MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

PROPERTIES AND SOLUTIONS OF TRIANGLE

Examples

1. In triangle ABC< D is on AC such that AD=BC and BD=DC, $\angle DBC = 2x$ and $\angle BAD = 3x$ where each angle is in degree. Then find x

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2. In a circle of radius r, chords of length $aandbcm$ subtend angles $\theta and 3\theta$, respectively, at the center. Show that $r = a\sqrt{\frac{a}{3a-b}}cm$

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3. Perpendiculars are drawn from the angles A, B and C of an acute-angled triangle onthe opposite sides, and produced to meet the circumscribing circle. If these produced parts are $\alpha., \beta, \gamma,$ respectively, then show that, then show that $\frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma}=2(\tan A+\tan B+\tan C)$.

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4. D, E, F are three points on the sides BC, CA, AB, respectively, such that $\angle ADB=\angle BEC=\angle CFA=\theta$. A', B', C' are the points of intersections of the lines AD, BE, CF inside the triangle. Show that area of $\Delta A'B'C'=4\Delta\cos^2\theta$, where Δ is the area of ΔABC

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5. In $ABC,$ as semicircle is inscribed, which lies on the side $\cdot$ If x is the lengthof the angle bisector through angle $C,$ then prove that the radius of the semicircle is $x \sin\left(\frac{C}{2}\right)$.

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6. Given the base of a triangle, the opposite angle A, and the product k^2 of other two sides, show that it is not possible for a to be less than $2k\sin\frac{A}{2}$

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7. If in a triangle of base 'a', the ratio of the other two sides is r (<1).Show that the altitude of the triangle is less than or equal to $\frac{ar}{1-r^2}$

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8. Let ABC be a triangle with incentre I. If P and Q are the feet of the perpendiculars from A to BI and CI, respectively, then prove that $\frac{AP}{BI}+\frac{AQ}{Cl}=\cot.\ \frac{A}{2}$

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9. Let O be the circumcentre and H be the orthocentre of an acute angled triangle ABC. If $A > B > C$, then show that $Ar(\Delta BOH) = Ar(\Delta AOH) + Ar(\Delta COH)$

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10. If I is the incenter of ΔABC and $R_1, R_2,$ and R_3 are, respectively, the radii of the circumcircle of the triangle IBC, ICA, and IAB, then prove that $R_1R_2R_3 = 2rR^2$

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11. Show that the line joining the incenter to the circumcentre of triangle ABC is inclined to the side BC at an angle $\tan^{-1}\left(\frac{\cos B + \cos C - 1}{\sin C - \sin B}\right)$

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12. In a ΔABC, the median to the side BC is of length $\frac{1}{\sqrt{11-6\sqrt{3}}}$ and it divides the $\angle A$ into angles 30° and $45 \circ$. Find the length of the side BC.

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13. Three circles touch each other externally. The tangents at their point of contact meet at a point whose distance from a point of contact is 4. Then, the ratio of their product of radii to the sum of the radii is

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14. Let ABC be a triangle with incentre I and inradius r. Let D, E, F be the feet of the perpendiculars from I to the sides BC, CA and AB, respectively, If r_2 and r_3 are the radii of circles inscribed in the quadrilaterls AFIE, BDIF and CEID respectively, then prove that $\frac{r_1}{r-r_1}+\frac{r_2}{r-r_2}+\frac{r_3}{r-r_3}=\frac{r_1r_2r_3}{(r-r_1)(r-r_2)(r-r_3)}$

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15. In convex quadrilateral $ABCD, AB = a, BC = b, CD = c, DA = d$. This quadrilateral is such that a circle can be inscribed in it and a circle can also be circumscribed about it. Prove that $\frac{\tan^2 A}{2} = \frac{bc}{ad}$.

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16. If an a triangle $ABC, b = 3cand\ C - B = 90^0,$ then find the value of $\tan B$

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17. In a triangle ABC if $BC = 1andAC = 2,$ then what is the maximum possible value of angle A?

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18. The perimeter of a triangle ABC is saix times the arithmetic mean of the sines of its angles. If the side $a is 1$ then find angle $A\cdot$

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19. If $A = 75^0, b = 45^0,$ then prove that $b + c\sqrt{2} = 2a$

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20. If the base angles of triangle are $\frac{22}{12} and 112\frac{1}{2^0}$, then prove that the altitude of the triangle is equal to $\frac{1}{2}$ of its base.

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21. If a^2, b^2, c^2 are in A.P., then prove that $\tan A, \tan B, \tan C$ are in H.P.

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22. In any triangle ABC, prove that:
$$\frac{a^2 \sin(B-C)}{\sin B + s \in C} + \frac{b^2 \sin(C-A)}{\sin C + s \in A} + \frac{c^2 \sin(A-B)}{\sin A + s \in B} = 0$$

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23. In any triangle. if $\frac{a^2 - b^2}{a^2 + b^2} = \frac{\sin(A-B)}{\sin(A+B)}$, then prove that the triangle is either right angled or isosceles.

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24. ABCD is a trapezium such that $AB \mid \mid CDandCB$ is perpendicular to them. If $\angle ADB = \theta, BC = p, andCD = q$, show that $AB = \frac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

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25. In a triangle $ABC, \angle c = 60^0 and \angle A = 75^0$. If D is a point on AC such that the area of the $BCD, the \angle ABD$

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26. In a scalene triangle ABC, D is a point on the side AB such that $CD^2 = ADDB$, sin $s \in AS \in B = \frac{\sin^2 C}{2}$ then prove that CD is internal bisector of $\angle C\cdot$

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27. In a triangle ABC, $\angle A = 60^\circ$ and $b : c = (\sqrt{3} + 1) : 2$, then find the value of $(\angle B - \angle C)$

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28. If the median AD of triangle ABC makes an angle $\frac{\pi}{4}$ with the side BC, then find the value of $|\cot B - \cot C|\cdot$

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29. The base of a triangle is divided into three equal parts. If t_1, t_2, t_3 are the tangents of the angles subtended by these parts at the opposite vertex, prove that $\left(\frac{1}{t_1}+\frac{1}{t_2}\right)\left(\frac{1}{t_2}+\frac{1}{t_3}\right)=4\left(1+\frac{1}{t22}\right)\cdot$

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30. In ΔABC, prove that $(a-b)^2\cos^2.\ \frac{C}{2}+(a+b)^2\sin^2.\ \frac{C}{2}=c^2$

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31. In $ABC, \ =$ if $(a+b+c)(a-b+c)=3ac,$ then find $\angle B\cdot$

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32. If $a = \sqrt{3}, b = \frac{1}{2}(\sqrt{6} + \sqrt{2}), \text{ and } c = \sqrt{2}$, then find $\angle A$

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33. The sides of a triangle are $x^2 + x + 1, 2x + 1 \text{ and } x^2 - 1.$ Prove that the greatest angle is 120^0

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34. If the angles A,B,C of a triangle are in A.P. and sides a,b,c, are in G.P., then prove that a^2, b^2, c^2 are in A.P.

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35. Let $a, bandc$ be the three sides of a triangle, then prove that the equation $b^2x^2 + (b^2 = c^2 - \alpha^2)x + c^2 = 0$ has imaginary roots.

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36. Let $a \leq b \leq c$ be the lengths of the sides of a triangle. If `a^2+b^2

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37. In a triangle ABC, if the sides a,b,c, are roots of $x^3 - 11x^2 + 38x - 40 = 0$, then find the value of $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$

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38. If in a triangle $ABC, \angle C = 60^0$, then prove that $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$.

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39. In a triangle, if the angles $A, B, andC$ are in A.P. show that $2\frac{\cos 1}{2}(A-C)=\frac{a+c}{\sqrt{a^2-ac+c^2}}$

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40. If $a=9, b=4andc=8$ then find the distance between the middle point of BC and the foot of the perpendicular form $A\cdot$

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41. Three parallel chords of a circle have lengths 2,3,4 units and subtend angles $\alpha, \beta, \alpha+\beta$ at the centre, respectively `(alpha

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42. In a cyclic quadrilateral PQRS, PQ= 2 units, QR= 5 units, RS=3 units and $\angle PQR=60^0,$ then what is the measure of SP?

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43. For any triangle ABC, prove that $a(b\cos C - osB) = b^2 - c^2$

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44. If in a triangle a $\frac{\cos^2 C}{2} + \frac{\cos^2 A}{2} = \frac{3b}{2}$, then find the relation between the sides of the triangle.

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45. Prove that $(b+c)\cos A + (c+a)\cos B + (a+b)\cos C = 2s$.

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46. If $\frac{\cos A}{2} = \sqrt{\frac{b+c}{2c}}$, then prove that $a^2 + b^2 = c^2$.

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47. If the cotangents of half the angles of a triangle are in A.P., then prove that the sides are in A.P.

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48. If the sides $a, bandCofABC$ are in $A\dot{P};$ prove that $2\frac{\sin A}{2}\frac{\sin C}{2}=\frac{\sin B}{2}a\frac{\cos^2 C}{2}+\frac{\cos^2 A}{2}=\frac{3b}{2}$

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49. Prove that $\left(\frac{\cot A}{2}+\frac{\cot B}{2}\right)\left(a\frac{\sin^2 B}{2}+b\frac{\sin^2 A}{2}\right)=ot\frac{C}{2}$

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50. Find the value of tan A, if area of $\Delta ABCisa^2-(b-c)^2$.

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51. Prove that $a^2 \sin 2B + b^2 \sin 2A = 4\Delta$

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52. Prove that $\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2} = \sin^2 A$

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53. If the sides of a triangle are $17, 25 and 28,$ then find the greatest length of the altitude.

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54. In equilateral triangle ABC with interior point D, if the perpendicular distances from D to the sides of 4,5, and 6, respectively, are given, then find the area of ABC.

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55. If area of a triangle is 2 sq. units, then find the value of the product of the arithmetic mean of the lengths of the sides of a triangle and harmonic mean of the lengths of the altitudes of the triangle.

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56. A triangle has sides 6,7, and 8. The line through its incenter parallel to the shortest side is drawn to meet the other two sides at P and Q. Then find the length of the segment PQ.

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57. Each side of triangle ABC is divided into three equal parts. Find the ratio of the are of hexagon $PQRSTU$ to the area of the triangle ABC.

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58. The two adjacent sides of a cyclic quadrilateral are $2and5$ and the angle between them is 60^0. If the area of the quadrilateral is $4\sqrt{3}$, find the remaining two sides.

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59. In triangle $ABC, a:b:c = 4:5:6.$ The ratio of the radius of the circumcircle to that of the incircle is____.

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60. Given a triangle ABC with sides a=7, b=8 and c=5. Find the value of expression $(\sin A + \sin B + \sin C)\left(\frac{\cot A}{2} + \frac{\cot B}{2} + \frac{\cot C}{2}\right)$

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61. If $b = 3, c = 4, and B = \frac{\pi}{3},$ then find the number of triangles that can be constructed.

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62. If $A = 30^0, a = 7, and b = 8$ in $ABC,$ then find the number of triangles that can be constructed.

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63. If in triangle ABC, $\left(a = \left(1 + \sqrt{3}\right)cm, b = 2cm, and \angle C = 60^0\right.$, then find the other two angles and the third side.

