



## MATHS

### BOOKS - CENGAGE MATHS (HINGLISH)

## MONOTONICITY AND MAXIMA MINIMA OF FUNCTIONS

### Examples

1. Check the nature of the following differentiable functions (i)  $f(x) = e^x + \sin x$ ,  $x \in \mathbb{R}^+$  (ii)  $f(x) = \sin x + \tan x - 2x$ ,  $x \in (0, \pi/2)$

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2. Prove that the function  $f(x) = (\log)_e(x^2 + 1) - e^{-x} + 1$  is strictly increasing  $\forall x \in \mathbb{R}$ .

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3. Find the least value of  $k$  for which the function  $x^2 + kx + 1$  is an increasing function in the interval  $\mathbb{R}$

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4. Find the values of  $a$  if  $f(x) = 2e^x - ae^{-x} + (2a + 1)x - 3$  is increasing for all values of  $x$ .

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5. If  $f(x)$  and  $g(x)$  are differentiable and increasing functions then which of the following functions always increases?

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6.

Let

$$g(x) = (f(x))^3 - 3(f(x))^2 + 4f(x) + 5x + 3 \sin x + 4 \cos x \quad \forall x \in \mathbb{R}.$$

Then prove that  $g$  is increasing whenever  $f$  is increasing.



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7. Find the complete set of values of  $\alpha$  for which the function

$$f(x) = \begin{cases} x + 1, & x < 1 \\ \alpha, & x = 1 \\ x^2 - x + 3, & x > 1 \end{cases}$$

is strictly increasing

$$x^2 - x + 3, \quad x > 1$$



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8. Prove that function  $f(x) = \begin{cases} -2x^3 + 3x^2 - 6x + 5, & x < 0 \\ -x^2 - x + 1, & x \geq 0 \end{cases}$  is decreasing for all  $x$ .



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9. If  $f, g, h(x)$  is an increasing function, then which of the following is not possible?  $f(x), g(x), \text{ and } h(x)$  are increasing  $f(x) \text{ and } g(x)$  are decreasing and  $h(x)$  is increasing  $f(x), g(x), \text{ and } h(x)$  are decreasing

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10. Let  $f(x) \text{ and } g(x)$  be two continuous functions defined from  $\mathbb{R} \rightarrow \mathbb{R}$ , such that  $f(x_1) > f(x_2) \text{ and } g(x_1) < g(x_2)$

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11. Let  $f: [0, \infty) \rightarrow [0, \infty)$  and  $g: [0, \infty) \rightarrow [0, \infty)$  be non-increasing and non-decreasing functions, respectively, and  $h(x) = g(f(x))$ . If  $f$  and  $g$  are differentiable functions,  $h(x) = g(f(x))$ . If  $f$  and  $g$  are differentiable for all points in their respective domains and  $h(0) = 0$ , then show  $h(x)$  is always, identically zero.

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12. Find the values of  $p$  if  $f(x) = \cos x - 2px$  is invertible.

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13. Find the critical points(s) and stationary points (s) of the function

$$f(x) = (x - 2)^{2/3}(2x + 1)$$

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14. Separate the intervals of monotonicity of the following function:

(i)  $f(x) = 3x^4 - 8x^3 - 6x^2 + 24x + 7$

(ii)  $F(x) = -\sin^3 x + 3\sin^2 x + 5, x \in (-\pi/2, \pi/2)$

(iii)  $f(x) = (2^x - 1)(2^x - 2)^2$

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15. अंतराल ज्ञात कीजिए जिन पर

$$f(x) = \frac{4 \sin x - 2x - x \cos x}{2 + \cos x}$$

से प्रदत्त फलन  $f(x)$  निरंतर वर्धमान (ii) निरंतर ह्रासमान है।



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16. Find the interval of monotonicity of the function  $f(x) = |x - 1|x^2$ .



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17. Find the intervals of decrease and increase for the function

$$f(x) = \cos\left(\frac{\pi}{x}\right)$$



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18. Let  $g(x) = f(x) + f(1 - x)$  and  $f^x > 0 \forall x \in (0, 1)$ . Find the intervals of increase and decrease of  $g(x)$ .



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19. Find the range of the function  $f(x) = x \sin x - \frac{1}{2} \sin^2 x$  for  $x \in \left(0, \frac{\pi}{2}\right)$



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20. Find the range of  $f(x) = \frac{1}{\pi} \sin^{-1} x + \tan^{-1} x + \frac{x+1}{x^2+2x+5}$



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21. Find the number of roots of the equation

$$\log_e(1+x) - \frac{\tan^{-1} x}{1+x} = 0$$



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22. Find the number of roots of the function

$$f(x) = \frac{1}{(x+1)^3} - 3x + \sin x.$$

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23. Prove that  $\log_e(1+x) < xf$  or  $x > 0$

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24. Let  $f$  and  $g$  be differentiable on  $\mathbb{R}$  and suppose  $f(0) = g(0)$  and  $f'(x) \leq g'(x)$  for all  $x \geq 0$ . Then show that  $f(x) \leq g(x)$  for all  $x \geq 0$ .

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25. Show that  $1 + \xi n(x + \sqrt{x^2 + 1}) \geq \sqrt{1 + x^2}$  for all  $x \geq 0$ .

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26. If  $a, b > 0$  and  $0$

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27. Prove that  $|\cos \alpha - \cos \beta| \leq |\alpha - \beta|$

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28. If  $P(1) = 0$  and  $\frac{dP(x)}{dx}$ ,  $\sin x + 2x \geq \frac{3x(x+1)}{\pi}$ . Explain the identity, if any, used in the proof.

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29. For  $0 < x < \frac{\pi}{2}$ , prove that  $x > \sin x$  and hence  $\cos(\sin x) > \sin(\cos x)$

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30. Prove that  $e^x \geq 1 + x$  and hence  $e^x + \sqrt{1 + e^{2x}} \geq (1 + x) + \sqrt{2 + 2x + x^2} \forall x \in \mathbb{R}$



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31. Separate the intervals of concavity of the following functions

(i)  $f(x) = \sin^{-1} x$ , (ii)  $f(x) = x + \sin x$



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32. If graph of the function  $f(x) = 3x^4 + 2x^3 + ax^2 - x + 2$  is concave upward for all real  $x$ , then find values of  $a$ ,



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33. Prove that for any two numbers  $x_1$  and  $x_2$

$$\frac{2e^{x_1} + e^{x_2}}{3} > e^{\frac{2x_1 + x_2}{3}}$$



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34. Prove that in triangle ABC,  $\sin A + \sin B + \sin C \leq \frac{3\sqrt{3}}{2}$

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35. Prove that  $a_1^m + a_2^m + \dots + a_n^m < \frac{a_1 + a_2 + \dots + a_n}{(n)^m}$

If  $0 < m < 1$  and  $a_i > 0$  for all  $i$ .

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36. Find the points of inflection for  $f(x) = \sin x$   $f(x) = 3x^4 - 4x^3$

$f(x) = x^{\frac{1}{3}}$

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37. Find the coordinates of the point of inflection of the curve  $f(x) = e^{-x^2}$

$x^2$

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38. The function  $y = \frac{ax + b}{x - 1}(x - 4)$  has turning point at  $P(2, -1)$ . Then find the values of  $a$  and  $b$ .

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39. How many critical points does the function  $f(x) = (x-1)|x-3| - 4x + 12$  have? How many of these are points of local extrema?

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40. Discuss the extremum of the function  $f(x) = (x-1)(x-2)(x-3)$ . How many critical points does  $f(x)$  have?

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41. Find the points of extrema of the function  $f(x) = 2 \sec x + 3 \operatorname{cosec} x$

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42. Prove that for all  $a, b$  in the function  $f(x) = 3x^4 - 4x^3 + 6x^2 + ax + b$  has exactly one extremum.

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43. The function  $f(x) = (x^2 - 4)^n(x^2 - x + 1)$ ,  $n \in \mathbb{N}$ , assumes a local minimum value at  $x = 2$ . Then find the possible values of  $n$

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44. Test  $f(x) = \{x\}$  for the existence of a local maximum and minimum at  $x=1$ , where  $\{ \cdot \}$  represents the fractional part function.

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45. Let  $f(x) = \begin{cases} x^3 + x^2 + 10x, & x < 0 \\ -3 \sin x, & x \geq 0 \end{cases}$ . Investigate  $x=0$  for local maxima/minima.

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46. Let  $f(x) = \frac{a}{x} + x^2$ . If it has a maximum at  $x = -3$ , then find the value of  $a$ .

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47. Discuss the extremum of  $f(x) = \sin x(1 + \cos x)$ ,  $x \in \left(0, \frac{\pi}{2}\right)$

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48. Find the points at which the function  $f$  given by  $f(x) = (x - 2)^4(x + 1)^3$  has local maxima and local minima and points

of inflexion.



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49. Discuss the extremum of  $f(x) = 40(3x^4 - 8x^3 - 18x^2 + 60)$ .



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50. Discuss extrema of the function

$$f(x) = \int_1^x 2(x-1)(x-2)^3 + 3(x-1)^2(x-2)^2 dx$$



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51. Let  $f(x) = \sin^3 x + \lambda \sin^2 x$ ,  $-\pi/2$



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52. Let  $f(x) = \sin^3 x + \lambda \sin^2 x$ ,  $-\pi/2$



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53. Discuss the extrema of  $f(x) = \frac{x}{1 + x \tan x}$ ,  $x \in \left(0, \frac{\pi}{2}\right)$



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54. Find the point of extremum of the function  $f(x) = \frac{12}{x^x}$ . Draw the graph of the function and hence find the range of function. Also determine which is bigger,  $\frac{1}{(\pi)^{\frac{1}{e}}}$  or  $\frac{1}{(e)^{\frac{1}{\pi}}}$ ?



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55. Discuss the extrema of the following functions

(i)  $f(x) = |x|$ , (ii)  $f(x) = e^{-|x|}$ , (iii)  $f(x) = x^{2/3}$



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56. If  $f(x) = \begin{cases} x^2, & x \leq 0 \\ \end{cases}$ . Investigate the functions at  $x$  for maxima/minima

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57. Discuss the extremum of  $f(x) = 2x + 3x^{\frac{2}{3}}$

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58.  $f(x) = \begin{cases} \frac{\cos(\pi x)}{2}, & x > 0 \\ x + a, & x \leq 0 \end{cases}$  Find the values of  $a$  if  $x = 0$  is a point of maxima.

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59. let  $f(x) = -x^3 + \frac{b^3 - b^2 + b - 1}{b^2 + 3b + 2}$  if  $x$  is 0 to 1 and  $f(x) = 2x - 3$  if  $x$  is 1 to 3. All possible real values of  $b$  such that  $f(x)$  has the smallest value at  $x = 1$  are



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60.  $f(x) = |ax - b| + c|x| \forall x \in (-\infty, \infty)$ , where  $a > 0, b > 0, c > 0$ . Find the condition if  $f(x)$  attains the minimum value only at one point.



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61. Let  $f(x) = 2x^3 = 9x^2 + 12x + 6$ . Discuss the global maxima and minima of  $f(x) \in [0, 2]$  and  $(1, 3)$  and, hence, find the range of  $f(x)$  for corresponding intervals.



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62. Find the absolute maximum and absolute minimum values of  $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$  in  $[0, 3]$



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63. Find the range of the function  $f(x) = 4 \cos^3 x - 8 \cos(2)x + 1$ .

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64. Find the range of the function  $f(x) = 2\sqrt{x-2} + \sqrt{4-x}$

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65. A function  $y=f(x)$  is represented parametrically as following

$$x = \phi(t) = t^5 - 20t + 7$$

$$y = \psi(t) = 4t^3 - 3t^2 - 18t + 3$$

where  $t$  in  $[-2,2]$

Find the intervals of monotonicity and also find the points of extrema. Also find the range of function.

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66. Find how many roots of the equations  $x^4 + 2x^2 - 8x + 3 = 0$ .



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67. Draw the graph of  $y = \frac{x^2}{\sqrt{x+1}}$



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68. Draw the graph of  $y = xe^x$ . Also

(i) Find the range of the function

(ii) Find the point of inflection.

(iii) Find the values of  $k$  if  $1xe^x = k$  has two distinct real roots



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69. Minimum integral value of  $k$  for which the equation  $e^x = kx^2$  has exactly three real distinct solution,

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70. Draw the graph of  $f(x) = \log_e(\sqrt{1-x^2})$ . Find the range of the function. Also find the values of  $k$  if  $(x)$  has two distinct real roots.

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71. Draw the graph of  $f(x) = \frac{x^2 - 5x + 6}{x^2 - x}$

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72. Find the value of  $a$  if  $x^3 - 3x + a = 0$  has three distinct real roots.

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73. Prove that there exist exactly two non-similar isosceles triangles  $ABC$  such that  $\tan A + \tan B + \tan C = 100$ .

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74. If  $t$  is a real number satisfying the equation  $2t^3 - 9t^2 + 30 - a = 0$ , then find the values of the parameter  $a$  for which the equation  $x + \frac{1}{x} = t$  gives six real and distinct values of  $x$ .

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75. The tangent to the parabola  $y = x^2$  has been drawn so that the abscissa  $x_0$  of the point of tangency belongs to the interval  $[1, 2]$ . Find  $x_0$  for which the triangle bounded by the tangent, the axis of ordinates, and the straight line  $y = x^2$  has the greatest area.

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76. Find the point  $(\alpha, \beta)$  on the ellipse  $4x^2 + 3y^2 = 12$ , in the first quadrant, so that the area enclosed by the lines  $y = x, y = \beta, x = \alpha$ , and the x-axis is maximum.



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77.  $LL'$  is the latus sectum of the parabola  $y^2 = 4ax$  and  $PP'$  is a double ordinate drawn between the vertex and the latus rectum. Show that the area of the trapezium  $PP'LL'$  is maximum when the distance  $PP'$  from the vertex is  $a/9$ .



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78. Find the points on the curve  $5x^2 - 8xy + 5y^2 = 4$  whose distance from the origin is maximum or minimum.



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79. Rectangles are inscribed inside a semi-circle of radius  $r$ . Find the rectangle with maximum area.



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**80.** A running track of 440 ft is to be laid out enclosing a football field, the shape of which is a rectangle with a semi-circle at each end. If the area of the rectangular portion is to be maximum, then find the length of its sides.



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**81.** If the sum of the lengths of the hypotenuse and another side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between these sides is  $\frac{\pi}{3}$ .



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**82.** The tangent to the parabola  $y = x^2$  has been drawn so that the abscissa  $x_0$  of the point of tangency belongs to the interval  $[1,2]$ . Find  $x_0$  for which the triangle bounded by the tangent, the axis of ordinates, and the straight line  $y = x^2$  has the greatest area.



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83. Find the point  $(\alpha, \beta)$  on the ellipse  $4x^2 + 3y^2 = 12$ , in the first quadrant, so that the area enclosed by the lines  $y = x, y = \beta, x = \alpha$ , and the x-axis is maximum.

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84.  $LL'$  is the latus rectum of the parabola  $y^2 = 4ax$  and  $PP'$  is a double ordinate drawn between the vertex and the latus rectum. Show that the area of the trapezium  $PP'LL'$  is maximum when the distance  $PP'$  from the vertex is  $a/9$ .

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85. Find the points on the curve  $5x^2 - 8xy + 5y^2 = 4$  whose distance from the origin is maximum or minimum.

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**86.** A sheet of area  $40m^2$  is used to make an open tank with square base. Find the dimensions of the base such that the volume of this tank is maximum.

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**87.** A sheet of area  $40m^2$  is used to make an open tank with square base. Find the dimensions of the base such that the volume of this tank is maximum.

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**88.** The latest edge of a regular hexagonal pyramid is  $1cm$ . If the volume is maximum, then find its height.

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89. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius  $R$  is  $\frac{2R}{\sqrt{3}}$ .

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90. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height  $h$  and semi vertical angle is one-third that of the cone and the greatest volume of cylinder is  $\frac{4}{27}\pi h^3 \tan^2 \alpha$ .

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91. Find the possible values of  $a$  such that  $f(x) = e^{2x} - (a + 1)e^x + 2x$  is monotonically increasing for  $x \in R$ .

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92.

Of

$$f(x) = \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right) - \log(x^2 + x + 1) (\lambda^2 - 5\lambda + 3)x + 10$$

is a decreasing function for all  $x \in R$ , find the permissible values of  $\lambda$ .


