MATHS

BOOKS - CENGAGE MATHS (HINGLISH)

VECTOR ALGEBRA

Solved Examples And Exercises

1. In a trapezium, vector $\vec{B}C = \alpha \vec{A}D^{\cdot}$ We will then find that $\vec{p} = \vec{A}C + \vec{B}D$ is collinear with$\vec{A}D^{\cdot}$ If $\vec{p} = \mu \vec{A}D,$ then which of the following is true? a. $\mu = \alpha + 2$ b. $\mu + \alpha = 2$ c. $\alpha = \mu + 1$ d. $\mu = \alpha + 1$

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2. If the vectors $\vec{a} and \vec{b}$ are linearly idependent satisfying $\left(\sqrt{3}\tan\theta + 1\right)\vec{a} + \left(\sqrt{3}sec\theta - 2\right)\vec{b} = 0,$ then the most general values of θ

are a. $n\pi - \frac{\pi}{6}, n \in Z$ b. $2n\pi \pm \frac{11\pi}{6}, n \in Z$ c. $n\pi \pm \frac{\pi}{6}, n \in Z$ d. $2n\pi + \frac{11\pi}{6}, n \in Z$

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3. Given three non-zero, non-coplanar vectors $\vec{a}, \vec{b}, and \vec{\cdot}\ \vec{r}_1 = p\vec{a} + q\vec{b} + \vec{c} and \vec{r}_2 = \vec{a} + p\vec{b} + q\vec{\cdot}$ If the vectors $\vec{r}_1() + 2\vec{r}_2 and 2\vec{r}_1 + \vec{r}_2$ are collinear, then (P, q) is a. $(0, 0)$ b. $(1, -1)$ c. $(-1, 1)$ d. $(1, 1)$

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4. Let $\vec{r}_1, \vec{r}_2, \vec{r}_3, , \vec{r}_n$ be the position vectors of points P_1, P_2, P_3, P_n relative to the origin $\dot{O}$ If the vector equation $a_1\vec{r}_1 + a_2\vec{r}_2 + \ + a_n\vec{r}_n = 0$ hold, then a similar equation will also hold w.r.t. to any other origin provided a. $a_1 + a_2 + \dot{\ } + a_n = n$ b. $a_1 + a_2 + \dot{\ } + a_n = 1$ c. $a_1 + a_2 + \dot{\ } + a_n = 0$ d. $a_1 = a_2 = a_3 \dot{+} a_n = 0$

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5. In triangle ABC, $\angle A = 30^0$, H is the orthocenter and D is the midpoint of BC. Segment HD is produced to T such that $HD = DT$ The length AT is equal to

(a). $2BC$

(b). $3BC$

(c). $\frac{4}{2}BC$

(d). none of these

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6. If $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = a\vec{\delta} and \vec{\beta} + \vec{\gamma} + \vec{\delta} = b\vec{\alpha}, \vec{\alpha} and \vec{\delta}$ are non-colliner, then $\vec{\alpha} + \vec{\beta} + \vec{\gamma} + \vec{\delta}$ equals a. $a\vec{\alpha}$ b. $b\vec{\delta}$ c. 0 d. $(a + b)\vec{\gamma}$

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7. Given three vectors $\vec{a} = 6\hat{i} - 3\hat{j}, \vec{b} = 2\hat{i} - 6\hat{j} and \vec{c} = -2\hat{i} + 21\hat{j}$ such that $\vec{\alpha} = \vec{a} + \vec{b} + \vec{c}$ Then the resolution of the vector $\vec{\alpha}$ into components with respect to $\vec{a} and \vec{b}$ is given by a. $3\vec{a} - 2\vec{b}$ b. $3\vec{b} - 2\vec{a}$ c. $2\vec{a} - 3\vec{b}$ d. $\vec{a} - 2\vec{b}$

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8. Let us define the length of a vector $a\hat{i} + b\hat{j} + c\hat{k} as |a| + |b| + |c|$ This definition coincides with the usual definition of length of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is and only if a. $a = b = c = 0$ b. any two of $a, b, and c$ are zero c. any one of $a, b, and c$ is zero d. $a + b + c = 0$

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9. Vectors $\vec{a} = -4\hat{i} + 3\hat{k}; \vec{b} = 14\hat{i} + 2\hat{j} - 5\hat{k}$ are laid off from one point. Vector $\hat{d}$, which is being laid of from the same point dividing the angle between vectors $\vec{a} and \vec{b}$ in equal halves and having the magnitude $\sqrt{6}$, is a. $\hat{i} + \hat{j} + 2\hat{k}$ b. $\hat{i} - \hat{j} + 2\hat{k}$ c. $\hat{i} + \hat{j} - 2\hat{k}$ d. $2\hat{i} - \hat{j} - 2\hat{k}$

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10. Vectors $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 4\hat{k},$ are so placed that the end point of one vector is the starting point of the next vector. Then the vector are (A) not coplanar (B) coplanar but cannot form a triangle (C) coplanar and form a triangle (D) coplanar and can form a right angled triangle

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11. The position vectors of the vertices $A, B, and C$ of a triangle are $\hat{i} + \hat{j}, \hat{j} + \hat{k} and \hat{i} + \hat{k}$, respectively. Find the unite vector $\hat{r}$ lying in the plane of ABC and perpendicular to $IA, where I$ is the incentre of the triangle.

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12. A ship is sailing towards the north at a speed of 1.25 m/s. The current is taking it towards the east at the rate of 1 m/s and a sailor is climbing a vertical pole on the ship at the rate of 0.5 m/s. Find the velocity of the sailor in space.

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13. Given four points $P_1, P_2, P_3 and P_4$ on the coordinate plane with origin O which satisfy the condition $\left(\overrightarrow{OP}\right)_{n-1} + \left(\overrightarrow{OP}\right)_{n+1} = \frac{3}{2}\overrightarrow{OP}_n$ (i) If P1 and P2 lie on the curve xy=1 , then prove that P3 does not lie on the curve (ii) If P1,P2,P3 lie on a circle $x^2 + y^2 = 1$, then prove that P4 also lies on this circle.

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14. $ABCD$ is a tetrahedron and O is any point. If the lines joining O to the vrticfes meet the opposite faces at $P, Q, R and S,$ prove that $\frac{OP}{AP} + \frac{OQ}{BQ} + \frac{OR}{CR} + \frac{OS}{DS} = 1.$

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15. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors and $\vec{A} = (p + 4q)\vec{a} = (2p + q + 1)\vec{b} and \vec{B} = (- 2p + q + 2)\vec{a} + (2p - 3q - 1)\vec{b}$,a n d if $3\vec{A} = 2\vec{B}$, then determine p and q.

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16. If $\vec{a}, \vec{b} and \vec{c}$ are any three non-coplanar vectors, then prove that points $l_1\vec{a} + m_1\vec{b} + n_1\vec{c}, l_2\vec{a} + m_2\vec{b} + n_2\vec{c}, l_3\vec{a} + m_3\vec{b} + n_3\vec{c}, l_4\vec{a} + m_4\vec{b} + n_4\vec{c}$ are coplanar if $\begin{bmatrix} l_1 & l_2 & l_3 & l_4 \\ m_1 & m_2 & m_3 & m_4 \\ n_1 & n_2 & n_3 & n_4 \\ 1 & 1 & 1 & 1 \end{bmatrix} = 0$

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17. If $\vec{a}, \vec{b} and \vec{c}$ are three non-zero non-coplanar vectors, then find the linear relation between the following four vectors:

$\vec{a} - 2\vec{b} + 3\vec{c}, 2\vec{a} - 3\vec{b} + 4\vec{c}, 3\vec{a} - 4\vec{b} + 5\vec{c}, 7\vec{a} - 11\vec{b} + 15\vec{\cdot}$

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18. Let a, b, c be distinct non-negative numbers and the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}, c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, and then prove that the quadratic equation $ax^2 + 2cx + b = 0$ has equal roots.

