MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

PARABOLA

Examples

1. Find the equation of parabola

(i) having focus at (0,-3) its directrix is y = 3.

(ii) having end points of latus rectum (5,10) and (5,10) and which opens towards right.

(iii) having vertex at origin and focus at (0,2)

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2. An arch is in the from of a parabola with its axis vertical. The arch is 12 m high and 6 m wide at the base. How wide is it 6 m from the vertex of the parabola ?

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3. A beam is supported at its ends by supports which are 12 metres apart. Since the load is concentrated at its centre, there is a deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola. How far from the centre is the

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4. Find the coordinates of a point the parabola $y^2 = 8x$ whose distance from the focus is 10.

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5. M is the foot of the perpendicular from a point P on a parabola $y^2 = 4ax$ to its directrix and SPM is an equilateral triangle, where S is the focus. Then find SP .

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6. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, such that one vertex of this triangle coincides with the vertex of the parabola. Then find the side length of this triangle.

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7. Find the equation of the chord of the parabola $y^2 = 8x$ having slope 2 if midpoint of the chord lies on the line x=4.

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8. Find the locus of midpoint of family of chords $\lambda x + y = 5(\lambda$ is parameter) of the parabola $x^2 = 20y$

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9. Find the position of points P(1,3) w.r.t. parabolas $y^2 = 4x$ and $x^2 = 8y.$

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10. The point $(a, 2a)$ is an interior point of the region bounded by the parabola $y^2 = 16x$ and the double ordinate through the focus. then find the values of $a\cdot$

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11. Find the locus of the middle points of the chords of the parabola $y^2 = 4ax$ which subtend a right angle at the vertex of the parabola.

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12. In the following figure, find the locus of centroid of triangle PAB, where AP perpendicular to PB.

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13. A squadrilateral is inscribed in a parabola $y^2 = 4ax$ and three of its sides pass through fixed points on the axis. Show that the fourth side also passes through a fixed point on the axis of the parabola.

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14. Find the equation of parabola

(i) having its vertex at A(1,0) and focus at S(3,0)

(ii) having its focus at S(2,5) and one of the extremities of latus rectum is A (4,5)

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15. $y^2 + 2y - x + 5 = 0$ represents a parabola. Find its vertex, equation of axis, equation of latus rectum, coordinates of the focus, equation of the directrix, extremities of the latus rectum, and the length of the latus rectum.

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16. The parametric equation of a parabola is $x = t^2 + 1, y = 2t + 1.$ Then find the equation of the directrix.

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17. Find the points on the parabola $y^2 - 2y - 4x = 0$ whose focal length is 6.

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18. Find the value of P such that the vertex of $y = x^2 + 2px + 13$ is 4 units above the x-axis.

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19. Find the equation of the parabola which has axis parallel to the y-axis and which passes through the points $(0, 2), (-1, 0), and (1, 6)$.

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20. Prove that the focal distance of the point (x, y) on the parabola $x^2 - 8x + 16y = 0$ is $|y + 5|$

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21. If the focus of a parabola is (2, 3) and its latus rectum is 8, then find the locus of the vertex of the parabola.

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22. Prove that the locus of the center of the circle which touches the given circle externally and the given line is a parabola.

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23. In triangle ABC, base BC is fixed. Then prove that the locus of vertex A such that tan B+tan C= Constant is parabola.

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24. Consider a square with vertices at $(1, 1), (-1, 1), (-1, -1), and (1, -1)\cdot$ Set S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S and find its area.

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25. Find the value of λ if the equation $9x^2 + 4y^2 + 2\lambda xy + 4x - 2y + 3 = 0$ represents a parabola.

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26. Does equation $(5x - 5)^2 + (5y + 10)^2 = (3x + 4y + 5)^2$ represents a parabola ?

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27. Find the equation of the parabola having focus (1, 1) and vertex at $(-3, -3)$.

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28. Find the value of λ if the equation $(x - 1)^2 + (y - 2)^2 = \lambda(x + y + 3)^2$ represents a parabola. Also, find its focus, the equation of its directrix, the equation of its axis, the coordinates of its vertex, the equation of its latus rectum, the length of the latus rectum, and the extremities of the latus rectum.

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29. Show that the curve whose parametric coordinates are $x = t^2 + t + l, y = t^2 - t + 1$ represents a parabola.

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30. If $(2, -8)$ is at an end of a focal chord of the parabola $y^2 = 32x$, then find the other end of the chord.

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31. Let S is the focus of the parabola $y^2 = 4ax$ and X the foot of the directrix, PP' is a double ordinate of the curve and PX meets the curve again in Q. Prove that $P'Q$ passes through focus.

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32. Length of the focal chord of the parabola $y^2 = 4ax$ at a distance p from the vertex is:

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33. If AB is a focal chord of $x^2 - 2x + y - 2 = 0$ whose focus is S and $AS = l_1$, then find BS.

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34. Circles are drawn with diameter being any focal chord of the parabola $y^2 - 4x - y - 4 = 0$ with always touch a fixed line. Find its equation.

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35. Find the equation of the tangent to the parabola $y^2 = 8x$ having slope 2 and also find the point of contact.

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36. A tangent to the parabola $y^2 = 8x$ makes an angle of 45^0 with the straight line $y = 3x + 5$. Then find one of the points of contact.

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37. Show that $x \cos \alpha + a \sin^2 \alpha = p$ touches the parabola $y^2 = 4ax$ if $p \cos \alpha + a \sin^2 \alpha = 0$ and that the point of contact is $(a \tan^2 \alpha, -2a \tan \alpha)$.

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38. The parabola $y^2 = 4x$ and the circle having its center at 6, 5) intersect at right angle. Then find the possible points of intersection of these curves.

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39. Find the equation of tangents of the parabola $y^2 = 12x$, which passes through the point (2, 5).

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40. The tangents to the parabola $y^2 = 4ax$ at the vertex V and any point P meet at Q. If S is the focus, then prove that $SP\dot{S}Q$, and SV are in GP.

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41. The equation of the common tangent to the parabolas $y^2 = 4ax$and $x^2 = 4by$ is given by

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42. If a tangent to the parabola $y^2 = 4ax$ meets the x-axis at T and intersects the tangents at vertex A at P, and rectangle $TAPQ$ is completed, then find the locus of point Q.

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43. Two tangent are drawn from the point $(-2, -1)$ to parabola $y^2 = 4x$. if α is the angle between these tangents, then find the value of $\tan\alpha$.

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44. If two tangents drawn from the point (α, β) to the parabola $y^2 = 4x$ are such that the slope of one tangent is double of the other, then prove that $\alpha = \frac{2}{9}\beta^2$.

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45. If the tangent at the point $P(2, 4)$ to the parabola $y^2 = 8x$ meets the parabola $y^2 = 8x + 5$ at $QandR$, then find the midpoint of chord QR.

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46. The locus of foot of the perpendiculars drawn from the vertex on a variable tangent to the parabola $y^2 = 4ax$ is

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47. Find the equation of the tangent to the parabola $y = x^2 - 2x + 3$ at point (2, 3).

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48. Find the equation of the tangent to the parabola $x = y^2 + 3y + 2$ having slope 1.

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49. Find the equation of tangents drawn to the parabola $y = x^2 - 3x + 2$ from the point $(1, -1)$.

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50. Find the shortest distance between the line $y = x - 2$ and the parabola $y = x^2 + 3x + 2$.

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51. If the lines $L_1 and L_2$ are tangents to $4x^2 - 4x - 24y + 49 = 0$ and are normals for $x^2 + y^2 = 72,$ then find the slopes of L_1 and L_2.

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52. Tangent are drawn from the point $(-1, 2)$ on the parabola $y^2 = 4x$. Find the length that these tangents will intercept on the line $x = 2$.

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53. Tangents are drawn to the parabola $y^2 = 4ax$ at the point where the line $lx + my + n = 0$ meets this parabola. Find the point of intersection of these tangents.

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54. If the chord of contact of tangents from a point P to the parabola $y^2 = 4ax$ touches the parabola $x^2 = 4by$, then find the locus of P.

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55. From a variable point on the tangent at the vertex of a parabola $y^2 = 4ax$, a perpendicular is drawn to its chord of contact. Show that these variable perpendicular lines pass through a fixed point on the axis of the parabola.

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56. Find the points of contact Q and R of a tangent from the point $P(2, 3)$ on the parabola $y^2 = 4x$.

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57. Tangents are drawn from any point on the line $x + 4a = 0$ to the parabola $y^2 = 4ax\cdot$ Then find the angle subtended by the chord of contact at the vertex.

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58. Two straight lines $(y - b) = m_1(x + a)$ and $(y - b) = m_2(x + a)$ are the tangents of $y^2 = 4ax\cdot$ Prove $m_1 m_2 = -1.$

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59. Mutually perpendicular tangents $TAandTB$ are drawn to $y^2 = 4ax$. Then find the minimum length of $AB\cdot$

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60. Tangent $PAandPB$ are drawn from the point P on the directrix of the parabola $(x-2)^2+(y-3)^2=\frac{(5x-12y+3)^2}{160}$. Find the least radius of the circumcircle of triangle $PAB\cdot$

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61. Tangents are drawn to the parabola $(x-3)^2+(y-4)^2=\frac{(3x-4y-6)^2}{25}$ at the extremities of the chord $2x-3y-18=0$. Find the angle between the tangents.

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62. Let $3x-y-8=0$ be the equation of tangent to a parabola at the point (7, 13). If the focus of the parabola is at (-1,-1). Its directrix is

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63. Find the locus of the point of intersection of tangents in the parabola $x^2 = 4ax \cdot$ which are inclined at an angle θ to each other. Which intercept constant length c on the tangent at the vertex. such that the area of ABR is constant c, where $AandB$ are the points of intersection of tangents with the y-axis and R is a point of intersection of tangents.

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64. Find the equations of normal to the parabola $y^2 = 4ax$ at the ends of the latus rectum.

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65. If $y = x + 2$ is normal to the parabola $y^2 = 4ax$, then find the value of $a \cdot$

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66. Find the equation of line which is normal to the parabola $x^2 = 4y$ and touches the parabola $y^2 = 12x$.

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67. Find the equation of normal to the parabola $y = x^2 - x - 1$ which has equal intercept on the axes. Also find the point where this normal meets the curve again.

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68. Prove that the length of the intercept on the normal at the point $P(at^2, 2at)$ of the parabola $y^2 = 4ax$ made by the circle described on the line joining the focus and P as diameter is $a\sqrt{1+t^2}$.

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69. A normal chord of the parabola $y^2 = 4ax$ subtends a right angle at the vertex if its slope is

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70. How many normals can be drawn to parabola $y^2 = 4x$ from point (15, 12)? Find their equation. Also, find corresponding feet of normals on the parabola.

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71. Three normals are drawn from the point (7, 14) to the parabola $x^2 - 8x - 16y = 0$. Find the coordinates of the feet of the normals.

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72. Find the minimum distance between the curves $y^2 = 4xandx^2 + y^2 - 12x + 31 = 0$

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73. If normals drawn at three different point on the parabola $y^2 = 4ax$ pass through the point (h,k), then show that h $h > 2a$.

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74. IF three distinct normals to the parabola $y^2 - 2y = 4x - 9$ meet at point (h,k), then prove that $h > 4$.

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75. In the parabola $y^2 = 4ax$, then tangent at P whose abscissa is equal to the latus rectum meets its axis at T, and normal P cuts the curve again at Q. Show that $PT : PQ = 4 : 5$.

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76. Find the length of normal chord which subtends an angle of 90^0 at the vertex of the parabola $y^2 = 4x$.

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77. Prove that the locus of the point of intersection of the normals at the ends of a system of parallel chords of a parabola is a straight line which is a normal to the curve.

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78. Find the locus of the midpoint of normal chord of parabola $y^2 = 4ax$.

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79. If the angle between the normal to the parabola $y^2 = 4ax$ at point P and the focal chord passing through P is 60°, then find the slope of the tangent at point P.

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80. A parabola mirror is kept along $y^2 = 4x$ and two light rays parallel to its axis are reflected along one straight line. If one of the incident light rays is at 3 units distance from the axis, then find the distance of the other incident ray from the axis.

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81. If two of the three feet of normals drawn from a point to the parabola $y^2 = 4x$ are (1, 2) and $(1, -2)$, then find the third foot.

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82. If the normals from any point to the parabola $y^2 = 4x$ cut the line $x = 2$ at points whose ordinates are in AP, then prove that the slopes of tangents at the co-normal points are in GP.

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83. Find the locus of thepoint of intersection of two normals to a parabolas which are at right angles to one another.

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84. $P(t_1)$ and $Q(t_2)$ are points t_1 and t_2 on the parabola $y^2 = 4ax$. The normals at P and Q meet on the parabola. Show that the middle point of PQ lies on the parabola $y^2 = 2a(x + 2a)$.

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85. Normals are drawn at points $A, B,$ and C on the parabola $y^2 = 4x$ which intersect at P. The locus of the point P if the slope of the line joining the feet of two of them is 2, is

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86. prove that for a suitable point P on the axis of the parabola, chord AB through the point P can be drawn such that $\left[\left(\frac{1}{AP^2}\right) + \left(\frac{1}{BP^2}\right)\right]$ is same for all positions of the chord.

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87. A parabola of latus rectum l touches a fixed equal parabola. The axes of two parabolas are parallel. Then find the locus of the vertex of the moving parabola.

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88. The area of the trapezium whose vertices lie on the parabola $y^2 = 4x$ and its diagonals pass through (1,0) and having length $\frac{25}{4}$ units each is

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89. Find the radius of the largest circle, which passes through the focus of the parabola $y^2 = 4(x + y)$ and is also contained in it.

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90. The vertices $A, BandC$ of a variable right triangle lie on a parabola $y^2 = 4x$. If the vertex B containing the right angle always remains at the point (1, 2), then find the locus of the centroid of triangle ABC.

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91. Tangents are drawn to the parabola at three distinct points. Prove that these tangent lines always make a triangle and that the locus of the orthocentre of the triangle is the directrix of the parabola.

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92. A movable parabola touches x-axis and y-axis at (0,1) and (1,0). Then the locus of the focus of the parabola is :

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93. Two lines are drawn at right angles, one being a tangent to $y^2 = 4ax$ and the other $x^2 = 4by$. Then find the locus of their point of intersection.

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94. The tangent and normal at $P(t)$, for all real positive t, to the parabola $y^2 = 4ax$ meet the axis of the parabola in T and G respectively, then the angle at which the tangent at P to the parabola is inclined to the tangent at P to the circle passing through the points P, T and G is

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95. If the normals at P, Q, R of the parabola $y^2 = 4ax$ meet in O and S be its focus, then prove that . $SP.\ SQ.\ SR = a.\ (SO)^2$.

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96. The shortest distance between the parabolas $2y^2 = 2x - 1$ and $2x^2 = 2y - 1$ is $2\sqrt{2}$ (b) $\frac{1}{2}\sqrt{2}$ (c) 4 (d) $\sqrt{\frac{36}{5}}$

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97. If two chords drawn from the point $A(4, 4)$ to the parabola $x^2 = 4y$ are bisected by the line $y = mx$, the interval in which m lies is $(-2\sqrt{2}, 2\sqrt{2})$ $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ $(-\infty, -2\sqrt{2} - 2) \cup (2\sqrt{2} - 2, \infty)$ none of these

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98. Tangent is drawn at any point (p, q) on the parabola $y^2 = 4ax$. Tangents are drawn from any point on this tangant to the circle $x^2 + y^2 = a^2$, such that the chords of contact pass through a fixed point (r, s). Then p, q, r and s can hold the relation $r^2q = 4p^2s$ (b) $rq^2 = 4ps^2$ $rq^2 = -4ps^2$ (d) $r^2q = -4p^2s$

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99. Length of the shortest normal chord of the parabola $y^2 = 4ax$ is

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100. Find the equation of parabola (i) having focus at (0,-3) its directrix is y = 3. (ii) having end points of latus rectum (5,10) and (5,-10) and which opens towards right. (iii) having vertex at origin and focus at (0,2)

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102. Find the coordinates of a point the parabola $y^2 = 8x$ whose distance from the focus is 10.