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64. In $ABC, sides b, c$ and angle B are given such that a has two valus $a_1 and a_2$. Then prove that $|a_1 - a_2| = 2\sqrt{b^2 - c^2 \sin^2 B}$

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65. In $ABC, a, c and A$ are given and b_1, b_2 are two values of the third side b such that $b_2 = 2b_1$. Then prove that $\sin A = \sqrt{\frac{9a^2 - c^2}{8c^2}}$

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66. O is the circumcenter of $ABC and R_1, R_2, R_3$ are respectively, the radii of the circumcircles of the triangle OBC, OCA and OAB. Prove that $\frac{a}{R_1} + \frac{b}{R_2} + \frac{c}{R_3}, \frac{abc}{R_3}$

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67. In $ABC, C = 60^0 and B = 45^0$. Line joining vertex A of triangle and its circumcenter (O) meets the side $BC \in D$ Find the ratio $BD : DC$ Find the ratio $AO : OD$

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68. The diameters of the circumcirle of triangle ABC drawn from A,B and C meet BC, CA and AB, respectively, in L,M and N. Prove that $\frac{1}{AL}+\frac{1}{BM}+\frac{1}{CN}=\frac{2}{R}$

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69. Find the lengths of chords of the circumcircle of triangle ABC, made by its altitudes

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70. Let ABC be a triangle with $\angle B=90^0$. Let AD be the bisector of $\angle A$ with D on BC. Suppose AC=6cm and the area of the triangle ADC is $10cm^2$. Find the length of BD.

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71. If the distances of the vertices of a triangle =ABC from the points of contacts of the incercle with sides are $\alpha, \beta and \gamma$ then prove that $r^2 = \frac{\alpha\beta\gamma}{\alpha = \beta + \gamma}$

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72. If $x, y and z$ are the distances of incenter from the vertices of the triangle ABC , respectively, then prove that $\frac{abc}{xyz} = \frac{\cot A}{2}\frac{\cot B}{2}\frac{\cot C}{2}$

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73. Prove that $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$

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74. Prove that $\frac{\mathrm{a\,c\,o\,s}A + b\cos B + \mathrm{os}C}{a + b + c} = \frac{r}{R}.$

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75. Incircle of ΔABC touches the sides BC, CA and AB at D, E and F, respectively. Let r_1 be the radius of incircle of ΔBDF. Then prove that $r_1 = \frac{1}{2}\frac{(s-b)\sin B}{\left(1+\sin.\ \frac{B}{2}\right)}$

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76. In an acute angle triange ABC, a semicircle with radius r_a is constructed with its base on BC and tangent to the other two sides r_b and r_c are defined similarly. If r is the radius of the incircle of triangle ABC then prove that $\frac{2}{r} = \frac{1}{r_a} + \frac{1}{r_b} + \frac{1}{r_c}$

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77. Let the incircle with center I of ΔABC touch sides BC, CA, AB at D,E and F, respectively. Let a circle is drawn touching ID, IF and incircle of

ΔABC having radius r_2. Similarly r_1 and r_3 are defined. Prove that $\frac{r_1}{r-r_1}\cdot\frac{r_2}{r-r_2}\cdot\frac{r_3}{r-r_3}=\frac{a+b+c}{8R}$

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78. In ΔABC, the bisector of the angle A meets the sides BC at D and the circumscirbed circle at E. Prove that $DE=\frac{a^2\sec.\frac{A}{2}}{2(b+c)}$

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79. Let I be the incetre of ΔABC having inradius r. Al, BI and Ci intersect incircle at D, E and F respectively. Prove that area of ΔDEF is $\frac{r^2}{2}\left(\cos.\frac{A}{2}+\cos.\frac{B}{2}+\cos.\frac{C}{2}\right)$

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80. In ΔABC, the bisectors of the angles A, B and C are extended to intersect the circumcircle at D,E and F respectively. Prove that

$$AD\cos.\ \frac{A}{2} + BE\cos.\ \frac{B}{2} + CF\cos.\ \frac{C}{2} = 2R(\sin A + \sin B + \sin C)$$

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81. Given a right triangle with $\angle A = 90^\circ$. Let M be the mid-point of BC. If the inradii of the triangle ABM and ACM are r_1 and r_2 then find the range of $\frac{r_1}{r_2}$

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82. Prove that the distance between the circumcenter and the incenter of triangle ABC is $\sqrt{R^2 - 2Rr}$

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83. Prove that $a\cos A + b\cos B + osC \leq s\cdot$

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84. If Δ is the area of a triangle with side lengths $a, b, c,$ then show that as $\Delta \leq \frac{1}{4}\sqrt{(a+b+c)abc}$ Also, show that the equality occurs in the above inequality if and only if $a = b = c$.

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85. If in ΔABC, the distance of the vertices from the orthocenter are x,y, and z then prove that $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\frac{abc}{xyz}$

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86. ABC is an acute angled triangle with circumcenter O and orthocentre H. If AO=AH, then find the angle A.

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87. In a acute angled triangle ABC, proint D, E and F are the feet of the perpendiculars from A,B and C onto BC, AC and AB, respectively. H is orthocentre. If $\sin A = \frac{3}{5} and BC = 39$, then find the length of AH

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88. Prove that the distance between the circumcenter and the orthocenter of triangle ABC is $R\sqrt{1 - 8\cos A \cos B \cos C}$

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89. Let ABC be an acute angled triangle whose orthocentre is at H. If altitude from A is produced to meet the circumcircle of triangle ABC at D, then prove $HD = 4R \cos B \cos C$

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90. In ABC, let L, M, N be the feet of the altitudes. The prove that $\sin(\angle MLN) + \sin(\angle LMN) + \sin(\angle MNL) = 4\sin A\sin B\sin C$

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91. The lengths of the medians through acute angles of a right-angled triangle are 3 and 4. Find the area of the triangle.

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92. Two medians drawn from the acute angles of a right angled triangle intersect at an angle $\frac{\pi}{6}$. If the length of the hypotenuse of the triangle is $3units$, then the area of the triangle (in sq. units) is $\sqrt{3}$ (b) 3 (c) $\sqrt{2}$ (d) 9

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93. Prove that $r_1 + r_2 + r_3 - r = 4R$

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94. If in a triangle $r_1 = r_2 + r_3 + r$, prove that the triangle is right angled.

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95. Prove that $\frac{r_{1+r_2}}{1} = 2R$

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96. Prove that $(r+r_1)\tan\left(\frac{B-C}{2}\right) + (r+r_2)\tan\left(\frac{C-A}{2}\right) + (r+r_3)\tan\left(\frac{A-B}{2}\right)$

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97. If the distance between incenter and one of the excenter of an equilateral triangle is 4 units, then find the inradius of the triangle.

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98. If I_1, I_2, I_3 are the centers of escribed circles of ΔABC, show that the area of $\Delta I_1 I_2 I_3$ is (abc)/(2r)`

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99. Prove that the sum of the radii of the radii of the circles, which are, respectively, inscribed and circumscribed about a polygon of n sides, whose side length is a, is $\frac{1}{2} a \frac{\cot \pi}{2n}$.

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100. If the area of the circle is A_1 and the area of the regular pentagon inscribed in the circle is A_2, then find the ratio $\frac{A_1}{A_2}$.

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101. Prove that the area of a regular polygon hawing $2n$ sides, inscribed in a circle, is the geometric mean of the areas of the inscribed and circumscribed polygons of n sides.

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Exercise 5 1

1. Find the value of $\frac{a^2+b^2+c^2}{R^2}$ in any right-angled triangle.

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2. Let the angles $A, BandC$ of triangle ABC be in $A\dot{P}$. and let $b: c$ be $\sqrt{3}: \sqrt{2}$. Find angle A.

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3. In a triangle ABC, if $(\sqrt{3} - 1)a = 2b, A = 3B$, then $\angle C$ is

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4. In a triangle ABC, if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and the side $a = 2$, then area of triangle is

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5. In triangle ABC, if $\cos^2 A + \cos^2 B - \cos^2 C = 1$, then identify the type of the triangle

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6. Prove that $b^2 \cos 2A - a^2 \cos 2B = b^2 - a^2$

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7. In any triangle ABC, prove that following :

$$\frac{c}{a+b} = \frac{1 - \tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)}{1 + \tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)}$$

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8. For any triangle ABC, prove that

$(b^2c^2)\backslash \cot A\backslash \ +\backslash \ (c^2a^2)\backslash \cot B\backslash \ +\backslash \ (a^2b^2)\backslash \cot C\backslash \ =\backslash \ 0$

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9. In a triangle ABC, prove that $\frac{b+c}{a} \leq \cos ec.\ \frac{A}{2}$

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10. In any triangle ABC, prove that: $\frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}=\frac{a^2+b^2}{a^2+c^2}$

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11. In a triangle ABC, if a, b, c are in A.P. and $\frac{b}{c}\sin 2C+\frac{c}{b}\sin 2B+\frac{b}{a}\sin 2A+\frac{a}{b}\sin 2B=2$, then find the value of sin B

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12. Prove that $a\cos A+b\cos B+c\cos C=4R\sin A\sin B\sin C$.

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Exercise 5 2

1. If the sides of a triangle are a, b and $\sqrt{a^2 + ab + b^2}$, then find the greatest angle

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2. If the segments joining the points $A(a, b) and B(c, d)$ subtends an angle θ at the origin, prove that : $\theta = \frac{ac + bd}{(a^2 + b^2)(c^2 + d^2)}$

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3. If x, $y > 0$, then prove that the triangle whose sides are given by $3x + 4y, 4x + 3y, \text{ and } 5x + 5y$ units is obtuse angled.

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4. In ΔABC, angle A is 120°, $BC + CA = 20, \text{ and } AB + BC = 21$ Find the length of the side BC

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5. In $\Delta ABC, AB=1, BC=1, \text{ and } AC=1/\sqrt{2}.$ In $\Delta MNP, MN=1, NP=1, \text{ and } \angle MNP=2\angle ABC.$ Find the side MP

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6. If in a triangle $ABC, \dfrac{bc}{2\cos A}=b^2+c^2-2bc\cos A$ then prove that the triangle must be isosceless

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7. With usual notations, if in a triangle $ABC\dfrac{b+c}{11}=\dfrac{c+a}{12}=\dfrac{a+b}{13}$, then prove that: $\dfrac{\cos A}{7}=\dfrac{\cos B}{19}=\dfrac{\cos C}{25}$

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8. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smalles one. Determine the sides of the triangle.

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Exercise 5 3

1. In ΔABC, prove that $c\cos(A-\alpha)+a\cos(C+\alpha)=b\cos\alpha$

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2. Prove that $\dfrac{\cos C+\cos A}{c+a}+\dfrac{\cos B}{b}=\dfrac{1}{b}$

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3. Prove that

$a(b^2+c^2)\cos A+b(c^2+a^2)\cos B+c(a^2+b^2)\cos C=3abc$

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Exercise 5 4

1. In a triangle ABC if $b+c=3a$ then find the value of $\cot\left(\frac{B}{2}\right)\cot\left(\frac{C}{2}\right)$

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2. Prove that $bc\cos^2.\frac{A}{2}+ca\cos^2.\frac{B}{2}+ab\cos^2.\frac{C}{2}=s^2$

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3. If in ΔABC, $\tan.\frac{A}{2}=\frac{5}{6}$ and $\tan.\frac{C}{2}=\frac{2}{5}$, then prove that a, b, and c are in A.P.