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19. A pyramid with vertex at point P has a regular hexagonal base $ABCDEF$, Positive vector of points A and B are $\hat{i}$ and $\hat{i} + 2\hat{j}$ The centre of base has the position vector $\hat{i} + \hat{j} + \sqrt{3}\hat{k}^{\cdot}$ Altitude drawn from P on the base meets the diagonal AD at point $\dot{G}$ find the all possible position vectors of $\dot{G}$ It is given that the volume of the pyramid is $6\sqrt{3}$ cubic units and AP is 5 units.

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20. A straight line L cuts the lines $AB, ACandAD$ of a parallelogram $ABCD$ at points $B_1, C_1andD_1,$ respectively. If $\left(\vec{AB}\right)_1, \lambda_1\vec{AB}, \left(\vec{AD}\right)_1 = \lambda_2\vec{AD}and\left(\vec{AC}\right)_1 = \lambda_3\vec{AC},$ then prove that $\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}.$

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21. $A, B, CandD$ have position vectors $\vec{a}, \vec{b}, \vec{c}and\vec{d},$ respectively, such that $\vec{a} - \vec{b} = 2\left(\vec{d} - \vec{c}\right)^{\cdot}$ Then a. $ABandCD$ bisect each other b. $BDandAC$ bisect each other c. $ABandCD$ trisect each other d. $BDandAC$ trisect each other

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22. If $\vec{a}and\vec{b}$ are two unit vectors and θ is the angle between them, then the unit vector along the angular bisector of $\vec{a}$ and $\vec{b}$ will be given by a. $\frac{\vec{a} - \vec{b}}{\cos(\theta/2)}$ b. $\frac{\vec{a} + \vec{b}}{2\cos(\theta/2)}$ c. $\frac{\vec{a} - \vec{b}}{2\cos(\theta/2)}$ d. none of these

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23. $ABCD$ is a quadrilateral. E is the point of intersection of the line joining the midpoints of the opposite sides. If O is any point and $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D = x\vec{O}E, thenx$ is equal to a. 3 b. 9 c. 7 d. 4

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24. If vectors $\vec{A}B = -3\hat{i} + 4\hat{k}and\vec{A}C = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a $\text{Delta}ABC$, then the length of the median through Ais a. $\sqrt{14}$ b. $\sqrt{18}$ c. $\sqrt{29}$ d. $\sqrt{5}$

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25. $ABCD$ parallelogram, and A_1andB_1 are the midpoints of sides $BCandCD$, respectivley . If $\vec{\forall}_1 + \vec{A}B_1 = \lambda\vec{A}C, then\lambda$ is equal to a. $\frac{1}{2}$ b. 1 c. $\frac{3}{2}$ d. 2 e. $\frac{2}{3}$

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26. The position vectors of the points $PandQ$ with respect to the origin O are $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} - 2\hat{k}$, respectively. If M is a point on PQ, such that OM is the bisector of $\angle POQ$, then $\vec{O}M$ is a. $2\left(\hat{i} - \hat{j} + \hat{k}\right)$ b. $2\hat{i} + \hat{j} - 2\hat{k}$ c. $2\left(-\hat{i} + \hat{j} - \hat{k}\right)$ d. $2\left(\hat{i} + \hat{j} + \hat{k}\right)$

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27. If G is the centroid of triangle $ABC, then\vec{G}A + \vec{G}B + \vec{G}C$ is equal to a. $\vec{0}$ b. $3\vec{G}A$ c. $3\vec{G}B$ d. $3\vec{G}C$

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28. Let ABC be triangle, the position vecrtors of whose vertices are respectively $\hat{i} + 2\hat{j} + 4\hat{k}$, -$2\hat{i} + 2\hat{j} + \hat{k} and 2\hat{i} + 4\hat{j} - 3\hat{k}$. Then $\mathrm{Delta}ABC$ is a. isosceles b. equilateral c. right angled d. none of these

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29. If $\left|\vec{a}+\vec{b}\right| < \left|\vec{a}-\vec{b}\right|$, then the angle between $\vec{a} and \vec{b}$ can lie in the interval a. $(\pi/2, \pi/2)$ b. $(0, \pi)$ c. $(\pi/2, 3\pi/2)$ d. $(0, 2\pi)$

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30. $'I'$ is the incentre of triangle ABC whose corresponding sides are $a, b, c,$ rspectively. $a\vec{IA}+b\vec{IB}+c\vec{IC}$ is always equal to a. $\vec{0}$ b. $(a+b+c)\vec{BC}$ c. $\left(\vec{a}+\vec{b}+\vec{c}\right)\vec{AC}$ d. $(a+b+c)\vec{AB}$

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31. Let $x^2+3y^2=3$ be the equation of an ellipse in the $x-y$ plane. $A and B$ are two points whose position vectors are $-\sqrt{3}\hat{i} and -\sqrt{3}\hat{i}+2\hat{k}$ Then the position vector of a point P on the ellipse such that $\angle APB=\pi/4$ is a. $\pm\hat{j}$ b. $\pm\left(\hat{i}+\hat{j}\right)$ c. $\pm\hat{i}$ d. none of these

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32. If $\vec{x} and \vec{y}$ are two non-collinear vectors and ABC isa triangle with side lengths $a, b, and c$ satisfying $(20a - 15b)\vec{x} + (15b - 12c)\vec{y} + (12c - 20a)\left(\vec{\times} x\vec{y}\right) = 0,$ then triangle ABC is a. an acute-angled triangle b. an obtuse-angled triangle c. a right-angled triangle d. an isosceles triangle

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33. If $\hat{i} - 3\hat{j} + 5\hat{k}$ bisects the angle between $\hat{a} and - \hat{i} + 2\hat{j} + 2\hat{k}, where \hat{a}$ is a unit vector, then a. $\hat{a} = \frac{1}{105}\left(41\hat{i} + 88\hat{j} - 40\hat{k}\right)$ b. $\hat{a} = \frac{1}{105}\left(41\hat{i} + 88\hat{j} + 40\hat{k}\right)$ c. $\hat{a} = \frac{1}{105}\left(- 41\hat{i} + 88\hat{j} - 40\hat{k}\right)$ d. $\hat{a} = \frac{1}{105}\left(41\hat{i} - 88\hat{j} - 40\hat{k}\right)$

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34. If $4\hat{i} + 7\hat{j} + 8\hat{k}, 2\hat{i} + 3\hat{j} + 24 and 2\hat{i} + 5\hat{j} + 7\hat{k}$ are the position vectors of the vertices $A, B and C,$ respectively, of triangle ABC , then the position vecrtor of the point where the bisector of angle A meets BC is a. $\frac{2}{3}\left(-6\hat{i}-8\hat{j}-\hat{k}\right)$ b. $\frac{2}{3}\left(6\hat{i}+8\hat{j}+6\hat{k}\right)$ c. $\frac{1}{3}\left(6\hat{i}+13\hat{j}+18\hat{k}\right)$ d. $\frac{1}{3}\left(5\hat{j}+12\hat{k}\right)$

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35. If $\vec{b}$ is a vector whose initial point divides thejoin of $5\hat{i} and 5\hat{j}$ in the ratio $k:1$ and whose terminal point is the origin and $\left|\vec{b}\right| \le \sqrt{37}, then k$ lies in the interval a. $[-6, -1/6]$ b. $(-\infty, -6] \cup [-1/6, \infty)$ c. $[0, 6]$ d. none of these

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36. Find the value of λ so that the points P, Q, R and S on the sides OA, OB, OC and $AB,$ respectively, of a regular tetrahedron $OABC$ are coplanar. It is given that $\frac{OP}{OA}=\frac{1}{3}, \frac{OQ}{OB}=\frac{1}{2}, \frac{OR}{OC}=\frac{1}{3}$ and $\frac{OS}{AB}=\lambda$ (A) $\lambda=\frac{1}{2}$ (B) $\lambda=-1$ (C) $\lambda=0$ (D) for no value of λ

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37. A uni-modular tangent vector on the curve $x = t^2 + 2, y = 4t - 5, z = 2t^2 - 6t$ at t=2 is a. $\frac{1}{3}\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$ b. $\frac{1}{3}\left(\hat{i} - \hat{j} - \hat{k}\right)$ c. $\frac{1}{6}\left(2\hat{i} + \hat{j} + \hat{k}\right)$ d. $\frac{2}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$

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38. If $\vec{x} and \vec{y}$ are two non-collinear vectors and a, b, and c represent the sides of a ABC satisfying $(a - b)\vec{x} + (b - c)\vec{y} + (c - a)\left(\vec{\times} x\vec{y}\right) = 0,$ then ABC is (where $\vec{\times} x\vec{y}$ is perpendicular to the plane of $x and y$) a. an acute-angled triangle b. an obtuse-angled triangle c. a right-angled triangle d. a scalene triangle

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39. The position vectors of points $A and B$ w.r.t. the origin are $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$ respectively. Determine vector $\vec{OP}$ which bisects angle $AOB,$ where P is a point on AB

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40. What is the unit vector parallel to $\vec{a} = 3\hat{i} + 4\hat{j} - 2\hat{k}$? What vector should be added to $\vec{a}$ so that the resultant is the unit vector $\hat{i}$?