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104. Find the position of points P(1,3) w.r.t. parabolas $y^2 = 4x$ and $x^2 = 8y$.

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105. Find the equation of parabola

(i) having its vertex at A(1,0) and focus at S(3,0)

(ii) having its focus at S(2,5) and one of the extremities of latus rectum is A (4,5)

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106. Does equation $(5x-5)^2+(5y+10)^2=(3x+4y+5)^2$ represents a parabola ?

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107. Let y=3x-8 be the equation of the tangent at the point (7, 13) lying on a parabola whose focus is at (-1,-1). Find the equation of directrix and the length of the latus rectum of the parabola.

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108. Find the equation of line which is normal to the parabola $x^2 = 4y$ and touches the parabola $y^2 = 12x$.

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109. How many normals can be drawn to parabola $y^2 = 4x$ from point (15, 12)? Find their equation. Also, find corresponding feet of normals on the parabola.

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111. If the angle between the normal to the parabola $y^2 = 4ax$ at point P and the focal chord passing through P is 60°, then find the slope of the tangent at point P.

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Exercise 5.1

1. Find the angle made by a double ordinate of length 2a at the vertex of the parabola $y^2 = ax$.

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2. If focal distance of a point P on the parabola $y^2 = 4ax$ whose abscissa is 5 10, then find the value of a.

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3. Analyse the following equations if they represent parabola(s) or part of parabola(s) ?

(a) $y = x|x|$, (b) $x = \sqrt{-y}$

(c) $x^2 = y^4$, (d) $x = e^t, 2t = \log_e y$

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4. Find the range of values of λ for which the point $(\lambda, -1)$ is exterior to both the parabolas $y^2 = |x|$.

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5. The locus of a point on the variable parabola $y^2 = 4ax,$ whose distance from the focus is always equal to $k,$ is equal to (a is parameter)

$4x^2 + y^2 - 4kx = 0$ $x^2 + y^2 - 4kx = 0$ $2x^2 + 4y^2 - 9kx = 0$

$4x^2 - y^2 + 4kx = 0$

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6. Find the locus of the midpoint of chords of the parabola $y^2 = 4ax$ that pass through the point $(3a, a)$.

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7. If chord BC subtends right angle at the vertex A of the parabola $y^2 = 4x$ with $AB = \sqrt{5}$ then find the area of triangle ABC.

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8. PQ is a chord ofthe parabola $y^2 = 4x$ whose perpendicular bisector meets the axis at M and the ordinate of the midpoint PQ meets the axis at N. Then the length MN is equal to

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9. LOL' and MOM' are two chords of parabola $y^2 = 4ax$ with vertex A passing through a point O on its axis. Prove that the radical axis of the circles described on LL' and MM' as diameters passes though the vertex of the parabola.

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10. If focal distance of a point P on the parabola $y^2 = 4ax$ whose abscissa is 5 10, then find the value of a.

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11. If chord BC subtends right angle at the vertex A of the parabola $y^2 = 4x$ with $AB = \sqrt{5}$ then find the area of triangle ABC.

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Exercise 5.2

1. If the focus and vertex of a parabola are the points (0, 2) and (0, 4), respectively, then find the equation

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2. Find the equation of parabola whose focus is (0,1) and the directrix is x+2=0. Also find the vertex of the parabola.

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3. Find the vertex, focus and directrix of the parabola $x^2 = 2(2x + y)$.

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4. The vertex of a parabola is (2, 2) and the coordinats of its two extremities of latus rectum are $(-2, 0)$ and (6, 0). Then find the equation of the parabola.

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5. A parabola passes through the point the point (1,2), (2,1), (3,4) and (4,3). Find the equation of the axis of parabola.

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6. Find the length of the common chord of the parabola $x^2 = 4(x+3)$ and the circle $x^2 + y^2 + 4x = 0$.

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7. The equation of the latus rectum of a parabola is $x + y = 8$ and the equation of the tangent at the vertex is $x + y = 12$. Then find the length of the latus rectum.

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8. Find the length of the latus rectum of the parabola whose focus is at (2, 3) and directrix is the line $x - 4y + 3 = 0$.

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9. If (a, b) is the midpoint of a chord passing through the vertex of the parabola $y^2 = 4(x + 1)$, then prove that $2(a + 1) = b^2$

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10. Show that the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio $1 : 2$ is parabola. Find the vertex of this parabola.

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11. Plot the region in the first quadrant in which points are nearer to the origin than to the line $x = 3$.

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12. Prove that the locus of a point, which moves so that its distance from a fixed line is equal to the length of the tangent drawn from it to a given circle, is a parabola.

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13. Prove that the locus of the center of a circle, which intercepts a chord of given length $2a$ on the axis of x and passes through a given point on the axis of y distant b from the origin, is a parabola.

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14. Find the equation of the parabola whose focus is $S(-1, 1)$ and directrix is $4x + 3y - 24 = 0$. Also find its axis, the vertex, the length, and the equation of the latus rectum.

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15. The axis of parabola is along the line y=x and the distance of its vertex and focus from origin are $\sqrt{2}$ and $2\sqrt{2}$ respectively. If vertex and focus both lie in the first quadrant, then the equation of the parabola is :

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16. Find the equation of parabola whose focus is (0,1) and the directrix is x+2=0. Also find the vertex of the parabola.

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17. Find the vertex, focus and directrix of the parabola $x^2 = 2(2x + y)$.

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Exercise 5.3

1. If $t_1 and t_2$ are the ends of a focal chord of the parabola $y^2 = 4ax,$ then prove that the roots of the equation $t_1x^2 + ax + t_2 = 0$ are real.

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2. If the line passing through the focus S of the parabola $y = ax^2 + bx + c$ meets the parabola at $P and Q$ and if $SP = 4$ and $SQ = 6$, then find the value of $a\cdot$

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3. If a focal chord of $y^2 = 4ax$ makes an angle $\alpha \in [\pi/4, \pi/2]$ with the positive direction of the x-axis, then find the maximum length of this focal shord.

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4. If the length of focal chord of $y^2 = 4ax$ is l, then find the angle between the axis of the parabola and the focal chord.

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5. If length of focal chord PQ is l, and p is the perpendicular distance of PQ from the vertex of the parabola, then prove that $l \propto \frac{1}{p^2}$.

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6. Circle drawn having its diameter equal to the focal distance of any point lying on the parabola $x^2 - 4x + 6y + 10 = 0$ will touch a fixed line. Find its equation.

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7. A circle is drawn to pass through the extremities of the latus rectum of the parabola $y^2 = 8x \cdot$ It is given that this circle also touches the directrix of the parabola. Find the radius of this circle.

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Exercise 5.4

1. Find the point on the curve $y^2 = ax$ the tangent at which makes an angle of 45^0 with the x-axis.

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2. Find the equation of the straight lines touching both $x^2 + y^2 = 2a^2$ and $y^2 = 8ax \cdot$

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3. Find the angle at which the parabolas $y^2 = 4x$ and $x^2 = 32y$ intersect.

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4. If the line $y = 3x + c$ touches the parabola $y^2 = 12x$ at point P , then find the equation of the tangent at point Q where PQ is a focal chord.

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5. If the line $x + y = a$ touches the parabola $y = x - x^2$, then find the value of $a\cdot$

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6. Find the slopes of the tangents to the parabola $y^2 = 8x$ which are normal to the circle $x^2 + y^2 + 6x + 8y - 24 = 0.$

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7. Find the equation of the tangent to the parabola $9x^2 + 12x + 18y - 14 = 0$ which passes through the point (0, 1).

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8. Find the locus of the point from which the two tangents drawn to the parabola $y^2 = 4ax$ are such that the slope of one is thrice that of the other.

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9. From an external point P, a pair of tangents is drawn to the parabola $y^2 = 4x$. If $\theta_1 and th\eta_2$ are the inclinations of these tangents with the x-axis such that $\theta_1 + \theta_2 = \frac{\pi}{4}$, then find the locus of P.

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10. Show that the common tangents to the parabola $y^2 = 4x$ and the circle $x^2 + y^2 + 2x = 0$ form an equilateral triangle.

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11. TP and TQ are tangents to the parabola $y^2 = 4ax$ at $PandQ$, respectively. If the chord PQ passes through the fixed point $(-a, b)$, then find the locus of T.

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12. At any point P on the parabola $y^2 - 2y - 4x + 5 = 0$ a tangent is drawn which meets the directrix at Q. Find the locus of point R which divides QP externally in the ratio $\frac{1}{2} : 1$

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13. If the distance of the point $(\alpha, 2)$ from its chord of contact w.r.t. the parabola $y^2 = 4x$ is 4, then find the value of α.

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Exercise 5.5

1. If the tangents at the points $PandQ$ on the parabola $y^2 = 4ax$ meet at $T, andS$ is its focus, the prove that $SP, ST, andSQ$ are in GP.

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2. If PQ is the focal chord of parabola $y = x^2 - 2x + 3$ such that $P \equiv (2, 3)$, then find slope of tangent at Q.

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3. If there exists at least one point on the circle $x^2 + y^2 = a^2$ from which two perpendicular tangents can be drawn to parabola $y^2 = 2x$, then find the values of a.

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4. Find the angle between the tangents drawn to $y^2 = 4x$, where it is intersected by the line $y = x - 1$.

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5. Find the angle between the tangents drawn from the origin to the parabolas $y^2 = 4a(x - a)$

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6. Find the locus of the point of intersection of the perpendicular tangents of the curve $y^2 + 4y - 6x - 2 = 0$.

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7. A tangent is drawn to the parabola $y^2 = 4ax$ at P such that it cuts the y-axis at Q. A line perpendicular to this tangents is drawn through Q which cuts the axis of the parabola at R. If the rectangle $PQRS$ is completed, then find the locus of S.

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8. Let y=x+1 is axis of parabola, y+x-4=0 is tangent of same parabola at its vertex and y=2x+3 is one of its tangents. Then find the focus of the parabola.

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9. Let y=x+1 is axis of parabola, y+x-4=0 is tangent of same parabola at its vertex and y=2x+3 is one of its tangents. Then find the focus of the parabola.

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Exercise 5.6

1. Prove that the chord $y - x\sqrt{2} + 4a\sqrt{2} = 0$ is a normal chord of the parabola $y^2 = 4ax$. Also find the point on the parabola when the given chord is normal to the parabola.

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2. Find the equation of normal to parabola $y = x^2 - 3x - 4$

(a) at point (3,-4)

(b) having slope 5.

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3. If $y = 2x + 3$ is a tangent to the parabola $y^2 = 24x$, then find its distance from the parallel normal.

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4. whatever be the value of θ, the line $y = (x - 11)\cos\theta - \cos 3\theta$ is always normal to the parabola

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5. Find the locus of the midpoints of the portion of the normal to the parabola $y^2 = 4ax$ intercepted between the curve and the axis.

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6. If the parabolas $y^2 = 4ax$ and $y^2 = 4c(x - b)$ have a common normal other than the x-axis $(a, b, c$ being distinct positive real numbers), then prove that $\frac{b}{a - c} > 2.$

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7. Three normals are drawn from the point (c, 0) to the curve $y^2 = x.$ Show that c must be greater than 1/2. One normal is always the axis. Find c for which the other two normals are perpendicular to each other.

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8. Prove that for $\theta \in R,$ the line $y = (x - 11)\cos\theta - \cos 3\theta$ is always normal to the parabola $y^2 = 16x.$

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Exercise 5.7

1. If the normal to the parabola $y^2 = 4ax$ at point t_1 cuts the parabola again at point t_2 , then prove that $t22 \geq 8.$

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2. Find the angle at which normal at point $P(at^2, 2at)$ to the parabola meets the parabola again at point Q.

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3. If normal to parabola $y^2 = 4ax$ at point $P(at^2, 2at)$ intersects the parabola again at Q, such that sum of ordinates of the points P and Q is 3, then find the length of latus ectum in terms of t.

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4. If tangents are drawn to $y^2 = 4ax$ from any point P on the parabola $y^2 = a(x+b)$, then show that the normals drawn at their point for contact meet on a fixed line.

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5. If line x-2y-1=0 intersects parabola $y^2 = 4x$ at P and Q, then find the point of intersection of normals at P and Q.

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6. Find the locus of the point of intersection of the normals at the end of the focal chord of the parabola $y^2 = 4ax$.

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7. If incident ray from point (-2,4) parallel to the axis of the parabola $y^2 = 4x$ strikes the parabola, then find the equation of the reflected ray.

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8. Let L_1, L_2 and L_3 be the three normals to the parabola $y^2 = 4ax$ from point P inclined at the angle θ_1, θ_2 and θ_3 with x-axis, respectively. Then find the locus of point P given that $\theta_1 + \theta_2 + \theta_3 = \alpha$ (constant).

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9. If line x-2y-1=0 intersects parabola $y^2 = 4x$ at P and Q, then find the point of intersection of normals at P and Q.

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Exercise (Single)

1. Which one of the following equation represent parametric equation to a parabolic curve? $x=3\cos t; y=4\sin t$ $x^2-2=2\cos t; y=4\frac{\cos^2 t}{2}$ $\sqrt{x}=\tan t; \sqrt{y}=\sec t$ $x=\sqrt{1-\sin t}; y=\frac{\sin t}{2}+\frac{\cos t}{2}$

A. $x=3\cos t, y=4\sin t$

B. $x^2-2=2\cos t, y=4\cos^2\frac{t}{2}$

C. $\sqrt{x}=\tan t, \sqrt{y}=\sec t$

D. $x=\sqrt{1-\sin t}, y=\sin\frac{t}{2}+\cos\frac{t}{2}$

Answer: B

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2. A point $P(x, y)$ moves in the xy-plane such that $x=a\cos^2\theta$ and $y=2a\sin\theta$, where θ is a parameter. The locus of the point P is a/an circle (b) aellipse unbounded parabola (d) part of the parabola

A. circle

B. ellipse

C. unbounded parabola

D. part of the parabola

Answer: D

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3. A line L passing through the focus of the parabola $y^2 = 4(x-1)$ intersects the parabola at two distinct points. If m is the slope of the line L, then `-11m in R` (d) none of these

A. $-1 < m < 1$

B. $m < \ -1 \text{ or } m <$

C. $\min R$

D. none of these

Answer: D

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4. The circle $x^2 + y^2 + 2\lambda x = 0, \lambda \in R$, touches the parabola $y^2 = 4x$ externally. Then,

A. $\lambda > 0$

B. $\lambda < 0$

C. $\lambda > 1$

D. none of these

Answer: A

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5. A set of parallel chords of the parabola $y^2 = 4ax$ have their midpoint on any straight line through the vertex any straight line through the focus a straight line parallel to the axis another parabola

A. any straight line through the vertex

B. any straight line through the focus

C. a straight line parallel to the axis

D. another parabola

Answer: C

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6. If the points A (1,3) and B (5,5) lying on a parabola are equidistant from focus, then the slope of the directrix is

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 2

D. -2

Answer: A

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7. The radius of the circle whose centre is $(-4, 0)$ and which cuts the parabola $y^2 = 8x$ at A and B such that the common chord AB subtends a right angle at the vertex of the parabola is equal to

A. $4\sqrt{13}$

B. $3\sqrt{5}$

C. $3\sqrt{2}$

D. $2\sqrt{5}$

Answer: A

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8. The circle $x^2 + y^2 = 5$ meets the parabola $y^2 = 4x$ at P and Q . Then the length PQ is equal to

