

India's Number 1 Education App

MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

METHODS OF DIFFERENTIATION

Single Correct Answer Type

1. The right hand derivative of $f(x)=[x]tan\pi xatx=7$ is (where [.] denotes the greatest integer function) 0 b. 7π c. -7π d. none of these

A. 0

B. 7π

 $\mathsf{C.} - 7\pi$

D. None of these

Answer: B

2. If
$$f(x-y)=f(x)$$
, $g(y)-f(y)$, $g(x)$ and $g(x-y)=g(x)$, $g(y)+f(x)$, $f(y)$ for all $x\in R$. If right hand derivative at $x=0$ exists for $f(x)$. Find the derivative of $g(x)$ at x =0

$$A. - 1$$

B. 0

C. 1

D. none of these

Answer: B



Watch Video Solution

3. If
$$xe^{xy}-y=\sin^2 x$$
 then $\dfrac{dy}{dx}$ at x = 0 is

A. 0

$$C. -1$$

D. none of these

Answer: B



- **4.** Let f,g and h are differentiable functions. If f(0)=1; g(0)=2; h(0)=3 and the derivatives of their wise products at x=0 are (fg)'(0)=6; (gh)'(0)=4 and (hf)'(0)=5 then compute the value of (fgh)'(0).
 - A. 2
 - B. 4
 - C. 8
 - D. 16

Answer: C



Watch Video Solution

- **5.** If for a continuous function $f,\,f(0)=f(1)=0,\,f'(1)=2andy(x)=f(e^x)e^{f(x)} \ \ , \ \ {\rm then} \ \ y'(0) \ \ {\rm is}$ equal to a. 1 b. 2 c. 0 d. none of these
 - A. 1
 - B. 2
 - C. 0
 - D. none of these

Answer: B



6. The derivative of
$$\cos\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right) - 2\cos^{-1}\sqrt{\frac{1-x}{2}}\right)$$
 w.r.t. x

is

A.
$$1-rac{1}{\sqrt{1-x^2}}$$

B.
$$1 - \frac{1}{\sqrt{1+r^2}}$$

$$\mathsf{C.}\,2 - \frac{1}{\sqrt{1-x^2}}$$

D. 2
$$-\frac{1}{\sqrt{1+x^2}}$$

Answer: A



Watch Video Solution

7. If $y = \frac{x^2}{2} + \frac{1}{2} x \sqrt{x^2 + 1} + \ln \sqrt{x + \sqrt{x^2 + 1}}$ then the value of $xy' + \log y'$ is

A. y

B. 2y

C. 0

D. -2y

Answer: B



Watch Video Solution

8. Let $g(x) = f(x)\sin x$, where f(x) is a twice differentiable function on

$$(\,-\infty,\infty)$$
 such that $f(\,-\pi)=1.\,$ The value of $\left|g^{\,-\pi}
ight|$ equals ______

A. 1

B. 2

 $\mathsf{C.}-2$

D. 0

Answer: C



9. If
$$f(x) = \frac{\log_e(\log_e x)}{\log_e x}$$
, then $f'(x)$ at $x = e$ is

B. 1

 $\mathsf{C}.\,e$

D.1/e

Answer: D



Watch Video Solution

Let
$$g(x) = e^{f(\,x\,)} \, and f(x+1) = x + f(x) \, orall \, x \in R$$

 $2\Big(1+rac{1}{3}+rac{1}{5}+rac{1}{2n-1}\Big)\,n$ 1

A. $2\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}\right)$

B. $2\left(1+\frac{1}{3}+\frac{1}{5}+...+\frac{1}{2n-n}\right)$

 $g(n) \in I^+, then rac{g'\left(n+rac{1}{2}
ight)}{g\left(n+rac{1}{2}
ight)} - rac{g'\left(rac{1}{2}
ight)}{g\left(rac{1}{2}
ight)} = 0$

If

 $2\left(1+rac{1}{2}+rac{1}{3}+\ +rac{1}{n}
ight)$

C. n

D. 1

Answer: C



Watch Video Solution

11.
$$rac{d}{dx} \Bigl[\cos^{-1} \Bigl(x \sqrt{x} - \sqrt{(1-x)ig(1-x^2ig)} \Bigr) \Bigr] =$$

