



India's Number 1 Education App

MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

PRINCIPLE OF MATHEMATICAL INDUCTION

Examples

1. Prove the following by using the principle of mathematical induction

for all
$$n\in N$$
: $1^3+2^3+3^3++n^3=\left(rac{n(n+1)}{2}
ight)^2$



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2. Using the principle of mathematical induction prove that

$$1 + rac{1}{1+2} + rac{1}{1+2+3} + rac{1}{1+2+3+4} + rac{1}{1+2+3++n} = rac{2n}{n+1}$$

for all $n \in N$

3.
$$1.3 + 2.3^2 + 3.3^3 + \ldots + n.3^3 = \frac{(2n-1)3^{n+1} + 3}{4}$$



- **4.** Using principle of mathematical induction, prove that for all $n \in N, n(n+1)(n+5)$ is a multiple of 3.
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- **5.** Prove the following by using the principle of mathematical induction for all $n \in N$: $3^{2n+2}-8n-9$ is divisible by 8.
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6. Using the principle of mathematical induction , prove that for $n \in N$,



 $41^n - 14^n$ is a multiple of 27.

7. Prove the following by using the principle of mathematical induction for all $n \in N$: $(2n+7) < (n+3)^2$.



8. Using the principle of mathematical induction , prove that for $n\in N, rac{1}{n+1}+rac{1}{n+2}+rac{1}{n+3}+\ldots\ldots+rac{1}{3n+1}>1.$



9. A sequence a_1,a_2,a_3,\ldots is defined by letting $a_1=3$ and $a_k=7a_{k-1}$, for all natural numbers $k\leq 2$. Show that $a_n=3\cdot 7^{n-1}$ for natural

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numbers.

10.

 $A_n = a_1 + a_2 + \ldots + a_n, B_n = b_1 + b_2 + b_3 + \ldots + b_n, D_n = c_1 + c_2$

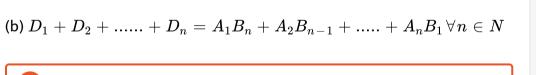
and $c_n=a_1b_n+a_2b_{n-1}+\ldots\ldots+a_nb_1Aan\in N.$ Using mathematical

Let

Use

that

induction, prove that (a)



Let $U_1 = 1, U_2 = 1 \text{ and } U_{n+2} = U_{n+1} + U_n \text{ for } n \ge 1.$

to

such

induction

 $D_n = a_1 B_n + a_2 B_{n-1} + \dots + a_n B_1 = b_1 A_n + b_2 A_{n-1} + \dots + b_n A_1 \, \forall n$

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mathematical

$$U_n = rac{1}{\sqrt{5}} \left\{ \left(rac{1+\sqrt{5}}{2}
ight)^n - \left(rac{1-\sqrt{5}}{2}
ight)^n
ight\}$$
 for all $n \geq 1$.

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12. If p is a fixed positive integer, prove by induction that $p^{n+1}+(p+1)^{2n-1}$ is divisible by P^2+p+1 for all $n\in N.$



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13. Let $0 < A_i\pi$ for $i=1,2,\ldots n$. Use mathematical induction to prove $\tan A_1 + \sin A_2 + \ldots + \sin A_n \leq n \sin \left(\frac{A_1 + A_2 + \ldots + A_n}{n}\right)$ where $n \geq 1$ is a natural number. [You may use the fact that $p\sin x + (1-p)\sin y \leq \sin[px + (1-p)y],$ where $0 \leq p \leq 1$ and $0 < x,y < \pi.$



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Exercise

1.
$$1.3 + 3.5 + 5.7 + \ldots + (2n-1)(2n+1) = \frac{n(4n^2 + 6n - 1)}{3}$$



2. Using the principle of mathematical induction prove that $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$



for all $n \in N$

- **3.** Using the principle of mathematical induction, prove that $\left(2^{3n}-1
 ight)$ is divisible by 7 for all $n\in N$
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4. Using the principle of mathematical induction. Prove that (x^n-y^n) is divisible by (x-y) for all $n\in N$.

5. Using principle of mathematical induction prove that $\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}}$ for all natural numbers $n \geq 2$.



6. Prove by the principle of mathematical induction that $\frac{n^5}{5}+\frac{n^3}{3}+\frac{7n}{15}$ is a natural number for all $n\in N$.



7. Using principle of mathematical induction, prove that $7^{4^n}-1$ is divisible by 2^{2n+3} for any natural number n.

A.
$$2^{2n}$$

$$\mathsf{B.}\ 2^{2k+5}$$

\boldsymbol{c}	2^{2n}

D.
$$2^{n+3}$$

Answer: B



8. Prove by mathematical induction that n^5 and n have the same unit digit for any natural number n.



9. A sequence b_0,b_1,b_2,\ldots is defined by letting $b_0=5$ and $b_k=4+b_{k-1}$, for all natural number k. Show that $b_n=5+4n$, for all natural number n using mathematical induction.