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41. ABCD is a quadrilateral and E is the point of intersection of the lines joining the middle points of opposite side. Show that the resultant of $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ = 4 $\overrightarrow{OE}$, where O is any point.

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42. $ABCD$ is a parallelogram. If $LandM$ are the mid-points of $BCandDC$ respectively, then express $\vec{A}Land\vec{A}M$ in terms of $\vec{A}Band\vec{A}D$. Also, prove that $\vec{A}L + \vec{A}M = \frac{3}{2}\vec{A}C$.

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43. If $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are four vectors in three-dimensional space with the same initial point and such that $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$, show that terminals $A, B, C and D$ of these vectors are coplanar. Find the point at which $AC and BD$ meet. Find the ratio in which P divides $AC and BD$.

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44. Find the vector of magnitude 3, bisecting the angle between the vectors $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

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45. If $\vec{a} and \vec{b}$ are two vectors of magnitude 1 inclined at 120^{0}, then find the angle between $\vec{b} and \vec{b} - \vec{a}$.

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46. If $\vec{r}_1, \vec{r}_2, \vec{r}_3$ are the position vectors of the collinear points and scalar $p and q$ exist such that $\vec{r}_3 = p\vec{r}_1 + q\vec{r}_2$, then show that $p + q = 1$.

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47. Examine the following vector for linear independence: (1) $\vec{i} + \vec{j} + \vec{k}, 2\vec{i} + 3\vec{j} - \vec{k}, -\vec{i} - 2\vec{j} + 2\vec{k}$ (2) $3\vec{i} + \vec{j} - \vec{k}, 2\vec{i} - \vec{j} + 7\vec{k}, 7\vec{i} - \vec{j} + 13\vec{k}$

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48. Show that the vectors $2\vec{a} - \vec{b} + 3\vec{c}, \vec{a} + \vec{b} - 2\vec{c} and \vec{a} + \vec{b} - 3\vec{c}$ are non-coplanar vectors (where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors)

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49. Let $\vec{a}, \vec{b} and \vec{c}$ be three units vectors such that $2\vec{a} + 4\vec{b} + 5\vec{c} = 0$. Then which of the following statement is true? a. $\vec{a}$ is parallel to $\vec{b}$ b. $\vec{a}$ is perpendicular to $\vec{b}$ c. $\vec{a}$ is neither parallel nor perpendicular to $\vec{b}$ d. none of these

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50. Four non –zero vectors will always be a. linearly dependent b. linearly independent c. either a or b d. none of these

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51. A boat moves in still water with a velocity which is k times less than the river flow velocity. Find the angle to the stream direction at which the boat should be rowed to minimize drifting.

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52. In a triangle $PQR, SandT$ are points on $QRandPR,$ respectively, such that $QS = 3SRandPT = 4TR^{\cdot}$ Let M be the point of intersection of $PSandQT^{\cdot}$ Determine the ratio $QM:MT$ using the vector method .

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53. In a quadrilateral $PQRS, \vec{PQ} = \vec{a}, \vec{QR}, \vec{b}, \vec{SP} = \vec{a} - \vec{b}, M$ is the midpoint of $\vec{Q}RandX$ is a point on SM such that $SX = \frac{4}{5}SM^{\cdot}$ Prove that $P, XandR$ are collinear.

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54. If $D, EandF$ are three points on the sides $BC, CAandAB,$ respectively, of a triangle ABC such that the $\frac{BD}{CD}, \frac{CE}{AE}, \frac{AF}{BF} = -1$

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55. Sow that $x_1\hat{i}+y_1\hat{j}+z_1\hat{k}, x_2\hat{i}+y_2\hat{j}+z_2\hat{k}, and x_3\hat{i}+y_3\hat{j}+z_3\hat{k},$ are non-coplanar if $|x_1|>|y_1|+|z_1|, |y_2|>|x_2|+|z_2| and |z_3|>|x_3|+|y_3|.$

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56. The position vector of the points $P and Q$ are $5\hat{i}+7\hat{j}-2\hat{k}$ and $-3\hat{i}+3\hat{j}+6\hat{k}$, respectively. Vector $\vec{A}=3\hat{i}-\hat{j}+\hat{k}$ passes through point P and vector $\vec{B}=-3\hat{i}+2\hat{j}+4\hat{k}$ passes through point Q . A third vector $2\hat{i}+7\hat{j}-5\hat{k}$ intersects vectors $A and B$ Find the position vectors of points of intersection.

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57. Consider the vectors $\hat{i}+\cos(\beta-\alpha)\hat{j}+\cos(\gamma-\alpha)\hat{k}, \cos(\alpha-\beta)\hat{i}+\hat{j}+\cos(\gamma-\beta)\hat{k} and \cos(\alpha-\gamma)\hat{i}+\cos(\beta-\gamma)\hat{k}$ where $\alpha, \beta,$ and γ are different angles. If these vectors are coplanar, show that a is independent of α, β and γ

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58. If $\vec{A} and \vec{B}$ are two vectors and k any scalar quantity greater than zero, then prove that $\left|\vec{A}+\vec{B}\right|^2 \leq (1+k)\left|\vec{A}\right|^2 + \left(1+\frac{1}{k}\right)\left|\vec{B}\right|^2.$

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59. The vectors $x\hat{i}+(x+1)\hat{j}+(x+2)\hat{k}, (x+3)\hat{i}+(x+4)\hat{j}+(x+5)\hat{k} and (x+6)\hat{i}+(x+7)\hat{j}+(x+8$ are coplanar if x is equal to a. 1 b. -3 c. 4 d. 0

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60. $\vec{A}$ isa vector with direction cosines $\cos\alpha, \cos\beta and \cos\gamma$. Assuming the $y-z$ plane as a mirror, the directin cosines of the reflected image of $\vec{A}$ in the plane are a. $\cos\alpha, \cos\beta, \cos\gamma$ b. $\cos\alpha, -\cos\beta, \cos\gamma$ c. $-\cos\alpha, \cos\beta, \cos\gamma$ d. $-\cos\alpha, -\cos\beta, -\cos\gamma$

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61. The vector $\vec{a}$ has the components $2p$ and 1 w.r.t. a rectangular Cartesian system. This system is rotated through a certain angel about the origin in the counterclockwise sense. If, with respect to a new system, $\vec{a}$ has components $(p+1) and 1$, then p is equal to a. -4 b. $-1/3$ c. 1 d. 2

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62. The sides of a parallelogram are $2\hat{i}+4\hat{j}-5\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$. The unit vector parallel to one of the diagonals is a. $\frac{1}{7}\left(3\hat{i}+6\hat{j}-2\hat{k}\right)$ b. $\frac{1}{7}\left(3\hat{i}-6\hat{j}-2\hat{k}\right)$ c. $\frac{1}{\sqrt{69}}\left(\hat{i}+6\hat{j}+8\hat{k}\right)$ d. $\frac{1}{\sqrt{69}}\left(-\hat{i}-2\hat{j}+8\hat{k}\right)$

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63. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vector and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c}, \lambda\vec{b}+\mu\vec{c} and (2\lambda-1)\vec{c}$ are coplanar when a. $\mu \in R$ b.