A. 2

B. $2\sqrt{2}$

C. 4

D. none of these

Answer: C

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9. If $y_1, y_2,$ and y_3 are the ordinates of the vertices of a triangle inscribed in the parabola $y^2 = 4ax$, then its area is

A. $\frac{1}{2a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$

B. $\frac{1}{4a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$

C. $\frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$

D. none of these

Answer: C

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10. let P be the point $(1, 0)$ and Q be a point on the locus $y^2 = 8x$. The locus of the midpoint of PQ is

A. $y^2 + 4x + 2 = 0$

B. $y^2 - 4x + 2 = 0$

C. $x^2 - 4y + 2 = 0$

D. $x^2 + 4y + 2 = 0$

Answer: B

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11. An equilateral triangle SAB in inscribed in the parabola $y^2 = 4ax$ having it's focus at S. If chord lies towards the left of S, then the side length of this triangle is

A. $2a(2-\sqrt{3})$

B. $4a(2-\sqrt{3})$

C. $a(2-\sqrt{3})$

D. $8a(2-\sqrt{3})$

Answer: B

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12. C is the centre of the circle with centre $(0, 1)$ and radius unity. $y = ax^2$ is a parabola. The set of the values of $'a'$ for which they meet at a point other than the origin, is

A. $a > 0$

B. $a \in (0, 1/2)$

C. (1/4, 1/2)

D. $(1/2, \infty)$

Answer: D

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13. $P(x, y)$ is a variable point on the parabola $y^2 = 4ax$ and $Q(x+c, y+c)$ is another variable point, where c is a constant. The locus of the midpoint of PQ is a/n parabola (b) hyperbola hyperbola (d) circle

A. parabola

B. ellipse

C. hyperbola

D. circle

Answer: A

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14. AB is a chord of the parabola $y^2 = 4ax$ with its vertex at A. BC is drawn perpendicular to AB meeting the axis at C.The projecton of BC on the axis of the parabola is

A. a

B. 2a

C. 4a

D. 8a

Answer: C

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15. Set of value of α for which the point $(\alpha, 1)$ lies inside the circle $x^2 + y^2 - 4 = 0$ and parabola $y^2 = 4x$ is

A. $|\alpha| < \sqrt{3}$

B. $|\alpha| < 2$

C. $\frac{1}{4} < \alpha < \sqrt{3}$

D. none of these

Answer: C

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16. If X is the foot of the directrix on axis of the parabola. PP' is a double ordinate of the curve and PX meets the curve again in Q. Then prove that P' Q passes through fixed point which is

A. vertex

B. focus

C. midpoint of vertex and focus

D. none of these

Answer: B

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17. A water jet from a function reaches it maximum height of 4 m at a distance 0.5 m from the vertical passing through the point O of water outlet. The height of the jet above the horizontal OX at a distance of 0.75 m from the point O is 5 m (b) 6 m (c) 3 m (d) 7 m

A. 5 m

B. 6 m

C. 3 m

D. 7 m

Answer: C

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18. Area of the triangle formed by the vertex, focus and one end of latusrectum of the parabola $(x+2)^2 = -12(y-1)$ is

A. 36

B. 18

C. 9

D. 6

Answer: C

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19. The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is:

A. xy=105/64

B. xy=3/4

C. xy=35/16

D. xy=64/105

Answer: A

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20. Two parabola have the same focus. If their directrices are the x-axis and the y-axis respectively, then the slope of their common chord is :

A. ± 1

B. $4/3$

C. $3/4$

D. none of these

Answer: A

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21. The locus of the point $\left(\sqrt{3h}, \sqrt{\sqrt{3}k+2}\right)$ if it lies on the line $x - y - 1 = 0$ is straight line (b) a circle a parabola (d) none of these

A. a straight line

B. a circle

C. a parabola

D. none of these

Answer: C

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22. A circle touches the x-axis and also touches the circle with center $(0, 3)$ and radius 2. The locus of the center

A. a circle

B. an ellipse

C. a parabola

D. a hyperbola

Answer: C

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23. If parabolas $y^2 = \lambda x$ and $25\left[(x-3)^2 + (y+2)^2\right] = (3x - 4y - 2)^2$ are equal, then the value of λ is

A. 9

B. 3

C. 7

D. 6

Answer: D

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24. The length of the latus rectum of the parabola whose focus is $\left(\frac{u^2}{2g}\sin 2\alpha, -\frac{u^2}{2g}\cos 2\alpha\right)$ and directrix is $y = \frac{u^2}{2g}$ is $\frac{u^2}{g}\cos^2\alpha$ (b) $\frac{u^2}{g}\cos^2 2\alpha$ $\frac{2u^2}{g}\cos^2 2\alpha$ (d) $\frac{2u^2}{g}\cos^2\alpha$

A. $\frac{u^2}{g}\cos^2\alpha$

B. $\frac{u^2}{g}\cos 2\alpha$

C. $\frac{2u^2}{g}\cos 2\alpha$

D. $\frac{2u^2}{g}\cos^2\alpha$

Answer: D

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25. The graph of the curve $x^2+y^2-2xy-8x-8y+32=0$ falls wholly in the first quadrant (b) second quadrant third quadrant (d) none of these

A. first quadrant

B. second quadrant

C. third quadrant

D. none of these

Answer: A

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26. The vertex of the parabola whose parametric equation is $x = t^2 - t + 1, y = t^2 + t + 1; t \in R,$ is (1, 1) (b) (2, 2) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (d) $(3, 3)$

A. (1,1)

B. (2,2)

C. (1/2, 1/2)

D. (3,3)

Answer: A

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27. If the line $y - \sqrt{3}x + 3 = 0$ cut the parabola $y^2 = x + 2$ at P and Q , then $AP\dot{A}Q$ is equal to

A. $\frac{2\left(\sqrt{3}+2\right)}{3}$

B. $\frac{4\sqrt{3}}{2}$

C. $\left(4\frac{2-\sqrt{2}}{3}\right)$

D. $\frac{4\left(\sqrt{3}+2\right)}{3}$

Answer: D

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28. A line is drawn from A(-2,0) to intersect the curve $y^2 = 4x$ in P and Q in the first quadrant such that $\frac{1}{AP} + \frac{1}{AQ} < \frac{1}{4}$, then slope of the line always be :

A. $> \sqrt{3}$

B. $< 1/\sqrt{3}$

C. $> \sqrt{2}$

D. $> 1/\sqrt{3}$

Answer: A

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29. The length of the chord of the parabola $y^2 = x$ which is bisected at the point (2, 1) is

A. $2\sqrt{3}$

B. $4\sqrt{3}$

C. $3\sqrt{2}$

D. $2\sqrt{5}$

Answer: D

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30. If a line $y = 3x + 1$ cuts the parabola $x^2 - 4x - 4y + 20 = 0$ at $A and B$, then the tangent of the angle subtended by line segment AB at the origin is $\frac{8\sqrt{3}}{205}$ (b) $\frac{8\sqrt{3}}{209}$ $\frac{8\sqrt{3}}{215}$ (d) none of these

A. $8\sqrt{3}/205$

B. $8\sqrt{3}/209$

C. $8\sqrt{3}/215$

D. none of these

Answer: B

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31. If P be a point on the parabola $y^2 = 3(2x - 3)$ and M is the foot of perpendicular drawn from the point P on the directrix of the parabola, then length of each sides of an equilateral triangle SMP(where S is the focus of the parabola), is

A. 2

B. 4

C. 6

D. 8

Answer: C

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32. A parabola $y = ax^2 + bx + c$ crosses the x-axis at $(\alpha, 0)(\beta, 0)$ both to the right of the origin. A circle also passes through these two points. The length of a tangent from the origin to the circle is: $\sqrt{\frac{bc}{a}}$ (b) ac^2 (d) $\sqrt{\frac{c}{a}}$

A. $\sqrt{bc/a}$

B. ac^2

C. b/a

D. $\sqrt{c/a}$

Answer: D

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33. The number of common chords of the parabolas $x = y^2 - 6y + 11$ and $y = x^2 - 6x + 11$ is 1 (b) 2 (c) 4 (d) 6

A. 1

B. 2

C. 4

D. 6

Answer: D

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34. Two parabola have the same focus. If their directrices are the x-axis and the y-axis respectively, then the slope of their common chord is :

A. -1

B. $-1/2$

C. $-\sqrt{3}/2$

D. none of these

Answer: A

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35. PSQ is a focal chord of a parabola whose focus is S and vertex is A. PA, QA, are produced to meet the dirrecterix in R and T. Then $\angle RST$ is equal to

A. 30°

B. 90°

C. 60°

D. 45°

Answer: B

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36. If PSQ is a focal chord of the parabola $y^2 = 8x$ such that $SP = 6$, then the length of SQ is 6 (b) 4 (c) 3 (d) none of these

A. 6

B. 4

C. 3

D. none of these

Answer: C

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37. The triangle PQR of area 'A' is inscribed in the parabola $y^2 = 4ax$ such that the vertex P lies at the vertex pf the parabola and base QR is a focal chord.The modulus of the difference of the ordinates of the points Q and R is :

A. A/2a

B. A/a

C. 2A/a

D. 4A/a

Answer: C

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38. If A_1B_1 and A_2B_2 are two focal chords of the parabola $y^2 = 4ax$, then the chords A_1A_2 and B_1B_2 intersect on directrix (b) axis tangent at vertex (d) none of these

A. directrix

B. axis

C. tangent at vertex

D. none of these

Answer: A

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39. If $a and c$ are the lengths of segments of any focal chord of the parabola $y^2 = bx, (b > 0)$, then the roots of the equation $ax^2 + bx + c = 0$ are real and distinct (b) real and equal imaginary (d) none of these

A. real and distinct

B. real and equal

C. imaginary

D. none of these

Answer: C

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40. If $x = mx + c$ touches the parabola $y^2 = 4a(x + a)$, then $c = \frac{a}{m}$ (b) $c = am + \frac{a}{m}$ $c = a + \frac{a}{m}$ (d) none of these

A. $c = \frac{1}{m}$

B. $c = am + \frac{a}{m}$

C. $c = a + \frac{a}{m}$

D. none of these

Answer: B

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41. The area of the triangle formed by the tangent and the normal to the parabola $y^2 = 4ax$, both drawn at the same end of the latus rectum, and the axis of the parabola is $2\sqrt{2}a^2$ (b) $2a^2$ $4a^2$ (d) none of these

A. $2\sqrt{2}a^2$

B. $2a^2$

C. $4a^2$

D. none of these

Answer: C

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42. Parabola $y^2 = 4a(x - c_1)$ and $x^2 = 4a(y - c_2)$ where c_1 and c_2 are variables, touch each other. Locus of their point of contact is

A. $xy = 2a^2$

B. $xy = 4a^2$

C. $xy = a^2$

D. none of these

Answer: B

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43. Let $y = f(x)$ be a parabola, having its axis parallel to the y-axis, which is touched by the line $y = x$ at $x = 1$. Then, $2f(0) = 1 - f'(0)$ (b) $f(0) + f'(0) + f^0 = 1$ $f'(1) = 1$ (d) $f'(0) = f'(1)$

A. 2f(0)=1-f'(0)

B. f(0)+f'(0)+f'(0)=1

C. f'(1)=1

D. none of these

Answer: A

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44. If $y = 2x - 3$ is tangent to the parabola $y^2 = 4a\left(x - \frac{1}{3}\right)$, then a is equal to $\frac{22}{3}$ (b) -1 (c) $\frac{14}{3}$ (d) $\frac{-14}{3}$

A. $\frac{22}{3}$

B. -1

C. $\frac{14}{3}$

D. $\frac{14}{3}$

Answer: D

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45. The locus of the center of a circle which cuts orthogonally the parabola $y^2 = 4x$ at (1,2) is a curve

A. (3,4)

B. (4,3)

C. (5,3)

D. (2,4)

Answer: A

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46. If the parabola $y = ax^2 - 6x + b$ passes through $(0, 2)$ and has its tangent at $x = \frac{3}{2}$ parallel to the x-axis, then $a = 2, b = -2$ (b) $a = 2, b = 2$ $a = -2, b = 2$ (d) $a = -2, b = -2$

A. a=2, b=-2

B. a=2, b=2

C. a=-2, b=2

D. a=-2, b=-2

Answer: B

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47. Double ordinate AB of the parabola $y^2 = 4ax$ subtends an angle $\frac{\pi}{2}$ at the focus of the parabola. Then the tangents drawn to the parabola at $AandB$ will intersect at $(-4a, 0)$ (b) $(-2a, 0)$ $(-3a, 0)$ (d) none of these

A. (-4a,0)

B. (-2a,0)

C. (-3a,0)

D. none of these

Answer: A

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48. The tangent to $y^2 = ax$ make angles θ_1 and θ_2 with the x-axis. If $\cos\theta_1\cos\theta_2 = \lambda$, then the locus of their point of intersection is

A. $x^2 = \lambda^2\left[(x-a)^2 + 4y^2\right]$

B. $x^2 = \lambda^2\left[(\pm a)^2 + y^2\right]$

C. $x^2 = \lambda^2\left[(x-a)^2 + y^2\right]$

D. $4x^2 = \lambda^2\left[(x-a)^2 + y^2\right]$

Answer: C

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49. A tangent is drawn to the parabola $y^2 = 4ax$ at the point P whose abscissa lies in the interval (1, 4). The maximum possible area f the triangle formed by the tangent at $P,$ the ordinates of the point $P,$ and the x-axis is equal to 8 (b) 16 (c) 24 (d) 32

A. 8

B. 16

C. 24

D. 32

Answer: B

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50. The straight lines joining any point P on the parabola $y^2 = 4ax$ to the vertex and perpendicular from the focus to the tangent at P intersect at R. Then the equation of the locus of R is $x^2 + 2y^2 - ax = 0$ $2x^2 + y^2 - 2ax = 0$ $2x^2 + 2y^2 - ay = 0$ (d) $2x^2 + y^2 - 2ay = 0$

A. $x^2 + 2y^2 - ax = 0$

B. $2x^2 + y^2 - ax = 0$

C. $2x^2 + 2y^2 - ay = 0$

D. $2x^2 + y^2 - ay = 0$

Answer: B

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51. Through the vertex O of the parabola $y^2 = 4ax$, two chords $OPandOQ$ are drawn and the circles on OP and OQ as diameters intersect at R. If θ_1, θ_2 , and φ are the angles made with the axis by the tangents at P and Q on the parabola and by $OR,$ then value of $\cot\theta_1 + \cot\theta_2$ is $-2\tan\varphi$ (b) $-2\tan(\pi - \varphi)$ 0 (d) $2\cot\varphi$

A. $-2\tan\phi$

B. $-2\tan(\pi - \phi)$

C. 0

D. $2\cot\phi$

Answer: A

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52. AB is a double ordinate of the parabola $y^2 = 4ax\cdot$ Tangents drawn to the parabola at $AandB$ meet the y-axis at A_1andB_1 , respectively. If the area of trapezium $\forall_1 B_1B$ is equal to $12a^2,$ then the angle subtended by A_1B_1 at the focus of the parabola is equal to $2\tan^{-1}(3)$ (b) $\tan^{-1}(3)$ $2\tan^{-1}(2)$ (d) $\tan^{-1}(2)$

A. $2\tan^{-1}$ (3)

B. $\tan^{-1}$ (3)

C. $2\tan^{-1}$ (2)

D. $\tan^{-1}$ (2)

Answer: C

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53. If the locus of the middle of point of contact of tangent drawn to the parabola $y^2 = 8x$ and the foot of perpendicular drawn from its focus to the tangents is a conic, then the length of latus rectum of this conic is $\frac{9}{4}$ (b) 9 (c) 18 (d) $\frac{9}{2}$

A. $9/4$

B. 9

C. 18

D. $9/2$

Answer: B

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54. If the bisector of angle APB, where $PAandPB$ are the tangents to the parabola $y^2 = 4ax$, is equally, inclined to the coordinate axes, then the point P lies on the tangent at vertex of the parabola directrix of the parabola circle with center at the origin and radius a the line of the latus rectum.