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93. Let  $f(x) = x^3 + ax^2 + bx + 5$  be an increasing function on the set  $R$ . Then,  $a$  and  $b$  satisfy (a)  $a^2 - 3b - 15 > 0$  (b)  $a^2 - 3b + 15 > 0$  (c)  $a^2 - 3b + 15 < 0$  (d)  $a > 0$  and  $b > 0$


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94. Prove that  $\left[ \lim_{x \rightarrow 0} \frac{\sin x \cdot \tan x}{x^2} \right] = 1$ , where  $[.]$  represents greatest integer function.


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95. Using the relation  $\sqrt{2(1-\cos x)}$



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96. If  $f'(x) > 0$ ,  $f(x) > 0 \forall x \in (0, 1)$  and  $f(0) = 0$ ,  $f(1) = 1$ , then prove that  $f(x)f^{-1}(x)$



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97. Discuss the monotonicity of  $Q(x)$ , where  $Q(x) = 2f\left(\frac{x^2}{2}\right) + f(6 - x^2) \forall x \in R$  It is given that  $f' > 0 \forall x \in R$ . Find also the point of maxima and minima of  $Q(x)$ .



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98. Prove that  $\left(\frac{\tan^{-1}(1)}{e}\right)^2 + \frac{2e}{\sqrt{e^2 + 1}}$



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99. Prove that  $\sin^2 \theta$

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100.  $F(x) = x^3 - 3x^2 + 6$  and  
 $g(x) = \max \{F(t) : x + 1 \leq t \leq x + 2\} - 3 \leq x < 0$  and  $1 - x$  for  
 $x \geq 0$  Find continuity and differentiability of  $g(x)$  for  $x \in [-3, 1]$

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101. Show that  $5x \leq 8 \sin x - \sin 2x \leq 6x$  or  $x \leq x \leq \frac{\pi}{3}$

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102. Let  $f(x), x \geq 0$ , be a non-negative continuous function. If  
 $f'(x) \cos x \leq f(x) \sin x \forall x \geq 0$ , then find  $f\left(\frac{5\pi}{3}\right)$



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103. If  $ax^2 + \frac{b}{x} \geq c$  for all positive  $x$  where  $a > 0$  and  $b > 0$ , show that  $27ab^2 \geq 4c^3$ .



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104. Prove that  $\sin x + 2x \geq \frac{3x(x+1)}{\pi}$ ,  $\forall x \in \left[0, \frac{\pi}{2}\right]$  (Justify the inequality, if any used).



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105. Find the set of values of  $p$  for which line  $2px - 4y + 2\pi - 9 = 0$  intersect the curve  $y = \cos^{-1}$  at three distinct points.



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**106.** From a fixed point  $A$  on the circumference of a circle of radius  $r$ , the perpendicular  $AY$  falls on the tangent at  $P$ . Find the maximum area of triangle  $APY$ .



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**107.**  $P$  and  $Q$  are two points on a circle of centre  $C$  and radius  $\alpha$ . The angle  $PCQ$  being  $2\theta$ , find the value of  $\sin \theta$  when the radius of the circle inscribed in the triangle  $CPQ$  is maximum.



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**108.** The lower corner of a leaf in a book is folded over so as to reach the inner edge of the page. Show that the fraction of the width folded over when the area of the folded part is minimum is  $2/3$ .



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**109.** Let  $A(p^2, -p)$ ,  $B(q^2, q)$ ,  $C(r^2, -r)$  be the vertices of triangle ABC. A parallelogram AFDE is drawn with D, E, and F on the line segments BC, CA and AB, respectively. Using calculus, show that the maximum area of such a parallelogram is  $\frac{1}{2}(p+q)(q+r)(p-r)$ .

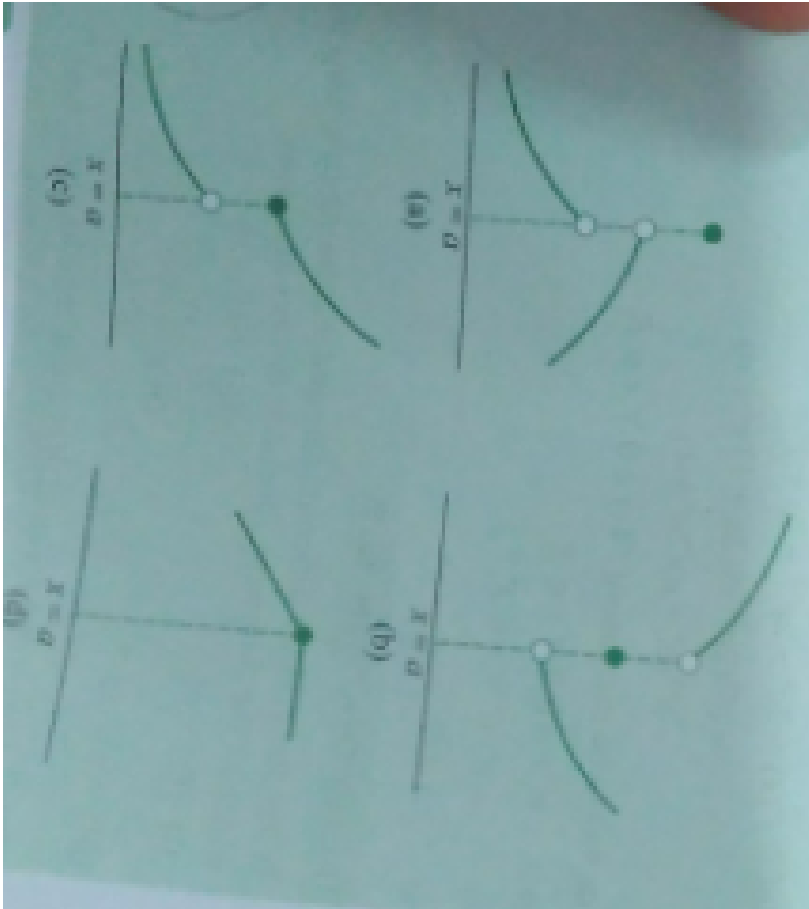
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**110.** A window of perimeter  $P$  (including the base of the arch) is in the form of a rectangle surrounded by a semi-circle. The semi-circular portion is fitted with the colored glass while the rectangular part is fitted with the clear glass that transmits three times as much light per square meter as the colored glass does. What is the ratio for the sides of the rectangle so that the window transmits the maximum light?

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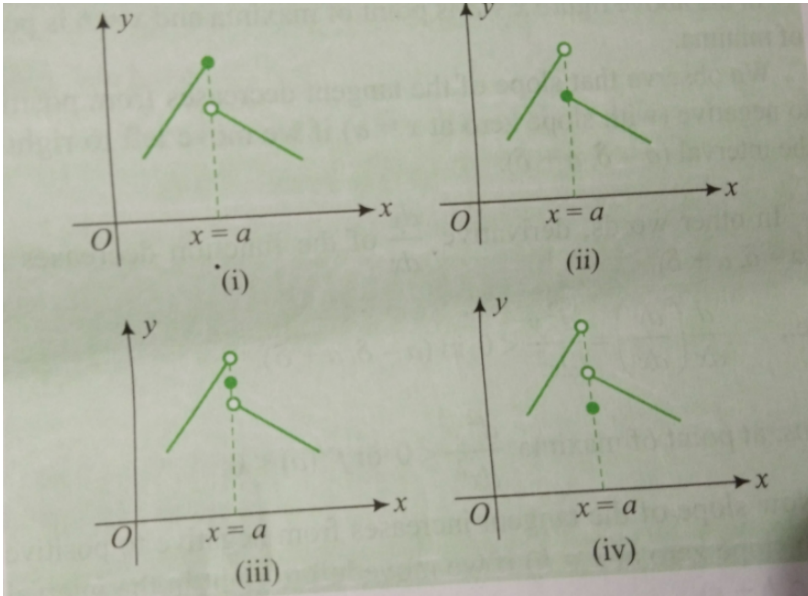
Illustration

1. For each of the following graph, comment whether  $f(x)$  is increasing or decreasing or neither increasing nor decreasing at  $x=a$ .



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2. Consider the following graphs of the functions. Check each for the extrema at  $x=a$



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### Exercise 6 1

1. Prove that the following functions are strictly increasing:

$$f(x) = \log(1+x) - \frac{2x}{2+x}$$

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2. Separate the intervals of monotonicity for the following function:

(a)  $f(x) = -2x^3 - 9x^2 - 12x + 1$

(b)  $f(x) = x^2 e^{-x}$

(c)  $f(x) = \sin x + \cos x, x \in (0, 2\pi)$

(d)  $f(x) = 3 \cos^4 x + \cos x, x \in (0, 2\pi)$

(e)  $f(x) = (\log_e x)^2 + (\log_e x)$



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3. Discuss monotonicity of  $f(x) = \frac{x}{\sin x}$  and  $g(x) = x/(\tan x)$ , where



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4. A function  $y = f(x)$  is given by  $x = \frac{1}{1+t^2}$  and  $y = \frac{1}{t(1+t^2)}$  for all

$t > 0$  then  $f$  is



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5. Find the value of  $a$  for which the function  $(a + 2)x^3 - 3ax^2 + 9ax - 1$  decreases monotonically for all real  $x$ .

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6. Find the value of  $a$  in order that  $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$  decreases for all real values of  $x$ .

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7. Discuss the monotonicity of function  $f(x) = 2\log|x - 1| - x^2 + 2x + 3$ .

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8. If  $f(x) = \sin x + \log_e|\sec x + \tan x| - 2x$  for  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  then check the monotonicity of  $f(x)$





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9. Find the interval of the monotonicity of the function  $f(x) =$

$$\log_e \left( \frac{\log_e x}{x} \right)$$



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10. Let  $g(x) = f(\log x) + f(2 - \log x)$  and  $f^x < 0 \forall x \in (0, 3)$ . Then find the interval in which  $g(x)$  increases.



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## Exercise 6 2

1. Find the range of  $f(x) = \tan^{-1} x - \frac{1}{2} \log_e x \in \frac{1}{\sqrt{3}}, (\sqrt{3})$



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2. Find the range of  $f(x) = \frac{\sin x}{x} + \frac{x}{\tan x} \in \left(0, \frac{\pi}{2}\right)$

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3. Range of  $f(x) = \sin^{-1} x + \cos^{-1} x + \cot^{-1} x$

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4. Find the number of solution of the equation

$$x^3 + 2x + \cos x + \tan x = 0 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

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5. Show that  $\frac{x}{(1+x)} \langle 1n(1+x) f \text{ or } x \rangle 0$

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6. For  $x > 0$

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7. Show that  $\tan^{-1} x > \frac{x}{1 + \frac{x^2}{3}}$  if  $x \in (0, \infty)$ .

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8. Show that  $2 \sin x + \tan x \geq 3x$ , where  $0 \leq x < \frac{\pi}{2}$

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9. Prove that  $f(x) = \frac{\sin x}{x}$  is monotonically decreasing in  $\left[0, \frac{\pi}{2}\right]$ . Hence, prove that  $\frac{\sin(2x)}{2x} > \frac{\sin x}{x}$

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10. For  $\lambda > 0$



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### Exercise 6.3

1. Show that graph of the function  $f(x) = \log_e(x - 2) - \frac{1}{x}$  always concave downwards.



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2. Separate the interval of concavity of  $y = x \log_e x - \frac{x^2}{2} + \frac{1}{2}$



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3. Consider  $f(x) = \cos 2x + 2x\lambda^2 + (2\lambda + 1)(\lambda - 1)x^2$ ,  $\lambda \in \mathbb{R}$

If  $\alpha \neq \beta$  and  $\frac{f(\alpha + \beta)}{2} < \frac{f(\alpha) + f(\beta)}{2}$  for  $\alpha$  and  $\beta$  then find the

values of  $\lambda$

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4. Find the values of  $x$  where function  $f(x) = (\sin x + \cos x)e^x$  in  $(0, 2\pi)$  has point of inflection

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5. Prove that 
$$\frac{a_1^m + a_2^m + \dots + a_n^m}{n} < \left( \frac{a_1 + a_2 + \dots + a_n}{n} \right)^m$$
 if  $0 < m < 1$  and  $a_i > 0$  for all  $i$ .

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6. Find the least value of  $\sec A + \sec B + \sec C$  in an acute angled triangle.

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## Exercise 6 4

1. Find the critical (stationary ) points of the function  $f(x) = \frac{x^5}{20} - \frac{x^4}{12}$   
+5Name these points .Also find the point of inflection



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2. The curve  $f(x) = \frac{x^2 + ax + 6}{x - 10}$  has a stationary point at  $(4, 1)$  . Find the values of  $a$  and  $b$  . Also, show that  $f(x)$  has point of maxima at this point.



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3. If the function  $f(x) = ax e^{bx^2}$  has maximum value at  $x=2$  such that  $f(2) = 1$  , then find the values of  $a$  and  $b$



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4. Discuss the extremum of  $f(x) = \frac{1}{3} \left( x + \frac{1}{x} \right)$

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5. Discuss the extremum of  $f(x) = 1 + 2 \sin x + 3 \cos^2 x$ ,  $x \leq x \leq \frac{2\pi}{3}$

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6. Discuss the extremum of

$$f(x) = \sin x + \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x, 0 \leq x \leq \pi.$$

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7. Let  $f(x) = -\sin^3 x + 3 \sin^2 x + 5 \cos x$  on  $\left[ 0, \frac{\pi}{2} \right]$ . Find the local maximum and local minimum of  $f(x)$ .

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8. discuss the extremum of  $f(\theta) = \sin^p \theta \cos^q \theta$ ,  $p, q > 0$ ,  $0 < \theta < \frac{\pi}{2}$

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9. Find the maximum and minimum values of the function

$y = (\log)_e (3x^4 - 2x^3 - 6x^2 + 6x + 1) \forall x \in (0, 2)$  Given that

$(3x^4 - 2x^3 - 6x^2 + 6x + 1) > 0 \forall x \in (0, 2)$

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10. Discuss the extremum of  $f(x) = x(x^2 - 4)^{-\frac{1}{3}}$

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11. Discuss the maxima and minima of the function  $f(x) = x^{\frac{2}{3}} - x^{\frac{4}{3}}$ .

Draw the graph of  $y = f(x)$  and find the range of  $f(x)$ .

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12. Discuss the extremum of  $f(x) = |x^2 - 2|, -1 \leq x$

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13. Discuss the extremum of

$$f(x) = \begin{cases} 1 + \sin x, & x < 0 \\ x^2 - x + 1, & x \geq 0 \end{cases} \text{ at } x = 0$$

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14. Find the minimum value of  $|x| + \left|x + \frac{1}{2}\right| + |x - 3| + \left|x - \frac{5}{2}\right|$ .

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15. Let  $f(x)$  be defined as  $f(x) = (\tan^{-1} x)^2 - 5x^2, 0$

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16.

Let

$$f(x) = \begin{cases} x^3 - x^2 + 10x - 5, & x \leq 1 \\ -2x + (\log)_2(b^2 - 2), & x > 1 \end{cases}$$
 Find

the values of  $b$  for which  $f(x)$  has the greatest value at  $x = 1$ .


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### Exercise 6 5

1. Draw the graph of  $f(x) = \frac{x^2 - x + 1}{x^2 + x + 1}$ .


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2. Discuss the number of roots of the equation  $e(k - x \log x) = 1$  for different value of  $k$ .


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3. Draw the graph of  $y = (x + 1)^{\frac{2}{3}} + (x - 1)^{\frac{2}{3}}$



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4. Draw the graph of  $f(X) = \log_e(1 - \log_e x)$ . Find the point of inflection



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5. Draw and graph of  $f(x) = \frac{4 \log_e x}{x^2}$ . Also find the range.



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## Exercise 6 6

1. A private telephone company serving a small community makes a profit of Rs. 12.00 per subscriber, if it has 725 subscribers. It decides to reduce



the rate by a fixed sum for each subscriber over 725, thereby reducing the profit by 1 paise per subscriber. Thus, there will be profit of Rs. 11.99 on each of the 726 subscribers, Rs. 11.98 on each of the 727 subscribers, etc. What is the number of subscribers which will give the company the maximum profit?

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2. एक वृत्त और एक वर्ग के परिमाण का योग  $k$  है, जहां  $k$  एक अचर है सिद्ध कीजिए कि उनके क्षेत्रफल का योग निम्नतम है जब वर्ग की भुजा वृत्त की त्रिज्या की दुगुनी है।

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3. A figure is bounded by the curves  $y = x^2 + 1$ ,  $y = 0$ ,  $x = 0$ , and  $x = 1$ . At what point  $(a, b)$  should a tangent be drawn to curve  $y = x^2 + 1$  for it to cut off a trapezium of greatest area from the figure?

