# MATHS

## BOOKS - CENGAGE MATHS (HINGLISH)

## RELATIONS AND FUNCTIONS

### Examples

**1.** If sets A = (-3, 2] and B = (-1, 5], then find the following sets

(i) $A \cap B$ (ii) $A \cup B$ (iii) $A - B$ (iv) $B - A$

Watch Video Solution

**2.** Find the value of $x^2$ for the given values of x.

$(i)x < 3(ii)x > -1(iii)x \geq 2(iv)x < -1$

Watch Video Solution

**3.** Find all possible values of the following expressions :

$(i)\sqrt{x^2 - 4}(ii)\sqrt{9 - x^2}(iii)\sqrt{x^2 - 2x + 10}$

Watch Video Solution

**4.** Find the value of $1/x$ for the given values of $x$ $x > 3$ (ii) $x < -2$ (iii) $x \in (-1, 3) - \{0\}$

Watch Video Solution

**5.** Find all the possible values of the following expressions: $\dfrac{1}{x^2 + 2}$ (ii) $\dfrac{1}{x^2 - 2x + 3}$ (iii) $\dfrac{1}{x^2 - x - 1}$

Watch Video Solution

**6.** Find the values of x for which expression $\sqrt{1-\sqrt{1-\sqrt{1-x^2}}}$ is meaningful.

Watch Video Solution

**7.** Solve $x^2 - x - 2 > 0.$

Watch Video Solution

**8.** Solve $x^2 - x - 1 < 0.$

Watch Video Solution

**9.** Solve $(x-1)(x-2)(1-2x) > 0.$

Watch Video Solution

**10.** Solve $\frac{2}{x} < 3$.

Watch Video Solution

**11.** Solve $\frac{x - 2}{x + 2} > \frac{2x - 3}{4x - 1}$.

Watch Video Solution

**12.** Solve $x > \sqrt{1 - x}$

Watch Video Solution

**13.** Solve $x(x + 2)^2(x - 1)^5(2x - 3)(x - 3)^4 \geq 0$.

Watch Video Solution

**14.** Solve $x\left(2^x - 1\right)^{3^x - 9} \wedge 5(x - 3) < 0$.

Watch Video Solution

**15.** Solve $\left(2^{x} - 1\right)\left(3^{x} - 9\right)(\sin x - \cos x)\left(5^{x} - 1\right) < 0, \ -\pi/2 < x < 2\pi.$

Watch Video Solution

**16.** Find the value of $x$ for which following expressions are defined: $\frac{1}{\sqrt{x - |x|}}$ (ii) $\frac{1}{\sqrt{x + |x|}}$

Watch Video Solution

**17.** For $2 < x < 4$ find the values of |x|.

(ii) For $-3 \leq x \leq \ -1$, find the values of |x|.

(iii) For $-3 \leq x < 1$, find the values of |x|

(iv) For $-5 < x < 7$ find the values of |x-2|

(v) For $1 \leq x \leq 5$ find fthe values of |2x -7|

Watch Video Solution

**18.** Solve the following :

(i) $|x-2| = (x-2)$ (ii) $|x+3| = -x-3$

(iii) $\left|x^2-x\right| = x^2-x$ (iv) $\left|x^2-x-2\right| = 2+x-x^2$

Watch Video Solution

**19.** Prove that

$\sqrt{x^2+2x+1}-\sqrt{x^2-2x+1} = \{-2, x < -12x, -1 \leq x \leq 12, x > 1$

Watch Video Solution

**20.** For $x \in R$, find all possible values of $|x-3|-2$ (ii) $4-|2x+3|$

Watch Video Solution

**21.** Find the possible values of $\sqrt{|x|-2}$ (ii) $\sqrt{3-|x-1|}$ (iii) $\sqrt{4-\sqrt{x^2}}$

Watch Video Solution

**22.** Solve $|x - 3| + |x - 2| = 1$.

Watch Video Solution

**23.** Solve: $\dfrac{|x + 3| + x}{x + 2} > 1$

Watch Video Solution

**24.** Solve $|3x - 2| \leq \dfrac{1}{2}$.

Watch Video Solution

**25.** Solve $||x - 1| - 5| \geq 2$.

Watch Video Solution

**26.** Solve: $\dfrac{-1}{|x| - 2} \geq 1.$

Watch Video Solution

**27.** Solve $|x - 1| + |x - 2| \geq 4$

Watch Video Solution

**28.** Solve $|\sin x + \cos x| = |\sin x| + |\cos x|, x \in [0, 2\pi].$

Watch Video Solution

**29.** Solve: $\left|-2x^2 + 1 + e^x + \sin x\right| = \left|2x^2 - 1\right| + e^x + \left|\sin x\right|, x \in [0, 2\pi]$

Watch Video Solution

**30.** Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by $\left\{\left(a\dot{b}\right): a, b \in A, b\right.$is exactly divisible by a}.(i) Write R in roster form(ii) Find the domain of R(iii) Find the range of R.

Watch Video Solution

**31.** If $R = \left\{(x, y): x, y \in W, x^2 + y^2 = 25\right\}$, then find the domain and range or R.

Watch Video Solution

**32.** If $R_1 = \{(x, y) \mid y = 2x + 7,\ \text{where } x \in R \text{ and } -5 \leq x \leq 5\}$ is a relation. Then find the domain and Range of $R_1$.

Watch Video Solution

**33.** Show that the relation R in the set R of real numbers, defined as $R = \left\{(a, b) : a \leq b^2\right\}$ is neither reflexive nor symmetric nor transitive.

Watch Video Solution

**34.** Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b|$ is even $\}$, is an equivalence relation.

Watch Video Solution

**35.** Show that the relation R in the set A of points in a plane given by $R = \{(P, Q) :$ distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set o

Watch Video Solution

**36.** Show that the relation R defined in the set A of all triangles as $R = \left\{\left(T_1, T_2\right): T_1\right.$ is similar to $T_2\}$, is equivalence relation.

Watch Video Solution

**37.** Given a non-empty set X, consider P(X) which is the set of all subsets of X. Define the relation R in P(X) as follows: For subsets A, B in P(X), ARB if and only if A B. Is R an equivalence relation on P(X)? Justify you answer

Watch Video Solution

**38.** Examine each of the following relations given below and state in each case, giving reasons whether it is a function or not ?

(i) $R = \{(4, 1), (5, 1), (6, 7)\}$

(ii) $R = \{(2, 3), (2, 5), (3, 3), (6, 6)\}$

(iii) $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)\}$

(iv) $R = \{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1)\}$

Watch Video Solution

Watch Video Solution

**39.** If A is set of different triangles in the plane and B is set of all positive real numbers. A relation R is defined from set A to set B such that every element of set A is associated with some number in set B which is measure of area of triangle. Is this relation as function?

Watch Video Solution

**40.** A relation R is defined from N to N as $R = \{(ab, a + b) : a, b \in N\}$. Is R a function from N to N ? Justify your answer.

Watch Video Solution

**41.** Set A has m distinct elements and set B has n distinct elements. Then how many different mappings from set A to set B can be formed?

Watch Video Solution

**42.** Write explicit functions of $y$ defined by the following equations and also find the domains of definitions of the given implicit functions: $x + |y| = 2y$ (b) $e^y - e^{-y} = 2x$ $10^x + 10^y = 10$ (d) $x^2 - \sin^{-1}y = \frac{\pi}{2}$

Watch Video Solution

**43.** Find the domain and range of the following functions.

(i) $f(x) = \sqrt{2x - 3}$ (ii) $f(x) = \frac{1}{x - 2}$

(iii) $f(x) = x^2 + 3$ (iv) $f(x) = \frac{1}{x^2 + 2}$

Watch Video Solution

**44.** Find the domain and range function $f(x) = \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$.

Watch Video Solution

**45.** Find the values of x for which the following functions are identical.

(i) $f(x) = x$ and $g(x) = \frac{1}{1/x}$

(ii) $f(x) = \frac{\sqrt{9 - x^2}}{\sqrt{x - 2}}$ and $g(x) = \sqrt{\frac{9 - x^2}{x - 2}}$

Watch Video Solution

**46.** ABCD is a square of side $l$. A line parallel to the diagonal BD at a distance 'x' from the vertex A cuts two adjacent sides. Express the area of the segment of the square with A at a vertex, as a function of x. Find this area at $x = 1/\sqrt{2}$ and at $x = 2$, when $l = 2$.

View Text Solution

**47.** The relation f is defined by $f(x) = \begin{cases} 3x + 2, & 0 \le x \le 2 \\ x^3, & 2 \le x \le 5 \end{cases}$.

The relation g is defined by $g(x) = \begin{cases} 3x + 2, & 0 \le x \le 1 \\ x^3, & 1 \le x \le 5 \end{cases}$

Show that f is a function and g is not a function

Watch Video Solution

**48.** If $f:[-3,4] \to R, f(x) = 2x,$ and $g:[-2,6] \to R, g(x) = x^2$. Then find function $(f+g)(x)$.

Watch Video Solution

**49.** If $f(x) = \begin{cases} x^3, & x < 1 \\ 2x - 1, & x \geq 1 \end{cases}$ and $g(x) = \begin{cases} 3x, & x \leq 2 \\ x^2, & x > 2 \end{cases}$ then find $(f - g)(x)$.

Watch Video Solution

**50.** Check the nature of the following function.

(i) $f(x) = \sin x, x \in R$ (ii) $f(x) = \sin x, x \in N$

Watch Video Solution

**51.** Check the nature of the function $f(x) = x^3 + x + 1, x \in R$ using analytical method and differentiation method.

Watch Video Solution

**52.** Let $f: R \to R$ where, $f(x) = \frac{x^2 + 4x + 7}{x^2 + x + 1}$ Is $f(x)$ one-one?

Watch Video Solution

**53.** Let $f: R \to R$ where $f(x) = \sin x$. Show that $f$ is into. Also find the codomain if $f$ is onto.

Watch Video Solution

**54.** Let $f: N\vec{Z}$ be a function defined as $f(x) = x - 1000$. Show that $f$ is an into function.

Watch Video Solution

**55.** If the function $f : R\vec{A}$ given by $f(x) = \dfrac{x^2}{x^2 + 1}$ is surjection, then find $A$.

Watch Video Solution

**56.** Let $A = \{x : -1 \le x \le 1\} = B$ be a function $f : A \to B$. Then find the nature of each of the following functions.

(i) $f(x) = |x|$ (ii) $f(x) = x|x|$

(iii) $f(x) = x^3$ (iv) $f(x) = \sin\dfrac{\pi x}{2}$

Watch Video Solution

**57.** If $f : R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$, then check the nature of the function.

Watch Video Solution

**58.** If $f:[0, \infty) \rightarrow [0, 1)$, and $f(x) = \dfrac{x}{1 + x}$ then check the nature of the function.

Watch Video Solution

**59.** If the functions $f(x)$ and $g(x)$ are defined on $R \rightarrow R$ such that $f(x) = \{0, x \in$ retional and $x, x \in$ irrational ; $g(x) = \{0, x \in$ irratinal and $x, x \in$ rational then $(f - g)(x)$ is

Watch Video Solution

**60.** Show that $f:R\vec{R}$ defined by $f(x) = (x - 1)(x - 2)(x - 3)$ is surjective but not injective.

Watch Video Solution

**61.** If $f: R \to R$ be a function such that $f(x) = \{x|x| - 4; x \in Q, x|x| - \sqrt{3}; x \notin Q$ then f(x) is

Watch Video Solution

**62.** If $f: R \to R$ be a function such that $f(x) = x^3 + x^2 + 3x + \sin x$, then discuss the nature of the function.

Watch Video Solution

**63.** If function f(x) is defined from set A to B, such that $n(A) = 3$ and $n(B) = 5$. Then find the number of one-one functions and number of onto functions that can be formed.

Watch Video Solution

**64.** Find the range of $f(x) = x^2 - x - 3$.

Watch Video Solution

**65.** Find the domain and range of the following

(i) $f(x) = \sqrt{x^2 - 3x + 2}$ (ii) $f(x) = \sqrt{x^2 - 4x + 6}$

Watch Video Solution

**66.** Find the range of $f(x)\dfrac{x^2 - x + 1}{x^2 + x + 1}$

Watch Video Solution

**67.** Find the complete set of values of $a$ such that $\dfrac{x^2 - x}{1 - ax}$ attains all real values.

Watch Video Solution

**68.** Find the domain of the function $f(x) = \dfrac{1}{1 + 2\sin x}$

Watch Video Solution

**69.** Find domain for $f(x) = \sqrt{\cos(\sin x)}$

Watch Video Solution

**70.** Find the range of $f(x) = \sin^2 x - \sin x + 1.$

Watch Video Solution

**71.** Find the range of $f(x) = \dfrac{1}{2\cos x - 1}$

Watch Video Solution

**72.** Find the value of $x$ for which function are identical.

$f(x) = \cos x \text{ and } g(x) = \dfrac{1}{\sqrt{1 + \tan^2 x}}$

Watch Video Solution

Watch Video Solution

**73.** Find the range of the function $f(x) = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$.

Watch Video Solution

**74.** if:$f(x) = \frac{\sin x}{\sqrt{1 + \tan^2 x}} - \frac{\cos x}{\sqrt{1 + \cot^2 x}}$, then find the range of $f(x)$

Watch Video Solution

**75.** Find the range of $f(x) = |\sin x| + |\cos x|, x \in R$.

Watch Video Solution

**76.** Find the range of $f(\theta) = 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$

Watch Video Solution

**77.** Solve $\sin x > \frac{1}{2}$ or find the domain of $f(x) = \frac{1}{\sqrt{1 + 2\sin x}}$

Watch Video Solution

**78.** Find the number of solutions of $\sin x = \frac{x}{10}$

Watch Video Solution

**79.** Find the number of solutions of the equation $\sin x = x^2 + x + 1$.

Watch Video Solution

**80.** Find the domain and range of the function $f(x) = \sin^{-1}\frac{x^2}{2}$

Watch Video Solution

**81.** Domain of the function f(x)=$\sin^{-1}\left(1+3x+2x^2\right)$

Watch Video Solution

**82.** Find the values of x for which the following pair of functions are identical.

(i) $f(x)=\tan^{-1}x+\cot^{-1}x$ and $g(x)=\sin^{-1}x+\cos^{-1}x$

(ii) $f(x)=\cos\left(\cos^{-1}x\right)$ and $g(x)=\cos^{-1}(\cos x)$

Watch Video Solution

**83.** Find the domain and range of the function $f(x)=\sin^{-1}\left(\left(1+e^x\right)^{-1}\right)$.

Watch Video Solution

**84.** Find the domain for $f(x)=\sin^{-1}\left(\frac{1+x^2}{2x}\right)$

Watch Video Solution

Watch Video Solution

**85.** Find the range of $f(x) = \sin^{-1}x + \tan^{-1}x + \cos^{-1}x$.

Watch Video Solution

**86.** Find the domain of $f(x) = \sqrt{\cos^{-1}x - \sin^{-1}x}$

Watch Video Solution

**87.** Find the range of $\tan^{-1}\left(\frac{2x}{1 + x^2}\right)$

Watch Video Solution

**88.** Find the range of $\cot^{-1}\left(2x - x^2\right)$

Watch Video Solution

**89.** Find the range of $f(x) = \cos^{-1}\left(\frac{\sqrt{1+2x^2}}{1}\right)$

Watch Video Solution

**90.** Find the domain of $f(x) = \sqrt{\left(\frac{1-5^x}{7^{-1}-7}\right)}$

Watch Video Solution

**91.** Find the domain of $f(x) = \sqrt{(0.625)^{4-3x} - (1.6)^{x(x+8)}}$

Watch Video Solution

**92.** Is the pair of the functions $e\sqrt{\log_e x}$ and $\sqrt{x}$ identical ?

Watch Video Solution

**93.** Find the domain and range of following functions

(i) $f(x) = \log_e(\sin x)$

(ii) $f(x) = \log_3\left(5 - 4x - x^2\right)$

Watch Video Solution

**94.** Range of the function : $f(x) = \log_2\left(\dfrac{\pi + 2\sin^{-1}\left(\frac{3-x}{7}\right)}{\pi}\right)$

Watch Video Solution

**95.** Find the domain of $f(x) = (\log)_{10}(\log)_2(\log)_{\frac{2}{\pi}}\left(\tan^{-1}x\right)^{-1}$

Watch Video Solution

**96.** Find the domain and range of $f(x) = \sqrt{(\log)_3\{\cos(\sin x)\}}$

Watch Video Solution

Watch Video Solution

**97.** Find the domain of $f(x) = \sin^{-1}\left\{(\log)_9\left(\frac{x^2}{4}\right)\right\}$

Watch Video Solution

**98.** Find the domain of function $f(x) = (\log)_4\left[(\log)_5\left\{(\log)_3\left(18x - x^2 - 77\right\}\right]\right.$

Watch Video Solution

**99.** Let $x = \in \left(0, \frac{\pi}{2}\right)$. Then find the domain of the function $f(x) = \frac{1}{-(\log)_{\sin x}\tan x}$

Watch Video Solution

**100.** Find the domain of $f(x) = \sqrt{(\log)_{0.4}\left(\frac{x-1}{x+5}\right)}$

Watch Video Solution

**101.** Find the range of $f(x) = \log_e x - \dfrac{(\log_e x)^2}{|\log_e x|}$.

Watch Video Solution

**102.** If $f(x) = \sin(\log)_e \left\{ \dfrac{\sqrt{4 - x^2}}{1 - x} \right\}$, then the domain of $f(x)$ is _____ and its range is __________.

Watch Video Solution

**103.** Find the domain of the function $f(x) = \dfrac{1}{[x]^2 - 7[x] - 8}$, where [.] represents the greatest integer function.

Watch Video Solution

**104.** Find the domain of $f(x) = \sqrt{([x] - 1)} + \sqrt{(4 - [x])}$ (where [ ] represents the greatest integer function).

Watch Video Solution

**105.** Find the domain and range of $f(x) = \sin^{-1}[x] wher[]$ represents the greatest function).

Watch Video Solution

**106.** Solve If $\left[\cos^{-1}x\right] + \left[\cot^{-1}x\right] = 0,$ where [.] denotes the greatest integer function, then the complete set of values of x is.

Watch Video Solution

**107.** Write the piecewise definition of the following functions.

(i) $f(x) = \left[\sqrt{x}\right]$ (ii) $f(x) = \left[\tan^{-1}x\right]$ (iii) $f(x) = \left[\log_e x\right]$

In each case [.] denotes the greatest integer function.

View Text Solution

108. The range of $f(x) = [\sin x \mid [\cos x[\tan x[\sec x]]]], x \in \left(0, \frac{\pi}{4}\right)$, *where* [.] denotes the greatest integer function less than or equal to $x$, is (0,1) (b) - $\{1, 0, 1\}$ $\{1\}$ (d) none of these

Watch Video Solution

109. The range of $f(x) = [1 + \sin x] + \left[2 + s \in \frac{2}{x}\right] + \left[3 + s \in \frac{x}{3}\right] + + \left[n + s \in \frac{x}{n}\right] \forall x \in [0, \pi]$, where [.] denotes the greatest integer function, is,

$\left\{\frac{n + n - 2^2}{2}, \frac{n(n+1)}{2}\right\}$ $\left\{\frac{n(n+1)}{2}\right\}$ $\left\{\frac{n^2 + n - 2^{\square}}{2}, \frac{n(n+1)}{2} \frac{n^2 + n + 2}{2}\right\}$ $\left[\frac{n(n+1)}{2}, \frac{n^2 + n + 2}{2}\right]$

Watch Video Solution

**110.** Solve $x^2 - 4 - [x] = 0$ (where [] denotes the greatest integer function).

Watch Video Solution

**111.** Find the domain and range of $f(f) = \log\{x\}, where\{\}$ represents the fractional part function).

Watch Video Solution

**112.** Find the domain and range of $f(x) = \sin^{-1}(x - [x])$, where [.] represents the greatest integer function.

Watch Video Solution

**113.** Write the function $f(x) = \{\sin x\}$ where {.} denotes the fractional part function) in piecewise definition.

Watch Video Solution

**114.** Solve $2[x] = x + \{x\}, whre[]and\{\}$ denote the greatest integer function and the fractional part function, respectively.

Watch Video Solution

**115.** Find the range of $f(x) = \dfrac{x - [x]}{1 - [x] + x'}, where[]$ represents the greatest integer function.

Watch Video Solution

**116.** The domain of the function $f(x) = \dfrac{1}{\sqrt{\{\sin x\} + \{\sin(\pi + x)\}}}$ where $\{.\}$ denotes the fractional part, is $[0, \pi]$ (b) $(2n + 1)\dfrac{\pi}{2}, n \in Z$ $(0, \pi)$ (d) none of these

Watch Video Solution

**117.** Solve : $[x]^2 = x + 2\{x\}$, where [.] and {.} denote the greatest integer and the fractional part functions, respectively.

Watch Video Solution

**118.** Solve the system of equations in $x, y$ and $z$ satisfying the following equations $x + [y] + \{z\} = 3.1, y + [z] + \{x\} = 4.3$ and $z + [x] + \{y\} = 5.4$

Watch Video Solution

**119.** If $f(x) = [x], 0 \leq \{x\} < 0.5$ and $f(x) = [x] + 1, 0.5 < \{x\} < 1$ then prove that f (x) = -f(-x) (where[.] and{.} represent the greatest integer function and the fractional part function, respectively).

Watch Video Solution

**120.** Verify that $xsgnx = |x|\ |x|sgnx = x\ x(sgnx)(sgnx) = x$

Watch Video Solution

Watch Video Solution

**121.** For the following functions write the piecewise definition and draw the graph

(i) $f(x) = \text{sgn}\left(\log_e x\right)$ (ii) $f(x) = \text{sgn}(\sin x)$

Watch Video Solution

**122.** Find the range of the following

(i) $f(x) = \text{sgn}\left(x^2\right)$ (ii) $f(x) = \text{sgn}\left(x^2 - 2x + 3\right)$

Watch Video Solution

**123.** If $f : \vec{R}\vec{R}$ are two given functions, then prove that

$2m \in$ if $\dot{(x)} - g(x), 0 = f(x) - |g(x) - f(x)|$

Watch Video Solution

**124.** Draw the graph of the function $f(x) = max\dot{\sin x}, \cos 2\dot{x}, x \in [0, 2\pi]$

Write the equivalent definition of $f(x)$ and find the range of the function.

Watch Video Solution

**125.** Which of the following function is (are) even, odd, or neither?

$f(x) = x^2 \sin x$ $\quad f(x) = \sqrt{1 + x + x^2} - \sqrt{1 - x + x^2}$ $\quad f(x) = \log\left(\frac{1 - x}{1 + x}\right)$

$f(x) = \log\left(x + \sqrt{1 + x^2}\right)$ $f(x) = \sin x - \cos x$ $f(x) = \frac{e^x + e^{-x}}{2}$

Watch Video Solution

**126.** If $f(x) = \left(h_1(x) - h_1(-x)\right)\left(h_2(x) - h_2(-x)\right)\left(h_{2n+1}(-x) and f(200) = 0,\right.$

then prove that $f(x)$ is many one function.

Watch Video Solution

**127.** Find whether the given function is even or odd: $f(x) = \begin{cases} x\dfrac{\sin x + \tan x}{\left[\dfrac{x}{\pi}\right] - \dfrac{1}{2}}; \end{cases}$

whether [] denotes the greatest integer function.

Watch Video Solution

**128.** if`f(x)={x^3+x^2,for0lt=xlt=2x+2,for2

Watch Video Solution

**129.** Prove that period of function $f(x) = \sin x, x \in R$ is $2\pi$.

Watch Video Solution

**130.** Verify that the period of function $f(x) = \sin^{10}x$ is $\pi$.

Watch Video Solution

**131.** Prove that function $f(x) = \cos\sqrt{x}$ is non-periodic.

Watch Video Solution

**132.** Find the period of the following functions

(i) $f(x) = |\sin 3x|$

(ii) $f(x) = 2\operatorname{cosec}(5x - 6) + 7$

(iii) $f(x) = x - [x - 2.6]$, where [.] represents the greatest integer function.

Watch Video Solution

**133.** The fundamental period of the function

$f(x) = 4\cos^4\left(\dfrac{x - \pi}{4\pi^2}\right) - 2\cos\left(\dfrac{x - \pi}{2\pi^2}\right)$ is equal to :

Watch Video Solution

**134.** Find the period of the following.

(i) $f(x) = \frac{2^x}{2^{[x]}}$, where [.] represents the greatest integer function.

(ii) $f(x) = e^{\sin x}$

(iii) $f(x) = \sin^{-1}(\sin 3x)$

(iv) $f(x) = \sqrt{\sin x}$

(v) $f(x) = \tan\left(\frac{\pi}{2}[x]\right)$, where [.] represents greatest integer function.

Watch Video Solution

**135.** Period of $f(x) = \sin((\cos x) + x)$ is

Watch Video Solution

**136.** In each of the following cases find the period of the function if it is periodic.

(i) $f(x) = \sin\frac{\pi x}{\sqrt{2}} + \cos\frac{\pi x}{\sqrt{3}}$ (ii) $f(x) = \sin\frac{\pi x}{\sqrt{3}} + \cos\frac{\pi x}{2\sqrt{3}}$

Watch Video Solution

**137.** Find the period of

(i) $f(x) = \sin\pi x + \{x/3\}$, where {.} represents the fractional part.

(ii) $f(x) = |\sin 7x| - \cos^4 \frac{3x}{4} + \tan \frac{2x}{3}$

Watch Video Solution

**138.** Find the period $f(x) = \sin x + \{x\}$, where {x} is the fractional part of $x$

Watch Video Solution

**139.** If $f(x) = \sin x + \cos ax$ is a periodic function, show that $a$ is a rational number

Watch Video Solution

**140.** Find the period of the following function

(i) $f(x) = |\sin x| + |\cos x|$

(ii) $f(x) = \cos(\cos x) + \cos(\sin x)$

(iii) $f(x) = \dfrac{|\sin x + \cos x|}{|\sin x| + |\cos x|}$

Watch Video Solution

**141.** For what integral value of n is $3\pi$ the period of the function $\cos(nx)\sin\left(\dfrac{5x}{n}\right)$?

Watch Video Solution

**142.** Let $f: \{2, 3, 4, 5\} \rightarrow \{3, 4, 5, 9\}$ and $g: \{3, 4, 5, 9\} \rightarrow \{7, 11, 15\}$ be functions defined as $f(2) = 3, f(3) = 4, f(4) = f(5) = 5, g(3) = g(4) = 7,$ and $g(5) = g(9) = 11.$ Find $g$

Watch Video Solution

**143.** Let f(x) and g(x) be bijective functions where $f:\{a, b, c, d\} \rightarrow \{1, 2, 3, 4\}$ and $g:\{3, 4, 5, 6\} \rightarrow \{w, x, y, z\}$, respectively. Then, find the number of elements in the range set of $g(f(x))$.

Watch Video Solution

**144.** Suppose that $g(x) = 1 + \sqrt{x}$ and $f(g(x)) = 3 + 2\sqrt{x} + x$. Then find the function $f(x)$.

Watch Video Solution

**145.** The function $f(x)$ is defined in $[0, 1]$. Find the domain of $f(\tan x)$.

Watch Video Solution

**146.** $f(x) = \{x + 1, x < 0, x^2, x \geq 0$ and $g(x) = \{x^3, x < 1, 2x - 1, x \geq 1$ Then find $f(g(x))$ and find its domain and range.

Watch Video Solution

**147.** If $f(x) = -1 + |x-1|, -1 \leq x \leq 3$ and $g(x) = 2 - |x+1|, -2 \leq x \leq 2,$ then find $fog(x)$ and $gof(x)$.

Watch Video Solution

**148.** Two functions are defined as under : $f(x) = \begin{cases} x+1 & x \leq 1 \\ 2x+1 & 1 < x \leq 2 \end{cases}$ and $g(x) = \begin{cases} x^2 & -1 \leq x \leq 2 \\ x+2 & 2 \leq x \leq 3 \end{cases}$ Find $fog$ and $gof$

Watch Video Solution

**149.** Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be two functions. Then; if gof is onto then g is onto; if gof is one one then f is one-one and if gof is onto and g is one one then f is onto and if gof is one one and f is onto then g is one one.

Watch Video Solution

Watch Video Solution

**150.** Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be two functions. Then; if gof is onto then g is onto; if gof is one one then f is one-one and if gof is onto and g is one one then f is onto and if gof is one one and f is onto then g is one one.

Watch Video Solution

**151.** Which of the following functions has inverse function? $f: Z \vec{d}ef \in edbyf(x) = x + 2$ $f: Z \vec{d}ef \in edbyf(x) = 2x$ $f: Z \vec{d}ef \in edbyf(x) = x$ $f: Z \vec{d}ef \in edbyf(x) = |x|$

Watch Video Solution

**152.** Let $f: R \rightarrow [1, \infty), f(x) = x^2 - 4x + 5$. Then find the largest possible intervals for which $f^{-1}(x)$ is defined and find corresponding $f^{-1}(x)$.

Watch Video Solution

**153.** Let $A = R - \{3\}, B = R - \{1\}$, and let $f : \vec{AB}$ be defined by $f(x) = \dfrac{x - 2}{x - 3}$ is $f$ invertible? Explain.

Watch Video Solution

**154.** Let $f : R \to R$ be defined by $f(x) = e^x - e^{-x}$. Prove that $f(x)$ is invertible. Also find the inverse function.

Watch Video Solution

**155.** Find the inverse of $f(x) = \{x, \; < 1x^2, 1 \le x \le 48\sqrt{x}, x > 4$

Watch Video Solution

**156.** Find the inverse of the function $f : [-1, 1] \to [-1, 1], f(x) = x^2 \times sgn(x)$.

Watch Video Solution

**157.** Find the inverse of the function

$f:\left[-\frac{\pi}{2}-\tan^{-1}\frac{3}{2}, \frac{\pi}{2}-\tan^{-1}\frac{3}{4}\right] \rightarrow [-1, 1],$

$f(x) = 3\cos x + 4\sin x + 7.$

Watch Video Solution

**158.** If $f(x) = 3x - 2 and (gof)^{-1}(x) = x - 2,$ then find the function $g(x)$

Watch Video Solution

**159.** Let $f(x) = x + f(x - 1)$ where $x \varepsilon R$. If $F(0) = 1$ find $f(100)$

Watch Video Solution

**160.** The function f(x) is defined for all real x. If $f(a+b) = f(ab) \forall a$ and $b$ and $f\left(-\frac{1}{2}\right) = -\frac{1}{2}$ then find the value of $f(1005)$.

Watch Video Solution

**161.** Let a function $f(x) satiiex + f(2x) + f(2-x) + f(1+x) = \forall x \in R^{\cdot}$ Then find the value of $f(0)^{\cdot}$

Watch Video Solution

**162.** Let $f$ be a function satisfying of $x^{\cdot}$ Then $f(xy) = \frac{f(x)}{y}$ for all positive real numbers $xandy^{\cdot}$ If $f(30) = 20$, then find the value of $f(40)^{\cdot}$

Watch Video Solution

**163.** If $f(x)$ is a polynomial function satisfying $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(4) = 65, thenf \in df(6)$

Watch Video Solution

**164.** Let $f(x) = \frac{9^x}{9^x + 3}$ . Show $f(x) + f(1 - x) = 1$ and, hence, evaluate. $f\left(\frac{1}{1996}\right) + f\left(\frac{2}{1996}\right) + f\left(\frac{3}{1996}\right) + \quad + f\left(\frac{1995}{1996}\right)$

Watch Video Solution

**165.** Consider a real-valued function f(x) satisfying $2f(xy) = (f(x))^y + (f(y))^x \forall x, y \in Rand(1) = a, wherea \neq 1.$ Prove that $(a - 1)$ $\sum_{i=1}^{n} f(i) = a^{n+1} - a$

Watch Video Solution

**166.** Let $f$ be a real-valued function such that $f(x) + 2f\left(\frac{2002}{x}\right) = 3x$. Then find $f(x)$.

Watch Video Solution

**167.** If $f: R\vec{R}$ is an odd function such that $f(1 + x) = 1 + f(x)$ $x^2 f\left(\frac{1}{x}\right) = f(x), x \neq 0$ then find $f(x)$.

Watch Video Solution

**168.** Let $f: R^+ \vec{R}$ be a function which satisfies $f(x)f(y) = f(xy) + 2\left(\frac{1}{x} + \frac{1}{y} + 1\right)$ for $x, y > 0$. Then find $f(x)$.

Watch Video Solution

**169.** A continuous function $f(x)onR \rightarrow$ satisfies the relation $f(x) + f(2x + y) + 5xy = f(3x - y) + 2x^2 + 1f$ or $\forall x, y \in RThenf \in df(x)$

Watch Video Solution

**170.** If for all real values of $uandv, 2f(u)\cos v = (u + v) + f(u - v)$, prove that for all real values of $x$, $f(x) + f(-x) = 2a\cos x$ $f(\pi - x) + f(-x) = 0$ $f(\pi - x) + f(x) = 2b\sin x$ Deduce that $f(x) = a\cos x + b\sin x, wherea, b$ are arbitrary constants.

Watch Video Solution

**171.** Prove that $f(x)givenbyf(x + y) = f(x) + f(y) \forall x \in R$ is an odd function.

Watch Video Solution

**172.** If $f(x+y)=f(x)f(y)$ for all real $x, y and f(0)\neq 0,$ then prove that the function $g(x)=\dfrac{f(x)}{1+\{f(x)\}^2}$ is an even function.

Watch Video Solution

**173.** Let $f(x)$ be periodic and $k$ be a positive real number such that $f(x+k)+f(x)=0 f$ or $all x \in R$ Prove that $f(x)$ is periodic with period $2k$

Watch Video Solution

**174.** If $f(x)$ satisfies the relation $f(x)+f(x+4)=f(x+2)+f(x+6)$ for all $x$, then prove that $f(x)$ is periodic and find its period.

Watch Video Solution

**175.** An odd function is symmetric about the vertical line $x = a, (a > 0), and$ if $\sum_{r=0}^{\infty}\left[f(1 + 4r]^r = 8,\right.$ then find the value of $f(1)$.

Watch Video Solution

**176.** Check whether the function defined by $f(x + \lambda) = 1 + \sqrt{2f(x) - f^2(x)}$ $\forall x \in R$ is periodic or not. If yes, then find its period $(\lambda > 0)$.

Watch Video Solution

**177.** Draw the graph of $y = \log_e(-x)$, $-\log_e x, y = \left|\log_e x\right|, y = \log_e|x|$ and $y = \left|\log_e|x|\right|$ transforming the graph of $y = \log_e x$.

View Text Solution

**178.** Draw the graph of $y = |||x| - 2| - 3|$ by transforming the graph of $y = |x|$

View Text Solution

View Text Solution

**179.** Consider the function $f(x) = \{2x + 3, x \leq 1$ and $-x^2 + 6, x > 1$ Then draw the graph of the function $y = f(x), y = f(|x|), y = |f(x)|,$ and $y = |f(x)|$.

Watch Video Solution

**180.** Sketch the curve $|y| = (x - 1)(x - 2)$.

Watch Video Solution

**181.** Draw the graph and find the points of discontinuity $f(x) = [2\cos x]$, $x \in [0, 2\pi]$. ([.] represents the greatest integer function.)