A. tangent at vertex of the parabola

B. directrix of the parabola

C. circle with center at the origin and radius

D. the line of latus rectum

Answer: D

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55. From a point A(t) on the parabola $y^2 = 4ax$, a focal chord and a tangent are drawn. Two circles are drawn in which one circle is drawn taking focal chord AB as diameter and other is drawn by taking the intercept of tangent between point A and point of the circles is

A. the line joining focus and p

B. the line joining focus and A

C. tangent to the parabola at point A

D. none of these

Answer: C

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56. The point of intersection of the tangents of the parabola $y^2 = 4x$ drawn at the end point of the chord x+y=2 lies on

A. x-2y=0

B. x+2y=0

C. y-x=0

D. x+y=0

Answer: C

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57. The angle between tangents to the parabola $y^2 = 4ax$ at the points where it intersects with teine $x - y - a = 0$ is $(a > 0)$

A. $\pi / 3$

B. $\pi / 4$

C. $\pi / 6$

D. $\pi / 2$

Answer: D

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58. $y = x + 2$ is any tangent to the parabola $y^2 = 8x$. The point P on this tangent is such that the other tangent from it which is perpendicular to it is $(2, 4)$ (b) $(- 2, 0)$ $(- 1, 1)$ (d) $(2, 0)$

A. (2,4)

B. (-2,0)

C. (-1,1)

D. (2,0)

Answer: B

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59. If $y = m_1x + c$ and $y = m_2x + c$ are two tangents to the parabola $y^2 + 4a(x + a) = 0$, then $m_1 + m_2 = 0$ (b) $1 + m_1 + m_2 = 0$ $m_1m_2 - 1 = 0$ (d) $1 + m_1m_2 = 0$

A. $m_1 + m_2 = 0$

B. $1 + m_1 + m_2 = 0$

C. $m_1m_2 - 1 = 0$

D. $1 + m_1m_2 = 0$

Answer: D

Watch Video Solution

60. The angle between the tangents to the curve $y = x^2 - 5x + 6$ at the point (2, 0) and (3, 0) is $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{4}$

Answer: A

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61. Two mutually perpendicular tangents of the parabola $y^2 = 4ax$ meet the axis at P_1 and P_2. If S is the focal of the parabola, Then $\frac{1}{SP_1} + \frac{1}{SP_2}$ is equal to

A. $\frac{1}{2a}$

B. $\frac{1}{a}$

C. $\frac{2}{a}$

D. $\frac{4}{a}$

Answer: B

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62. Radius of the circle that passes through the origin and touches the parabola $y^2 = 4ax$ at the point $(a, 2a)$ is $\frac{5}{\sqrt{2}}a$ (b) $2\sqrt{2}a$ $\sqrt{\frac{5}{2}}a$ (d) $\frac{3}{\sqrt{2}}a$

A. $\frac{5}{\sqrt{2}}a$

B. $2\sqrt{2}a$

C. $\sqrt{\frac{5}{2}}a$

D. $\frac{3}{\sqrt{2}}a$

Answer: A

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63. The mirror image of the parabola $y^2 = 4x$ in the tangent to the parabola at the point (1, 2) is:

A. $(x-1)^2 = 4(y+1)$

B. $(x+1)^2 = 4(y+1)$

C. $(x+1)^2 = 4(y-1)$

D. $(x-1)^2 = 4(y-1)$

Answer: C

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64. Consider the parabola $y^2 = 4x$. Let $A \equiv (4, -4)$ and $B \equiv (9, 6)$ be two fixed points on the parabola. Let C be a moving point on the parabola between $A and B$ such that the area of the triangle ABC is maximum. Then the coordinates of C are $\left(\frac{1}{4}, 1\right)$ (b) $(4, 4)$ $\left(3, \frac{2}{\sqrt{3}}\right)$ (d) $(3, -2\sqrt{3})$

A. (1/4,1)

B. (4,4)

C. $\left(3,\ 2\sqrt{3}\right)$

D. $\left(3,\ -2\sqrt{3}\right)$

Answer: A

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65. A line of slope `lambda(0

A. 2λ

B. $\frac{2\lambda}{-1+\lambda^2}$

C. $\frac{1-\lambda^2}{1+\lambda^2}$

D. none of these

Answer: B

Watch Video Solution

66. The tangent at any point P onthe parabola $y^2 = 4ax$ intersects the y-axis at Q. Then tangent to the circumcircle of triangle $PQS(S$ is the focus) at Q is a line parallel to x-axis y-axis a line parallel to y-axis (d) none of these

A. a line parallel to axis

B. y-axis

C. a line parallel to y-axis

D. none of these

Answer: B

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67. If $P(t^2, 26), t \in [0, 2]$, is an arbitrary point on the parabola $y^2 = 4x, Q$ is the foot of perpendicular from focus S on the tangent at P, then the maximum area of PQS is 1 (b) 2 (c) $\frac{5}{16}$ (d) 5

A. 1

B. 2

C. $5/16$

D. 5

Answer: D

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68. The minimum area of circle which touches the parabolas $y = x^2 + 1$ and $y^2 = x - 1$ is

A. $\frac{9\pi}{16}$ sq. unit

B. $\frac{9\pi}{32}$ sq. unit

C. $\frac{9\pi}{8}$ sq. unit

D. $\frac{9\pi}{4}$ sq. unit

Answer: B

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69. If the tangents and normals at the extremities of a focal chord of a parabola intersect at (x_1, y_1) and (x_2, y_2), respectively, then $x_1 = y^2$ (b) $x_1 = y_1$ $y_1 = y_2$ (d) $x_2 = y_1$

A. $x_1 = y_2$

B. $x_1 = y_1$

C. $y_1 = y_2$

D. $x_2 = y_1$

Answer: C

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70. At what point on the parabola $y^2 = 4x$ the normal makes equal angle with the axes? $(4, 4)$ (b) $(9, 6)$ (d) $(4, -4)$ (d) $(1, \pm 2)$

A. (4,4)

B. (9,6)

C. (4,-4)

D. $(1, \pm 2)$

Answer: D

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71. The line $2x + y + \lambda = 0$ is a normal to the parabola $y^2 = -8x$, is λ =

A. 12

B. -12

C. 24

D. -24

Answer: C

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72. If two normals to a parabola $y^2 = 4ax$ intersect at right angles then the chord joining their feet pass through a fixed point whose co-ordinates are:

A. (-2a,0)

B. (a,0)

C. (2a,0)

D. none of these

Answer: B

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73. The equation of the line that passes through $(10, -1)$ and is perpendicular to $y = \frac{x^2}{4} - 2$ is $4x + y = 39$ (b) $2x + y = 19$ $x + y = 9$ (d) $x + 2y = 8$

A. 4x+y=39

B. 2x+y=19

C. x+y=9

D. x+2y=8

Answer: D

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74. Tongent and normal drawn to a parabola at $A(at^2, 2at), t \neq 0$ meet the x-axis at point $BandD$, respectively. If the rectangle $ABCD$ is completed, then the locus of C is $y = 2a$ (b) $y + 2a = c\ x = 2a$ (d) none of these

A. y=2a

B. $x = 2a - \frac{y^2}{4a}$

C. x=2a

D. none of these

Answer: B

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75. The radius of the circle touching the parabola $y^2 = x$ at (1, 1) and having the directrix of $y^2 = x$ as its normal is $\frac{5\sqrt{5}}{8}$ (b) $\frac{10\sqrt{5}}{3}$ $\frac{5\sqrt{5}}{4}$ (d) none of these

A. $5\sqrt{5}/8$

B. $10\sqrt{5}/3$

C. $5\sqrt{5}/4$

D. none of these

Answer: C

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76. If two different tangents of $y^2 = 4x$ are the normals to $x^2 = 4by$, then $|b| > \frac{1}{2\sqrt{2}}$ (b) $|b| < \frac{1}{2\sqrt{2}}$ $|b| > \frac{1}{\sqrt{2}}$ (d) $|b| < \frac{1}{\sqrt{2}}$

A. $|b| > 1/2\sqrt{2}$

B. $|b| < 1/2\sqrt{2}$

C. $|b| < 1/\sqrt{2}$

D. $|b| < 1/\sqrt{2}$

Answer: B

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77. Maximum number of common normals of $y^2 = 4ax$ and $x^2 = 4by$ is

____

A. 3

B. 4

C. 6

D. 5

Answer: D

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78. If line PQ, where equation is $y=2x+k$, is a normal to the parabola whose vertex is $(-2,3)$ and the axis parallel to the x-axis with latus rectum equal to 2, then the value of k is $\frac{58}{8}$ (b) $\frac{50}{8}$ (c) 1 (d) -1

A. $58/8$

B. $50/8$

C. 1

D. -1

Answer: C

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79. $\min\left[\left(x_1-x^2\right)^2+\left(3+\sqrt{1-x12}-\sqrt{4x_2}\right)\right], \forall x_1, x_2 \in R,$ is $4\sqrt{5}+1$ (b) $3-2\sqrt{2}$ $\sqrt{5}+1$ (d) $\sqrt{5}-1$

A. $4\sqrt{1}$

B. $3-2\sqrt{2}$

C. $\sqrt{5}+1$

D. $\sqrt{5}-1$

Answer: B

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80. If the normals to the parabola $y^2=4ax$ at three points $\left(ap^2, 2ap\right)$, and $\left(aq^2, 2aq\right)$ are concurrent, then the common root of equations $Px^2+qx+r=0$ and $a(b-c)x^2+b(c-a)x+c(a-b)=0$ is p (b) q (c) r (d) 1

A. p

B. q

C. r

D. 1

Answer: D

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81. Normals $AO, \forall_1 and \forall_2$ are drawn to the parabola $y^2 = 8x$ from the point $A(h, 0)$. If triangle OA_1A_2 is equilateral then the possible value of h is 26 (b) 24 (c) 28 (d) none of these

A. 26

B. 24

C. 28

D. none of these

Answer: C

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82. If the normals to the parabola $y^2 = 4ax$ at the ends of the latus rectum meet the parabola at $QandQ'$, then $\mathbb{Q}'$ is $10a$ (b) $4a$ (c) $20c$ (d) $12a$

A. 10a

B. 4a

C. 20a

D. 12a

Answer: D

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83. From a point $(\sin\theta, \cos\theta)$, if three normals can be drawn to the parabola $y^2 = 4ax$ then the value of a is

A. (1/2,1)

B. [-1/2,0)

C. [1/2,1]

D. $\left(-\frac{1}{2}, 0 \cup (0, 12)\right)$

Answer: D

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84. If the normals at $P(t_1)$ and $Q(t_2)$ on the parabola meet on the same parabola, then

A. $t_1t_2 = -1$

B. $t_2 = -t_1 - \frac{2}{t_1}$

C. $t_1t_2 = 1$

D. $t_1t_2 = 2$

Answer: D

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85. If the normals to the parabola $y^2 = 4ax$ at P meets the curve again at Q and if PQ and the normal at Q make angle α andβ , respectively, with the x-axis, then $tan\alpha(\tan\alpha + \tan\beta)$ has the value equal to 0 (b) -2 (c) $-\frac{1}{2}$ (d) -1

A. 0

B. -2

C. $-1/2$

D. -1

Answer: B

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86. PQ is a normal chord of the parabola $y^2 = 4ax$ at P, A being the vertex of the parabola. Through P a line is drawn parallel to AQ meeting the x-axis in R. Then the length of AR is : (A) equal to the length of the latus rectum (B) equal to the focal distance of the point P (C) equal to the twice of the focal distance of the point P (D) equal to the distance of the point P from the directrix.

A. equal to the length of the latus rectum

B. equal to the focal distance of the point P

C. equal to twice focal distance of the point P

D. equal to the distance of the point P from the directrix

Answer: C

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87. $P, Q,$ and R are the feet of the normals drawn to a parabola $(y-3)^2 = 8(x-2)$. A circle cuts the above parabola at points $P, Q, R, andS$. Then this circle always passes through the point. $(2, 3)$ (b) $(3, 2)$ (c) $(0, 3)$ (d) $(2, 0)$

A. (2,3)

B. (3,2)

C. (0,3)

D. (2,0)

Answer: A

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88. Normals at two points $(x_1y_1)and(x_2, y_2)$ of the parabola $y^2 = 4x$ meet again on the parabola, where $x_1 + x_2 = 4$. Then $|y_1 + y_2|$ is equal to $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) none of these

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $4\sqrt{2}$

D. none of these

Answer: C

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89. The endpoints of two normal chords of a parabola are concyclic. Then the tangents at the feet of the normals will intersect at tangent at vertex of the parabola axis of the parabola directrix of the parabola none of these

A. tangent at vertex of the parabola

B. axis of the parabola

C. directrix of the parabola

D. none of these

Answer: B

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90. If normal at point P on the parabola $y^2 = 4ax, (a > 0)$, meets it again at Q in such a way that OQ is of minimum length, where O is the vertex of parabola, then OPQ is a right angled triangle an obtuse angled triangle an acute angle triangle none of these

A. a right-angled triangle

B. an obtuse-angled triangle

C. an acute-angled triangle

D. none of these

Answer: A

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91. The set of points on the axis of the parabola $(x-1)^2 = 8(y+2)$ from where three distinct normals can be drawn to the parabola is the set (h,k) of points satisfying

A. $h > 2$

B. $h > 1$

C. $k > 2$

D. none of these

Answer: C

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92. Tangent and normal are drawn at the point $P \equiv (16, 16)$ of the parabola $y^2 = 16x$ which cut the axis of the parabola at the points $AandB$, rerspectively. If the center of the circle through $P, A, andB$ is C , then the angle between PC and the axis of x is $\frac{\tan^{-1} 1}{2}$ (b) $\tan^{-1} 2$ $\frac{\tan^{-1} 3}{4}$ (d) $\frac{\tan^{-1} 4}{3}$

A. $\frac{\tan^{-1}(1)}{2}$

B. $\tan^{-1} 2$

C. $\frac{\tan^{-1}(3)}{4}$

D. $\frac{\tan^{-1}(4)}{3}$

Answer: D

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93. In parabola y^2=4x, From the point (15,12), three normals are drawn to the three co-normals point is

A. (16/3.0)

B. (4,0)

C. (26/3.0)

D. (6,0)

Answer: C

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94. The line $x - y = 1$ intersects the parabola $y^2 = 4x$ at A and B . Normals at $AandB$ intersect at C. If D is the point at which line CD is normal to the parabola, then the coordinates of D are $(4, -4)$ (b) $(4, 4)$ $(-4, -4)$ (d) none of these

A. (4,-4)

B. (4,4)

C. (-4,-4)

D. none of these

Answer: B

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95. If normal are drawn from a point $P(h, k)$ to the parabola $y^2 = 4ax$, then the sum of the intercepts which the normals cut-off from the axis of the parabola is $(h + c)$ (b) $3(h + a)$ $2(h + a)$ (d) none of these

A. (h+a)

B. 3(h+a)

C. 2(h+a)

D. none of these

Answer: C

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96. The circle $x^2 + y^2 + 2\lambda x = 0, \lambda \in R$, touches the parabola $y^2 = 4x$ externally. Then,

A. $\lambda > 0$

B. $\lambda < 0$

C. $\lambda > 1$

D. none of these

Answer: A

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97. The radius of the circle whose centre is (-4,0) and which cuts the parabola $y^2 = 8x$ at A and B such that the common chord AB subtends a right angle at the vertex of the parabola is equal to

A. $4\sqrt{13}$

B. $3\sqrt{5}$

C. $3\sqrt{2}$

D. $2\sqrt{5}$

Answer: A

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98. If normal at point P on parabola $y^2 = 4ax, (a > 0)$, meets it again at Q in such a way that OQ is of minimum length, where O is the vertex of parabola, then ΔOPQ is

A. a right-angled triangle

B. an obtuse-angled triangle

C. an acute-angled triangle

D. none of these

Answer: A

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Exercise (Multiple)

1. If the focus of the parabola $x^2 - ky + 3 = 0$ is (0,2), then a values of k is (are) 4 (b) 6 (c) 3 (d) 2

A. 4

B. 6

C. 3

D. 2

Answer: B::D

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2. If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of k is $\frac{1}{8}$ (b) 8 (c) 4 (d) $\frac{1}{4}$

A. -8

B. $1/8$

C. $1/4$

D. 4

Answer: A::D

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3. The extremities of latus rectum of a parabola are (1, 1) and $(1, -1)$. Then the equation of the parabola can be $y^2 = 2x - 1$ (b) $y^2 = 1 - 2x$ $y^2 = 2x - 3$ (d) $y^2 = 2x - 4$

A. $y^2 + 2x - 1$

B. $y^2 = 1 - 2x$

C. $y^2 = 3 - 2x$

D. $y^2 = 2x - 4$

Answer: A::C

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4. The value(s) of a for which two curves $y = ax^2 + ax + \frac{1}{24}$ and $x = ay^2 + ay + \frac{1}{24}$ touch each other is/are

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $\frac{3}{2}$

D. $\frac{1}{2}$

Answer: A::C

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5. In which of the following cases, a unique parabola will be obtained ?