$\lambda = \frac{1}{2}$ c. $\lambda = 0$ d. no value of λ

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64. If points $\hat{i} + \hat{j}, \hat{i} - \hat{j} and p\hat{i} + q\hat{j} + r\hat{k}$ are collinear, then a. $p = 1$ b. $r = 0$ c. qR d. $q \neq 1$

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65. If the vectors $\hat{i} - \hat{j}, \hat{j} + \hat{k} and \vec{a}$ form a triangle, then $\vec{a}$ may be a. $-\hat{i} - \hat{k}$ b. $\hat{i} - 2\hat{j} - \hat{k}$ c. $2\hat{i} + \hat{j} + \hat{k}$ d. $\hat{i} + \hat{k}$

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66. If the resultant of three forces $\vec{F}_1 = p\hat{i} + 3\hat{j} - \hat{k}, \vec{F}_2 = 6\hat{i} - \hat{k} and \vec{F}_3 = -5\hat{i} + \hat{j} + 2\hat{k}$ acting on a parricle has magnitude equal to 5 units, then the value of p is a. -6 b. -4 c. 2 d. 4

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67. $\vec{a}, \vec{b}, \vec{c}$ are three coplanar unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$. If three vectors $\vec{p}, \vec{q}, and \vec{r}$ are parallel to $\vec{a}, \vec{b}, and \vec{c}$, respectively, and have integral but different magnitudes, then among the following options, $\left|\vec{p} + \vec{q} + \vec{r}\right|$ can take a value equal to a. 1 b. 0 c. $\sqrt{3}$ d. 2

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68. The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i} + (4x - 2)\dot{\hat{j}} + 2\hat{k}$. Then value of x are $-\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 2

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69. Prove that point $\hat{i}$ +2$\hat{j}$ - 3$\hat{k}$,2$\hat{i}$ - $\hat{j}$ + $\hat{k}$ and 2$\hat{i}$ + 5$\hat{j}$ - $\hat{k}$ from a triangle in space.

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70. Show that the point A, B and C with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k} = 2\hat{i}$ $j + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$, respectively from the vertices of a right angled triangle.

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71. If $2\vec{AC} = 3\vec{CB}$, then prove that $2\vec{OA} = 3\vec{CB}$ then prove that $2\vec{OA} + 3\vec{OB}$ $= 5\vec{OC}$ where O is the origin.

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72. Fined the unit vector in the direction of vector $\vec{PQ}$, where P and Q are the points (1,2,3) and (4,5,6), respectively.

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73. For given vector, $\vec{a} = 2\hat{i}\ j + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

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74. If the projections of vector $\vec{a}$ on x -, y - and z -axes are 2, 1 and 2 units ,respectively, find the angle at which vector $\vec{a}$ is inclined to the z -axis.

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75. Find a vector in the direction of the vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.

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76. If $\vec{a}$,$\vec{b}$,$\vec{c}$,$\vec{d}$ are the position vector of point A, B, C and D , respectively referred to the same origin O such that no three of these point are collinear and $\vec{a} + \vec{c} = \vec{b} + \vec{d}$, than prove that quadrilateral $ABCD$ is a parallelogram.

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77. Show that the points $A(6, -7, 0), B(16, -19, -4), C(0, 3, -6) and D(2, -5, 10)$ are such that $ABandCD$ interesect at the point $P(1, -1, 2)$.

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78. Statement 1: The direction cosines of one of the angular bisectors of two intersecting line having direction cosines as $l_1, m_1, n_1 and l_2, m_2, n_2$ are proportional to $l_1 + l_2, m_1 + m_2, n_1 + n_2$. Statement 2: The angle between the two intersection lines having direction cosines as $l_1, m_1, n_1 and l_2, m_2, n_2$ is given by $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$.

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79. Statement 1: In $\text{Delta}ABC, \vec{A}B + \vec{B}C + \vec{C}A = 0$ Statement 2: If $\vec{O}A = \vec{a}, \vec{O}B = \vec{b}, then\vec{A}B = \vec{a} + \vec{b}$

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80. Statement 1: If $\vec{u}and\vec{v}$ are unit vectors inclined at an angle $\alpha and\vec{x}$ is a unit vector bisecting the angle between them, then $\vec{x} = (\vec{u} + \vec{v})/\left(2\sin(\alpha/2)^{\cdot}\right.$ Statement 2: If $\text{Delta}ABC$ is an isosceles triangle with $AB = AC = 1,$ then the vector representing the bisector of angel A is given by $\vec{A}D = (\vec{A}B + \vec{A}C)/2.$

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81. Statement 1: If $\cos\alpha, \cos\beta, and\cos\gamma$ are the direction cosines of any line segment, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$ Statement 2: If $\cos\alpha, \cos\beta, and\cos\gamma$ are the direction cosines of any line segment, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 1.$

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82. A vector has components p and 1 with respect to a rectangular Cartesian system. The axes are rotted through an angel αabout the origin the anticlockwise sense. Statement 1: IF the vector has component $p+2$ and 1 with respect to the new system, then $p=-1$. Statement 2: Magnitude of the original vector and new vector remains the same.

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83. Statement 1: if three points $P, QandR$ have position vectors $\vec{a}, \vec{b}, and\vec{c}$, respectively, and $2\vec{a}+3\vec{b}-5\vec{c}=0,$ then the points $P, Q, andR$ must be collinear. Statement 2: If for three points $A, B, andC, \vec{A}B=\lambda\vec{A}C,$ then points $A, B, andC$ must be collinear.

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84. In a four-dimensional space where unit vectors along the axes are $\hat{i}, \hat{j}, \hat{k} and \hat{l}, and \vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are four non-zero vectors such that no vector can be expressed as a linear combination of others and $(\lambda - 1)\left(\vec{a}_1 - \vec{a}_2\right) + \mu\left(\vec{a}_2 + \vec{a}_3\right) + \gamma\left(\vec{a}_3 + \vec{a}_4 - 2\vec{a}_2\right) + \vec{a}_3 + \delta\vec{a}_4 = 0,$ then a. $\lambda = 1$ b. $\mu = -2/3$ c. $\gamma = 2/3$ d. $\delta = 1/3$

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85. Let ABC be a triangle, the position vectors of whose vertices are $7\hat{j} + 10\hat{k}, -\hat{i} + 6\hat{j} + 6\hat{k} and - 4\hat{i} + 9\hat{j} + 6\hat{k} Then \text{Delta} ABC$ is a. isosceles b. equilateral c. right angled d. none of these

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86. If non-zero vectors $\vec{a} and \vec{b}$ are equally inclined to coplanar vector $\vec{c}, then \vec{c}$ can be a. $\frac{|\vec{a}|}{|\vec{a}| + 2|\vec{b}|}a + \frac{|\vec{b}|}{|\vec{a}| + |\vec{b}|}\vec{b}$ b. $\frac{|\vec{b}|}{|\vec{a}| + |\vec{b}|}a + \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|}\vec{b}$ c.

$\frac{|\vec{a}|}{|\vec{a}|+2|\vec{b}|}a+\frac{|\vec{b}|}{|\vec{a}|+2|\vec{b}|}\vec{b}$ d. $\frac{|\vec{b}|}{2|\vec{a}|+|\vec{b}|}a+\frac{|\vec{a}|}{2|\vec{a}|+|\vec{b}|}\vec{b}$

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87. If $A(-4, 0, 3) and B(14, 2, -5)$, then which one of the following points lie on the bisector of the angle between $\vec{O}A and \vec{O}B(O$ is the origin of reference)? a. $(2, 2, 4)$ b. $(2, 11, 5)$ c. $(-3, -3, -6)$ d. $(1, 1, 2)$

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88. Prove that the sum of three vectors determined by the medians of a triangle directed from the vertices is zero.

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89. Prove that the resultant of two forces acting at point O and represented by $\vec{O}B$ and $\vec{O}C$ is given by $2\vec{O}D$,where D is the midpoint of

BC.

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90. Two forces $\vec{AB}$ and $\vec{AD}$ are acting at vertex A of a quadrilateral ABCD and two forces $\vec{CB}$ and $\vec{CD}$ at C prove that their resultant is given by $4\vec{EF}$, where E and F are the midpoints of AC and BD, respectively.

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91. ABC is a triangle and P any point on BC. if $\vec{PQ}$ is the sum of $\vec{AP} + \vec{PB} + \vec{PC}$, show that ABPQ is a parallelogram and Q , therefore , is a fixed point.