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4. Prove that $(b+c-a)\left(\cot.\frac{B}{2}+\cot.\frac{C}{2}\right)=2a\cot.\frac{A}{2}$

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5. If $\sin^2\left(\frac{A}{2}\right), \sin^2\left(\frac{B}{2}\right)$, and $\sin^2\left(\frac{C}{2}\right)$ are in $H.P.$, then prove that the sides of triangle are in $H.P.$

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Exercise 5 5

1. If $c^2=a^2+b^2$, then prove that $4s(s-a)(s-b)(s-c)=a^2b^2$

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2. If the sides of a triangle are in the ratio $3:7:8$, then find $R:r$

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3. In triangle ABC, if a = 2 and bc = 9, then prove that $R = 9/2\Delta$

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4. In ΔABC, if lengths of medians BE and CF are 12 and 9 respectively, find the maximum value of Δ

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5. Let the lengths of the altitudes drawn from the vertices of ΔABC to the opposite sides are 2, 2 and 3. If the area of $\Delta ABC \text{ is } \Delta$, then find the area of triangle

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6. A triangle with integral sides has perimeter 8 cm. Then find the area of the triangle

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7. The sides of a triangle are in A.P. and its area is $\frac{3}{5}$ th of an equilateral triangle of the same perimeter. Find the greatest angle of the triangle

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Exercise 5 6

1. In which of the following cases, there exists a triangle ABC?

(a) $b \sin A = a,\ A < \pi / 2$

(b) $b \sin A > a,\ A > \pi / 2$

(c) $b \sin A > a,\ A < \pi / 2$

(d) $b \sin A < a,\ A < \pi / 2,\ b > a$

(e) $b \sin A < a,\ A > \pi / 2,\ b = a$

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2. If in $\Delta ABC, b = 3cm, c = 4cm$ and the length of the perpendicular from A to the side BC is 2 cm, then how many such triangle are possible ?

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3. In a triangle $ABC, \frac{a}{b} = \frac{2}{3}$ and $\sec^2 A = \frac{8}{5}$. Find the number of triangle satisfying these conditions

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4. In a triangle, the lengths of the two larger sides are 10 and 9, respectively. If the angles are in A.P., then the length of the third side can be $5 - \sqrt{6}$ (b) $3\sqrt{3}$ (c) 5 (d) $5 + \sqrt{6}$

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5. If a, b and A are given in a triangle and c_1, c_2 are possible values of the third side, then prove that $c_1^2 + c_2^2 - 2c_1c_2 \cos 2A = 4a^2 \cos^2 A$

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6. In $\Delta ABC, a, b$ and A are given and c_1, c_2 are two values of the third side c. Prove that the sum of the area of two triangles with sides a, b, c_1 and a, bc_2 is $\frac{1}{2}b^2 \sin 2A$

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Exercise 5 7

1. Let f, g and h be the lengths of the perpendiculars from the circumcenter of ΔABC on the sides a, b, and c, respectively. Prove that $\frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \frac{1}{4}\frac{abc}{fgh}$

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2. If AD, BE, CF are the diameters of circumcircle of ΔABC, then prove that area of hexagon AFBDCE is 2Δ

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3. If the sides of triangle are in the ratio $3:5:7$, then prove that the minimum distance of the circumcentre from the side of triangle is half the circmradius

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4. If circumradius of triangle ABC is 4 cm, then prove that sum of perpendicular distances from circumcentre to the sides of triangle cannot exceed 6 cm

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Exercise 5 8

1. If the incircle of the triangle ABC passes through its circumcenter, then find the value of $4\sin.\frac{A}{2}\sin.\frac{B}{2}\sin.\frac{C}{2}$

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2. In $\Delta ABC, a=10, A=\frac{2\pi}{3}$, and circle through B and C passes through the incenter. Find the radius of this circle

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3. Let ABC be a triangle with $\angle BAC = 2\pi/3$ and $AB = x$ such that (AB) (AC) = 1. If x varies, then find the longest possible length of the angle bisector AD

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4. If the incircle of the ΔABC touches its sides at L, M and N as shown in the figure and if x, y, z be thecircumradii of the triangles MIN, NIL and LIM respectively, where I is the incentre, then the product xyz is equal to:

(A) Rr^2 (B)rR^2

(C) $\frac{1}{2}Rr^2$ (D) $\frac{1}{2}rR^2$

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5. In a triangle ABC, CD is the bisector of the angle C. If $\cos\left(\frac{C}{2}\right)$ has the value $\frac{1}{3}$ and $l(CD) = 6$, then $\left(\frac{1}{a} + \frac{1}{b}\right)$ has the value equal to -

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6. In ΔABC, $\angle A = \frac{\pi}{3}$ and its inradius of 6 units. Find the radius of the circle touching the sides AB, AC internally and the incircle of ΔABC externally

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7. In triangle ABC, prove that the maximum value of $\frac{\tan A}{2}\frac{\tan B}{2}\frac{\tan C}{2} is \frac{R}{2s}$

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Exercise 5 9

1. Line joining vertex A of triangle ABC and orthocenter (H) meets the side BC in D. Then prove that

(a) $BD : DC = \tan C : \tan B$

(b) $AH : HD = (\tan B + \tan C) : \tan A$

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2. In a triangle ABC, $\angle A = 30^2, BC = 2 + \sqrt{5}$, then find the distance of the vertex A from the orthocenter

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3. If the perimeter of the triangle formed by feet of altitudes of the triangle ABC is equal to four times the circumradius of ΔABC, then identify the type of ΔABC

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4. AD, BE and CF are the medians of triangle ABC whose centroid is G. If the points A, F, G and E are concyclic, then prove that $2a^2 = b^2 + c^2$

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5. In an acute angle triangle ABC, AD, BE and CF are the altitudes, then $\frac{EF}{a} + \frac{FD}{b} + \frac{DE}{c}$ is equal to -

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Exercise 5 10

1. In ΔABC, if $r_1 < r_2 < r_3$, then find the order of lengths of the sides

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2. The exradii $r_1, r_2, \text{ and } r_3 \text{ of } \Delta ABC$ are in H.P. show that its sides a, b, and c are in A.P.

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3. If in ΔABC, $(a-b)(s-c) = (b-c)(s-a)$, prove that r_1, r_2, r_3 are in A.P.

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4. Prove that $2R\cos A = 2R + r - r_1$

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5. If the lengths of the perpendiculars from the vertices of a triangle ABC on the opposite sides are p_1, p_2, p_3 then prove that $\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} = \frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$.

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6. Prove that $r_1 r_2 + r_2 r_3 + r_3 r_1 = \frac{1}{4}(a + b + c)^2$

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7. In any triangle ABC, find the least value of $\frac{r_1 + r_2 + r_3}{r}$

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8. Prove that $\frac{r_1 - r}{a} + \frac{r_2 - r}{b} = \frac{c}{r_3}$

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Exercise 5 11

1. Regular pentagons are inscribed in two circles of radius $5$and 2 units respectively. The ratio of their areas is

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2. Let A be a point inside a regular polygon of 10 sides. Let $p_1, p_2 \ldots., p_{10}$ be the distances of A from the sides of the polygon. If each side is of length 2 units, then find the value of $p_1 + p_2 + \ldots + p_{10}$

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3. Let $A_1, A_2, \ldots. A_n$ be the vertices of an n-sided regular polygon such that ,$\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$. Find the value of n.

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4. I_n is the area of n sided refular polygon inscribed in a circle unit radius and O_n be the area of the polygon circumscribing the given circle, prove that $I_n = \frac{O_n}{2}\left(1+\sqrt{1-\left(\frac{2I_n}{n}\right)^2}\right)$

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Exercise Single

1. In ΔABC, $\frac{\sin A(a-b\cos C)}{\sin C(c-b\cos A)} =$

A. -2

B. -1

C. 0

D. 1

Answer: D

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2. If in a triangle ABC, $\frac{1+\cos A}{a}+\frac{1+\cos B}{b}+\frac{1+\cos C}{c}=\frac{k^2(1+\cos A)(1+\cos B)(1+\cos$

, then k is equal to

A. $\frac{1}{2\sqrt{2}R}$

B. 2R

C. $\frac{1}{R}$

D. none of these

Answer: B

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3. In triangle $ABC, 2ac\sin\left(\frac{1}{2}(A-B+C)\right)$ is equal to $a^2+b^2-c^2$ (b) $c^2+a^2-b^2$ $b^2-c^2-a^2$ (d) $c^2-a^2-b^2$

A. $a^2 + b^2 - c^2$

B. $c^2 + a^2 - b^2$

C. $b^2 - c^2 - a^2$

D. $c^2 - a^2 - b^2$

Answer: B

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4. If the angles of a triangle are in the ratio $4:1:1$, then the ratio of the longest side to the perimeter is

A. $\sqrt{3}:(2+\sqrt{3})$

B. $1:6$

C. $1:2+\sqrt{3}$

D. $2:3$

Answer: A

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5. Which of the following pieces of data does NOT uniquely determine an acute-angled triangle $ABC(R$ being the radius of the circumcircle)?

A. $a, \sin A, \sin B$

B. a, b, c

C. $a, \sin B, R$

D. $a, \sin A, R$

Answer: D

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6. The sides of a triangle are in the ratio $1 : \sqrt{3} : 2$. Then the angles are in the ratio

A. $1 : 3 : 5$

B. $2:3:4$

C. $3:2:1$

D. $1:2:3$

Answer: D

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7. In $ABC, a=5, b=12, c=90^0 and D$ is a point on AB so that $\angle BCD=45^0$. Then which of the following is not true? $CD=\frac{60\sqrt{2}}{17}$ (b) $BD=\frac{65}{17}$ $AD=\frac{60\sqrt{2}}{17}$ (d) none of these

A. $CD=\frac{60\sqrt{2}}{17}$

B. $BD=\frac{65}{17}$

C. $AD=\frac{60\sqrt{2}}{17}$

D. none of these

Answer: C

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8. In $\Delta ABC, (a+b+c)(b+c-a) = kbc$ if

A. $k < 0$

B. $k > 0$

C. $0 < k < 4$

D. $k < 4$

Answer: C

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9. Let D be the middle point of the side BC of a triangle ABC. If the triangle ADC is equilateral, then $a^2 : b^2 : c^2$ is equal to

A. $1 : 4 : 3$

B. $4 : 1 : 3$

C. $4:3:1$

D. $3:4:1$

Answer: B

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10. In a triangle ABC, the altitude from A is not less than BC and the altitude from B is not less than AC. The triangle

A. right angled

B. isosceles

C. obtuse angled

D. equilateral

Answer: A

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11. In ΔABC, if $\frac{\sin A}{c \sin B} + \frac{\sin B}{c} + \frac{\sin C}{b} = \frac{c}{ab} + \frac{b}{ac} + \frac{a}{bc}$, then the value of angle A is

A. 120°

B. 90°

C. 60°

D. 30°

Answer: B

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12. If in ΔABC, sides a, b, c are in A.P. then

A. $B > 60^\circ$

B. $B < 60^\circ$

C. $B \leq 60^\circ$

D. $B = |A - C|$

Answer: C

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13. In a triangle ABC, AD is the altitude from A. If $b > c$. $\angle C = 23^\circ$ and $AD = \dfrac{abc}{b^2 - c^2}$, then $\angle B =$

A. 83°

B. 97°

C. 113°

D. 127°

Answer: C

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14. If the sides a, b, c, of a triangle ABC form successive terms of G.P.with common ratio $r(>1)$ then which of the following is correct ?