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4. Find the minimum length of radius vector of the curve  $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$

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5. Find the point at which the slope of the tangent of the function  $f(x) = e^x \cos x$  attains minima, when  $x \in [0, 2\pi]$ .

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6. An electric light is placed directly over the centre of a circular plot of lawn 100 m in diameter. Assuming that the intensity of light varies directly as the sine of the angle at which it strikes an illuminated surface and inversely as the square of its distance from its surface, how should the light be hung in order that the intensity may be as great as possible at the circumference of the plot?

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7. A swimmer  $S$  is in the sea at a distance  $d$  km from the closest point  $A$  on a straight shore. The house of the swimmer is on the shore at distance  $L$  km. The swimmer can swim at a speed of  $u$  km/h per hour and walk at a speed of  $v$  km/h,  $u > v$ .



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8. Let  $(h, k)$  be a fixed point, where  $h > 0, k > 0$ . A straight line passing through this point cuts the positive direction of the coordinate axes at the point  $P$  and  $Q$ . Find the minimum area of triangle  $OPQ$ ,  $O$  being the origin.



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9. A point  $P$  is given on the circumference of a circle of radius  $r$ . Chords  $QR$  are parallel to the tangent at  $P$ . Determine the maximum possible area of triangle  $PQR$ .



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## Exercise 6 7

1. A metal box with a square base and vertical sides is to contain  $1024 \text{ cm}^3$  of water, the material for the top and bottom costs Rs 5 per  $\text{cm}^2$  and the material for the sides costs Rs 2.50 per  $\text{cm}^2$ . Find the least cost of the box.



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2.  $100 \text{ cm}^2$  आयतन वाले डिब्बे सभी बंद बेलनकार ( लंब वृतीय ) डिब्बों में से न्यूनतम पृष्ठ क्षेत्रफल वाले डिब्बे की विमाएँ ज्ञात कीजिए



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3. The lateral edge of a regular rectangular pyramid is  $acmlong$ . The lateral edge makes an angle  $\alpha$  with the plane of the base. Find the value of  $\alpha$  for which the volume of the pyramid is greatest.



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4. Show that the cone of the greatest volume which can be inscribed in a given sphere has an altitude equal to  $\frac{2}{3}$  of the diameter of the sphere.

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5. A regular square based pyramid is inscribed in a sphere of given radius  $R$  so that all vertices of the pyramid belong to the sphere. Find the greatest value of the volume of the pyramid.

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### Exercise Single

1. A function is matched below against an interval where it is supposed to be increasing. Which of the following parts is incorrectly matched?

Interval, Function  $[2, \infty)$  ,  $2x^3 - 3x^2 - 12x + 6$   $(-\infty, \infty)$  ,

$$x^3 = 3x^2 + 3x + 3 \quad (-\infty - 4) \quad , \quad x^3 + 6x^2 + 6 \quad \left(-\infty, \frac{1}{3}\right) \quad ,$$

$$3x^2 - 2x + 1$$

A.  $[2, \infty)$ ,  $2x^3 - 3x^2 - 12x + 6$

B.  $[-\infty, \infty)$ ,  $x^3 - 3x^2 - 3x + 3$

C.  $[-\infty, -4)$ ,  $x^3 - 6x^2 + 6$

D.  $\left[-\infty, \frac{1}{3}\right]$ ,  $3x^2 - 2x + 1$

**Answer: 4**



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2. On which of the following intervals is the function  $x^{100} + \sin x - 1$  decreasing? (a)  $\left(0, \frac{\pi}{2}\right)$  (b)  $(0, 1)$  (c)  $\left(\frac{\pi}{2}, \pi\right)$  (d) none of these

A.  $(0, \pi/2)$

B.  $(0,1)$

C.  $(\pi/2, \pi)$

D. None of these

**Answer: 4**



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3. The function  $f(x) = \tan^{-1}(\sin x + \cos x)$  is an increasing function in

(a)  $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$  (b)  $\left(0, \frac{\pi}{2}\right)$  (c)  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  (d)  $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

A.  $(0, \pi/2)$

B.  $\left(0, \frac{\pi}{2}\right)$

C.  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

D.  $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

**Answer: 1**



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4. Consider the following statements in S and R S: Both  $\sin x$  and  $\cos x$  are decreasing function in the interval  $\left(\frac{\pi}{2}, \pi\right)$  R: If a differentiable function decreases in an interval  $(a, b)$ , then its derivative also decrease in  $(a, b)$ . Which of the following is true? Both S and R are wrong. Both S and R are correct, but R is not the correct explanation of S. S is correct and R is the correct explanation for S. S is correct and R is wrong.

A. Both S and R are wrong

B. Both S and R are correct , but R is not the correct explanation of S.

C. S is correct and R is the correct explanation for S.

D. S is correct and R is wrong.

**Answer: 4**



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5. The length of the longest interval in which the function  $3 \sin x - 4 \sin^3 x$  is increasing is  $\frac{\pi}{3}$  (b)  $\frac{\pi}{2}$  (c)  $\frac{3\pi}{2}$  (d)  $\pi$



A.  $\frac{\pi}{3}$

B.  $\frac{\pi}{2}$

C.  $\frac{\pi}{2}$

D.  $(\pi)$

**Answer: 1**



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6. The function  $x^x$  decreases in the interval (0, e) (b) (0, 1)  $\left(0, \frac{1}{e}\right)$  (d)

none of these

A. (0,e)

B. (0,1)

C.  $\left(0, \frac{1}{e}\right)$

D. none of these

**Answer: 3**



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7. Let  $f(x) = x\sqrt{4ax - x^2}$ , ( $a > 0$ ) Then  $f(x)$  is

- A. increasing in  $(0, 3a)$  decreasing in  $(3a, 4a)$
- B. increasing in  $(a, 4a)$ , decreasing in  $(5a, \infty)$
- C. increasing in  $(0, 4a)$
- D. none of these

Answer: 1



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8. Function  $f(x) = |x| - |x - 1|$  is monotonically increasing when  
 $x < 0$  (b)  $x > 1$   $x < 1$  (d) 0

- A.  $x < 0$
- B.  $x > 0$

C.  $x < 0$

D.  $0 < x < 1$

**Answer: 4**



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9. If  $f'(x) = |x| - \{x\}$ , where  $\{x\}$  denotes the fractional part of  $x$ , then

$f(x)$  is decreasing in (a)  $\left(-\frac{1}{2}, 0\right)$  (b)  $\left(-\frac{1}{2}, 2\right)$  (c)  $\left(-\frac{1}{2}, 2\right)$  (d)  $\left(\frac{1}{2}, \infty\right)$

A.  $\left(\frac{-1}{2}, 0\right)$

B.  $\left(\frac{-1}{2}, 2\right)$

C.  $\left(\frac{-1}{2}, 2\right)$

D.  $\left(\frac{1}{2}, \infty\right)$

**Answer: 1**



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10. The length of the largest continuous interval in which the function

$f(x) = 4x - \tan 2x$  is monotonic is (a)  $\frac{\pi}{2}$  (b)  $\frac{\pi}{4}$  (c)  $\frac{\pi}{8}$  (d)  $\frac{\pi}{16}$

A.  $\pi/2$

B.  $\pi/4$

C.  $\pi/8$

D.  $\pi/16$

**Answer: 2**



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11.  $f(x) = (x - 2)|x - 3|$  is monotonically increasing in

(a)  $(-\infty, \frac{5}{2}) \cup (3, \infty)$  (b)  $(\frac{5}{2}, \infty)$  (c)  $(2, \infty)$  (d)  $(-\infty, 3)$

A.  $(-\infty, 5/2) \cup (3, \infty)$

B.  $5/2, \infty)$

C.  $(2, \infty)$

D.  $(-\infty, 3)$

**Answer: 1**



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12.  $f(x) = (x - 8)^4(x - 9)^5, 0 \leq x \leq 10$ , monotonically decreases in

A.  $\left(\frac{76}{9}, 10\right)$

B.  $\left(8, \frac{76}{9}\right)$

C.  $[0, 8)$

D.  $\left(\frac{76}{9}, 10\right)$

**Answer: 2**



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13. For all  $x \in (0, 1)$   $e^x < 1 + x$  (b)  $(\log)_e(1+x) < x < x(\log)_e x$

A.  $e^x < 1 + x$

B.  $\log e^{1+x} < x$

C.  $\sin x > x$

D.  $\log_e x > x$

**Answer: 2**

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14. If  $f(x) = xe^{x(x-1)}$ , then  $f(x)$  is increasing on  $\left[-\frac{1}{2}, 1\right]$   
decreasing on  $R$  increasing on  $R$  (d) decreasing on  $\left[-\frac{1}{2}, 1\right]$

A. increasing on  $[-1/2, 1]$

B. decreasing on  $[-1/2, \infty]$

C. increasing on  $R$

D. decreasing on  $[-1/2, \infty]$

**Answer: 1**



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15. If  $f(x) = kx^3 - 9x^2 + 9x + 3$  monotonically increasing in  $R$ , then  $k < 3$  (b)  $k \leq 2$   $k \geq 3$  (d) none of these

A.  $k < 3$

B.  $k \leq 2$

C.  $k \geq 3$

D. none of these

Answer: 3



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16. If the function  $f(x) = \frac{K \sin x + 2 \cos x}{\sin x + \cos x}$  is strictly increasing for all values of  $x$ , then  $K < 1$  (b)  $K > 1$   $K < 2$  (d)  $K > 2$

A.  $k < 1$

B.  $k \leq 2$

C.  $k \geq 3$

D. none of these

**Answer: 4**



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17. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x) = ax + 3 \sin x + 4 \cos x$

Then  $f(x)$  is invertible if

A.  $k < 1$

B.  $k > 1$

C.  $k < 2$

D.  $k > 2$

**Answer: 4**



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18. Which of the following statement is always true? If  $f(x)$  is increasing, the  $f^{-1}(x)$  is decreasing. If  $f(x)$  is increasing, then  $\frac{1}{f(x)}$  is also increasing. If  $f$  and  $g$  are positive functions and  $f$  is increasing and  $g$  is decreasing, then  $\frac{f}{g}$  is a decreasing function. If  $f$  and  $g$  are positive functions and  $f$  is decreasing and  $g$  is increasing, the  $\frac{f}{g}$  is a decreasing function.

A. If  $f(x)$  is increasing then  $f^{-1}(x)$  is also decreasing

B. If  $f(x)$  is increasing then  $1/f(x)$  is also increasing

C. If  $f$  and  $g$  are positive functions and  $f$  is increasing and  $g$  is decreasing then  $f/g$  is decreasing function

D. If  $f$  and  $g$  are positive functions and  $f$  is decreasing and  $g$  is increasing then  $f/g$  is decreasing function

**Answer: 4**



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19. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function for all values of  $x$  and has the property that  $f(x)$  and  $f'(x)$  has opposite signs for all value of  $x$ . Then,  $f(x)$  is an increasing function  $f(x)$  is an decreasing function  $f^2(x)$  is an decreasing function  $|f(x)|$  is an increasing function

- A.  $f(x)$  is an iincreasing function
- B.  $f(x)$  is a decreasing function
- C.  $f^2(x)$  is a decreasing function
- D.  $|f(x)|$  is an increasing function

**Answer: 3**



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20. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function  $\forall x \in \mathbb{R}$ . If the tangent drawn to the curve at any point  $x \in (a, b)$  always lies below the curve, then

$$f'(x) < 0, f''(x) < 0 \forall x \in (a, b) \qquad f'(x) > 0, f''(x) > 0 \forall x \in (a, b)$$

$$f'(x) > 0, f''(x) > 0 \forall x \in (a, b) \text{ none of these}$$

A.  $f'(x) > 0, f''(x) < 0 \forall x \in (a, b)$

B.  $f'(x) < 0, f''(x) < 0 \forall x \in (a, b)$

C.  $f'(x) > 0, f''(x) > 0 \forall x \in (a, b)$

D.

**Answer: 3**

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21. Let  $f(x)$  be a function such that  $f(x) = (\log)_{\frac{1}{2}} [(\log)_3(\sin x + a)]$ . If

$f(x)$  is decreasing for all real values of  $x$ , then  $a \in (1, 4)$  (b)  $a \in (4, \infty)$

$a \in (2, 3)$  (d)  $a \in (2, \infty)$

A.  $a \in (1, 4)$

B.  $a \in (4, \infty)$

C.  $a \in (2, 3)$

D.  $a \in (2, \infty)$

**Answer: 2**



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**22.** If  $f(x) = x^3 + 4x^2 + \lambda x + 1$  is a monotonically decreasing function of  $x$  in the largest possible interval  $\left(-2, -\frac{2}{3}\right)$ . Then  $\lambda = 4$  (b)  $\lambda = 2$   
 $\lambda = -1$  (d)  $\lambda$  has no real value

A.  $\lambda = 4$

B.  $\lambda = 2$

C.  $\lambda = -1$

D.  $\lambda$  has no real value

**Answer: 1**



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23.  $f(x) = |x \log_e x|$  monotonically decreases in (a)  $\left(0, \frac{1}{e}\right)$  (b)  $\left(\frac{1}{e}, 1\right)$  (c)  $(1, \infty)$  (d)  $\left(\frac{1}{e}, \infty\right)$

A.  $(0, 1/e)$

B.  $(1/e, 1)$

C.  $(1, \infty)$

D.  $(1/e, \infty)$

**Answer: 2**



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24. The set of value(s) of  $a$  for which the function

$f(x) = \frac{ax^3}{3} + (a+2)x^2 + (a-1)x + 2$  possesses a negative point of

inflection is (a)  $(-\infty, -2) \cup (0, \infty)$  (b)  $\left\{-\frac{4}{5}\right\}$  (c)  $(-2, 0)$  (d) empty set

A.  $(-\infty, -2) \cup (0, \infty)$

B.  $\left\{-\frac{4}{5}\right\}$

C.  $(-2, 0)$

D. empty set

**Answer: 1**



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25. The maximum value of the function  $f(x) = \frac{(1+x)^{0.6}}{1+x^{0.6}}$  in the interval  $[0, 1]$  is

A.  $2^{0.4}$

B.  $2^{-0.4}$

C. 1

D.  $2^{0.6}$

**Answer: 3**



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26. Suppose that  $f$  is a polynomial of degree 3 and that  $f'' \neq 0$  at any of the stationary point. Then  $f$  has exactly one stationary point  $f$  must have no stationary point  $f$  must have exactly two stationary points  $f$  has either zero or two stationary points.

- A.  $f$  has exactly one stationary point
- B.  $f$  must have no stationary point
- C.  $f$  must have exactly two stationary points
- D.  $f$  has either zero or two stationary points

**Answer: 4**

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27. A function  $g(x)$  is defined as  $g(x) = \frac{1}{4}f(2x^2 - 1) + \frac{1}{2}f(1 - x^2)$  and  $f(x)$  is an increasing function.