A. Focus and equation of tangent at vertex are given.

B. Focus and vertex are given

C. Equation of directrix and vertex are given.

D. Equation of directrix and equation of tangent at vertex are given.

Answer: A::B::C

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6. A quadrilateral is inscribed in a parabola . Then,

A. the quadrilateral may be cyclic

B. diagonals of the quadrilateral may be equal

C. all possible pairs of adjacent sides may be perpendicular

D. none of these

Answer: A::B

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7. The locus of the midpoint of the focal distance of a variable point moving on theparabola $y^2 = 4ax$ is a parabola whose latus rectum is half the latus rectum of the original parabola vertex is $\left(\frac{a}{2}, 0\right)$ directrix is y-axis. focus has coordinates (a, 0)

A. latus rectum is half the latus rectum of the original parabola

B. vertex is (a/2,0)

C. directrix is y-axis

D. focus has coordinates (a,0)

Answer: A::B::C::D

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8. A square has one vertex at the vertex of the parabola $y^2 = 4ax$ and the diagonal through the vertex lies along the axis of the parabola. If the ends of the other diagonal lie on the parabola, the coordinates of the vertices of the square are $(4a, 4a)$ (b) $(4a, -4a)$ $(0, 0)$ (d) $(8a, 0)$

A. (4a,4a)

B. (4a,-4a)

C. (0,0)

D. (8a,0)

Answer: A::B::C

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9. If two distinct chords of a parabola $y^2 = 4ax$, passing through (a,2a) are bisected by the line x+y=1 ,then length of latus rectum can be

A. 2

B. 1

C. 4

D. 3

Answer: A::B::D

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10. Let PQ be a chord of the parabola $y^2 = 4x$. A circle drawn with PQ as a diameter passes through the vertex V of theparabola. If $ar(\Delta PVQ) = 20$ sq unit then the coordinates of P are

A. (16,8)

B. (16,-8)

C. (9,6)

D. (9,-6)

Answer: A::B

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11. If the parabola $x^2 = ay$ makes an intercept of length $\sqrt{40}$ unit on the line $y - 2x = 1$ then a is equal to

A. -1

B. -2

C. 1

D. 2

Answer: B::C

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12. The equation of the directrix of the parabola with vertex at the origin and having the axis along the x-axis and a common tangent of slope 2 with the circle $x^2 + y^2 = 5$ is (are) $x = 10$ (b) $x = 20$ $x = -10$ (d) $x = -20$

A. x=10

B. x=20

C. x=-10

D. x=-20

Answer: A::C

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13. Tangent is drawn at any point (x_1, y_1) other than the vertex on the parabola $y^2 = 4ax$. If tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of contact pass through a fixed point (x_2, y_2), then $x_1 a, x_2$ in GP (b) $\frac{y_1}{2}, a, y_2$ are in GP $-4, \frac{y_1}{y_2}, x_1 / x_2$ are in GP (d) $x_1 x_2 + y_1 y_2 = a^2$

A. x_1, a, x_2 are in GP

B. $\frac{y_1}{2}, a, y_2$ are in GP

C. $-4\frac{y_1}{y_2}, \frac{x_1}{x_2}$ are in GP

D. $x_1x_2 + y_1y_2 = a^2$

Answer: B::C::D

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14. The parabola $y^2 = 4x$ and the circle having its center at 6, 5) intersect at right angle. Then find the possible points of intersection of these curves.

A. (9,6)

B. $(2, \sqrt{8})$

C. (4,4)

D. $(3, 2\sqrt{3})$

Answer: A::C

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15. Which of the following line can be tangent to the parabola $y^2 = 8x$?

$x - y + 2 = 0$ (b) $9x - 3y + 2 = 0$ $x + 2y + 8 = 0$ (d) $x + 3y + 12 = 0$

A. x-y+2=0

B. 9x-3y+2=0

C. x+2y+8=0

D. x+3y+12=0

Answer: A::B::C

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16. If the line $k^2(x-1) + k(y-2) + 1 = 0$ touches the parabola $y^2 - 4x - 4y + 8 = 0$, then k can be

A. -3

B. $-\sqrt{5}$

C. $\frac{7}{19}$

D. 1000

Answer: A::B::C::D

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17. The equation of the line that touches the curves $y = x|x|$ and $x^2 + (y^2 - 2)^2 = 4$, where $x \neq 0,$ is $y = 4\sqrt{5}x + 20$ (b) $y = 4\sqrt{3}x - 12$ $y = 0$ (d) $y = -4\sqrt{5}x - 20$

A. $y = 4\sqrt{5}x + 20$

B. $y = 4\sqrt{3}x - 12$

C. y=0

D. $y = -4\sqrt{5}x - 20$

Answer: A::B::C

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18. The equations of the common tangents to the parabola $y = x^2$ and $y = -(x-2)^2$ is/are :

A. y=4(x-1)

B. y=0

C. y=-4(x-1)

D. y=-30x-50

Answer: A::B

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19. The line $x + y + 2 = 0$ is a tangent to a parabola at point A, intersect the directrix at B and tangent at vertex at C respectively. The focus of parabola is $S(2, 0)$. Then

A. CS is perpendicular to AB

B. $AC \cdot BC = CS^2$

C. $AC \cdot BC = 8$

D. AC=BC

Answer: A::B::C

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20. Which of the following line can be normal to parabola $y^2 = 12x$? $x + y - 9 = 0$ (b) $2x - y - 32 = 0$ $2x + y - 36 = 0$ (d) $3x - y - 72 = 0$

A. x+y-9=0

B. 2x-y-32=0

C. 2x+y-36=0

D. 3x-y-99=0

Answer: A::C::D

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21. A normal drawn to the parabola $= 4ax$ meets the curve again at Q such that the angle subtended by PQ at the vertex is 90^0. Then the coordinates of P can be $\left(8a, 4\sqrt{2}a\right)$ (b) $(8a, 4a)$ $\left(2a, \ -2\sqrt{2}a\right)$ (d) $\left(2a, 2\sqrt{2}a\right)$

A. $\left(8a, 4\sqrt{2}a\right)$

B. (8a,4a)

C. $\left(2a, \ -2\sqrt{2}a\right)$

D. $\left(2a, 2\sqrt{2}a\right)$

Answer: C::D

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22. A circle is drawn having centre at $C(0, 2)$ and passing through focus (S) of the parabola $y^2 = 8x$, if radius (CS) intersects the parabola at point P, then

A. distance of point P from directrix is $(8 - 4\sqrt{2})$

B. distance of point C from point P is $(6\sqrt{2} - 8)$

C. angle subtended by intercept made by circle on directrix at its centre is $\frac{\pi}{2}$

D. point P is the midpoint of C and S

Answer: A::B::C

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23. From any point P on the parabola $y^2 = 4ax$, perpebdicular PN is drawn on the meeting it at N. Normal at P meets the axis in G. For what value/values of t, the point N divides SG internally in the ratio 1 : 3, where S is the focus ?

A. $\sqrt{\frac{3}{5}}$

B. $\sqrt{-\frac{5}{3}}$

C. $\sqrt{-\frac{3}{5}}$

D. $\sqrt{\frac{5}{3}}$

Answer: B::D

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24. Let C_1 and C_2 be parabolas $x^2 = y - 1$ and $y^2 = x - 1$ respectively. Let P be any point on C_1 and Q be any point C_2. Let P_1 and Q_1 be the reflection of P and Q, respectively w.r.t the line y = x then prove that P_1 lies on C_2 and Q_1 lies on C_1 and $PQ \geq [PP_1, QQ_1]$. Hence or otherwise , determine points P_0 and Q_0 on the parabolas C_1 and C_2 respectively such that $P_0Q_0 \leq PQ$ for all pairs of points (P,Q) with P on C_1 and Q on C_2

A. P_1 lies on C_2 and Q_1 lies on C_1

B. $PQ \geq \min\{PP_1, QQ_1\}$

C. point P_0 on C_1 such that $P_0Q_0 \leq PQ$ for all pairs of points (P,Q) is $\left(\frac{1}{3}, \frac{10}{9}\right)$

D. point Q_0 on C_2 such that $P_0Q_0 \leq PQ$ for all pairs of points (P,Q) is $\left(\frac{10}{9}, \frac{1}{3}\right)$

Answer: A::B

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25. The value(s) of a for which two curves $y = ax^2 + ax + \frac{1}{24}$ and $x = ay^2 + ay + \frac{1}{24}$ touch each other is/are

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $\frac{3}{2}$

D. $\frac{1}{2}$

Answer: A::C

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26. From any point P on the parabola $y^2 = 4ax$, perpebdicular PN is drawn on the meeting it at N. Normal at P meets the axis in G. For what value/values of t, the point N divides SG internally in the ratio 1 : 3, where S is the focus ?

A. $\sqrt{\frac{3}{5}}$

B. $\sqrt{-\frac{5}{3}}$

C. $\sqrt{-\frac{3}{5}}$

D. $\sqrt{\frac{5}{3}}$

Answer: B::D

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Exercise (Comprehension)

1. A tangent is drawn at any point P(t) on the parabola $y^2 = 8x$ and on it is takes a point $Q(\alpha, \beta)$ from which a pair of tangent QA and OB are drawn to the circle $x^2 + y^2 = 8$. Using this information, answer the following questions :

The locus of the point of concurrecy of the chord of contact AB of the circle $x^2 + y^2 = 4$ is

A. $y^2 - 2x = 0$

B. $y^2 - x^2 = 4$

C. $y^2 + 4x = 0$

D. $y^2 - 2x^2 = 4$

Answer: C

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2. A tangent is drawn at any point P(t) on the parabola $y^2 = 8x$ and on it is takes a point $Q(\alpha, \beta)$ from which a pair of tangent QA and QB are drawn to the circle $x^2 + y^2 = 8$. Using this information, answer the following questions : The point from which perpendicular tangents can be drawn both the given circle and the parabola is

A. $(4, \pm\sqrt{3})$

B. $(-1, \sqrt{2})$

C. $(-\sqrt{2}, -\sqrt{2})$

D. $(-2, \pm 2\sqrt{3})$

Answer: D

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3. A tangent is drawn at any point P(t) on the parabola $y^2 = 8x$ and on it is takes a point $Q(\alpha, \beta)$ from which a pair of tangent QA and OB are drawn to the circle $x^2 + y^2 = 8$. Using this information, answer the following questions :

The locus of circumcenter of ΔAQB id t=2 is

A. x-2y+2=0

B. x+2y-4=0

C. x-2y-4=0

D. x+2y+4=0

Answer: A

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4. Tangent to the parabola $y = x^2 + ax + 1$ at the point of intersection of the y-axis also touches the circle $x^2 + y^2 = r^2$. Also, no point of the parabola is below the x-axis. The radius of circle when a attains its maximum value is

A. $1/\sqrt{10}$

B. $1/\sqrt{5}$

C. 1

D. $\sqrt{5}$

Answer: B

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5. Tangent to the parabola $y = x^2 + ax + 1$ at the point of intersection of the y-axis also touches the circle $x^2 + y^2 = r^2$. Also, no point of the parabola is below the x-axis.

The slope of the tangents when the radius of the circle is maximum is

A. -1

B. 1

C. 0

D. 2

Answer: C

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6. Tangent to the parabola $y = x^2 + ax + 1$ at the point of intersection of the y-axis also touches the circle $x^2 + y^2 = r^2$. Also, no point of the parabola is below the x-axis.

The minimum area bounded by the tangent and the coordinate axes is

A. 1

B. $1/3$

C. $1/2$

D. $1/4$

Answer: D

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7. The locus of the circumcenter of a variable triangle having sides the y-axis, y=2, and lx+my=1, where (1,m) lies on the parabola $y^2 = 4x$, is a curve C.

The coordinates of the vertex of this curve C is

A. $(-2, 3/2)$

B. $(-2, -3/2)$

C. $(2, 3/2)$

D. $(2, -3/2)$

Answer: A

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8. The locus of the circumcenter of a variable triangle having sides the y-axis, y=2, and lx+my=1, where (1,m) lies on the parabola $y^2 = 4x$, is a curve C.

The length of the smallest chord of this C is

A. $1/4$

B. $1/12$

C. $1/8$

D. $1/16$

Answer: C

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9. The locus of the circumcenter of a variable triangle having sides the y-axis, y=2, and lx+my=1, where (1,m) lies on the parabola $y^2 = 4x$, is a curve C.

The curve C is symmetric about the line

A. $x = 3/2$

B. $y = -3/2$

C. $x = -3/2$

D. $y = 3/2$

Answer: D

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10. y=x is tangent to the parabola $y = ax^2 + c$.

If a=2, then the value of c is

A. 1

B. $-1/2$

C. $1/2$

D. $1/8$

Answer: D

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11. y=x is tangent to the parabola $y = ax^2 + c$.

If (1,1) is the point of contact, then a is

A. $1/4$

B. $1/3$

C. $1/2$

D. $1/6$

Answer: C

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12. y=x is tangent to the parabola $y = ax^2 + c$.

If c=2, then the point of contact is

A. (3,3)

B. (2,2)

C. (6,6)

D. (4,4)

Answer: D

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13. If l, m are variable real numbers such that $5l^2 + 6m^2 - 4lm + 3l = 0$ and the variable line $lx + my = 1$ always touches a fixed parabola $P(x, y) = 0$ whose axis parallel to x-axis , then A) vertex of P(x,y) is (-5/3, 4/3) B) Latus Rectum is equal to 4 C) Directrix of P(x,y) is parallel to y-axis

(D) AB is a focal chord of P(x,y) =0 then $\frac{1}{SA}+\frac{1}{SB}=\frac{1}{2}$ where S is the focus

A. $(-5/3, 4/3)$

B. $(-7/4, 3/4)$

C. $(-7/4, 3/4)$

D. $(1/2, -3/4)$

Answer: A

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14. If l and m are variable real number such that $5l^2+6m^2-4lm+3l=0$, then the variable line lx+my=1 always touches a fixed parabola, whose axes is parallel to the x-axis.

The focus of the parabola is

A. $(1/6, -7/6)$

B. $(1/3, 3/4)$

C. $(3/2, -3/2)$

D. $(-3/4, 3/4)$

Answer: B

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15. If l and m are variable real number such that $5l^2 + 6m^2 - 4lm + 3l = 0$, then the variable line lx+my=1 always touches a fixed parabola, whose axes is parallel to the x-axis.