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92. If vector $\vec{a} + \vec{b}$ bisects the angle between $\vec{a}$ and $\vec{b}$, then prove that $|\vec{a}|$ = $|\vec{b}|$.

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93. ABCDE is a pentagon .prove that the resultant of force $\vec{A}B$, $\vec{A}E$,$\vec{B}C$,$\vec{D}C$,$\vec{E}D$ and $\vec{A}C$,is 3$\vec{A}C$.

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94. if $\vec{A}o + \vec{O}B = \vec{B}O + \vec{O}C$,than prove that B is the midpoint of AC.

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95. A unit vector of modulus 2 is equally inclined to x - and y -axes at an angle $\pi/3$. Find the length of projection of the vector on the z -axis.

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96. Let $\vec{a}, \vec{b} and \vec{c}$ be unit vectors such that $\vec{a} + \vec{b} - \vec{c} = 0$. If the area of triangle formed by vectors $\vec{a} and \vec{b} is A$, then what is the value of $4A^2$?

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97. If the resultant of three forces $\vec{F}_1 = p\hat{i} + 3\hat{j} - \hat{k}, \vec{F}_2 = 6\hat{i} - \hat{k} and \vec{F}_3 = -5\hat{i} + \hat{j} + 2\hat{k}$ acting on a parricle has magnitude equal to 5 units, then the value of p is a. -6 b. -4 c. 2 d. 4

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98. Statement 1: Let $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ be the position vectors of four points $A, B, C and D$ and $3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = 0$. Then points $A, B, C, and D$ are coplanar. Statement 2: Three non-zero, linearly dependent coinitial vector $\left(\vec{PQ}, \vec{PR} and \vec{PS}\right)$ are coplanar. Then $\vec{PQ} = \lambda\vec{PR} + \mu\vec{PS}, where \lambda and \mu$ are scalars.

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99. Statement 1:Let $A(\vec{a}), B(\vec{b}) and C(\vec{c})$ be three points such that $\vec{a} = 2\hat{i} + \hat{k}, \vec{b} = 3\hat{i} - \hat{j} + 3\hat{k} and \vec{c} = -\hat{i} + 7\hat{j} - 5\hat{k}$ Then $OABC$ is a tetrahedron. Statement 2: Let $A(\vec{a}), B(\vec{b}) and C(\vec{c})$ be three points such that vectors $\vec{a}, \vec{b} and \vec{c}$ are non-coplanar. Then $OABC$ is a tetrahedron where O is the origin.

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100. Statement 1: If $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b}\right|$, then $\vec{a}$ and $\vec{b}$ are perpendicular to each other. Statement 2: If the diagonal of a parallelogram are equal magnitude, then the parallelogram is a rectangle.

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101. Statement 1: $\vec{a} = 3\vec{i} + p\vec{j} + 3\vec{k}$ and $\vec{b} = 2\vec{i} + 3\vec{j} + q\vec{k}$ are parallel vectors if $p = 9/2 and q = 2.$ Statement 2: if

$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k} and \vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$ are parallel, then $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$.

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102. The position vectors of the vertices $A, B and C$ of a triangle are three unit vectors $\vec{a}, \vec{b}, and \vec{c}$, respectively. A vector $\vec{d}$ is such that $\vec{d}\dot{\vec{a}} = \vec{d}\dot{\vec{b}} = \vec{d}\dot{\vec{c}} and \vec{d} = \lambda\left(\vec{b} + \vec{c}\right)^{.}$ Then triangle ABC is a. acute angled b. obtuse angled c. right angled d. none of these

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103. $a and b$ form the consecutive sides of a regular hexagon $ABCDEF^{.}$ Column I, Column II If $\vec{CD} = x\vec{a} + y\vec{b},$ then, p. $x = -2$ If $\vec{CE} = x\vec{a} + y\vec{b},$ then, q.$x = -1$ If $\vec{AE} = x\vec{a} + y\vec{b},$ then, r. $y = 1$ $\vec{AD} = -x\vec{b},$ then, s.$y = 2$

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104. Column I, Column II Collinear vectors, p.$\vec{a}$ Coinitial vectors, q. $\vec{b}$ Equal vectors, r. $\vec{c}$ Unlike vectors (same intitial point), s. $\vec{d}$

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105. Statement 1: $|\vec{a}| = 3, |\vec{b}| = 4 and |\vec{a} + \vec{b}| = 5, then |\vec{a} - \vec{b}| = 5.$

Statement 2: The length of the diagonals of a rectangle is the same.

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106. A man travelling towards east at 8km/h finds that the wind seems to blow directly from the north On doubling the speed, he finds that it appears to come from the north-east. Find the velocity of the wind.

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107. OABCDE is a regular hexagon of side 2 units in the XY-plane in the first quadrant. O being the origin and OA taken along the x-axis. A point P is taken on a line parallel to the z-axis through the centre of the hexagon at a distance of 3 unit from O in the positive Z direction. Then find vector AP.

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108. If $\vec{a} = 7\hat{i} - 4\hat{k} and \vec{b} = -2\hat{i} - \hat{j} + 2\hat{k}$, determine vector $\vec{c}$ along the internal bisector of the angle between of the angle between vectors $\vec{a} and \vec{b} such that \left|\vec{c}\right| = 5\sqrt{6}$

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109. Find a unit vector $\vec{c}$ if $-\vec{i} + \vec{j} - \vec{k}$ bisects the angle between $\vec{c}$ and $3\vec{i} + 4\vec{j}$.

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110. The vectors $2i + 3\hat{j}, 5\hat{i} + 6\hat{j}$ and $8\hat{i} + \lambda\hat{j}$ have initial points at (1, 1). Find the value of λ so that the vectors terminate on one straight line.

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111. If $\vec{a}, \vec{b} and \vec{c}$ are three non-zero vectors, no two of which ar collinear, $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a},$ then find the value of $\left|\vec{a} + 2\vec{b} + 6\vec{c}\right|^{.}$

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112. i. Prove that the points $\vec{a} - 2\vec{b} + 3\vec{c}, 2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-7\vec{b} + 10\vec{c}$ are are collinear, where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar. ii. Prove that the points $A(1, 2, 3), B(3, 4, 7),$ and $C(-3, -2, -5)$ are collinear. find the ratio in which point C divides AB.

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113. Check whether the given three vectors are coplanar or non-coplanar.

$-2\hat{i}-2\hat{j}+4\hat{k},\ -2\hat{i}+4\hat{j}, 4\hat{i}-2\hat{j}-2\hat{k}$

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114. Prove that the four points $6\hat{i}-7\hat{j}, 16\hat{i}-19\hat{j}-4\hat{k}, 3\hat{j}-6\hat{k} and 2\hat{i}+5\hat{j}+10\hat{5}$ form a tetrahedron in space.

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115. If $\vec{a} and \vec{b}$ are two non-collinear vectors, show that points $l_1\vec{a}+m_1\vec{b}, l_2\vec{a}+m_2\vec{b}$ and $l_3\vec{a}+m_3\vec{b}$ are collinear if $\left|l_1 l_2 l_3 m_1 m_2 m_3 111\right|=0.$

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116. Show, by vector methods, that the angularbisectors of a triangle are concurrent and find an expression for the position vector of the point of concurrency in terms of the position vectors of the vertices.

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117. Let $\vec{A}(t) = f_1(t)\hat{i} + f_2(t)\hat{j}$ and $\vec{B}(t) = g(t)\hat{i} + g_2(t)\hat{j}, t \in [0, 1], f_1, f_2, g_1 g_2$ are continuous functions. If $\vec{A}(t)$ and $\vec{B}(t)$ are non-zero vectors for all t and $\vec{A}(0) = 2\hat{i} + 3\hat{j}, \vec{A}(1) = 6\hat{i} + 2\hat{j}, \vec{B}(0) = 3\hat{i} + 2\hat{i}$ and $\vec{B}(1) = 2\hat{j} + 6\hat{j}$

Then,show that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some t.

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118. Find the least positive integral value of x for which the angel between vectors $\vec{a} = x\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = 2x\hat{i} + x\hat{j} - \hat{k}$ is acute.