A.
$$\dfrac{1}{\sqrt{1-x^2}}-\dfrac{1}{2\sqrt{x-x^2}}$$

$$\text{B.} \ \frac{-1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$$

C.
$$\dfrac{1}{\sqrt{1-x^2}}+\dfrac{1}{2\sqrt{x-x^2}}$$

D.
$$\frac{1}{\sqrt{1-x^2}}$$

Answer: B



12. If
$$tig(1+x^2ig)=x \ ext{and} \ x^2+t^2=y, \ ext{then at} \ x=2 \ ext{the value of} \ rac{dy}{dx}$$
 is equal to

13. if $x=rac{1+t}{t^3},y=rac{3}{2t^2}+rac{2}{t}$ satisfies $f(x)\cdot\left\{rac{dy}{dx}
ight\}^3=1+rac{dy}{dx}$ then

A.
$$\frac{24}{5}$$
B. $\frac{101}{125}$

D.
$$\frac{358}{125}$$

c. $\frac{488}{155}$

Answer: C



f(x) is:

B.
$$\frac{x^2}{1 \perp X^2}$$

$$\mathsf{C.}\,x+x+\frac{1}{x}$$

D.
$$x-\frac{1}{x}$$

Answer: A



Watch Video Solution

- **14.** Let $y=x^3-8x+7$ and x=f(t). If $\frac{dy}{dt}=2$ and x=3 at t=0, then $\frac{dx}{dt}$ at t=0 is given by 1 (b) $\frac{19}{2}$ (c) $\frac{2}{19}$ (d) none of these
 - A. 1
 - B. $\frac{19}{2}$
 - c. $\frac{2}{19}$
 - D. None of these

Answer: C



15. If
$$x = \sec \theta - \cos \theta$$
 and $y = \sec^n \theta - \cos^n \theta$ then show that $(du)^2$

$$z^2$$
 .

 $ig(x^2+4ig)igg(rac{dy}{dx}igg)^2=n^2ig(y^2+4ig)$



A. 8



B. 16



D. 49

Answer: C



Watch Video Solution

16. The derivative of the function represented parametrically as x=2t=|t|, $y=t^3+t^2|t|a$ = 0 is a. -1 b. 1 c. 0 d. does not exist

A.
$$-1$$

B. 0

D. does not exist

Answer: B



Watch Video Solution

17. If
$$y= an^{-1}igg(rac{u}{\sqrt{1-u^2}}igg)$$
 and $x=\sec^{-1}igg(rac{1}{2u^2-1}igg)$, $u\inigg(0,rac{1}{\sqrt{2}}igg)\cupigg(rac{1}{\sqrt{2}},1igg)$, prove that $2rac{dy}{dx}+1=0$.

A. y

B. xy

C. 0

D. 1

Answer: C



18. The differential coefficient of
$$\sin^{-1}\!\left(\frac{5\cos x - 4s \in x}{\sqrt{41}}\right)$$
 is -2 b. -1 c.

$$A.-2$$

$$\mathsf{B.}-1$$

Answer: D



Watch Video Solution

19. $xy=(x+y)6nandrac{dy}{dx}=rac{y}{x}the \ \cap \ = \ 1$ b.2 c. 3 d. 4

Answer: B



Watch Video Solution

- **20.** If $x+y=3e^2therac{d}{dx}(x^y)=0f\,\, ext{or}\,\,x=\,e^2\,\mathsf{b.}\,e^e\,\mathsf{c.}\,e\,\mathsf{d.}\,2e^2$
 - A. e
 - $B.e^2$
 - $\mathsf{C}.\,e^e$
 - D. $2e^2$

Answer: B



21. If
$$f(x)=(x-1)^{100}(x-2)^{2\,(\,99\,)}\,(x-3)^{3\,(\,98\,)}\dots(x-100)^{100},$$
 then the value of $\frac{f'(101)}{f(101)}$ is

C. 3030

D. 1250

B. 2575

Answer: A



View Text Solution

22. The function $f\!:\!R\stackrel{
ightarrow}{R}$ satisfies $f\!\left(x^2
ight)\!\stackrel{\cdot}{f}^x=f'\!\left(x
ight)\!\stackrel{\cdot}{f}'\!\left(x^2
ight)$ for all real x-

Given that f(1)=1 and $f^1=8$, then the value of $f^{\,\prime}(1)+f^1$ is 2 b. 4 c.