Watch Video Solution

**182.** Find the range of $f(x) = \sqrt{\sin(\cos x)} + \sqrt{\cos(\sin x)}$.

Watch Video Solution

Watch Video Solution

**183.** Let $g(x) = \sqrt{x - 2k}, \forall 2k \le x < 2(k + 1)$, where $k \in$ integer. Check whether $g(x)$ is periodic or not.

Watch Video Solution

**184.** Let $f(x) = x^2 - 2x, x \in R, and g(x) = f(f(x) - 1) + f(5 - (x))^{\cdot}$ Show that $g(w) \ge o \forall x \in R^{\cdot}$

Watch Video Solution

**185.** If $fandg$ are two distinct linear functions defined on $R$ such that they map $\{ - 1, 1]$ onto $[0, 2]$ and $h : R - \{ - 1, 0, 1\}\vec{R}$ defined by $h(x) = \dfrac{f(x)}{g(x)}$, then show that $\left|h(h(x)) + h\left(h\left(\dfrac{1}{x}\right)\right)\right| > 2.$

Watch Video Solution

**186.** Let $f(x) = (\log)_2(\log)_3(\log)_4(\log)_5\left(s \in x + a^2\right)^{.}$ Find the set of values of $a$ for which the domain of $f(x) is R^{.}$

Watch Video Solution

**187.** If $f$ is polynomial function satisfying $2 + f(x)f(y) = f(x) + f(y) + f(xy) \forall x, y \in R$ and if $f(2) = 5,$ then find the value of $f(f(2))^{.}$

Watch Video Solution

**188.** Let $f: X \rightarrow Y$ be a function defined by f(x) = a sin ( x + $\frac{\pi}{4}$) + c. If f is both one-one and onto, then find the set X and Y

Watch Video Solution

**189.** Let $f: R\vec{R}, f(x) = \dfrac{x - a}{(x - b)(x - c)}, b > \cdot$ If $f$ is onto, then prove that $a \in (b, c)^{\cdot}$

Watch Video Solution

**190.** If p, q are positive integers, f is a function defined for positive numbers and attains only positive values such that $f(xf(y)) = x^p y^q$, then prove that $p^2 = q$.

Watch Video Solution

**191.** If $f: R\vec{0}, \infty isafunctionsuchthatf(x - 1) + f(x + 1) = \sqrt{3}f(x)$, then prove that $f(x)$ is periodic and find its period.

Watch Video Solution

**192.** If $a, b$ are two fixed positive integers such that $f(a + x) = b + \left[b^3 + 1 - 3b^2 f(x) + 3b\{f(x)\}^2 - \{f(x)\}^3\right]^{\frac{1}{3}}$ for all real $x$, then prove that $f(x)$ is periodic and find its period.

Watch Video Solution

**193.** Let f(x, y) be a periodic function satisfying $f(x, y) = f(2x + 2y, 2y - 2x)$ for all x, y; Define $g(x) = f\left(2^x, 0\right)$. Show that g(x) is a periodic function with period 12.

Watch Video Solution

**194.** Consider the function $f(x) = \begin{cases} x - [x] - \frac{1}{2} & x \notin \\ 0 & x \in I \end{cases}$ where [.] denotes the fractional integral function and I is the set of integers. Then find $g(x) \max . \left[x^2, f(x), |x|\right\}, \ -2 \leq x \leq 2.$

Watch Video Solution

**195.** Let $f(x)$ be defined on $[-2, 2]$ and be given by

$$f(x) = \begin{cases} -1, & -2 \le x \le 0 \\ x-1, & 0 < x \le 2 \end{cases} \text{ and } g(x) = f(|x|) + |f(x)|.$$

Then find $g(x)$.

Watch Video Solution

**196.** If $f(x) = \dfrac{a^x}{a^x + \sqrt{a,}}, (a > 0),$ then find the value of $\sum_{r=1}^{2n1} 2f\left(\dfrac{r}{2n}\right)$

Watch Video Solution

## Exercise 1.1

**1.** If sets $A = [-4, 1]$ and $B = [0, 3),$ then find the following sets:

(a) $A \cap B$ (b) $A \cup B$ (c) $A - B$

(d) $B - A$ (e) $(A \cup B)'$ (f) $(A \cap B)'$

Watch Video Solution

Watch Video Solution

2. Find the value of $x^2$ for the following values of x:

(a) $[-5, -1]$ (b) $(3, 6)$ (c) $(-2, 3]$

Watch Video Solution

3. Find the values of 1/x for the following values fo x:

(a) $(2, 5)$ (b) $[-5, -1)$ (c) $(3, \infty)$

(d) $(-\infty, -2]$ (e) $[-3, 4]$

Watch Video Solution

4. Find all possible values ( range) of the following quadratic expressions when $x \in R$ and when $x \in [-3, 2]$

(a) $4x^2 + 28x + 41$

(b) $1 + 6x - x^2$

Watch Video Solution

**5.** Find all possible values of expressions $\frac{2+x^2}{4-x^2}$

Watch Video Solution

**6.** Solve $\frac{x(3-4x)(x+1)}{2x-5} < 0$

Watch Video Solution

**7.** Solve $\frac{(2x+3)(4-3x)^3(x-4)}{(x-2)^2x^5} \leq 0$

Watch Video Solution

**8.** Solve $\frac{5x+1}{(x+1)^2}$

Watch Video Solution

**9.** Find the number of integal values of x satisfying

$\sqrt{-x^2 + 10x - 16} < x - 2$

Watch Video Solution

**10.** Find all the possible values of $f(x) = \frac{1 - x^2}{x^2 + 3}$

Watch Video Solution

**11.** Find the values of x for which the following function is defined:

$f(x) = \sqrt{\frac{1}{|x - 2| - (x - 2)}}$

Watch Video Solution

**12.** Solve |4-|x-1||=3

Watch Video Solution

**13.** Find all values of f(x) for which f(x) $= x + \sqrt{x^2}$

Watch Video Solution

**14.** Solve the following :

(a) $1 \leq |x - 2| \leq 3$ (b) $0 < |x - 3| \leq 5$

(c ) $|x - 2| + |2x - 3| = |x - 1|$ (d) $\left|\dfrac{x - 3}{x + 1}\right| \leq 1$

Watch Video Solution

**15.** Find all possible values of expression $\sqrt{1 - \sqrt{x^2 - 6x + 9}}$.

Watch Video Solution

## Exercise 1.2

**1.** (a) If $n(A) = 6$ and $n(A \times B) = 42$ then find $n(B)$

(b) If some of the elements of $A \times B$ are $(x, p), (p, q), (r, s)$. Then find the minimum value of $n(A \times B)$.

Watch Video Solution

**2.** Let $A = \left\{1, 2, 3, \ddot{} 14\right\}$ . Define a relation on a set A by $R = \{(x, y) : 3x - y = 0. \text{ where } x, y \in A\}$ . Depict this relationship using an arrow diagram. Write down its domain, co-domain and range.

Watch Video Solution

**3.** Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing (1, 2) is (A) 1 (B) 2 (C) 3 (D) 4

Watch Video Solution

**4.** Let a relation $R_1$ on the set R of real numbers be defined as $(a, b) \in R_{11} + ab > 0$ for all $a, b \in R$ Show that $R_1$ is reflexive and symmetric but not transitive.

Watch Video Solution

**5.** Let $Z$ be the set of all integers and $R$ be the relation on $Z$ defined as $R = \{(a, b); a, \ b \ \in Z,$ and $(a - b)$ is divisible by 5. $\}$ . Prove that $R$ is an equivalence relation.

Watch Video Solution

## Exercise 1.3

**1.** Find the domain of the following functions

(a) $f(x) = \dfrac{1}{\sqrt{x - 2}}$ (b) $f(x) = \dfrac{1}{x^3 - x}$

(c ) $f(x) = \sqrt[3]{x^2 - 2}$

Watch Video Solution

**2.** Find the range of the following functions.

(a) $f(x) = 5 - 7x$ (b) $f(x) = 5 - x^2$

(c ) $f(x) = \dfrac{x^2}{x^2 + 1}$

Watch Video Solution

**3.** Find the domain and range of $f(x) = \dfrac{2 - 5x}{3x - 4}$.

Watch Video Solution

**4.** Find the domain and range of $f(x) = \sqrt{4 - 16x^2}$.

Watch Video Solution

**5.** Find the range fo the function $f(x) = \dfrac{x^2 - x - 6}{x - 3}$

Watch Video Solution

6. If the relation $f(x) = \begin{Bmatrix} 2x - 3, & x \le 2 \\ x^3 - a, & x \ge 2 \end{Bmatrix}$ is a function, then find the value of $a$.

A. 5

B. 7

C. 6

D. 8

**Answer: B**

Watch Video Solution

7. If the relation $f(x) = \begin{cases} 1, & x \in Q \\ 2, & x \notin Q \end{cases}$ where Q is set of rational numbers, then find the value $f(\pi) + f\left(\frac{22}{7}\right)$.

A. 1

B. 2

C. 3

D. 4

**Answer: C**

Watch Video Solution

**8.** Let $f(x) = \begin{cases} x^2 - 4x + 3, & x < 3 \\ x - 4, & x \geq 3 \end{cases}$ and

$g(x) = \begin{cases} x - 3, & x < 4 \\ x^2 + 2x + 2, & x \geq 4 \end{cases}.$

Describe the function $f/g$ and find its domain.

Watch Video Solution

**9.** Which of the following functions is/are identical to $|x - 2|$ ?

A. $f(x) = \sqrt{x^2 - 4x + 4}$

B. $g(x) = |x| - |2|$

C. $h(x) = \dfrac{|x - 2|^2}{|x - 2|}$

D. $t(x) = \left|\dfrac{x^2 - x - 2}{x + 1}\right|$

**Answer: A**

Watch Video Solution

## Exercise 1.4

**1.** Which of the following function from Z to itself are bijections?

A. $f(x) = x^3$

B. $f(x) = x + 2$

C. $f(x) = 2x + 1$

D. $f(x) = x^2 + x$

**Answer: B**

Watch Video Solution

**2.** If $f: N\vec{Z}f(n) = \begin{cases} \frac{n-1}{2}, whe \cap isodd - \frac{n}{2}, ident \quad \text{if} \quad ythewhe \cap iseven \end{cases}$

Watch Video Solution

**3.** If $f: R\vec{R}$ is given by $f(x) = \frac{x^2 - 4}{x^2 + 1}$, identify the type of function.

A. many-one & into

B. many-one & onto

C. one-one & into

D. one-one & onto

**Answer: A**

Watch Video Solution

**4.** If $f : R \rightarrow S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$ then find the set $S$.

A. $(-1, 3)$

B. $[-1, 3)$

C. $(-1, 3\}$

D. $[-1, 3]$

**Answer: D**

Watch Video Solution

**5.** Let $g : R0, \overrightarrow{\frac{\pi}{3}}$ be defined by $g(x) = \cos^{-1}\left(\frac{x^2 - k}{1 + x^2}\right)$. Then find the possible values of $k$ for which $g$ is a subjective function.

Watch Video Solution

**6.** Identify the type of the function $f : R \rightarrow R$,

$f(x) = e^{x^2} + \cos x$.

Watch Video Solution

**7.** Let a function $f : R \rightarrow R$ be defined by $f(x) = 2x + \cos x + \sin x$ for $x \in R$.

Then find the nature of $f(x)$.

Watch Video Solution

**8.** If $f : R \rightarrow R$ given by $f(x) = x^3 + px^2 + qx + r$, is then find the condition for which $f(x)$ is one-one.

Watch Video Solution

## Exercise 1.5

**1.** The entire graph of the equation $y = x^2 + kx - x + 9$ in strictly above the $x - a\xi s$ if and only if $k < 7$ (b) `-5-5` (d) none of these

Watch Video Solution

**2.** Find the range of $f(x) = \dfrac{x^2 + 34x - 71}{x^3 + 2x - 7}$

Watch Video Solution

**3.** Find the range of $f(x)\sqrt{x - 1} + \sqrt{5 - 1}$

Watch Video Solution

**4.** If $f(x) = \sqrt{x^2 + ax + 4}$ is defined for all $x,$ then find the values of $a$

Watch Video Solution

**5.** Find the domain and range of $f(x) = \sqrt{3 - 2x - x^2}$

Watch Video Solution

## Exercise 1.6

**1.** Find the domain of $f(x) = \sqrt{\sin x} + \sqrt{16 - x^2}$

Watch Video Solution

**2.** Solve (a) $\tan x < 2 \cos x \leq -\frac{1}{2}$

Watch Video Solution

**3.** Prove that the least positive value of $x$, satisfying $\tan x = x + 1, liesinthe \in terval\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

Watch Video Solution

Watch Video Solution

**4.** Find the range of $f(x) = \sec\left(\frac{\pi}{4}\cos^2 x\right)$, where $x \in R$

A. $[1, 2]$

B. $\left[1, \sqrt{2}\right]$

C. $\left[1, \frac{2}{\sqrt{3}}\right]$

D. $\left[-\sqrt{2}, 1\right]$

**Answer: B**

Watch Video Solution

**5.** If $x \in [1, 2]$, then find the range of $f(x) = \tan x$

Watch Video Solution

**6.** Find the range of $f(x) = \dfrac{1}{1 - 3\sqrt{1 - \sin^2 x}}$

Watch Video Solution

**7.** Find the range of $f(x) = \dfrac{2\sin^2 x + 2\sin x + 3}{\sin^2 x + \sin x + 1}$

Watch Video Solution

**8.** Draw the graph of $y = (\sin 2x)\sqrt{1 + \tan^2 x}$, find its domain and range.

Watch Video Solution

## Exercise 1.7

**1.** Find the domain of the following following functions:

(a) $f(x) = \dfrac{\sin^{-1}}{x}$

(b) $f(x) = \sin^{-1}(|x - 1| - 2)$

(c ) $f(x) = \cos^{-1}\left(1 + 3x + 2x^2\right)$

(d ) $f(x) = \dfrac{\sin^{-1}(x - 3)}{\sqrt{9 - x^2}}$

(e ) $f(x) = \cos^{-1}\left(\dfrac{6 - 3x}{4}\right) + \operatorname{cosec}^{-1}\left(\dfrac{x - 1}{2}\right)$

(f) $f(x) = \sqrt{\sec^{-1}\left(\dfrac{2 - |x|}{4}\right)}$

View Text Solution

**2.** Find the range of $f(x) = \tan^{-1}\sqrt{\left(x^2 - 2x + 2\right)}$

Watch Video Solution

**3.** Find the range of the function $f(x) = \cot^{-1}(\log)_{0.5}\left(x^4 - 2x^2 + 3\right)$

Watch Video Solution

**4.** The domain of definition of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real-valued $x$ is $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (c) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ (d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

Watch Video Solution

**5.** Find the domain and range of

$f(x) = \sin^{-1}x + \tan^{-1}x + \sec^{-1}x.$

Watch Video Solution

## Exercise 1.8

**1.** Find the domain of the function :

$f(x) = \sqrt{4^x + 8\left(\frac{2}{3}\right)^{(2x-2)} - 13 - 2^{2(x-1)}}$

Watch Video Solution

**2.** Find the domain of the function : $f(x) = \sin^{-1}\left((\log)_2 x\right)$

Watch Video Solution

**3.** Find the domain of the function : $f(x) = (\log)_{(x-4)}\left(x^2 - 11x + 24\right)$

Watch Video Solution

**4.** Find the domain of the function : $f(x) = \dfrac{3}{4 - x^2} + (\log)_{10}\left(x^3 - x\right)$

Watch Video Solution

**5.** Find the domain $f(x) = \sqrt{\dfrac{\log_{0.3}|x - 2|}{|x|}}$.

Watch Video Solution

**6.** Find the domain of the following functions :

$$f(x) = \sqrt{\log_{10}\left(\frac{\log_{10}x}{2\left(3 - \log_{10}x\right)}\right)}$$

Watch Video Solution

**7.** Find the domain of the function : $f(x) = \frac{1}{\sqrt{(\log)_{\frac{1}{2}}\left(x^2 - 7x + 13\right)}}$

Watch Video Solution

**8.** Find the range of $f(x) = (\log)_2\left(\frac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}}\right)$

Watch Video Solution

**9.** Find the value of $x$ in $[-\pi, \pi]$ for which

$f(x) = \sqrt{(\log)_2\left(4\sin^2 x - 2\sqrt{3}\sin x - 2\sin x + \sqrt{3} + 1\right)}$ is defined.

Watch Video Solution

## Exercise 1.9

**1.** Solve $[x]^2 - 5[x] + 6 = 0$.

Watch Video Solution

**2.** If $y = 3[x] + 1 = 4[x - 1] - 10$, then find the value of $[x + 2y]$.

Watch Video Solution

**3.** Find the domain of $f(x) = \dfrac{1}{\sqrt{x - [x]}}$ (b) $f(x) = \dfrac{1}{\log[x]}$ $f(x) = \log\{x\}$

Watch Video Solution

**4.** Find the domain of $f(x) = \dfrac{1}{\sqrt{|[|x| - 1]| - 5}}$

Watch Video Solution

**5.** Find the domain of $f(x)\dfrac{\sqrt{(1 - \sin x)}}{(\log)_5(1 - 4)^2} + \cos^{-1}(1 - \{x\})$.

Watch Video Solution

**6.** Find the range of $f(x) = \cos\left((\log)_e\{x\}\right)$.

Watch Video Solution

**7.** Find the domain and range of $f(x) = \cos^{-1}\sqrt{(\log)_{[x]}\left(\frac{|x|}{x}\right)}$

Watch Video Solution

**8.** Find the range of $f(x) = (\log)_{[x-1]}\sin x$

Watch Video Solution

**9.** Solve $(x - 2)[x] = \{x\} - 1.$

Watch Video Solution

**10.** Find the domain of the function $f(x) = \dfrac{1}{8\{x\}^2 - 6\{x\} + 1}$.

Watch Video Solution

**11.** Write the equivalent definition and draw the graphs of the following functions.

(a) $f(x) = sgn\left(\log_e|x|\right)$

(b) $f(x) = sgn\left(x^3 - x\right)$

Watch Video Solution

**12.** Consider the function: $f(x) = max1, |x - 1|, \text{ min } \{4, |3x - 1|\}\} \forall x \in R$ .

Then find the value of $f(3)$

Watch Video Solution

**13.** Find the equivalent definition of

$f(x) = maxx^2, ( - x)^2, 2x(1 - x)whre0 \leq x \leq 1$

Watch Video Solution

## Exercise 1.10

**1.** Identify the type of the functions: $f(x) = \{g(x) - g( - x)\}^3$

A. Odd

B. Even

C. Neither

D. Both

**Answer: A**

Watch Video Solution

**2.** Identify the type of the functions: $f(x) = \log\left(\frac{x^4 + x^2 + 1}{x^2 + x + 1}\right)$

A. Odd

B. Even

C. Neither

D. Both

**Answer: B**

Watch Video Solution

**3.** Identify the following functions : $f(x) = xg(x)g(-x) + \tan(\sin x)$

A. Odd

B. Even

C. Neither

D. Both

**Answer: A**

Watch Video Solution

**4.** Identify the following functions: $f(x) = \cos[x] + \left[\frac{\sin x}{2}\right]$ where [.] denotes the greatest integer function.

A. Odd

B. Even

C. Neither

D. Both

**Answer: C**

Watch Video Solution

**5.** Identify the following functions: `f(x)={x|x|,xlt=-1[x+1]+[1-x],-x|x|xlt=1-1

Watch Video Solution

**6.** Let the function $f(x) = x^2 + x + s \in x - \cos x + \log(1 + |x|)$ be defined on the interval $[0, 1]$ .Define functions $g(x) and h(x) \in [-1, 0]$ satisfying $g(-x) = -f(x) and h(-x) = f(x) \forall x \in [0, 1]$

Watch Video Solution

## Exercise 1.11

**1.** Which of the following functions is not periodic?

(a) $|\sin 3x| + \sin^2 x$ (b) $\cos\sqrt{x} + \cos^2 x$

(c ) $\cos 4x + \tan^2 x$ (d ) $\cos 2x + \sin x$

Watch Video Solution

**2.** Which of the following function/functions is/are periodic ?

(a) $sgn\left(e^{-x}\right)$ (b) $\sin x + |\sin x|$

(c) $\min(\sin x, |x|)$ (d) $\frac{x}{x}$

Watch Video Solution

**3.** Find the period of

(a) $\frac{|\sin 4x| + |\cos 4x|}{|\sin 4x - \cos 4x| + |\sin 4x + \cos 4x|}$

(b) $f(x) = \sin\frac{\pi x}{n!} - \cos\frac{\pi x}{(n+1)!}$

(c) $f(x) = \sin x + \tan\frac{x}{2} + \sin\frac{x}{2^2} + \tan\frac{x}{2^3} + \dots + \sin\frac{x}{2^{n-1}} + \tan\frac{x}{2^n}$

View Text Solution

**4.** Match the column

| Column I (Function) | Column II (Period) |
|---|---|
| p. $f(x) = \sin^3 x + \cos^4 x$ | a. $\pi/2$ |
| q. $f(x) = \cos^4 x + \sin^4 x$ | b. $\pi$ |
| r. $f(x) = \sin^3 x + \cos^3 x$ | c. $2\pi$ |
| s. $f(x) = \cos^4 x - \sin^4 x$ | |

View Text Solution

**5.** Let $[x]$ denotes the greatest integer less than or equal to $x$ . If the function $f(x) = \tan\left(\sqrt{[n]}x\right)$ has period $\frac{\pi}{3}$ then find the value of $n$

Watch Video Solution

**6.** If $f(x) = \lambda|\sin x| + \lambda^2|\cos x| + g(\lambda)$ has a period $= \frac{\pi}{2}$ then find the value of $\lambda$

Watch Video Solution

**7.** Find the fundamental period of $f(x) = \cos x \cos 2x \cos 3x$.

Watch Video Solution

## Exercise 1.12

**1.** Let $f(x) = 1 + |x|, x < -1 [x], x \geq -1, where[\cdot]$ denotes the greatest integer function.Then $f\{f(-2.3)\}$ is equal to

Watch Video Solution

**2.** If $f(x) = \log\left[\dfrac{1+x}{1-x}\right]$, then prove that $f\left[\dfrac{2x}{1+x^2}\right] = 2f(x)$.

Watch Video Solution

**3.** Let $f(x) = \dfrac{\alpha x}{x+1}$ Then the value of $\alpha$ for which $f(f(x) = x$ is

Watch Video Solution

4. If the domain of $y = f(x) is[ - 3, 2]$, then find the domain of $g(x) = f(|[x]|), wher[]$ denotes the greatest integer function.

Watch Video Solution

5. Let $f$ be a function defined on [0,2]. The prove that the domain of function $g(x) i9x^2 - 1$.

Watch Video Solution

6. A function $f$ has domain $[ - 1, 2]$ and rang $[0, 1]$. Find the domain and range of the function $g$ defined by $g(x) = 1 - f(x + 1)$.

Watch Video Solution

**7.** Let $f(x) = \tan x and g(f(x)) = f\left(x - \frac{\pi}{4}\right)$, where $f(x) and g(x)$ are real valued functions. Prove that $f(g)(x)) = \text{t }an\left(\frac{x+1}{x+1}\right)$.

Watch Video Solution

**8.** Let $g(x) = 1 = x - [x]$ and $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$ then for all $x$, $f(g(x))$ is equal to

(i) $x$

(ii) $1$

(iii) $f(x)$

(iv) $g(x)$

Watch Video Solution

**9.** $f(x) = \begin{cases} \log_e x, & 0 < x < 1 \\ x^2 - 1, & x \geq 1 \end{cases}$ and $g(x) = \begin{cases} x + 1, & x < 2 \\ x^2 - 1, & x \geq 2 \end{cases}$.

Then find $g(f(x))$.

Watch Video Solution

## Exercise 1.13

**1.** The inverse of the function $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} + 2$ is given by

Watch Video Solution

**2.** Find the inverse of the function: $f: R \vec{-\infty, 1} givenby f(x) = 1 - 2^{-x}$

Watch Video Solution

**3.** Find the inverse of the function: $f:(2, 3) \rightarrow (0, 1)$ defined by $f(x) = x - [x]$, where[.] represents the greatest integer function

Watch Video Solution

**4.** Find the inverse of the function: $f: Z \rightarrow Z$ defined by $f(x) = [x + 1]$, where [.] denotes the greatest integer function.

Watch Video Solution

**5.** Find the inverse of the function: $f(x) = \{x^3 - 1, , x < 2x^2 + 3, x \geq 2$

Watch Video Solution

**6.** Find the inverse of the function: $f:[ - 1, \overrightarrow{]-1, 1} def \in edbyf(x) = x|x|$

Watch Video Solution

**7.** Find the inverse of the function: $f:(-\infty, 1]\overrightarrow{\frac{1}{2}}, \infty, where f(x) = 2^{x(x-2)}$

Watch Video Solution

## Exercise 1.14

**1.** If $f(x+y+1) = \left\{\sqrt{f(x)} + \sqrt{f(y)}\right\}^2$ and $f(0) = 1 \forall x, y \in R, determ \in ef(n), n \in \dot{N}$

Watch Video Solution

**2.** Let $g(x)$ be a function such that $g(a+b) = g(a)\dot{g}(b) \forall a, b \in \dot{R}$ If zero is not an element in the range of $g$, then find the value of $g(x)g(\dot{-}\dot{x})$

Watch Video Solution

**3.** If $f(x+2a)=f(x-2a), thenprovethatf(x)isperiodi \cdot$

Watch Video Solution

**4.** If $f(x+f(y))=f(x)+y\,\forall x, y \in Randf(0)=1,$ then find the value of $f(7)^{\cdot}$

Watch Video Solution

**5.** If $f: R^{+}\vec{R}, f(x)+3xf\left(\frac{1}{x}\right)=2(x+1), thenf \in df(x)^{\cdot}$

Watch Video Solution

**6.** $f: R\vec{R}, f\left(x^{2}=x+3\right)+2f\left(x^{2}-3x+5\right)=6x^{2}-10x+17\,\forall x \in R,$ then find the function $f(x)^{\cdot}$

Watch Video Solution

7. Consider $f: R^{+} \vec{R} such that f(3) = 1$ for $a \in R^{+} and f(x) f(y) + f\left(\frac{3}{x}\right) f\left(\frac{3}{y}\right) = 2f(xy) \forall x, y \in R^{+}$. Then find $f(x)$.

Watch Video Solution

8. Determine all functions $f: R \vec{s} uchthat f(x - f(y)) = f(f(y)) + xf(y) + f(x) - 1 \forall x, y \geq \in R$.

Watch Video Solution

9. Determine the function satisfying $f^2(x + y) = f^2(x) + f^2(y) \forall x, y \in R$.

Watch Video Solution

10. If $f: R\vec{R}$ is a function satisfying the property $f(2x + 3) + f(2x + 7) = 2 \forall x \in R$, then find the fundamental period of $f(x)$.

Watch Video Solution

Watch Video Solution

11. If $f(x)$ is an even function and satisfies the relation $x^2 f(x) - 2f\left(\frac{1}{x}\right) = g(x), where g(x)$ is an odd function, then find the value of $f(5)$

Watch Video Solution

12. If $f(a - x) = f(a + x)$ and $f(b - x) = f(b + x)$ for all real x, where $a, b(a > b > 0)$ are constants, then prove that $f(x)$ is a periodic function.

Watch Video Solution

13. A real-valued functin $f(x)$ satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$, where a given constant and $f(0) = 1$. Then prove that $f(x)$ is symmetrical about point (a, 0).

Watch Video Solution

## Exercise 1.15

**1.** Draw the graph of $y = \sin|x|$.

Watch Video Solution

**2.** Draw the graph of the function: $|f(x)| = \tan x$

Watch Video Solution

**3.** Draw the graph of the function: $f(x) = \left|x^2 - 3\left|x\right| + 2\right|$

Watch Video Solution

**4.** Draw the graph of the function: $f(x) = -|x - 1|^{\frac{1}{2}}$

Watch Video Solution

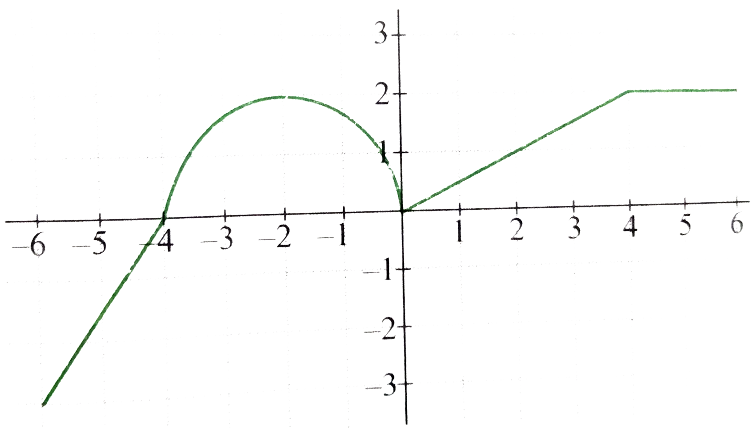
**5.** Find the total number of solutions of $\sin\pi x = | \in |x| |$

Watch Video Solution

**6.** Draw the graph of the function: Solve $\left|\frac{x^2}{x - 1}\right| \leq 1$ using the graphical method.

Watch Video Solution

**7.** Given the graph of $f(x)$, draw the graph each one of the following functions :



(a) $y = f(x) + 3$ (b) $y = -f(x) + 2$

(c ) $y = f(x + 1) - 2$ (d) $y = -f(x - 1)$

(e ) $y = f(-x)$ (f) $y = f(|x|)$

(g) $y = f(1 - x)$

View Text Solution

**8.** Draw the graph and find the points of discontinuity for $f(x) = \left[x^2 - x - 1\right], x \in [-1, 2]$ ([.] represents the greatest integer function).

Watch Video Solution

## Exercise (Single)

**1.** The relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on a set A={1, 2, 3} is

A. Reflexive but not symmetric

B. Reflexive but not transitive

C. Symmetric and transitive

D. Neither symmetric nor transitive

**Answer: A**

Watch Video Solution

**2.** Let $P = \{(x, y) \mid x^2 + y^2 = 1, x, y \in R\}$. Then, R, is

A. Reflexive

B. Symmetric

C. Transitive

D. Anti-symmetric

**Answer: B**

Watch Video Solution

**3.** Let R be an equivalence relation on a finite set A having n elements. Then the number of ordered pairs in R is

A. Less than n

B. Greater than or equal to n

C. Less than or equal to n

D. None of these

**Answer: B**

Watch Video Solution

**4.** A relation R on the set of complex numbers is defined by $z_1Rz_2$ if and oly if $\frac{z_1 - z_2}{z_1 + z_2}$ is real Show that R is an equivalence relation.

A. R is reflexive

B. R is symmetric

C. R is transitive

D. R is not equivalence

**Answer: D**

Watch Video Solution

**5.** Which one of the following relations on R is an equivalence relation?

A. $aR_1b \Leftrightarrow |a| = |b|$

B. $aR_2b \Leftrightarrow a \geq b$

C. $aR_3b \Leftrightarrow a \text{ divides } b$

D. $aR_4b \Leftrightarrow a < b$

**Answer: A**

Watch Video Solution

**6.** Let $R$ be the relation on the set R of all real numbers defined by a $Rb$ Iff $|a - b| \leq 1.$ Then $R$ is

A. Reflexive and symmetric

B. Symmetric only

C. Transitive only

D. None of these

**Answer: A**

Watch Video Solution

**7.** The function $f: N\vec{N}(N$ is the set of natural numbers) defined by $f(n) = 2n + 3is$ (a) surjective only (b) injective only (c) bijective (d) none of these

A. surjective only

B. injective only

C. bijective

D. none of these

**Answer: B**

Watch Video Solution

**8.** $f: N \rightarrow N$, where $f(x) = x - (-1)^x$, Then $f$ is

A. one-one and into

B. many-one and into

C. one-one and onto

D. many-one and onto

**Answer: C**

Watch Video Solution

**9.** If S be the set of all triangles and $f: S \rightarrow R^{+}, f(\Delta) =$ Area of $\Delta$, then f is-

A. injective but not surjective

B. surjective but not injective

C. injective as well as surjective

D. neither injective nor surjective

**Answer: B**

Watch Video Solution

**10.** The function $f: ( - \infty, \ - 1)\overrightarrow{0, e^{5}}$ defined by $f(x) = e^{x} \wedge (3 - 3x + 2)$ is many one and onto many one and into one-one and onto one-one and into

A. many-one and onto

B. many-one and into

C. one-one and onto

D. one-one and into

**Answer: D**

Watch Video Solution

**11.** Let $f: N \rightarrow N$ be defined by $f(x) = x^2 + x + 1, x \in N$. Then is $f$ is

A. one-one and onto

B. many-one onto

C. one-one but not onto

D. none of these

**Answer: C**

Watch Video Solution

**12.** Let $X = \{a_1, a_2, , a_6\} and Y = \{b_1, b_2, b_3\}^{\cdot}$ The number of functions $f$ from $x \to y$ such that it is onto and there are exactly three elements $x \in X$ such that $f(x) = b_1$ is

A. 75

B. 90

C. 100

D. 120

**Answer: D**

**Watch Video Solution**

**13.** Which of the following functions is an injective (one-one) function in its respective domain? (A) $f(x) = 2x + \sin 3x$ (B) $x.[x]$ , (where $[.]$ denotes the G.I.F) (C) $f(x) = \dfrac{2^x - 1}{4^x + 1}$ (D) $f(x) = \dfrac{2^x + 1}{4^x - 1}$

A. $f(x) = 2x + \sin 3x$

B. $f(x) = x \cdot [x]$, (where [.] denotes the G.I.F)

C. $f(x) = \dfrac{2^x - 1}{4^x + 1}$

D. $f(x) = \dfrac{2^x + 1}{4^x - 1}$

**Answer: D**

Watch Video Solution

**14.** Given the function $f(x) = \dfrac{a^x + a^{-x}}{2} (wherea > 2) \dot{}Thenf(x + y) + f(x - y) =$ $2f(x)\dot{f}(y)$ (b) $f(x)\dot{f}(y)$ $\dfrac{f(x)}{f(y)}$ (d) none of these

Watch Video Solution

**15.** If $f(x) = \cos(\log x)$, then $f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right] =$

A. -1

B. $1/2$

C. -2

D. 0

**Answer: D**

Watch Video Solution

**16.** The domain of the function $f(x) = \frac{1}{\sqrt{{}^{10}C_{x-1} - 3 \times {}^{10}C_x}}$ is

A. $\{9, 10, 11\}$

B. $\{9, 10, 12\}$

C. all natural numbers

D. $\{9, 10\}$

**Answer: D**

Watch Video Solution

**17.** The domain of the function $f(x) = \dfrac{\sin^{-1}(3 - x)}{\ln(|x| - 2)}$ is

A. $[2, 4]$

B. $(2, 3) \cup (3, 4]$

C. $[2, \infty)$

D. $(-\infty, -3) \cup [2, \infty)$

**Answer: B**

Watch Video Solution

**18.** The domain of $f(x) = \dfrac{\log_2(x + 3)}{x^2 + 3x + 2}$ is

A. $R - \{-1, -2\}$

B. $(-2, \infty)$

C. $R - \{-1, -2, -3\}$

D. $(-3, \infty) - \{-1, -2\}$

**Answer: D**

Watch Video Solution

**19.** The domain of the function $f(x) = \sqrt{x^2 - [x]^2}$, where $[x]$ is the greatest integer less than or equal to $x$, is $R$ (b) $[0, +\infty]$ $(-\infty, 0)$ (d) none of these

A. R

B. $[0, +\infty)$

C. $(-\infty, 0]$

D. none of these

**Answer: D**

Watch Video Solution

**20.** The domain of the function $f(x) = \log_{3+x}\left(x^2 - 1\right)$ is

A. $(-3, -1) \cup (1, \infty)$

B. $[-3, -1) \cup [1, \infty)$

C. $(-3, -2) \cup (-2, -1) \cup (1, \infty)$

D. $(-3, -2) \cup (1, \infty)$

**Answer: C**

Watch Video Solution

**21.** Domain of the function, $f(x) = \left[\log_{10}\left(\frac{5x - x^2}{4}\right)\right]^{\frac{1}{2}}$ is

A. $-\infty < x < \infty$

B. $1 \le x \le 4$

C. $4 \le x \le 16$

D. $-1 \le x \le 1$

**Answer: B**

Watch Video Solution

22. The domain of $f(x) = \log|\log x|$ is

A. $(0, \infty)$

B. $(1, \infty)$

C. $(0, 1) \cup (1, \infty)$

D. $(-\infty, 1)$

**Answer: C**

Watch Video Solution

23. If $x^3 f(x) = \sqrt{1 + \cos 2x} + |f(x)|, \frac{-3\pi}{4} < x < \frac{-\pi}{2}$ and $f(x) = \frac{\alpha \cos x}{1 + x^3}$, then the value of $\alpha$ is

A. 2

B. $-\sqrt{2}$

C. $\sqrt{2}$

D. 1

**Answer: B**

Watch Video Solution

**24.** The function $f(x) = \frac{\sec^{-1}x}{\sqrt{x - [x]}}$, where $[x]$ denotes the greatest integer less than or equal to $x$, is defined for all $x \in R$ (b) $R - \{(-1, 1) \cup \{n|n \in Z\}\}$ $R^{\pm}(0, 1)$ (d) $R^{\pm}\{n \mid n \in N\}$

A. R

B. $R - \{(-1, 1) \cup \{n \mid n \in Z\}\}$

C. $R^{+} - (0, 1)$

D. $R^{+} - \{n \mid n \in N\}$

**Answer: B**

Watch Video Solution

**25.** The domain of definition of the function $f(x)$ given by the equation $2^{y} = 2$ is `0

A. $0 < x \le 1$

B. $0 \le x \le 1$

C. $-\infty < x \le 0$

D. $-\infty < x < 1$

**Answer: D**

Watch Video Solution

**26.** The domain of $f(x) = \cos^{-1}\left(\frac{2 - |x|}{4}\right) + [l\log(3 - x)]^{1}$ is $[-2, 6]$ (b) $[-6, 2) \cup (2, 3)$ $[-6, 2]$ (d) $[-2, 2] \cup (2, 3)$

A. $[-2, 6]$

B. $[-6, 2) \cup (2, 3)$

C. $[-6, 2]$

D. $[-2, 2] \cup (2, 3)$

**Answer: B**

Watch Video Solution

**27.** The domain of the function $f(x) = \sqrt{\log\left(\frac{1}{|\sin x|}\right)}$

A. $R - \{-\pi, \pi\}$

B. $R - \{n\pi \mid n \in Z\}$

C. $R - \{2n\pi \mid n \in z\}$

D. $(-\infty, \infty)$

**Answer: B**

Watch Video Solution

**28.** Domain of definition of the function $f(x) = \log_2\left(-\log_{\frac{1}{2}}\left(1 + x^{-4}\right) - 1\right)$ is

A. $(0, 1)$

B. $(0, 1]$

C. $[1, \infty)$

D. $(1, \infty)$

**Answer: A**

Watch Video Solution

**29.** The number of real solutions of the $(\log)_{0.5}|x| = 2|x|$ is

A. 1

B. 2

C. 0

D. none of these

**Answer: B**

Watch Video Solution

**30.** Let $f: R \xrightarrow{} 0, \frac{\pi}{2}$ be defined by $f(x) = \tan^{-1}\left(x^2 + x + a\right)^{\cdot}$ Then the set of values of $a$ for which $f$ is onto is $(0, \infty)$ (b) $[2, 1]$ (c) $\left[\frac{1}{4}, \infty\right]$ (d) none of these

A. $[0, \infty)$

B. $[2, 1]$

C. $\left[\frac{1}{4}, \infty\right)$

D. none of these

**Answer: C**

Watch Video Solution

**31.** The domain of the function $f(x) = \sqrt{\text{In}_{(|x|-1)}\left(x^2 + 4x + 4\right)}$ is

A. $[-3, -1] \cup [1, 2]$

B. $(-2, -1) \cup [2, \infty)$

C. $(-\infty, -3] \cup (-2, -1) \cup (2, \infty)$

D. None of these

**Answer: C**

Watch Video Solution

**32.** The domain of $f(x) = 1n\left(ax^3 + (a+b)x^2 + (b+c)x + c\right)$, where $a > 0, b^2 - 4ac = 0, is(where[.]$ represents greatest integer function).