The directrix of the parabola is

A. 6x+7=0

B. 4x+11=0

C. 3x+11=0

D. none of these

Answer: C

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16. Consider the parabola whose focus is at (0,0) and tangent at vertex is

x-y+1=0

The length of latus rectum is

A. $4\sqrt{2}$

B. $2\sqrt{2}$

C. $8\sqrt{2}$

D. $3\sqrt{2}$

Answer: B

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17. Consider the parabola whose focus is at (0,0) and tangent at vertex is

x-y+1=0

The length of the chord of parabola on the x-axis is

A. $4\sqrt{2}$

B. $2\sqrt{2}$

C. $8\sqrt{2}$

D. $3\sqrt{2}$

Answer: A

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18. Consider the parabola whose focus is at (0,0) and tangent at vertex is x-y+1=0

Tangents drawn to the parabola at the extremities of the chord 3x+2y=0 intersect at angle

A. $\pi / 6$

B. $\pi / 3$

C. $\pi / 2$

D. none of these

Answer: C

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19. the tangent to a parabola are $x - y = 0$ and $x + y = 0$ If the focus of the parabola is F(2,3) then the equation of tangent at vertex is

A. 4x-6y+5=0

B. 4x-6y+3=0

C. 4x-6y+1=0

D. 4x-6y+3/2=0

Answer: A

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20. Two tangents on a parabola are x-y=0 and x+y=0.

S(2,3) is the focus of the parabola.

The length of latus rectum of the parabola is

A. $6/\sqrt{3}$

B. $10/\sqrt{13}$

C. $2/\sqrt{13}$

D. none of these

Answer: B

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21. Two tangents on a parabola are x-y=0 and x+y=0.

S(2,3) is the focus of the parabola.

If P and Q are ends of the focal chord of the parabola, then $\frac{1}{SP}+\frac{1}{SQ}=$

A. $2\sqrt{13}/3$

B. $2\sqrt{13}$

C. $2\sqrt{13}/5$

D. none of these

Answer: C

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22. $y^2 = 4x$ and $y^2 = -8(x - a)$ intersect at points A and C. Points O(0,0), A,B (a,0), and c are concyclic.

The length of the common chord of the parabolas is

A. $2\sqrt{6}$

B. $4\sqrt{3}$

C. $6\sqrt{5}$

D. $8\sqrt{2}$

Answer: D

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23. $y^2 = 4x$ and $y^2 = -8(x - a)$ intersect at points A and C. Points O(0,0), A,B (a,0), and c are concyclic.

The area of cyclic quadrilateral OABC is

A. $24\sqrt{3}$

B. $48\sqrt{2}$

C. $12\sqrt{6}$

D. $18\sqrt{5}$

Answer: B

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24. $y^2 = 4x$ and $y^2 = -8(x - a)$ intersect at points A and C. Points O(0,0), A,B (a,0), and c are concyclic.

Tangents to the parabola $y^2 = 4x$ at A and C intersect at point D and tangents to the parabola $y^2 = -8(x - a)$ intersect at point E. Then the area of quadrilateral DAEC is

A. $96\sqrt{2}$

B. $48\sqrt{3}$

C. $54\sqrt{5}$

D. $36\sqrt{6}$

Answer: A

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25. PQ is the double ordinate of the parabola $y^2 = 4x$ which passes through the focus S. ΔPQA is an isosceles right angle triangle, where A is on the axis of the parabola to the right of focus. Line PA meets the parabola at C and QA meets the parabola at B.

The of trapezium PBCQ is

A. 96 sq. units

B. 64 sq. units

C. 72 sq. units

D. 48 sq. units

Answer: B

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26. PQ is the double ordinate of the parabola $y^2 = 4x$ which passes through the focus S. ΔPQA is an isosceles right angle triangle, where A is on the axis of the parabola to the right of focus. Line PA meets the parabola at C and QA meets the parabola at B.

The circumradius of trapezium PBCQ is

A. $6\sqrt{5}$

B. $3\sqrt{6}$

C. $\sqrt{10}$

D. $5\sqrt{3}$

Answer: C

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27. PQ is the double ordinate of the parabola $y^2 = 4x$ which passes through the focus S. ΔPQA is an isosceles right angle triangle, where A is on the axis of the parabola to the right of focus. Line PA meets the parabola at C and QA meets the parabola at B. The ratio of the inradius of ΔABC and that of ΔPAQ is

A. $2:1$

B. $3:2$

C. $4:3$

D. $3:1$

Answer: D

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28. Consider the inequation $9^x - a3^x - a + 3 \leq 0$, where a is real parameter.

The given inequality has at least one negative soluiton for $a \in$

A. $(-\infty, 2)$

B. $(3, \infty)$

C. $(-2, \infty)$

D. (2,3)

Answer: D

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29. Consider the inequation $9^x - a3^x - a + 3 \leq 0$, where a is real parameter.

The given inequality has at least one real solutions for $a \in$.

A. $(-\infty, -2)$

B. $(3, \infty)$

C. $(2, \infty)$

D. $[-2, \infty)$

Answer: C

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30. Consider the inequation $9^x - a3^x - a + 3 \leq 0$, where a is real parameter.

The given inequality has at least one real solutions for $a \in$.

A. $(-\infty, 3)$

B. $[2, \infty)$

C. $(3, \infty)$

D. $[-2, \infty)$

Answer: B

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31. Consider one sides AB of a square ABCD in order on line $y = 2x - 17$, and other two vertices C, D on $y = x^2$

The minimum intercept of line CD on the y-axis is

A. 3

B. 4

C. 2

D. 6

Answer: A

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32. Consider one sides AB of a square ABCD in order on line $y = 2x - 17$, and other two vertices C, D on $y = x^2$

The maximum possible area of square ABCD is

A. 1180

B. 1250

C. 1280

D. none

Answer: C

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33. Tangent to the parabola $y = x^2 + ax + 1$ at the point of intersection of the y-axis also touches the circle $x^2 + y^2 = r^2$. Also, no point of the parabola is below the x-axis.

The radius of circle when a attains its maximum value is

A. $1/\sqrt{10}$

B. $1/\sqrt{5}$

C. 1

D. $\sqrt{5}$

Answer: B

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34. Tangent to the parabola $y = x^2 + ax + 1$ at the point of intersection of the y-axis also touches the circle $x^2 + y^2 = r^2$. Also, no point of the parabola is below the x-axis.

The slope of the tangents when the radius of the circle is maximum is

A. -1

B. 1

C. 0

D. 2

Answer: C

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35. Tangent to the parabola $y = x^2 + ax + 1$ at the point of intersection of the y-axis also touches the circle $x^2 + y^2 = r^2$. Also, no point of the parabola is below the x-axis.

The minimum area bounded by the tangent and the coordinate axes is

A. 1

B. $1/3$

C. $1/2$

D. $1/4$

Answer: D

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36. The locus of the circumcenter of a variable triangle having sides the y-axis, y=2, and lx+my=1, where (1,m) lies on the parabola $y^2 = 4x$, is a curve C.

The coordinates of the vertex of this curve C is

A. $(-2, 3/2)$

B. $(-2, -3/2)$

C. $(2, 3/2)$

D. $(2, -3/2)$

Answer: A

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37. The locus of the circumcenter of a variable triangle having sides the y-axis, y=2, and lx+my=1, where (1,m) lies on the parabola $y^2 = 4x$, is a curve C.

The length of the smallest chord of this C is

A. $1/4$

B. $1/12$

C. $1/8$

D. $1/16$

Answer: C

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38. The locus of the circumcenter of a variable triangle having sides the y-axis, y=2, and lx+my=1, where (1,m) lies on the parabola $y^2 = 4x$, is a curve C.

The curve C is symmetric about the line

A. $x = 3/2$

B. $y = -3/2$

C. $x = -3/2$

D. $y = 3/2$

Answer: D

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39. y=x is tangent to the parabola $y = ax^2 + c$.

If (1,1) is the point of contact, then a is

A. $1/4$

B. $1/3$

C. $1/2$

D. $1/6$

Answer: C

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40. y=x is tangent to the parabola $y = ax^2 + c$.

If c=2, then the point of contact is

A. (3,3)

B. (2,2)

C. (6,6)

D. (4,4)

Answer: D

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41. Consider the parabola whose focus is at (0,0) and tangent at vertex is x-y+1=0

Tangents drawn to the parabola at the extremities of the chord 3x+2y=0 intersect at angle

A. $\pi/6$

B. $\pi/3$

C. $\pi/2$

D. none of these

Answer: C

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Exercise (Matrix)

1. Consider the parabola $(x-1)^2+(y-2)^2=\frac{(12x-5y+3)^2}{169}$ and match the following lists :

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2. Consider the parabola $y^2=12x$ and match the following lists :

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3. Match the following lists :

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4. Match the following lists and and then choose the correct code.

A. $\begin{matrix} a & b & c & d \\ p & r & q & q \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ q & p & r & s \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ s & p & q & r \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ r & s & q & p \end{matrix}$

Answer: B

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Exercise (Numerical)

1. If the length of the latus rectum rectum of the parabola $169\left\{(x-1)^2+(y-3)^2\right\}=(5x-12y+17)^2$ is L then the value of 13L/4 is ________.

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2. A circle is drawn through the point of intersection of the parabola $y = x^2 - 5x + 4$ and the x-axis such that origin lies outside it. The length of a tangent to the circle from the origin is _______ .

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3. The focal chord of $y^2 = 16x$ is tangent to $(x - 6)^2 + y^2 = 2.$

Then the possible value of the square of slope of this chord is ________ .

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4. Two tangent are drawn from the point $(-2, -1)$ to parabola $y^2 = 4x$. if α is the angle between these tangents, then find the value of $\tan \alpha$.

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5. The equation of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is ax+by+c=0. Then the value of a+b+c is __________ .

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6. If the point P(4, -2) is the one end of the focal chord PQ of the parabola $y^2 = x,$ then the slope of the tangent at Q, is

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7. If the line x+y=6 is a normal to the parabola $y^2 = 8x$ at point (a,b), then the value of a+b is __________ .

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8. The locus of the midpoints of the portion of the normal to the parabola $y^2 = 16x$ intercepted between the curve and the axis is another parabola whose latus rectum is ________ .

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9. Consider the locus of center of the circle which touches the circle $x^2 + y^2 = 4$ externally and the line x=4. The distance of the vertex of the locus from the otigin is ________ .

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10. If on a given base BC[B(0,0) and C(2,0)], a triangle is described such that the sum of the base angles is 4, then the equation of the locus of the opposite vertex A is parabola whose directrix is y=k. The value of k is ________ .

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11. PQ is any focal chord of the parabola $y^2 = 8$x. Then the length of PQ can never be less than ________ .

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12. The length of focal chord to the parabola $y^2 = 12x$ drawn from the point (3,6) on is _________ .

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13. From the point (-1,2), tangent lines are to the parabola $y^2 = 4x$. If the area of the triangle formed by the chord of contact and the tangents is A, then the value of $A / \sqrt{2}$ is __________ .

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14. Line y=2x-b cuts the parabola $y = x^2 - 4x$ at points A and B. Then the value of b for which $\angle AOB$ is a right is (where O is origin) ________ .

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15. A line through the origin intersects the parabola $5y = 2x^2 - 9x + 10$ at two points whose x-coordinates add up to 17.

Then the slope of the line is ________ .

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16. If the circle $(x - 6)^2 + y^2 = r^2$ and the parabola $y^2 = 4x$ have maximum number of common chords, then the least integral value of r is ________ .

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17. The slope of line which belongs to family (1+ l) x + (1-l)y + 2(1-l) = 0 and makes shortest intercept on $x^2 = 4y - 4$

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18. If 3x+4y+k=0 represents the equation of tangent at the vertex of the parabola $16x^2 - 24xy^2 + 14x + 2y + 7 = 0$, then the value of k is _______ .

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19. Normals at $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) to the parabola $y^2 = 4x$ are concurrent at point P. If $y_1y_2 + y_2y_3 + y_3y_1 = x_1x_2x_3$, then locus of point P is part of parabola, length of whose latus rectum is ________.

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20. Foot of perpendicular from point P on the parabola $y^2 = 4ax$ to the axis is N. A straight line is drawn parallel to the axis which bisects PN and cuts the curve at Q. If NQ meets the tangent at the vertex A at a point T, then $\frac{PN}{AT}$=__________.

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21. Points A, B, C lie on the parabola $y^2 = 4ax$ The tangents to the parabola at A, B and C, taken in pair, intersect at points P, Q and R. Determine the ratio of the areas of the $\triangle ABC$ and $\triangle PQR$

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22. Normals are drawn from a point P with slopes m_1, m_2 and m_3 are drawn from the point p not from the parabola $y^2 = 4x$. For $m_1 m_2 = \alpha$, if the locus of the point P is a part of the parabola itself, then the value of α is (a) 1 (b)-2 (c) 2 (d) -1

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23. If the length of the latus rectum rectum of the parabola $169\left\{(x-1)^2+(y-3)^2\right\}=(5x-12y+17)^2$ is L then the value of 13L/4 is ________.

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24. If the line x+y=6 is a normal to the parabola $y^2=8x$ at point (a,b), then the value of a+b is __________ .

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25. Consider the locus of center of the circle which touches the circle $x^2+y^2=4$ externally and the line x=4. The distance of the vertex of the locus from the otigin is _________ .

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26. Line y=2x-b cuts the parabola $y = x^2 - 4x$ at points A and B. Then the value of b for which $\angle AOB$ is a right is (where O is origin) _______ .

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27. A line through the origin intersects the parabola $5y = 2x^2 - 9x + 10$ at two points whose x-coordinates add up to 17.

Then the slope of the line is ________ .

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28. If 3x+4y+k=0 represents the equation of tangent at the vertex of the parabola $16x^2 - 24xy^2 + 14x + 2y + 7 = 0$, then the value of k is _______ .

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JEE Main Previous Year

1. If two tangents drawn from a point P to the parabola y2 = 4x are at right angles, then the locus of P is (1) $2x+1=0$ (2) $x=1$ (3) $2x1=0$ (4) $x=1$

A. 2x-1=0

B. x=1

C. 2x+1=0

D. x=-1

Answer: D

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2. Given : A circle, $2x^2+2y^2=5$ and a parabola, $y^2=4\sqrt{5}x$. Statement - I : An equation of a common tangent to these curves is $y=$ x+$\sqrt{5}$ Statement - II : If the line, $y=mx+\dfrac{\sqrt{5}}{m}(m\neq 0)$ is their common tangent, then m satisfies $m^4-3m^2+2=0.$ (1) Statement - I is True; Statement -II is true; Statement-II is not a correct explanation for

Statement-I (2) Statement -I is True; Statement -II is False. (3) Statement -I is False; Statement -II is True (4) Statement -I is True; Statement -II is True; Statement-II is a correct explanation for Statement-I

A. Statement 1 is true , statement 2 is true , statement 2 is a correct explanation for statement 1.

B. Statement 1 is true , statement 2 is true , statement 2 is not correct explanation for statement 1.

C. Statement 1 is true , statement 2 is false.

D. Statement 1 is false , statement 2 is true.

Answer: B

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3. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

A. $1/2$

B. $3/2$

C. $1/8$

D. $2/3$

Answer: A

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4. Let O be the vertex and Q be any point on the parabola,$x^2 = 8y$. It the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is :

A. $x^2 = y$

B. $y^2 = x$

C. $y^2 = 2x$

D. $x^2 = 2y$

Answer: D

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5. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y+6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is : (1) $x^2 + y^2 - 4x + 8y + 12 = 0$ (2) $x^2 + y^2 - x + 4y - 12 = 0$ (3) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$ (4) $x^2 + y^2 - 4x + 9y + 18 = 0$

A. $x^2 + y^2 - x + 4y - 12 = 0$

B. $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

C. $x^3 + y^2 - 4x + 9y - 18 = 0$

D. $x^2 + y^2 - 4x + 8y - 12 = 0$

Answer: D

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6. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines $y = |x|$ is :

A. $4(\sqrt{2} + 1)$

B. $2(\sqrt{2} + 1)$

C. $2(\sqrt{2} - 1)$

D. $4(\sqrt{2} - 1)$

Answer: D

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7. If the tangent at (1,7) to curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is

A. 95

B. 195

C. 185

D. 85

Answer: A

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8. Tangent and normal are drawn at P(16,16) on the parabola $y^2 = 16x$ which intersect the axis of the parabola at A and B respectively. If C is the centre of the circle through the points P,A and B and $\angle CPB = \theta$ then the value of $\tan\theta$ is

A. $4/3$

B. $1/2$

C. 2

D. 3

Answer: C

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JEE Advenced Single Answer Type

1. Let (x,y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0,0) and (x,y) n the ratio 1:3. Then the locus of P is :

A. $x^2 = y$

B. $y^2 = 2x$

C. $y^2 = x$

D. $x^2 = 2y$

Answer: C

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2. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at P, Q andthe parabola at R, S. Then area of quadrilateral $PQRS$ is

A. 3

B. 6

C. 9

D. 15

Answer: D

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3. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose:

A. vertex is $(2a/3, 0)$

B. directrix is x=0

C. latus rectum is $2a/3$

D. focus is (a,0)

Answer: A::D

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4. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be (A) $-\frac{1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

A. $-1/r$

B. $1/r$

C. $2/r$

D. $-2/r$

Answer: C::D

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5. Let L be a normal to the parabola $y^2 = 4x$.If L passes through the point (9,6) then L is given by

A. y-x+3=0

B. y+3x-33=0

C. y+x-15=0

D. y-2x+12=0

Answer: A::C::D

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6. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is 32 , then which of the following is (are) the coordinates of P?