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119. If vectors $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k} and \vec{c} = \text{lambdai} + \hat{j} + 2\hat{k}$ are coplanar, then find the value of $(\lambda - 4)^{\cdot}$

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120. Find the values of λ such that $x, y, z \neq (0, 0, 0) and \left(\hat{i} + \hat{j} + 3\hat{k}\right)x + \left(3\hat{i} - 3\hat{j} + \hat{k}\right)y + \left(-4\hat{i} + 5\hat{j}\right)z = \lambda\left(x\hat{i} + y\hat{j} + z\hat{k}\right.$, where $\hat{i}, \hat{j}, \hat{k}$ are unit vector along coordinate axes.

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121. A vector has component $A_1, A_2 and A_3$ in a right -handed rectangular Cartesian coordinate system $OXYZ^{\cdot}$ The coordinate system is rotated about the x-axis through an angel $\pi/2$. Find the component of A in the new coordinate system in terms of $A_1, A_2, and A_3^{\cdot}$

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122. The position vectors of the point $A, B, CandDare 3\hat{i} - 2\hat{j} - \hat{k}, 2\hat{i} + 3\hat{j} - 4\hat{k}, -\hat{i} + \hat{j} + 2\hat{k}$ and $4\hat{i} + 5\hat{j} + \lambda\hat{k}$, respectively. If the points $A, B, CandD$ lie on a plane, find the value of λ.

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123. Let $OACB$ be a parallelogram with O at the origin and OC a diagonal. Let D be the midpoint of OA. using vector methods prove that $BDandCO$ intersect in the same ratio. Determine this ratio.

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124. In a triangle $ABC, DandE$ are points on $BCandAC$, respectivley, such that $BD = 2DCandAE = 3EC$. Let P be the point of intersection of $ADandBE$. Find BP/PE using the vector method.

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125. Prove, by vector method or otherwise, that the point of intersection of the diagonals of a trapezium lies on the line passing through the midpoint of the parallel sides (you may assume that the trapezium is not a parallelogram).

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126. If the resultant of two forces is equal in magnitude to one of the components and perpendicular to it direction, find the other components using the vector method.

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127. The axes of coordinates are rotated about the z-axis though an angle of $\pi/4$ in the anticlockwise direction and the components of a vector are $2\sqrt{2},\ 3\sqrt{2}, 4.$ Prove that the components of the same vector in the original system are -1,5,4.

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128. Three coinitial vectors of magnitudes a, 2a and 3a meet at a point and their directions are along the diagonals if three adjacent faces if a cube. Determined their resultant R. Also prove that the sum of the three vectors determinate by the diagonals of three adjacent faces of a cube passing through the same corner, the vectors being directed from the corner, is twice the vector determined by the diagonal of the cube.

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129. If two side of a triangle are $\hat{i} + 2\hat{j} and \hat{i} + \hat{k}$, then find the length of the third side.

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130. If in parallelogram ABCD, diagonal vectors are $\vec{AC} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{BD} = -6\hat{i} + 7\hat{j} - 2\hat{k}$, then find the adjacent side vectors $\vec{AB}$ and $\vec{AD}$

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131. Find the resultant of vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k} and \vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$ Find the unit vector in the direction of the resultant vector.

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132. Check whether the three vectors $2\hat{i} + 2\hat{j} + 3\hat{k}, - 3\hat{i} + 3\hat{j} + 2\hat{k} and 3\hat{i} + 4\hat{k}$ from a triangle or not

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133. The midpoint of two opposite sides of a quadrilateral and the midpoint of the diagonals are the vertices of a parallelogram. Prove that using vectors.

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134. The lines joining the vertices of a tetrahedron to the centroids of opposite faces are concurrent.

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135. Find the angle of vector $\vec{a} = 6\hat{i} + 2\hat{j} - 3\hat{k}$ with x -axis.

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136. If the vectors $\vec{\alpha} = a\hat{i} + a\hat{j} + c\hat{k}, \vec{\beta} = \hat{i} + \hat{k} and \vec{\gamma} = c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar, then prove that c is the geometric mean of $a and b$.

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137. The points with position vectors $60i + 3j, 40i - 8j, ai - 52j$ are collinear if a. $a = -40$ b. $a = 40$ c. $a = 20$ d. none of these

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138. Lett α, β and γ be distinct real numbers. The points whose position vector's are $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$; $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$ and $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$

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139. Let $\vec{a} = \vec{i} - \vec{k}, \vec{b} = x\vec{i} + \vec{j} + (1 - x)\vec{k}$ and $\vec{c} = y\vec{i} + x\vec{j} + (1 + x - y)\vec{k}$. Then $\left[\vec{a}\vec{b}\vec{c}\right]$ depends on (A) only x (B) only y (C) Neither x nor y (D) both x and y

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140. In a $\triangle OAB$,E is the mid point of OB and D is the point on AB such that $AD:DB = 2:1$ If OD and AE intersect at P then determine the ratio of $OP:PD$ using vector methods

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141. If $\vec{a}, \vec{b}$ are two non-collinear vectors, prove that the points with position vectors $\vec{a} + \vec{b}, \vec{a} - \vec{b}$ and $\vec{a} + \lambda\vec{b}$ are collinear for all real values of $\dot{\lambda}$

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142. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors & $\left|\vec{c}\right| = \sqrt{3}$, then ordered pair (α, β) is $(1, 1)$ (b) $(1, -1)$ $(-1, 1)$ (d) $(-1, -1)$

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143. The number of distinct real values of λ , for which the vectors $\lambda^2\hat{i} + \hat{j} + k, \hat{i} - \lambda^2\hat{j} + \hat{k} and \hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar is a. zero b. one c. two d. three

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144. If $\vec{AO} + \vec{OB} = \vec{BO} + \vec{OC}$, then $A, BnadC$ are (where O is the origin) a. coplanar b. collinear c. non-collinear d. none of these

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145. Find a vector magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}^{\cdot}$

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146. Show that the points $A(1, -2, -8), B(5, 0, -2) and C(1, 3, 7)$ are collinear, and find the ratio in which B divides $AC^{\cdot}$

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147. The position vectors of $PandQ$ are $5\hat{i} + 4\hat{j} + a\hat{k}$ and $-\hat{i} + 2\hat{j} - 2\hat{k}$, respectively. If the distance between them is 7, then find the value of $a^{\cdot}$

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148. Given three points are $A(-3, -2, 0), B(3, -3, 1) and C(5, 0, 2)$. Then find a vector having the same direction as that of $\vec{AB}$ and magnitude equal to $\left|\vec{AC}\right|$.

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149. Let $ABCD$ be a p[arallelogram whose diagonals intersect at P and let O be the origin. Then prove that $\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OP}$.

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150. If $ABCD$ is quadrilateral and $E and F$ are the mid-points of $AC and BD$ respectively, prove that $\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 4\ \vec{EF}$.

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151. If $ABCD$ is a rhombus whose diagonals cut at the origin O, then proved that $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D + \vec{O}$.

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152. Let $D, EandF$ be the middle points of the sides $BC, CAandAB$, respectively of a triangle ABC. Then prove that $\vec{A}D + \vec{B}E + \vec{C}F = \vec{0}$.

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153. Consider the set of eight vector $V = \left\{a\hat{i} + b\hat{j} + c\hat{k}; a, bc \in \{-1, 1\}\right\}$. Three non-coplanar vectors can be chosen from V is 2^p ways. Then p is______.

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154. Find the direction cosines of the vector joining the points $A(1, 2, -3)a \cap B(-1-2, 1)$ directed from $A \rightarrow B^{\cdot}$

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155. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}^{\cdot}$

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156. The median AD of the triangle ABC is bisected at E and BE meets AC at F. Find AF:FC.

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157. Vectors $\vec{a}$ and $\vec{b}$ are non-collinear. Find for what value of n vectors $\vec{c} = (n-2)\vec{a} + \vec{b}$ and $\vec{d} = (2n+1)\vec{a} - \vec{b}$ are collinear?

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158. Prove that the necessary and sufficient condition for any four points in three-dimensional space to be coplanar is that there exists a liner relation connecting their position vectors such that the algebraic sum of the coefficients (not all zero) in it is zero.