A. $A > \pi/3$

B. $B \geq \pi/3$

C. $C < \pi/3$

D. $A < B < \pi/3$

Answer: D

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15. In triangle ABC, $b^2 \sin 2C + c^2 \sin 2B = 2bc$ where $b = 20, c = 21,$ then inradius =

A. 4

B. 6

C. 8

D. 9

Answer: B

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16. In ΔABC if $AB = x, BC = x + 1, \angle C = \frac{\pi}{3}$, then the less integer value of x is

A. 6

B. 7

C. 8

D. none of these

Answer: B

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17. If one side of a triangle is double the other, and the angles on opposite sides differ by 60°, then the triangle is

A. equilateral

B. obtus angled

C. right angled

D. acute angled

Answer: C

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18. If the hypotenuse of a right-angled triangle is four times the length of the perpendicular drawn from the opposite vertex to it, then the difference of the two acute angles will be 60^0 (b) 15^0 (c) 75^0 (d) 30^0

A. 60°

B. 15°

C. 75°

D. 30°

Answer: A

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19. If P is a point on the altitude AD of the triangle ABC such the $\angle CBP = \frac{B}{3}$, then AP is equal to $2a\frac{\sin C}{3}$ (b) $2b\frac{\sin C}{3}$ $2c\frac{\sin B}{3}$ (d) $2c\frac{\sin C}{3}$

A. $2a \sin. \frac{C}{3}$

B. $2b \sin. \frac{C}{3}$

C. $2c \sin. \frac{B}{3}$

D. $2c \sin. \frac{C}{3}$

Answer: C

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20. With usual notations, in triangle ABC, $a\cos(B-C)+b\cos(C-A)+c\cos(A-B)$ is equal to

A. $\frac{abc}{R^2}$

B. $\frac{abc}{4R^2}$

C. $\frac{4abc}{R^2}$

D. $\frac{abc}{2R^2}$

Answer: A

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21. If in $\Delta ABC, 8R^2 = a^2 + b^2 + c^2$, then the triangle ABC is

A. right angled

B. isosceles

C. equilateral

D. none of these

Answer: A

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22. Let ABC be a triangle with $\angle A = 45^0$. Let P be a point on side BC with PB=3 and PC=5. If O is circumcenter of triangle ABC, then length OP is

A. $\sqrt{18}$

B. $\sqrt{17}$

C. $\sqrt{19}$

D. $\sqrt{15}$

Answer: B

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23. In any triangle ABC, $\dfrac{a^2+b^2+c^2}{R^2}$ has the maximum value of

A. 3

B. 6

C. 9

D. none of these

Answer: C

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24. In triangle ABC, $R(b+c) = a\sqrt{bc}$, where R is the circumradius of the triangle. Then the triangle is

A. isosceles but not right

B. right but not isosceles

C. right isosceles

D. equilateral

Answer: C

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25. In $\Delta ABC, \quad \text{if} \ b^2 + c^2 = 2a^2$, then value of $\dfrac{\cot A}{\cot B + \cot C}$ is

A. $\dfrac{1}{2}$

B. $\dfrac{3}{2}$

C. $\dfrac{5}{2}$

D. $\dfrac{5}{3}$

Answer: A

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26. If $\sin\theta \ \text{and} \ -\cos\theta$ are the roots of the equation $ax^2 - bx - c = 0$, where a, b, and c are the sides of a triangle ABC, then $\cos B$ is equal to

A. $1 - \dfrac{c}{2a}$

B. $1 - \dfrac{c}{a}$

C. $1 + \dfrac{c}{2a}$

D. $1 + \dfrac{c}{3a}$

Answer: C

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27. If D is the mid-point of the side BC of triangle ABC and AD is perpendicular to AC , then $3b^2 = a^2 - c$ (b) $3a^2 = b^2 3c^2$ $b^2 = a^2 - c^2$ (d) $a^2 + b^2 = 5c^2$

A. $3b^2 = a^2 - c^2$

B. $3a^2 = b^2 - 3c^2$

C. $b^2 = a^2 - c^2$

D. $a^2 + b^2 = 5c^2$

Answer: A

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28. In a triangle ABC, if $\cot A : \cot B : \cot C = 30 : 19 : 6$ then the sides a, b, c are

A. in A.P.

B. in G.P.

C. in H.P.

D. none of these

Answer: A

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29. In ΔABC, P is an interior point such that $\angle PAB = 10^\circ, \angle PBA = 20^\circ, \angle PCA = 30^\circ, \angle PAC = 40^\circ$ then ΔABC is

A. isosceles

B. right angled

C. equilateral

D. obtuse angled

Answer: A

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30. In ΔABC, if AB = c is fixed, and $\cos A + \cos B + 2\cos C = 2$ then the locus of vertex C is

A. ellipse

B. hyperbola

C. circle

D. parabola

Answer: A

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31. If in ABC, $A = \frac{\pi}{7}$, $B = \frac{2\pi}{7}$, $C = \frac{4\pi}{7}$ then $a^2 + b^2 + c^2$ must be R^2

(b) $3R^2$ (c) $4R^2$ (d) $7R^2$

A. R^2

B. $3R^2$

C. $4R^2$

D. $7R^2$

Answer: D

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32. In ΔABC, $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}$ is equal to

A. $\frac{\Delta}{r^2}$

B. $\frac{(a+b+c)^2}{abc}2R$

C. $\frac{\Delta}{r}$

D. $\frac{\Delta}{Rr}$

Answer: A

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33. In ΔABC, $\left(\cot.\ \frac{A}{2}+\cot.\ \frac{B}{2}\right)\left(a\frac{\sin.\ ^2(B)}{2}+b\frac{\sin.\ ^2(A)}{2}\right)=$

A. $\cot C$

B. $c\cot C$

C. $\cot.\ \frac{C}{2}$

D. $c\cot.\ \frac{C}{2}$

Answer: D

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34. In a right-angled isosceles triangle, the ratio of the circumradius and inradius is

A. $2(\sqrt{2}+1):1$

B. $(\sqrt{2}+1):1$

C. $2:1$

D. $\sqrt{2}:1$

Answer: B

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35. In the given figure, what is the radius of the inscribed semicircle having base on AB ?

A. $3/2$

B. $5/2$

C. $7/5$

D. none of these

Answer: A

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36. In ΔABC, $A = \frac{2\pi}{3}$, $b - c = 3\sqrt{3}cm$ and area of $\Delta ABC = \frac{9\sqrt{3}}{2}cm^2$, then BC =

A. $6\sqrt{3}cm$

B. 9 cm

C. 18 cm

D. 27 cm

Answer: B

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37. In triangle ABC, let $\angle C = \pi/2$. If r is the inradius and R is circumradius of the triangle, then $2(r+R)$ is equal to

A. $a+b$

B. $b+c$

C. $c+a$

D. $a+b+c$

Answer: A

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38. In the given figure, AB is the diameter of the circle, centered at O. If $\angle COA = 60^\circ$, $AB = 2r$, $AC = d$, and $CD = l$, then l is equal to

A. $d\sqrt{3}$

B. $d/\sqrt{3}$

C. 3d

D. $\sqrt{3}d/2$

Answer: A

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39. In triangle ABC, if P, Q, R divides sides BC, AC, and AB, respectively, in the raito $k:1$ (in order). If the ratio $\left(\frac{\text{area } \Delta PQR}{\text{area } \Delta ABC}\right)$ is $\frac{1}{3}$, then k is equal to

A. $1/3$

B. 2

C. 3

D. none of these

Answer: B

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40. If the angles of a traingle are 30° and 45° and the included side is $(\sqrt{3}+1)$ cm, then the area of the triangle is

A. $\frac{\sqrt{3}+1}{2}$ sq. units

B. $(\sqrt{3}+1)$ sq. units

C. $2(\sqrt{3}-1)$ sq. units

D. $\frac{2\sqrt{3}-1}{2}$ sq. units

Answer: A

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41. In triangle ABC , base BC and area of triangle are fixed. The locus of the centroid of triangle ABC is a straight line that is parallel to side BC right bisector of side BC perpendicular to BC inclined at an angle $\sin^{-1}\left(\frac{\sqrt{}}{BC}\right)$ to side BC

A. parallel to side BC

B. right bisector of side BC

C. prependicular to BC

D. inclined at an angle $\sin^{-1}(\sqrt{\Delta}/BC)$ to side BC

Answer: A

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42. Let the area of triangle ABC be $(\sqrt{3}-1)/2, b=2$ and $c=(\sqrt{3}-1)$, and $\angle A$ be acute. The measure of the angle C is

A. 15°

B. 30°

C. 60°

D. 75°

Answer: A

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43. In $\Delta ABC, \Delta = 6, abc = 60, r = 1$. Then the value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ is nearly

A. 0.5

B. 0.6

C. 0.4

D. 0.8

Answer: D

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44. Triangle ABC is isosceles with $AB = AC$ and $BC = 65cm.$ P is a point on BC such that the perpendiculardistances from P to

AB and AC are $24cm$ and $36cm$, respectively. The area of triangle ABC (in sq cm is)

A. 1254

B. 1950

C. 2535

D. 5070

Answer: C

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45. In an equilateral triangle, the inradius, circumradius, and one of the exradii are in the ratio

A. $2:4:5$

B. $1:2:3$

C. $1:2:4$

D. $2:4:3$

Answer: B

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46. In triangle ABC, if $\cos A + \cos B + \cos C = \frac{7}{4}$, then $\frac{R}{r}$ is equal to

A. $\frac{3}{4}$

B. $\frac{4}{3}$

C. $\frac{2}{3}$

D. $\frac{3}{2}$

Answer: B

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47. If two sides of a triangle are roots of the equation $x^2 - 7x + 8 = 0$ and the angle between these sides is 60° then the product of inradius and circumradius of the triangle is

A. $\frac{8}{7}$

B. $\frac{5}{3}$

C. $\frac{5\sqrt{2}}{3}$

D. 8

Answer: B

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48. Given $b = 2, c = \sqrt{3}, \angle A = 30^\circ$, then inradius of ΔABC is

A. $\frac{\sqrt{3}-1}{2}$

B. $\frac{\sqrt{3}+1}{2}$

C. $\frac{\sqrt{3}-1}{4}$

D. none of these

Answer: A

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49. In triangle ABC, if $A - B = 120^2$ and $R = 8r$, where R and r have their usual meaning, then cos C equals

A. $3/4$

B. $2/3$

C. $5/6$

D. $7/8$

Answer: D

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50. ABC is an equilateral triangle of side $4cm$. If R, r and h are the circumradius, inradius, and altitude, respectively, then $\frac{R+r}{h}$ is equal to

A. 4

B. 2

C. 1

D. 3

Answer: C

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51. A circle is inscribed in a triangle ABC touching the side AB at D such that $AD = 5, BD = 3, \quad \text{if} \quad \angle A = 60^0$ then length BC equals. 9 (b) $\frac{120}{13}$ (c) 13(d) 12

A. 9

B. $\frac{120}{13}$

C. 13

D. 12

Answer: C

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52. The rational number which equals the number 2.357 with recurring decimal is $\frac{2355}{1001}$ b. $\frac{2379}{997}$ c. $\frac{2355}{999}$ d. none of these