Then  $g(x)$  is increasing in the interval.  $(-1, 1)$

$\left(-\sqrt{\frac{2}{3}}, 0\right) \cup \left(\sqrt{\frac{2}{3}}, \infty\right)$   $\left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$  (d) none of these

A.  $(-1, 1)$

B.  $-\left(\frac{\sqrt{2}}{3}, 0\right) \cup \left(\frac{\sqrt{2}}{3}, \infty\right)$

C.  $-\frac{\sqrt{2}}{3} \cdot \frac{\sqrt{2}}{3}$

D. none of these

**Answer: 2**



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28. If  $\varphi(x)$  is a polynomial function and  $\varphi'(x) > \varphi(x) \forall x \geq 1$  and  $\varphi(1) = 0$ , then  $\varphi(x) \geq 0 \forall x \geq 1 \Rightarrow \varphi(x)$

A.  $\phi(x) \geq 0 \forall x \geq 1$

B.  $\phi(x) < 0 \forall x \geq 1$

C.  $\phi(x) = 0 \forall x \geq 1$

D. none of these

**Answer: 1**





A.  $f \circ g(u) < f \circ g(v)$

B.  $g \circ f(u) < g \circ f(v)$

C.  $g \circ f(u) > g \circ f(v)$

D.  $f \circ g(u) < f \circ g(v)$

**Answer: 4**



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**31.** The number of solutions of the equation

$$x^3 + 2x^2 + 5x + 2 \cos x = 0 \text{ in } (0, 2\pi] \text{ is}$$

A. one

B. two

C. three

D. zero

**Answer: 4**



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32. Let  $f(x) = \cos \pi x + 10x + 3x^2 + x^3$ ,  $-2 \leq x \leq 3$ . The absolute minimum value of  $f(x)$  is 0 (b)  $-15$  (c)  $3 - 2\pi$  none of these

A. 0

B. -15

C.  $3 - 2\pi$

D. none of these

Answer: 2



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33. The global maximum value of

$f(x) = (\log)_{10}(4x^3 - 12x^2 + 11x - 3)$ ,  $x \in [2, 3]$ , is

A.  $-\frac{3}{2}\log_{10}^3$

B.  $1 + \log 10^3$

C.  $\log 10^3$

D.  $\frac{3}{2} \log 10^3$

**Answer: 2**



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34.  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x)$  is differentiable such that  $f(x) = k(x^5 + x)$ . ( $k \neq 0$ ) Then  $f(x)$  is

A. increasing

B. decreasing on  $[-1/2, \infty]$

C. either increasing or decreasing

D. non monotonic

**Answer: 3**



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35. The value of  $a$  for which the function  $f(x) = a \sin x + \left(\frac{1}{3}\right) \sin 3x$  has an extremum at  $x = \frac{\pi}{3}$  is 1 (b)  $-1$  (c)  $0$  (d)  $2$

A. 1

B. -1

C. 0

D. 2

Answer: 5



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36. If  $f(x) = a \log|x| + bx^2 + x$  has its extremum values at  $x = -1$  and  $x = 2$ , then  $a = 2, b = -1$   $a = 2, b = -1/2$   $a = -2, b = 1/2$  (d) none of these

A.  $a=2, b=-1$

B.  $a=2, b=-1/2$

C.  $a=-2, b=1/2$

D. none of these

**Answer: 2**



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37. If a function  $f(x)$  has  $f'(a) = 0$  and  $f''(a) = 0$ , then  $x = a$  is a maximum for  $f(x)$   $x = a$  is a minimum for  $f(x)$  it is difficult to say (a) and (b)  $f(x)$  is necessarily a constant function.

A.  $x=a$  is a maximum for  $f(x)$

B.  $x=a$  is a minimum for  $f(x)$

C. it is difficult to say (a) and (b)

D.  $f(x)$  is necessarily a constant function

**Answer: 3**



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38. The function  $f(x) = \sin^4 x + \cos^4 x \in \text{creases}$  if

- A. It is monotonic increasing  $\forall x$  in  $\mathbb{R}$ .
- B.  $f(x)$  fails to exist for three distinct real values of  $x$
- C.  $f(x)$  changes its sign twice as  $x$  varies from  $-\infty \rightarrow \infty$
- D. The function attains its extreme values at  $x_1$  and  $x_2$  such that

$$x_1 x_2 > 0$$

Answer: 3



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39. If  $f(x) = \sin^4 x + \cos^4 x$  increases, if

- A.  $0 < x < \pi/8$
- B.  $\pi/40 < x < 3\pi/8$

C.  $3\pi/8 < x < 5\pi/8$

D.  $5\pi/8 < x < 3\pi/4$

**Answer: 2**



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**40.** If  $f(x) = x^5 - 5x^4 + 5x^3 - 10$  has local maximum and minimum at  $x = p$  and  $x = q$ , respectively, then  $(p, q) \equiv (0, 1)$  (b)  $(1, 3)$  (c)  $(1, 0)$  (d) none of these

A.  $(0, 1)$

B.  $(1, 3)$

C.  $(1, 0)$

D. none of these

**Answer: 2**



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41. Let  $P(x) = a_0 + a_1x^2 + a_2x^4 + \dots + a_nx^{2n}$  be a polynomial in a real variable  $x$  with  $a_0 < 0$

- A. neither a maximum nor a minimum
- B. only one maximum
- C. only one minimum
- D. only one maximum and only one minimum

**Answer: 3**



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42. Let  $f(x) = \begin{cases} |x| & \text{for } 0 < |x| \\ 2 & \text{for } x=0 \end{cases}$

- A. a local maximum
- B. no local maximum
- C. a local minimum

D. no extremum

**Answer: 1**

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43. If  $f(x) = x^3 + bx^2 + cx + d$  and  $\Delta < 0$

- A.  $f(x)$  is strictly increasing function
- B.  $f(x)$  has local maxima
- C.  $f(x)$  is a strictly decreasing function
- D.  $f(x)$  is bounded

**Answer: 1**

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44. If  $f(x) = \begin{cases} (\sin(x^2 - 3x)), & x \leq 0 \\ e^x + 5x^2, & x > 0 \end{cases}$

- A.  $f(x)$  has a local minima
- B.  $f(x)$  has a local maxima
- C.  $f(x)$  has point of inflection
- D. none of these

**Answer: 1**



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45. The greatest value of  $f(x) = \cos(xe^{[x]} + 7x^2 - 3x)$ ,  $x \in [-1, \infty]$ , is (where  $[.]$  represents the greatest integer function). -1 (b) 1 (c) 0 (d) none of these

- A. -1
- B. 1

C. 0

D. none of these

**Answer: 2**



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**46.** The function  $f(x) = (4\sin^2 x - 1)^n (x^2 - x + 1)$ ,  $n \in \mathbb{N}$ , has a local minimum at  $x = \frac{\pi}{6}$ . Then  $n$  is any even number  $n$  is an odd number  $n$  is odd prime number  $n$  is any natural number

A.  $n$  is any even integer

B.  $n$  is an odd integer

C.  $n$  is odd prime number

D.  $n$  is any natural number

**Answer: 1**



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47. The true set of real values of  $x$  for which the function  $f(x) = x \ln x - x + 1$  is positive is

- A.  $(1, \infty)$
- B.  $(1/e, \infty)$
- C.  $[e, \infty)$
- D.  $(0, 1) \cup (1, \infty)$

**Answer: 4**



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48. All possible value of  $f(x) = (x + 1)^{\frac{1}{3}} - (x - 1)^{\frac{1}{3}}$  on  $[0,1]$  is 1 (b) 2 (c)

3 (d)  $\frac{1}{3}$

- A. 1
- B. 2

C. 3

D.  $\frac{1}{3}$

**Answer: 2**



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49. The function  $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$  is increasing in  $(0, \infty)$  decreasing in  $(0, \infty)$  increasing in  $(0, \frac{\pi}{e})$ , decreasing in  $(\frac{\pi}{e}, \infty)$  decreasing in  $(0, \frac{\pi}{e})$ , increasing in  $(\frac{\pi}{e}, \infty)$

A. increasing in  $(0, \infty)$

B. decreasing oin  $(0, \infty)$

C. increasing in  $(0, \pi/e)$ , *decrea sin g*  $\in (\pi/e, \infty)$

D. decreasing in  $(0, \pi/e)$  increasing in  $(\pi/e, \infty)$

**Answer: 2**



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50. If the function  $f(x) = 2x^3 - 9ax^2 + 12x^2x + 1$ , where  $a > 0$ , attains its maximum and minimum at  $p$  and  $q$ , respectively, such that  $p^2 = q$ , then  $a$  equal to 1 (b) 2 (c)  $\frac{1}{2}$  (d) 3

A. 1

B. 2

C.  $\frac{1}{2}$

D. 3

**Answer: 2**



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51. Let  $f(x) = \begin{cases} x + 2, & -1 \leq x < 0 \\ 1, & x = 0 \\ \frac{x}{2}, & 0 < x \leq 1 \end{cases}$

A. a point of minima

- B. a point of maxima
- C. both points of minima and maxima
- D. neither a point of minima nor that of maxima

**Answer: 4**



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52. If  $f(x) = \begin{cases} \sin^{-1}(\sin x), & x > 0 \\ \frac{\pi}{2}, & x = 0, \end{cases}$  then  $\cos^{-1}(\cos x), x < 0$

- A.  $x=0$  is a point of maxima
- B.  $x=0$  is a point of minima
- C.  $x=0$  is a point of intersection
- D. none of these

**Answer: 1**



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53. A function  $f$  is defined by  $f(x) = |x|^m |x - 1|^n \forall x \in R$ . The local maximum value of the function is  $(m, n \in N)$ , 1 (b)  $m^n \wedge m$

$$\frac{m^m n^n}{(m+n)^{m+n}} \quad (d) \quad \frac{(mn)^{mn}}{(m+n)^{m+n}}$$

A. 1

B.  $m^n n^m$

C.  $\frac{m^m n^n}{(m+n)^{m+n}}$

D.  $\frac{mn^{mn}}{m+n^{m+n}}$

Answer: 3



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54. Let the function  $f(x)$  be defined as follows:

$$f(x) = \begin{cases} x^3 + x^2 - 10x & -1 \leq x \leq 0 \\ \cos x & 0 \leq x \leq \pi/2 \\ 1 + \sin x & \pi/2 \leq x \leq \pi \end{cases}$$

- A. a local minimum at  $x=\pi/2$
- B. a global maximum at  $x=\pi/2$
- C. a absolute maximum at  $x=-1$
- D. a absolute maximum at  $x=\pi$

**Answer: C**

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55. Consider the function  $f: (-\infty, \infty) \rightarrow (-\infty, \infty)$  defined by

$$f(x) = \frac{x^2 - a}{x^2 + a}, a > 0 \text{ which of the following is not true?}$$

- A. Maximum value of  $f$  is not attained even though  $f$  is bounded
- B.  $f(x)$  is increasing on  $(0, \infty)$  and has minimum at  $x=0$
- C.  $f(x)$  is decreasing on  $(-\infty, 0)$  and has minimum at  $x=0$
- D.  $f(x)$  is increasing on  $(-\infty, \infty)$  and has neither a local maximum nor a local minimum at  $x=0$

**Answer: 4**



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56.  $F(x) = 4 \tan x - \tan^2 x + \tan^3 x, x \neq n\pi + \frac{\pi}{2}$

- A. is monotonically increasing
- B. is monotonically decreasing
- C. has a point of maxima
- D. has a point of minima

**Answer: 1**



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57. Let  $h(x) = x^{\frac{m}{n}}$  for  $x \in R$ , where  $m$  and  $n$  are odd numbers and  $\neq 0$

- A. no local extremums

B. one local maximum

C. one local minimum

D. none of these

**Answer: 1**



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58. The greatest value of the function  $f(x) = \frac{\sin 2x}{\sin\left(x + \frac{\pi}{4}\right)}$  on the interval  $\left(0, \frac{\pi}{2}\right)$  is  $\frac{1}{\sqrt{2}}$  (b)  $\sqrt{2}$  (c) 1 (d)  $-\sqrt{2}$

A.  $\frac{1}{\sqrt{2}}$

B.  $\sqrt{2}$

C. 1

D.  $-\sqrt{2}$

**Answer: 3**



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59. The minimum value of  $e^{2x^2-2x+1} \sin^{2x}$  is (a)  $e$  (b)  $\frac{1}{e}$  (c) 1 (d) 0

A.  $e$

B.  $1/e$

C. 1

D. 0

**Answer: 3**



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60. The maximum value of  $x^4 e^{-x^2}$  is (a)  $e^2$  (b)  $e^{-2}$  (c)  $12e^{-2}$  (d)  $4e^{-2}$

A.  $e^2$

B.  $e^{-2}$

C.  $12e^{-2}$

D.  $4e^{-2}$

**Answer: 4**



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61. If  $a^2x^4 + b^2y^4 = c^6$ , then the maximum value of  $xy$  is  $\frac{c^2}{\sqrt{ab}}$  (b)  $\frac{c^3}{ab}$

$\frac{c^3}{\sqrt{2ab}}$  (d)  $\frac{c^3}{2ab}$

A.  $\frac{c^2}{\sqrt{ab}}$

B.  $\frac{c^3}{\sqrt{ab}}$

C.  $\frac{c^3}{\sqrt{2ab}}$

D.  $\frac{c^3}{\sqrt{2ab}}$

**Answer: 3**



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62. The least natural number  $a$  for which  $x + ax^{-2} > 2 \forall x \in (0, \infty)$  is 1

(b) 2 (c) 5 (d) none of these

A. 1

B. 2

C. 5

D. none of these

**Answer: 2**



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63.  $f(x) = \begin{cases} 4x - x^3 + \ln(a^2 - 3a + 3), & 0 \leq x < 3 \\ 3x - 18, & x \geq 3 \end{cases}$

complete set of values of  $a$  such that  $f(x)$  as a local minima at  $x = 3$  is

$[-1, 2]$   $(-\infty, 1) \cup (2, \infty)$   $[1, 2]$  (d)  $(-\infty, -1) \cup (2, \infty)$

A.  $[-1, 2]$

B.  $(-\infty, 1) \cup (2, \infty)$

C.  $[1, 2]$

D.  $(-\infty, -1) \cup (2, \infty)$

**Answer: 2**



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**64.**

$f(x) = \begin{cases} 2 - |x^2 + 5x + 6|, & x \neq 2a^2 + 1, \\ -2, & x = 2a^2 + 1 \end{cases}$  Then the range of  $f(x)$  is  $[a, 2]$  if

so that  $f(x)$  has maxima at  $x = -2$ , is (a)  $|a| \geq 1$  (b)  $|a| < 1$  (c)  $a > 1$  (d)

$a < 1$

A.  $|a| \geq 1$

B.  $|a| \leq 1$

C.  $a > 1$

D.  $a < 1$

**Answer: 4**



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65. A differentiable function  $f(x)$  has a relative minimum at  $x = 0$ . Then the function  $f = f(x) + ax + b$  has a relative minimum at  $x = 0$  for all  $a$  and all  $b$  (b) all  $b$  if  $a = 0$  all  $b > 0$  (d) all  $a > 0$

A. all  $a$  and all  $b$

B. all  $b$  if  $a = 0$

C. all  $b > 0$

D. all  $a > 0$

Answer: 2



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66. if  $f(x) = 4x^3 - x^2 - 2x + 1$  and

$g(x) = \{ \min \{f(t) : 0 \leq t \leq x; 0 \leq x \leq 1, 3 - x : 1\}$  then

$g(1/4) + g(3/4) + g(5/4)$  is equal to

A.  $7/4$

B.  $9/4$

C.  $13/4$

D.  $5/2$

**Answer: 4**



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67. If  $f: \vec{R} \rightarrow \vec{R}$  and  $g: \vec{R} \rightarrow \vec{R}$  are two functions such that  $f(x) + f^x = -xg(x)f'(x)$  and  $g(x) > 0 \forall x \in R$ . Then the function  $f^2(x) + f'(x)^2$  has a maxima at  $x = 0$  a minima at  $x = 0$  a point of inflexion at  $x = 0$  none of these

A. a maxima at  $x = 0$

B. a minima at  $x = 0$

C. a point of inflexion at  $x = 0$

D. a point of inflexion at  $x = 0$

**Answer: 1**



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68. If  $A > 0$ ,  $B > 0$ , and  $A + B = \frac{\pi}{3}$  then the maximum value of  $\tan A \tan B$  is

A.  $\frac{1}{\sqrt{3}}$

B.  $\frac{1}{3}$

C. 3

D.  $\sqrt{3}$

**Answer: 2**



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69. If  $f(x) = \frac{t + 3x - x^2}{x - 4}$ , where  $t$  is a parameter that has minimum and maximum, then the range of values of  $t$  is (a)  $(0, 4)$  (b)  $(0, \infty)$  (c)  $(-\infty, 4)$  (d)  $(4, \infty)$

A.  $(0, 4)$

B.  $(0, \infty)$

C.  $-(\infty, 4)$

D.  $(4, \infty)$

**Answer: 3**



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70. The least value of  $a$  for which the equation  $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = a$  has at least one solution in the interval  $\left(0, \frac{\pi}{2}\right)$  is

A. 9

B. 4

C. 8

D. 1

**Answer: A**



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71. If  $f(x) = -x^3 - 3x^2 - 2x + a$ ,  $a \in R$  then the real values of  $x$  satisfying  $f(x^2 + 1) > f(2x^2 + 2x + 3)$  will be

A.  $(-\infty, \infty)$

B.  $(0, \infty)$

C.  $(-\infty, 0)$

D.  $\phi$

Answer: 1



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72. which of the following is the greatest?