A. 2

6 d.8

B. 4

D. 8

Answer: C



Watch Video Solution

23. The second derivative of a single valued function parametrically represented by $x=\phi(t)$ and $y=\psi(t)$ (where $\phi(t)$ and $\psi(t)$ are different function and $\phi'(t)\neq 0$) is given by

$$\begin{array}{l} \mathsf{A.} \ \frac{d^2y}{dx^2} = \frac{\left(\frac{dx}{dt}\right)\left(\frac{d^2y}{dt^2}\right) - \left(\frac{d^2x}{dt^2}\right)\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)^3} \\ \mathsf{B.} \ \frac{d^2y}{dx^2} = \frac{\left(\frac{dx}{dt}\right)\left(\frac{d^2y}{dt^2}\right) - \left(\frac{d^2x}{dt^2}\right)\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)^2} \\ \mathsf{C.} \ \frac{d^2y}{dx^2} = \frac{\left(\frac{d^2x}{dt}\right)\left(\frac{dy}{dt}\right) - \frac{dx}{dt}\left(\frac{d^2y}{dt^2}\right)}{\left(\frac{dx}{dt}\right)^3} \\ \mathsf{D.} \ \frac{d^2y}{dx^2} = \frac{\left(\frac{d^2x}{dt}\right)\left(\frac{dy}{dt}\right) - \left(\frac{d^2y}{dt^2}\right)\left(\frac{dy}{dt}\right)}{\left(\frac{dy}{dt}\right)^3} \end{array}$$

Answer: A



Watch Video Solution

24. For the curve $\sin x + \sin y = 1$ lying in first quadrant. If $\lim_{x \to 0} x^{\alpha} \frac{d^2y}{dx^2}$ exists and non-zero than $2\alpha =$

A. 3

B. 4

C. 5

D. 1

Answer: A



$$\mathsf{B.}-\frac{1}{2}$$

A. 1

A. $7 \left(\frac{d^2 y}{dx^2} \right)^2$

 $\mathrm{B.}\,5{\left(\frac{d^2y}{dx^2}\right)^2}$

 $\mathsf{C.}\, 3 \bigg(\frac{d^2 y}{dx^2} \bigg)^2$

D. $\left(\frac{d^2y}{dx^2}\right)^2$

Watch Video Solution

 $f^{-1} = \text{inverse of } y = f(x)$

26. If f(1) = 3, f'(1) = 2, f''(1) = 4, then $(f^-)''(3) = 6$ (where

Answer: C

 $\mathsf{C}.-2$

Answer: B





27. If the third derivative of
$$\frac{x^4}{(x-1)(x-2)}$$
 is $\frac{-12k}{(x-2)^4}+\frac{6}{(x-1)^4},$ then the value of k is

B. 4

Answer: C



Watch Video Solution

28. $(a+bx)e^{rac{y}{x}}=x$, Prove that $x^3rac{d^2y}{dx^2}=\left(xrac{dy}{dx}-y
ight)^2$

A.
$$\left(\frac{dy}{dx} + x\right)^2$$

D.
$$\left(xrac{dy}{dx}+y
ight)^2$$

B. $\left(x\frac{dy}{dx} - y\right)^2$

C. $\left(\frac{dy}{dx} - y\right)^2$

Answer: B

Watch Video Solution

A. $\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} + \frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}}$

B. $\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} + \frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}}$

