MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

PRINCIPLE OF MATHEMATICAL INDUCTION

Examples

1. Prove the following by using the principle of mathematical induction for all $n \in N$:$1^3 + 2^3 + 3^3 + \vdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

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2. Using the principle of mathematical induction prove that $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \, + \frac{1}{1+2+3+ \, + n} = \frac{2n}{n+}$ for all $n \in N$

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3. $1.3 + 2.3^2 + 3.3^3 + \ldots + n.3^3 = \dfrac{(2n-1)3^{n+1}+3}{4}$

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4. Using principle of mathematical induction, prove that for all $n \in N, n(n+1)(n+5)$ is a multiple of 3.

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5. Prove the following by using the principle of mathematical induction for all $n \in N: 3^{2n+2} - 8n - 9$is divisible by 8.

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6. Using the principle of mathematical induction , prove that for $n \in N$, $41^n - 14^n$ is a multiple of 27.

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7. Prove the following by using the principle of mathematical induction for all $n \in N$:$(2n + 7) < (n + 3)^2$.

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8. Using the principle of mathematical induction , prove that for $n \in N, \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots\dots + \frac{1}{3n+1} > 1$.

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9. A sequence $a_1, a_2, a_3, \dots$ is defined by letting $a_1 = 3$ and $a_k = 7a_{k-1}$, for all natural numbers $k \leq 2$. Show that $a_n = 3 \cdot 7^{n-1}$ for natural numbers.

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10. Let

$A_n = a_1 + a_2 + \ldots\ldots + a_n, B_n = b_1 + b_2 + b_3 + \ldots. + b_n, D_n = c_1 + c_2$

and $c_n = a_1 b_n + a_2 b_{n-1} + \ldots\ldots + a_n b_1 Aan \in N$. Using mathematical induction , prove that

(a)

$D_n = a_1 B_n + a_2 B_{n-1} + \ldots.. + a_n B_1 = b_1 A_n + b_2 A_{n-1} + \ldots\ldots + b_n A_1 \forall n$

(b) $D_1 + D_2 + \ldots\ldots + D_n = A_1 B_n + A_2 B_{n-1} + \ldots.. + A_n B_1 \forall n \in N$

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11. Let $U_1 = 1, U_2 = 1$ and $U_{n+2} = U_{n+1} + U_n$ for $n \geq 1$. Use mathematical induction to such that :

$U_n = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right\}$ for all $n \geq 1$.

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12. If p is a fixed positive integer, prove by induction that $p^{n+1} + (p+1)^{2n-1}$ is divisible by $P^2 + p + 1$ for all $n \in N$.

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13. Let $0 < A_i\pi$ for $i = 1, 2, \ldots\ldots n$. Use mathematical induction to prove that $\sin A_1 + \sin A_2 + \ldots\ldots + \sin A_n \leq n \sin\left(\frac{A_1 + A_2 + \ldots\ldots + A_n}{n}\right)$ where $n \geq 1$ is a natural number. [You may use the fact that $p \sin x + (1-p)\sin y \leq \sin[px + (1-p)y]$, where $0 \leq p \leq 1$ and $0 \leq x, y \leq \pi$.

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Exercise

1. $1.3 + 3.5 + 5.7 + \ldots + (2n-1)(2n+1) = \frac{n(4n^2 + 6n - 1)}{3}$

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2. Using the principle of mathematical induction prove that $\frac{1}{1.\ 2.\ 3} + \frac{1}{2.\ 3.\ 4} + \frac{1}{3.\ 4.\ 5} + \ + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$ for all $n \in N$

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3. Using the principle of mathematical induction, prove that $(2^{3n} - 1)$ is divisible by 7 for all $n \in N\cdot$

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4. Using the principle of mathematical induction. Prove that $(x^n - y^n)$ is divisible by (x-y) for all $n \in N$.

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5. Using principle of mathematical induction prove that $\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots\dots + \frac{1}{\sqrt{n}}$ for all natural numbers $n \geq 2$.

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6. Prove by the principle of mathematical induction that $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a natural number for all $n \in N$.

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7. Using principle of mathematical induction, prove that $7^{4^n} - 1$ is divisible by 2^{2n+3} for any natural number n.

A. 2^{2n}

B. 2^{2k+5}

C. 3^{2n}

D. 2^{n+3}

Answer: B

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8. Prove by mathematical induction that n^5 and n have the same unit digit for any natural number n.

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9. A sequence $b_0, b_1, b_2, \ldots$ is defined by letting $b_0 = 5$ and $b_k = 4 + b_{k-1}$, for all natural number k. Show that $b_n = 5 + 4n$, for all natural number n using mathematical induction.

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