A.  $\log_2 3$

B.  $\log_3 5$

C.  $\log_4 7$

D.  $\log_5 9$

**Answer: 1**



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**73.** If the equation  $4x^3 + 5x + k = 0 (k \in \mathbb{R})$  has a negative real root then

A.  $k=0$

B.  $-\infty < k < 0$

C.  $0 < k < \infty$

D.  $-\infty < k < \infty$

**Answer: 3**



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74. A tangent is drawn to the ellipse  $\frac{x^2}{27} + y^2 = 1$  at  $(3\sqrt{3} \cos \theta, \frac{\pi}{2})$ .

Then find the value of  $\theta$  such that the sum of intercepts on the axes made by this tangent is minimum.

A.  $\frac{\pi}{3}$

B.  $\frac{\pi}{6}$

C.  $\frac{\pi}{8}$

D.  $\frac{\pi}{4}$

Answer: 2



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75. The largest term in the sequence  $a_n = \frac{n^2}{n^3 + 200}$  is given by  $\frac{529}{49}$  (b)

$\frac{8}{89}$   $\frac{49}{543}$  (d) none of these

A.  $\frac{529}{49}$

B.  $\frac{8}{89}$

C.  $\frac{49}{543}$

D. none of these

**Answer: 3**



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76. A factory  $D$  is to be connected by a road with a straight railway line on which a town  $A$  is situated. The distance  $DB$  of the factory to the railway line is  $5\sqrt{3}km$ . Length  $AB$  of the railway line is 20km. Freight charges on the road are twice the charges on the railway. The point  $P$  (A P

A.  $BP=5$  km

B.  $AP=5$  km

C.  $BP=7.5$  km

D. none of these

**Answer: 1**



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77. The volume of the greatest cylinder which can be inscribed in a cone of height 30 cm and semi-vertical angle  $30^\circ$  is  $4000\frac{\pi}{\sqrt{3}}$  (b)  $400\frac{\pi}{3}cm^3$

$4000\frac{\pi}{\sqrt{3}cm^3}$  (d) none of these

A.  $4000\pi / 3cm^3$

B.  $400\pi / 3cm^3$

C.  $4000\pi / \sqrt{3}cm^3$

D.  $4000\pi / 3cm^3$  none of these

Answer: 1



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78. A rectangle of the greatest area is inscribed in a trapezium  $ABCD$ , one of whose non-parallel sides  $AB$  is perpendicular to the base, so that one of the rectangles die lies on the larger base of the trapezium. The

base of trapezium are 6cm and 10 cm and  $AB$  is 8 cm long. Then the maximum area of the rectangle is  $24cm^2$  (b)  $48cm^2$   $36cm^2$  (d) none of these

A.  $24cm^2$

B.  $48cm^2$

C.  $36cm^2$

D. none of these

**Answer: 2**



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79. A bell tent consists of a conical portion above a cylindrical portion near the ground. For a given volume and a circular base of a given radius, the amount of the canvas used is a minimum when the semi-vertical angle of the cone is  $\frac{\cos^{-1} 2}{3}$  (b)  $\frac{\sin^{-1} 2}{3}$   $\frac{\cos^{-1} 1}{3}$  (d) none of these

A.  $\cos^{-1} 2/3$

B.  $\sin^{-1} 2/3$

C.  $\cos^{-1} 1/3$

D. none of these

**Answer: 1**



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**80.** A rectangle is inscribed in an equilateral triangle of side length  $2a$  units. The maximum area of this rectangle can be  $\sqrt{3}a^2$  (b)  $\frac{\sqrt{3}a^2}{4} a^2$  (d)  $\frac{\sqrt{3}a^2}{2}$

A.  $\sqrt{3a^2}$

B.  $\frac{\sqrt{3a^2}}{4}$

C.  $a^2$

D.  $\frac{\sqrt{3a^2}}{2}$

**Answer: 4**

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81. Tangents are drawn to  $x^2 + y^2 = 16$  from the point  $P(0, h)$ . These tangents meet the  $x$ -axis at  $A$  and  $B$ . If the area of triangle  $PAB$  is minimum, then  $h = 12\sqrt{2}$  (b)  $h = 6\sqrt{2}$  (c)  $h = 8\sqrt{2}$  (d)  $h = 4\sqrt{2}$

A.  $h = 12\sqrt{2}$

B.  $h = 6\sqrt{2}$

C.  $h = 8\sqrt{2}$

D.  $h = 4\sqrt{2}$

Answer: 4

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82. The largest area of the trapezium inscribed in a semi-circle of radius  $R$ , if the lower base is on the diameter, is  $\frac{3\sqrt{3}}{4}R^2$  (b)  $\frac{\sqrt{3}}{2}R^2$  (c)  $\frac{3\sqrt{3}}{8}R^2$  (d)  $R^2$

A.  $\left(\frac{3\sqrt{3}}{4}R^2\right)$

B.  $\left(\frac{\sqrt{3}}{2}R^2\right)$

C.  $\left(\frac{3\sqrt{3}}{8}R^2\right)$

D.  $R^2$

**Answer: 1**



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**83.** In the formula  $\angle A + \angle B + \angle C = 180^\circ$ , if  $\angle A = 90^\circ$  and  $\angle B = 55^\circ$ , then  $\angle C =$  \_\_\_\_\_

A.  $\pi/4$

B.  $\pi/6$

C.  $\pi/3$

D. none of these

**Answer: 3**



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84. Two runner A and B start at the origin and run along positive x axis ,with B running three times as fast as A. An obsever, standeing one unit above the origin , keeps A and B in view.Then the maximum angle theta of sight between the observer's view of A and B is

A.  $\pi / 8$

B.  $\pi / 6$

C.  $\pi / 3$

D.  $\pi / 4$

**Answer: 2**



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85. The fuel charges for running a train are proportional to the square of the speed generated in km/h, and the cost is Rs. 48 at 16 km/h. If the fixed

charges amount to Rs. 300/h, the most economical speed is 60 km/h (b)

40 km/h 48 km/h (d) 36 km/h

A. 60  $km/h$

B. 40  $km/h$

C. 48  $km/h$

D. 36  $km/h$

**Answer: 2**



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**86.** A cylindrical gas container is closed at the top and open at the bottom. If the iron plate of the top is  $\frac{5}{4}$  times as thick as the plate forming the cylindrical sides, the ratio of the radius to the height of the cylinder using minimum material for the same capacity is 3:4 (b) 5:6 (c) 4:5 (d) none of these

A. 3:4

B. 5 : 6

C. 4 : 5

D. none of these

**Answer: 3**



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**87.** Prove that the least perimeter of an isosceles triangle in which a circle of radius  $r$  can be inscribed is  $6\sqrt{3}r$ .

A.  $4\sqrt{3}r$

B.  $2\sqrt{3}r$

C.  $6\sqrt{3}r$

D.  $8\sqrt{3}r$

**Answer: 3**



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88. A given right cone has volume  $p$ , and the largest right circular cylinder that can be inscribed in the cone has volume  $q$ . Then  $p:q$  is 9:4  
(b) 8:3 (c) 7:2 (d) none of these

A. 9:4

B. 8:3

C. 7:2

D. none of these

**Answer: 1**



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89. Find the cosine of the angle at the vertex of an isosceles triangle having the greatest area for the given constant length  $e$  of the median drawn to its lateral side.

A. 0.4

B. 0.5

C. 0.6

D. 0.8

**Answer: 4**



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**90.** A box, constructed from a rectangular metal sheet, is 21 cm by 16cm by cutting equal squares of sides  $x$  from the corners of the sheet and then turning up the projected portions. The value of  $x$  os that volume of the box is maximum is 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C.

D. 3

**Answer: 3**



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**91.** The vertices of a triangle are  $(0,0)$ ,  $(x, \cos x)$ , and  $(\sin^3 x, 0)$ , where  $x \in (0, \frac{\pi}{2})$ .

A.  $3 \frac{\sqrt{3}}{32}$

B.  $\frac{\sqrt{3}}{32}$

C.  $\frac{4}{32}$

D.  $6 \frac{\sqrt{3}}{32}$

**Answer: 1**



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**92.** The maximum area of the rectangle whose sides pass through the vertices of a given rectangle of sides  $a$  and  $b$  is  $2(ab) - \frac{1}{2}(a + b)^2$

$$\frac{1}{2}(a^2 + b^2) \text{ (d) none of these}$$

A.  $2(ab)$

B.  $\frac{1}{2}(a + b)^2$

C.  $\frac{1}{2}(a + b)^2$

D. none of these

**Answer: 2**



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**93.** The base of prism is equilateral triangle. The distance from the centre of one base to one of the vertices of the other base is  $l$ . Then altitude of the prism for which the volume is greatest is  $\frac{l}{2}$  (b)  $\frac{l}{\sqrt{3}}$  (c)  $\frac{l}{3}$  (d)  $\frac{l}{4}$

A.  $\frac{l}{2}$

B.  $\frac{l}{\sqrt{3}}$

C.  $\frac{l}{3}$

D.  $\frac{l}{4}$

Answer: 2



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## Exercise Multiple

1. Let  $f(x) = \begin{cases} x^2 + 3x, & -1 \leq x < 0 \\ -\sin x, & 0 \leq x < \pi/2 \\ -1 - \cos x, & \frac{\pi}{2} \leq x \leq \pi \end{cases}$  . Draw the graph of the

function and find the following

(a) Range of the function

(b) Point of inflection

(c) Point of local minima

A.  $f(x)$  has global minimum value -2

B. global maximum value occurs at  $x=0$

C. global maximum value occurs at  $x=\pi$

D.  $x = \pi/2$  is point of local minima

**Answer: 1,2,3,4**



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2. Let  $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$ . Then,  $f$  increases on  $[1, \infty]$   $f$  decreases on  $[1, \infty]$   $f$  has a minimum at  $x = 1$   $f$  has neither maximum nor minimum

A.  $f$  increases on  $[1, \infty]$

B.  $f$  decreases on  $[1, \infty]$

C.  $f$  has a minimum at  $x=1$

D.  $f$  has neither maximum nor minimum

**Answer: 1,3**



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3. Let  $f(x) = 2x - \sin x$  and  $g(x) = 3\sqrt{x}$ . Then

- A. range of  $g \circ f$  is  $\mathbb{R}$
- B.  $g \circ f$  is one-one
- C. both  $f$  and  $g$  are one-one
- D. both  $f$  and  $g$  are onto

**Answer: 1,2,3,4**



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4. Show that  $f(x) = 2x + \cot^{-1} x + \log(\sqrt{1+x^2} - x)$  is increasing in  $\mathbb{R}$

- A. increases in  $[0, \infty)$
- B. idecreases in  $[0, \infty)$
- C. neither increases nor decreases in  $[0, \infty)$
- D. increases in  $(-\infty, \infty)$

Answer: 1,4

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5. Let  $f(x) = |x^2 - 3x - 4|$ ,  $-1 \leq x \leq 4$ . Then

A.  $f(x)$  is monotonically increasing in  $[-1, 3/2]$

B.  $f(x)$  is monotonically decreasing in  $(3/2, 4]$

C. the maximum value of  $f(x)$  is  $\frac{25}{4}$

D. the minimum value of  $f(x)$  is 0

Answer: 1,2,3,4

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6. If  $f(x) = \int_0^x \frac{\sin t}{t} dt$ ,  $x > 0$ , then

A.  $f(x)$  has a local maxima at  $x = n\pi$  ( $n = 2k, k \in I^+$ )



B.  $f(x)$  has a local minimum at  $x = n\pi$  ( $n = 2k, k \in I^+$ )

C.  $f(x)$  has neither maxima nor minima at  $x = n\pi$  ( $n \in I^+$ )

D.  $f(x)$  has local maxima at  $x = n\pi$  ( $n = 2k - 1, k \in I^+$ )

**Answer: 2,4**



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7. The values of parameter  $a$  for which the point of minimum of the

function  $f(x) = 1 + a^2x - x^3$  satisfies the inequality

$$\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0 \text{ are } (2\sqrt{3}, 3\sqrt{3}) \text{ (b) } -3\sqrt{3}, -2\sqrt{3}) \text{ (c) } (-2\sqrt{3}, 3\sqrt{3})$$

(d)  $(-2\sqrt{2}, 2\sqrt{3})$

A.  $2\sqrt{3}, 3\sqrt{3}$

B.  $-3\sqrt{3}, -2\sqrt{3}$

C.  $-2\sqrt{3}, 3\sqrt{3}$

D.  $-3\sqrt{2}, 2\sqrt{3}$

**Answer: 1,2**



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8. Let  $f(x) = ax^2 - b|x|$ , where  $a$  and  $b$  are constant. Then at  $x=0$ ,  $f(x)$  has

A. a maxima whenever  $a > 0, b > 0$

B. a maxima whenever  $a > 0, b < 0$

C. minima whenever  $a > 0, b < 0$

D. neither a maxima nor a minima whenever  $a > 0, b < 0$

**Answer: 1,3**



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9. The function  $y = \frac{2x - 1}{x - 2}, (x \neq 2)$

A. is its own inverse

B. decreases at all values of  $x$  in the domain

C. has a graph entirely above the  $x$  axis

D. is unbounded

**Answer: 1,2,4**



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**10.**

Let

$f(x) = ax^2 + bx + c$  and  $f(-1) < 1$ ,  $f(1) > -1$ ,  $f(3) < -4$  and  $a \neq 0$

, then

A.  $g(f(x + 1)) > f(f(x - 1))$

B.  $f(g(x - 1)) > f(f(x + 1))$

C.  $g(f(x + 1)) < g(f(x - 1))$

D.  $g(f(x + 1)) < g(g(x - 1))$

Answer: 2,3



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11. If  $f(x) = x^3 - x^2 + 100x + 2002$ , then  $f(1000) > f(1001)$

$$f\left(\frac{1}{2000}\right) > f\left(\frac{1}{2001}\right) \quad f(x-1) > f(x-2) \quad f(2x-3) > f(2x)$$

A.  $f(1000) < f(1001)$

B.  $f\left(\frac{1}{2000}\right) > f\left(\frac{1}{2001}\right)$

C.  $f(x-1) > f(x-2)$

D.  $f(2x-3) > f(2x)$

Answer: 2,3



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12. If  $f'(x) = g(x)(x-a)^2$ , where  $g(a) \neq 0$  and  $g$  is continuous at  $x = a$ ,

then :

A.  $f$  is increasing in the neighborhood of  $a$  if  $g(x) > 0$

B.  $f$  is increasing in the neighborhood of  $a$  if  $g(x) < 0$

C.  $f$  is decreasing in the neighborhood of  $a$  if  $g(x) > 0$

D.  $f$  is decreasing in the neighborhood of  $a$  if  $g(x) < 0$

**Answer: 1,4**

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**13.** The value of  $a$  for which the function

$f(x) = (4a - 3)(x + \log 5) + 2(a - 7) \frac{\cot x}{2} \frac{\sin^2 x}{2}$  does not possess

critical points is  $\left(-\infty, -\frac{4}{3}\right)$  (b)  $(-\infty, -1)$  [1,  $\infty$ ) (d)  $(2, \infty)$

A.  $(-\infty, -\frac{4}{3})$

B.  $(-\infty, -1)$

C.  $[1, \infty)$

D.  $(2, \infty)$

**Answer: 1,4**



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14. Let  $f(x) = (x^2 - 1)^{n+1} + (x^2 + x + 1)$ . Then  $f(x)$  has local extremum at  $x = 1$ , when  $n$  is (A)  $n = 2$  (C)  $n = 4$  (B)  $n = 3$  (D)  $n = 5$

A. a maxima at  $x=1$  if  $n$  is odd

B. a maxima at  $x=1$  if  $n$  is even

C. a minima  $x=1$  if  $n$  is even

D. a minima at  $x=2$  if  $n$  is even

**Answer: 1,3,4**



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15. Let  $f(x)=\sin x+ax+b$ . Then which of the following is/are true?

A.  $f(x) = 0$  has only one real root which is positive if  $a > 1, b < 0$

B.  $f(x) = 0$  has only one real root which is negative if  $a > 1, b > 0$

C.  $f(x) = 0$  has only one real root which is negative if  $a < -1, b < 0$

D. None of these

**Answer: 1,2,3**



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16. The function  $\frac{\sin(x+a)}{\sin(x+b)}$  has no maxima or minima if

$b - a = n\pi, n \in I$   $b - a = (2n + 1)\pi, n \in I$   $b - a = 2n\pi, n \in I$  (d)

none of these

A.  $b - a = n\pi, n \in I$

B.  $b - a = (2n + 1)\pi, n \in I$

C.  $b - a = 2n\pi, n \in I$

D. none of these

**Answer: 1,2,3**



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17. Consider  $f(x) = ax^4 + cx^2 + dx + e$  has no point of inflection. Then which of the following is/are possible?