C. $\frac{2}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} + \frac{2}{\left(\frac{d^2y}{dx^2}\right)^{2/3}}$

29. If $R=rac{\left[1+\left(rac{dy}{dx}
ight)^2
ight]^{-3/2}}{d^2y}$, then $R^{2/3}$ can be put in the form of

 $rac{1}{\left(rac{d^2y}{J_{-}^2}
ight)^{2/3}} + rac{1}{\left(rac{d^2x}{J_{-}^2}
ight)^{2/3}} \hspace{1.5cm} ext{b.} \hspace{1.5cm} rac{1}{\left(rac{d^2y}{dx^2}
ight)^{2/3}} - rac{1}{\left(rac{d^2x}{dy^2}
ight)^{2/3}}$

 $rac{2}{\left(rac{d^2y}{12}
ight)^{2/3}}+rac{2}{\left(rac{d^2x}{2}
ight)^{2/3}}\, {\sf d.}\, rac{1}{\left(rac{d^2y}{2}
ight)^{2/3}}rac{1}{\left(rac{d^2x}{2}
ight)^{2/3}}$

c.

A. 1/2

B.5/2

C. - 3/2

D. 2

Answer: C



D. $\frac{1}{\left(rac{d^2y}{dx^2}
ight)^{2\,/\,3}}$. $\frac{1}{\left(rac{d^2y}{dy^2}
ight)^{2\,/\,3}}$

30. $x = 2\cos t - \cos 2t, y = 2\sin t - \sin 2t$



31. If
$$y^3-y=2x$$
, $thenigg(x^2-rac{1}{27}igg)rac{d^2y}{dx^2}+xrac{dy}{d}=\ y$ b. $rac{y}{3}$ c. $rac{y}{9}$ d. $rac{y}{27}$

A. y

B. $\frac{y}{3}$

C.
$$\frac{y}{9}$$
D. $\frac{y}{27}$

Answer: C



Watch Video Solution

32. Let
$$f(x)=rac{g(x)}{r}whenx
eq 0$$
 and $f(0)=0.$

If

$$g(0) = g^{\,\prime}(0) = 0$$
and $g^0 = 17$ then $f(0) = \,3/4$ b. $-1/2$ c. $17/3$ d. $17/2$

A. 3/4

B. - 1/2

C.17/3

D. 17/2

Answer: D

33. Let
$$f:(-\infty,\infty)\overrightarrow{0,\infty}$$
 be a continuous function such that $f(x+y)=f(x)+f(y)+f(x)f(y),\ \forall x\in R.$ Also $f'(0)=1.$ Then $[f(2)]$ equal ([.] represents the greatest integer function) 5 b. 6 c. 7 d. 8

B. 6

D. 8

C. 7

Answer: B



34.

Watch Video Solution

If f(3)=4 and f(5)=52, then f'(x) is equal to

Let $f \colon R o R$ be a function satisfying $f(x+y) = f(x) = \lambda xy + 3x^2y^2$ for all $x, y \in R$

A.
$$10x$$

B. - 10x

 $\mathsf{C}.\,20x$

D. 128x

Answer: B



Watch Video Solution

$$f(xy)=f(x)f(y)-f(x)-f(y)+2.$$
 If differentiable on $R-\{0\}andf(2)=5, f'(x)=rac{f(x)-1}{x}\dot{\lambda}then\lambda=2'f(1)$ b. $3f'(1)$ c. $rac{1}{2}f'(1)$ d. $f'(1)$

the

equation

35. A function $f: R\overline{1, \infty}$ satisfies

B.
$$3f'(1)$$

A. 2f'(1)

C.
$$\frac{1}{2}f'(1)$$

D.
$$f'(1)$$

Answer: D



Watch Video Solution

36. Let $\frac{f(x+y)-f(x)}{2}=\frac{f(y)-a}{2}+xy$ for all real x and y. If f(x) is differentiable and f'(0) exists for all real permissible value of a and is equal to $\sqrt{5a-1-a^2}$. Then f(x) is positive for all real x f(x) is negative for all real x f(x)=0 has real roots Nothing can be said about the sign of f(x)

- A. f(x) is positive for all real x
- B. f(x) is negative for all real x
- C. f(x)=0 has real roots
- D. nothing can be said about the sign of f(x)