A. $(-1, \infty)\sim\left\{-\frac{b}{2a}\right\}$

B. $(1, \infty)\sim\left\{-\frac{b}{2a}\right\}$

C. $(-1, 1) \sim \left\{ -\frac{b}{2a} \right\}$

D. None of these

**Answer: A**

Watch Video Solution

**33.** The domain of the function $f(x) = \frac{1}{\sqrt{4x - \left|x^2 - 10x + 9\right|}}$ is $\left(7 - \sqrt{40}, 7 + \sqrt{40}\right)$(b) $\left(0, 7 + \sqrt{40}\right)$(c)$\left(7 - \sqrt{40}, \infty\right)$ (d) none of these

A. $\left(7 - \sqrt{40}, 7 + \sqrt{40}\right)$

B. $\left(0, 7 + \sqrt{40}\right)$

C. $\left(7 - \sqrt{40}, \infty\right)$

D. none of these

**Answer: D**

Watch Video Solution

**34.** The domain of the function $f(x) = \dfrac{1}{\sqrt{|\cos x| + \cos x}}$ is

A. $[-2n\pi, 2n\pi], n \in Z$

B. $\left(2n\pi, \overline{2n+1\pi}\right), n \in Z$

C. $\left(\dfrac{(4n+1)\pi}{2}, \dfrac{(4n+3)\pi}{2}\right), n \in Z$

D. $\left(\dfrac{(4n-1)\pi}{2}, \dfrac{(4n+1)\pi}{2}\right), n \in Z$

**Answer: D**

Watch Video Solution

**35.** The exhaustive domain of the following function is $f(x) = \sqrt{x^{12} - x^9 + x^4 - x + 1}$ $[0, 1]$ (b) $[1, \infty]$ $[-\infty, 1]$ (d) $R$

A. $[0, 1]$

B. $[1, \infty)$

C. $(-\infty, 1]$

D. R

**Answer: D**

Watch Video Solution

**36.** The domain of the function $f(x) = \sqrt{\sin x + \cos x} + \sqrt{7x - x^2 - 6}$ is

A. $[1, 6]$

B. $\left[1, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 6\right]$

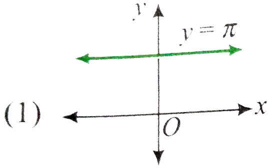
C. $[1, \pi] \cup \left[\frac{7\pi}{4}, 6\right]$

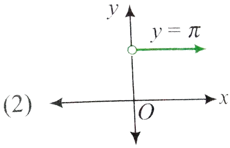
D. None of these

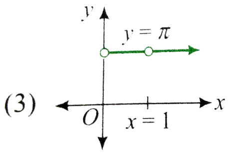
**Answer: B**

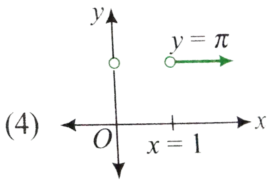
Watch Video Solution

**37.** 29. Which one of following best represents the graph of $y = x^{\log_x \pi}$

A. 

B. 

C. 

D. 

**Answer: C**

Watch Video Solution

**38.** If $x$ is real, then the value of the expression $\dfrac{x^2+14x+9}{x^2+2x+3}$ lies between

A. [4, 5]

B. [-4, 5]

C. [-5, 4]

D. none of these

**Answer: C**

Watch Video Solution

39. The range of the function $f(x) = |x - 1| + |x - 2|, \ -1 \leq x \leq 3,$ is

A. [1, 3]

B. [1, 5]

C. [3, 5]

D. None of these

**Answer: B**

Watch Video Solution

40. The function $f: R \to R$ is defined by $f(x) = \cos^2 x + \sin^4 x$ for $x \in R$.

Then the range of $f(x)$ is

A. $\left(\frac{3}{4}, 1\right]$

B. $\left[\frac{3}{4}, 1\right)$

C. $\left[\frac{3}{4}, 1\right]$

D. $\left(\frac{3}{4}, 1\right)$

**Answer: C**

Watch Video Solution

41. The range of $f9x\Big) = \Big[|s \in x| + \; | \cos x|]$ Where [.] denotes the greatest integer function, is {0} (b) {0,1} (c) {1} (d) none of these

A. {0}

B. {0, 1}

C. {1}

D. None of these

**Answer: C**

Watch Video Solution

**42.** The range of function $f(x) = {}^{7-x}P_{x-3} is$ {1,2,3} (b) $\{1, 2, 3, 4, 5, 6\}$ $\{1, 2, 3, 4\}$ (d) $\{1, 2, 3, 4, 5\}$

A. $\{1, 2, 3\}$

B. $\{1, 2, 3, 4, 5, 6\}$

C. $\{1, 2, 3, 4\}$

D. $\{1, 2, 3, 4, 5\}$

**Answer: A**

Watch Video Solution

**43.** The range of $f(x) = \sin^{-1}\left(\frac{x^2+1}{x^2+2}\right)$ is $\left[0, \frac{\pi}{2}\right]$ (b) $\left(0, \frac{\pi}{6}\right)$ (c) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (d)

none of these

A. $[0, \pi/2]$

B. $(0, \pi/6)$

C. $[\pi/6, \pi/2)$

D. None of these

**Answer: C**

Watch Video Solution

**44.** The range of the function $f(x) = \frac{e^x - e^{|x|}}{e^x + e^{|x|}}$ is

A. $(-\infty, \infty)$

B. $[0, 1)$

C. $(-1, 0]$

D. $( - 1, 1)$

**Answer: C**

Watch Video Solution

**45.** Domain (D) and range (R) of $f(x) = \sin^{-1}\left(\cos^{-1}[x]\right)$, where [.] denotes the greatest integer function, is $D \equiv x \in [1, 2], R \in \{0\}$ D $\equiv x \in 90, 1], R \equiv \{ - 1, 0, 1\}$ $\equiv x \in [ - 1, 1], R \equiv \left\{0, \sin^{-1}\left(\frac{\pi}{2}\right), \sin^{-1}(\pi)\right\}$ $\equiv x \in [ - 1, 1], R \equiv \left\{ - \frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

A. $D \equiv x \in [1, 2), R \equiv \{0\}$

B. $D \equiv x \in [0, 1], R = \{ - 1, 0, 1\}$

C. $D \equiv x \in [ - 1, 1], R \equiv \left\{0, \sin^{-1}\left(\frac{\pi}{2}\right), \sin^{-1}(\pi)\right\}$

D. $D \equiv x \in [ - 1, 1], R \equiv \left\{ - \frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

**Answer: A**

Watch Video Solution

**46.** The range of the function $f$ defined by $f(x) = \left[\frac{1}{\sin\{x\}}\right]$ (where [.] and $\{.\}$, respectively, denote the greatest integer and the fractional part functions) is I, the set of integers N, the set of natural number W, the set of whole numbers {1,2,3,4,...}

A. I, the set of integers

B. N, the set of natural numbers

C. W, the set of whole numbers

D. $\{1, 2, 3, 4, \ldots\}$

**Answer: D**

Watch Video Solution

**47.** Range ofthe function $f(x) = \cos(K\sin x)$ is $[-1, 1]$, then the least positive integral value of K will be

A. 1

B. 2

C. 3

D. 4

**Answer: D**

Watch Video Solution

**48.** Let $f(x) = \sqrt{|x| - \{x\}}$, where $\{.\}$ denotes the fractional part of x an X,Y and its domain and range respectively, then

A. $x \in \left(-\infty, \frac{1}{2}\right]$ and $Y \in \left[\frac{1}{2}, \infty\right)$

B. $x \in \left(-\infty, -\frac{1}{2}\right] \cup [0, \infty)$ and $Y \in \left[\frac{1}{2}, \infty\right)$

C. $X \in \left(-\infty, -\frac{1}{2}\right] \cup [0, \infty)$ and $Y \in [0, \infty)$

D. None of these

**Answer: C**

Watch Video Solution

**49.** The range of $f(x) = \cos^{-1}\left(\frac{1 + x^3}{x2}\right) + \sqrt{2 - x^2}$ is $\left\{0, 1 + \frac{\pi}{2}\right\}$ (b) $\{0, 1 + \pi)$ $\left\{1, 1 + \frac{\pi}{2}\right\}$ (d) $\{1, 1 + \pi\}$

A. $\left\{0, 1 + \frac{\pi}{2}\right\}$

B. $\{0, 1 + \pi\}$

C. $\left\{1, 1 + \frac{\pi}{2}\right\}$

D. $\{1, 1 + \pi\}$

**Answer: D**

Watch Video Solution

**50.** The range of the following function is

$$f(x) = \sqrt{(1 - \cos x)\sqrt{(1 - \cos x)\sqrt{(1 - \cos x)\sqrt{\ldots\infty}}}}$$

A. [0, 1]

B. [0, 1/2]

C. [0, 2)

D. None of these

**Answer: C**

Watch Video Solution

**51.** The range of $f(x) = (x + 1)(x + 2)(x + 3)(x + 4) + 5$ for $x \in [-6, 6]$ is

A. [4, 5045]

B. [0, 5045]

C. [-20, 5045]

D. None of these

**Answer: A**

Watch Video Solution

**52.** The range of $f(x) = \sec^{-1}\left((\log)_3 \tan x + (\log)_{\tan x} 3\right)$ is

A. $\left[\frac{\pi}{3}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$

B. $\left[0, \frac{\pi}{2}\right)$

C. $\left(\frac{2\pi}{3}, \pi\right]$

D. None of these

**Answer: A**

Watch Video Solution

**53.** The domain of definition of the function $f(x) = \{x\}^{\{x\}} + [x]^{[x]}$ is where $\{.\}$ represents fractional part and $[.]$ represent greatest integral function). $R - I$ (b) $R - [0, 1]$ $R - \{I \cup (0, 1)\}$ (d) $I \cup (0, 1)$

A. $R - I$

B. $R - \{0, 1)$

C. $R - \{I \cup (0, 1)\}$

D. $I \cup (0, 1)$

**Answer: C**

Watch Video Solution

**54.** 49. If $\left[x^2 - 2x + a\right] = 0$has no solution then

A. $-\infty < a < 1$

B. $2 \leq a < \infty$

C. $1 < a < 2$

D. $a \in R$

**Answer: B**

Watch Video Solution

**55.** If $[x]$ and $\{x\}$ represetnt the integral and fractional parts of $x$, respectively, then the value of $\sum_{r=1}^{2000} \frac{\{x+r\}}{2000}$ is $x$ (b) $[x]$ (c) $\{x\}$ (d) $x+2001$

A. x

B. [x]

C. {x}

D. x+2001

**Answer: C**

Watch Video Solution

**56.** If $f(x) = \dfrac{\sin([x]\pi)}{x^2 + x + 1}$ , where [.] denotes the greatest integer function, then $fisoneone$ $fis \neg one - oneandnon - constant$ $fisaconstantfunction$ $noneofthese$

A. $f$ is one-one

B. $f$ is not one-one and non-constant

C. $f$ is a constant function

D. None of these

**Answer: C**

Watch Video Solution

**57.** Let $f(x) = \left([a]^2 - 5[a] + 4\right)x^3 - \left(6\{a\}^2 - 5\{a\} + 1\right)x - (\tan x)x sgn x$ be an even function for all $x \in R$. Then the sum of all possible values of $a$ is (where $[.] and \{.\}$ denote greatest integer function and fractional part function, respectively). $\dfrac{17}{6}$ (b) $\dfrac{53}{6}$ (c) $\dfrac{31}{3}$ (d) $\dfrac{35}{3}$

A. $\frac{17}{6}$

B. $\frac{53}{6}$

C. $\frac{31}{3}$

D. $\frac{35}{3}$

**Answer: D**

Watch Video Solution

**58.** The solution set for $[x]\{x\} = 1$ (where {x} and [x] are respectively, fractional part function and greatest integer function) is $R^{\pm}(0, 1)$ (b) $r^{\pm}\{1\}$ $\left\{m + \frac{1}{m}m \in I - \{0\}\right\}$ $\left\{m + \frac{1}{m}m \in I - \{1\}\right\}$

A. $R^{+} - (0, 1)$

B. $R^{+} - \{1\}$

C. $\left\{m + \frac{1}{m}/m \in I - \{0\}\right\}$

D. $\left\{m + \frac{1}{m}/m \in N - \{1\}\right\}$

**Answer: D**

Watch Video Solution

**59.** Let [x] represent the greatest integer less than or equal to $x$ If $\left[\sqrt{n^2+\lambda}\right] = \left[n^2+1\right] + 2$ , where $\lambda, n \in N$, then $\lambda$ can assume $(2n+4)d \Leftrightarrow erentvalus$ $(2n+5)d \Leftrightarrow erentvalus$ $(2n+3)d \Leftrightarrow erentvalus$ $(2n+6)d \Leftrightarrow erentvalus$

A. $(2n+4)$ different values

B. $(2n+5)$ different values

C. $(2n+3)$ different values

D. $(2n+6)$ different values

**Answer: B**

Watch Video Solution

**60.** The number of roots of $x^2 - 2 = [\sin x]$, $where[.]$ stands for the greatest integer function is

A. 0

B. 1

C. 2

D. 3

**Answer: C**

Watch Video Solution

**61.** The domain of $f(x) = \sin^{-1}\left[2x^2 - 3\right]$, $where[.]$ denotes the greatest integer function, is

A. $\left(-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right)$

B. $\left(-\sqrt{\frac{3}{2}}, -1\right] \cup \left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$

C. $\left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$

D. $\left(-\sqrt{\frac{5}{2}}, -1\right] \cup \left[1, \sqrt{\frac{5}{2}}\right)$

**Answer: D**

Watch Video Solution

62. The domain of $f(x) = \sqrt{2\{x\}^2 - 3\{x\} + 1}$, where {.} denotes the fractional part in $[-1, 1]$ is

A. $[-1, 1] \sim \left(\frac{1}{2}, 1\right)$

B. $\left[-1, -\frac{1}{2}\right] \cup \left[0, \frac{1}{2}\right] \cup \{1\}$

C. $\left[-1, \frac{1}{2}\right]$

D. $\left[-\frac{1}{2}, 1\right]$

**Answer: B**

Watch Video Solution

**63.** The range of $\sin^{-1}\left[x^2+\frac{1}{2}\right]+\cos^{-1}\left[x^2-\frac{1}{2}\right]$, where [.] denotes the greatest integer function, is $\left\{\frac{\pi}{2},\pi\right\}$ (b) $\{\pi\}$ (c) $\left\{\frac{\pi}{2}\right\}$ (d) none of these

A. $\left\{\frac{\pi}{2},\pi\right\}$

B. $\{\pi\}$

C. $\left\{\frac{\pi}{2}\right\}$

D. None of these

**Answer: B**

Watch Video Solution

**64.** Let $f(x)=e^{e^{(|x|)\sin x}}$ and $g(x)=e^{e^{(|x|)\sin x}}, x\in R,$ where { } and [ ] denote the fractional and integral part functions, respectively. Also,

$h(x) = \log(f(x)) + \log(g(x))$ Then for real $x$, $h(x)$ is an odd function an even function neither an odd nor an even function both odd and even function

A. an odd function

B. an even function

C. neither an odd nor an even function

D. both odd and even function

**Answer: A**

Watch Video Solution

**65.** The number of solutions of the equation $[y + [y]] = 2\cos x$, where $y = \frac{1}{3}[\sin x + [\sin x + [\sin x]]]$ (where [.] denotes the greatest integer function) is

A. 4

B. 2

C. 3

D. 0

**Answer: D**

Watch Video Solution

**66.** The function $f(x) = \sin\left(\log\left(x + \sqrt{1 + x^2}\right)\right)$ is (a) even function (b) odd function (c) neither even nor odd (d) periodic function

A. even function

B. odd function

C. neither even nor odd

D. periodic function

**Answer: B**

Watch Video Solution

**67.** If $f(x) = x^m n, n \in N,$ is an even function, then $m$ is even integer (b) odd integer any integer (d) $f(x)$ - $evenis \neg possible$

A. even integer

B. odd integer

C. any integer

D. $f(x)$-even is not possible

**Answer: A**

Watch Video Solution

**68.** If $f(x) = \left\{x^2 \sin\left(\frac{\pi x}{2}\right), |x| < 1; x|x|, |x| \geq 1\right.$ then $f(x)$ is

A. an even function

B. an odd function

C. a periodic function

D. None of these

**Answer: B**

Watch Video Solution

**69.** If the graph of the function $f(x) = \dfrac{a^x - 1}{x^n\left(a^x + 1\right)}$ is symmetrical about the y axis , then n equals

A. 2

B. $\dfrac{2}{3}$

C. $\dfrac{4}{3}$

D. $-\dfrac{1}{3}$

**Answer: D**

Watch Video Solution

**70.** If $f: R \rightarrow$ is an invertible function such that $f(x) and f^{-1}(x)$ are symmetric about the line $y = -x$, then $f(x) is odd$ $f(x) and f^{-1}(x)$ may not be symmetric about the line $y = x$ $f(x)$ may not be odd $none of these$

A. $f(x)$ is odd

B. $f(x)$ and $f^{-1}(x)$ may not be symmetric about the line $y = x$

C. $f(x)$ may not be odd

D. None of these

**Answer: A**

Watch Video Solution

**71.** If $f(x) = ax^7 + bx^3 + cx - 5$ ,a , b , c are real constants, and $f(-7) = 7$, then the range of $f(7) + 17\cos x$

A. [-34, 0]

B. [0, 34]

C. [-34, 34]

D. None of these

**Answer: A**

Watch Video Solution

**72.** If $g:[-2,2]\vec{R}$, where $f(x)=x^3+\tan x+\left[\frac{x^2+1}{P}\right]$ is an odd function, then the value of parametric P, where [.] denotes the greatest integer function, is

A. $-5 < P < 5$

B. $P < 5$

C. $P > 5$

D. None of these

**Answer: C**

Watch Video Solution

Watch Video Solution

**73.** Let $f : [-1, 10] \rightarrow R, where f(x) = \sin x + \left[\frac{x^2}{a}\right]$, be an odd function. Then the set of values of parameter $a$ is/are

A. $(-10, 10) \sim \{0\}$

B. $(0, 10)$

C. $[100, \infty)$

D. $(100, \infty)$

**Answer: D**

Watch Video Solution

**74.** $f(x) = \frac{\cos x}{\left[\frac{2x}{\pi}\right] + \frac{1}{2}}$, where $x$ is not an integral multiple of $\pi$ and [.] denotes the greatest integer function, is an odd function an even function neither odd nor even none of these

A. an odd function

B. an even function

C. neither odd nor even

D. None of these

**Answer: A**

Watch Video Solution

**75.** Let $f(x) = \begin{cases} \sin x + \cos x, & 0 < x < \frac{\pi}{2} \\ a, & x = \pi/2 \\ \tan^2 x + \operatorname{cosec} x, & \pi/2 < x < \pi \end{cases}$

Then its odd extension is

A. $\begin{cases} -\tan^2 x - \operatorname{cosec} x, & -\pi < x < -\frac{\pi}{2} \\ -a, & x = -\frac{\pi}{2} \\ -\sin x + \cos x, & -\frac{\pi}{2} < x < 0 \end{cases}$

B. $\begin{cases} -\tan^2 x + \text{cosec}x, & -\pi < x < -\frac{\pi}{2} \\ -a, & x = -\frac{\pi}{2} \\ \sin x - \cos x, & -\frac{\pi}{2} < x < 0 \end{cases}$

C. $\begin{cases} -\tan^2 x + \text{cosec}x, & -\pi < x < -\frac{\pi}{2} \\ a, & x = -\frac{\pi}{2} \\ \sin x - \cos x, & -\frac{\pi}{2} < x < 0 \end{cases}$

D. $\begin{cases} \tan^2 x + \text{cosec}x, & -\pi < x < -\frac{\pi}{2} \\ -a, & x = -\frac{\pi}{2} \\ \sin x + \cos x, & -\frac{\pi}{2} < x < 0 \end{cases}$

**Answer: B**

Watch Video Solution

**76.** The period of the function $\left|\sin^3\left(\frac{x}{2}\right)\right| + \left|\cos^5\left(\frac{x}{5}\right)\right|$ is

A. $2\pi$

B. $10\pi$

C. $8\pi$

D. $5\pi$

**Answer: B**

Watch Video Solution

**77.** If $f$ is periodic, $g$ is polynomial function and $f(g(x))$ is periodic and $g(2) = 3, g(4) = 7$ then $g(6)$ is

A. 13

B. 15

C. 11

D. None of these

**Answer: C**

Watch Video Solution

**78.** The period of function $2^{\{x\}} + \sin\pi x + 3^{\{x/2\}} + \cos\pi x$ (where {x} denotes the fractional part of x) is

A. 2

B. 1

C. 3

D. None of these

**Answer: A**

Watch Video Solution

**79.** The period of the function $f(x) = [6x + 7] + \cos \pi x - 6x$, where [.] denotes the greatest integer function is:

A. 3

B. $2\pi$

C. 2

D. None of these

**Answer: C**

Watch Video Solution

**80.** If $f(x)$ and $g(x)$ are periodic functions with periods 7 and 11, respectively, then the period of $f(x) = f(x)g\left(\frac{x}{5}\right) - g(x)f\left(\frac{x}{3}\right)$ is

A. 177

B. 222

C. 433

D. 1155

**Answer: D**

Watch Video Solution

**81.** The period of the function $f(x) = c^{\left(\sin^2 x\right) + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)}$ is (where $c$ is constant)

A. 1

B. $\frac{\pi}{2}$

C. $\pi$

D. None of these

**Answer: D**

Watch Video Solution

**82.** Let $f(x) = \left\{(0.1)^{3[x]}\right\}$. (where [.] denotes greatest integer function and denotes fractional part). If $f(x+T) = f(x) \forall x \in 0$, where T is a fixed positive number then the least x value of T is

A. 2

B. 4

C. 6

D. None of these

**Answer: B**

Watch Video Solution

**83.** If the period of $\dfrac{\cos(\sin(nx))}{\tan\left(\frac{x}{n}\right)}$, $n \in N, is 6\pi$ , then $n =$ 3 (b) 2 (c) 6 (d) 1

A. 3

B. 2

C. 6

D. 1

**Answer: C**

Watch Video Solution

**84.** The period of $f(x) = [x] + [2x] + [3x] + [4x] + [nx] - \frac{n(n+1)}{2}x$, where $n \in N$, is (where [.] represents greatest integer function). $n$ (b) 1 (c) $\frac{1}{n}$ (d) none of these

A. n

B. 1

C. $\frac{1}{n}$

D. none of these

**Answer: B**

Watch Video Solution

**85.** If $f(x) = (-1)\left[\frac{2}{\pi}\right], g(x) = |\sin x| - |\cos x|, and \varphi(x) = f(x)g(x)$ (where [.] denotes the greatest integer function), then the respective fundamental periods of $f(x), g(x), and \varphi(x)$ are $\pi, \pi, \pi$ (b) $\pi, 2\pi, \pi$ $\pi, \pi, \frac{\pi}{2}$ (d) $\pi, \frac{\pi}{2}, \pi$

A. $\pi, \pi, \pi$

B. $\pi, 2\pi, \pi$

C. $\pi, \pi, \frac{\pi}{2}$

D. $\pi, \frac{\pi}{2}, \pi$

**Answer: C**

Watch Video Solution

**86.** If $f(x) = \frac{1}{x}, g(x) = \frac{1}{x^2}$, and $h(x) = x^2, then\ f(x) = x^2, x \neq 0, \left( h(g(x)) = \frac{1}{x^2}\right.$

$h(g(x)) = \frac{1}{x^2}, x \neq 0, fog(x) = x^2$ $fog(x) = x^2, x \neq 0, h(g(x)) = (g(x))^2, x \neq 0$

none of these

A. $fog(x) = x^2, x \neq 0, h(g(x)) = \frac{1}{x^2}$

B. $h(g(x)) = \frac{1}{x^2}, x \neq 0, fog(x) = x^2$

C. $fog(x) = x^2, x \neq 0, h(g(x)) = (g(x))^2, x \neq 0$

D. None of these

**Answer: C**

Watch Video Solution

**87.** If $f(x) = \begin{cases} x^2, & \text{for } x \geq 0 \\ x, & \text{for } x < 0 \end{cases}$, then fof(x) is given by

A. $x^2 \text{ for } x \geq 0, x \text{ for } x < 0$

B. $x^4 \text{ for } x \geq 0, x^2 \text{ for } x < 0$

C. $x^4 \text{ for } x \geq 0, -x^2 \text{ for } x < 0$

D. $x^4 \text{ for } x \geq 0, x \text{ for } x < 0$

**Answer: D**

Watch Video Solution

88. Let $f(x) = \sin x$ and $g(x) = (\log)_e|x|$ If the ranges of the composition functions $fog$ an $dgof$ are $R_1$ and$R_2$, respectively, then `R_1-{u :-1lt=u<1},R_2={v :-oo `R_1-{u :-oo `R_1-{u :-1 `R_1-{u :-1lt=ult=1},R_2={v :-oo

A. $R_1 = \{u: -1 \leq u < 1\}. R_2 = \{v: -\infty < v < 0\}$

B. $R_1 = \{u: -\infty < u < 0\}. R_2 = \{v: -\infty < v < 0\}$

C. $R_1 = \{u: -1 < u < 1\}. R_2 = \{v: -\infty < v < 0\}$

D. $R_1 = \{u: -1 \leq u \leq 1\}. R_2 = \{v: -\infty < v \leq 0\}$

**Answer: D**

Watch Video Solution

89. If $f(x) = \{x, \xi srational 1 - x, \xi sirrational, then f(f(x))$ is $x \forall x \in R$ (b) $\{x, \xi sirrational 1 - x, \xi srational$ $\{x, \xi srational 1 - x, \xi sirrational$ (d) none of these

A. $x \forall x \in R$

B. $f(x) = \begin{cases} x, & x \text{ is irrational} \\ 1 - x, & x \text{ is rational} \end{cases}$

C. $f(x) = \begin{cases} x, & x \text{ is rational} \\ 1 - x, & x \text{ is irrational} \end{cases}$

D. None of these

**Answer: A**

Watch Video Solution

**90.** If $f$ and $g$ are one-one functions, then

A. $f + g$ is one-one

B. $fg$ is one-one

C. $fog$ is one-one

D. None of these

**Answer: C**

Watch Video Solution

Watch Video Solution

**91.** The domain of $f(x) is (0, 1)^{\cdot}$ Then the domain of $\left(f\left(e^{x}\right) + f(1n|x|)\right.$ is

A. $(-1, e)$

B. $(1, e)$

C. $(-e, -1)$

D. $(-e, 1)$

**Answer: C**

Watch Video Solution

**92.** Let $h(x) = |kx + 5|$,the domain of f(x) be [-5,7], the domain of f(h(xx)) be [-6,1],and the range of h(x) be the same as the domain of f(x) Then the value of $k$ is.