A.  $a > 0, c < 0$

B.  $a < 0, c > 0$

C.  $a, c < 0$

D.  $a, c > 0$

**Answer: 3,4**



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18. Let  $f(x) = \begin{cases} \frac{(x-1)(6x-1)}{2x-1}, & \text{if } x \neq \frac{1}{2} \\ 0, & \text{if } x = \frac{1}{2} \end{cases}$ . Then at  $x = \frac{1}{2}$ , which of the following is/are not true?  $f$  has a local maxima  $f$



has a local minima  $f$  has an inflection point.  $f$  has a removable discontinuity.

- A.  $f$  has a local maxima
- B.  $f$  has a local minima
- C.  $f$  has an inflection point
- D.  $f$  has a removable discontinuity

**Answer: 1,2,4**



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**19.** In which of the following graphs is  $x = c$  point of inflection?

A. 

B. 

C. 

D. 

**Answer: 1,2,4**



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20. Let  $f(x)$  be an increasing function defined on  $(0, \infty)$ . If  $f(2a^2 + a + 1) > f(3a^2 - 4a + 1)$ , then the possible integers in the range of  $a$  is/are 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

**Answer: 2,3,4**



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21. If  $f(x) = (\sin^2 x - 1)^n$ , then  $x = \frac{\pi}{2}$  is a point of local maximum, if  $n$  is odd local minimum, if  $n$  is odd local maximum, if  $n$  is even local minimum, if  $n$  is even

A. local maximum , if  $n$  is odd

B. local minimum, if  $n$  is odd

C. local maximum if  $n$  is even

D. local minimum if  $n$  is even

**Answer: 1,4**



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22. For the cubic function  $f(x) = 2x^3 + 9x^2 + 12x + 1$ , which one of the following statement/statements hold good?  $f(x)$  is non-monotonic.

$f(x)$  increases in  $(-\infty, -2) \cup (-1, \infty)$  and decreases in

$(-2, -1)$   $f: \mathbb{R} \rightarrow \mathbb{R}$  is bijective. Inflection point occurs at  $x = -\frac{3}{2}$ .

A.  $f(x)$  is non monotonic

B.  $f(x)$  increases in  $(-\infty, -2) \cup (-1, \infty)$  and decreases in  $(-2, -1)$

C.  $f: \mathbb{R} \rightarrow \mathbb{R}$  is objective

D. O inflection point occurs at  $x = -3/2$

**Answer: 1,2,4**

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**23.** Let  $f(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x$ , where  $a_i$ 's are real and  $f(x) = 0$  has a positive root  $\alpha_0$ . Then  $f'(x) = 0$  has a positive root  $\alpha_1$  such that

A.  $f(x)=0$  has a root  $\alpha_1$  such that  $0 < \alpha_1 < \alpha_0$

B.  $f(x) = 0$  has at least two real roots

C.  $f(x) = 0$  has at least one real root

D. none of these

**Answer: 1,2,3**



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**24.** If  $f(x)$  and  $g(x)$  are two positive and increasing functions, then which of the following is not always true?  $[f(x)]^{g(x)}$  is always increasing  
 $[f(x)]^{g(x)}$  is decreasing, when  $f(x) < 1$   $[f(x)]^{g(x)}$  is increasing, then  $f(x) > 1$ . If  $f(x) > 1$ , then  $[f(x)]^{g(x)}$  is increasing.

- A.  $[f(x)]^{g(x)}$  is always increasing
- B.  $[f(x)]^{g(x)}$  is decreasing when  $f(x) < 1$
- C. If  $[f(x)]^{g(x)}$  is increasing then  $f(x) > 1$
- D. If  $f(x) > 1$ , then  $[f(x)]^{g(x)}$  is increasing

**Answer: 1,2,3**



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25. An extremum of the function

$$f(x) = \frac{2-x}{\pi} \cos \pi(x+3) + \frac{1}{\pi^2} \sin \pi(x+3), 0x < 4,$$

occurs at  $x = 1$  (b)  $x = 2$   $x = 3$  (d)  $x = \pi$

A.  $x=1$

B.  $x=2$

C.  $x=3$

D.  $x=\pi$

Answer: 1,3



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26. For the function  $f(x) = x^4(12(\log)_e x - 7)$ , the point (1,7) is the point of inflection.  $x = e^{\frac{1}{3}}$  is the point of minima the graph is concave downwards in (0,1) the graph is concave upwards in  $(1, \infty)$

A. the point (1,7) is the point of minima

B.  $x=e^{1/3}$  is the point of minima

C. the graph is concave downwards in  $(0,1)$

D. the graph is concave upwards in  $(1, \infty)$

**Answer: 1,2,3,4**



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27. Let  $f(x) = \log(2x - x^2) + \frac{\sin(\pi x)}{2}$ . Then which of the following is/are true? Graph of  $f$  is symmetrical about the line  $x = 1$  Maximum value of  $f$  is 1. Absolute minimum value of  $f$  does not exist. none of these

A. Graph of  $f$  is symmetrical about the line  $x=1$

B. maximum value of  $f$  is 1

C. absolute minimum value of  $f$  does not exist

D. none of these

**Answer: 1,2,3**

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28. Which of the following hold(s) good for the function  $f(x) = 2x - 3x^{\frac{2}{3}}$ ?  $f(x)$  has two points of extremum.  $f(x)$  is convave upward  $\forall x \in R$ .  $f(x)$  is non-differentiable function.  $f(x)$  is continuous function.

- A.  $f(x)$  has two points of extremum
- B.  $f(x)$  is convave upward  $\forall x \in R$
- C.  $f(x)$  is non differentiable function
- D.  $f(x)$  is continuous function

**Answer: 1,2,3,4**

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29. For the function  $f(x) = \frac{e^x}{1 + e^x}$ , which of the following hold good?  $f$  is monotonic in its entire domain. Maximum of  $f$  is not attained even



though  $f$  is bounded  $f$  has a point of inflection.  $f$  has one asymptote.

- A.  $f$  is monotonic in its entire domain
- B. maximum of  $f$  is not attained even though
- C.  $f$  is bounded
- D.  $f$  has a point of inflection

**Answer: 1,2,3**



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**30.** Which of the following is true about point of extremum  $x = a$  of function  $y = f(x)$ ? At  $x = a$ , function  $y = f(x)$  may be discontinuous.

At  $x = a$ , function  $y = f(x)$  may be continuous but non-differentiable.

At  $x = a$ , function  $y = f(x)$  may have point of inflection. none of these

- A. At  $x = a$ , function  $y = f(x)$  may be discontinuous
- B. At  $x = a$  function  $y = f(x)$  may be continuous but non differentiable
- C. At  $x = a$  function  $y = f(x)$  may have point of inflection

D. none of these

Answer: 1,2,3



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31. which of the following function has point of extremum at  $x=0$ ?

A.  $f(x) = e^{-|x|}$

B.  $f(x) = \sin |x|$

C.  $f(x) = \begin{cases} x^2 + 4x + 3 & x < 0 \\ -x & x \geq 0 \end{cases}$

D.  $f(x) = \begin{cases} x^2 + 4x + 3 & x < 0 \\ -x & x \geq 0 \end{cases}$

Answer: 1,2,4



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32. Which of the following function/functions has/have point of inflection?  $f(x) = x^{\frac{6}{7}}$  (b)  $f(x) = x^6$   $f(x) = \cos x + 2x$  (d)  $f(x) = x|x|$

A.  $f(x)=x^{6/7}$

B.  $f(x) = x^6$

C.  $f(x)=\cos x+2x$

D.  $f(x)=x|x|$

**Answer: 3,4**



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33. The function  $f(x)=x^2 + \frac{\lambda}{x}$  has a

A. minimum at  $x=2$  if  $\lambda=16$

B. maximum at  $x=2$  if  $\lambda=16$

C. maximum for no real value of  $\lambda$

D. point of inflection at  $x=1$  if  $\lambda=-1$

**Answer: 1,3,4**



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**34.** The function  $f(x) = x^{\frac{1}{3}}(x - 1)$  has two inflection points has one point of extremum is non-differentiable has range  $\left[ -3x2^{-\frac{8}{3}}, \infty \right)$

A. has two inflection points

B. has one point of extremum

C. is non differentiable

D. has range  $\left[ -3 \times 2^{-8/3}, \infty \right)$

**Answer: 1,2,3,4**



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**35.** Let f be function  $f(x) = \cos x - \left(1 - \frac{X^2}{2}\right)$ . Then

- A.  $f(x)$  is an increasing function in  $(0, \infty)$
- B.  $f(x)$  is a decreasing function in  $(-\infty, \infty)$
- C.  $f(x)$  is an increasing function in  $(-\infty, \infty)$
- D.  $f(x)$  is a decreasing function in  $(-\infty, 0)$

**Answer: 1,4**

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**36.** Consider the function  $f(x) = x \cos x - \sin x$ , then identify the statement which is correct

- A.  $f$  is odd
- B.  $f$  is monotonic decreasing at  $x=0$
- C.  $f$  has point of inflection at  $x=0$
- D.  $f$  has a maxima at  $x = \pi$

**Answer: 1,2,3**



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37. If  $f(x) = \frac{x^2}{2 - 2 \cos x}$ ;  $g(x) = \frac{x^2}{6x - 6 \sin x}$  where  $0 < x < 1$ , then

(A) both 'f' and 'g' are increasing functions

- A. f is increasing function
- B. g is increasing function
- C. f is decreasing function
- D. g is decreasing function

Answer: 1,4



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38. Find the greatest value of  $f(x) \frac{1}{2ax - x^2 - 5a^2} \in [-3, 5]$

depending upon the parameter  $a$ .

- A. f(5) if a =1

B.  $f(-3)$  if  $a = 1$

C.  $f(5)$  if  $a < 1$

D.  $f(-3)$  if  $a > 1$

**Answer: 1,2,3,4**



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39. For any acute angled  $\triangle ABC$ ,  $\frac{\sin A}{A} + \frac{\sin B}{B} + \frac{\sin C}{C}$  can

A. 1

B. 2

C. 3

D. 4

**Answer: 1,2**



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40. Let  $f(x)$  be a non negative continuous and bounded function for all  $x \geq 0$ . If  $(\cos x)f(x) < (\sin x - \cos x)f(x) \forall x \geq 0$ , then which of the following is/are correct?

A.  $f(6) + f(5) > 0$

B.  $x^2 - 3x + 2 + f(7) = 0$  has 2 distinct solution

C.  $f(5)f(7)-f(5)=0$

D.  $\lim_{x \rightarrow 6} \frac{f(x) - \sin(\pi x)}{x - 6} = 1$

Answer: 2,3



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### Exercise Comprehension

1.  $f(x) = \sin^{-1} x + x^2 - 3x + \frac{x^3}{3}, x \in [0, 1]$

A.  $f(x)$  has a point of maxima



B.  $f(x)$  has a point of minima

C.  $f(x)$  is increasing

D.  $f(x)$  is decreasing

**Answer: 2**



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2. which of the following is true for  $x \in [0, 1]$ ?

A.  $\sin^{-1} x + x^2 - x \frac{9 - x^2}{3} \leq 0$

B.  $\sin^{-1} x + x^2 - x \frac{9 - x^2}{3} \geq 0$

C.  $\sin^{-1} x + x^2 - x \frac{9 - x^2}{3} \leq 0$

D.  $\sin^{-1} x + x^2 - x \frac{9 - x^2}{3} \geq 0$

**Answer: 1**



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3. Let  $f'(\sin x) < 0$  and  $f''(\sin x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$  and  $g(x) = f(\sin x) + f(\cos x)$ , then

- A. g' is increasing
- B. g' is decreasing
- C. g' has a point of minima
- D. g' has a point of maxima

**Answer: 1**



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4. Let  $f'(\sin x) < 0$  and  $f''(\sin x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$  and  $g(x) = f(\sin x) + f(\cos x)$

which of the following is true?

- A. g(x) is decreasing in  $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$
- B. g(x) increasing in  $\left(0, \frac{\pi}{4}\right)$

C.  $g(x)$  is nonotonically increasing in  $\left(0, \frac{\pi}{2}\right)$

D. none of these

**Answer: 4**



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5. Let  $f'(\sin x) < 0$  and  $f''(\sin x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$  and  $g(x)$

$=f(\sin x)+f(\cos x)$

$$\text{consider function } f(x) = \begin{cases} -x^2 + 4x + a & x \leq 3 \\ ax + b & 3 < x < 4 \\ -\frac{b}{4}x + 6 & x \geq 4 \end{cases}$$

which of the following is true?

A.  $f(x)$  is discounting function for all ordered pairs  $(a,b)$

B.  $f(x)$  is contionuous for finite number of ordered pairs  $(,ab)$

C.  $f(x)$  can be differentiable

D.  $f(x)$  is continous for infinite ordered pairs  $(a,b)$

**Answer: 4**



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6. Let  $f'(\sin x) < 0$  and  $f''(\sin x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$  and  $g(x) = f(\sin x) + f(\cos x)$

If  $x = 3$  is the only point of minima in its neighborhood and  $x = 4$  is neither a point of maxima nor a point minima, then which of the following can be true?

A.  $a > 0, b < 0$

B.  $a < 0, b < 0$

C.  $a > 0, b \in \mathbb{R}$

D. none of these

Answer: 1



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7. Let  $f'(\sin x) < 0$  and  $f''(\sin x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$  and  $g(x) = f(\sin x) + f(\cos x)$

If  $x=4$  is the only point of maxima in its neighborhood but  $x=3$  is neither a point of maxima nor a point of minima then which of the following can be true?

A.  $a < 0, b > 0$

B.  $a > 0, b < 0$

C.  $a < 0, b < 0$

D. not possible

**Answer: 4**



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8. Let  $f'(\sin x) < 0$  and  $f''(\sin x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$  and  $g(x) = f(\sin x) + f(\cos x)$

If  $x=3$  is a point of minima and  $x=4$  is a point of maxima then which of the following is true?