Answer: A



37. Let f(3)=4 and f'(3)=5. Then $\lim_{x\to 3}\ [f(x)]$ (where [.] denotes the greatest integer function) is

A. 3

B. 4

C. 5

D. non-existent

Answer: D



38. Let f(x) be a function which is differentiable any number of times and $fig(2x^2-1ig)=2x^3f(x),\ orall x\in R.$ Then $f^{\,(\,2010\,)}(0)=$ (Here

$$f^{\,(\,n\,)}\,(x)=n^{th}$$
 order derivative of f at x)

A.
$$-1$$

B. 1

C. 0

D. data is insufficient

Answer: C



View Text Solution

39. If
$$f(x) = \begin{vmatrix} (x-a)^4 & (x-a)^3 & 1 \\ (x-b)^4 & (x-b)^3 & 1 \\ (x-c)^4 & (x-c)^3 & 1 \end{vmatrix}$$
 $f'(x) = \lambda \begin{vmatrix} (x-a)^4 & (x-a)^3 & 1 \\ (x-b)^4 & (x-b)^3 & 1 \\ (x-c)^4 & (x-c)^3 & 1 \end{vmatrix}$. Find the value of λ

$$J(x) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$(x-b)^3$$
 1

then

$$(x-c)^3$$





Answer: C



40. Suppose
$$\begin{vmatrix} f'(x) & f(x) \\ f''(x) & f'(x) \end{vmatrix} = 0$$
 where f(x) is continuously differentiable function with $f'(x) \neq 0$ and satisfies f(0) = 1 and f'(0) = 2 then $\lim_{x \to 0} \frac{f(x) - 1}{x}$ is

D. 0

Answer: B



Watch Video Solution

41. A nonzero polynomial with real coefficient has the property that $f(x)=f'(x)\dot{f}'(x)$. If a is the leading coefficient of f(x), then the value of 1/2a) is____

A.
$$1/3$$

B. 6

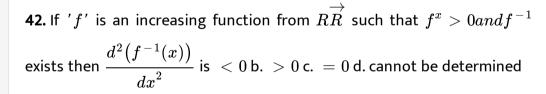
C. 12

D.1/18

Answer: D



Watch Video Solution



A. It 0

B. gt 0

C. 0

D. cannot be determined

Answer: A

43. Vertices of a variable acute angled triangle ABC lies on a fixed circle.

Also a, b, c and A, B, C are lengths of sides and angles of triangle ABC, respectively. If x_1, x_2 and x_3 are distances of orthocentre from A, B and C, respectively, then the maximum value of $\left(\frac{dx_1}{da} + \frac{dx_2}{db} + \frac{dx_3}{dc}\right)$ is

A.
$$-\sqrt{3}$$

$$\mathsf{B.} - 3\sqrt{3}$$

C.
$$\sqrt{3}$$

D.
$$3\sqrt{3}$$

Answer: B



View Text Solution

44. In a question a student was given to find the derivative of the product of two functions fandg. The student y misstate thought (fg)'=f'g'

for his question $f(x)=x^3$ and he got the correct answer. Given that g(4)=1. Then which of the following is false? $g(5)=rac{1}{8}$ b. f'(x)<0 c.

f(0) < 0 d. none of these

A.
$$g(5) = \frac{1}{8}$$

$$\mathsf{B}.\,f'(x)<0$$

C.
$$f(0) < 0$$

D. None of these

Answer: A



45.
$$f$$
 is a strictly monotonic differentiable function with $f'(x)=rac{1}{\sqrt{1+x^3}}$. If g is the inverse of f , then $g^x=rac{2x^2}{2\sqrt{1+x^3}}$ b.

$$rac{2g^2(x)}{2\sqrt{1+g^2(x)}}$$
 c. $rac{3}{2}g^2(x)$ d. $rac{x^2}{\sqrt{1+x^3}}$