A. 1

B. 2

C. 3

D. 4

**Answer: B**

Watch Video Solution

**93.** If $f(x) = \sin x + \cos x$and $g(x) = x^2 - 1$, then $g(f(x))$is invertible in the domain .

A. $\left[0, \frac{\pi}{2}\right]$

B. $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

C. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

D. $[0, \pi]$

**Answer: B**

Watch Video Solution

Watch Video Solution

**94.** If the function $f:[1,\infty) \rightarrow [1,\infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is

A. $\left(\frac{1}{2}\right)^{x(x-1)}$

B. $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

C. $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$

D. not defined

**Answer: B**

Watch Video Solution

**95.** Let $f(x) = (x+1)^2 - 1, x \geq -1$. Then the set $\{x : f(x) = f^{-1}(x)\}$ is $\left\{0, 1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}$ (b) $\{0, 1, -1 \{0, 1, 1\}$ (d) $empty$

A. $\left\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}$

B. $\{0, 1, -1\}$

C. $\{0, -1\}$

D. empty

**Answer: C**

Watch Video Solution

**96.** if $f : [1, \infty) \rightarrow [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$ then $f^{-1}(x)$ equals to

A. $\frac{\left(x + \sqrt{x^2 - 4}\right)}{2}$

B. $\frac{x}{1 + x^2}$

C. $\frac{\left(x - \sqrt{x^2 - 4}\right)}{2}$

D. $1 + \sqrt{x^2 - 4}$

**Answer: A**

Watch Video Solution

Watch Video Solution

**97.** Suppose $f(x)=(x+1)^2 f$ or $x \geq -1$. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line $y=x$, then $g(x)$ equal. $a - \sqrt{x} - 1, x \geq 0$ (b) $\dfrac{1}{(x+1)^2}, x > 1$ $\sqrt{x+1}, x \geq -1$ (d) $\sqrt{x} - 1, x \geq 0$

A. $1 - \sqrt{x} - 1, x \geq 0$

B. $\dfrac{1}{(x+1)^2}, x > -1$

C. $\sqrt{x+1}, x \geq -1$

D. $\sqrt{x} - 1, x \geq 0$

**Answer: D**

Watch Video Solution

**98.** Let $f:\left[-\frac{\pi}{3}, \frac{2\pi}{3}\right] \overrightarrow{0, 4}$ be a function defined as $f(x)=\sqrt{3}\sin x - \cos x + 2$. Then $f^{-1}(x)$ is given by $\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{\pi}{6}$ $\sin^{-1}\left(\frac{x-2}{2}\right) + \frac{\pi}{6}$

$\frac{2\pi}{3} + \cos^{-1}\left(\frac{x-2}{2}\right)$ (d) none of these

A. $\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{\pi}{6}$

B. $\sin^{-1}\left(\frac{x-2}{2}\right) + \frac{\pi}{6}$

C. $\frac{2\pi}{3} + \cos^{-1}\left(\frac{x-2}{2}\right)$

D. None of these

**Answer: B**

Watch Video Solution

**99.** Which of the following functions is the inverse of itself? $f(x) = \frac{1-x}{1+x}$

(b) $f(x) = 5^{\log x}$ $f(x) = 2^{x(x-1)}$ (d) None of these

A. $f(x) = \frac{1-x}{1+x}$

B. $f(x) = 5^{\log x}$

C. $f(x) = 2^{x(x-1)}$

D. None of these

**Answer: A**

Watch Video Solution

**100.** If $g(x) = x^2 + x - 2 and \frac{1}{2} gof(x) = 2x^2 - 5x + 2$, then which is not a possible $f(x)$? $2x - 3$ (b) $-2x + 2$ $x - 3$ (d) None of these

A. $2x - 3$

B. $-2x + 2$

C. $x - 3$

D. None of these

**Answer: C**

Watch Video Solution

**101.** Let $f: X\vec{y}f(x) = s \in x + \cos x + 2\sqrt{2}$ be invertible. Then which $X\vec{Y}$ is not possible? $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]\overrightarrow{\sqrt{2}, 3\sqrt{2}}$ $\left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]\overrightarrow{\sqrt{2}, 3\sqrt{2}}$ $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right]\overrightarrow{\sqrt{2}, 3\sqrt{2}}$ none of these

A. $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$

B. $\left[-\frac{3\pi}{4}, \frac{\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$

C. $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$

D. None of these

**Answer: C**

Watch Video Solution

**102.** If $f(x)$ is an invertible function and $g(x) = 2f(x) + 5,$ then the value of $g^{-1}(x) is\ 2f^{-1}(x) - 5$ (b) $\frac{1}{2f^{-1}(x) + 5}$ $\frac{1}{2}f^{-1}(x) + 5$ (d) $f^{-1}\left(\frac{x - 5}{2}\right)$

A. $2f^{-1}(x) - 5$

B. $\dfrac{1}{2f^{-1}(x) + 5}$

C. $\dfrac{1}{2}f^{-1}(x) + 5$

D. $f^{-1}\left(\dfrac{x - 5}{2}\right)$

**Answer: D**

Watch Video Solution

**103.** Let $f(x) = [x] + \sqrt{\{x\}}$, where [.] denotes the integral part of x and {x} denotes the fractional part of x. Then $f^{-1}(x)$ is

A. $[x] + \sqrt{\{x\}}$

B. $[x] + \{x\}^2$

C. $[x]^2 + \{x\}$

D. $\{x\} + \sqrt{\{x\}}$

**Answer: B**

Watch Video Solution

**104.** If $f$ is a function such that $f(0) = 2, f(1) = 3, and f(x + 2) = 2f(x) - f(x + 1)$ for every real $x$, then $f(5)$ is

A. 7

B. 13

C. 1

D. 5

**Answer: B**

Watch Video Solution

**105.** A function $f(x)$ satisfies the functional equation $x^2 f(x) + f(1 - x) = 2x - x^4$ for all real $x$. $f(x)$ must be

A. $x^2$

B. $1 - x^2$

C. $1 + x^2$

D. $x^2 + x + 1$

**Answer: B**

Watch Video Solution

**106.** If $f(x)$ is a polynomial satisfying $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) and f(3) = 28, then f(4)$ is equal to

A. 63

B. 65

C. 17

D. none of these

**Answer: B**

Watch Video Solution

**107.** If $f\left(2x + \frac{y}{8}, 2x - \frac{y}{8}\right) = xy$ , then $f(m, n) = 0$ only when $m = n$ only when

$m \neq n\ onlywhenm = -n$ (d) $f$ or $allmandn$

A. only when $m = n$

B. only when $m \neq n$

C. only when $m = -n$

D. for all m and n

**Answer: D**

Watch Video Solution

**108.** Let $f: R \rightarrow R$ be a function such that $f(0) = 1$ and for any $x, y \in R, f(xy + 1) = f(x)f(y) - f(y) - x + 2.$ Then $f$ is

A. one-one and onto

B. one-one but not onto

C. many one but onto

D. many one and into

**Answer: A**

Watch Video Solution

**109.** If $f(x+y)=f(x)+f(y)-xy-1, \ \forall x$, y in R and $f(1)=1$, then the number of solution of f(n)=n , $n \in N$, is

A. 0

B. 1

C. 2

D. more than 2

**Answer: B**

Watch Video Solution

**110.** The function $f$ satisfies the functional equation $3f(x) + 2f\left(\frac{x+59}{x1}\right) = 10x + 30$ for all real $x \neq 1$. The value of $f(7) is$ If $f(x + y) = f(x) + f(y) - xy - 1$, $\forall x$, y in R and f(1)=1,*thenthevmberofsolutionof* f(n)=n, n in N ,` is

A. 8

B. 4

C. -8

D. 11

**Answer: B**

Watch Video Solution

**111.** Let $f: R \rightarrow R$ be a continuous and differentiable function such that $\left(f\left(x^2+1\right)\right)^{\sqrt{x}} = 5f$ or $\forall x \in (0, \infty)$, then the value of $\left(f\left(\frac{16+y^2}{y^2}\right)\right)^{\frac{4}{\sqrt{y}}}$ for each y in $(0, \infty)$ is equal to

A. 5

B. 25

C. 125

D. 625

**Answer: B**

Watch Video Solution

**112.** Let $g(x) = f(x) - 1.$ If $f(x) + f(1 - x) = 2 \forall x \in R,$ then $g(x)$ is symmetrical about. the origin (b) $thel \in ex = \frac{1}{2}$ the point (1,0) (d) the point $\left(\frac{1}{2}, 0\right)$

A. the orgin

B. the line$x = \frac{1}{2}$

C. the point (1, 0)

D. the point $\left(\frac{1}{2}, 0\right)$

**Answer: D**

Watch Video Solution

**113.** If $f(x+1)+f(x-1)=2f(x) and f(0), \; = 0,$ then $f(n), n \in N,$ is $nf(1)$ (b) $\{f(1)\}^n$ $0$ (d) none of these

A. $nf(1)$

B. $\{f(1)\}^n$

C. 0

D. none of these

**Answer: A**

Watch Video Solution

**114.** If $f\left(x+\frac{1}{2}\right)+f\left(x-\frac{1}{2}\right)=f(x)f \text{ or } all x \in R,$ then the period of $f(x)$ is 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

**Answer: C**

Watch Video Solution

**115.** If $af(x+1)+bf\left(\frac{1}{x+1}\right)=x, x \neq -1, a \neq b$, then $f(2)$ is equal to

A. $\frac{2a+b}{2\left(a^2-b^2\right)}$

B. $\frac{a}{a^2-b^2}$

C. $\frac{a+2b}{a^2-b^2}$

D. none of these

**Answer: A**

Watch Video Solution

**116.** If $f(3x+2)+f(3x+29) = \ \forall x \in R,$ then the period of $f(x)$ is

A. 7

B. 8

C. 10

D. none of these

**Answer: D**

Watch Video Solution

**117.** If the graph of $y=f(x)$ is symmetrical about the lines $x=1 and x=2,$ then which of the following is true? $f(x+1)=f(x)$ (b) $f(x+3)=f(x)$ $f(x+2)=f(x)$ (d) None of these

A. $f(x+1)=f(x)$

B. $f(x+3) = f(x)$

C. $f(x+2) = f(x)$

D. none of these

**Answer: C**

Watch Video Solution

**118.** If $f(x) =$ maximum $\left\{x^3, x^2, \frac{1}{64}\right\} \forall x \in [0, \infty)$ then

A. $f(x) = \begin{cases} x^2, & 0 \le x \le 1 \\ x^3, & x > 1 \end{cases}$

B. $f(x) = \begin{cases} \frac{1}{64}, & 0 \le x \le \frac{1}{4} \\ x^2, & \frac{1}{4} < x \le 1 \\ x^3, & x > 1 \end{cases}$

C. $f(x) = \begin{cases} \frac{1}{64}, & 0 \leq x \leq \frac{1}{8} \\ x^2, & \frac{1}{8} < x \leq 1 \\ x^3, & x > 1 \end{cases}$

D. $f(x) = \begin{cases} \frac{1}{64}, & 0 \leq x \leq \frac{1}{8} \\ x^3, & x > 1/8 \end{cases}$

**Answer: C**

Watch Video Solution

**119.** The equation $||x - 2| + a| = 4$ can have four distinct real solutions for $x$ if $a$ belongs to the interval $(-\infty, -4)$ (b) $(-\infty, 0)$ $(4, \infty)$ (d) none of these

A. $(-\infty, -4)$

B. $(-\infty, 0]$

C. $[4, \infty)$

D. none of these

**Answer: A**

Watch Video Solution

**120.** Number of integral values of k for which the equation $4\cos^{-1}(-|x|) = k$ has exactly two solutions, is:

A. 4

B. 5

C. 6

D. 7

**Answer: C**

Watch Video Solution

**121.** If $f(x)$ is a real-valued function defined as $f(x) = In(1 - \sin x)$, then the graph of $f(x)$ is

A. symmetric about the line $x = \pi$

B. symmetric about the y-axis

C. symmetric and the line $x = \frac{\pi}{2}$

D. symmetric about the origin

**Answer: C**

Watch Video Solution

**122.** Let $f(x) = x + 2|x + 1| + x - 1 \mid \dot{I}ff(x) = k$ has exactly one real solution, then the value of $k$ is3 (b) 0 (c) 1(d) 2

A. 3

B. 0

C. 1

D. 2

**Answer: A**

Watch Video Solution

**123.** The number of solutions of $2\cos x = |\sin x|, 0 \leq x \leq 4\pi,$ is

A. 0

B. 2

C. 4

D. infinite

**Answer: C**

Watch Video Solution

**124.** Let $f_1(x) = \{x, x \leq x \leq 1$ and $1x > 1$ and $0$,otherwise $f_2(x) = f_1(-x)$ for all x abd $f_3(x) = -f_2(x)$ for all x and $f_4(x) = -f_3(-x)$ for all x Which of the following is necessarily true?

A. $f_4(x) = f_1(x)$ for all x

B. $f_1(x) = -f_3(-x)$ for all x

C. $f_2(-x) = f_4(x)$ for all x

D. $f_1(x) + f_3(x) = 0$ for all x

**Answer: B**

Watch Video Solution

**125.** If $\log_4\left(\frac{2f(x)}{1 - f(x)}\right) = x, \quad \text{then} \quad (f(2010) + f(-2009))$ is equal to

A. 0

B. -1

C. 1

D. 2

**Answer: C**

Watch Video Solution

## Exercise (Multiple)

**1.** Let $f(x) = \sec^{-1}\left[1 + \cos^2 x\right]$, where [.] denotes the greatest integer function. Then the

A. domain of $f$ is R

B. domain of $f$ is $[1, 2]$

C. domain of $f$ is $[1, 2]$

D. range of $f$ is $\left\{\sec^{-1}1, \sec^{-1}2\right\}$

**Answer: A::B**

Watch Video Solution

**2.** Let $f: R \rightarrow [-1, \infty]$ and $f(x) = \ln([|\sin 2x| + |\cos 2x|])$ (where[.] is greatest integer function), then -

A. $f(x)$ has range Z

B. Range of $f(x)$ is singleton set

C. $f(x)$ is invertible in $\left[0, \frac{\pi}{4}\right]$

D. $f(x)$ is into function

**Answer: B::D**

Watch Video Solution

**3.** If $f: R\vec{N} \cup \{0\}$, where $f$ (area of triangle joining points $P(5, 0), Q(8, 4) and R(x, y)$ such that angle PRQ is a right angle $=$ number of triangles, then which of the following is true? $f(5) = 4$ (b) $f(7) = 0$ $f(6, 25) = 2$ (d) $f(x) is \neg$

A. $f(5) = 4$

B. $f(7) = 0$

C. $f(6.25) = 2$

D. $f(4.5) = 4$

**Answer: A::B::C::D**

Watch Video Solution

**4.** The domain of the function

$$f(x) = \log_e\left\{\log_{|\sin x|}\left(x^2 - 8x + 23\right) - \frac{3}{\log_2|\sin x|}\right\}$$

contains which of the following interval (s) ?

A. $(3, \pi)$

B. $\left(\pi, \frac{3\pi}{2}\right)$

C. $\left(\frac{3\pi}{2}, 5\right)$

D. None of these

**Answer: A::B::C**

Watch Video Solution

**5.** Let $f(x) = sgn\left(\cot^{-1}x\right) + \tan\left(\frac{\pi}{2}[x]\right)$, where $[x]$ is the greatest integer function less than or equal to $x$, then which of the following alternatives is/are true? $f(x)$ is many-one but not an even function. $f(x)$ is a periodic function. $f(x)$ is a bounded function. The graph of $f(x)$ remains above the x-axis.

A. $f(x)$ is many-one but not an even function.

B. $f(x)$ is a periodic function.

C. $f(x)$ is a bounded function.

D. The graph of $f(x)$ remains above the x-axis.

**Answer: A::B::C::D**

Watch Video Solution

**6.** $f(x) = \sqrt{1 - \sin^2 x} + \sqrt{1 + \tan^2 x}$ then

A. fundamental period of $f(x)$ is $\pi$

B. range of $f(x)$ is $[2, \infty)$

C. domain of $f(x)$ is R

D. $f(x) = 2$ has 3 solution in $[0, 2\pi]$

**Answer: A::B::D**

Watch Video Solution

**7.** If the following functions are defined from $[-1, 1] \to [-1, 1]$, select those which are not objective. $\sin\left(s \in^{-1} x\right)$ (b) $\frac{2}{\pi}\sin^{-1}(\sin x)$ $(sgn(x))1N\left(e^x\right)$ (d) $x^3(sgn(x))$

A. $\sin\left(\sin^{-1} x\right)$

B. $\frac{2}{\pi}\sin^{-1}(\sin x)$

C. $(sgn(x))In\left(e^x\right)$

D. $x^3(sgn(x))$

**Answer: B::C::D**

Watch Video Solution

8. Let $f(x) = \{x^2 - 4x + 3, x < 3x - 4, x \geq 3$ $andg(x) = \{x - 3, x < 4x^2 + 2x + 2, x \geq 4$ then which of the following is/are true? $(f + g)(3.5) = 0$ $f(g(3)) = 3$ $(fg)(2) = 1$ (d) $(f - g)(4) = 0$

A. $(f + g)(3.5) = 0$

B. $f(g(3)) = 3$

C. $(fg)(2) = 1$

D. $(f - g)(4) = 0$

**Answer: A::B::C**

Watch Video Solution

9. Let $f(x) = \max(1 + s \in x, 1, 1 - \cos x), x \in [0, 2\pi], andg(x) = \max\{1, |x - 1|\}, x \in R$

Then $g(f(0)) = 1$ (b) $g(f(1)) = 1$ $f(f(1)) = 1$ (d) $f(g(0)) + 1\sin 1$

A. $g(f(0)) = 1$

B. $g(f(1)) = 1$

C. $f(f(1)) = 1$

D. $f(g(0)) = 1 + \sin 1$

**Answer: A::B::D**

Watch Video Solution

**10.** Consider the function $y = f(x)$ satisfying the condition $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}(x \neq 0)$. Then the

A. domain of $f(x)$ is R

B. domain of $f$ is $R - (-2, 2)$

C. range of $f(x)$ is $[-2, \infty)$

D. range of $f(x)$ is $[2, \infty)$

**Answer: B::D**

Watch Video Solution

**11.** Consider the real-valued function satisfying $2f(\sin x) + f(\cos x) = x$ then the domain of $f(x) is R$ domain of $f(x) is[-1, 1]$ range of $f(x)$ is $\left[-\frac{2\pi}{3}, \frac{\pi}{3}\right]$ range of $f(x) is R$

A. domain of $f(x)$ is R

B. domain of $f(x)$ is $[-1, 1]$

C. range of $f(x)$ is $\left[-\frac{2\pi}{3}, \frac{\pi}{3}\right]$

D. range of $f(x)$ is R

**Answer: B::C**

Watch Video Solution

**12.** If $f: R^+ \to R^+$ is a polynomial function satisfying the functional equation $f(f(x)) = 6x = f(x), then f(17)$ is equal to

A. 17

B. 51

C. 34

D. -34

**Answer: C**

Watch Video Solution

**13.** $f(x) = x^2 - 2ax + a(a+1), f : [a, \infty) \rightarrow [a, \infty)$ If one of the solution of the equation $f(x) = f^{-1}(x)$ is

A. 5051

B. 5048

C. 5052

D. 5050

**Answer: B::D**

Watch Video Solution

**14.** Which of the following function is/are periodic?

A. $f(x) = \begin{cases} 1, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$

B. $f(x) = \begin{cases} x - [x], & 2n \leq x < 2n + 1 \\ \frac{1}{2}, & 2n + 1 \leq x < 2n + 2 \end{cases}$

where [.] denotes the greatest integer function $n \in Z$

C. $f(x) = (-1)^{\left[\frac{2x}{\pi}\right]}$, where [.] denotes the greatest integer function

D. $f(x) = x - [x + 3] + \tan\left(\frac{\pi x}{2}\right)$, where [.] denotes the greatest integer function, and a is a rational number

**Answer: A::B::C::D**

Watch Video Solution

15. Let $f(x) = \frac{3}{4}x + 1, f^n(x)$ be defined as $f^2(x) = f(f(x))$, and for $n \geq 2, f^{n+1}(x) = f\left(f^n(x)\right)$. If $\lambda = \lim_{n \to \infty} f^n(x)$, then

A. $\lambda$ is independent of x

B. $\lambda$ is a linear polynomial in x

C. the line $y = \lambda$ has slope 0

D. the line $4y = \lambda$ touches the unit circle with center at the origin.

**Answer: A::C::D**

Watch Video Solution

16. If the fundamental period of function $f(x) = \sin x + \cos\left(\sqrt{4 - a^2}\right)x$ is $4\pi$, then the value of a is/are

A. $\frac{\sqrt{15}}{2}$

B. $-\frac{\sqrt{15}}{2}$

C. $\frac{\sqrt{7}}{2}$

D. $-\frac{\sqrt{7}}{2}$

**Answer: A::B::C::D**

Watch Video Solution

**17.** $f(x) = \sin^{-1}\left[e^x\right] + \sin^{-1}\left[e^{-x}\right]$ where [.] greatest integer function then

A. domain of $f(x)$ is $\left(-\log_e 2, \log_e 2\right)$

B. range of $f(x) = \{\pi\}$

C. Range of $f(x)$ is $\left\{\frac{\pi}{2}, \pi\right\}$

D. $f(x) = \cos^{-1}x$ has only one solution

**Answer: A::C**

Watch Video Solution

**18.** $[2x] - 2[x] = \lambda$ where [.] represents greatest integer function and {.} represents fractional part of a real number then

A. $\lambda = 1 \forall x \in R$

B. $\lambda = 0 \forall x \in R$

C. $\lambda = 1 \forall \{x\} \geq \frac{1}{2}$

D. $\lambda = 0 \forall \{x\} < \frac{1}{2}$

**Answer: C::D**

Watch Video Solution

**19.** The set of all values of x satisfying $\{x\} = x[\times]$ where $[\times]$ represents greatest integer function $\{\times\}$ represents fractional part of x

A. 0

B. $-\frac{1}{2}$

C. $-1 < x < 1$

D. Both A and B

**Answer: D**

View Text Solution

**20.** The function 'g' defined by $g(x) = \sin\left(\sin^{-1}\sqrt{\{x\}}\right) + \cos\left(\sin^{-1}\sqrt{\{x\}}\right) - 1$ (where {x} denotes the functional part function) is (1) an even function (2) a periodic function (3) an odd function (4) neither even nor odd

A. an even function

B. periodic function

C. odd function

D. Neither even nor odd

**Answer: A::B**

Watch Video Solution

21. If the function / satisfies the relation $f(x+y)+f(x-y)=2f(x), f(y)\forall x, y \in R and f(0)\neq 0$, then $f(x) is a \neq ven function$ $f(x) is an odd function$ $If f(2)=a, then f(-2)=a$ $If f(4)=b, then f(-4)=-b$

A. $f(x)$ is an even function

B. $f(x)$ is an odd function

C. If $f(2)=a$, then $f(-2)=a$

D. If $f(4)=b$, then $f(-4)=-b$

**Answer: A::C**

Watch Video Solution

22. Let $f(x)+f(y)=f\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)[f(x)$ is not identically zero]. Then $f\left(4x^3-3x\right)+3f(x)=0$ $f\left(4x^3-3x\right)=3f(x)$ $f\left(2x\sqrt{1-x^2}+2f(x)=0\right.$ $f\left(2x\sqrt{1-x^2}=2f(x)\right.$

A. $f\left(4x^3 - 3x\right) + 3f(x) = 0$

B. $f\left(4x^3 - 3x\right) = 3f(x)$

C. $f\left(2x\sqrt{1 - x^2}\right) + 2f(x) = 0$

D. $f\left(2x\sqrt{1 - x^2}\right) = 2f(x)$

**Answer: A::D**

Watch Video Solution

**23.** Let $f: R\vec{R}$ be a function defined by $f(x+1) = \dfrac{f(x) - 5}{f(x) - 3} \forall x \in R$. Then which of the following statement(s) is/are ture? $f(2008) = f(2004)$ $f(2006) = f(2010)$ $f(2006) = f(2002)$ $f(2006) = f(2018$

A. $f(2008) = f(2004)$

B. $f(2006) = f(2010)$

C. $f(2006) = f(2002)$

D. $f(2006) = f(2018)$

**Answer: A::B::C::D**

Watch Video Solution

**24.** Let a function $f(x), x \neq 0$ be such that

$$f(x) + f\left(\frac{1}{x}\right) = f(x) \cdot f\left(\frac{1}{x}\right) \text{ then } f(x) \text{ can be}$$

A. $1 - x^{2013}$

B. $\sqrt{|x|} + 1$

C. $\dfrac{\pi}{2\tan^{-1}|x|}$

D. $\dfrac{2}{1 + k \, \text{In} \, |x|}$

**Answer: A::B::C::D**

View Text Solution

**25.** Let f be a differential function such that $f(x) = f(2 - x)$ and $g(x) = f(1 + x)$ then (1) g(x) is an odd function (2) g(x) is an even function

(3) graph of f(x) is symmetrical about the line x= 1 (4) $f'(1) = 0$

A. g(x) is an odd function

B. g(x) is an even function

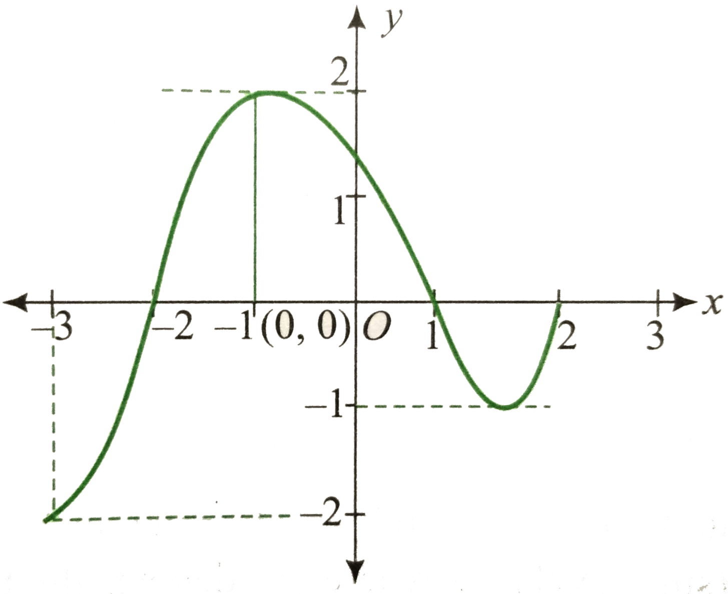
C. Graph of $f(x)$ is symmetrical about the line $x = 1$

D. $f'(1) = 0$

**Answer: B::C::D**

Watch Video Solution

**26.** The figure illustrates the graph of the function $y = f(x)$ defined in [-3, 2].




Identify the correct statement(s)?

A. Range of $y = f( - |x|)$ is $[ - 2, 2]$

B. Domain of $y = f(|x|)$ is $[ - 2, 2]$

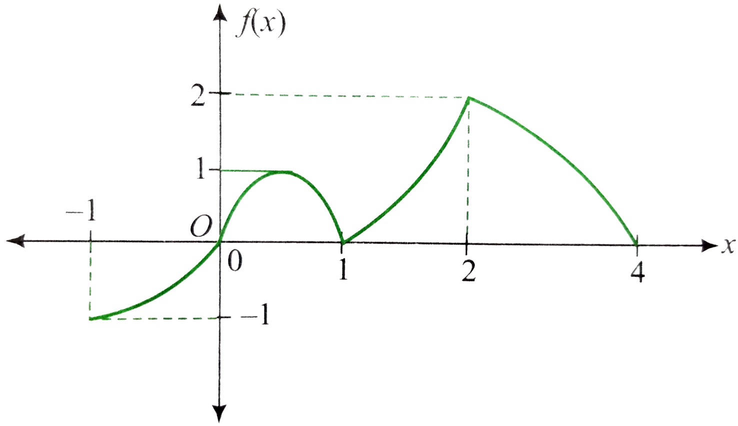
C. Domain of $y = f|x| + 1)$ is $[ - 1, 1]$

D. Range of $y = f(|x| + 1)$ is $[ - 1, 0]$

**Answer: A::B::C::D**

Watch Video Solution

**27.** If graph of a function $f(x)$ which is defined in [-1, 4] is shown in the following figure then identify the correct statement(s).




A. domain of $f(|x| - 1)$ is $[-5, 5]$

B. range of $f(|x| + 1)$ is $[0, 2]$

C. range of $f(-|x|)$ is $[-1, 0]$

D. domain of $f(|x|)$ is $[-3, 3]$

**Answer: A::B::C**

Watch Video Solution

Exercise (Comprehension)

**1.** Consider the functions

$$f(x) = \begin{cases} x+1, & x \le 1 \\ 2x+1, & 1 < x \le 2 \end{cases} \text{ and } g(x) = \begin{cases} x^2, & -1 \le x < 2 \\ x+2, & 2 \le x \le 3 \end{cases}$$

The domain of the function $f(g(x))$ is

A. $\left[0, \sqrt{2}\right]$

B. $[-1, 2]$

C. $\left[-1, \sqrt{2}\right]$

D. None of these

**Answer: C**

Watch Video Solution

**2.** Consider the functions

$$f(x) = \begin{cases} x+1, & x \le 1 \\ 2x+1, & 1 < x \le 2 \end{cases} \text{ and } g(x) = \begin{cases} x^2, & -1 \le x < 2 \\ x+2, & 2 \le x \le 3 \end{cases}$$

The range of the function $f(g(x))$ is

A. $[1, 5]$

B. $[2, 3]$

C. $[1, 2] \cup [3, 5]$

D. None of these

**Answer: C**

Watch Video Solution

**3.** If the function f(x) ={x+1 if $x \leq 1$ , 2x+1 if $1 < x \leq 2$ and g(x) = {$x^2$ , -$1 \leq x \leq 2$ , x+2 $2 \leq x \leq 3$ then the number of roots of the equation f(g(x))=2

A. 1

B. 2

C. 4

D. None of these

**Answer: B**

Watch Video Solution

4. Consider the function $f(x)$ satisfying the identity

$$f(x) + f\left(\frac{x-1}{x}\right) = 1 + x \,\forall x \in R - \{0, 1\}, \text{ and } g(x) = 2f(x) - x + 1.$$

The domain of $y = \sqrt{g(x)}$ is

A. $\left(-\infty, \frac{1-\sqrt{5}}{2}\right] \cup \left[1, \frac{1+\sqrt{5}}{2}\right]$

B. $\left(-\infty, \frac{1-\sqrt{5}}{2}\right] \cup (0, 1) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$

C. $\left[\frac{-1-\sqrt{5}}{2}, 0\right] \cup \left[\frac{-1+\sqrt{5}}{2}, 1\right)$

D. None of these

**Answer: B**

View Text Solution

**5.** Consider the function f(x) satisfyig the identity

$$f(x) + f\left(\frac{x - 1}{x}\right) = 1 + x, \ \forall x \in R - \{0, 1\} \text{ and } g(x) = 2f(x) - x + 1$$

A. $(-\infty, 5]$

B. $[1, \infty)$

C. $(-\infty, 1) \cup [5, \infty)$

D. None of these

**Answer: C**

Watch Video Solution

**6.** Consider the function f(x) satisfyig the identity

$$f(x) + f\left(\frac{x - 1}{x}\right) = 1 + x, \ \forall x \in R - \{0, 1\} \text{ and } g(x) = 2f(x) - x + 1$$

A. 2

B. 1

C. 3

D. 0

**Answer: D**

Watch Video Solution

**7.** If $(f(x))^2 + f\left(\frac{1-x}{1+x}\right) = 64x \,\forall \in D_f$ then

A. $4x^{2/3}\left(\frac{1+x}{1-x}\right)^{1/3}$

B. $x^{1/3}\left(\frac{1-x}{1+x}\right)^{1/3}$

C. $x^{1/3}\left(\frac{1-x}{1+x}\right)^{1/3}$

D. $x\left(\frac{1+x}{1-x}\right)^{1/3}$

**Answer: A**

Watch Video Solution

**8.** If $(f(x))^2 \times f\left(\frac{1-x}{1+x}\right) = 64x \, \forall x \in D_f,$ then

The domain of $f(x)$ is

A. $[0, \infty)$

B. $R - \{1\}$

C. $(-\infty, \infty)$

D. None of these

**Answer: B**

Watch Video Solution

**9.** If $(f(x))^2 \times f\left(\frac{1-x}{1+x}\right) = 64x \, \forall x \in D_f,$ then

The value of $f(9/7)$ is

A. $8(7/9)^{2/3}$

B. $4(9/7)^{1/3}$

C. $-8(9/7)^{2/3}$

D. None of these

**Answer: C**

Watch Video Solution

**10.** $f(x) = \begin{cases} x-1, & -1 \leq x \leq 0 \\ x^2, & 0 \leq x \leq 1 \end{cases}$ and $g(x) = \sin x$

Consider the functions $h_1(x) = f(|g(x)|)$ and $h_2(x) = |f(g(x))|$.

Which of the following is not true about $h_1(x)$?

A. It is a periodic function with period $\pi$.

B. The range is [0, 1].

C. The domain is R.

D. None of these

**Answer: D**

Watch Video Solution

**11.** $f(x) = \begin{cases} x - 1, & -1 \le x \le 0 \\ x^2, & 0 \le x \le 1 \end{cases}$ and $g(x) = \sin x$

Consider the functions $h_1(x) = f(|g(x)|)$ and $h_2(x) = |f(g(x))|$.

Which of the following is not true about $h_2(x)$?

A. The domain is R

B. It is periodic with period $2\pi$.

C. The range is [0, 1].

D. None of these

**Answer: C**

Watch Video Solution

**12.** $f(x) = \begin{cases} x - 1, & -1 \le x \le 0 \\ x^2, & 0 \le x \le 1 \end{cases}$ and $g(x) = \sin x$

Consider the functions $h_1(x) = f(|g(x)|)$ and $h_2(x) = |f(g(x))|$.

If for $h_1(x)$ and $h_2(x)$ are identical functions, then which of the following is not true?

A. Domain of $h_1(x)$ and $h_2(x)$ is $x \in [2n\pi, (2n+1)\pi], n \in Z.$

B. Range of $h_1(x)$ and $h_2(x)$ is $[0, 1]$

C. Period of $h_1(x)$ and $h_2(x)$ is $\pi$

D. None of these

**Answer: C**

View Text Solution

**13.** If $a_0 = x, a_{n+1} = f\left(a_n\right)$, where $n = 0, 1, 2, \ldots,$ then answer thefollowing questions. If $f(x) = m\sqrt{a - x^m}, x < 0, m \le 2, m \in N$,then

A. $a_n = x, n = 2k + 1,$ where k is an integer

B. $a_n = f(x)$ if $n = 2k,$ where k is an integer

C. The inverse of $a_n$ exists for any value of n and m

D. None of these

**Answer: D**

Watch Video Solution

**14.** If $a_0 = x, a_{n+1} = f\left(a_n\right)$, where $n = 0, 1, 2, \ldots,$ then answer the following questions.

If $f(x) = \frac{1}{1 - x}$, then which of the following is not true?

A. $a_n = \frac{1}{1 - x}$ if $n = 3k + 1$

B. $a_n = \frac{x - 1}{x}$ if $n = 3k + 2$

C. $a_n = x$ if $n = 3k$

D. None of these

**Answer: D**

Watch Video Solution

**15.** If $a_0 = x, a_{n+1} = f(a_n)$, where $n = 0, 1, 2, \ldots,$ then answer the following questions.

If $f: R \to R$ is given by $f(x) = 3 + 4x$ and $a_n = A + Bx$, then which of the following is not true?

A. $A + B + 1 = 2^{2n+1}$

B. $|A - B| = 1$

C. $\lim h \to \infty \dfrac{A}{B} = -1$

D. None of these

**Answer: C**

Watch Video Solution

**16.** Let $f(x) = f_1(x) - 2f_2(x)$, where ,where $f_1(x) = \left\{\begin{pmatrix} \min\{x^2, |x|\} & |x| \le 1 \\ \max\{x^2, |x|\} & |x| \le 1 \end{pmatrix}\right.$

and $f_2(x) = \left\{\begin{pmatrix} \min\{x^2, |x|\} & |x| < 1 \\ \{x^2, |x|\} & |x| \le 1 \end{pmatrix}\right.$ and let

$g(x) = \left\{\begin{pmatrix} \min \{f(t): -3 \le t \le x, -3 \le x \le 0\} \\ \max \{f(t): 0 \le t \le x, 0 \le x \le 3\} \end{pmatrix}\right.$ for $-3 \le x \le -1$ the range of $g(x)$ is

A. $[-1, 3]$

B. $[-1, -15]$

C. $[-1, 9]$

D. None of these

**Answer: A**

Watch Video Solution

**17.** Let $f(x) = f_1(x) - 2f_2(x)$, where

where $f(x) = \begin{cases} \min \left\{x^2, |x|\right\}, & |x| \le 1 \\ \max \left\{x^2, |x|\right\}, & |x| > 1 \end{cases}$

and $f_2(x) = \begin{cases} \min \left\{x^2, |x|\right\}, & |x| > 1 \\ \max \left\{x^2, |x|\right\}, & |x| \le 1 \end{cases}$

and let $g(x) = \begin{cases} \min\{f(t): -3 \le t \le x, \ -3 \le x < 0\} \\ \max\{f(t): 0 \le t \le x, \ 0 \le x \le 3\} \end{cases}$

For $x \in (-1, 0)$, $f(x) + g(x)$ is

A. $x^2 - 2x + 1$

B. $x^2 + 2x - 1$

C. $x^2 + 2x + 1$

D. $x^2 - 2x - 1$

**Answer: B**

View Text Solution

**18.** Let $f(x) = f_1(x) - 2f_2(x)$, where ,where $f_1(x) = \left\{\begin{pmatrix} \min\left\{x^2, |x|\right\} & |x| \le 1 \\ \max\left\{x^2, |x|\right\} & |x| \le 1 \end{pmatrix}\right.$

and $f_2(x) = \left\{\begin{pmatrix} \min\left\{x^2, |x|\right\} & |x| < 1 \\ \left\{x^2, |x|\right\} & |x| \le 1 \end{pmatrix}\right.$ and let

$g(x) = \left\{ \begin{pmatrix} \min \{f(t): -3 \leq t \leq x, \ -3 \leq x \leq 0\} \\ \max \{f(t): 0 \leq t \leq x, 0 \leq x \leq 3\} \end{pmatrix} \right.$ for $-3 \leq x \leq -1$ the range of $g(x)$ is

A. 1 point

B. 2 points

C. 3 points

D. None of these

**Answer: A**

Watch Video Solution

**19.** Let $f(x) = \begin{cases} 2x + a, & x \geq -1 \\ bx^2 + 3, & x < -1 \end{cases}$

and $g(x) = \begin{cases} x + 4, & 0 \leq x \leq 4 \\ -3x - 2, & -2 < x < 0 \end{cases}$

$g(f(x))$ is not defined if

A. $a \in (10, \infty), b \in (5, \infty)$

B. $a \in (4, 10), b \in (5, \infty)$

C. $a \in (10, \infty), b \in (0, 1)$

D. $a \in (4, 10), b \in (1, 5)$

**Answer: A**

View Text Solution

**20.** Let $f(x) = \begin{cases} 2x + a, & x \geq -1 \\ bx^2 + 3, & x < -1 \end{cases}$

and $g(x) = \begin{cases} x + 4, & 0 \leq x \leq 4 \\ -3x - 2, & -2 < x < 0 \end{cases}$

If the domain of $g(f(x))$ is $[-1, 4]$, then

A. $a = 1, b > 5$

B. $a = 2, b > 7$

C. $a = 2, b > 10$

D. $a = 0, b \in R$

**Answer: D**

View Text Solution

**21.** Let $f(x) = \begin{cases} 2x + a, & x \geq -1 \\ bx^2 + 3, & x < -1 \end{cases}$

and $g(x) = \begin{cases} x + 4, & 0 \leq x \leq 4 \\ -3x - 2, & -2 < x < 0 \end{cases}$

If $a = 2$ and $b = 3$, then the range of $g(f(x))$ is

A. $(-2, 8]$

B. $(0, 8]$

C. $[4, 8]$

D. $[-1, 8]$

**Answer: C**

Watch Video Solution

**22.** Let $f: R \rightarrow R$ is a function satisfying $f(2 - x) = f(2 + x)$ and $f(20 - x) = f(x), \forall x \in R$. On the basis of above information, answer the following questions If $f(0) = 5$, then minimum possible number of values of x satisfying $f(x) = 5$, for $x \in [10, 170]$ is

A. 21

B. 12

C. 11

D. 22

**Answer: C**

Watch Video Solution

**23.** Let $f: R \rightarrow R$ be a function satisfying $f(2 - x) = f(2 + x)$ and $f(20 - x) = f(x) \forall x \in R$. For this function $f$, answer the following.