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159. Points $A(\vec{a}), B(\vec{b}), C(\vec{c}) and D(\vec{d})$ are relates as $x\vec{a} + y\vec{b} + z\vec{c} + w\vec{d} = 0$ and $x + y + z + w = 0, where x, y, z, and w$ are scalars (sum of any two of $x, y, znadw$ is not zero). Prove that if $A, B, CandD$ are concylic, then $|xy|\left|\vec{a} - \vec{b}\right|^2 = |wz|\left|\vec{c} - \vec{d}\right|^{2^{\cdot}}$

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160. If $\vec{a}, \vec{b} and \vec{c}$ are non-coplanar vectors, prove that the four points $2\vec{a} + 3\vec{b} - \vec{c}, \vec{a} - 2\vec{b} + 3\vec{c}, 3\vec{a} + 4\vec{b} - 2\vec{c} and \vec{a} - 6\vec{b} + 6\vec{c}$ are coplanar.

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161. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

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162. Let $\vec{a}, \vec{b} and \vec{c}$ be unit vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{x}, \vec{a}\dot{} \vec{x} = 1, \vec{b}\dot{} \vec{x} = \frac{3}{2}, |\vec{x}| = 2.$ Then find the angel between and $\dot{\times}$

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163. Let $\vec{A} and \vec{B}$ be two non-parallel unit vectors in a plane. If $\left(\alpha\vec{A} + \vec{B}\right)$ bisects the internal angle between $\vec{A} and \vec{B},$ then find the value of α.

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164. If the vectors $3\vec{p} + \vec{q}; 5p - 3\vec{q} and 2\vec{p} + \vec{q}; 4\vec{p} - 2\vec{q}$ are pairs of mutually perpendicular vectors, then find the angle between vectors $\vec{p} and \vec{q}$.

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165. $P(1, 0, -1), Q(2, 0, -3), R(-1, 2, 0) and S(, -2, -1),$ then find the projection length of $\vec{P}Qon\vec{R}S$.

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166. A, B, C, D are any four points, prove that $\vec{A}B\vec{C}D + \vec{B}C\vec{A}D + \vec{C}A\vec{B}D = 0.$

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167. Let $\hat{u} = \hat{i} + \hat{j}, \hat{v} = \hat{i} - \hat{j} and \hat{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\hat{u}.\hat{n} = 0 and \hat{v}.\hat{n} = 0,$ then find the value of $\left|\hat{w}.\hat{n}\right|.$

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168. If the angel between unit vectors $\vec{a} and \vec{b} 60^0$, then find the value of $\left|\vec{a}-\vec{b}\right|^{\cdot}$

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169. $\vec{a}+\vec{b}+\vec{c}=\vec{0}, \left|\vec{a}\right|=3, \left|\vec{b}\right|=5, \left|\vec{c}\right|=9$,find the angle between $\vec{a}$ and $\vec{c}$.

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170. Constant forces $P_1=\hat{i}+\hat{j}+\hat{k}, P_2=-\hat{i}+2\hat{j}-\hat{k} and P_3=-\hat{j}-\hat{k}$ act on a particle at a point $A^{\cdot}$ Determine the work done when particle is displaced from position $A\left(4\hat{i}-3\hat{j}-2\hat{k}\right) \rightarrow B\left(6\hat{i}+\hat{j}-3\hat{k}\right)^{\cdot}$

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171. If $\vec{a}, and\vec{b}$ are unit vectors , then find the greatest value of $\left|\vec{a} + \vec{b}\right| + \left|\vec{a} - \vec{b}\right|^{\cdot}$

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172. Let G_1, G_2andG_3 be the centroids of the triangular faces $OBC, OCAandOAB,$ respectively, of a tetrahedron $OABC^{\cdot}$ If V_1 denotes the volumes of the tetrahedron $OABCandV_2$ that of the parallelepiped with OG_1, OG_2andOG_3 as three concurrent edges, then prove that $4V_1 = 9V_1^{\cdot}$

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173. Prove that $\hat{i} \times \left(\vec{a} \times \hat{i}\right)\hat{j} \times \left(\vec{a} \times \hat{j}\right) + \hat{k} \times \left(\vec{a} \times \hat{k}\right) = 2\vec{a}^{\cdot}$

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174. If $\hat{i} \times \left[\left(\vec{a} - \hat{j}\right) \times \hat{i}\right] + \hat{j} \times \left[\left(\vec{a} - \hat{k}\right) \times \hat{j}\right] + \hat{k} \times \left[\left(\vec{a} - \hat{i}\right) \times \hat{k}\right] = 0,$ then find vector $\vec{a}$.

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175. Let $\vec{a}, \vec{b}, and \vec{c}$ be any three vectors, then prove that $\left[\vec{a} \times \vec{b}\vec{b} \times \vec{c}\vec{c} \times \vec{a}\right] = \left[\vec{a}\vec{b}\vec{c}\right]^2.$

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176. If $\left[\vec{a}\vec{b}\vec{c}\right] = 2,$ then find the value of $\left[\left(\vec{a} + 2\vec{b} - \vec{c}\right)\left(\vec{a} - \vec{b}\right)\left(\vec{a} - \vec{b} - \vec{c}\right)\right].$

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177. If $\vec{a}, \vec{b}, and \vec{c}$ are mutually perpendicular vectors and $\vec{a} = \alpha\left(\vec{a} \times \vec{b}\right) + \beta\left(\vec{b} \times \vec{c}\right) + \gamma\left(\vec{c} \times \vec{a}\right) and \left[\vec{a}\vec{b}\vec{c}\right] = 1,$ then find the value of $\alpha + \beta + \dot{\gamma}$

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178. If $a, b and c$ are non-cop0lanar vector, then that prove $\left|\left(\dot{\vec{a}\vec{d}}\right)\left(\vec{b} \times \vec{c}\right) + \left(\dot{\vec{b}\vec{d}}\right)\left(\vec{c} \times \vec{a}\right) + \left(\dot{\vec{c}\vec{d}}\right)\left(\vec{a} \times \vec{b}\right)\right|$ is independent of $d, where e$ is a unit vector.

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179. Prove that vectors $\vec{u} = \left(al + a_1l_1\right)\hat{i} + \left(am + a_1m_1\right)\hat{j} + \left(an + a_1n_1\right)\hat{k}$

$\vec{v} = \left(bl + b_1l_1\right)\hat{i} + \left(bm + b_1m_1\right)\hat{j} + \left(bn + b_1n_1\right)\hat{k}$

$\vec{w} = \left(bl + b_1l_1\right)\hat{i} + \left(bm + b_1m_1\right)\hat{j} + \left(bn + b_1n_1\right)\hat{k}$ are coplanar.

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180. For any four vectors, prove that $\left(\vec{b}\times\vec{c}\right)\dot{\vec{a}}\times\vec{d}+\left(\vec{c}\times\vec{a}\right)\dot{\vec{b}}\times\vec{d}+\left(\vec{a}\times\vec{b}\right)\dot{\vec{c}}\times\vec{d}=0.$

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181. If $\vec{b} and \vec{c}$ are two-noncollinear vectors such that $\vec{a} \mid \mid \left(\vec{b}\times\vec{c}\right)$, then prove that $\left(\vec{a}\times\vec{b}\right).\left(\vec{a}\times\vec{c}\right)$ is equal to $|\vec{a}|^2\left(\vec{b}\dot{}\vec{c}\right)^{.}$

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182. If the vectors A, B, C of a triangle ABC are $(1, 2, 3), (-1, 0, 0), (0, 1, 2)$, respectively then find $\angle ABC^{.}$

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183. Let $\vec{a}, \vec{b} and \vec{c}$ be pairwise mutually perpendicular vectors, such that $|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 2.$ Then find the length of $\vec{a} + \vec{b} + \vec{c}$

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184. Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is a perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a},$ for any two non-zero vectors $\vec{a} and \vec{b}^{\cdot}$

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185. If $|\vec{a}| = 3, |\vec{b}| = 4 and$ the angle between $a and b$ is $120°$, then find the value of $|4\vec{a} + 3\vec{b}|^{\cdot}$

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186. If $\vec{a}, \vec{b}, and \vec{c}$ be three non-coplanar vector and $a^{'}, b^{'} and c'$ constitute the reciprocal system of vectors, then prove that

$\vec{r} = \left(\vec{r}\dot{\vec{a}}'\right)\vec{a} + \left(\vec{r}\dot{\vec{b}}'\right)\vec{b} + \left(\vec{r}\dot{\vec{c}}'\right)\vec{c}\ \vec{r} = \left(\vec{r}\dot{\vec{a}}'\right)\vec{a}' + \left(\vec{r}\dot{\vec{b}}'\right)\vec{b}' + \left(\vec{r}\dot{\vec{c}}'\right)\vec{c}'$

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187. Find $\left|\vec{a}\right| and \left|\vec{b}\right|$, if $\left(\vec{a} + \vec{b}\right)\dot{\vec{a}} - \vec{b} = 8, \left|\vec{a}\right| = 8\left|\vec{b}\right|^{\cdot}$

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188. Let $\vec{a}, \vec{b}, and \vec{c} and \vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors, then prove that $\vec{a}' \times \vec{b}' + \vec{b}' \times \vec{c}' + \vec{c}' \times \vec{a}' = \dfrac{\vec{a} + \vec{b} + \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}$.