A. $\frac{25}{9}$

B. $\frac{25}{3}$

C. $\frac{25}{18}$

D. $\frac{10}{3}$

Answer: B

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53. Let AD be a median of the ΔABC. If AE and AF are medians of the triangle ABD and ADC, respectively, and AD = m_1, $AE = m_2$, $AF = m_3$, then $a^2/8$ is equal to

A. $m_2^2 + m_3^2 - 2m_1^2$

B. $m_1^2 + m_2^2 - 2m_3^2$

C. $m_1^2 + m_3^2 - 2m_2^2$

D. none of these

Answer: A

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54. For a triangle ABC, $R = \frac{5}{2}$ and $r = 1$. Let D, E and F be the feet of the perpendiculars from incentre I to BC, CA and AB, respectively. Then the value of $\frac{(IA)(IB)(IC)}{(ID)(IE)(IF)}$ is equal to _____

A. $\frac{5}{2}$

B. $\frac{5}{4}$

C. $\frac{1}{10}$

D. $\frac{1}{5}$

Answer: C

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55. In triangle $ABC, \angle A = 60^0, \angle B = 40^0, and \angle C = 80^0.$ If P is the center of the circumcircle of triangle ABC with radius unity, then the radius of the circumcircle of triangle BPC is 1 (b) $\sqrt{3}$ (c) 2 (d) $\sqrt{3}\ 2$

A. 1

B. $\sqrt{3}$

C. 2

D. $\sqrt{3}/2$

Answer: A

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56. If H is the othrocenter of an acute angled triangle ABC whose circumcircle is $x^2 + y^2 = 16,$ then circumdiameter of the triangle HBC is 1 (b) 2 (c) 4 (d) 8

A. 1

B. 2

C. 4

D. 8

Answer: D

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57. In triangle ABC, the line joining the circumcenter and incenter is parallel to side AC, then $\cos A + \cos C$ is equal to

A. $\frac{1}{2}$

B. 1

C. $\sqrt{3}$

D. 2

Answer: B

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58. In triangle ABC, line joining the circumcenter and orthocenter is parallel to side AC, then the value of tan A tan C is equal to

A. $\sqrt{3}$

B. 3

C. $3\sqrt{3}$

D. none of these

Answer: B

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59. In triangle ABC, $\angle C = \frac{2\pi}{3}$ and CD is the internal angle bisector of $\angle C$, meeting the side $AB at D$. If Length CD is 1, the H.M. of a and b is equal to:

A. 1

B. 2

C. 3

D. 4

Answer: B

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60. In the given figure ΔABC is equilateral on side AB produced. We choose a point such that A lies between P and B. We now denote 'a' as the length of sides of ΔABC, r_1 as the radius of incircle ΔPAC and r_2 as the ex-radius of ΔPBC with respect to side BC. Then $r_1 + r_2$ is equal to

A. $\frac{1}{2}$

B. $\frac{3}{2}a$

C. $\frac{\sqrt{3}}{2}a$

D. $a\sqrt{2}$

Answer: C

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61. A variable triangle ABC is circumscribed about a fixed circle of unit radius. Side BC always touches the circle at D and has fixed direction. If B and C vary in such a way that (BD) (CD)=2, then locus of vertex A will be a straight line. parallel to side BC perpendicular to side BC making an angle $\left(\frac{\pi}{6}\right)$ with BC making an angle $\sin^{-1}\left(\frac{2}{3}\right)$ with BC

A. parallel to side BC

B. perpendicular to side BC

C. making an angle $(\pi/6)$ with BC

D. making an angle $\sin^{-1}(2/3)$ with BC

Answer: A

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62. In ΔABC, if $a=10$ and $b\cot B+c\cot C=2(r+R)$ then the maximum area of ΔABC will be

A. 50

B. $\sqrt{50}$

C. 25

D. 5

Answer: C

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63. Let C be incircle of ΔABC. If the tangents of lengths t_1, t_2 and t_3 are drawn inside the given triangle parallel to side a,b, and c, respectively, then $\frac{t_1}{a} + \frac{t_2}{b} + \frac{t_3}{c}$ is equal to

A. 0

B. 1

C. 2

D. 3

Answer: B

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64. A park is in the form of a rectangle $120m x 100m$. At the centre of the park there is a circular lawn. The area of park excluding lawn is $8700m^2$. Find the radius of the circular lawn. $\left(Use \pi \frac{22}{7}\right)$

A. c^2

B. $\frac{c^2}{2}$

C. $\frac{c^2}{4}$

D. none of these

Answer: C

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65. In triangle ABC, if $r_1 = 2r_2 = 3r_3$, then $a : b$ is equal to

A. $\frac{5}{4}$

B. $\frac{4}{5}$

C. $\frac{7}{4}$

D. $\frac{4}{7}$

Answer: A

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66. If in a triangle, $\left(1-\frac{r_1}{r_2}\right)\left(1-\frac{r_1}{r_3}\right)=2$, then the triangle is

A. right angled

B. isosceles

C. equilateral

D. none of these

Answer: A

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67. If in a triangle $\frac{r}{r_1}=\frac{r_2}{r_3}$, then

A. $A=90^{\circ}$

B. $B=90^{\circ}$

C. $C=90^{\circ}$

D. none of these

Answer: C

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68. In ΔABC, I is the incentre, Area of $\Delta IBC, \Delta IAC$ and ΔIAB are, respectively, Δ_1, Δ_2 and Δ_3. If the values of Δ_1, Δ_2 and Δ_3 are in A.P., then the altitudes of the ΔABC are in

A. A.P.

B. G.P.

C. H.P.

D. none of these

Answer: C

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69. In an acute angled triangle ABC, $r + r_1 = r_2 + r_3$ and $\angle B > \frac{\pi}{3}$, then

A. $b + 2c < 2a < 2b + 2c$

B. $b + 4 < 4a < 2b + 4c$

C. $b + 4c < 4a < 4b + 4c$

D. $b + 3c < 3a < 3b + 3c$

Answer: D

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70. If in triangle $ABC, \sum \frac{\sin A}{2} = \frac{6}{5} and \sum II_1 = 9$ (where $I_1, I_2 and I_3$ are excenters and I is incenter, then circumradius R is equal to $\frac{15}{8}$ (b) $\frac{15}{4}$ (c) $\frac{15}{2}$ (d) $\frac{4}{12}$

A. $\frac{15}{8}$

B. $\frac{15}{4}$

C. $\frac{15}{2}$

D. $\frac{4}{12}$

Answer: A

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71. The radii r_1, r_2, r_3 of the escribed circles of the triangle ABC are in H.P. If the area of the triangle is $24cm^2$ and its perimeter is 24 cm, then the length of its largest side is

A. 10

B. 9

C. 8

D. none of these

Answer: A

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72. In ΔABC with usual notations, if $r=1, r_1=7 \text{ and } R=3, \quad \text{then} \quad \Delta ABC$ is

A. equilateral

B. acute angled which is not equilateral

C. obtuse angled

D. right angled

Answer: D

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73. Which of the following expresses the circumference of a circle inscribed in a sector OAB with radius $RandAB=2a$? $2\pi\frac{Ra}{R+a}$ (b) $\frac{2\pi R^2}{a}$ $2\pi(r-a)^2$ (d) $2\pi\frac{R}{R-a}$

A. $2\pi\frac{Ra}{R+a}$

B. $\dfrac{2\pi R^2}{a}$

C. $2\pi(R-a)^2$

D. $2\pi\dfrac{R}{R-a}$

Answer: A

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74. In ABC, the median AD divides $\angle BAC$ such that $\angle BAD : \angle CAD = 2 : 1$. Then $\cos\left(\frac{A}{3}\right)$ is equal to $\frac{\sin B}{2\sin C}$ (b) $\frac{\sin C}{2\sin B}$ $\frac{2\sin B}{\sin C}$ (d) $noneofthese$

A. $\dfrac{\sin B}{2\sin C}$

B. $\dfrac{\sin C}{2\sin B}$

C. $\dfrac{2\sin B}{\sin C}$

D. none of these

Answer: A

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75. The area of the circle and the area of a regular polygon of n sides and of perimeter equal to that of the circle are in the ratio of $\tan\left(\frac{\pi}{n}\right):\frac{\pi}{n}$ (b) $\cos\left(\frac{\pi}{n}\right):\frac{\pi}{n}$ $\frac{\sin\pi}{n}:\frac{\pi}{n}$ (d) $\cot\left(\frac{\pi}{n}\right):\frac{\pi}{n}$

A. $\tan\left(\frac{\pi}{n}\right):\frac{\pi}{n}$

B. $\cos\left(\frac{\pi}{n}\right):\frac{\pi}{n}$

C. $\sin.\ \frac{\pi}{n}:\frac{\pi}{n}$

D. $\cot\left(\frac{\pi}{n}\right):\frac{\pi}{n}$

Answer: A

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76. The ratio of the area of a regular polygon of n sides inscribed in a circle to that of the polygon of same number of sides circumscribing the same is 3:4. Then the value of n is 6 (b) 4 (c) 8 (d) 12

A. 6

B. 4

C. 8

D. 12

Answer: A

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77. In any triangle, the minimum value of $r_1r_2r_3/r^3$ is equal to

A. 1

B. 9

C. 27

D. none of these

Answer: C

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78. If R_1 is the circumradius of the pedal triangle of a given triangle ABC, and R_2 is the circumradius of the pedal triangle of the pedal triangle formed, and so on R_3, R_4...., then the value of $\sum_{i=1}^{\infty} R_i$, where R (circumradius) of ΔABC is 5 is

A. 8

B. 10

C. 12

D. 15

Answer: B

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79. A sector $OABO$ of central angle θ is constructed in a circle with centre O and of radius 6. The radius of the circle that is circumscribed about the triangle OAB, is $6\frac{\cos\theta}{2}$ (b) $6\frac{\sec\theta}{2}$ $3\frac{\sec\theta}{2}$ (d) $3\left(\frac{\cos\theta}{2}+2\right)$

A. $6\cos.\frac{\theta}{2}$

B. $6\sec.\frac{\theta}{2}$

C. $3\sec.\frac{\theta}{2}$

D. $3\left(\cos.\frac{\theta}{2}+2\right)$

Answer: C

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80. There is a point P inside an equilateral ΔABC of side a whose distances from vertices A, B and C are 3, 4 and 5, respectively. Rotate the triangle and P through 60° about C. Let A go to A' and P to P'. Then the area of $\Delta PAP'$ (in sq. units) is

A. 8

B. 12

C. 16

D. 6

Answer: D

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Exercise Multiple

1. The sides of ΔABC satisfy the equation $2a^2 + 4b^2 + c^2 = 4ab + 2ac.$

Then

A. the triangle is isosceles

B. the triangle is obtuse

C. $B = \cos^{-1}(7/8)$

D. $A = \cos^{-1}(1/4)$

Answer: A::C::D

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2. If sides of triangle ABC are a, b, and c such that $2b = a + c$, then

A. $\frac{b}{c} > \frac{2}{3}$

B. $\frac{b}{c} > \frac{1}{3}$

C. $\frac{b}{c} < 2$

D. $\frac{b}{c} < \frac{3}{2}$

Answer: A::C

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3. If the sines of the angle A and B of a triangle ABC satisfy the equation $c^2x^2 - c(a+b)x + ab = 0$, then the triangle