The graph of $y = f(x)$ is not symmetrial about

A. symmetrical about $x = 2$

B. symmetrical about $x = 10$

C. symmetrical about $x = 8$

D. None of these

**Answer: C**

Watch Video Solution

**24.** Let $f: R \rightarrow R$ be a function satisfying $f(2 - x) = f(2 + x)$ and $f(20 - x) = f(x) \forall x \in R$. For this function $f$, answer the following.

If $f(2) \neq f(6)$, then the

A. fundamental period of $f(x)$ is $1$

B. fundamental period of $f(x)$ may be $1$

C. period of $f(x)$ cannot be 1

D. fundamental period of $f(x)$ is $8$

**Answer: C**

Watch Video Solution

**25.** Consider two functions

$$f(x) = \begin{cases} [x], & -2 \leq x \leq -1 \\ |x| + 1, & -1 < x \leq 2 \end{cases} \text{ and } g(x) = \begin{cases} [x], & -\pi \leq x < 0 \\ \sin x, & 0 \leq x \leq \pi \end{cases}$$

where [.] denotes the greatest integer function.

The number of integral points in the range of $g(f(x))$ is

A. $[0, 2]$

B. $[-2, 0]$

C. $[-2, 2]$

D. $[-2, 2]$

**Answer: C**

Watch Video Solution

**26.** Consider two functions

$$f(x) = \begin{cases} [x], & -2 \le x \le -1 \\ |x| + 1, & -1 < x \le 2 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} [x], & -\pi \le x < 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$$

where [.] denotes the greatest integer function.

The number of integral points in the range of $g(f(x))$ is

A. $[\sin 3, \sin 1]$

B. $[\sin 3, 1] \cup \{-2, -1, 0\}$

C. $[\sin 3, 1] \cup \{-2, -1\}$

D. $[\sin 1, 1]$

**Answer: C**

Watch Video Solution

**27.** Consider two functions

$$f(x) = \begin{cases} [x], & -2 \le x \le -1 \\ |x| + 1, & -1 < x \le 2 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} [x], & -\pi \le x < 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$$

where [.] denotes the greatest integer function.

The number of integral points in the range of $g(f(x))$ is

A. 2

B. 4

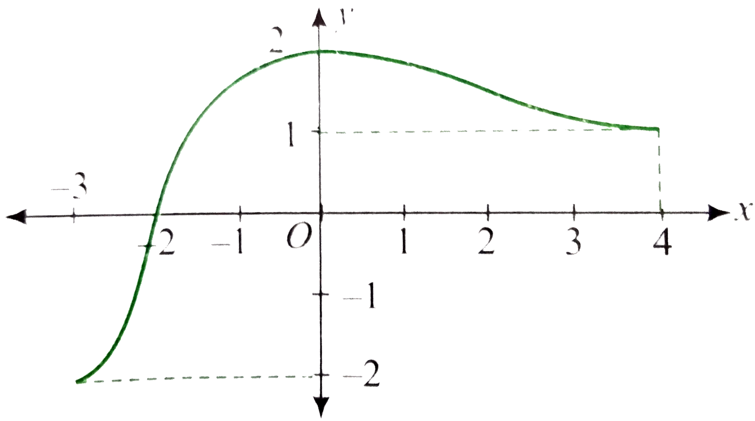
C. 3

D. 5

**Answer: B**

Watch Video Solution

**28.** Consider a function f whose domain is [-3, 4] and range is [-2, 2] with following graph.



Domain and range of $g(x) = f(|x|)$ is [a, b] and [c, d] respectively, then $(b - a + c + d)$ is

A. 11

B. 10

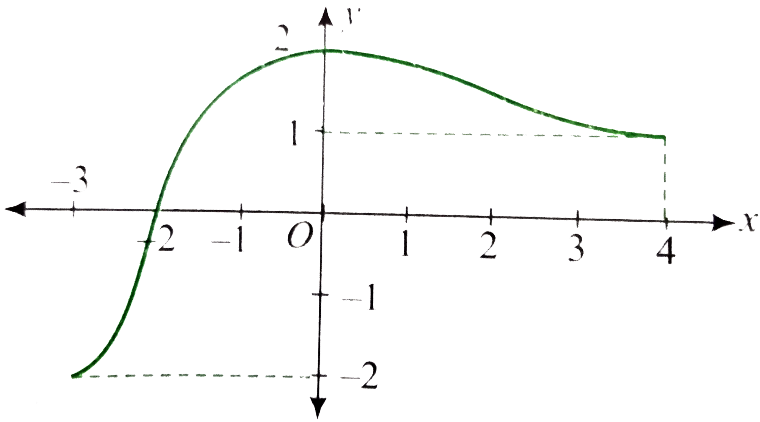
C. 8

D. 7

**Answer: A**

Watch Video Solution

**29.** Consider a function f whose domain is [-3, 4] and range is [-2, 2] with following graph.




If $h(x) = \left|f(x) - \frac{3}{2}\right|$ has range [e, f] and n be number of real solutions of $h(x) = \frac{1}{4}$, then $(n + e + 2f)$ is

A. 8

B. 9

C. 10

D. 11

**Answer: D**

View Text Solution

**30.** Consider a differentiable $f: R \to R$ for which $f(1) = 2$ and $f(x + y) = 2^x f(y) + 4^y f(x) \forall x, y \in R.$

The value of f(4) is

A. 160

B. 240

C. 200

D. None of these

**Answer: A**

Watch Video Solution

**31.** Consider a differentiable $f: R \to R$ for which $f(1) = 2$ and $f(x + y) = 2^x f(y) + 4^y f(x) \forall x, y \in R.$

The minimum value of $f(x)$ is

A. 1

B. $-\frac{1}{2}$

C. $-\frac{1}{4}$

D. None of these

**Answer: C**

Watch Video Solution

**32.** Consider a differentiable $f: R \to R$ for which $f(1) = 2$ and $f(x+y) = 2^x f(y) + 4^y f(x) \forall x, y \in R$.

The number of solutions of $f(x) = 2$ is

A. 0

B. 1

C. 2

D. infinite

**Answer: B**

Watch Video Solution

## Exercise (Matrix)

**1.** The function $f(x)$ is defined on the interval [0, 1]. Now, match the following lists:

| **Lisr I: Function** | **List II: Domain** |
|---|---|
| **a.** $f(\tan x)$ | **p.** $\left[2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right], n \in Z$ |
| **b.** $f(\sin x)$ | **q.** $\left[2n\pi, 2n\pi + \frac{\pi}{6}\right] \cup \left[2n\pi + \frac{5\pi}{6}, (2n+1)\pi\right]$ $n \in Z$ |
| **c.** $f(\cos x)$ | **r.** $[2n\pi, (2n+1)\pi], n \in Z$ |
| **d.** $f(2\sin x)$ | **s.** $\left[n\pi, n\pi + \frac{\pi}{4}\right], n \in Z$ |

View Text Solution

**2.** Match the following lists:

| List I: Function | List II: Type of function |
|---|---|
| **a.** $f(x) = \{(\operatorname{sgn} x)^{\operatorname{sgn} x}\}^n$; $x \neq 0$, $n$ is an odd integer | **p.** odd function |
| **b.** $f(x) = \dfrac{x}{e^x - 1} + \dfrac{x}{2} + 1$ | **q.** even function |
| **c.** $f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$ | **r.** neither odd nor even function |
| **d.** $f(x) = \max\{\tan x, \cot x\}$ | **s.** periodic |

View Text Solution

**3.** Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$. Then match the expressions/statements in List I with expression /statements in List II.

| List I | List II |
|---|---|
| **a.** If $-1 < x < 1$, then $f(x)$ satisfies | **p.** $0 < f(x) < 1$ |
| **b.** If $1 < x < 2$, then $f(x)$ satisfies | **q.** $f(x) < 0$ |
| **c.** If $3 < x < 5$, then $f(x)$ satisfies | **r.** $f(x) > 0$ |
| **d.** If $x > 5$, then $f(x)$ satisfies | **s.** $f(x) < 1$ |

View Text Solution

View Text Solution

**4.** Match the following lists:

| List I: Function | List II: Values of $x$ for which both the functions in any option of List I are identical |
|---|---|
| **a.** $f(x) = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$, $g(x) = 2\tan^{-1}x$ | **p.** $x \in \{-1, 1\}$ |
| **b.** $f(x) = \sin^{-1}(\sin x)$ and $g(x) = \sin(\sin^{-1}x)$ | **q.** $x \in [-1, 1]$ |
| **c.** $f(x) = \log_{x^2} 25$ and $g(x) = \log_x 5$ | **r.** $x \in (-1, 1)$ |
| **d.** $f(x) = \sec^{-1}x + \text{cosec}^{-1}x$, $g(x) = \sin^{-1}x + \cos^{-1}x$ | **s.** $x \in (0, 1)$ |

View Text Solution

**5.** Match the following lists:

| List I | List II |
|---|---|
| **a.** $f: R \to \left[\frac{3\pi}{4}, \pi\right)$ and $f(x) = \cot^{-1}(2x - x^2 - 2)$. Then $f(x)$ is | **p.** one-one |
| **b.** $f: R \to R$ and $f(x) = e^x \sin x$. Then $f(x)$ is | **q.** into |
| **c.** $f: R^+ \to [4, \infty]$ and $f(x) = 4 + 3x^2$. Then $f(x)$ is | **r.** many-one |
| **d.** $f: X \to X$ and $f(f(x)) = x \ \forall\ x \in X$. Then $f(x)$ is | **s.** onto |

View Text Solution

**6.** Match the following lists:

| List I: Function | List II: Fundamental Period |
|---|---|
| **a.** $f(x) = \cos(\lvert\sin x\rvert - \lvert\cos x\rvert)$ | **p.** $\pi$ |
| **b.** $f(x) = \cos(\tan x + \cot x) \times \cos(\tan x - \cot x)$ | **q.** $\pi/2$ |
| **c.** $f(x) = \sin^{-1}(\sin x) + e^{\tan x}$ | **r.** $4\pi$ |
| **d.** $f(x) = \sin^3 x \sin 3x$ | **s.** $2\pi$ |

View Text Solution

**7.** { . } denotes the fractional part function and [.] denotes the greatest integer function. Now, match the following lists:

| List I: Function | List II: Period |
|---|---|
| **a.** $f(x) = e^{\cos^4 \pi x + x - [x] + \cos^2 \pi x}$ | **p.** 1/3 |
| **b.** $f(x) = \cos 2\pi\{2x\} + \sin 2\pi\{2x\}$ | **q.** 1/4 |
| **c.** $f(x) = \sin 3\pi\{x\} + \tan \pi[x]$ | **r.** 1/2 |
| **d.** $f(x) = 3x - [3x + a] - b$, where $a, b \in R^+$ | **s.** 1 |

Watch Video Solution

**8.** Match the following lists and then choose the correct code.

| List I: Function | List II: Range |
|---|---|
| **a.** $f(x) = \log_3(5 + 4x - x^2)$ | **p.** Function not defined |
| **b.** $f(x) = \log_3(x^2 - 4x - 5)$ | **q.** $[0, \infty)$ |
| **c.** $f(x) = \log_3(x^2 - 4x + 5)$ | **r.** $(-\infty, 2]$ |
| **d.** $f(x) = \log_3(4x - 5 - x^2)$ | **s.** $R$ |

A. $\begin{matrix} a & b & c & d \\ p & r & s & q \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ r & s & q & p \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ r & q & s & p \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ p & q & s & r \end{matrix}$

**Answer: B**

Watch Video Solution

**9.** Match the following lists and then choose the correct code.

| List I: Equation | List II: Number of roots |
|---|---|
| **a.** $x^2 \tan x = 1, x \in [0, 2\pi]$ | **p.** 5 |
| **b.** $2^{\cos x} = \lvert\sin x\rvert, x \in [0, 2\pi]$ | **q.** 2 |
| **c.** If $f(x)$ is a polynomial of degree 5 with real coefficients such that $f(\lvert x \rvert) = 0$ has 8 real roots, then the number of roots of $f(x) = 0$ | **r.** 3 |
| **d.** $7^{\lvert x \rvert}(\lvert 5 - \lvert x \rvert\rvert) = 1$ | **s.** 4 |

A. $\begin{matrix} a & b & c & d \\ q & s & r & p \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ p & s & r & s \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ q & s & p & s \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ s & p & q & r \end{matrix}$

**Answer: C**

View Text Solution

## Exercise (Numerical)

**1.** Let $f$ be a real-valued invertible function such that $f\left(\frac{-32x}{x-2}\right) = 5x - 2, x \neq 2$. Then value of $f^{-1}(13)$ is_______

Watch Video Solution

**2.** Let $f(x) = 3x^2 - 7x + c$, where $c$ is a variable coefficient and $x > \frac{7}{6}$. Then the value of $[c]$ such that $f(x)$ touches $f^{-1}(x)$ is (where [.] represents greatest integer function)________

Watch Video Solution

**3.** The number of points on the real line where the function f(x)-log- Ix-3)

is not defined is $f(x) = \log_{|x^2-1|}|x - 3|$ is not defined is

Watch Video Solution

**4.** The number of integral values of $x$ for which

$$\frac{\left(\frac{\pi}{2^{\tan \wedge ((-1)x-4)}}\right)(x - 4)(x - 10)}{x! - (x - 1)!} < 0is\_\_\_\_$$

Watch Video Solution

**5.** If $f(x) = \left\{x\cos x + (\log)_e\left(\frac{1 - x}{1 + x}\right)a; x = 0; x \neq 0\right.$ is odd, then $a\_\_\_\_,$

Watch Video Solution

**6.** The number of integers in the range of the function

$f(x) = \left|4\frac{(\sqrt{\cos x} - \sqrt{\sin x})(\sqrt{\cos x} + \sqrt{\sin x})}{(\cos x + \sin x)}\right|$ is ________.

Watch Video Solution

**7.** The number of integers in the domain of function, satisfying

$f(x) + f(x^{-1}) = \frac{x^3 + 1}{x}, is\_\_\_$

Watch Video Solution

**8.** If a polynomial function $f(x)$ satisfies $f(f(f(x)) = 8x + 21,$ where $pandq$ are real numbers, then $p + q$ is equal to ______

Watch Video Solution

**9.** If $f(x)$ is an odd function, $f(1) = 3, f(x + 2) = f(x) + f(2),$ then the value of $f(3)$ is_______

Watch Video Solution

**10.** Let $f: R\vec{R}$ be a continuous onto function satisfying $f(x) + f(-x) = 0 \forall x \in R^{\cdot}$ If $f(-3) = 2 and f(5) = 4 \in [-5, 5],$ then the minimum number of roots of the equation $f(x) = 0$ is

Watch Video Solution

**11.** The set of all real values of x for which the funciton $f(x) = \sqrt{\sin x + \cos x} + \sqrt{7x - x^2 - 6}$ takes real values is

Watch Video Solution

**12.** Suppose that $f$ is an even, periodic function with period $2, and that f(x) = x$ for all $x$ in the interval $[0, 1]$ . The values of $[10f(3.14)]$ is(where [.] represents the greatest integer function) _____

Watch Video Solution

**13.** If $f(x) = \sqrt{4 - x^2} + \sqrt{x^2 - 1}$ , then the maximum value of $(f(x))^2$ is ___________

Watch Video Solution

**14.** The function $f(x) = \dfrac{x + 1}{x^3 + 1}$ can be written as the sum of an even function $g(x)$ and an odd function $h(x)$ . Then the value of $|g(0)|$ is__________

Watch Video Solution

**15.** If $T$ is the period of the function $f(x) = [8x + 7] + |\tan 2\pi x + \cot 2\pi x| - 8x]$ (where [.] denotes the greatest integer function), then the value of $\frac{1}{T}$ is

__________

Watch Video Solution

**16.** An even polynomial function $f(x)$ satisfies a relation $f(2x)\left(1 - f\left(\frac{1}{2x}\right)\right) + f \cdot 16^2 y\Big) = f(-2) - f(4xy) \forall x, y \in R - \{0\} and f(4) = -255, f($

Then the value of $|(f(2) + 1)/2|$ is________

Watch Video Solution

**17.** If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right) and g\left(\frac{5}{4}\right) = 1,$ then $(gof)(x)$ is __________

Watch Video Solution

**18.** Let $E = \{1, 2, 3, 4, \}$ and $F = \{1, 2\}$. Then the number of onto functions from E to F, is _____.

Watch Video Solution

**19.** The function of $f$ is continuous and has the property $f(f(x)) = 1 - x$.

Then the value of $f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right)$ is

Watch Video Solution

**20.** A function $f$ from integers to integers is defined as

$$f(n) = \begin{cases} n + 3, & n \in odd \\ n/2, & n \in even \end{cases}$$

Suppose $k \in odd$ and $f(f(f(k))) = 27$. Then the value of k is _______

Watch Video Solution

**21.** If $\theta$ is the fundamental period of the function $f(x) = \sin^{99}x + \sin^{99}\left(x + \frac{2\pi}{3}\right) + \sin^{99}\left(x + \frac{4\pi}{3}\right)$, then the complex number $z = |z|(\cos\theta + i\sin\theta)$ lies in the quadrant number.

Watch Video Solution

**22.** If $x = \frac{4}{9}$ satisfies the equation $(\log)_a\left(x^2 - x + 2\right) > (\log)_a\left( - x^2 + 2x + 3\right)$, then the sum of all possible distinct values of $[x]$ is (where[.] represetns the greatest integer function)

___

Watch Video Solution

**23.** If $4^x - 2^{x+2} + 5 + ||b - 1| - 3| - \sin y \mid , x, y, b \in R,$ then the possible value of $b$ is________

Watch Video Solution

**24.** If $f : N \rightarrow N,$ and $x_2 > x_1 \Rightarrow f\left(x_2\right) > f(x) \forall x_1, x_2 \in N$ and $f(f(n)) = 3n \forall n \in N,$ t

_______.

Watch Video Solution

**25.** The number of integral values of $a$ for which $f(x) = \log\left((\log)_{\frac{1}{3}}\left((\log)_7(\sin x + a)\right)\right)$ is defined for every real value of $x$ is

_________

Watch Video Solution

**26.** Let $f(x) = \sin^{23}x - \cos^{22}x and g(x) = 1 + \frac{1}{2}\tan^{-1}|x|$ . Then the number of values of $x$ in the interval $[-10\pi, 8\pi]$ satisfying the equation $f(x) = sgn(g(x))$ is __________

Watch Video Solution

**27.** Suppose that $f(x)$ is a function of the form

$f(x) = \dfrac{ax^8 + bx^6 + cx^4 + dx^2 + 15x + 1}{x}$, $(x \neq 0)$. If $f(5) = 2$, then the value of $f(-5)$ is ________.

Watch Video Solution

**28.** If $f:(2, -\infty) \to [8, \infty)$ is a surjective function defined by $f(x) = x^2 - (p-2)x + 3p - 2, p \in R$ then sum of values of p is $m + \sqrt{n}$, where $m, n \in N$. Find the value of $\dfrac{n}{m}$.

Watch Video Solution

**29.** Period of the function

$f(x) = \sin\left(\dfrac{x}{2}\right)\cos\left[\dfrac{x}{2}\right] - \cos\left(\dfrac{x}{2}\right)\sin\left[\dfrac{x}{2}\right]$, where [.] denotes the greatest integer function, is ________.

Watch Video Solution

**30.** If the interval x satisfying the equation

$$[x] + [-x] = \frac{\log_3(x-2)}{\left|\log_3(x-2)\right|} \text{ is } (a, b), \text{ then } a + b = \_\_\_\_\_\_.$$

Watch Video Solution

**31.** Let $f(x)$ be a polynomial of degree 5 such that $f(|x|) = 0$ has 8 real distinct , Then number of real roots of $f(x) = 0$ is _______.

Watch Video Solution

## JEE Previous Year

**1.** For real x, let $f(x) = x^3 + 5x + 1$ , then (1) f is oneone but not onto R (2) f is onto R but not oneone (3) f is oneone and onto R (4) f is neither oneone nor onto R

A. $f$ is one-one but not onto R

B. $f$ is onto R but not one-one

C. $f$ is one-one and onto R

D. $f$ is neither one-one nor onto R

**Answer: C**

Watch Video Solution

**2.** Let $f : [-1, \infty] \in [-1, \infty]$ be a function given $f(x) = (x+1)^2 - 1, x \geq -1$

Statement-1: The set $\left[x : f(x) = f^{-1}(x)\right] = \{0, 1\}$

Statement-2: f is a bijection.

A. Statement 1 is ture, statement 2 is true, statement 2 is a correct explanation for statement 1.

B. Statement 1 is ture, statement 2 is true, statement 2 is not a correct explanation for statement 1.

C. Statement 1 is ture, statement 2 is false.

D. Statement 1 is false, statement 2 is true.

**Answer: C**

Watch Video Solution

**3.** Consider the following relations: R = {(x, y) | x, y are real numbers and x = wy for some rational number w};

$S = \left\{ \left( \frac{m}{n}, \frac{p}{q} \right) \text{m , n , pandqa r ei n t e g e r ss u c ht h a tn ,q} \neq 0\text{andq m} = \text{p n} \right\}$

. Then (1) neither R nor S is an equivalence relation (2) S is an equivalence relation but R is not an equivalence relation (3) R and S both are equivalence relations (4) R is an equivalence relation but S is not an equivalence relation

A. R and S both are equivalence relations.

B. R is an equivalence relation but S is not an equivalence relation.

C. Neither R nor S is an equivalence relation.

D. S is an equivalence relation but R is not an equivalence relation.

**Answer: D**

Watch Video Solution

**4.** Let R be the set of real numbers.

Statement 1:$A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.

Statement 2: $B = \{x, y\} \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R.

A. Statement 1 is false, statement 2 is true.

B. Statement 1 is ture, statement 2 is true, statement 2 is a correct explanation for statement 1.

C. Statement 1 is true, statement 2 is true, statement 2 is not a correct explanation for statement 1.

D. Statement 1 is false, statement 2 is false.

**Answer: D**

Watch Video Solution

5. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is: (1) $( - \infty, \infty)$ (2) $(0, \infty$ (3) $( - \infty, 0)$ (4) $( - \infty, \infty)\text{-}\{0\}$

A. $( - \infty, \infty)\sim\{0\}$

B. $( - \infty, \infty)$

C. $(0, \infty)$

D. (-oo,0)`

**Answer: D**

Watch Video Solution

6. If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where [x] denotes the greatest integer $\leq x$) has no integral solution, then all

possible values of a lie in the interval: (1) (-2,-1) (2) $(\infty, -2) \cup (2, \infty)$ (3) $(-1, 0) \cup (0, 1)$ (4) (1,2)

A. $(-1, 0) \cup (0, 1)$

B. $(1, 2)$

C. $(-2, -1)$

D. $(-\infty, -2) \cup (2, \infty)$

**Answer: A**

Watch Video Solution

**7.** If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0,$ and $S = \{x \in R : f(x) = f(-x)\}$ ; then S: (1) is an empty set. (2) contains exactly one element. (3) contains exactly two elements. (4) contains more than two elements

A. contains exactly one element

B. contains exactly two elements

C. contains more than two elements

D. is an empty set

**Answer: B**

Watch Video Solution

**8.** The function $f : R \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$ is

A. neither injective nor surjective.

B. invertible.

C. injective but not surjective.

D. Surjective but not injective.

**Answer: D**

Watch Video Solution

9. Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + B + c = 3$ and $f(x + y) = f(x) + f(y) + xy, \forall x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to

A. 255

B. 330

C. 165

D. 190

**Answer: B**

Watch Video Solution

10. Let $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$ be given by $f(x) = (\log(\sec x + \tan x))^3$ Then which of the following is wrong?

A. $f(x)$ is an odd function

B. $f(x)$ is a one-one function

C. $f(x)$ is an onto function

D. $f(x)$ is an event function

**Answer: A::B::C**

Watch Video Solution

**11.** Let $f(x) = \sin\left[\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right]$ for all $x \in \mathbb{R}$

A. Range of $f$ is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

B. Range of fog is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

C. $\lim x \to 0 \frac{f(x)}{g(x)} = \frac{\pi}{6}$

D. There is an $x \in R$ such that $(gof)(x) = 1$

**Answer: A::B::C**

Watch Video Solution

12. Let

$E_1 = \left\{x \in R : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$ and $E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)\right.$ i

(Here, the inverse trigonometric function $\sin^{-1}x$ assumes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.)

Let $f : E_1 \to R$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$ and $g : E_2 \to R$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$.

| List I | List II |
| --- | --- |
| a. The range of $f$ is | p. $\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$ |
| b. The range of $g$ contains | q. $(0, 1)$ |
| c. The domain of $f$ contains | r. $[-1/2, 1/2]$ |
| d. the domain of $g$ is | s. $(-\infty, 0) \cup (0, \infty)$ |
| | t. $\left(-\infty, \frac{e}{e-1}\right]$ |
| | u. $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$ |

The correct option is

A. $a \rightarrow s, b \rightarrow q, c \rightarrow p, d \rightarrow p$

B. $a \rightarrow r, b \rightarrow r, c \rightarrow u, d \rightarrow t$

C. $a \rightarrow s, b \rightarrow q, c \rightarrow p, d \rightarrow u$

D. $a \rightarrow s, b \rightarrow r, c \rightarrow u, d \rightarrow t$

**Answer: A**

View Text Solution

## Solved Examples And Exercises

**1.** Find the inverse of the function: $f:(-\infty, 1] \rightarrow \left[\frac{1}{2}, \infty\right], wheref(x) = 2^{x(x-2)}$

Watch Video Solution

**2.** Find the value of $x$ for which function are identical. $f(x) = xandg(x) = \frac{1}{1/x}$

Watch Video Solution

**3.** Find the value of $x$ for which function are identical. $f(x) = \cos xandg(x) = \frac{1}{\sqrt{1 + \tan^2 x}}$

Watch Video Solution

**4.** Find the value of $x$ for which function are identical.

$f(x) = \frac{\sqrt{9 - x^2}}{\sqrt{x - 2}} andg(x) = \sqrt{\frac{9 - x^2}{x - 2}}$

Watch Video Solution

**5.** Find the inverse of the function: $f: R \rightarrow (-\infty, 1) givenbyf(x) = 1 - 2^{-x}$

Watch Video Solution

**6.** $f: (2, 3)\overrightarrow{0, 1} def \in edbyf(x) = x - [x], where[.]$ represents the greatest integer function.

Watch Video Solution

**7.** Find the inverse of the function: $f(x) = \{x^3 - 1, , x < 2x^2 + 3, x \geq 2$

Watch Video Solution

**8.** Find the inverse of the function: $f:[-1,1] \to [-1,1]$defined by $f(x) = x|x|$

Watch Video Solution

**9.** If $f(x+y+1) = \left\{\sqrt{f(x)} + \sqrt{f(y)}\right\}^2$ and $f(0) = 1 \forall x, y \in R, determ \in ef(n), n \in N$

Watch Video Solution

**10.** Let $f(x) = \dfrac{9^x}{9^x + 3}$. Show $f(x) + f(1-x) = 1$ and, hence, evaluate.

$$f\left(\frac{1}{1996}\right) + f\left(\frac{2}{1996}\right) + f\left(\frac{3}{1996}\right) + \; + f\left(\frac{1995}{1996}\right)$$

Watch Video Solution

**11.** If $f(x+2a) = f(x-2a), thenprovethatf(x)isperiodi \cdot$

Watch Video Solution

**12.** Let $g(x)$ be a function such that $g(a+b) = g(a)g(b) \forall a, b \in R$. If zero is not an element in the range of $g$, then find the value of $g(x)g(-x)$.

Watch Video Solution

**13.** Find the value of $x$ for which function are identical. $f(x) = \tan^{-1}x + \frac{\tan^{-1}1}{x} and g(x) = \sin^{-1}x + \cos^{-1}x$

Watch Video Solution

**14.** The period of $f(x) = [x] + [2x] + [3x] + [4x] + [nx] - \frac{n(n+1)}{2}x$, where $n \in N$, is (where [.] represents greatest integer function). $n$ (b) 1 (c) $\frac{1}{n}$ (d) none of these

Watch Video Solution

**15.** Plot $y = |x|, y = |x - 2|,$ and $y = |x + 2|$

Watch Video Solution

**16.** If $f\left(x + \frac{1}{2}\right) + f\left(x - \frac{1}{2}\right) = f(x)f \text{ or } allx \in R,$ then the period of $f(x)$ is 1 (b) 2 (c) 3 (d) 4

Watch Video Solution

**17.** If for all real values of $uandv, 2f(u)\cos v = (u + v) + f(u - v),$ prove that for all real values of $x,$ $f(x) + f( - x) = 2a\cos x^{\cdot}$ $f(\pi - x) + f( - x) = 0$ $f(\pi - x) + f(x) = 2b\sin x^{\cdot}$ Deduce that $f(x) = a\cos x + b\sin x, wherea, b$ are arbitrary constants.

Watch Video Solution

**18.** If the period of $\frac{\cos(\sin(nx))}{\tan\left(\frac{x}{n}\right)}, n \in N, is 6\pi$, then $n =$ 3 (b) 2 (c) 6 (d) 1

Watch Video Solution

**19.** If $f: X \rightarrow [1, \infty)$ is a function defined as $f(x) = 1 + 3x^3$, find the superset of all the sets $X$ such that $f(x)$ is one-one.

Watch Video Solution

**20.** Find the period (if periodic) of the following function ([.] denotes the greatest integer functions): $f(x) = \frac{\tan\pi}{2}[x]$

Watch Video Solution

**21.** Solve $(x - 2)[x] = \{x\} - 1$, (where $[x] and \{x\}$ denote the greatest integer function less than or equal to $x$ and the fractional part function, respectively).

Watch Video Solution

**22.** The domain of $f(x) = \ln\left(ax^3 + (a+b)x^2 + (b+c)x + c\right)$, where $a > 0, b^2 - 4ac = 0, is(where[.]$ represents greatest integer function).

Watch Video Solution

**23.** If $f: R^{+} \vec{R}, f(x) + 3xf\left(\frac{1}{x}\right) = 2(x+1), then f \in df(x)^{\cdot}$

Watch Video Solution

**24.** Write the equivalent (piecewise) definition of $f(x) = sgn(\sin x)^{\cdot}$

Watch Video Solution

**25.** If $f(x) = \{x^2$, for $x \geq 1x$ ,for $x < 0$,t h e n$fof(x)$ is given by

Watch Video Solution

**26.** If $f(x + f(y)) = f(x) + y \forall x, y \in Randf(0) = 1$, then find the value of $f(7)$

Watch Video Solution

**27.** Let $f: R \rightarrow R$, where $f(x) = \sin x$, Show that f is into.

Watch Video Solution

**28.** Draw the graph of the function: $f(x) = \log|x|$

Watch Video Solution

**29.** If the graph of $y = f(x)$ is symmetrical about the lines $x = 1 and x = 2$, then which of the following is true? (a)$f(x + 1) = f(x)$ (b) $f(x + 3) = f(x)$ (c) $f(x + 2) = f(x)$ (d) None of these

Watch Video Solution

**30.** Find the domain of $f(x)\dfrac{\sqrt{(1 - \sin x)}}{(\log)_5\left(1 - 4x^2\right)} + \cos^{-1}(1 - \{x\})$.

Watch Video Solution

**31.** Sketch the curve $|y| = (x - 1)(x - 2)$.

Watch Video Solution

**32.** If $f(x)$ is a real-valued function defined as $f(x) = In(1 - \sin x)$, then the graph of $f(x)$ is (A) symmetric about the line $x = \pi$ (B) symmetric about the y-axis (C) symmetric and the line $x = \frac{\pi}{2}$ (D) symmetric about the origin

Watch Video Solution

**33.** Find the range of $f(x) = \cos\left((\log)_e\{x\}\right)$.

Watch Video Solution

**34.** Plot $y = \sin x and y = 2\sin x$.

Watch Video Solution

**35.** If $f : x \to y$, where $x$ and $y$ are sets containing natural numbers, $f(x) = \frac{x+5}{x+2}$ then the number of elements in the domain and range of $f(x)$ are, respectively. (a) $1 \text{ and } 1$ (b) $2 \text{ and } 1$ (c) $2 \text{ and } 2$ (d) $1 \text{ and } 2$

Watch Video Solution

**36.** Find the domain and range of $f(x) = \cos^{-1}\sqrt{(\log)_{[x]}\left(\frac{|x|}{x}\right)}$

Watch Video Solution

**37.** Consider the function $f(x) = \{2x + 3, x \leq 1 - x^2 + 6, x > 1$

Watch Video Solution

**38.** Find the range of $f(x) = (\log)_{[x-1]}\sin x$

Watch Video Solution

**39.** Plot $y = \sin x and y = \sin\left(\frac{x}{2}\right)$

Watch Video Solution

**40.** In the questions, $[x] and \{x\}$ represent the greatest integer function and the fractional part function, respectively. If $y = 3[x] + 1 = 4[x - 1] - 10$, then find the value of $[x + 2y]$.

