)^{f(x)}$ is equal to

View Text Solution