A. $\left(4, 2\sqrt{2}\right)$

B. $(9, 3\sqrt{2})$

C. $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$

D. $(1, \sqrt{2})$

Answer: A::D

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7. Let P be the point on parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ let Q be the point on the circle dividing the line segment SP internally. Then

A. $SP = 2\sqrt{5}$

B. $SQ : QP = (\sqrt{5} + 1) : 2$

C. the x-intercept of the normal to the parabola at P is 6

D. the slope of the tangent to the circle at Q is $\frac{1}{2}$

Answer: A::C::D

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8. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the quadrant. Let the tangent to the circle C_1 at P touches other tqo circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3. respectively. If Q_2 and Q_3 lie on the y-axis, then

A. $Q_2Q_3 = 12$

B. $R_2R_3 = 4\sqrt{6}$

C. area of the triangle OR_2R_3 is $6\sqrt{2}$

D. area of the triangle PQ_2Q_3 is $4\sqrt{2}$

Answer: A::B::C

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9. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation 2x+y=p, and midpoint (h,k), then which of the following is (are) possible value(s) of p,h and k ?

A. p=5, h=4, k=-3

B. p=-1, h=1, k=-3

C. p=-2, h=2, k=-4

D. p=2, h=3, k=-4

Answer: D

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10. Let PQ be a focal chord of the parabola $y^2 = 4ax$ The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a, a > 0.$ Length of chord PQ is

A. 7a

B. 5a

C. 2a

D. 3a

Answer: B

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11. Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at point lying on the line

$y = 2x + a, a < 0.$

If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, then $\tan\theta =$

A. $2\sqrt{7}/3$

B. $-2\sqrt{7}/3$

C. $2\sqrt{5}/3$

D. $-2\sqrt{5}/3$

Answer: D

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12. Let a, r, s, t be non-zero real numbers. Let $P(at^2, 2at), Q, R(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points onthe parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K isthe point (2a, 0). The value of r is

A. $-\frac{1}{t}$

B. $\frac{t^2+1}{t}$

C. $\frac{1}{t}$

D. $\frac{t^2-1}{t}$

Answer: D

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13. Let a, r, s, t be non-zero real numbers. Let $P(at^2, 2at)$, $Q(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K the point (2a,0).

If st=1, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

A. $\dfrac{(t^2+1)^2}{2t^3}$

B. $\dfrac{a(t^2+1)^2}{2t^3}$

C. $\dfrac{a(t^2+1)^2}{t^3}$

D. $\dfrac{a(t^2+2)^2}{t^3}$

Answer: B

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14. A line L : y = mx + 3 meets y-axis at E (0, 3) and the arc of the parabola $y^2 = 16x \; 0 \leq y \leq 6$ at the point art $F(x_0, y_0)$. The tangent to the parabola at $F(X_0, Y_0)$ intersects the y-axis at $G(0, y)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum P) m= Q) = Maximum area of $\triangle \; EFG$ is (R) $y_0 =$ (S) $y_1 =$

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15. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point P($\frac{1}{2}$,2) on the parabola and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of latus rectum. $\frac{\Delta_1}{\Delta_2}$ is :

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16. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is -

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17. Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the line $x + y + 4 = 0$. If A and B are the points of intersection of C with the line $y = -5$, then the distance between A and B is

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18. Â·If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x - 3)^2(y + 2)^2 = r^2$, then the value of r^2 is

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Single Correct Answer Type

1. The equation $x^2 - 2xy + y^2 + 3x + 2 = 0$ represents

A. A parabola

B. An ellipse

C. A hyperbola

D. A circle

Answer: A

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2. The length of the latus rectum of $3x^2 - 4y + 6x - 3 = 0$ is

A. 3

B. 2

C. $\frac{4}{3}$

D. $\frac{3}{4}$

Answer: C

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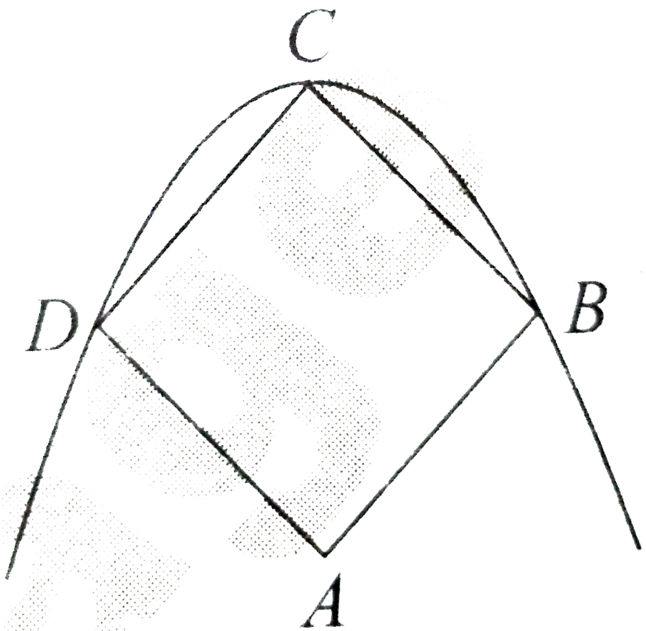
3. In the adjacent figure a parabola is drawn to pass through the vertices B,C and D of the square ABCD. If $A(2, 1), C(2, 3)$, then focus of this parabola is



A. $\left(1, \frac{11}{4}\right)$

B. $\left(2, \frac{11}{4}\right)$

C. $\left(3, \frac{13}{4}\right)$

D. $\left(2, \frac{13}{4}\right)$

Answer: B

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4. Length of the latus rectum of the parabola $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is

A. $a\sqrt{2}$

B. $\frac{a}{\sqrt{2}}$

C. a

D. 2a

Answer: A

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5. Consider the parabola $x^2 + 4y = 0$. Let $P(a, b)$ be any fixed point inside the parabola and let S be the focus of the parabola. Then the minimum value at $SQ + PQ$ as point Q moves on the parabola is

A. $|1 - a|$

B. $|ab| + 1$

C. $\sqrt{a^2 + b^2}$

D. $1 - b$

Answer: D

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6. If the points (2,3) and (3,2) on a parabola are equidistant from the focus, then the slope of its tangent at vertex is

A. 1

B. -1

C. 0

D. ∞

Answer: B

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7. Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points on the parabola $y^2 = 4ax$. If the circle with chord AB as a dimater touches the parabola, then $|y_1 - y_2|$ is equal to

A. $4a$

B. $8a$

C. $6\sqrt{2}a$

D. not a constant

Answer: B

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8. $y=\sqrt{3}x+\lambda$ is drawn through focus S of the parabola $y^2=8x+16$. If two intersection points of the given line and the parabola are A and B such that perpendicular bisector of AB intersects the x-axis at P then length of PS is

A. $8/7$

B. $7/17$

C. $8\sqrt{3}$

D. $16/3$

Answer: D

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9. If the point $(2a, a)$ lies inside the parabola $x^2-2x-4y+3=0$, then a lies in the interval

A. $\left[\frac{1}{2}, \frac{3}{2}\right]$

B. $\left(\frac{1}{2}, \frac{3}{2}\right)$

C. $(1, 3)$

D. $\left(\frac{-3}{2}, \frac{-1}{2}\right)$

Answer: B

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10. If AFB is a focal chord of the parabola $y^2 = 4ax$ such that $AF = 4$ and $FB = 5$ then the latus-rectum of the parabola is equal to

A. 80

B. $\frac{9}{80}$

C. 9

D. $\frac{80}{9}$

Answer: D

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11. Length of the focal chord of the parabola $(y+3)^2 = -8(x-1)$ which lies at a distance 2 units from the vertex of the parabola is

A. 8

B. $6\sqrt{2}$

C. 9

D. $5\sqrt{3}$

Answer: A

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12. Let $A(0, 2)$, B and C be points on parabola $y^2 + x + 4$ such that $\angle CBA \frac{\pi}{2}$. Then the range of ordinate of C is

A. $(-\infty, 0) \cup (4, \infty)$

B. $(-\infty, 0] \cup [4, \infty)$

C. $[0, 4]$

D. $(-\infty, 0) \cup [4, \infty)$

Answer: B

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13. $lx + my = 1$ is the equation of the chord PQ of $y^2 = 4x$ whose focus is S. If PS and QS meet the parabola again at R and T respectively, then slope of RT is

A. $-\frac{1}{m}$

B. $\frac{l}{m}$

C. $\frac{2}{m}$

D. none of these

Answer: A

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14. A line from $(-1, 0)$ intersects the parabola $x^2 = 4y$ at A and B. Then the locus of centroid of ΔOAB is (where O is origin)

A. $3x^2 - 2x = 4y$

B. $3y^2 - 2y = 4x$

C. $3x^2 + 2x = 4y$

D. none of these

Answer: C

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15. All the three vertices of an equilateral triangle lie on the parabola $y = x^2$, and one of its sides has a slope of 2. Then the sum of the x-coordinates of the three vertices is

A. $\frac{5}{9}$

B. $\frac{9}{13}$

C. $\frac{6}{11}$

D. None of these

Answer: C

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16. Find the equations of the chords of the parabola $y^2 = 4ax$ which pass through the point (- 6a, 0) and which subtends an angle of 45^0 at the vertex.

A. $\pm\frac{2}{7}$

B. $\pm\frac{3}{8}$

C. $\pm\frac{7}{2}$

D. $\pm\frac{5}{6}$

Answer: A

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17. Two equal circles of largest radii have following property:

(i) They intersect each other orthogonally,

(ii) They touch both the curves $4(y+2)=x^2$ and $4(2-y)=x^2$ in the region $x \in \left[-2\sqrt{2}, 2\sqrt{2}\right]$. Then radius of this circle is

A. $\sqrt{2}$

B. $\sqrt{3}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{3}{2}$

Answer: A

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18. A and B are two points on the parabola $y^2=4ax$ with vertex O. if OA is perpendicular to OB and they have lengths r_1 and r_2 respectively, then the valye of $\dfrac{r_1^{4/3} r_2^{4/3}}{r_1^{2/3} + r_2^{2/3}}$ is

A. $16a^2$

B. a^2

C. $4a$

D. None of these

Answer: A

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19. A line $ax + by + c = 0$ through the point $A(-2, 0)$ intersects the curve $y^2 = 4a$ in P and Q such that $\dfrac{1}{AP} + \dfrac{1}{AQ} = \dfrac{1}{4}$ (P,Q are in 1st quadrant). The value of $\sqrt{a^2 + b^2 + c^2}$ is

A. 2

B. 4

C. 6

D. 8

Answer: B

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20. Suppose a parabola $y = x^2 - ax - 1$ intersects the coordinate axes at three points A,B and C, respectively. The circumcircle of ΔABC intersects the y-axis again at the point $D(0, t)$. Then the value of t is

A. $1/2$

B. 1

C. $3/2$

D. 2

Answer: B

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21. The line $x - b + \lambda y = 0$ cuts the parabola $y^2 = 4ax(a > 0)$ at $P(t_1)$ and $Q(t_2)$. If $b \in [2a, 4a]$ then range of t_1t_2 where $\lambda \in R$ is

A. $[-4, -2]$

B. $[2, 4]$

C. $[4, 16]$

D. $[-16, -4]$

Answer: A

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22. If the parabola $y = (a-b)x^2 + (b-c)x + (c-a)$ touches x- axis then the line $ax + by + c = 0$ passes through a fixed point

A. always passes through a fixed point

B. represents the family of parallel lines

C. is always perpendicular to x-axis

D. always has negative slope

Answer: A

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23. A normal to parabola, whose inclination is 30°, cuts it again at an angle of

A. $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

B. $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

C. $\tan^{-1}(2\sqrt{3})$

D. $\tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

Answer: D

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24. If $(-2, 5)$ and (3,7) are the points of intersection of the tangent and normal at a point on a parabola with the axis of the parabola, then the focal distance of that point is

A. $\frac{\sqrt{29}}{2}$

B. $\frac{5}{2}$

C. $\sqrt{29}$

D. $\frac{2}{5}$

Answer: A

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25. The angle of intersection between the curves $x^2 = 4(y+1)$ and $x^2 = -4(y+1)$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. 0

D. $\frac{\pi}{2}$

Answer: C

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26. The parabolas $y^2 = 4ac$ and $x^2 = 4by$ intersect orthogonally at point $P(x_1, y_1)$ where $x_1, y_1 \neq 0$ provided (A) $b = a^2$ (B) $b = a^3$ (C) $b^3 = a^2$ (D) none of these

A. $b = a^2$

B. $b = a^3$

C. $b^3 = a^2$

D. None of these

Answer: D

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27. Sum of slopes of common tangent to $y = \frac{x^2}{4} - 3x + 10$ and $y = 2 - \frac{x^2}{4}$ is

A. -6

B. -3

C. $1/2$

D. none of these

Answer: B

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28. The slope of normal to be parabola $y = \frac{x^2}{4} - 2$ drawn through the point $(10, -1)$ is

A. -2

B. $-\sqrt{3}$

C. $-\frac{1}{2}$

D. $-\frac{5}{3}$

Answer: C

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29. The tangent and normal at the point $P(4, 4)$ to the parabola, $y^2 = 4x$ intersect the x-axis at the points Q and R, respectively. Then the circumcentre of the ΔPQR is

A. $(2, 0)$

B. $(2, 1)$

C. $(1, 0)$

D. $(1, 2)$

Answer: C

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30. The point on the parabola $y^2 = 8x$ at which the normal is inclined at 60° to the x-axis has the co-ordinates as

A. $(6, -4\sqrt{3})$

B. $(6, 4\sqrt{3})$

C. $(-6, -4\sqrt{3})$

D. $(-6, 4\sqrt{3})$

Answer: A

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31. If two distinct chords of a parabola $y^2 = 4ax$, passing through (a,2a) are bisected by the line x+y=1 ,then length of latus rectum can be

A. 9

B. 3

C. 4

D. 5

Answer: B

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32. From an external point P, a pair of tangents is drawn to the parabola $y^2 = 4x$. If $\theta_1 andth\eta_2$ are the inclinations of these tangents with the x-axis such that $\theta_1 + \theta_2 = \frac{\pi}{4}$, then find the locus of P.