Watch Video Solution

**41.** Plot $y = \sin x and y = \sin 2x$.

Watch Video Solution

**42.** Let $f(x) = x + 2|x + 1| + 2|x - 1|$. If $f(x) = k$ has exactly one real solution, then the value of $k$ is (a)3 (b) 0 (c)1 (d) 2

Watch Video Solution

**43.** Sketch the curve $y = \log|x|$

Watch Video Solution

**44.** Find the domain of $f(x) = \frac{1}{\sqrt{x - [x]}}$ (b) $f(x) = \frac{1}{\log[x]}$ $f(x) = \log\{x\}$

Watch Video Solution

**45.** The domain of $f(x) = \sin^{-1}\left[2x^2 - 3\right]$, *where*[.] denotes the greatest integer function, is $\left(-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right)$ $\left(-\sqrt{\frac{3}{2}}, -1\right) \cup \left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$ $\left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$ $\left(-\sqrt{\frac{5}{2}}, -1\right) \cup \left(1, \sqrt{\frac{5}{2}}\right)$

Watch Video Solution

**46.** Find the domain of $f(x) = \frac{1}{\sqrt{|[|x| - 1]| - 5}}$

Watch Video Solution

**47.** Sketch the graph for $y = |\sin x|$

Watch Video Solution

**48.** Draw the graph for $y = |\log x|$

Watch Video Solution

**49.** Find the domain of $f(x) = \dfrac{1}{\sqrt{|[|x| - 1]| - 5}}$

Watch Video Solution

**50.** Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be two one-one and onto functions such that they are mirror images of each other about the line $y = a$. If $h(x) = f(x) + g(x)$, then $h(x)$ is (A) one-one onto (B) one-one into (D) many-one into (C) many-one onto

Watch Video Solution

51. Plot the curve $y = (\log)_e(-x)$.

Watch Video Solution

52. Find the domain of the function : $f(x) = \frac{3}{4 - x^2} + (\log)_{10}\left(x^3 - x\right)$

Watch Video Solution

53. Let a function $f(x)$ satisfies $f(x) + f(2x) + f(2 - x) + f(1 + x) = x, \forall x \in R$.

Then find the value of $f(0)$.

Watch Video Solution

54. Find the domain of the function : $f(x) = \sqrt{\frac{(\log)_{0.3}|x - 2|}{|x|}}$

Watch Video Solution

**55.** Find the domain of the function : $f(x) = \sqrt{(\log)_{10}\left\{\frac{(\log)_{10}x}{2\left(3 - (\log)_{10}x\right)}\right\}}$

Watch Video Solution

**56.** Let $f$ be a function satisfying of $x^{\cdot}$ Then $f(xy) = \frac{f(x)}{y}$ for all positive real numbers $xandy^{\cdot}$ If $f(30) = 20,$ then find the value of $f(40)^{\cdot}$

Watch Video Solution

**57.** Find the domain of the function : $f(x) = \frac{1}{\sqrt{(\log)_{\frac{1}{2}}\left(x^2 - 7x + 13\right)}}$

Watch Video Solution

**58.** If $f(x)$ is a polynomial function satisfying $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(4) = 65, thenf \in df(6)$

Watch Video Solution

**59.** Let $f: R\vec{R}$ be a continuous onto function satisfying $f(x) + f(-x) = 0 \forall x \in R$ If $f(-3) = 2andf(5) = 4 \in [-5, 5]$, then the minimum number of roots of the equation $f(x) = 0$ is

Watch Video Solution

**60.** Find the range of $f(x) = (\log)_e x - \frac{\left((\log)_e x\right)^2}{\left|(\log)_e x\right|}$

Watch Video Solution

**61.** Let $f$ be a real-valued function such that $f(x) + 2f\left(\frac{2002}{x}\right) = 3x$. Then find $f(x)$.

Watch Video Solution

**62.** If $f(x)$ is an odd function, $f(1) = 3, f(x+2) = f(x) + f(2),$ then the value of $f(3)$ is_______

Watch Video Solution

**63.** Find the domain of the function : $f(x) = \sqrt{4^x + 8\left(\frac{2}{3}\right)^{(x-2)} - 13 - 2^{2(x-1)}}$

Watch Video Solution

**64.** The graph of $(y - x)$against$(y + x)$ is shown. fig which one of the following shows the graph of $y$ against $x$? fig (b) fig (c) fig (d) fig

Watch Video Solution

**65.** If $f: R\vec{R}$ is an odd function such that $f(1 + x) = 1 + f(x)$ $x^2 f\left(\frac{1}{x}\right) = f(x), x \neq 0$ then find $f(x)$

Watch Video Solution

**66.** Find the domain of the function : $f(x) = \sin^{-1}\left((\log)_2 x\right)$

Watch Video Solution

**67.** Each question has four choices, a,b,c and d,out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. if both the statements are true and statement 2 is the correct explanation of statement 1. If both the statements are true but statement 2 is not the correct explanation of statement 1. If statement is True and statement2 is false. If statement1 is false and statement2 is true. Statement 1: $f: N\vec{R}, f(x) = \sin x$ is a one-one function. Statement 2: The period of $\sin \xi s 2\pi and 2\pi$ is an irrational number.

Watch Video Solution

**68.** Let $f: R^{+} \to R$ be a function which satisfies $f(x)\dot{f}(y) = f(xy) + 2\left(\frac{1}{x} + \frac{1}{y} + 1\right)$ for $x, y > 0$. Then find $f(x)^{\cdot}$

Watch Video Solution

**69.** Find the domain of the function : $f(x) = (\log)_{(x-4)}\left(x^2 - 11x + 24\right)$

Watch Video Solution

**70.** If $g:[-2,2] \to R$, where $f(x) = x^3 + \tan x + \left[\frac{x^2+1}{P}\right]$ is an odd function, then the range of parametric P, where [.] denotes the greatest integer function, is

Watch Video Solution

**71.** A continuous function $f(x) on \overrightarrow{R}$ satisfies the relation $f(x) + f(2x+y) + 5xy = f(3x-y) + 2x^2 + 1f$ or $\forall x, y \in R Then \dot{f} \in d\dot{f}(x)$

Watch Video Solution

**72.** Find the range of $f(x) = (\log)_2\left(\frac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}}\right)$

A. [1,2]

B. (1,2)

C. {1,2}

D. none of these

**Answer: A**

Watch Video Solution

**73.** Prove that $f(x) given by f(x+y) = f(x) + f(y) \forall x \in R$ is an odd function.

Watch Video Solution

**74.** If $f(x) = \frac{\sin([x]\pi)}{x^2 + x + 1}$ where [.] denotes the greatest integer function, then (A) f is one-one (B) f is not one-one and not constant (C) f is a constant function (D) none of these

Watch Video Solution

**75.** Find the value of $x$ in $[-\pi, \pi]$ for which $f(x) = \sqrt{(\log)_2\left(4\sin^2 x - 2\sqrt{3}\sin x - 2\sin x + \sqrt{3} + 1\right)}$ is defined.

Watch Video Solution

**76.** If $f(x+y)=f(x)f(y)$ for all real $x, yandf(0) \neq 0,$ then prove that the function $g(x)=\dfrac{f(x)}{1+\{f(x)\}^2}$ is an even function.

Watch Video Solution

**77.** Let $f(x)$ be defined for all $x>0$ and be continuous. Let $f(x)$ satisfies $f\left(\dfrac{x}{y}\right)=f(x)-f(y)$ for all $x, y$ and $f(e)=1.$ Then (a) $f(x)$ is bounded (b) $f\left(\dfrac{1}{x}\right)\vec{0}$ as $x\vec{0}$ (c) $f(x)$ is bounded (d) $f(x)=(\log)_e x$

Watch Video Solution

**78.** Find the domain of $f(x)=\sqrt{x-4-2\sqrt{(x-5)}}-\sqrt{x-4+2\sqrt{(x-5)}}$

Watch Video Solution

**79.** Let $f(x)$ be periodic and $k$ be a positive real number such that $f(x+k)+f(x)=0f\text{ or } allx \in \dot{R}$ Prove that $f(x)$ is periodic with period $2\dot{k}$

Watch Video Solution

**80.** Find the domain of the following functions: $f(x)=\frac{x-3}{(x+3)\sqrt{x^2-4}}$

Watch Video Solution

**81.** Let $S$ be the set of all triangles and $R^{+}$ be the set of positive real numbers. Then the function $f: S \rightarrow R^{+}, f(\Delta)=areaof\Delta, where \in S,$ is injective but not surjective. surjective but not injective injective as well as surjective neither injective nor surjective

Watch Video Solution

**82.** Check whether the function defined by $f(x+\lambda)=1+\sqrt{2f(x)-f^2(x)}$ $\forall x \in R$ is periodic or not. If yes, then find its period $(\lambda > 0)$.

Watch Video Solution

**83.** Solve $(x-1)^2(x+4) < 0$

Watch Video Solution

**84.** If $f(x) = \{x, x \text{is rational } 1-x, x \text{ is irrational}$ ,then $f(f(x))$ is

Watch Video Solution

**85.** An odd function is symmetric about the vertical line $x=a, (a > 0), and$ if $\sum_{r=0}^{\infty}\left[f(1+4r]^r = 8,\right.$ then find the value of $f(1)$.

Watch Video Solution

**86.** Solve $x > \sqrt{(1 - x)}$

Watch Video Solution

**87.** The range of $f(x) = \cos^{-1}\left(\frac{1 + x^2}{2x}\right) + \sqrt{2 - x^2}$ is $\left\{0, 1 + \frac{\pi}{2}\right\}$ (b) $\{0, 1 + \pi)$ $\left\{1, 1 + \frac{\pi}{2}\right\}$ (d) $\{1, 1 + \pi\}$

Watch Video Solution

**88.** If $f(x) = \lambda|\sin x| + \lambda^2|\cos x| + g(\lambda)$ has a period = $\frac{\pi}{2}$ then find the value of $\lambda$

Watch Video Solution

**89.** Find the domain of $f(x) = \sqrt{1 - \sqrt{1 - \sqrt{1 - x^2}}}$

Watch Video Solution

**90.** If $f(x) = (\log)_e\left(\frac{x^2+e}{x^2+1}\right)$, then the range of $f(x)$

Watch Video Solution

**91.** If $f(x)$ satisfies the relation $f(x) + f(x+4) = f(x+2) + f(x+6)$ for all$x$, then prove that $f(x)$ is periodic and find its period.

Watch Video Solution

**92.** Solve $(x - 1)|x + 1|\cos x > 0, f$ or $x \in [-\pi, \pi]$

Watch Video Solution

**93.** The range of $f(x) = \left[|\sin x| + |\cos x|\right]$ Where [.] denotes the greatest integer function, is {0} (b) {0,1} (c) {1} (d) none of these

Watch Video Solution

**94.** Solve $(2x + 1)(x - 3)(x + 7) < 0$

Watch Video Solution

**95.** Which of the following pair(s) of function have same graphs?

$f(x) = \frac{\sec x}{\cos x} - \frac{\tan x}{\cot x}, g(x) = \frac{\cos x}{\sec x} + \frac{\sin x}{cosecx}$

Watch Video Solution

**96.** If the function $f: (1, \infty) \rightarrow (1, \infty)$ is defined by $f(x) = 2^{x(x-1)}, then f^{-1}(x)$ is $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}\left(1 + \sqrt{1 + 4(\log)_2 x}\right)$ $\frac{1}{2}\left(1 - \sqrt{1 + (\log)_2 x}\right.$ (d) not defined

Watch Video Solution

**97.** Solve $\frac{2}{x} < 3$

Watch Video Solution

**98.** Given the graph of $y = f(x)$, which of the following is graph of $y = f(1 - x)$

Watch Video Solution

**99.** The domain of the function $f(x) = \dfrac{1}{\sqrt{4x - \left|x^2 - 10x + 9\right|}}$ is

Watch Video Solution

**100.** Solve $\dfrac{2x - 3}{3x - 5} \geq 3$

Watch Video Solution

**101.** Draw the graph of the function: Solve $\left|\frac{x^2}{x - 1}\right| \leq 1$ using the graphical method.

Watch Video Solution

**102.** The number of solutions of $2\cos x = |\sin x|, 0 \leq x \leq 4\pi,$ is 0 (b) 2 (c) 4 (d) infinite

Watch Video Solution

**103.** Write the equivalent definition and draw the graphs of the following functions. $f(x) = sgn\left((\log)_e|x|\right)$

Watch Video Solution

**104.** Find the total number of solutions of $\sin\pi x = |\ln|x\ |\ |$

Watch Video Solution

Watch Video Solution

**105.** The number of roots of the equation $x\sin x = 1, x \in [-2\pi, 0) \cup (0, 2\pi]$ is 2 (b) 3 (c) 4 (d) 0

Watch Video Solution

**106.** Find the domain of $f(x) = \sqrt{(\log)_{0.4}\left(\frac{x-1}{x+5}\right)}$

Watch Video Solution

**107.** Draw the graph of the function: $f(x) = -|x-1|^{\frac{1}{2}}$

Watch Video Solution

**108.** The number of solutions of $\tan x - mx = 0, m > 1,$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is (A) 1 (B) 2 (C) 3 (D) $m$

Watch Video Solution

**109.** Let $x \in \left(0, \frac{\pi}{2}\right)$. Then find the domain of the function $f(x) = \frac{1}{\sqrt{\left(-(\log)_{\sin x}\tan x\right)}}$

Watch Video Solution

**110.** Draw the graph of the function: $f(x) = \left|x^2 - 3\left|x\right| + 2\right|$

Watch Video Solution

**111.** If $af(x+1) + bf\left(\frac{1}{x+1}\right) = x, x \neq -1, a \neq b, \text{then} f(2)$ is equal to $\frac{2a}{2\left(a^2 - b^2\right)}$ (b) $\frac{a}{a^2 - b^2}$ $\frac{a+2b}{a^2 - b^2}$ (d) none of these

Watch Video Solution

**112.** Find the domain of $f(x) = \sqrt{(0.625)^{4-3x} - (1.6)^{x(x+8)}}$

Watch Video Solution

**113.** Draw the graph of the function: $|f(x)| = \tan x$

Watch Video Solution

**114.** If $x = \frac{4}{9}$ satisfies the equation $(\log)_a\left(x^2 - x + 2\right) > (\log)_a\left(-x^2 + 2x + 3\right)$, then the sum of all possible distinct values of $[x]$ is (where[.] represetns the greatest integer function)

___

Watch Video Solution

**115.** Find the domain and range of $f(x) = \sqrt{(\log)_3\{\cos(\sin x)\}}$

Watch Video Solution

**116.** Draw the graph of the function: $f(x) = ||x - 2| - 3|$

Watch Video Solution

**117.** If $4^{x} - 2^{x+2} + 5 + ||b - 1| - 3| = |\sin y|, x, y, b \in R,$ then the possible value of $b$ is________

Watch Video Solution

**118.** Find the domain of $f(x) = (\log)_{10}(\log)_2(\log)_{\frac{2}{\pi}}\left(\tan^{-1}x\right)^{-1}$

Watch Video Solution

**119.** A function $f$ from integers to integers is defined as $f(x) = \left\{n + 3, n \in odd\frac{n}{2}, n \in even\right.$ suppose $k \in$ odd and $f(f(f(k))) = 27$ .

Then the sum of digits of $k$ is________

Watch Video Solution

**120.** Consider a real-valued function f(x) satisfying $2f(xy) = (f(x))^y + (f(y))^x \forall x, y \in R and f(1) = a, where a \neq 1$. Prove that $(a - 1)$

$$\sum_{i=1}^{n} f(i) = a^{n+1} - a$$

Watch Video Solution

**121.** Find the number of solution of $2^x + 3^x + 4^x - 5^x = 0$

Watch Video Solution

**122.** If $\theta$ is the fundamental period of the function $f(x) = \sin^{99}x + \sin^{99}\left(x + \frac{2\pi}{3}\right) + \sin^{99}\left(x + \frac{4\pi}{3}\right)$, then the complex number $z = |z|(\cos\theta + i\sin\theta)$ lies in the quadrant number.

Watch Video Solution

**123.** Let $f(x) = x + f(x - 1)f$ or $\forall x \in RIff(0) = 1, f \in df(100)$

Watch Video Solution

**124.** Solve $\log_x\left(x^2 - 1\right) \leq 0$

Watch Video Solution

**125.** The function of $f$ is continuous and has the property $f(f(x)) = 1 - x$

Then the value of $f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right)$ is

Watch Video Solution

**126.** Find the domain of function

$f(x) = (\log)_4\left[(\log)_5\left\{(\log)_3\left(18x - x^2 - 77\right\}\right]\right.$

Watch Video Solution

**127.** If the domain of $y = f(x) is[-3, 2]$, then find the domain of $g(x) = f(|[x]|), wher[]$ denotes the greatest integer function.

Watch Video Solution

**128.** The number of integral values of $x$ satisfying the inequality $\left(\frac{3}{4}\right)^{6x+10-x^2} < \frac{27}{64} is\_\_\_\_\_$

Watch Video Solution

**129.** Let $f$ be a function defined on [0,2]. Then find the domain of function $g(x) = f\left(9x^2 - 1\right)$

A. $co_2$

B. null

C. null

D. null

Watch Video Solution

**130.** Find the domain of $f(x) = \sin^{-1}\left\{(\log)_9\left(\frac{x^2}{4}\right)\right\}$

Watch Video Solution

**131.** If $\left[\cot^{-1}x\right] + \left[\cos^{-1}x\right] = 0$ , where [] denotes the greatest integer functions, then the complete set of values of $x$ is $(\cos 1, 1)$ (b) $\cos 1, \cos 1)$ $(\cot 1, 1)$ (d) none of these

Watch Video Solution

**132.** Draw the graph of the function $f(x) = max\dot{\sin}x, \cos 2x, x \in [0, 2\pi\dot{]}$ Write the equivalent definition of $f(x)$ and find the range of the function.

Watch Video Solution

**133.** Let $f(x) = \{1 + |x|, x < -1[x], x \geq -1$ , where [.] denotes the greatest integer function. The find the value of $f\{f(-2, 3)\}^{.}$

Watch Video Solution

**134.** Let $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\vec{R}$ be given by $f(x) = (\log(\sec x + \tan x))^3$ then $f(x)$ is an odd function $f(x)$ is a one-one function $f(x)$ is an onto function $f(x)$ is an even function

Watch Video Solution

**135.** If $f(x) = \log\left[\frac{1 + x}{1 - x}\right]$, then prove that $f\left[\frac{2x}{1 + x^2}\right] = 2f(x)^{.}$

Watch Video Solution

**136.** If $f : R \to R$ and $g : R \to R$ are two given functions, then prove that $2\min . \{f(x) - g(x), 0\} = f(x) - g(x) - |g(x) - f(x)|$

Watch Video Solution

**137.** The range of the function $f(x) = \frac{x^2 + x + 2}{x^2 + x + 1}, x \in R,$ is $(1, \infty)$ (b) $\left(1, \frac{11}{7}\right)$ $\left(1, \frac{7}{3}\right)$ (d) $\left(1, \frac{7}{5}\right)$

Watch Video Solution

**138.** Find the range of the function $f(x) = \cot^{-1}(\log)_{0.5}\left(x^4 - 2x^2 + 3\right)$

Watch Video Solution

**139.** If $f(x) = \sin x + \cos x$and $g(x) = x^2 - 1$, then $g(f(x))$is invertible in the domain .

Watch Video Solution

Watch Video Solution

**140.** Find the domain of $f(x) = \sqrt{\left(\frac{1 - 5^x}{7^{-x} - 7}\right)}$

Watch Video Solution

**141.** If $f$ is the greatest integer function and $g$ is the modulus function, then find the value of $(g0f)\left(-\frac{5}{3}\right) - (fog)\left(-\frac{5}{3}\right)$.

Watch Video Solution

**142.** If the functions $f(x) and g(x)$ are defined on $R \rightarrow R$ such that $f(x) = \{0, x \in rational x, x \in irrational$ $and g(x) = \{O, x \in irrational x, x \in rational$ then $(f - g)(x) is$ (a)one-one and onto (b)neither one-one nor onto (c)one-one but not onto (d)onto but not one-one

Watch Video Solution

**143.** Suppose that $g(x) = 1 + \sqrt{x}$ and $f(g(x)) = 3 + 2\sqrt{x} + x$ Then find the function $f(x)$

Watch Video Solution

**144.** Let $f: R, \vec{R} where f(x) = \dfrac{x^2 + 4x + 7}{x^2 + x + 1}$. Is $f(x) one one$?

Watch Video Solution

**145.** X and Y are two sets and $f: X \rightarrow Y$ If $\{f(c) = y; c \subset X, y \subset Y\}$ and $\{f^{-1}(d) = x; d \subset Y, x \subset X,$ then the true statement is $(a) f\left(f^{-1}(b)\right) = b$ $(b) f^{-1}(f(a)) = a$ $(c) f\left(f^{-1}(b)\right) = b, b \subset y$ $(d) f^{-1}(f(a)) = a, a \subset x$

Watch Video Solution

**146.** The function $f(x)$ is defined in $[0, 1]$. Find the domain of $f(tanx)$

Watch Video Solution

**147.** Find the equivalent definition of $f(x) = \max\left\{x^2, (1-x)^2, 2x(1-x)\right\}$ where $0 \le x \le 1$

Watch Video Solution

**148.** If $y = f(x) = \frac{(x+2)}{(x-1)}, then$ (a)$x = f(y)$ (b) $f(1) = 3$ (c)$y$ increases with $x$for $x < 1$ (d)$f$ is a rational function of $x$

Watch Video Solution

**149.** Consider the function: $f(x) = max1, |x-1|, \min\{4, |3x-1|\}\} \forall x \in R$. Then find the value of $f(3)$

Watch Video Solution

Watch Video Solution

**150.** Let $g(x)$ be a function defined on $[-1, 1]$. If the area of the equilateral triangle with two of its vertices at $(0, 0)$a n d$(x, g(x))$ is (a)$\frac{\sqrt{3}}{4}$, then the function $g(x)$ is (b)$g(x) = \pm\sqrt{1-x^2}$ (c)$g(x) = \sqrt{1-x^2}$ (d)$g(x) = -\sqrt{1-x^2}$ $g(x) = \sqrt{1+x^2}$

Watch Video Solution

**151.** Find the domain of the function: $f(x) = \cos^{-1}\left(1 + 3x + 2x^2\right)$

Watch Video Solution

**152.** Find the period of $f(x) = \sin x + \frac{\tan x}{2} + \frac{\sin x}{2^2} + tan\frac{x}{2^3} + \quad + \frac{\sin x}{2^{n-1}} + \frac{\tan x}{2^n}$

Watch Video Solution

**153.** Find the domain of the function: $f(x) = \dfrac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$

Watch Video Solution

**154.** If $f(x) = \cos\left[\pi^2\right]x + \cos\left[-\pi^2\right]x$, where $[x]$ stands for the greatest integer function, then (a)$f\left(\frac{\pi}{2}\right) = -1$ (b) $f(\pi) = 1$ (c)$f(-\pi) = 0$ (d) $f\left(\frac{\pi}{4}\right) = 1$

Watch Video Solution

**155.** Find the period of $f(x) = \dfrac{\sin(\pi x)}{n!} - \dfrac{\cos(\pi x)}{(n+1)!}$

Watch Video Solution

**156.** Find the domain of the function: $f(x) = \dfrac{\sin^{-1}x}{x}$

Watch Video Solution

**157.** If $f(x) = 3x - 5$, then $f^{-1}(x)$ is given by (a) $\frac{1}{(3x - 5)}$ (b) $\frac{(x + 5)}{3}$ (c)does not exist because $f$ is not one-one (d)does not exist because $f$ is not onto

Watch Video Solution

**158.** if `f(x)={ lnx, 0=1`

Watch Video Solution

**159.** Find the domain of the function: $f(x) = \sin^{-1}(|x - 1| - 2)$

Watch Video Solution

**160.** Find the natural number $a$ for which $\sum_{k=1}^{n} f(a + k) = 16\left(2^n - 1\right)$, where the function $f$ satisfies the relation $f(x + y) = f(x)f(y)$ for all natural number $x, yand, further, f(1) = 2$.

Watch Video Solution

Watch Video Solution

**161.** Let $f(x) = \tan x and g(f(x)) = f\left(x - \frac{\pi}{4}\right),$ where $f(x) and g(x)$ are real valued functions. Prove that $f(g(x)) = \tan\left(\frac{x+1}{x+1}\right)^{\cdot}$

Watch Video Solution

**162.** Find the range of $f(x) = \tan^{-1}\sqrt{\left(x^2 - 2x + 2\right)}$

Watch Video Solution

**163.** The domain of $f(x) is (0, 1)^{\cdot}$ Then the domain of $\left(f\left(e^x\right) + f(1n|x|)\right.$ is $(-1, e)$ (b) $(1, e)$ $(-e, -1)$ (d) $(-e, 1)$

Watch Video Solution

**164.** If $f(x) = 3x - 2 and (gof)^{-1}(x) = x - 2,$ then find the function $g(x)^{\cdot}$

Watch Video Solution

**165.** Find the range of $f(x) = \sqrt{\cos^{-1}\sqrt{\left(1 - x^2\right)} - \sin^{-1}x}$

Watch Video Solution

**166.** The domain of $f(x) = \dfrac{1}{\sqrt{|\cos x| + \cos x}}$ is

Watch Video Solution

**167.** If $f(x) = \left(ax^2 + b\right)^3$, then find the function $g$ such that $f(g(x)) = g(f(x))$.

Watch Video Solution

**168.** Find the domain of the function:

$f(x) = \cos^{-1}\left(\dfrac{6 - 3x}{4}\right) + \operatorname{cosec}^{-1}\left(\dfrac{x - 1}{2}\right)$

Watch Video Solution

**169.** If $f(2x+3y, 2x-7y)=20x,$ then $f(x, y)$ equals :

Watch Video Solution

**170.** Solve the equation $x^2-x+1=\frac{1}{2}+\sqrt{x-\frac{3}{4}}, wherex \geq \frac{3}{4}\cdot$

Watch Video Solution

**171.** Find the domain of the function: $f(x)=\sqrt{\sec^{-1}\left(\frac{2-|x|}{4}\right)}$

Watch Video Solution

**172.** Let $X=\left\{a_1, a_2, , a_6\right\} andY=\left\{b_1, b_2, b_3\right\}^{\cdot}$ The number of functions $f$ from $x \rightarrow y$ such that it is onto and there are exactly three elements $\xi nX$

such that $f(x) = b_1$ is 75 (b) 90 (c) 100 (d) 120

Watch Video Solution

**173.** Find the inverse of $f(x) = \{x, < 1x^2, 1 \leq x \leq 48\sqrt{x}, x > 4$

Watch Video Solution

**174.** Write the equivalent definition and draw the graphs of the following functions. $f(x) = sgn\left(x^3 - x\right)$

Watch Video Solution

**175.** The range of $f(x) = [\sin x + [\cos x + [\tan x + [\sec x]]]], x \in \left(0, \frac{\pi}{4}\right), where$ [.] denotes the greatest integer function less than or equal to $x$, is

Watch Video Solution

**176.** Which of the following functions has inverse function? $f: Z \rightarrow Z$ defined by$f(x) = x + 2$ $f: Z \rightarrow Z$defined by$f(x) = 2x$ $f: Z \rightarrow Z$defined byy $f(x) = x$ $f: Z \rightarrow Z$defined by$f(x) = |x|$

Watch Video Solution

**177.** Find the range of $f(x) = \cos^{-1}\left(\frac{\sqrt{1 + 2x^2}}{1 + x^2}\right)$

Watch Video Solution

**178.** If $f(3x + 2) + f(3x + 29) = 0x \in R,$ then the period of $f(x)$ is 7 (b) 8 (c) 10 (d) none of these

Watch Video Solution

**179.** A function $f$ has domain $[-1, 2]$ and rang $[0, 1]$. Find the domain and range of the function $g$ defined by $g(x) = 1 - f(x + 1)$

Watch Video Solution

**180.** Find the domain for $f(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$

Watch Video Solution

**181.** Let $f: R \to R$ be defined by $f(x) = \left(e^x - e^{-x}\right)/2$. Is $f(x)$ invertible? If so, find is inverse.

Watch Video Solution

**182.** Find the range of $\tan^{-1}\left(\frac{2x}{1+x^2}\right)$

Watch Video Solution

**183.** If $fandg$ are one-one functions, then (a)$f+g$ is one one (b)$fg$ is one one (c)$fog$ is one one (d)$noneofthese$

Watch Video Solution

**184.** Let $A=R-\{3\}, B=R-\{1\},$ and let $f: A\vec{B}$ be defined by $f(x)=\dfrac{x-2}{x-3}$ is $f$ invertible? Explain.

Watch Video Solution

**185.** Find the domain of $f(x)=\sqrt{\cos^{-1}x-\sin^{-1}x}$

Watch Video Solution

**186.** If $T$ is the period of the function $f(x)=[8x+7]+|\tan 2\pi x+\cot 2\pi x|-8x]$ (where [.] denotes the greatest integer function), then the value of $\dfrac{1}{T}$ is

__________

Watch Video Solution

**187.** Solve the equation $x^2 - x + 1 = \frac{1}{2} + \sqrt{x - \frac{3}{4}}, \text{ where} x \geq \frac{3}{4}$.

Watch Video Solution

**188.** Find the range of $f(x) = \sin^{-1}x + \tan^{-1}x + \cos^{-1}x$.

Watch Video Solution

**189.** Find the inverse of $f(x) = \{x, < 1x^2, 1 \leq x \leq 48\sqrt{x}, x > 4$

Watch Video Solution

**190.** The function $f(x) = \frac{x+1}{x^3+1}$ can be written as the sum of an even function $g(x)$ and an odd function $h(x)$ . Then the value of $|g(0)|$

is__________

Watch Video Solution

**191.** Find the domain of $f(x) = \sin^{-1}\left(\frac{x^2}{2}\right)$.

Watch Video Solution

**192.** Find the period of $\cos(\cos x) + \cos(\sin x)$.

Watch Video Solution

**193.** Draw the graph of $y = (\sin 2x)\sqrt{1 + \tan^2 x}$, find its domain and range.

Watch Video Solution

**194.** An even polynomial function $f(x)$ satisfies a relation

$f(2x)\left(1 - f\left(\frac{1}{2x}\right)\right) + f\left(16x^2y\right) = f( - 2) - f(4xy)\,\forall x, y \in R - \{0\} and f(4) = - 255, f($

Then the value of $|(f(2) + 1)/2|$ is________

Watch Video Solution

**195.** For what integral value of n is $3\pi$ the period of the function

$\cos(nx)\sin\left(\frac{5x}{n}\right)?$

Watch Video Solution

**196.** If $a, b$ and $c$ are non-zero rational numbers, then the sum of all the possible values of $\frac{|a|}{a} + \frac{|b|}{b} + \frac{|c|}{c}$ is __________

Watch Video Solution

**197.** Find the range of $f(x) = \dfrac{2\sin^2 x + 2\sin x + 3}{\sin^2 x + \sin x + 1}$

Watch Video Solution

**198.** Identify the following functions whether odd or even or neither: $f(x) = xg(x)g(-x) + \tan(\sin x)$

Watch Video Solution

**199.** Find the range of $f(x) = \dfrac{1}{1 - 3\sqrt{1 - \sin^2 x}}$

Watch Video Solution

**200.** Let $f: R \rightarrow R and g: R \rightarrow R$ be two one-one and onto function such that they are the mirror images of each other about the line $y = a$. If $h(x) = f(x) + g(x), then h(x)$ is (a)one-one and onto (b)only one-one and not onto (c)only onto but not one-one (d)neither one-one nor onto

Watch Video Solution

**201.** Identify the following functions whether odd or even or neither: $f(x) = \cos|x| + \left[\left|\frac{\sin x}{2}\right|\right]$ where [.] denotes the greatest integer function.

Watch Video Solution

**202.** If $x \in [1, 2]$, then find the range of $f(x) = \tan x$

Watch Video Solution

**203.** If $f(x) = (-1)\left[\frac{2}{\pi}\right], g(x) = |\sin x| - |\cos x|, and \varphi(x) = f(x)g(x)$ (where [.] denotes the greatest integer function), then the respective fundamental periods of $f(x), g(x), and \varphi(x)$ are $\pi, \pi, \pi$ (b) $\pi, 2\pi, \pi$ $\pi, \pi, \frac{\pi}{2}$ (d) $\pi, \frac{\pi}{2}, \pi$

Watch Video Solution

**204.** Which of the following function is (are) even, odd, or neither?

$f(x) = x^2\sin x$ $\quad f(x) = \sqrt{1 + x + x^2} - \sqrt{1 - x + x^2}$ $\quad f(x) = \log\left(\frac{1 - x}{1 + x}\right)$

$f(x) = \log\left(x + \sqrt{1 + x^2}\right)$ $f(x) = \sin x - \cos x$ $f(x) = \frac{e^x + e^{-x}}{2}$

Watch Video Solution

**205.** Find the number of solutions of the equation $\sin x = x^2 + x + 1$.

Watch Video Solution

**206.** Let $f(x) = sgn\left(\cot^{-1}x\right) + \tan\left(\frac{\pi}{2}[x]\right)$, where $[x]$ is the greatest integer function less than or equal to $x$, then which of the following alternatives is/are true? $f(x)$ is many-one but not an even function. $f(x)$ is a periodic function. $f(x)$ is a bounded function. The graph of $f(x)$ remains above the x-axis.

Watch Video Solution

**207.** Identify the following functions whether odd or even or neither: $f(x) = \{x|x|f \text{ or } x \leq -1; [1+x]+[1-x]f \text{ or } x\varepsilon(-1, 1); -x|x|f \text{ or } x \geq 1$

Watch Video Solution

**208.** Find the range of $f(x) = \sin^2 x - \sin x + 1$.

Watch Video Solution

**209.** Suppose that $f$ is an even, periodic function with period $2, andthatf(x) = x$ for all $x$ in the interval $[0, 1]$. The values of $[10f(3.14)]$ is(where [.] represents the greatest integer function) _____

Watch Video Solution

**210.** Find the period $f(x) = \sin x + \{x\},$ where {x} is the fractional part of $x$

Watch Video Solution

**211.** Find the range of $f(x) = \frac{1}{2\cos x - 1}$

Watch Video Solution

**212.** If $f(x) = \sin x + \cos ax$ is a periodic function, show that $a$ is a rational number

Watch Video Solution

**213.** The entire graph of the equation $y = x^2 + kx - x + 9$ in strictly above the $x - a\xi s$ if and only if (a)$k < 7$ (b) $-5 < k < 7$ (c)$k > -5$ (d) none of these

Watch Video Solution

**214.** if:$f(x) = \frac{\sin x}{\sqrt{1 + \tan^2 x}} - \frac{\cos x}{\sqrt{1 + \cot^2 x}}$, then find the range of $f(x)$

Watch Video Solution

215. Discuss whether the function $f(x) = \sin(\cos x + x)$ is period or not. If yes, then what is its period?

Watch Video Solution

216. The exhaustive domain of the following function is $f(x) = \sqrt{x^{12} - x^9 + x^4 - x + 1}$ (a)$[0, 1]$ (b) $[1, \infty]$ $[-\infty, 1]$ (d) $R$

Watch Video Solution

217. Find the range of $f(x) = |\sin x| + |\cos x|, x \in R$

Watch Video Solution

218. If $f : R\vec{R}$ is given by $f(x) = \dfrac{x^2 - 4}{x^2 + 1}$, identify the type of function.

Watch Video Solution

Watch Video Solution

**219.** If $f(x) = \left|x^2 - 5x + 6\right|$, then $f'(x)$ equals

Watch Video Solution

**220.** Find the range of $f(\theta) = 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$

Watch Video Solution

**221.** If $f: R\vec{S}$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1, ison \rightarrow$, then find the set $\dot{S}$

Watch Video Solution

**222.** Let $f(x) = x^2 andg(x) = \sin xf$ or $allx \in \dot{R}$ Then the set of all $x$ satisfying $(fogogof)(x) = (gogof)(x), where(fog)(x) = f(g(x)),$ is

$\pm\sqrt{n\pi}, n \in \{0, 1, 2, .\}$ $\pm\sqrt{n\pi}, n \in \{1, 2, .\}$ $\frac{\pi}{2} + 2n\pi, n \in \{, -2, -1, 0, 1, 2\}$ $2n\pi, n \in \{, -2, -1, 0, 1, 2, \}$

Watch Video Solution

**223.** Find the domain of $f(x) = \sqrt{\sin x} + \sqrt{16 - x^2}$

Watch Video Solution

**224.** Show that $f : R\vec{R}$ defined by $f(x) = (x - 1)(x - 2)(x - 3)$ is surjective but not injective.

Watch Video Solution

**225.** If $f(x)$ is a polynomial satisfying $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) andf(3) = 28, thenf(4)$ is equal to 63 (b) 65 (c) 17 (d) none of these

Watch Video Solution

Watch Video Solution

**226.** Solve (a) $\tan x < 2$ $(b)\cos x \leq -\frac{1}{2}$

Watch Video Solution

**227.** If the function $f: R\vec{A}$ given by $f(x) = \frac{x^2}{x^2 + 1}$ is surjection, then find $A$.

Watch Video Solution

**228.** Prove that the least positive value of $x$, satisfying $\tan x = x + 1, lies in the \in terval\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

Watch Video Solution

**229.** The values of $b and c$ for which the identity of $f(x + 1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d, are$ $b = 2, c = 1$ (b) $b = 4, c = -1$

$b = -1, c = 4$ (d) $b = -1, c = 1$

Watch Video Solution

**230.** Find the range of $f(x) = \sec\left(\frac{\pi}{4}\cos^2 x\right)$, where $-\infty < x < \infty$ is

Watch Video Solution

**231.** The number of values of $x$ for which $\left|\left|\left|x^2 - x + 4\right| - 2\right| - 3\right| = x^2 + x - 12$ is_____________

Watch Video Solution

**232.** Which of the following function from Z to itself are bijections? $f(x) = x^3$ (b) $f(x) = x + 2$ $f(x) = 2x + 1$ (d) $f(x) = x^2 + x$

Watch Video Solution

**233.** Find the domain and range of $f(f) = \log\{x\}, where\{\}$ represents the fractional part function).

Watch Video Solution

**234.** The domain of the function $f(x) = \sqrt{x^2 - [x]^2}$ , where $[x]$ is the greatest integer less than or equal to $x,$ is $R$ (b) $[0, +\infty]$ $(-\infty, 0)$ (d) none of these

Watch Video Solution

**235.** If $f: N \rightarrow Zf(n) = \begin{cases} \frac{n-1}{2}; \end{cases}$ when n is odd $= -\frac{n}{2};$ when n is even Identify the type of function

Watch Video Solution

**236.** Find the domain and range of $f(x) = \sin^{-1}[x] wher[]$ represents the greatest function).

Watch Video Solution

**237.** The range of the function $f(x) = |x - 1| + |x - 2|, \ -1 \le x \le 3,$ is [1,3] (b) [1,5] (c) [3,5] (d) none of these

Watch Video Solution

**238.** If $f: R \overset{\rightarrow}{\ }$ is a function such that $f(x) = x^3 + x^2 + 3x + \sin x,$ then identify the type of function.