A.
$$\dfrac{3x^2}{2\sqrt{1+x^3}}$$

A. 6

is 6 b. 9 c. 12 d. 15

C. 12

Watch Video Solution

 $\text{B.}\ \frac{3g^2(x)}{2\sqrt{1+g^2(x)}}$

 $\mathsf{C.}\ \frac{3}{2}g^2(x)$

D. $\dfrac{x^2}{\sqrt{1+x^3}}$

Answer: C

46. Suppose $f \colon \stackrel{\longrightarrow}{RR}^+$ be a differentiable function such that

 $3f(x+y)=f(x)f(y)\,orall x,y\in R$ with f(1)=6. Then the value of f(2)

- D. 15

Multiple Correct Answer Type

1. If
$$y=\cos^{-1}\sqrt{rac{\sqrt{1+x^2+1}}{2\sqrt{1+x^2}}}, thenrac{dy}{dx}isequa< o$$
 $rac{1}{2(1+x^2)}, x\in R$

(b)
$$\dfrac{1}{2(1+x^2)}, x>0$$
 $\dfrac{-1}{2(1+x^2)}, x<0$ (d) $\dfrac{1}{2(1+x^2)}, x<0$

A.
$$\dfrac{1}{2(1+x^2)}, x \in R$$

B.
$$\frac{1}{2(1+x^2)}, x>0$$

C.
$$\frac{1}{2(1+x^2)}$$
, $x < 0$

D.
$$\dfrac{-1}{2(1+x^2)}, x<0$$

Answer: B::D



Suppose that f(x) is differentiable invertible function 2.

$$f'(x)
eq 0$$
 $and h'(x) = f(x)$. Given that $f(1) = f'(1) = 1, h(1) = 0$

and g(x) is inverse of f(x) . Let $G(x) = x^2 g(x) - x h(g(x)) \, orall x \in R$ Which of the following is/are correct? $G^{\prime}(1)=2$ b. $G^{\prime}(1)=3$ c. $G^{1}=2$

$$\mathsf{d.}\,G^1=3$$

Answer: A::D



Watch Video Solution

given parametrically 3. curve by $x = t + t^3$ and $y = t^2$, where $t \in R$. For what vlaue(s) of t is

C. 3

B. 2

A. $\frac{1}{3}$

D. 1

Answer: A::D



Watch Video Solution

4. If
$$y = x^{(\log x)^{\log(\log x)}}$$
 , then $\frac{dy}{dx}$ is

A.
$$rac{y}{x} \Bigl(\ln x^{\log x - 1} \Bigr) + 2 \ln x \ln (\ln x)$$

$$\frac{x}{x}$$
 (mx) + 2 mx m(mx)

B.
$$\frac{y}{x}(\log x)^{\log(\log x)}\left(2\log(\log x)+1\right)$$

C.
$$\left[(\ln x)^2 + 2 \ln(\ln x)
ight]$$
D. $rac{y}{x} rac{\log y}{\log x} (2 \log(\log x) + 1)$

Answer: B::D



5. If $y=e^{-x}\cos x$ and $y_n+k_ny=0$ where $yn=\displaystyle\frac{d^ny}{dx^n}$ and k_n are constant $n\in N$ then

A.
$$k_4=4$$

B.
$$k_8 = -16$$

$$C. k_{12} = 20$$

D.
$$k_{16} = -24$$

Answer: A::B



Watch Video Solution

6. If y=y(x) and it follows the relation $e^{xy}+y\cos x=2$, then find (i) y'(0) and (ii) y(0).

A.
$$y'(0) = -1$$

B.
$$y''(0) = 2$$

$$C. y'(0) = 3/2$$

D.
$$y''(0) = -2$$

Answer: A::B



Watch Video Solution

7. A twice differentiable function f(x) is defined for all real numbers and

satisfies the following conditions f(0) = 2; f'(0) - 5 and f(0) = 3.

The function g(x) is defined by $g(x) = e^{ax} + f(x) \, orall \, x \in R$, where 'a' is

any constant If $g^{\,\prime}(0)+g(0)=0.$ Find the value(s) of 'a'

- A. 1
- B. 1
- C. 2

D.-2

Answer: A::D