A.  $a < 0, b > 0$

B.  $a > 0, b < 0$

C.  $a > 0, b > 0$

D. not possible

**Answer: 3**



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9. If  $\phi(x)$  is a differentiable real valued function satisfying  $\phi(x) + 2\phi' \leq 1$ ,

then it can be adjusted as

$$e^{2x}\phi(x) + 2e^{2x}\phi'(x) \leq e^{2x} \text{ or } \frac{d}{dx} \left( e^{2x}\phi(x) - \frac{e^{2x}}{2} \right) \leq 0 \text{ or } \frac{d}{dx} e^{2x} \left( \phi(x) - \frac{1}{2} \right) \leq 0$$

Here  $e^{2x}$  is called integrating factor which helps in creating single

differential coefficient as shown above. Answer the following question:

If  $p(1)=0$  and  $dP \frac{x}{dx} < P(x)$  for all  $x \geq 1$  then

A.  $P(x) > 0 \forall x > 1$

B.  $P(x)$  is a constant function

C.  $P(x) < 0 \forall x > 1$

D. none of these

**Answer: 1**

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10. If  $H(x_0)=0$  for some  $x=x_0$  and  $\frac{d}{dx}H(x) > 2cxH(x)$  for all  $x \geq x_0$

where  $c > 0$  then

A.  $H(x) = 0$  has root for  $x > x_0$

B.  $H(x) = 0$  has no root for  $x > x_0$

C.  $H(x)$  is a constant function

D. none of these

**Answer: 2**

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11. Let  $h(x) = f(x) - a(f(x))^3$  for every real number  $x$

$h(x)$  increase as  $f(x)$  increases for all real values of  $x$  if

A.  $a \in (0, 3)$

B.  $a \in (-2, 2)$

C.  $[3, \infty)$

D. none of these

**Answer: 1**

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12.  $H(x)$  increase as  $f(x)$  decreases for all real values of  $x$  if

A.  $a \in (0, 3)$

B.  $a \in (-2, 2)$



C.  $[3, \infty)$

D. none of these

**Answer: 4**



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13. If  $f(x)$  is strictly increasing function then  $h(x)$  is non monotonic function given

A.  $a \in (0, 3)$

B.  $a \in (-2, 2)$

C.  $(3, \infty)$

D.  $a \in (-\infty, 0) \cup (3, \infty)$

**Answer: 4**



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14. If the function  $f(x) = x^3 - 9x^2 + 24x + c$  has three real and distinct roots  $\alpha$ ,  $\beta$  and  $\gamma$ , then the value of  $[\alpha] + [\beta] + [\gamma]$  is

A. (-20, -16)

B. (-20,-18)

C. (-18,-16)

D. none of these

Answer: 1



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15.  $f(x) = x^3 - 9x^2 + 24x + c$  has real roots  $\alpha, \beta, \gamma$  then find the value of  $[\alpha] + [\beta] + [\gamma]$ . Where  $[\ ]$  represents greatest integer function

A. (-20, -16)

B. (-20,-18)

C. (-18,-16)

D. none of these

**Answer: 3**



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16. If  $[\alpha] + [\beta] + [\gamma] = 7$  then the values of  $c$ , where  $[.]$  represent the greatest integer function are,

A. (-20,-16)

B. (-20,-18)

C. (-18,-16)

D. none of these

**Answer: 2**



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17. Let  $f(x) = 4x^2 - 4ax + a^2 - 2a + 2$  and the global minimum value of  $f(x)$  for  $x$  in  $[0,2]$  is equal to 3

The number of values of  $a$  for which the global minimum value equal to 3 for  $x$  in  $[0,2]$  occurs at the endpoint of interval  $[0,2]$  is

A. 1

B. 2

C. 3

D. 0

**Answer: 2**



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18. Let  $f(x) = 4x^2 - 4ax + a^2 - 2a + 2$  and the global minimum value of  $f(x)$  for  $x$  in  $[0,2]$  is equal to 3

The number of values of  $a$  for which the global minimum value equal to 3 for  $x$  in  $[0,2]$  occurs for the value of  $x$  lying in  $(0,2)$  is

A. 1

B. 2

C. 3

D. 0

**Answer: 4**



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**19.** Let  $f(x) = 4x^2 - 4ax + a^2 - 2a + 2$  and the global minimum value of  $f(x)$  for  $x$  in  $[0,2]$  is equal to 3

The values of  $a$  for which  $f(x)$  is monotonic for  $x \in [0,2]$

A.  $a \leq 0$  or  $a \geq 4$

B.  $0 \leq a \leq 4$

C.  $a > 0$

D. none of these

**Answer: 1**



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20. Let  $f(x) = x^3 - 3(7 - a)x^2 - 3(9 - a^2)x + 2$

The values of parameter  $a$  if  $f(x)$  has a negative point of local minimum are

A.  $\pi$

B.  $(-3, 3)$

C.  $\left(-\infty, \frac{58}{14}\right)$

D. none of these

**Answer: 1**



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21. Let  $f(x) = x^3 - 3(7 - a)x^2 - 3(9 - a^2)x + 2$

The values of parameter  $a$  if  $f(x)$  has a positive point of local maxima are

A.  $\pi$

B.  $-\infty, -3 \cup \left(3, \frac{29}{7}\right)$

C.  $-\infty, \frac{58}{14}$

D. none of these

**Answer: 2**



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22. Let  $f(x) = x^3 - 3(7 - a)x^2 - 3(9 - a^2)x + 2$

The values of parameter  $a$  if  $f(x)$  has points of extrema which are opposite in sign are

A.  $\pi$

B.  $(-3,3)$

C.  $\left(-\infty, \frac{58}{14}\right)$

D. none of these

**Answer: 2**



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23. consider the function  $f(x) = 1\left(1 + \frac{1}{x}\right)^x$

The domain of  $f(x)$  is

A.  $(-1, 0) \cup (0, \infty)$

B.  $\mathbb{R} - \{0\}$

C.  $(-\infty, -1) \cup (0, \infty) \cup (0, \infty)$

D.  $(0, \infty)$

**Answer: 3**



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24. consider the function  $f(x) = 1 + \frac{1}{x}$

The function  $f(x)$

- A. has a maximum but no minima
- B. has a minima but no maxima
- C. has exactly one maxima and one minima
- D. is monotonic

**Answer: 4**



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25. consider the function  $f(x) = 1 + \frac{1}{(x)^x}$

The range of the function  $f(x)$  is

- A.  $(0, p\infty)$
- B.  $(-\infty, e)$
- C.  $(1, \infty)$

D.  $(1, e) \cup (e, \infty)$

**Answer: 4**



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26. consider the function  $f(x) = x + \cos x - a$

which of the following is not true about  $y = f(x)$ ?

- A. It is an increasing function
- B. It is a monotonic function
- C. It has infinite points of inflection s
- D. None of these

**Answer: 4**



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27. consider the function  $f(x) = x + \cos x - a$

values of  $a$  which  $f(x) = 0$  has exactly one positive root are

- A.  $(0, 1)$
- B.  $(-\infty, 1)$
- C.  $(-1, 1)$
- D.  $(1, \infty)$

**Answer: 4**



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28. consider the function  $f(x) = x + \cos x - a$

values of  $a$  for which  $f(x) = 0$  has exactly one negative root are

- A.  $(0, 1)$
- B.  $(-\infty, 1)$
- C.  $(-1, 1)$

D.  $(1, \infty)$

**Answer: 2**



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29. consider the function  $f(x) = 3x^4 + 4x^3 - 12x^2$

$Y = f(x)$  increase in the interval

A.  $(-1, 0) \cup (2, \infty)$

B.  $(-\infty, 0) \cup (1, 2)$

C.  $(-2, 0) \cup (1, \infty)$

D. none of these

**Answer: 3**



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30. consider the function  $f(x) = 3x^4 + 4x^3 - 12x^2$

The range of the function  $y=f(x)$  is

A.  $(-\infty, \infty)$

B.  $[-32, \infty)$

C.  $[0, \infty)$

D. none of these

**Answer: 2**



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31. consider the function  $f(x) = 3x^4 + 4x^3 - 12x^2$

The range of values of  $a$  for which  $f(x) = a$  has no real

A.  $(4, \infty)$

B.  $(10, \infty)$

C.  $(20, \infty)$

D. none of these

**Answer: 4**



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32. consider the function  $f: R \rightarrow R, f(x) = \frac{x^2 - 6x + 4}{x^2 + 2x + 4}$

$f(x)$  is

A. unbounded function

B. one one function

C. onto function

D. none of these

**Answer: 4**



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33. consider the function  $f: R \rightarrow R, f(x) = \frac{x^2 - 6x + 4}{x^2 + 2x + 4}$

which of the following is not true about  $f(x)$ ?

- A.  $f(x)$  has two points of extremum
- B.  $f(x)$  has only one asymptote
- C.  $f(x)$  is differentiable for all  $x$  in  $R$
- D. none of these

**Answer: 4**



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34. consider the function  $f: R \rightarrow R, f(x) = \frac{x^2 - 6x + 4}{x^2 + 2x + 4}$

Range of  $f(x)$  is

- A.  $(-\infty, -\frac{2}{3}] \cup [2, 0)$
- B.  $[\frac{-1}{3}, 5]$
- C.  $(-\infty, 2) \cup [\frac{7}{3}, \infty)$

D.  $(20, \infty)$

**Answer: 2**



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35. Consider a polynomial  $y = P(x)$  of the least degree passing through  $A(-1,1)$  and whose graph has two points of inflection  $B(1,2)$  and  $C$  with abscissa 0 at which the curve is inclined to the positive axis of abscissa at an angle of  $\sec^{-1} \sqrt{2}$

The value of  $P(2)$  is

A. -1

B.  $-\frac{3}{2}$

C.  $\frac{5}{2}$

D.  $\frac{7}{2}$

**Answer: 3**



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36. Consider a polynomial  $y = P(x)$  of the least degree passing through  $A(-1,1)$  and whose graph has two points of inflection  $B(1,2)$  and  $C$  with abscissa 0 at which the curve is inclined to the positive axis of abscissa at an angle of  $\sec^{-1} \sqrt{2}$

The value of  $P(0)$  is

A. 1

B. 0

C.  $\frac{3}{4}$

D.  $\frac{1}{2}$

**Answer: 4**



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37. Consider a polynomial  $y = P(x)$  of the least degree passing through  $A(-1,1)$  and whose graph has two points of inflection  $B(1,2)$  and  $C$  with

abscissa 0 at which the curve is inclined to the positive axis of abscissa at an angle of  $\sec^{-1} \sqrt{2}$

The equation  $P(x) = 0$  has

- A. three distinct real roots
- B. one real root
- C. three real roots such that one root is repeated
- D. none of these

**Answer: 3**



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**38.** Let  $f(x)$  be real valued continuous function on  $\mathbb{R}$  defined as  $f(x) = x^2 e^{-|x|}$

The values of  $k$  for which the equation  $x^2 e^{-|x|} = k$  has four real roots are

- A.  $0 < k < e$
- B.  $0 < k < \frac{8}{e^2}$
- C.  $0 < k < \frac{4}{e^2}$

D. none of these

**Answer: 3**



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**39.** Let  $f(x)$  be real valued continuous function on  $\mathbb{R}$  defined as  $f(x) = x^2 e^{-|x|}$

Number of points of inflection for  $y = f(x)$  is

A.  $y = f(x)$  has two points of maxima

B.  $y = f(x)$  has only one asymptote

C.  $f(x) = 0$  has three real roots

D. none of these

**Answer: 4**



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40. Let  $f(x)$  be real valued continuous function on  $\mathbb{R}$  defined as  $f(x) = x^2 e^{-|x|}$

Number of points of inflection for  $y = f(x)$  is

A. 1

B. 2

C. 3

D. 4

**Answer: 4**



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41.  $P(x)$  be a polynomial of degree 3 satisfying  $P(-1) = 10$ ,  $P(1) = -6$  and  $p(x)$  has maxima at  $x = -1$  and  $p(x)$  has minima at  $x=1$  then The value of  $P(2)$  is

A. -15

B. -16

C. -17

D. -22

**Answer: 3**



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42.  $P(x)$  be a polynomial of degree 3 satisfying  $P(-1) = 10$ ,  $P(1) = -6$  and  $p(x)$  has maxima at  $x = -1$  and  $p(x)$  has minima at  $x=1$  then The value of  $P(1)$  is

A. -12

B. -10

C. 15

D. 21

**Answer: 1**



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## Linked Comprehension Type

1. consider the graph of  $y=g(x)=f'(x)$  given that  $f(c) = 0$ , where  $y=f(x)$  is a polynomial funtion



The graph of  $y=f(x)$  will intersect the x axis

- A. twice
- B. once
- C. never
- D. none of these

**Answer: 2**



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2. consider the graph of  $y=g(x)=f'(x)$  given that  $f(c) = 0$ , where  $y=f(x)$  is a polynomial funtion



The equation  $f(x) = 0, a \leq x \leq b$ , has

- A. four real roots
- B. no real roots
- C. two distinct real roots
- D. at least three repeated roots

**Answer: 4**



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3. consider the graph of  $y=g(x)=f'(x)$  given that  $f(c) = 0$ , where  $y=f(x)$  is a polynomial function



The graph of  $y=f(x), a \leq x \leq b$  has

- A. two points of inflection
- B. one point of inflection

C. no point of inflection

D. none of these

**Answer: 2**



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4. consider the graph of  $y=g(x)=f'(x)$  given that  $f(c) = 0$ , where  $y=f(x)$  is a polynomial function



The function  $y = f(x)$ ,  $a \leq x \leq b$ , has

A. exactly one local maxima

B. one local minima and one maxima

C. exactly one local minima

D. none of these

**Answer: 4**



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5. consider the graph of  $y=g(x)=f'(x)$  given that  $f(c) = 0$ , where  $y=f(x)$  is a polynomial function



The equation  $f''(x)=0$

- A. has no real root
- B. has at least one real root
- C. has at least two distinct roots
- D. none of these

**Answer: 2**

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6. The graph of  $y =g(x) =f'(x)$  is as shown in the following figure analyse this graph and answer the following question



The graph of  $y=f(x)$  for  $a < x < b$  has

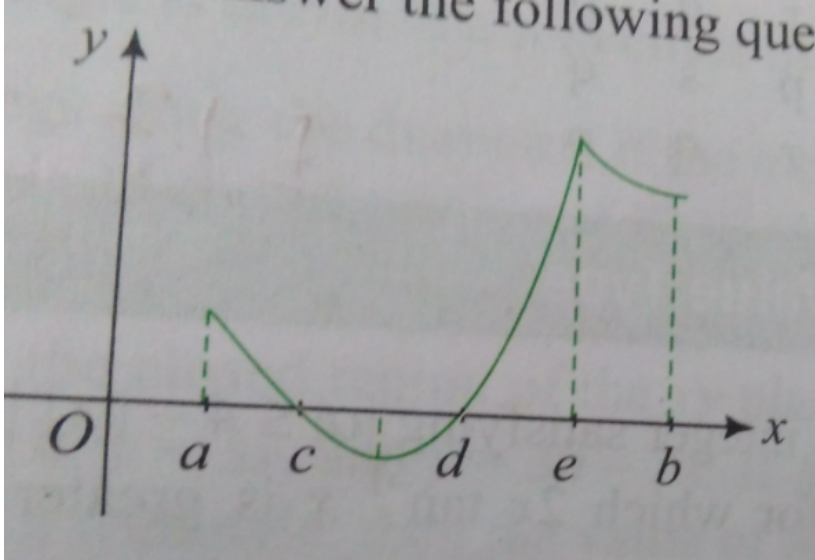
- A. no point of extremum
- B. one point of extrema
- C. two points extrema
- D. can't say anything

**Answer: 3**



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7. The graph of  $y = g(x) = f(x)$  is as shown in the following figure analyse this graph and answer the following question



Number of points of inflection on the graph of  $y = f(x)$  for  $a < x < b$  has

- A. 0
- B. 1
- C. 2
- D. can't say anything

**Answer: 3**



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8. The graph of  $y = g(x) = f(x)$  is as shown in the following figure analyse this graph and answer the following question



Which of the following is not true about the graph of  $y = f(x)$ ,  $a < x < b$

- A. always increasing
- B. discontinuous at one point
- C. first increases then decreases
- D. none of these

**Answer: 3**



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**Matrix Match Type**

1. consider function  $f(x) = x^4 - 14x^2 + 24x - 3$ . Now match the following lists:



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2. Match the following lists



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3. Let  $f(x) = (x - 1)^m(2 - x)^n$ ,  $mn \in N$  and  $m, n < 2$ . Then match the following lists:



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4. The function  $f(x) = \sqrt{ax^3 + bx^3 + cx + d}$  has its nonzero local minimum and maximum values at  $x=-2$  and  $x=2$  respectively. If  $a$  is a root of

$x^2 - x - 6 = 0$  then match the following lists:



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5. Match the following lists:



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6. Match the following lists :



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7. Match the following lists:



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8. Match the following lists:



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9.  $f(x)$  is polynomial function of degree 6 which  $\lim_{x \rightarrow 0} 1 + \frac{f(x)}{(x^3)^{1/x}} = e^2$

and has local maximum at  $x = 1$  and local minimum at  $x = 0$  and  $x = 2$ . Now match the following lists and then choose the correct code.



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10. Match the statement /expressions in List I with the open intervals in

List II



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11. Let  $f(x) = x + \log_e x - x \log_e x, x \in (0, \infty)$

List I contains information about zero of  $f(x)$ ,  $f'(x)$  and  $f''(x)$

List II contains information about the limiting behaviour of  $f(x)$ ,  $f'(x)$  and  $f''(x)$  at infinity

List III contains information about increasing /decreasing nature of  $f(x)$  and  $f'(x)$



which of the following options is the only CORRECT combination?

A. (iv)(i)(S)

B. (I)(ii)(R)

C. (III)(iv)(P)

D. (II)(ii)(S)

**Answer: 4**



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## Exercise Numerical

1. If  $\alpha$  is an integer satisfying  $|\alpha| \leq 4 - |[x]|$ , where  $x$  is a real number for which  $2x \tan^{-1} x$  is greater than or equal to  $\ln(1 + x^2)$ , then the number of maximum possible values of  $a$  (where  $[.]$  represents the greatest integer function) is \_\_\_\_\_

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2. From a given solid cone of height  $H$ , another inverted cone is carved whose height is  $h$  such that its volume is maximum. Then the ratio  $\frac{H}{h}$  is

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3. Let  $f(x) = \begin{cases} x^3 + x^2 + 3x + \sin x \left( 3 + \frac{\sin(1)}{x} \right) & x \neq 0 \\ 0 & x = 0 \end{cases}$

Then the number of points where  $f(x)$  attains its minimum value is \_\_\_\_\_.