Watch Video Solution

**239.** Find the domain of the following functions: $f(x) = \sqrt{2 - x} - \dfrac{1}{\sqrt{9 - x^2}}$

Watch Video Solution

Watch Video Solution

**240.** Which of the following functions is the inverse of itself? $f(x) = \frac{1 - x}{1 + x}$

(b) $f(x) = 5^{\log x}$ $f(x) = 2^{x(x-1)}$ (d) None of these

Watch Video Solution

**241.** If $f: \vec{RR}$ is given by $f(x) = \frac{x^2 - 4}{x^2 + 1}$, identify the type of function.

Watch Video Solution

**242.** Find the domain of $f(x) = \sqrt{([x] - 1)} + \sqrt{(4 - [x])}$ (where [ ] represents the greatest integer function).

Watch Video Solution

**243.** The function $f$ satisfies the functional equation $3f(x) + 2f\left(\frac{x + 59}{x - 1}\right) = 10x + 30$ for all real $x \neq 1$. The value of $f(7) is$ 8 (b) 4 (c) -8 (d) 11

Watch Video Solution

**244.** Let $f(x) = ax + bandg(x) = cx + d, a \neq 0.$ Assume $a = 1, b = 2.$ If $(fog)(x) = (gof)(x)$ for all $x,$ what can you say about $candd$?

Watch Video Solution

**245.** Solve $x^2 - 4 - [x] = 0$ (where [] denotes the greatest integer function).

Watch Video Solution

**246.** If $f(x) = \{x^2 \frac{\sin(\pi x)}{2}, |x| < 1x|x|, |x| \geq 1, then f(x) is$ an even function (b) an odd function a periodic function (d) none of these

Watch Video Solution

**247.** Let $f(x) and g(x)$ be bijective functions where $f: \{1, b, c, d\} \rightarrow 1, 2, 3, 4 and g: \{3, 4, 5, 6\} \rightarrow 2, x, y, z,$ respectively. Then, find the number of elements in the range of $g(f(x))\}$.

Watch Video Solution

**248.** If $a < b < c$, then find the range of $f(x) = |x - a| + |x - b| + |x - c|$

Watch Video Solution

**249.** The function $f:(-\infty, -1) \rightarrow \left(0, e^5\right)$ defined by $f(x) = e^{x^3 - 3x + 2}$ is (a)many one and onto (b)many one and into (c)one-one and onto (d)one-one and into

Watch Video Solution

**250.** Let $f:\{2, 3, 4, 5\} \rightarrow \{3, 4, 5, 9\}$a n d$g:\{3, 4, 5, 9\} \rightarrow \{7, 11, 15\}$ be functions defined at f(2)=3,f(3)=4,f(4)=f(5)=5,g(3)=g(4)=7,a n dg(5)=g(9)=11. Find $g(f(x))$

Watch Video Solution

**251.** Find the set of real value(s) of $a$ for which the equation $|2x + 3| + |2x - 3| = ax + 6$ has more than two solutions.

Watch Video Solution

**252.** Which of the following function/functions is/are periodic ?

(a) $sgn\left(e^{-x}\right)$ (b) $\sin x + |\sin x|$

(c ) $\min(\sin x, |x|)$ (d) $\frac{x}{x}$

Watch Video Solution

**253.** If $f(x) = \frac{1}{x}, g(x) = \frac{1}{x^2},$ and $h(x) = x^2$, then

(A) $f(g(x)) = x^2, x \neq 0, h(g(x)) = \frac{1}{x^2}$

(B) $h(g(x)) = \frac{1}{x^2}, x \neq 0, fog(x) = x^2$

(C) $fog(x) = x^2, x \neq 0, h(g(x)) = (g(x))^2, x \neq 0$

(D) none of these

Watch Video Solution

**254.** Find the domain of the function $f(x) = \frac{1}{1 + 2\sin x}$

Watch Video Solution

**255.** Find the period (if periodic) of the following function ([.] denotes the greatest integer functions): $f(x) = \dfrac{|\sin x + \cos x|}{|\sin x| + |\cos x|}$

Watch Video Solution

**256.** Find the range of $f(x) = \sqrt{1 - \sqrt{x^2 - 6x + 9}}$

Watch Video Solution

**257.** If $[x]$ and $\{x\}$ represetnt the integral and fractional parts of $x$, respectively, then the value of $\sum_{r=1}^{2000} \dfrac{\{x + r\}}{2000}$ is (a)$x$ (b) $[x]$ (c) $\{x\}$ (d) $x + 2001$

Watch Video Solution

**258.** Find the period (if periodic) of the following function ([.] denotes the greatest integer functions): $f(x) = x - [x - b], b \in R$

Watch Video Solution

Watch Video Solution

**259.** Let $f(x) = 3x^2 - 7x + c$, where $c$ is a variable coefficient and $x > \frac{7}{6}$. Then the value of $[c]$ such that $f(x)$ touches $f^{-1}(x)$ is (where [.] represents greatest integer function)________

Watch Video Solution

**260.** Solve: $\left|-2x^2 + 1 + e^x + \sin x\right| = \left|2x^2 - 1\right| + e^x + |\sin x|, x \in [0, 2\pi]$

Watch Video Solution

**261.** Let $[x]$ denotes the greatest integer less than or equal to $x$. If the function $f(x) = \tan\left(\sqrt{[n]}x\right)$ has period $\frac{\pi}{3}$ then find the value of $n$

Watch Video Solution

**262.** The domain of the function $f(x) = \sin^{-1}\left\{(\log)_2\frac{x^2}{2}\right\}$ is given by__

Watch Video Solution

**263.** Solve $|\sin x + \cos x| = |\sin x| + |\cos x|, x \in [0, 2\pi]$

Watch Video Solution

**264.** Which of the following functions is not periodie? $|\sin 3x| + \sin^2 x$ $\cos\sqrt{x} + \cos^2 x$ $\cos 4x + \tan^2 x$ $\cos 2x + \sin x$

Watch Video Solution

**265.** Find the number of solutions of $\sin x = \frac{x}{10}$

Watch Video Solution

**266.** Let $f(x) = e^{e^{|x| sgn x}} and g(x) = e^{e^{|x| sgn x}}, x \in R$, where { } and [ ] denote the fractional and integral part functions, respectively. Also, $h(x) = \log(f(x)) + \log(g(x))$ Then for real $x$, $h(x)$ is (a)an odd function (b)an even function (c)neither an odd nor an even function (d)both odd and even function

Watch Video Solution

**267.** Solve $\sin x > \frac{1}{2}$ or find the domain of $f(x) = \frac{1}{\sqrt{1 + 2\sin x}}$

Watch Video Solution

**268.** The domain of the function $f(x) = \frac{x}{\sqrt{\sin(\ln x) - \cos(\ln x)}}$, $(n \in Z)$ is (a) $\left(e^{2n\pi}, e^{\left(3n + \frac{1}{2}\right)\pi}\right)$ (b) $\left(e^{\left(2n + \frac{1}{4}\right)\pi}, e^{\left(2n + \frac{5}{4}\right)\pi}\right)$ $\left(e^{\left(2n + \frac{1}{4}\right)\pi}, e^{\left(2n - \frac{3}{4}\right)\pi}\right)$ (d) none of these

Watch Video Solution

**269.** If $f(a - x) = f(a + x)$ and $f(b - x) = f(b + x)$ for all real x, where $a, b(a > b > 0)$ are constants, then prove that $f(x)$ is a periodic function.

Watch Video Solution

**270.** Let $f: N\vec{Z}$ be a function defined as $f(x) = x - 1000.$ Show that $f$ is an into function.

Watch Video Solution

**271.** If $f$ is a function such that $f(0) = 2, f(1) = 3, andf(x + 2) = 2f(x) - f(x + 1)$ for every real $x,$ then $f(5)$ is 7 (b) 13 (c) 1 (d) 5

Watch Video Solution

**272.** A real valued function f(x) satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$, where a is a given constant and f(0), f(2a-x) =

Watch Video Solution

**273.** Find the domain of $f(x) = \dfrac{1}{\sqrt{x + |x|}}$

Watch Video Solution

**274.** If $f: R \to R$ is a function satisfying the property $f(2x + 3) + f(2x + 7) = 2\,\forall x \in R$, then find the fundamental period of $f(x)$.

Watch Video Solution

**275.** If $f(n + 1) = \dfrac{2F(n) + 1}{2}, n = 1, 2, 3, \ldots. \text{ and } F(1) = 2.$ Then $F(101)$ equals 52 (b) 49 (c) 48 (d) 51

Watch Video Solution

**276.** Find the domain of $f(x) = \frac{1}{\sqrt{x - |x|}}$

Watch Video Solution

**277.** The domain of the function $f(x) = \frac{1}{\sqrt{\left(1^{10}C_{x-1} - 3X^{10}C_x\right)}}$ contains the points. (a) 9,10,11 (b) 9,10,12 (c) all natural numbers (d) none of these

Watch Video Solution

**278.** If $f(x)$ is an even function and satisfies the relation $x^2 \dot{f}(x) - 2f\left(\frac{1}{x}\right) = g(x), where g(x)$ is an odd function, then find the value of $\dot{f}(5)$

Watch Video Solution

**279.** Solve $||x - 1| - 5| \leq 2$

Watch Video Solution

**280.** Determine all functions $f: R \rightarrow R$ such that f(x-f(y))=f(f(y))+xf(y)+f(x)-1 $\forall x, y \geq 0 \in R^{\cdot}$

Watch Video Solution

**281.** The domain of $f(x) = \cos^{-1}\left(\frac{2 - |x|}{4}\right) + [\log(3 - x)]^{-1}$ is (a)$[-2, 6]$ (b) $[-6, 2) \cup (2, 3)$ $[-6, 2]$ (d) $[-2, 2] \cup (2, 3)$

Watch Video Solution

**282.** find the domain of $f(x) = \sqrt{\frac{1 - |x|}{|x| - 2}}$

Watch Video Solution

**283.** Determine the function satisfying $f^2(x+y)=f^2(x)+f^2(y)\,\forall x, y \in R$.

Watch Video Solution

**284.** The domain of the function $f(x)=\sqrt{\log\left(\frac{1}{|\sin x|}\right)}$ (a)$R-\{-\pi, \pi\}$ (b) $R-\{n\pi \mid n\pi Z\}$ (c)$R-\{2n\pi \mid n \in z\}$ (d) $(-\infty, \infty)$

Watch Video Solution

**285.** Find the domain and range of $f(x)=\sqrt{3-2x-x^2}$

Watch Video Solution

**286.** The range of $f(x)=\sin^{-1}\left(\frac{x^2+1}{x^2+2}\right)$ is $\left[0, \frac{\pi}{2}\right]$ (b) $\left(0, \frac{\pi}{6}\right)$ (c) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (d) none of these

Watch Video Solution

**287.** $f: R \rightarrow R, f\left(x^2 + x + 3\right) + 2f\left(x^2 - 3x + 5\right) = 6x^2 - 10x + 17 \,\forall x \in R,$

then find the function $f(x)$.

Watch Video Solution

**288.** Solve $|3x - 2| \leq \frac{1}{2}$

Watch Video Solution

**289.** The function $f(x) = \frac{\sec^{-1}x}{\sqrt{x - [x]}}$, where $[x]$ denotes the greatest integer less than or equal to $x$, is defined for all $x \in R$ (b) $R - \{(-1, 1) \cup \{n | n \in Z\}\}$ $R^{\pm}(0, 1)$ (d) $R^{\pm}\{n \mid n \in N\}$

Watch Video Solution

**290.** Consider $f: R^{+} \rightarrow R$such that$f(3) = 1$ for $a \in R^{+} and f(x)f(y) + f\left(\frac{3}{x}\right)f\left(\frac{3}{y}\right) = 2f(xy) \forall x, y \in R^{+}$ Then find $f(x)$

Watch Video Solution

**291.** Find the range of $f(x) = \sqrt{x - 1} + \sqrt{5 - x}$

Watch Video Solution

**292.** Identify the following functions whether odd or even or neither: $f(x) = \log\left(\frac{x^4 + x^2 + 1}{x^2 + x + 1}\right)$

Watch Video Solution

**293.** Number of solutions of the equation, $[y + [y]] = 2\cos x$ is: (where $y = 1/3)[\sin x + [\sin x + [\sin x]]]$ and $[] =$ greatest integer function) 0 (b) 1

(c) 2 (d) $\infty$

Watch Video Solution

**294.** If $f(x) = \sqrt{x^2 + ax + 4}$ is defined for all $x$, then find the value of $a$

Watch Video Solution

**295.** Identify the following functions whether odd or even or neither: f(x) = {g(x) - g( - x)}

Watch Video Solution

**296.** Find the complete set of values of $a$ such that $\frac{x^2 - x}{1 - ax}$ attains all real values.

Watch Video Solution

**297.** Let $f_1(x) = \{x, x \le x \le 1 \text{ and } 1x > 1 \text{ and } 0$,otherwise $f_2(x) = f_1(-x)$ for all x abd $f_3(x) = -f_2(x)$ for all x and $f_4(x) = -f_3(-x)$ for all x Which of the following is necessarily true?

Watch Video Solution

**298.** Let $g : R \to \left(0, \frac{\pi}{3}\right)$ be defined by $g(x) = \cos^{-1}\left(\frac{x^2 - k}{1 + x^2}\right)$. Then find the possible values of $k$ for which $g$ is a surjective function.

Watch Video Solution

**299.** Find the range of $f(x) = \frac{x^2 + 34x - 71}{x^2 + 2x - 7}$

Watch Video Solution

**300.** The domain of the following function is

$f(x) = (\log)_2\left( - (\log)_{\frac{1}{2}}\left(1 + \frac{1}{\left(x^{\frac{1}{4}}\right)} - 1\right)\right)$ (0, 1) (b) (0, 1) (1, ∞) (d) (1, ∞)

Watch Video Solution

**301.** Let $f:(-1, 1) \to B$ be a function defined by $f(x) = \tan^{-1}\left[\frac{2x}{1 - x^2}\right]$.

Then $f$ is both one-one and onto when $B$ is the interval. (a) $\left[0, \frac{\pi}{2}\right)$ (b) $\left(0, \frac{\pi}{2}\right)$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Watch Video Solution

**302.** Solve $\frac{|x + 3| + x}{x + 2} > 1$

Watch Video Solution

**303.** If

$f(x) = \left(h_1(x) - h_1(-x)\right)\left(h_2(x) - h_2(-x)\right)\ldots\left(h_{2n+1}(x) - h_{2n+1}(-x) and f(200) = 0,\right.$

then prove that $f(x)$ is many one function.

Watch Video Solution

**304.** The range of $f(x) = \sin^{-1}\left(\sqrt{x^2 + x + 1}\right) is \left(0, \frac{\pi}{2}\right)$ (b) $\left(0, \frac{\pi}{3}\right)$ (c) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

(d) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

Watch Video Solution

**305.** $Solve: |x - 1| + |x - 2| \geq 4.$

Watch Video Solution

**306.** The range of $f(x) = (x + 1)(x + 2)(x + 3)(x + 4) + 5$ for $x \in [-6, 6]$ is [4, 5045] (b) [0, 5045] $[-20, 5045]$ (d) none of these

Watch Video Solution

**307.** Which of the following function is (are) even, odd, or neither?

$f(x) = x^2\sin x$ $\quad f(x) = \sqrt{1 + x + x^2} - \sqrt{1 - x + x^2}$ $\quad f(x) = \log\left(\dfrac{1 - x}{1 + x}\right)$

$f(x) = \log\left(x + \sqrt{1 + x^2}\right)$ $f(x) = \sin x - \cos x$ $f(x) = \dfrac{e^x + e^{-x}}{2}$

Watch Video Solution

**308.** Verify that $x sgn x = |x|$; $|x| sgn x = x$; $x(sgn x)(sgn x) = x$

Watch Video Solution

**309.** Let $h(x) = |kx + 5|$,then domainof f(x) be [-5,7], the domain of f(h(x)) be $[-6, 1]$,and the range of h(x) be the same as the domain of f(x). Then the value of $k$ is. (a)1 (b) 2 (c) 3 (d) 4

Watch Video Solution

**310.** Find the domain of the following functions: $f(x) = \sqrt{x - \sqrt{1 - x^2}}$

Watch Video Solution

**311.** Find whether the given function is even or odd: $f(x) = x\dfrac{\sin x + \tan x}{\left[\dfrac{x+\pi}{\pi}\right] - \dfrac{1}{2}}$;

where [] denotes the greatest integer function.

Watch Video Solution

**312.** The range of the following function is

$f(x) = \sqrt{(1 - \cos x)\sqrt{(1 - \cos x)\sqrt{(1 - \cos x)\sqrt{\infty}}}}$ (0,1) (b) $\left(0, \dfrac{1}{2}\right)$ (c) $(0, 2)$ (d)

*noneofthese*

Watch Video Solution

Watch Video Solution

**313.** Find the range of $f(x) = sgn\left(x^2 - 2x + 3\right)^{\cdot}$

Watch Video Solution

**314.** Check whether the function $h(x) = \left(\sqrt{\sin x} - \sqrt{tan x}\right)\left(\sqrt{\sin x} + \sqrt{\tan x}\right)$ is whether odd or even.

Watch Video Solution

**315.** The sum of roots of the equation $\cos^{-1}(\cos x) = [x], [.]$ denotes the greatest integer function, is $2\pi + 3$ (b) $\pi + 3$ (c) $\pi - 3$ (d) $2\pi - 3$

Watch Video Solution

**316.** Find the range of $f(x) = [\sin\{x\}],$ where $\{\}$ represents the fractional part function and $[]$ represents the greatest integer function.

A. - 1

B. 0

C. 1

D. 0.5

Watch Video Solution

**317.** Let the function $f(x) = x^2 + x + s \in x - \cos x + \log(1 + |x|)$ be defined on the interval $[0, 1]$ .Define functions $g(x)andh(x) \in [-1, 0]$ satisfying $g(-x) = -f(x)andh(-x) = f(x) \forall x \in [0, 1]^{\cdot}$

Watch Video Solution

**318.** If $f(x) = \sin(\log)_e \left\{ \frac{\sqrt{4 - x^2}}{1 - x} \right\}$, then the domain of $f(x)$ is ____ and its range is ________.

Watch Video Solution

**319.** Solve the following $1 \leq |x - 2| \leq 3$

Watch Video Solution

**320.** if`f(x)={x^3+x^2,for0lt=xlt=2x+2,for2

Watch Video Solution

**321.** There are exactly two distinct linear functions, _____ and _____, which map [-1,1] onto [0,2].

Watch Video Solution

**322.** Solve the following $0 < |x - 3| \leq 5$

Watch Video Solution

**323.** If $f$ is an even function defined on the interval $(-5, 5)$, then four real values of $x$ satisfying the equation $f(x) = f\left(\frac{x+1}{x+2}\right)$ are ______, _________, ______ and______.

Watch Video Solution

**324.** Find the period (if periodic) of the following function $f(x) = e^{\log(\sin x)} + \tan^3 x - \operatorname{cosec}(3x - 5)$

Watch Video Solution

**325.** Solve the following $\left|\frac{x-3}{x+1}\right| \leq 1$

Watch Video Solution

**326.** Find the fundamental period of $f(x) = \cos x \cos 2x \cos 3x$

Watch Video Solution

**327.** If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right) and g\left(\frac{5}{4}\right) = 1,$ then $(gof)(x)$ is __________

Watch Video Solution

**328.** Solve the following $\left|\frac{x-3}{x+1}\right| \le 1$

Watch Video Solution

**329.** The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is

Watch Video Solution

**330.** Solve the system of equations in $x, y$ and $z$ satisfying the following equations $x + [y] + \{z\} = 3.1, y + [z] + \{x\} = 4.3$ and $z + [x] + \{y\} = 5.4$

Watch Video Solution

**331.** If $f(x) = \left(p - x^n\right)^{\frac{1}{n}}$ ,$p > 0$ and$n$ is a positive integer then $f[f(x)]$ is equal to

Watch Video Solution

**332.** Find the range of $f(x) = \frac{x - [x]}{1 - [x] + x'}, where[]$ represents the greatest integer function.

Watch Video Solution

**333.** the function $f(x) = \frac{x^2 + 4x + 30}{x^2 - 8x + 18}$ is not one-to-one.

Watch Video Solution

**334.** Solve $2[x] = x + \{x\}, whre[]and\{\}$ denote the greatest integer function and the fractional part function, respectively.

Watch Video Solution

**335.** If $f_1(x)$ and $f_2(x)$ are defined on domains $D_1$ and $D_2$,respectively, then $f_1(x) + f_2(x)$ is defined on $D_1 \cap D_2$.

Watch Video Solution

**336.** In the questions, $[x]and\{x\}$ represent the greatest integer function and the fractional part function, respectively. Solve: $[x]^2 - 5[x] + 6 = 0$.

Watch Video Solution

**337.** Let $R$ be the set of real numbers. If $f: R \rightarrow R$ is a function defined by $f(x) = x^2$, then $f$ is injective but not surjective surjective but not injective bijective none of these (a) injective but not surjective (b) surjective but not injective (c) bijective (d) non of these

Watch Video Solution

**338.** If $f(x) = [x], 0 \leq \{x\} < 0.5$ and $f(x) = [x] + 1, 0.5 < \{x\} < 1$ then prove that f (x) = -f(-x) (where[.] and{.} represent the greatest integer function and the fractional part function, respectively).

Watch Video Solution

**339.** Find the range of $f(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

Watch Video Solution

**340.** Statement 1 : For a continuous surjective function $f: R\vec{R}, f(x)$ can never be a periodic function. Statement 2: For a surjective function $f: R\vec{R}, f(x)$ to be periodic, it should necessarily be a discontinuous function.

Watch Video Solution

**341.** Find the domain and range of $f(x) = \sqrt{x^2 - 4x + 6}$

Watch Video Solution

**342.** Find the range of $f(x) = (\log)_e \sin x$

Watch Video Solution

**343.** Let $f(x) = (x + 1)^2 - 1, x \geq -1$. Then the set $\{x : f(x) = f^{-1}(x)\}$ is (a) $\left\{0, 1, \frac{-3 + i\sqrt{3}}{2}, \frac{-3 - i\sqrt{3}}{2}\right\}$ (b) $\{0, -1\}$ (c)$\{0, 1\}$ (d) *empty*

Watch Video Solution

**344.** Find the domain of the following functions: $f(x) = \sqrt{\left(\frac{2}{x^2 - x + 1} - \frac{1}{x + 1} - \frac{2x - 1}{x^3 + 1}\right)}$

Watch Video Solution

**345.** If $(\log)_3\left(x^2 - 6x + 11\right) \leq 1$, then the exhaustive range of values of $x$ is: $(-\infty, 2) \cup (4, \infty)$ (b) $(2, 4)$ $(-\infty, 1) \cup (1, 3) \cup (4, \infty)$ (d) none of these

Watch Video Solution

**346.** Find the domain of the following functions: $f(x) = \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1-x}{1+x}}$

Watch Video Solution

**347.** Given the function $f(x) = \frac{a^x + a^{-x}}{2}(where a > 2) Then f(x+y) + f(x-y) =$

(A) $2f(x).\ f(y)$ (B) $f(x).\ f(y)$ (C) $\frac{f(x)}{f(y)}$ (D) none of these

Watch Video Solution

**348.** Find the range of $f(x) = (\log)_3\left(5 - 4x - x^2\right)$

Watch Video Solution

**349.** The equation $||x-2| + a| = 4$ can have four distinct real solutions for $x$ if $a$ belongs to the interval $(-\infty, -4)$ (b) $(-\infty, 0)$ $(4, \infty)$ (d) none of these

Watch Video Solution

350. Find the range of the function $f(x) = 6^x + 3^x + 6^{-x} + 3^{-x} + 2$.

Watch Video Solution

351. The period of function $2^{\{x\}} + \sin\pi x + 3\left\{\frac{x}{2}\right\} + \cos 2\pi x$ (where $\{x\}$ denotes the fractional part of $(x)$ is 2 (b) 1 (c) 3 (d) none of these

Watch Video Solution

352. Find the domain and range of $f(x) = \sqrt{x^2 - 3x + 2}$

Watch Video Solution

353. If $f$ is periodic, $g$ is polynomial function and $f(g(x))$ is periodic and $g(2) = 3, g(4) = 7$ then $g(6)$ is

Watch Video Solution

**354.** Solve: $x\left(e^{x}-1\right)(x+2)(x-3)^{2} \le 0.$

Watch Video Solution

**355.** If $f(x)=x^{m}n, n \in N,$ is an even function, then $m$ is even integer (b) odd integer any integer (d) $f(x)$ - $evenis \neg possible$

Watch Video Solution

**356.** Find the range of $f(x)=\dfrac{x^{2}+1}{x^{2}+2}$

Watch Video Solution

**357.** The period of the function $\left|\dfrac{\sin^{3}x}{2}\right|+\left|\dfrac{\cos^{5}x}{5}\right|$ is

A. $2\pi$

B. $10\pi$

C. $8\pi$

D. $5\pi$

**Answer: B**

Watch Video Solution

**358.** The number of real solutions of the $(\log)_{0.5}|x| = 2|x|$ is 1 (b) 2 (c) 0 (d) none of these

Watch Video Solution

**359.** The domain of definition of the function $f(x) = \{x\}^{\{x\}} + [x]^{[x]}$ is where $\{.\}$ represents fractional part and $[.]$ represent greatest integral function). (a)$R - I$ (b) $R - [0, 1]$ $R - \{I \cup (0, 1)\}$ (d) $I \cup (0, 1)$

Watch Video Solution

**360.** If $f(x) = ma\xi\mu m\left\{x^3, x^2, \frac{1}{64}\right\} \forall x \in [0, \infty), then$ $f(x) = \left\{x^2, 0 \leq x \leq 1x^3, x > 0\right.$ `f(x)={1/(64),0lt=xlt=1/4x^2,1/41f(x)= {1/(64),0lt=xlt=1/8x^2,1/81f(x)={1/(64),0lt=xlt=1/8x^3,x >1/8`

Watch Video Solution

**361.** Statement 1: The function $f(x) = x^2 + \tan^{-1}x$ is a non-periodic function. Statement 2: The sum of two non-periodic functions is always non-periodic.

Watch Video Solution

**362.** The function $f(x) = \sin\left(\log\left(x + \sqrt{1 + x^2}\right)\right)$ is (a) even function (b) odd function (c) neither even nor odd (d) periodic function

Watch Video Solution

**363.** The function $f: N \rightarrow N(N$ is the set of natural numbers) defined by $f(n) = 2n + 3is$

A. surjective only

B. injective only

C. bijective

D. none of these

**Answer: B**

Watch Video Solution

**364.** The function $f: R - R$ is defined by $f(x) = \cos^2 x + \sin^4 x f$ or $x \in \dot{R}$ Then the range of $f(x)$ is $\left(\frac{3}{4}, 1\right]$ (b) $\left[\frac{3}{4}, 1\right)$ (c) $\left[\frac{3}{4}, 1\right]$ (d) $\left(\frac{3}{4}, 1\right)$

Watch Video Solution

**365.** If $x$ is real, then the value of the expression $\dfrac{x^2+14x+9}{x^2+2x+3}$ lies between

(a) $5 \text{ and } 4$

(b) $5 \text{ and } -4$

(c) $-5 \text{ and } 4$

(d) none of these

Watch Video Solution

**366.** The domain of $\sqrt{\log_{10}\left(\dfrac{5x-x^2}{4}\right)}$ is

Watch Video Solution

**367.** The domain of the function $f(x) = (\log)_{3+x}\left(x^2-1\right)$ is

Watch Video Solution

**368.** The domain of $f(x) = \log|\log x| is\ (0, \infty)$ (b) $(1, \infty)$ (c) $(0, 1) \cup (1, \infty)$ (d) $(-\infty, 1)$

Watch Video Solution

**369.** The domain of the function $f(x) = \dfrac{\sin^{-1}(3 - x)}{In(|x|) - 2}$ is

Watch Video Solution

**370.** Let $f: \left[-\dfrac{\pi}{3}, \dfrac{2\pi}{3}\right] \overrightarrow{0, 4}$ be a function defined as $f(x) = \sqrt{3}\sin x - \cos x + 2$. Then $f^{-1}(x)$ is given by $\sin^{-1}\left(\dfrac{x - 2}{2}\right) - \dfrac{\pi}{6}$ $\sin^{-1}\left(\dfrac{x - 2}{2}\right) + \dfrac{\pi}{6}$ $\dfrac{2\pi}{3} + \cos^{-1}\left(\dfrac{x - 2}{2}\right)$ (d) none of these

Watch Video Solution

**371.** The domain of $f(x) = \frac{(\log)_2(x + 3)}{x^2 + 3x + 2}$ is $R - \{-1, 2\}$ (b) $(-2, \infty)$ $R - \{-1, -2, -3\}$ (d) $(-3, \infty) - (-1, -2\}$

Watch Video Solution

**372.** Let $f : X \to y f(x) = s \in x + \cos x + 2\sqrt{2}$ be invertible. Then which $X \to Y$ is not possible? $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$ $\left[-\frac{3\pi}{4}, \frac{\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$ $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$ none of these

Watch Video Solution

**373.** If $f(x) = ax^7 + bx^3 + cx - 5$, $a, b, c$ are real constants, and $f(-7) = 7$, then the range of $f(7) + 17\cos x$ is $[-34, 0]$ (b) $[0, 34]$ $[-34, 34]$ (d) none of these

Watch Video Solution

**374.** If $g(x) = x^2 + x - 2$ and $\frac{1}{2}gof(x) = 2x^2 - 5x + 2$, then which is not a possible $f(x)$? (A)$2x - 3$ (B) $-2x + 2$ (C)$x - 3$ (D) None of these

Watch Video Solution

**375.** If $R \to R$ is an invertible function such that $f(x)$ and $f^{-1}(x)$ are symmetric about the line $y = -x$, then (a) $f(x)$ is odd (b) $f(x)$ and $f^{-1}(x)$ may be symmetric (c) $f(c)$ may not be odd (d) non of these

Watch Video Solution

**376.** Let $f: R \to R$ and $g: R \to R$ be two given functions such that $f$ is injective and $g$ is surjective. Then which of the following is injective? (a)$gof$ (b) $fog$ (c) $gog$ (d) none of these

Watch Video Solution

**377.** f: $N \to N$, where $f(x) = x - (-1)^x$. Then f is: (a)one-one and into (b)many-one and into (c)one-one and onto (d)many-one and onto

Watch Video Solution

**378.** The value of $f(x) = 3\sin\sqrt{\left(\frac{\pi^2}{16} - x^2\right)}$ lie in the interval____

Watch Video Solution

**379.** A function $f: IR \to IR$, where $IR$, is the set of real numbers, is defined by $f(x) = \frac{ax^2 + 6x - 8}{a + 6x - 8x^2}$ Find the interval of values of a for which is onto. Is the functions one-to-one for $a = 3$? Justify your answer.

Watch Video Solution

**380.** For any real number $x$, $[x]$ denotes the largest integer less than or equal to and $\{x\} = x - [x]$ The number of real solutions of $7[x] + 23\{x\} = 191$ is (A) 0 (B) 1 (c) 2 (D) 3

Watch Video Solution

**381.** The domain of the function $f(x) = (\log)_e\left\{(\log)_{|\sin x|}\left(x^2 - 8x + 23\right) - \left\{\frac{3}{(\log)_2|\sin x|}\right\}\right.$ contains which of the following interval(s)? $3, \pi)$ (b) $\left(\pi, \frac{3\pi}{2}\right)$ (c) $\left(\frac{3\pi}{2}, 5\right)$ (d) none of these

Watch Video Solution

**382.** Let f be an injective map. with domain (x, y, z and range (1, 2, 3), such that exactly one following statements is correct and the remaining are false : $f(x) = 1, f(y) \neq 1, f(z) \neq 2$ The value of $\left.f^{-1}\right)(1)$ is

Watch Video Solution

**383.** If $f(x) = x^9 - 6x^8 - 2x^7 + 12x^6 + x^4 - 7x^3 + 6x^2 + x - 3$, find $f(6)$.

Watch Video Solution

**384.** Draw the graph of $y = |x|^{\frac{1}{2}}$ for $-1 \leq x < 1$.

Watch Video Solution

**385.** Find the domain and range of the function $f(x) = \dfrac{x^2}{1 + x^2}$. Is the function one-to-one?

Watch Video Solution

**386.** Let $E = \{1, 2, 3, 4\} and F = \{1, 2\}$. If $N$ is the number of onto functions from $E \rightarrow F$, then the value of $N/2$ is

Watch Video Solution

**387.** The range of $f(x) = \sec^{-1}\left((\log)_3 \tan x + (\log)_{\tan x} 3\right)$ is

$\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$ ∪ $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$ $\left[0, \frac{\pi}{2}\right]$ (c) $\left(\frac{2\pi}{3}, \pi\right)$ (d) none of these

Watch Video Solution

**388.** Let $A$ be a set of $n$ distinct elements. Then the total number of distinct function from $A \to A$ is ______ and out of these, _____ are onto functions.