A. is acute angled

B. is right angled

C. is obtus angled

D. satisfies the equation $\sin A + \cos A = \frac{(a+b)}{c}$

Answer: B::D

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4. There exists triangle ABC satisfying

A. $\tan A + \tan B + \tan C = 0$

B. $\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sin C}{7}$

C. $(a+b)^2 = c^2 + ab$ and $\sqrt{2}(\sin A + \cos A) = \sqrt{3}$

D. $\sin A + \sin B = \frac{\sqrt{3}+1}{2}, \cos A \cos B = \frac{\sqrt{3}}{4} = \sin A \sin B$

Answer: C::D

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5. In triangle, ABC if $2a^2b^2 + 2b^2c^2 = a^4 + b^4 + c^4$, then angle B is equal to

A. 45°

B. 135°

C. 120°

D. 60°

Answer: A::B

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6. If in triangle ABC, a, c and angle A are given and $c \sin A < a < c$, then ($b_1 \text{ and } b_2$ are values of b)

A. $b_1 + b_2 = 2c \cos A$

B. $b_1 + b_2 = c \cos A$

C. $b_1 b_2 = c^2 - a^2$

D. $b_1 b_2 = c^2 + a^2$

Answer: A::C

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7. If area of $\Delta ABC(\Delta)$ and angle C are given and if c opposite to given angle is minimum, then

A. $a = \sqrt{\frac{2\Delta}{\sin C}}$

B. $b = \sqrt{\frac{2\Delta}{\sin C}}$

C. $a = \frac{4\Delta}{\sin C}$

D. $b = \frac{4\Delta}{\sin^2 C}$

Answer: A::B

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8. If Δ represents the area of acute angled triangle ABC $\sqrt{a^2b^2 - 4\Delta^2} + \sqrt{b^2c^2 - 4\Delta^2} + \sqrt{c^2a^2 - 4\Delta^2} =$

A. $a^2 + b^2 + c^2$

B. $\dfrac{a^2+b^2+c^2}{2}$

C. $ab\cos C + bc\cos A + ca\cos B$

D. $ab\sin C + bc\sin A + ca\sin B$

Answer: B::C

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9. Sides of ΔABC are in A.P. If $a < \min\{b, c\}$, then cos A may be equal to

A. $\dfrac{4b-3c}{2b}$

B. $\dfrac{3c-4b}{2c}$

C. $\dfrac{4c-3b}{2b}$

D. $\dfrac{4c-3b}{2c}$

Answer: A::D

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10. If the angles of a triangle are 30° and 45°, and the included side is $(\sqrt{3}+1)$ cm, then

A. area of the triangle is $\frac{1}{-}(\sqrt{3}+1)$ sq. units

B. area of the triangle is $\frac{1}{2}(\sqrt{3}-1)$ sq. units

C. ratio of greater side to smaller side is $\frac{\sqrt{3}+1}{\sqrt{2}}$

D. ratio of greater side to smaller side is $\frac{1}{4\sqrt{3}}$

Answer: A::C

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11. Lengths of the tangents from A,B and C to the incircle are in A.P., then

A. $r_1, r_2 r_3$ are in H.P

B. r_1, r_2, r_3 are in AP

C. a, b, c are in A.P

D. $\cos A = \frac{4c-3b}{2c}$

Answer: A::C::D

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12. CF is the internal bisector of angle C of $\angle ABC$, then CF is equal to

A. $\dfrac{2ab}{a+b}\cos.\ \dfrac{C}{2}$

B. $\dfrac{a+b}{2ab}\cos.\ \dfrac{C}{2}$

C. $\dfrac{b\sin A}{\sin\left(B+\frac{C}{2}\right)}$

D. none of these

Answer: A::C

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13. The incircle of ΔABC touches side BC at D. The difference between BD and CD (R is circumradius of ΔABC) is

A. $\left|4R \sin . \frac{A}{2} \sin . \frac{B-C}{2}\right|$

B. $\left|4R \cos . \frac{A}{2} \sin . \frac{B-C}{2}\right|$

C. $|b-c|$

D. $\left|\frac{b-c}{2}\right|$

Answer: A::C

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14. A circle of radius 4 cm is inscribed in ΔABC, which touches side BC at D. If BD = 6 cm, DC = 8 cm then

A. the triangle is necessarily acute angled

B. $\tan . \frac{A}{2} = \frac{4}{7}$

C. perimeter of the triangle ABC is 42 cm

D. area of ΔABC is $84 cm^2$

Answer: A::B::C::D

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15. If H is the orthocentre of triangle ABC, R = circumradius and $P = AH + BH + CH$, then

A. $P = 2(R + r)$

B. max. of P is 3R

C. min. of P is 3R

D. $P = 2(R - r)$

Answer: A::B

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16. Let ABC be an isosceles triangle with base BC. If r is the radius of the circle inscribsed in ΔABC and r_1 is the radius of the circle ecribed opposite to the angle A, then the product $r_1 r$ can be equal to (where R is the radius of the circumcircle of ΔABC)

A. $R^2 \sin^2 A$

B. $R^2 \sin^2 2B$

C. $\frac{1}{2}a^2$

D. $\frac{a^2}{4}$

Answer: A::B::D

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17. If inside a big circle exactly $n(n \leq 3)$ small circles, each of radius r, can be drawn in such a way that each small circle touches the big circle and also touches both its adjacent small circles, then the radius of big circle is $r\left(1+\cos ec\frac{\pi}{n}\right)$ (b) $\left(\frac{1+\frac{\tan \pi}{n}}{\frac{\cos \pi}{\pi}}\right)$ $r\left[1+\cos ec\frac{2\pi}{n}\right]$ (d) $\frac{r\left[s \in \frac{\pi}{2n}+\frac{\cos(2\pi)}{n}\right]^2}{\frac{\sin \pi}{n}}$

A. $r\left(1+\cos ec.\ \frac{\pi}{n}\right)$

B. $\left(\frac{1+\tan \pi / n}{\cos \pi / n}\right)$

C. $r\left[1+\cos ec.\ \frac{2\pi}{n}\right]$

D. $\frac{r\left[\sin.\ \frac{\pi}{2n}+\cos.\ \frac{2\pi}{n}\right]^2}{\sin\pi/n}$

Answer: A::D

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18. The area of a regular polygon of n sides is (where r is inradius, R is circumradius, and a is side of the triangle) $\frac{nR^2}{2}\sin\left(\frac{2\pi}{n}\right)$ (b) $nr^2\tan\left(\frac{\pi}{n}\right)$ $\frac{na^2}{4}\frac{\cot\pi}{n}$ (d) $nR^2\tan\left(\frac{\pi}{n}\right)$

A. $\frac{nR^2}{2}\sin\left(\frac{2\pi}{n}\right)$

B. $nr^2\tan\left(\frac{\pi}{n}\right)$

C. $\frac{na^2}{4}\cot.\ \frac{\pi}{n}$

D. $nR^2\tan\left(\frac{\pi}{n}\right)$

Answer: A::B::C

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19. In acute angled triangle ABC, AD is the altitude. Circle drawn with AD as its diameter cuts $ABandACatPandQ$, respectively. Length of PQ is equal to $/(2R)$ (b) $\frac{abc}{4R^2}$ $2R\sin A\sin B\sin C$ (d) $/R$

A. $\frac{\Delta}{2R}$

B. $\frac{abc}{4R^2}$

C. $2R\sin A\sin B\sin C$

D. $\frac{\Delta}{R}$

Answer: C::D

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20. If A is the area and 2s is the sum of the sides of a triangle, then

A. $A \leq \frac{s^2}{4}$

B. $A \leq \frac{s^2}{3\sqrt{3}}$

C. $A < \frac{s^2}{\sqrt{3}}$

D. none of these

Answer: A::B

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21. In ΔABC, internal angle bisector of $\angle A$ meet side BC in D. $DE \perp AD$ meet AC in E and AB in F. then

A. AE in H.M of b and c

B. $AD = \frac{2bc}{b+c} \cos . \frac{A}{2}$

C. $EF = \frac{4bc}{b+c} \sin . \frac{A}{2}$

D. ΔAEF is isosceles

Answer: A::B::C::D

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22. In a triangle ABC, AB = 5 , BC = 7, AC = 6. A point P is in the plane such that it is at distance '2' units from AB and 3 units form AC then its distance from BC

A. is $\frac{12\sqrt{6}-28}{7}$ when P is inside the trinagle

B. may be $\frac{12\sqrt{6}-8}{7}$ when P is outside the triangle

C. may be $\frac{12\sqrt{6}+14}{7}$ when P is inside the triangle

D. may be $\frac{12\sqrt{6}+14}{7}$ when P is outside the triangle

Answer: A::B::C

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23. The base BC of ΔABC is fixed and the vertex A moves, satisfying the condition $\cot.\frac{B}{2}+\cot.\frac{C}{2}=2\cot.\frac{A}{2}$, then

A. $b+c=a$

B. $b+c=2a$

C. vertex A moves along a straight line

D. vertex A moves along an ellipse

Answer: B::D

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24. If D, E and F be the middle points of the sides BC,CA and AB of the ΔABC, then $AD + BE + CF$ is

A. centroid of the triangle DEF is the same as that of ABC

B. orthocenter of the triangle DEF is the circumcentre of ABC

C. orthocenter of the triangle DEF is the incenter of ABC

D. centroid of the triangle DEF is not the same as that of ABC

Answer: A::B

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Exercise Comprehension

1. Given that $\Delta = 6, r_1 = 3, r_3 = 6$

Circumradius R is equal to

A. 2.5

B. 3.5

C. 1.5

D. none of these

Answer: A

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2. Given that $\Delta = 6, r_1 = 3, r_3 = 6$

Inradius is equal to

A. 2

B. 1

C. 1.5

D. 2.5

Answer: B

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3. Given that $\Delta = 6, r_1 = 2, r_2 = 3, r_3 = 6$ Difference between the greatest and the least angles is

A. $\cos^{-1}. \frac{4}{5}$

B. $\tan^{-1}. \frac{3}{4}$

C. $\cos^{-1}. \frac{3}{5}$

D. none of these

Answer: C

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4. Let a = 6, b = 3 and $\cos(A - B) = \frac{4}{5}$

Area (in sq. units) of the triangle is equal to

A. 9

B. 12

C. 11

D. 10

Answer: A

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5. Let a = 6, b = 3 and $\cos(A - B) = \frac{4}{5}$

Angle C is equal to

A. $\frac{3\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. none of these

Answer: C

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6. Let a = 6, b = 3 and $\cos(A - B) = \frac{4}{5}$

Value of $\sin A$ is equal to

A. $\frac{1}{2\sqrt{5}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{5}}$

D. $\frac{2}{\sqrt{5}}$

Answer: D

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7. Let ABC be an acute angled triangle with orthocenter H.D, E, and F are the feet of perpendicular from A,B, and C, respectively, on opposite sides. Also, let R be the circumradius of $\Delta ABC.$ Given $AH.\,CH=3$ and $(AH)^2+(BH)^2+(CH)^2=7$

Then answer the following

Value of $\dfrac{\cos A.\cos B:\cos C}{\cos^2 A+\cos^2 B+\cos^2 C}$ is

A. $\dfrac{3}{14R}$

B. $\dfrac{3}{7R}$

C. $\dfrac{7}{3R}$

D. $\dfrac{14}{3R}$

Answer: A

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8. Let ABC be an acute angled triangle with orthocenter H.D, E, and F are the feet of perpendicular from A,B, and C, respectively, on opposite sides.