A. $x - y + 1 = 0$

B. $x + y - 1 = 0$

C. $x - y - 1 = 0$

D. $x + y + 1 = 0$

Answer: C

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33. A variable parabola $y^2 = 4ax, a$ (where $a \neq -\frac{1}{4}$) being the parameter, meets the curve $y^2 + x - 2 = 0$ at two points. The locus of the point of intersecion of tangents at these points is

A. $x - 2y - 4 = 0$

B. $x - 4y + 2 = 0$

C. $x - 4y - 1 = 0$

D. $2x - y + 1 = 0$

Answer: B

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34. Let S is the focus of the parabola $y^2 = 4ax$ and X the foot of the directrix, PP' is a double ordinate of the curve and PX meets the curve again in Q. Prove that $P'Q$ passes through focus.

A. line

B. circle

C. parabola

D. none of these

Answer: A

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35. Let PQ be the latus rectum of the parabola $y^2 = 4x$ with vetex A. Minimum length of the projection of PQ on a tangent drawn in portion of Parabola PAQ is

A. $\sqrt{2}a$

B. $2a\sqrt{2}$

C. $2a$

D. $3a\sqrt{2}$

Answer: B

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36. Through the vertex O of the parabola $y^2 = 4ax$, a perpendicular is drawn to any tangent meeting it at P and the parabola at Q. Then OP, 2a and OQ are in

A. A.P.

B. G.P.

C. H.P.

D. none of these

Answer: B

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37. Tangents PQ and PR are drawn to the parabola $y^2 = 20(x+5)$ and $y^2 = 60(x+15)$, respectively such that $\angle RPQ = \frac{\pi}{2}$. Then the locus of point P is

A. $x + 10 = 0$

B. $x + 30 = 0$

C. $x + 40 = 0$

D. $x + 20 = 0$

Answer: D

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38. The locus of centroid of triangle formed by a tangent to the parabola $y^2 = 36x$ with coordinate axes is

A. $y^2 = -9x$

B. $y^2 + 3x = 0$

C. $y^2 = 3x$

D. $y^2 = 9x$

Answer: B

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39. PC is the normal at P to the parabola $y^2 = 4ax$, C being on the axis. CP is produced outwards to disothat $PQ = CP$; show that the locus of Q is a parabola.

A. $(a, 0)$

B. $(-a, 0)$

C. $(-2a, 0)$

D. $(2a, 0)$

Answer: D

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40. If three parabols touch all the lines $x = 0$, $y = 0$ and $x + y = 2$, then maximum area of the triangle formed by joining their foci is

A. $\sqrt{3}$

B. $\sqrt{6}$

C. $\frac{3\sqrt{3}}{4}$

D. $\frac{3\sqrt{3}}{2}$

Answer: D

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41. If $2x + 3y = \alpha, x - y = \beta$ and $kx + 15y = r$ are 3 concurrent normal of parabola $y^2 = \lambda x$ then value of k is

A. 3

B. 4

C. 5

D. 7

Answer: C

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42. Let $(2, 3)$ be the focus of a parabola and $x + y = 0$ and $x - y = 0$ be its two tangents. Then equation of its directrix will be

A. $2x - 3y = 0$

B. $3x + 4y = 0$

C. $x + y = 5$

D. $12x - 5y + 1 = 0$

Answer: A

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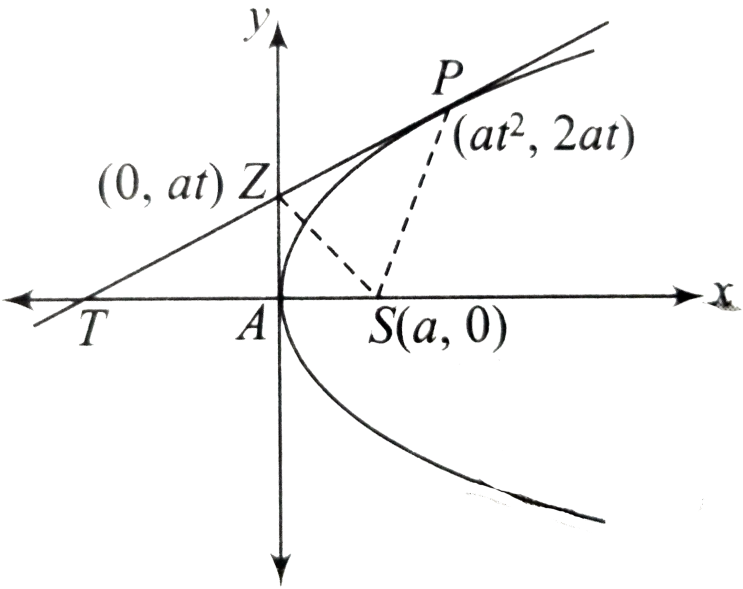
43. In the following figure, $AS = 4$ and $SP = 9$. The value of SZ is



A. 6

B. 5.5

C. 6.5

D. none of these

Answer: A

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44. TP and TQ are any two tangents to a parabola and the tangent at a third point R cuts them in P' and Q'. Prove that $\frac{TP'}{TP}+\frac{TQ'}{TQ}=1$

A. 1

B. 2

C. 3

D. none of these

Answer: A

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45. The distance of two points P and Q on the parabola $y^2=4ax$ from the focus S are 3 and 12 respectively. The distance of the point of intersection of the tangents at P and Q from the focus S is

A. 8

B. 6

C. 9

D. 12

Answer: B

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46. A parabola having directrix $x + y + 2 = 0$ touches a line $2x + y - 5 = 0$ at (2,1). Then the semi-latus rectum of the parabola, is

A. 8

B. $\frac{9}{\sqrt{2}}$

C. $\frac{10}{\sqrt{2}}$

D. $\frac{11}{\sqrt{2}}$

Answer: B

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Multiple Correct Answers Type

1. A family of curve S is given by $S \equiv x^2 + 2xy + y^2 - 4x(1-\lambda) - 4y(1+\lambda) + 4$, then $S = 0$ represents

A. pair of straight line $\forall \lambda \in R$

B. straight line for exactly one value of λ

C. parabola $\forall \lambda \in R - \{0\}$

D. ellipse for three values of λ

Answer: B::C

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2. The curves $x^2 + y^2 + 6x - 24y + 72 = 0$ and $x^2 - y^2 + 6x + 16y - 46 = 0$ intersect in four points P,Q,R and S lying on a parabola. Let A be the focus of the parabola, then

A. $AP + AQ + AR + AS = 20$

B. $AP + AQ + AR + AS = 40$

C. vertex of the parabola is at $(-3, 1)$

D. coordinates of A are $(-3, 1)$

Answer: B::C

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3. If the parabols $y^2 = 4kx(k > 0)$ and $y^2 = 4(x - 1)$ do not have a common normal other than the axis of parabola, then $k \in$

A. $(0, 1)$

B. $(2, \infty)$

C. $(3, \infty)$

D. $(0, \infty)$

Answer: A::B::C

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4. At a point P on the parabola $y^2 = 4ax$, tangent and normal are drawn. Tangent intersects the x-axis at Q and normal intersects the curve at R such that chord PQ subtends an angle of 90° at its vertex. Then

A. $PQ = 2a\sqrt{6}$

B. $PR = 6a\sqrt{3}$

C. area of $\Delta PQR = 18\sqrt{2}a^2$

D. $PQ = 3a\sqrt{2}$

Answer: A::B::C

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5. Let $y^2 - 5y + 3x + k = 0$ be a parabola, then

A. its latus rectum is least when $k = 1$

B. its latus rectum is independent of k

C. the line $y = 2x + 1$ will touch the parabola if $k = \frac{73}{16}$

D. $y = \frac{5}{2}$ is the only normal to the parabola whose slope is zero

Answer: B::C::D

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6. Let A,B and C be three distinct points on $y^2 = 8x$ such that normals at these points are concurrent at P. The slope of AB is 2 and abscissa of centroid of ΔABC is $\frac{4}{3}$. Which of the following is (are) correct?

A. Area of ΔABC is 8 sq. units

B. Coordinates of $P \equiv (6, 0)$

C. Angle between normals are $45^\circ, 45^\circ, 90^\circ$

D. Angle between normals are $30^\circ, 30^\circ, 60^\circ$

Answer: A::B::C

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7. If PQ and Rs are normal chords of the parabola $y^2 = 8x$ and the points P,Q,R,S are concyclic, then

A. tangents at P and R meet on X-axis

B. tangents at P and R meet on Y-axis

C. PR is parallel to Y-axis

D. PR is parallel to X-axis

Answer: A::C

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8. Tangents are drawn from (-2,0) to $y^2 = 8x$, then the radius of the circle that would touch these tangents and the corresponding chord of contact can be

A. $4(\sqrt{2}+1)$

B. $4(\sqrt{2}-1)$

C. $8\sqrt{2}$

D. none of these

Answer: A::B

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9. Given a parabola $y^2 = 4ax$ and the points $A(at^2, 2at), B(at^{-2}, 2at^{-1}), C\left(\frac{4a}{t^2}, \frac{4a}{t}\right) D\left(a\left(t+\frac{2}{t}\right)^2, -2a\left(t+\frac{2}{t}\right)\right.$

choose all the correct alternative.

A. AB is a focal chord

B. AD is a normal chord

C. Normals at A,C intersect on the parabola

D. Tangents at A,B intersect at 90° on the directrix.

Answer: A::B::C::D

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10. If a parabola touches the lines $y = x$ and $y = -x$ at $P(3, 3)$ and $Q(2, -2)$ respectively, then

A. focus is $\left(\frac{30}{13}, \frac{-6}{13}\right)$

B. equation of directrix is $x + 5y = 0$

C. equation of line through origin and focus is $x + 5y = 0$

D. equation of line through origin and parallel to axis is $x - 5y = 0$

Answer: A::B::C::D

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Comprehension Type

1. A parabola is drawn through two given points $A(1, 0)$ and $B(-1, 0)$ such that its directrix always touches the circle $x^2 + y^2 = 4$. Then, The locus of focus of the parabola is=

A. $\frac{x^2}{4} + \frac{y^2}{3} = 1$

B. $\frac{x^2}{4} + \frac{y^2}{5} = 1$

C. $\frac{x^2}{3} + \frac{y^2}{4} = 1$

D. $\frac{x^2}{5} + \frac{y^2}{4} = 1$

Answer: A

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2. A parabola is drawn through two given points $A(1, 0)$ and $B(-1, 0)$ such that its directrix always touches the circle $x^2 + y^2 = 4$. Then, The locus of focus of the parabola is=

A. $2 + \sqrt{3}$

B. $3 + \sqrt{3}$

C. $4 + \sqrt{3}$

D. $1 + \sqrt{3}$

Answer: A

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Matching Column Type

1. AB is a chord of $y^2 = 4x$ such that normals at A and B intersect at

List I	List II
(p) Length of AB	(1) 20
(q) Area of ΔABC	(2) $\frac{4}{\sqrt{13}}$
(r) Distance of origin from the line through AB	(3) $\sqrt{13}$
(s) The area bounded by the coordinate axes and the line through AB	(4) 4/3

$C(9, 6)$.

A. $\begin{matrix}(p) & (q) & (r) & (s). \\ 3 & 1 & 2 & 4\end{matrix}$

B. $\begin{matrix}(p) & (q) & (r) & (s). \\ 2 & 3 & 4 & 1\end{matrix}$

C. $\begin{matrix}(p) & (q) & (r) & (s). \\ 4 & 2 & 1 & 3\end{matrix}$

D. $\begin{matrix}(p) & (q) & (r) & (s). \\ 2 & 4 & 3 & 1\end{matrix}$

Answer: A

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Question Bank

1. If (α, β) is a point on parabola $y^2 = 4x$ which is nearest to the circle $x^2 + (y-12)^2 = 1$, then $(\alpha + \beta)$ is equal to

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2. The, focall chord of the parabola $(y-2)^2 = 16(x-1)$ is a tangent to the circle $x^2 + y^2 - 14x - 4y + 51 = 0$. Then slope of the focal'chord can be

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3. A chord PQ is a normal to the $y^2 = 4ax$ at P and subtendsa right angle at the verte $SQ = \lambda SP$, where S is the focus, then the value of λ is

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4. If radius of circle passing through the focus of parabola $x^2 = 4y$ and touches it at $M(6, 9)$ is $p\sqrt{10}(p \in N)$, then find p_i

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5. The equation of latus rectum of a parabola is $x + y = 8$ and cquation of the tangent at the vertex is $x + y = 12$. Then the length of latus rectum is

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6. Normals of parabola $y^2 = 4x$ at P and Q meet at $R(x_2, 0)$ and tangents at P and Q mect-at $T(x_1, 0)$. If $x_2 = 3$, then find the area of quadrilateral $PTQR$.

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7. Chord of the curve $3x^2 - y^2 - 2x + 4y = 0$, which subtends a right angle at the origin, always passes through vertex of a concave up parabola whose axis is parallel to y -axis and length of latus rectum is 8. If extremity of latus rectum of parabola is $L_1(\alpha, \beta), L_2(\gamma, \delta)$, then find $(\alpha + \beta + \gamma + \delta)$

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8. If the equation $\lambda(4x - 3)^2 + 4(2y - 7)^2 right = \mu(4x - 3y + 3)^2$ represents a parabola and λ, μ are least possible natural numbers, then $\lambda + \mu$ is equal to

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9. Tangents are drawn from any point.on directrix of $y^2 = 16x$ to parabola. If locus of midpoint of chords of contact is a parabola whose focus is (h, k), then $h + k$ is

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10. Absoulte value of y -intercept of the common tangent to the parabola $y^2 = 32x$ and $x^2 = 108y$ is

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11. Let $y = x + 1$ be the axis of parabola, $y + x - 4 = 0$ be the tangent of same parabola at its vertex and $y = 2x + 3$ be one of its tangent. If $S(\alpha, \beta)$ is focus of.parabola, then $9(\beta + \alpha)$ is equal to

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12. Let the parabola $y = ax^2 + bx + c$ has vertex at $M(4, 2)$ and $a \in [1, 3]$. If the difference between the extreme values of abc is equal to N, then find the digit at unit place of N.

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13. From the point $(4, 6)$, a pair of tangent lines is drawn to the parabola $y^2 = 8x$. The area of the triangle formed by these pairs of tangent lines and the chord of contact of the point $(4, 6)$ is

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14. If the normal to a parabola $y^2 = 4ax$ at P meets the curve again in Q and if PQ and the normal at Q make angles α and β, respectively with the x -axis, then $|\tan\alpha(\tan\alpha + \tan\beta)|$ has the value equal to

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15. A circle is drawn to pass through the extremities of the latus rectum of the parabola $y^2 = 20x$. It is given that this circle also touches the directrix of the parabola, Find the radius of this circle.

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16. If (-2,7) is the highest point on the graph of $y = -2x^2 - 4ax + k$, then |k| equals

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17. The tangent at $P(1, 2)$ to the parabola $y^2 = 4x$ meets the tangent at vertex H. If S is the focus of the parabola and A is the area of the circle circumscribing ΔSHP, then $[A]$ is (where [.] is greatest integer function)

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18. If three normals are drawn from the point $(6, 0)$ to the parabola $y^2 = 4ax$ of which two are mutually perpendicular, then length of its latus rectum is

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19. Square of the area of the triangle formed by end points of a focal chord PQ of length 32 units of the parabola $y^2 = 8x$ and its vertex is

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20. Let S be the set all points (x, y) satisfying $y^2 \leq 16x$. For points in S, let maximum and minimum values of $\frac{y+1}{x+2}$ be M and m, respectively, then $2(m + M)$. Is

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21. If the image of the parabola $y = x^2$ in the line $x + y = 1$ is $x + y^2 = ky$, then k is equal to

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