Watch Video Solution

**389.** If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right) and g\left(\frac{5}{4}\right) = 1$, then $(gof)(x)$ is ___________

Watch Video Solution

**390.** Each question has four choices, a,b,c and d,out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. if both the statements are true and statement 2 is the correct explanation of statement 1. If both the statements are true but statement 2 is not the correct explanation of statement 1. If statement is True and statement2 is false. If statement1 is false and statement2 is true. Consider the function $f(x) = \sin(kx) + \{x\}$, where $(x)$ represents the fractional part function.

Statement 1 : $f(x)$ is periodic for $k = m\pi$, where $m$ is a rational number

Statement 2 : The sum of two periodic functions is always periodic.

Watch Video Solution

**391.** Each question has four choices, a,b,c and d,out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. if both the statements are true and statement 2 is the correct explanation of statement 1. If both the statements are true but statement 2 is not the correct explanation of statement 1. If statement is True and statement2 is false. If statement1 is false and statement2 is true. Consider the function satisfying the relation if $f\left(\frac{2\tan x}{1+\tan^2 x}\right)=\left((1+\cos 2x)\frac{\sec^2 x+2\tan x}{2}\right.$

Statement 1: The range of $y=f(x) is R^{\cdot}$ Statement 2: Linear function has rang $R$ if domain is $R^{\cdot}$

Watch Video Solution

**392.** Let $f(x)=|x-1|^{\cdot}$ Then (a)$f\left(x^2\right)=(f(x))^2$ (b) $f(x+y)=f(x)+f(y)$ (c) $f(|x|)-|f(x)|$ (d) none of these

Watch Video Solution

**393.** Let $f:R\vec{R}, f(x)=\frac{x-a}{(x-b)(x-c)}, b>\cdot$ If $f$ is onto, then prove that $a\in(b,c)^{\cdot}$

Watch Video Solution

**394.** Show that there exists no polynomial $f(x)$ with integral coefficients which satisfy $f(a) = b, f(b) = c, f(c) = a,$ where $a, b, c,$ are distinct integers.

Watch Video Solution

**395.** Consider the function $f(x) = \begin{cases} x - [x] - \frac{1}{2} & x \notin \\ 0 & x \in I \end{cases}$ where [.] denotes the fractional integral function and I is the set of integers. Then find $g(x) \max . \left[x^2, f(x), |x|\right\}, \ -2 \leq x \leq 2.$

Watch Video Solution

**396.** The function $f$ satisfies the functional equation $3f(x) + 2f\left(\frac{x + 59}{x - 1}\right) = 10x + 30$ for all real $x \neq 1.$ The value of $f(7) is$ 8 (b) 4 (c) -8 (d) 11

Watch Video Solution

**397.** Let $f:[-1, 10]\vec{R}, where f(x) = \sin x + \left[\frac{x^2}{a}\right]$, be an odd function. Then the set of values of parameter $a$ is/are $(-10, 10)\sim\{0\}$ (b) $(0, 10)$ $(100, \infty)$ (d) $(100, \infty)$

Watch Video Solution

**398.** If $a, b$ are two fixed positive integers such that $f(a + x) = b + \left[b^3 + 1 - 3b^2 f(x) + 3b\{f(x)\}^2 - \{f(x)\}^3\right]^{\frac{1}{3}}$ for all real $x$, then prove that $f(x)$ is periodic and find its period.

Watch Video Solution

**399.** Let f(x, y) be a periodic function satisfying $f(x, y) = f(2x + 2y, 2y - 2x)$ for all x, y; Define $g(x) = f\left(2^x, 0\right)$. Show that g(x) is a periodic function with period 12.

Watch Video Solution

**400.** The domain of the function $f(x) = \dfrac{1}{\sqrt{\{\sin x\} + \{\sin(\pi + x)\}}}$ where $\{.\}$ denotes the fractional part, is (a)$[0, \pi]$ (b) $(2n + 1)\dfrac{\pi}{2}, n \in Z$ (c)$(0, \pi)$ (d) none of these

Watch Video Solution

**401.** Let $f: R \rightarrow \left[0, \dfrac{\pi}{2}\right)$ be defined by $f(x) = \tan^{-1}\left(x^2 + x + a\right)^{\cdot}$ Then the set of values of $a$ for which $f$ is onto is (a)$(0, \infty)$ (b) $[2, 1]$ (c) $\left[\dfrac{1}{4}, \infty\right]$ (d) none of these

Watch Video Solution

**402.** Let $f(x) = \left([a]^2 - 5[a] + 4\right)x^3 - \left(6\{a\}^2 - 5\{a\} + 1\right)x - (\tan)x\sin x$ be an even function for all $x \in R$ Then the sum of all possible values of $a$ (where $[.]$ and $\{.\}$ denot greatest integer function and fractional part function,respectively). (a) $\dfrac{17}{6}$ (b) $\dfrac{53}{6}$ (c) $\dfrac{31}{6}$ (d) $\dfrac{35}{3}$

Watch Video Solution

**403.** f(x)=$\dfrac{\cos x}{\left[2\dfrac{x}{\pi}\right]+\dfrac{1}{2}}$ where x is not an integral multiple of $\pi$ and [ . ] denotes the greatest integer function, is (a)an odd function (b)an even function (c)neither odd nor even (d)none of these

Watch Video Solution

**404.** If $f(x+y)=f(x)+f(y)-xy-1\ \forall x, y \in R and f(1)=1,$ then the number of solution of $f(n)=n, n \in N,$ is 0 (b) 1 (c) 2 (d) more than 2

Watch Video Solution

**405.** If $f(x)=\dfrac{a^x}{a^x+\sqrt{a,}}, (a>0),$ then find the value of $\sum_{r=1}^{2n1} 2f\left(\dfrac{r}{2n}\right)$

Watch Video Solution

**406.** If $f(x)$ is an invertible function and $g(x) = 2f(x) + 5$, then the value of $g^{-1}(x) is\ 2f^{-1}(x) - 5$ (b) $\frac{1}{2f^{-1}(x) + 5}$ $\frac{1}{2}f^{-1}(x) + 5$ (d) $f^{-1}\left(\frac{x-5}{2}\right)$

Watch Video Solution

**407.** The range of the function $f(x) = \frac{e^x - e^{|x|}}{e^x + e^{|x|}}$ is $(-\infty, \infty)$ (b) $[0, 1]$ $(-1, 0]$ (d) $(-1, 1)$

Watch Video Solution

**408.** If $f: [0, \infty] \to [0, \infty)$ and $f(x) = \frac{x}{1+x}$, then $f$ (a) one-one and onto (b)one-one but not onto (c)onto but not one-one (d)neither on-one nor onto

Watch Video Solution

**409.** The domain of definition of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real-valued $x$ is $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (c) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ (d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

Watch Video Solution

**410.** If $f(x) = \sqrt{4 - x^2} + \sqrt{x^2 - 1}$ , then the maximum value of $(f(x))^2$ is

___________

Watch Video Solution

**411.** If $F: [1, \infty)2, \overrightarrow{\infty}$ is given by $f(x) = x + \frac{1}{x}, then f^{-1}(x)$ equals. $\frac{x + \sqrt{x^2 - 4}}{2}$

(b) $\frac{x}{1 + x^2}$ (c) $\frac{x - \sqrt{x^2 - 4}}{2}$ (d)$1 + \sqrt{x^2 - 4}$

Watch Video Solution

**412.** The domain of definition of $f(x) = \frac{(\log)_2(x+3)}{x^2+3x+2}$ is $R - \{-1, -2\}$ (b) $(-2, \infty)$ $R - \{-1, -2, -3\}$ (d) $(-3, \infty) - \{-1, -2\}$

Watch Video Solution

**413.** The domain of definition of the function $f(x)$ given by the equation $2^x + 2^y = 2$ is

Watch Video Solution

**414.** Let $g(x) = 1 + x - [x]$ and $f(x) = \{-1, x < 00, x = 0f, x > 0.$ Then for all $x, f(g(x))$ is equal to (where [.] represents the greatest integer function). (a) $x$ (b) $1$ (c) $f(x)$ (d) $g(x)$

Watch Video Solution

**415.** Suppose $f(x) = (x+1)^2$ for $x \geq -1$. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line $y = x$, then $g(x)$ equal. (a) $1 - \sqrt{x} - 1, x \geq 0$ (b) $\frac{1}{(x+1)^2}, x > -1$ (c) $\sqrt{x+1}, x \geq -1$ (d) $\sqrt{x} - 1, x \geq 0$

Watch Video Solution

**416.** Let the function $f: R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$. Then $f$ is (a)one-to-one and onto (b)one-to-one but not onto (c)onto but not-one-to-one (d)neither one-to-one nor onto

Watch Video Solution

**417.** Let $E = \{1, 2, 3, 4\}$ and $F = \{1, 2\}$. If $N$ is the number of onto functions from $E \to F$, then the value of $N/2$ is

Watch Video Solution

**418.** Let $f(x) = \dfrac{\alpha x}{(x+1)}, x \neq -1$. The for what value of $\alpha$ is $f(f(x)) = x$ (a)$\sqrt{2}$ (b) $-\sqrt{2}$ (c) $1$ (d) $-1$

Watch Video Solution

**419.** Let $f(x) + f(y) = f\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$[$f(x)$ is not identically zero]. Then $f\left(4x^3 - 3x\right) + 3f(x) = 0$ $f\left(4x^3 - 3x\right) = 3f(x)$ $f\left(2x\sqrt{1-x^2} + 2f(x) = 0\right.$ $f\left(2x\sqrt{1-x^2} = 2f(x)\right.$

Watch Video Solution

**420.** Consider the real-valued function satisfying $2f(\sin x) + f(\cos x) = x$ then the (a)domain of $f(x) is R$ (b)domain of $f(x) is [-1, 1]$ (c)range of $f(x)$ is $\left[-\dfrac{2\pi}{3}, \dfrac{\pi}{3}\right]$ (d)range of $f(x) is R$

Watch Video Solution

**421.** If the function / satisfies the relation $f(x + y) + f(x - y) = 2f(x), f(y) \forall x, y \in Randf(0) \neq 0$, then

$f(x)isa \neq venfunction$ $f(x)isanoddfunction$ $Iff(2) = a, thenf(-2) = a$

$Iff(4) = b, thenf(-4) = -b$

Watch Video Solution

**422.** Consider the function $y = f(x)$ satisfying the condition $f\left(x + \frac{1}{x}\right) = x^2 + 1/x^2(x \neq 0)^{\cdot}$ Then the domain of $f(x)isR$ domain of $f(x)isR - (-2, 2)$ range of $f(x)is[-2, \infty]$ range of $f(x)is(2, \infty)$

Watch Video Solution

**423.** Which of the following functions are identical?

$f(x) = 1nx^2andg(x) = 21nx$ $f(x) = (\log)_x eandg(x) = \frac{1}{(\log)_e x}$

$f(x) = \sin\left(\cos^{-1}x\right)andg(x) = \cos\left(\sin^{-1}x\right)$ $noneofthese$

Watch Video Solution

**424.** Which of the following functions have the graph symmetrical about the origin? (a) $f(x)$ given by $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$ (b) $f(x)$ given by $f(x) + f(y) = f\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$ (c) $f(x)$ given by $f(x+y) = f(x) + f(y) \forall x, y \in R$ (d) none of these

Watch Video Solution

**425.** The range of function $f(x) = {}^{7-x}P_{x-3} is$ {1,2,3} (b) $\{1, 2, 3, 4, 5, 6\}$ $\{1, 2, 3, 4\}$ (d) $\{1, 2, 3, 4, 5\}$

Watch Video Solution

**426.** Let

$f(x) = \max\{1 + \sin x, 1, 1 - \cos x\}, x \in [0, 2\pi], and g(x) = \max\{1, |x - 1|\}, x \in R$.

Then (a)$g(f(0)) = 1$ (b) $g(f(1)) = 1$ (c)$f(f(1)) = 1$ (d) $f(g(0)) = 1\sin 1$

Watch Video Solution

**427.** If $f(x)$ satisfies the relation $f(x+y) = f(x) + f(y)$ for all $x, y \in Randf(1) = 5, then \quad f(x) isanoddfunction \quad f(x)$ is an even function $\sum_{n=1}^{m} f(r) = 5^{m+1}C_2 \sum_{n=1}^{m} f(r) = \frac{5m(m+2)}{3}$

Watch Video Solution

**428.** Let $f(x) = \{x^2 - 4x + 3, x < 3x - 4, x \geq 3$ $andg(x) = \{x - 3, x < 4x^2 + 2x + 2, x \geq 4$ then which of the following is/are true? (a)$(f+g)(3.5) = 0$ (b)$f(g(3)) = 3$ (c)$(fg)(2) = 1$ (d) $(f-g)(4) = 0$

Watch Video Solution

**429.** Let $f(x) = \frac{3}{4}x + 1, f^n(x)$ be defined as $f^2(x) = f(f(x))$, and for $n \geq 2, f^{n+1}(x) = f\left(f^n(x)\right)$. If $\lambda = (\lim)_{n \to \infty} f^n(x)$ , then (a)$\lambda$ is independent of $x$ (b)$\lambda$ is a linear polynomial in $x$ (c)the line $y = \lambda$ has slope $0$. (d)the line $4y = \lambda$ touches the unit circle with centre at the origin.

Watch Video Solution

**430.** If $f:\{1, 2, 3.... \} \rightarrow \{0, \pm 1, \pm 2...\}$ is defined by $f(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is} \end{cases}$ even , $-\left(\frac{n-1}{2}\right)$ if $n$ is odd } then $f^{-1}(-100)$ is

Watch Video Solution

**431.** If $f: R \rightarrow [0, \infty)$is a function such that $f(x-1)+f(x+1)=\sqrt{3}f(x),$ then prove that $f(x)$ is periodic and find its period.

Watch Video Solution

**432.** If p, q are positive integers, f is a function defined for positive numbers and attains only positive values such that $f(xf(y)) = x^p y^q$, then prove that $p^2 = q$.

Watch Video Solution

**433.** Each question has four choices, a,b,c and d,out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. if both the statements are true and statement 2 is the correct explanation of statement 1. If both the statements are true but statement 2 is not the correct explanation of statement 1. If statement is True and statement2 is false. If statement1 is false and statement2 is true. Statement 1: $f(x) = (\log)_e x$ cannot be expressed as the sum of odd and even function. Statement 2 : $f(x) = (\log)_e x$ in neither odd nor even function.

Watch Video Solution

**434.** Each question has four choices, a,b,c and d,out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. if both the statements are true and statement 2 is the correct explanation of statement 1. If both the statements are true but statement 2 is not the correct explanation of statement 1. If statement is True and statement2 is false. If statement1 is false and statement2 is true. Statement 1:

$f(x) = (\log)_e x$ cannot be expressed as the sum of odd and even function.

Statement 2 : $f(x) = (\log)_e x$ in neither odd nor even function.

Watch Video Solution

**435.** Each question has four choices, a,b,c and d,out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. if both the statements are true and statement 2 is the correct explanation of statement 1. If both the statements are true but statement 2 is not the correct explanation of statement 1. If statement is True and statement2 is false. If statement1 is false and statement2 is true. Consider the function satisfying the relation if $f\left(\frac{2\tan x}{1+\tan^2 x}\right) = \left((1+\cos 2x)\frac{\sec^2 x + 2\tan x}{2}\right.$

Statement 1: The range of $y = f(x) is R^{\cdot}$ Statement 2: Linear function has rang $R$ if domain is $R^{\cdot}$

Watch Video Solution

**436.** Each question has four choices, a,b,c and d,out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. if both the statements are true and statement 2 is the correct explanation of statement 1. If both the statements are true but statement 2 is not the correct explanation of statement 1. If statement is True and statement2 is false. If statement1 is false and statement2 is true. Statement 1 : $Ifg(x) = f(x) - 1, f(x) + f(1 - x) = 2 \forall x \in R, then(g(x)$ is symmetrical about the point $\left.\frac{1}{2}, 0\right)^{\cdot}$ Statement 2 : If $g(a - x) = - g(a + x) \forall x \in R,$ then $g(x)$ is symmetrical about the point $(a, 0)$

Watch Video Solution

**437.** Let $f(x) = (\log)_2(\log)_3(\log)_4(\log)_5\left(s \in x + a^2\right)^{\cdot}$ Find the set of values of $a$ for which the domain of $f(x) is R^{\cdot}$

Watch Video Solution

**438.** Let

$f(x) = \{x^2 - 4x + 3, , x < 3x - 4, x \geq 3 and g(x) = \{x - 3, x < 4x^2 + 2x + 2, x \geq 4$

Describe the function $\frac{f}{g}$ and find its domain.

Watch Video Solution

**439.** If $f and g$ are two distinct linear functions defined on $R$ such that they map $\{-1, 1]$ onto $[0, 2]$ and $h: R - \{-1, 0, 1\}\vec{R}$ defined by $h(x) = \frac{f(x)}{g(x)}$, then show that $\left|h(h(x)) + h\left(h\left(\frac{1}{x}\right)\right)\right| > 2.$

Watch Video Solution

**440.** Let $f(x) = x^2 - 2x, x \in R, and g(x) = f(f(x) - 1) + f(5 - f(x))^{\cdot}$ Show that $g(x) \geq 0 \forall x \in R^{\cdot}$

Watch Video Solution

**441.** Let $f: X \rightarrow Y$ be a function defined by f(x) = a sin ( x $+\frac{\pi}{4}$) + c. If f is both one-one and onto, then find the set X and Y

Watch Video Solution

**442.** If $f$ is polynomial function satisfying $2 + f(x)f(y) = f(x) + f(y) + f(xy) \forall x, y \in R$ and if $f(2) = 5$, then find the value of $f(f(2))$.

Watch Video Solution

**443.** Let $R = \{x, y): x, y \in R, x^2 + y^2 \leq 25 \}$ and $R' = \left\{(x, y): x, y \in R, y \geq \frac{4}{9}x^2\right\}$. Then find the domain and range of $R \cap R'$.

Watch Video Solution

**444.** A certain polynomial $P(x)x \in R$ when divided by $x - a, x - bandx - c$ leaves remainders$a, b, andc$ , resepectively. Then find remainder when $P(x)$ is divided by $(x - a)(x - b)(x - c)whereab, c$ are distinct.

Watch Video Solution

**445.** The period of the function $f(x) = (6x + 7) + \cos\pi x - 6x,$ where [.] denotes the greatest integer function is: 3 (b) $2\pi$ (c) 2 (d) none of these

Watch Video Solution

**446.** If the graph of the function $f(x) = \dfrac{a^x - 1}{x^n\left(a^x + 1\right)}$ is symmetrical about the y-a xi s ,then n equals 2 (b) $\dfrac{2}{3}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{1}{3}$

Watch Video Solution

**447.** The solution set for $[x]\{x\} = 1$ (where {x} and [x] are respectively, fractional part function and greatest integer function) is $R^{\pm}(0, 1)$ (b) $r^{\pm}\{1\}$ $\left\{m + \frac{1}{m}m \in I - \{0\}\right\}$ $\left\{m + \frac{1}{m}m \in N - \{1\}\right\}$

Watch Video Solution

**448.** Let $f: R\vec{R}$ be a continuous and differentiable function such that $\left(f\left(x^2 + 1\right)\right)^{\sqrt{x}} = 5f$ or $\forall x \in (0, \infty)$, then the value of $\left(f\left(\frac{16 + y^2}{y^2}\right)\right)^{\frac{4}{\sqrt{y}}}f$ or $eachy \in (0, \infty)$ is equal to 5 (b) 25 (c) 125 (d) 625

Watch Video Solution

**449.** The possible values of $a$ such that the equation $x^2 + 2ax + a = \sqrt{a^2 + x - \frac{1}{16} - \frac{1}{16}}, x \geq -a,$ has two distinct real roots are given by: $[0, 1]$ (b) $[-\infty, 0]$ $[0, \infty]$ (d) $\left(\frac{3}{4}, \infty\right)$

Watch Video Solution

Watch Video Solution

**450.** Let $g(x) = f(x) - 1.$ If $f(x) + f(1 - x) = 2\forall x \in R,$ then $g(x)$ is symmetrical about. (a)The origin (b) the line$x = \frac{1}{2}$ the point (1,0) (d) the point $\left(\frac{1}{2}, 0\right)$

Watch Video Solution

**451.** Domain (D) and range (R) of $f(x) = \sin^{-1}\left(\cos^{-1}[x]\right),$ where [.] denotes the greatest integer function, is

Watch Video Solution

**452.** If $f(x + 1) + f(x - 1) = 2f(x) and f(0), \ = 0,$ then $f(n), n \in N,$ is

(a)$nf(1)$

(b) $\{f(1)\}^n$

(c)0

(d) none of these

Watch Video Solution

**453.** The range of the function $f$ defined by $f(x) = \left[\dfrac{1}{\sin\{x\}}\right]$ (where [.] and $\{.\}$, respectively, denote the greatest integer and the fractional part functions) is

Watch Video Solution

**454.** If $a, b$ are two fixed positive integers such that $f(a + x) = b + \left[b^3 + 1 - 3b^2 f(x) + 3b\{f(x)\}^2 - \{f(x)\}^3\right]^{\frac{1}{3}}$ for all real $x$, then prove that $f(x)$ is periodic and find its period.

Watch Video Solution

**455.** If $f(x) = \cos\left((\log)_e x\right), then f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$ has value (a) $-1$ (b) $\frac{1}{2}$ (c) $-2$ (d) none of these

Watch Video Solution

**456.** If saties $|x - 1| + x - 2| + |x - 3| \geq 6,$ then (a) $0 \leq x \leq 4$ (b) $x \leq -2x \geq 4x \leq 0$ or $x \geq 4$ (d) none of these

Watch Video Solution

**457.** Which of the following functions is periodic?

Watch Video Solution

**458.** The domain of definition of the function $y = \frac{1}{(\log)_{10}(1 - x)} + \sqrt{x + 2}$ is

Watch Video Solution

**459.** Let $f(x) = \sin x$ and $g(x) = (\log)_e|x|$ If the ranges of the composition functions $fog$ and $gof$ are $R_1$ and $R_2$, respectively then (a) $R_1\{u: -1 \le u < 1\}, R_2 = \{v: -\infty v < 0\}$ (b) $R_1 = \{u: -\infty < u < 0\}, R_2 = \{v: -\infty < v < 0\}$ (c) $R_1 = \{u: -1 < u < 1\}, R_1 = \{v: -\infty V \le 0\}$

Watch Video Solution

**460.** If $a, b$ are two fixed positive integers such that $f(a + x) = b + \left[b^3 + 1 - 3b^2 f(x) + 3b\{f(x)\}^2 - \{f(x)\}^3\right]^{\frac{1}{3}}$ for all real $x$, then prove that $f(x)$ is periodic and find its period.

Watch Video Solution

**461.** Which of the following pairs of functions is/are identical? (a) $f(x) = \tan\left(\tan^{-1}x\right) and g(x) = \cot\left(\cot^{-1}x\right)$ (b)

$f(x) = sgn(x) and g(x) = sgn(sgn(x))$ (c)

$f(x) = \cot^2 x \cos^2 x and g(x) = \cot^2 x - \cos^2 x$ (d)$f(x) = e^{\ln \sec^{-1} x} and g(x) = \sec^{-1} x$

Watch Video Solution

**462.** Let $f(x) = \sec^{-1}\left[1 + \cos^2 x\right]$, where [.] denotes the greatest integer function. Then the

Watch Video Solution

**463.** Which of the following is/are not functions ([.]) and {.} denote the greatest integer and fractional part functions, respectively? (a) $\frac{1}{1n(1 - |x|)}$

(b) $\frac{x!}{\{x\}}$ (c)$x!\{x\}$ (d) $\frac{1n(x - 1)}{\sqrt{\left(1 - x^2\right)}}$

Watch Video Solution

**464.** $f(x) = x^2 - 2ax + a(a+1), f:[a, \infty)a, \infty \overrightarrow{.}$ If one of the solution of the equation $f(x) = f^{-1}(x) is 5049,$ then the other may be 5051 (b) 5048 (c) 5052 (d) 5050

Watch Video Solution

**465.** Let $f: R\vec{R}$ be a function defined by $f(x+1) = \frac{f(x) - 5}{f(x) - 3} \forall x \in R^{\cdot}$ Then which of the following statement(s) is/are ture? $f(2008) = f(2004)$ $f(2006) = f(2010)$ $f(2006) = f(2002)$ $f(2006) = f(2018$

Watch Video Solution

**466.** If $f: R^+ \to R^+$ is a polynomial function satisfying the functional equation $f(f(x)) = 6x - f(x), then f(17)$ is equal to 17 (b) 51 (c) 34 (d) -34

Watch Video Solution

**467.** Suppose that $f(x)$ is a function of the form $f(x) = \dfrac{ax^8 + bx^6 + cx^4 + dx^2 + 15x + 1}{x}, (x \neq 0) If f(5) = 2,$ then the value of $|f(-5)|/4$ is____

Watch Video Solution

**468.** If $f: R\vec{N} \cup \{0\},$ where $f$ (area of triangle joining points $P(5, 0), Q(8, 4) and R(x, y)$ such that angle PRQ is a right angle $=$ number of triangles, then which of the following is true? $f(5) = 4$ (b) $f(7) = 0$ $f(6, 25) = 2$ (d) $f(x) is \neg$

Watch Video Solution

**469.** If the following functions are defined from $[-1, 1] \rightarrow [-1, 1],$ select those which are not bijective. (a)$\sin\left(\sin^{-1}x\right)$ (b) $\dfrac{2}{\pi}\sin^{-1}(\sin x)$ (c) $(sgn(x))\ln\left(e^x\right)$ (d) $x^3(sgn(x))$

Watch Video Solution

**470.** If $f: N \to N$,and $x_2 > x_1 \Rightarrow f(x_2) > f(x_1)$ for $x_1, x_2 \in N$ a n d $f(f(n)) = 2n \forall n \in N,$ then $f(2)$=________`

Watch Video Solution

**471.** The number of integral values of $a$ for which $f(x) = \log\left((\log)_{\frac{1}{3}}\left((\log)_7(\sin x + a)\right)\right)$ is defined for every real value of $x$ is ________

Watch Video Solution

**472.** If the function $f: (1, \infty) \to (1, \infty)$ is defined by $f(x) = 2^{x(x-1)}, then f^{-1}(x)$ is $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}\left(1 + \sqrt{1 + 4(\log)_2 x}\right)$ $\frac{1}{2}\left(1 - \sqrt{1 + (\log)_2 x}\right.$ (d) not defined

Watch Video Solution

**473.** Let $f(x) = \sin^{23}x - \cos^{22}x and g(x) = 1 + \frac{1}{2}\tan^{-1}|x|$ . Then the number of values of $x$ in the interval $[-10\pi, 8\pi]$ satisfying the equation $f(x) = sgn(g(x))$ is ________

Watch Video Solution

**474.** The number of integral values of $x$ for which

$$\frac{\left(2^{\frac{\pi}{\tan^{-1}x}} - 4\right)(x-4)(x-10)}{x! - (x-1)!} < 0 is - - - - - - .$$

Watch Video Solution

**475.** If $f(x) = \left\{x\cos x + (\log)_e\left(\frac{1-x}{1+x}\right)a; x = 0; x \ne 0\right.$ is odd, then $a$ _ __ _,

Watch Video Solution

**476.** The number of integers in the range of the function $f(x) = \left|4\frac{\left(\sqrt{\cos x} - \sqrt{\sin x}\right)\left(\sqrt{\cos x} + \sqrt{\sin x}\right)}{\cos x + \sin x}\right|$ is_____

Watch Video Solution

**477.** Let $a > 2$ be a constant. If there are just 18 positive integers satisfying the inequality $(x - a)(x - 2a)\left(x - a^2\right) < 0$, then find the value of $a$.

Watch Video Solution

**478.** Let $f$ be a real-valued invertible function such that $f\left(\frac{2x - 3}{x - 2}\right) = 5x - 2, x \neq 2$. Then value of $f^{-1}(13)$ is_______

Watch Video Solution

**479.** Write explicit functions of $y$ defined by the following equations and also find the domains of definitions of the given implicit functions: $x + |y| = 2y$ (b) $e^y - e^{-y} = 2x$ (c)$10^x + 10^y = 10$ (d) $x^2 - \sin^{-1}y = \frac{\pi}{2}$

Watch Video Solution

**480.** Let $g(x) = \sqrt{x - 2k}, \forall 2k \leq x < 2(k + 1)$, where $k \in$ integer. Check whether $g(x)$ is periodic or not.

Watch Video Solution

**481.** Let $f(x) = (2\cos x - 1)(2\cos 2x - 1)\left(2\cos 2^2 x - 1\right)\left(2\cos 2^n x - 1\right)$, (where $\left.n \geq 1\right)$. Then prove that $f\left(\frac{2\pi k}{2^n \pm 1}\right) = 1 \forall k \in I$.

Watch Video Solution

**482.** The number of integers in the domain of function, satisfying $f(x)+f\left(x^{-1}\right)=\dfrac{x^3+1}{x}$, *is* _–_

Watch Video Solution

**483.** If a polynomial function $f(x)$ satisfies $f(f(f(x))=8x+21$, where *pandq* are real numbers, then $p+q$ is equal to ______

Watch Video Solution

**484.** If $f\left(2x+\dfrac{y}{8}, 2x-\dfrac{y}{8}\right)=xy$ , then $f(m,n)+f(n,m)=0$ only when $m=n$ only when $m\neq n$ only when$m=-n$ (d) $f$ or *allmandn*

Watch Video Solution

**485.** The total number of solutions of $[x]^2=x+2\{x\}$, where [.] and {.} denote the greatest integer and the fractional part functions, respectively, is equal to: 2 (b) 4 (c) 6 (d) none of these

Watch Video Solution

**486.** The range of

$f(x) = [1 + \sin x] + \left[2 + \sin\left(\frac{x}{2}\right)\right] + \left[3 + \sin\left(\frac{x}{3}\right)\right] + .... + \left[n + \sin\left(\frac{x}{n}\right)\right] \forall x \in [$

, where [.] denotes the greatest integer function, is,

Watch Video Solution

**487.** The domain of the function $f(x) = \sqrt{\log_{(|x|-1)}\left(x^2 + 4x + 4\right)}$ is (a) $(-3, -1) \cup (1, 2)$ (b)$(-2, -1) \cup (2, \infty)$ (c)$(-\infty, -3) \cup (-2, -1) \cup (2, \infty)$ (d)none of these

Watch Video Solution

**488.** The number of roots of $x^2 - 2 = [\sin x], where[.]$ stands for the greatest integer function is 0 (b) 1 (c) 2 (d) 3.

Watch Video Solution

**489.** Let $f(x) = \sqrt{|x| - \{x\}}(where\{.\}$ denotes the fractional part of $(x)andX, Y$ are its domain and range, respectively). Then (a) $X \in \left(-\infty, \frac{1}{2}\right)$ and $Y \in \left(\frac{1}{2}, \infty\right)$ (b)$X \in \left(-\infty \in, \frac{1}{2}\right) \cup [0, \infty)andY \in \left(\frac{1}{2}, \infty\right)$ (c) $X \in \left(-\infty, -\frac{1}{2}\right) \cup [0, \infty)andY \in [0, \infty)$ (d) none of these

Watch Video Solution

**490.** Let [x] represent the greatest integer less than or equal to $x$ If $\left[\sqrt{n^2 + \lambda}\right] = \left[\sqrt{n^2 + 1}\right] + 2$, where $\lambda, n \in N$, then $\lambda$ can assume (a) $2n + 4$ different values (b)$2n + 5$ different values (c)$2n + 3$ different values (d) $2n + 6$ different values

Watch Video Solution

**491.** The period of the function $f(x) = c^{\sin} \wedge 2x + \sin^2\left(x+\frac{\pi}{3}\right) + \cos x \cos\left(x+\frac{\pi}{3}\right)$ is (where $c$ is constant) 1 (b) $\frac{\pi}{2}$ (c) $\pi$ (d) cannot be determined

Watch Video Solution

**492.** If $f(x) and g(x)$ are periodic functions with periods 7 and 11, respectively, then the period of $f(x) = f(x)g\left(\frac{x}{5}\right) - g(x)f\left(\frac{x}{3}\right)$ is 177 (b) 222 (c) 433 (d) 1155

Watch Video Solution

**493.** The range of $\sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$, where [.] denotes the greatest integer function, is $\left\{\frac{\pi}{2}, \pi\right\}$ (b) $\{\pi\}$ (c) $\left\{\frac{\pi}{2}\right\}$ (d) none of these

Watch Video Solution

**494.** The domain of $f(x) = \sqrt{2\{x\}^2 - 3\{x\} + 1}$, where {.} denotes the fractional part in $[-1, 1]$ is (a) $[-1, 1] - \left(\frac{1}{2, 1}\right)$ (b) $\left[-1, -\frac{1}{2}\right] \cup \left[\frac{0, 1}{2}\right] \cup \{1\}$ (c) $\left[-1, \frac{1}{2}\right]$ (d) $\left[-\frac{1}{2}, 1\right]$

Watch Video Solution

**495.** If $g(f(x)) = |\sin x| and f(g(x)) = \left(\sin\sqrt{x}\right)^2$ , then $f(x) = \sin^2 x, g(x) = \sqrt{x}$ $f(x) = \sin x, g(x) = |x|$ $f\left(x = x^2, g(x) = \sin\sqrt{x}\right.$ $f and g$ cannot be determined

Watch Video Solution

**496.** Each question has four choices, a,b,c and d,out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. if both the statements are true and statement 2 is the correct explanation of statement 1. If both the statements are true but statement 2 is not the correct explanation of statement 1. If statement is True and statement2 is false. If statement1 is false and statement2 is true. Statement 1: If

$x \in [1, \sqrt{3}]$, then the range of $f(x) = \tan^{-1} x$ is $\left[\frac{\pi}{4}, \frac{\pi}{3}\right]$ Statement 2 : If $x \in [a, b]$, then the range of $f(x) is [f(a), f(b)]$

Watch Video Solution

**497.** Let $f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \quad + \frac{1}{n}$. Then $f(1) + f(2) + f(3) + \quad + f(n)$ is equal to

(a)$nf(n) - 1$

(b) $(n+1)f(n) - n$

(c)$(n+1)f(n) + n$

(d) $nf(n) + n$

Watch Video Solution

**498.** Statement-1 : Solution of $\left(1 + x\sqrt{x^2 + y^2}\right)dx + y\left(-1 + \sqrt{x^2 + y^2}\right)dy = 0$ is $x - \frac{y^2}{2} + \frac{1}{3}\left(x^2 + y^2\right)^{\frac{3}{2}} + c = 0$ Statement-2 : Solution of $(1 + xy)ydx + (1 - xy)xdy = 0$ is $\ln\left(\frac{x}{y}\right) - \frac{1}{xy} = c$ (A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for (B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation of STATEMENT-1 for STATEMENT-1 C) STATEMENT-1 is true, STATEMENT-2 is false D)STATEMENT-1 is false, STATEMENT-2 is true

Watch Video Solution