Also, let R be the circumradius of ΔABC. Given $AH.BH.CH = 3$ and $(AH)^2 + (BH)^2 + (CH)^2 = 7$

Then answer the following

Value of R is

A. 1

B. $\frac{3}{2}$

C. $\frac{5}{2}$

D. none

Answer: B

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9. Let ABC be an acute angled triangle with orthocenter H.D, E, and F are the feet of perpendicular from A,B, and C, respectively, on opposite sides. Also, let R be the circumradius of ΔABC. Given $AH.CH = 3$ and $(AH)^2 + (BH)^2 + (CH)^2 = 7$

Then answer the following

Value of $HD.\,HF$ is

A. $\dfrac{9}{64R^3}$

B. $\dfrac{9}{8R^3}$

C. $\dfrac{8}{9R^3}$

D. $\dfrac{64}{9R^3}$

Answer: B

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10. Let O be a point inside ΔABC such that

$\angle OAB = \angle OBC = \angle OCA = \theta$

$\cot A + \cot B + \cot C$ is equal to

A. $\tan^2\theta$

B. $\cot^2\theta$

C. $\tan\theta$

D. $\cot\theta$

Answer: D

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11. Let O be a point inside ΔABC such that

$\angle OAB = \angle OBC = \angle OCA = \theta$

$\cos ec^2 A + \cos ec^2 B + \cos ec^2 C$ is equal to

A. $\cot^2\theta$

B. $\cos ec^2\theta$

C. $\tan^2\theta$

D. $\sec^2\theta$

Answer: B

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12. Let O be a point inside ΔABC such that

$\angle OAB = \angle OBC = \angle OCA = \theta$

Area of ΔABC is equal to

A. $\left(\frac{a^2+b^2+c^2}{4}\right)\tan\theta$

B. $\left(\frac{a^2+b^2+c^2}{4}\right)\cot\theta$

C. $\left(\frac{a^2+b^2+c^2}{2}\right)\tan\theta$

D. $\left(\frac{a^2+b^2+c^2}{2}\right)\cot\theta$

Answer: A

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13. Given an isoceles triangle with equal side of length b and angle $\alpha < \pi/4$, then

the circumradius R is given by

A. $\frac{1}{2}b\cos ec\alpha$

B. $b\cos ec\alpha$

C. $2b$

D. none of these

Answer: A

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14. Given an isoceles triangle with equal side of length b and angle $\alpha < \pi/4$, then

the inradius r is given by

A. $\dfrac{b\sin 2\alpha}{2(1-\cos\alpha)}$

B. $\dfrac{b\sin 2\alpha}{2(1+\cos\alpha)}$

C. $\dfrac{b\sin\alpha}{2}$

D. $\dfrac{b\sin\alpha}{2(1+\sin\alpha)}$

Answer: B

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15. Given an isoceles triangle with equal side of length b and angle $\alpha < \pi/4$, then

the distance between circumcenter O and incenter I is

A. $\left|\dfrac{b\cos(3\alpha/2)}{2\sin\alpha\cos(\alpha/2)}\right|$

B. $\left|\dfrac{b\cos 3\alpha}{\sin 2\alpha}\right|$

C. $\left|\dfrac{b\cos 3\alpha}{\cos\alpha\sin(\alpha/2)}\right|$

D. $\left|\dfrac{b}{\sin\alpha\cos\alpha/2}\right|$

Answer: A

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16. Incircle of ΔABC touches the sides BC, AC and AB at D, E and F, respectively. Then answer the following question

$\angle DEF$ is equal to

A. $\dfrac{\pi - B}{2}$

B. $\pi - 2B$

C. $A - C$

D. none of these

Answer: A

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17. Incircle of ΔABC touches the sides BC, AC and AB at D, E and F, respectively. Then answer the following question

Area of ΔDEF is

A. $2r^2 \sin(2A)\sin(2B)\sin(2C)$

B. $2r^2 \cos.\ \frac{A}{2} \cos.\ \frac{B}{2} \cos.\ \frac{C}{2}$

C. $2r^2 \sin(A - B)\sin(B - C)\sin(C - A)$

D. none of these

Answer: B

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18. Incircle of ΔABC touches the sides BC, AC and AB at D, E and F, respectively. Then answer the following question

The length of side EF is

A. $r \sin . \frac{A}{2}$

B. $2r \sin . \frac{A}{2}$

C. $r \cos . \frac{A}{2}$

D. $2r \cos . \frac{A}{2}$

Answer: D

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19. Internal bisectors of ΔABC meet the circumcircle at point D, E, and F

Length of side eF is

A. $2R\cos.\ \dfrac{A}{2}$

B. $2R\sin\left(\dfrac{A}{2}\right)$

C. $R\cos\left(\dfrac{A}{2}\right)$

D. $2R\cos\left(\dfrac{B}{2}\right)\cos\left(\dfrac{C}{2}\right)$

Answer: A

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20. Internal bisectors of ΔABC meet the circumcircle at point D, E, and F

Area of ΔDEF is

A. $2R^2\cos^2\left(\dfrac{A}{2}\right)\cos^2\left(\dfrac{B}{2}\right)\cos^2\left(\dfrac{C}{2}\right)$

B. $2R^2\sin\left(\dfrac{A}{2}\right)\sin\left(\dfrac{B}{2}\right)\sin\left(\dfrac{C}{2}\right)$

C. $2R^2\sin^2\left(\dfrac{A}{2}\right)\sin^2\left(\dfrac{B}{2}\sin^2\left(\dfrac{C}{2}\right)\right.$

D. $2R^2\cos\left(\dfrac{A}{2}\right)\cos\left(\dfrac{B}{2}\right)\cos\left(\dfrac{C}{2}\right)$

Answer: D

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21. Internal bisectors of ΔABC meet the circumcircle at point D, E, and F

Ratio of area of triangle ABC and triangle DEF is

A. ≥ 1

B. ≤ 1

C. $\geq 1/2$

D. $\leq 1/2$

Answer: B

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22. The area of any cyclic quadrilateral ABCD is given by $A^2 = (s-a)(s-b)(s-c)(s-d)$, where $2s = a+b+ \ + c+d$, a, b, c and d are the sides of the quadrilateral

Now consider a cyclic quadrilateral ABCD of area 1 sq. unit and answer the following question

The minium perimeter of the quadrilateral is

A. 4

B. 2

C. 1

D. none of these

Answer: A

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23. The area of any cyclic quadrilateral ABCD is given by $A^2 = (s-a)(s-b)(s-c)(s-d)$, where $2s = a + b + \ + c + d$, a, b, c and d are the sides of the quadrilateral

Now consider a cyclic quadrilateral ABCD of area 1 sq. unit and answer the following question

The minimum value of the sum of the lenghts of diagonals is

A. $2\sqrt{2}$

B. 2

C. $\sqrt{2}$

D. none of these

Answer: A

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24. The area of any cyclic quadrilateral ABCD is given by $A^2=(s-a)(s-b)(s-c)(s-d)$, where $2s=a+b+\ +c+d,\ a,\ b,\ c$ and d are the sides of the quadrilateral

Now consider a cyclic quadrilateral ABCD of area 1 sq. unit and answer the following question

When the perimeter is minimum, the quadrilateral is necessarily

A. a square

B. a rectangle but not a square

C. a rhombus but not a square

D. none of these

Answer: A

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25. In $\Delta ABC, R, r, r_1, r_2, r_3$ denote the circumradius, inradius, the exradii opposite to the vertices A,B, C respectively. Given that $r_1 : r_2 : r_3 = 1 : 2 : 3$

The sides of the triangle are in the ratio

A. $1 : 2 : 3$

B. $3 : 5 : 7$

C. $1 : 5 : 9$

D. $5 : 8 : 9$

Answer: D

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26. In $\Delta ABC, R, r, r_1, r_2, r_3$ denote the circumradius, inradius, the exradii opposite to the vertices A,B, C respectively. Given that $r_1 : r_2 : r_3 = 1 : 2 : 3$

The value of $R : r$ is

A. $5 : 2$

B. $5 : 4$

C. $5 : 3$

D. $3 : 2$

Answer: A

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27. In $\Delta ABC, R, r, r_1, r_2, r_3$ denote the circumradius, inradius, the exradii opposite to the vertices A,B, C respectively. Given that

$r_1 : r_2 : r_3 = 1 : 2 : 3$

The greatest angle of the triangle is given by

A. $\cos^{-1}\left(\frac{1}{30}\right)$

B. $\cos^{-1}\left(\frac{1}{3}\right)$

C. $\cos^{-1}\left(\frac{1}{10}\right)$

D. $\cos^{-1}\left(\frac{1}{5}\right)$

Answer: C

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28. In ΔABC, P, Q, R are the feet of angle bisectors from the vertices to their opposite sides as shown in the figure. ΔPQR is constructed

If $\angle BAC = 120^\circ$, then measusred of $\angle RPQ$ will be

A. 60°

B. 90°

C. 120°

D. 150°

Answer: B

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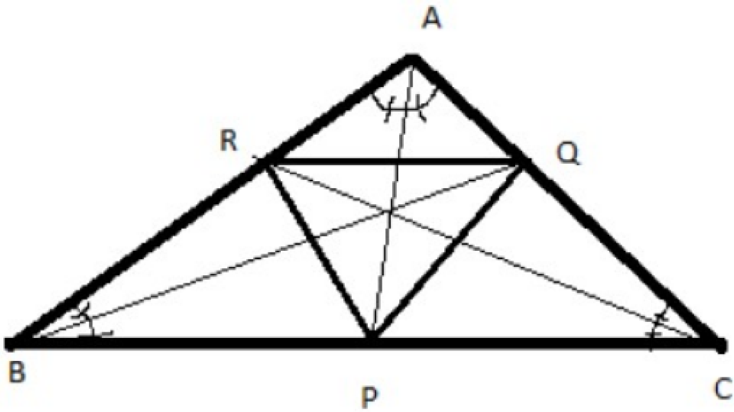
29. In ΔABC, P, Q, R are the feet of angle bisectors from the vertices to their opposite sides as shown in the figure. ΔPQR is constructed



If $AB = 7$ units, BC = 8 units, AC = 5 units, then the side PQ will be

A. $\frac{\sqrt{28}}{3}$ units

B. $\frac{\sqrt{88}}{3}$ units

C. $\frac{\sqrt{78}}{3}$ units

D. $\frac{\sqrt{84}}{3}$ units

Answer: D

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30. Let G be the centroid of triangle ABC and the circumcircle of triangle AGC touches the side AB at A

If $BC = 6$, $AC = 8$, then the length of side AB is equal to

A. $\frac{1}{2}$

B. $\frac{2}{\sqrt{3}}$

C. $5\sqrt{2}$

D. none of these

Answer: C

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31. Let G be the centroid of triangle ABC and the circumcircle of triangle AGC touches the side AB at A

If $\angle GAC = \frac{\pi}{3}$ and $a = 3b$, then sin C is equal to

A. $\frac{3}{4}$

B. $\frac{1}{2}$

C. $\frac{2}{\sqrt{3}}$

D. none of these

Answer: B

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32. Let G be the centroid of triangle ABC and the circumcircle of triangle AGC touches the side AB at A

If AC = 1, then the length of the median of triangle ABC through the vertex A is equal to

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{2}$

C. $\frac{2}{\sqrt{3}}$

D. $\frac{5}{\sqrt{2}}$

Answer: A

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33. The inradius in a right angled triangle with integer sides is r

If r = 4, the greatest perimeter (in units) is

A. 96

B. 90

C. 60

D. 48

Answer: B

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34. The inradius in a right angled triangle with integer sides is r

If r = 5, the greatest area (in sq. units) is

A. 150

B. 210

C. 330

D. 450

Answer: C

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Exercise Matrix

1.