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4. Let  $f(x)$  be a cubic polynomial which has local maximum at  $x = -1$  and  $f(x)$  has a local minimum at  $x = 1$ . If  $f(-1) = 10$  and  $f(3) = -22$ , then one fourth of the distance between its two horizontal tangents is \_\_\_\_\_

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5. Consider  $P(x)$  to be a polynomial of degree 5 having extremum at  $x = -1, 1$  and  $\lim_{x \rightarrow 0} p \frac{x}{x^3} - 2 = 4$ . Then the value of  $P(x)$  \_\_\_\_\_.

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6. If  $m$  is the minimum value of  $f(x, y) = x^2 - 4x + y^2 + 6y$  when  $x$  and  $y$  are subjected to the restrictions  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$ , then the value of  $|m|$  is \_\_\_\_\_

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7. For a cubic function  $y = f(x)$ ,  $f^x = 4x$  at each point  $(x, y)$  on it and it crosses the  $x$ -axis at  $(-2, 0)$  at an angle of  $45^\circ$  with positive direction of the  $x$ -axis. Then the value of  $\left| \frac{f(1)}{5} \right|$  is\_\_\_\_\_

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8. Number of integral values of  $b$  for which the equation  $\frac{x^3}{3} - x = b$  has three distinct solutions is\_\_\_\_\_

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9. Let  $f(x) = \begin{cases} x + 2, & x < -1 \\ x^2, & -1 \leq x < 1 \\ (x - 2)^2, & x \geq 1 \end{cases}$  Then number of times  $f'(x)$  changes its sign in  $(-\infty, \infty)$  is\_\_\_\_\_

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10. The number of nonzero integral values of  $a$  for which the function

$$f(x) = x^4 + ax^3 + \frac{3x^2}{2} + 1 \text{ is concave upward along the entire real}$$

line is \_\_\_\_\_



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11. Let  $f(x) = \begin{cases} x^{\frac{3}{5}}, & \text{if } x \leq 1 \\ -(x-2)^3, & \text{if } x > 1 \end{cases}$  Then the number of critical points on the graph of the function is \_\_\_



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12. A right triangle is drawn in a semicircle of radius  $\frac{1}{2}$  with one of its legs along the diameter. If the maximum area of the triangle is  $M$ , then the value of  $32\sqrt{3}Mc$  is \_\_\_\_\_



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13. A rectangle with one side lying along the x-axis is to be inscribed in the closed region of the  $xy$  plane bounded by the lines  $y = 0$ ,  $y = 3x$ , and  $y = 30 - 2x$ . If  $M$  is the largest area of such a rectangle, then the value of  $\frac{2M}{27}$  is \_\_\_\_\_

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14. The least integral value of  $x$  where  $f(x) = (\log)_{\frac{1}{2}}(x^2 - 2x - 3)$  is monotonically decreasing is \_\_\_\_\_

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15. The least area of a circle circumscribing any right triangle of area  $\frac{9}{\pi}$  is \_\_\_\_\_

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16. Let  $f(x) = \begin{cases} |x^2 - 3x| + a, & 0 \leq x < \frac{3}{2} \\ -2x + 3, & x \geq \frac{3}{2} \end{cases}$  If  $f(x)$  has a local maxima at  $x = \frac{3}{2}$ , then greatest value of  $|4a|$  is \_\_\_\_\_

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17. Let  $f(x) = 30 - 2x - x^3$ , the number of the positive integral values of  $x$  which does satisfy  $f(f(f(x))) > f(f(-x))$  is

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18. Let  $f(x) = \begin{cases} x(x-1)(x-2), & (0 \leq x < n) \\ \sin(\pi x), & (n \leq x \leq 2n) \end{cases}$   
least value of  $n$  for which  $f(x)$  has more points of minima than maxima in  $[0, 2n]$  is \_\_\_\_\_.

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19. Number of critical point of the function  $f(x) = x + \sqrt{|x|}$  is \_\_\_\_\_.

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20. consider  $f(x) = \frac{1}{1 + |x|} + \frac{1}{1 + |x - 1|}$  Let  $x_1$  and  $x_2$  be point  
where  $f(x)$  attains local minimum and global maximum respectively .If  
 $k = f(x_1) + f(x_2)$  then  $6k-9=$ \_\_\_\_\_.

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## Jee Main Previous Year

1. Given  $P(x) = x^4 + ax^3 + bx^2 + cx + d$  such that  $x=0$  is the only real  
root of  $P'(x) = 0$  . If  $P(-1) < P(1)$ , then  $\in$  the interval  $[-1,1]$

- A.  $P(-1)$  is the minimum and  $P(1)$  is the maximum of  $P$
- B.  $P(-1)$  is not minimum but  $P(1)$  is the maximum of  $P$
- C.  $P(-1)$  is not minimum and  $P(1)$  is not the maximum of  $P$
- D. neither  $P(-1)$  is the minimum nor  $P(1)$  is the maximum of  $P$

**Answer: 2**



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2. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = \begin{cases} k - 2x, & \text{if } x \leq -1 \\ -12x + 3, & \text{if } x > -1 \end{cases}$ . If  $f$  has a local minimum at  $x = -1$ , then a possible value of  $k$  is (1) 0 (2)  $-\frac{1}{2}$  (3)  $-1$  (4) 1

A. -1

B. 1

C. 0

D.  $\frac{1}{2}$

**Answer: 1**



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3. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function defined by  $f(x) = \frac{1}{e^x + 2e^{-x}}$ .

Statement-1:  $f(c) = \frac{1}{3}$ , for some  $c \in \mathbb{R}$ . Statement-2:

$0 < f(x) \leq \frac{1}{2\sqrt{2}}$ , for all  $x \in \mathbb{R}$ . (1) Statement-1 is true, Statement-2 is

true; Statement-2 is not the correct explanation for Statement-1 (2)

Statement-1 is true, Statement-2 is false (3) Statement-1 is false,

Statement-2 is true (4) Statement-1 is true, Statement-2 is true;

Statement-2 is the correct explanation for Statement-1

A. statement 1 is false statement 2 is true

B. statement 1 is true, statement 2 is true statement 2 is a correct explanation for statement 1

C. statement 1 is true statement 2 is true statement 2 is not a correct explanation for statement 2

D. statement 1 is true, statement 2 is false

**Answer: 2**

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4. Let  $a, b \in \mathbb{R}$  be such that the function  $f$  given by  $f(x) = \ln|x| + bx^2 + ax, x \neq 0$  has extreme values at  $x = 1$  and  $x = 2$ . Statement 1:  $f$  has local maximum at  $x = 1$  and at  $x = 2$ .

Statement 2:  $a = \frac{1}{2}$  and  $b = \frac{-1}{4}$  (1) Statement 1 is false, statement 2 is true (2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1 (3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1 (4) Statement 1 is true, statement 2 is false

A. statement 1 is false statement 2 is true

B. statement 1 is true statement 2 is true, statement 2 is a correct explanation for statement 1

C. statement 1 is true statement 2 is true, statement 2 is not a correct explanation for statement 1

D. statement 1 is true statement 2 is false

**Answer: 2**



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5. find the values  $k$  for which the quadratic equation  $2x^2 + Kx + 3 = 0$

has two real equal roots

- A. lies between 1 and 2
- B. lies between 2 and 3
- C. lies between -1 and 0
- D. does not exist

**Answer: 4**



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6. If  $x = -1$  and  $x = 2$  are extreme points of  $f(x) = \alpha \log|x| + \beta x^2 + x$ , then

A.  $\alpha = -6, \beta = \frac{1}{3}$

B.  $\alpha = -6, \beta = -\frac{1}{2}$

C.  $\alpha = 2, \beta = -\frac{1}{2}$

$$D. \alpha = 2, \beta = \frac{1}{2}$$

**Answer: C**



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7. Let  $f(x)$  be a polynomial of degree four having extreme values at  $x = 1$  and  $x = 2$ . If  $\lim_{x \rightarrow 0} \left[ 1 + \frac{f(x)}{x^2} \right] = 3$  then  $f(2)$  is equal to

A. -8

B. -4

C. 0

D. 4

**Answer: 3**



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8. A wire of length 2 units is cut into two parts which are bent respectively to form a square of  $side = x$  units and a circle of  $radius = r$  units. If the sum of the areas of the square and the circle so formed is minimum, then : (1)  $2x = (\pi + 4)r$  (2)  $(\pi + 4)x = \pi r$  (3)  $x = 2r$  (4)  $2x = r$

A.  $(4 - \pi)x = \pi r$

B.  $x=2r$

C.  $2x=r$

D.  $2x = (\pi + 4)r$

**Answer: 2**



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9. Twenty metres of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in  $sqm$ ) of the flower-bed is: 25 (2) 30 (3) 12.5 (4) 10

A. 30

B. 12.5

C. 10

D. 25

**Answer: 4**



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10. Let  $f(x) = x^2 + \left(\frac{1}{x^2}\right)$  and  $g(x) = x - \frac{1}{x} \quad \xi n R - \{-1, 0, 1\}$ . If  $h(x) = \left(\frac{f(x)}{g(x)}\right)$  then the local minimum value of  $h(x)$  is: (1) 3 (2)  $-3$  (3)  $-2\sqrt{2}$  (4)  $2\sqrt{2}$

A.  $2\sqrt{2}$

B. 3

C. -3

D.  $-2\sqrt{2}$

**Answer: 1**



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### Jee Advanced Previous Year

1. The function  $f: [0, 3] \rightarrow [1, 29]$ , defined by  $f(x) = 2x^3 - 15x^2 + 36x + 1$ , is one-one and onto but not one-one and onto neither one-one nor onto

- A. one-one and onto
- B. onto but not one -one
- C. one-one but not onto
- D. neither one-one nor on to

**Answer: 2**



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2. The number of points in  $(-\infty, \infty)$ , for which  $x^2 - x \sin x - \cos x = 0$ , is 6 (b) 4 (c) 2 (d) 0

A. 6

B. 4

C. 2

D. 0

**Answer: 3**



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3. If  $f: R \rightarrow R$  is a twice differentiable function such that  $f''(x) > 0$  for all  $x \in R$ , and  $f\left(\frac{1}{2}\right) = \frac{1}{2}$ ,  $f(1) = 1$ , then

A.  $0 < f'(1) \leq \frac{1}{2}$

B.  $f'(1) \leq 0$

C.  $f'(x1) <$



D.  $\frac{1}{2} < f'(1) \leq 1$

**Answer: 3**



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4. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8:15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume.

Then the length of the sides of the rectangular sheet are 24 (b) 32 (c) 45

(d) 60

A. 24

B. 32

C. 45

D. 60

**Answer: 1,3**



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5. The function  $f(x) = 2|x| + |x + 2| = ||x|2| - 2|x||$  has a local minimum or a local maximum at  $x = -2$  (b)  $-\frac{2}{3}$  (c) 2 (d)  $\frac{2}{3}$

A. -2

B.  $-\frac{2}{3}$

C. 2

D.  $\frac{2}{3}$

Answer: 1,2



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6. For every pair of continuous functions  $f, g: [0, 1] \rightarrow R$  such that  $\max \{f(x) : x \in [0, 1]\} = \max \{g(x) : x \in [0, 1]\}$  then which are the correct statements

A.  $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$  for some  $c \in [0,1]$

B.  $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$  for some  $c \in [0,1]$

C.  $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$  for some  $c \in [0,1]$

D.  $(f(c))^2 = (g(c))^2$  for some  $c \in [0,1]$

**Answer: 1,4**

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7. Let  $a \in \mathbb{R}$  and let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = x^5 - 5x + a$ . then

A.  $f(x)$  has three real roots if  $a > 4$

B.  $f(x)$  has only one real root if  $a > 4$

C.  $f(x)$  has three real roots if  $a < -4$

D.  $f(x)$  has three real roots if  $-4 < a < 4$

**Answer: 2,4**

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8. Let  $f: \mathbb{R} \rightarrow (0, \infty)$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  be twice differentiable functions such that  $f''$  and  $g''$  are continuous functions on  $\mathbb{R}$ . Suppose  $f(x) = g(x) = 0$ ,  $f''(2) \neq 0$  and  $g''(2) \neq 0$ . If  $\lim_{x \rightarrow 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1$

then

- A.  $f$  has a local minimum at  $x=2$
- B.  $f$  has a local maximum at  $x=2$
- C.  $f''(2) > f'(x)$
- D.  $f(x) - f''(x) = 0$  for at least one  $x$  in  $\mathbb{R}$

**Answer: 1,4**



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9. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is a differentiable function such that  $f(x) > 2f'(x)$  for all  $x \in \mathbb{R}$  and  $f(0) = 1$ , then

A.  $f(x) > e^{2x} \in (0, \infty)$

B.  $f(x)$  is decreasing in  $(0, \infty)$

C.  $f(x)$  is increasing in  $(0, \infty)$

D.  $f'(x) < e^{2x}$  in  $(0, \infty)$

**Answer: 1,3**

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10. If  $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$ , then

A.  $f'(x) = 0$  at exactly three points in  $(-\pi, \pi)$

B.  $f(x)$  attains its maximum at  $x=0$

C.  $f(x)$  attains its minimum at  $x=0$

D.  $f'(x) = 0$  at more than three points in  $(-\pi, \pi)$

**Answer: 2,4**

11. Let  $f: (0, \pi) \rightarrow \mathbb{R}$  be a twice differentiable function such that

$$\lim_{t \rightarrow x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x \text{ for all } x \in (0, \pi)$$

If  $f\left(\frac{\pi}{6}\right) = \left(-\frac{\pi}{12}\right)$  then which of the following statement (s) is (are)

TRUE?

A.  $f\left(x\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$

B.  $f(x) < \frac{x^4}{6} - x^2 f$  or  $\text{all } x \in (0, \pi)$

C. There exist  $\alpha \in (0, \pi)$  such that  $f'(\alpha) = 0$

D.  $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

Answer: 2,3,4

12. Let  $f[0, 1] \rightarrow \mathbb{R}$  (the set of all real numbers) be a function. Suppose the function  $f$  is twice differentiable,  $f(0) = f(1) = 0$ , and satisfies

$f'(x) - 2f''(x) + f(x) \leq e^x$ ,  $x \in [0, 1]$ . Which of the following is true for  $0 < x < 1$ ?

A.  $0 < f(x) < \infty$

B.  $-\frac{1}{2} < f(x) < \frac{1}{2}$

C.  $-\frac{1}{4} < f(x) < 1$

D.  $-\infty < f(x) < 0$

**Answer: 4**



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13. If the function  $d^{-x} f(x)$  assumes its minimum in the interval  $[0, 1]$  at  $x = 1/4$  which of the following is true?

A.  $f'(x) < f''(x)$ ,  $1/4 < x < 3/4$

B.  $f'(x) > f''(x)$ ,  $0 < x < 1/4$

C.  $f'(x) < f''(x)$ ,  $0 < x < 1/4$

D.  $f'(x) < f(X)$ ,  $3/4 < x < 1$

**Answer: 3**

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14. Let  $f$  be a function defined on  $\mathbb{R}$  (the set of all real numbers) such that  $f'(x) = 2010(x - 2009)(x - 2010)^2(x - 2011)^3(x - 2012)^4$ , for all  $x \in \mathbb{R}$ . If  $g$  is a function defined on  $\mathbb{R}$  with values in the interval  $(0, \infty)$  such that  $f(x) = \ln(g(x))$ , for all  $x \in \mathbb{R}$ , then the number of point is  $\mathbb{R}$  at which  $g$  has a local maximum is \_\_\_

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15. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined as  $f(x) = |x| + |x^2 - 1|$ . The total number of points at which  $f$  attains either a local maximum or a local minimum is \_\_\_\_\_

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16. Let  $p(x)$  be a real polynomial of least degree which has a local maximum at  $x = 1$  and a local minimum at  $x = 3$ . If  $p(1) = 6$  and  $p(3) = 2$ , then  $p'(0)$  is \_\_\_\_\_

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17. A cylindrical container is to be made from certain solid material with the following constraints: It has a fixed inner volume of  $Vm^3$ , has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2mm and is of radius equal to the outer radius of the container. If the volume the material used to make the container is minimum when the inner radius of the container is  $10mm$ . then the value of  $\frac{V}{250\pi}$  is

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