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2. In acute -angled triangle ABC

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3.

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4. Let O be the circumcenter, H be the orthocenter, I be the incenter, and I_1, I_2, I_3 be the excenters of acute-angled ΔABC

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5. In triangle ABC, AD is prependicular to BC and DE is perpendicular to AB

A. $\begin{array}{cccc} a & b & c & d \\ p & r & q & q \end{array}$

B. $\begin{array}{cccc} a & b & c & d \\ q & r & p & s \end{array}$

C. $\begin{array}{cccc} a & b & c & d \\ s & p & q & r \end{array}$

D. $\begin{array}{cccc} a & b & c & d \\ r & p & s & q \end{array}$

Answer: D

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6. In a triangle ABC, $a = 7, b = 8, c = 9$, BD is the median and BE the altitude from the vertex B. Match the following lists

$$\begin{array}{ll} a.\ BD = & p.\ 2 \\ b.\ BE = & q.\ 7 \\ c.\ ED = & r.\ \sqrt{45} \\ d.\ AE = & s.\ 6 \end{array}$$

A. $\begin{array}{cccc} a & b & c & d \\ p & r & q & q \end{array}$

B. $\begin{matrix} a & b & c & d \\ r & q & s & p \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ q & r & p & s \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ s & p & q & r \end{matrix}$

Answer: C

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Exercise Numerical

1. Suppose $\alpha, \beta, \gamma and \delta$ are the interior angles of regular pentagon, hexagon, decagon, and dodecagon, respectively, then the value of $|\cos\alpha\sec\beta\cos\gamma\cos ec\delta|$ is ________

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2. Let ABCDEFGHIJKL be a regular dodecagon. Then the value of $\frac{AB}{AF}+\frac{AF}{AB}$ is equal to ____

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3. In a $\Delta ABC, b = 12$ units, c = 5 units and $\Delta = 30$sq. units. If d is the distance between vertex A and incentre of the triangle then the value of d^2 is _____

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4. In ΔABC, if $r = 1, R = 3, \text{ and } s = 5$, then the value of $a^2 + b^2 + c^2$ is _____

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5. Consider a ΔABC in which the sides are $a = (n+1), b = (n+1), c = n$ with $\tan C = 4/3$, then the value of Δ is _____

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6. In $\Delta AEX, T$ is the midpoint of XE and P is the midpoint of ET. If ΔAPE is equilateral of side length equal to unity, then the vaue of $(AX)^2$ is ____

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7. In ΔABC, the incircle touches the sides BC, CA and AB, respectively, at D, E,and F. If the radius of the incircle is 4 units and BD, CE, and AF are consecutive integers, then the value of s, where s is a semi-perimeter of triangle, is _____

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8. The altitudes from the angular points A,B, and C on the opposite sides BC, CA and AB of ΔABC are 210, 195 and 182 respectively. Then the value of a is ___

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9. In ΔABC, If $\angle C = 3\angle A,\ BC = 27,\ \text{ and } \ AB = 48.$ Then the value of AC is ______

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10. The area of a right triangle is 6864 sq. units. If the ratio of its legs is $143 : 24$, then the value of r is _____

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11. In $\Delta ABC, \quad \text{if } \ \cos A + \sin A - \dfrac{2}{\cos B + \sin B} = 0,$ then the value of $\left(\dfrac{a+b}{c}\right)^4$ is

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12. In $\Delta ABC, \angle C = 2\angle A,$ and $AC = 2BC,$ then the value of $\dfrac{a^2 + b^2c^2}{R^2}$ (where R is circumradius of triangle) is ______

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13. In $\Delta ABC,$ if $b(b+c) = a^2$ and $c(c+a) = b^2,$ then $|\cos A. \cos B. \cos C|$ is______

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14. The sides of triangle ABC satisfy the relations $a + b - c = 2$ and $2ab - c^2 = 4$, then the square of the area of triangle is ______

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15. The lengths of the tangents drawn from the vertices A, B and C to the incicle of ΔABC are 5, 3 and 2, respectively. If the lengths of the parts of tangents within the triangle which are drawn parallel to the sides BC, CA and AB of the triangle to the incircle are α, β and $gamm$, respectively, then the value of $[\alpha + \beta + \gamma]$ (where $[.]$ respresents the greatest integer functin) is ____

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16. If a, b and c represent the lengths of sides of a triangle then the possible integeral value of $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$ is ____

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17. In triangle ABC, $\sin A \sin B + \sin B \sin C + \sin C \sin A = 9/4$ and $a = 2$, then the value of $\sqrt{3}\Delta$, where Δ is the area of triangle, is _____

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18. In a ΔABC, $AB = 52$, $BC = 56$, $CA = 60$. Let D be the foot of the altitude from A and E be theintersection of the internal angle bisector of $\angle BAC$ with BC. Find the length DE.

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19. Point D,E are taken on the side BC of an acute angled triangle ABC,, such that $BD = DE = EC$. If $\angle BAD = x$, $\angle DAE = y$ and $\angle EAC = z$ then the value of $\frac{\sin(x+y)\sin(y+z)}{\sin x \sin z}$ is _____

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20. For a triangle ABC, $R = \frac{5}{2}$ and $r = 1$. Let D, E and F be the feet of the perpendiculars from incentre I to BC, CA and AB, respectively. Then the value of $\frac{(IA)(IB)(IC)}{(ID)(IE)(IF)}$ is equal to ____

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21. Circumradius of ΔABC is 3 cm and its area is $6cm^2$. If DEF is the triangle formed by feet of the perpendicular drawn from A,B and C on the sides BC, CA and AB, respectively, then the perimeter of ΔDEF (in cm) is

_____

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22. The distance of incentre of the right-angled triangle ABC (right angled at A) from B and C are $\sqrt{10}$ and $\sqrt{5}$, respectively. The perimeter of the triangle is _____

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Jee Main Previous Year

1. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is There is a regular polygon with $\frac{r}{R}=\frac{1}{\sqrt{2}}$ (17) There is a regular polygon with $\frac{r}{R}=\frac{2}{3}$ (30) There is a regular polygon with $\frac{r}{R}=\frac{\sqrt{3}}{2}$ (47) There is a regular polygon with $\frac{r}{R}=\frac{1}{2}$ (60)

A. There is a regular polygon with $\frac{r}{R}=\frac{\sqrt{3}}{2}$

B. There is a regular polygon with $\frac{r}{R}=\frac{1}{2}$

C. There is a regular polygon with $\frac{r}{R}=\frac{1}{\sqrt{2}}$

D. There is a regular polygon with $\frac{r}{R}=\frac{2}{3}$

Answer: D

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2. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$ if $\angle ADB=\theta, Bc=p$ and $CD=q,$ then AB is equal to

A. $\left(\dfrac{p^2 + q^2 \sin\theta}{p\cos\theta + q\sin\theta}\right.$

B. $\dfrac{\left(p^2 + q^2\right)\cos\theta}{p\cos\theta + q\sin thet}$

C. $\dfrac{p^2 + q^2}{p^2\cos\theta + q^2\sin\theta}$

D. $\dfrac{\left(p^2 + q^2\right)\sin\theta}{(p\cos\theta + q\sin\theta)}$

Answer: A

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Jee Advanced Previous Year

1. Let ABC be a triangle such that $\angle ACB = \dfrac{\pi}{6}$ and let $a, bandc$ denote the lengths of the side opposite to $A, B, andC$ respectively. The value(s) of x for which $a = x^2 + x + 1, b = x^2 - 1, andc = 2x + 1$ is(are) $-\left(2 + \sqrt{3}\right)$ (b) $1 + \sqrt{3}\,2 + \sqrt{3}$ (d) $4\sqrt{3}$

A. $-\left(2 + \sqrt{3}\right)$

B. $1 + \sqrt{3}$

C. $2 + \sqrt{3}$

D. $4\sqrt{3}$

Answer: B

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2. If the angles A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is

A. $\frac{1}{2}$

B. $\frac{\sqrt{3}}{2}$

C. 1

D. $\sqrt{3}$

Answer: D

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3. Let PQR be a triangle of area Δ with $a = 2, b = 7/2$, and $c = 5/2$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R, respectively. Then $\dfrac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals

A. $\dfrac{3}{4\Delta}$

B. $\dfrac{45}{4\Delta}$

C. $\left(\dfrac{3}{4\Delta}\right)^2$

D. $\left(\dfrac{45}{4\Delta}\right)^2$

Answer: C

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4. In a triangle ABC with fixed base BC, the vertex A moves such that

$\cos B + \cos C = 4\sin^2 A/2$

If a, b and c denote the lengths of the sides of the triangle opposite to the angles A,B and C respectively, then

A. $b+c=4a$

B. $b+c=2a$

C. locus of point A is an ellipse

D. locus of point A is a pair of straight lines

Answer: B::C

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5. In a triangle PQR, P is the largest angle and $\cos P = 1/3$. Further the incircle of the triangle touches the sides PQ. QR and PR at N, L and M, respectively, such that the length of PN, QL, and RM are consecutive even integers. Then possible length (s) of the side(s) of the triangle is (are)

A. 16

B. 18

C. 24

D. 22

Answer: B::D

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6. In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z, respectively, and 2s = x + y + z. If $\frac{s-x}{4}=\frac{s-y}{3}=\frac{s-z}{2}$ of incircle of the triangle XYZ is $\frac{8\pi}{3}$

A. area of the triangle XYZ is $6\sqrt{6}$

B. the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

C. $\sin.\frac{X}{2}\sin.\frac{Y}{2}\sin.\frac{Z}{2}=\frac{4}{35}$

D. $\sin^2\left(\frac{X+Y}{2}\right)=\frac{3}{5}$

Answer: A::C::D

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7. In a triangle PQR, let $\angle PQR = 30^\circ$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE ?

A. $\angle QPR = 45^\circ$

B. The area of the triangle PQR is $25\sqrt{3}$ and $\angle QRP = 120^\circ$

C. The radius of the incircle of the triangle PQR is $10\sqrt{3} - 15$

D. The area of the circumcircle of the triangle PQR is 100π

Answer: B::C::D

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8. Match the statements/expressions in List I statements/expression in List II

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9. Let ABC and ABC' be two non-congruent triangles with sides $AB = 4$, $AC = AC' = 2\sqrt{2}$ and angle B $= 30^\circ$. The absolute value of the differnce between the area of these triangle is ______

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10. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chord subtend angles $\frac{\pi}{k}$ and $\frac{\pi}{2k}$ at the center, where $k > 0$, then the value of [k] is ____

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11. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C, respectivelu. Suppose $a = 6$, $b = 10$ and the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtus and if r denotes than radius of the incircle of the triangle, then the value of r^2 is ____

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