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189. If $\vec{a}, \vec{b}, and \vec{c}$ are three non-coplanar non-zero vecrtors, then prove that $\left(\vec{a}.\vec{a}\right)\vec{b} \times \vec{c} + \left(\vec{a}.\vec{b}\right)\vec{c} \times \vec{a} + \left(\vec{a}.\vec{c}\right)\vec{a} \times \vec{b} = \left[\vec{b}\vec{c}\vec{a}\right]\vec{a}$

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190. Find a set of vectors reciprocal to the set $-\hat{i}+\hat{j}+\hat{k}, \hat{i}-\hat{j}+\hat{k}, \hat{i}+\hat{j}+\hat{k}$.

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191. If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}\neq 0, where \vec{a}, \vec{b}, and \vec{c}$ are coplanar vectors, then for some scalar k prove that $\vec{a}+\vec{c}=k\vec{b}$.

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192. If $\vec{a}=2\vec{i}+3\vec{j}-\vec{k}, \vec{b}=-\vec{i}+2\vec{j}-4\vec{k} and \vec{c}=\vec{i}+\vec{j}+\vec{k}$, then find thevalue of $\left(\vec{a}\times\vec{b}\right)\cdot\vec{a}\times\vec{c}$.

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193. If the vectors $\vec{c}, \vec{a} = x\hat{i} + y\hat{j} + z\hat{k} and \vec{b} = \hat{j}$ are such that $\vec{a}, \vec{c} and \vec{b}$ form a right-handed system, then find $\vec{\cdot}$

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194. Given that $\vec{a}\dot{\vec{b}} = \vec{a}\dot{\vec{c}}, \vec{a} \times \vec{b} = \vec{a} \times \vec{c} and \vec{a}$ is not a zero vector. Show that $\vec{b} = \vec{\cdot}$

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195. If $\left|\vec{a}\right| = 5, \left|\vec{a} - \vec{b}\right| = 8 and \left|\vec{a} + \vec{b}\right| = 10$, then find $\left|\vec{b}\right|^{\cdot}$

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196. If A, B, C, D are four distinct point in space such that AB is not perpendicular to CD and satisfies

$\vec{A}B\vec{C}D = k\left(\left|\vec{A}D\right|^2 + \left|\vec{B}C\right|^2 - \left|\vec{A}C\right|^2 - \left|\vec{B}D\right|^2\right)$, then find the value of k

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197. If $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}, \vec{b} = m\hat{i} + n\hat{j} + 12\hat{k} and \vec{a} \times \vec{b} = \vec{0}$, then find (m, n)

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198. If $\left|\vec{a}\right| = 2\left|\vec{b}\right| = 5$ and $\left|\vec{a} \times \vec{b}\right| = 8$, then find the value of $\vec{a}.\vec{b}$

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199. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\vec{a} \times \vec{b}$ and given a geometrical interpretation of it.

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200. If $\vec{x} and \vec{y}$ are unit vectors and $|\vec{z}| = \frac{2}{7}$ such that $\vec{z} + \vec{z} \times \vec{x} = \vec{y}$, then find the angle θ between $\vec{x} and \vec{z}^{\cdot}$

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201. Prove that $\left[\vec{a} + \vec{b}\vec{b} + \vec{c}\vec{c} + \vec{a}\right] = 2\left[\vec{a}\vec{b}\vec{c}\right]^{\cdot}$

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202. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors and $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$. If the angel between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$, then find the value of $\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$.

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203. The position vectors of the four angular points of a tetrahedron are $A\left(\hat{j} + 2\hat{k}\right), B\left(3\hat{i} + \hat{k}\right), C\left(4\hat{i} + 3\hat{j} + 6\hat{k}\right) and D\left(2\hat{i} + 3\hat{j} + 2\hat{k}\right)^{\cdot}$ Find the volume of the tetrahedron $ABCD$

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204. If the vectors $2\hat{i} - 3\hat{j}, \hat{i} + \hat{j} - \hat{k} and 3\hat{i} - \hat{k}$ form three concurrent edges of a parallelepiped, then find the volume of the parallelepiped.

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205. If $\vec{u}, \vec{v} and \vec{w}$ are three non-cop0lanar vectors, then prove that $\left(\vec{u} + \vec{v} - \vec{w}\right)\vec{u} - \vec{v} \times \left(\vec{v} - \vec{w}\right) = \vec{u}\vec{v} \times \vec{w}$

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206. Find the value of a so that the volume of the parallelepiped formed by vectors $\hat{i} + a\hat{j} + k, \hat{j} + a\hat{k} and a\hat{i} + \hat{k}$ becomes minimum.

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207. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k},$ then find the vaue of

$$\left| \vec{a}\vec{a}\vec{a}\vec{b}\vec{a}\vec{c}\vec{b}\vec{a}\vec{b}\vec{a}\vec{b}\vec{a} \cdot \vec{a} \cdot \vec{a} \cdot \vec{a} \right|.$$

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208. Prove that $\left[\vec{l}\,\vec{m}\,\vec{n}\right]\left[\vec{a}\,\vec{b}\,\vec{c}\right] = \left| \vec{l}\,\vec{a}\,\vec{l}\,\vec{b}\,\vec{l}\,\vec{c}\vec{m}\vec{a}\vec{m}\vec{a}\vec{m}\vec{a}\vec{n}\vec{a}\vec{n}\vec{a}\vec{n}\vec{a} \right|.$

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209. Find the altitude of a parallelepiped whose three coterminous edtges are vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}, \vec{B} = 2\hat{i} + 4\hat{j} - \hat{k} and \vec{C} = \hat{i} + \hat{j} + 3\hat{k} with \vec{A} and \vec{B}$ as the sides of the base of the parallepiped.

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210. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}\times\vec{b}\right| = 2,$ then find the value of $\left[\vec{a}\vec{b}\vec{a}\times\vec{b}\right]$.

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211. Prove that

$$\vec{R} + \frac{\left[\vec{R}\vec{\beta}\times\left(\vec{\beta}\times\vec{\alpha}\right)\right]\vec{\alpha}}{\left|\vec{\alpha}\times\vec{\beta}\right|^2} + \frac{\left[\vec{R}\vec{\alpha}\times\left(\vec{\alpha}\times\vec{\beta}\right)\right]\vec{\beta}}{\left|\vec{\alpha}\times\vec{\beta}\right|^2} = \frac{\left[\vec{R}\vec{\alpha}\vec{\beta}\right]\left(\vec{\alpha}\times\vec{\beta}\right)}{\left|\vec{\alpha}\times\vec{\beta}\right|^2}$$

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212. If $\vec{a}, \vec{b}, and \vec{c}$ are non-coplanar unit vectors such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \frac{\vec{b}+\vec{c}}{\sqrt{2}}, \vec{b} and \vec{c}$ are non-parallel, then prove that the angel between $\vec{a} and \vec{b} is 3\pi/4.$

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213. If

$\vec{r}.\vec{a}=0, \vec{r}.\vec{b}=1$ and $\begin{bmatrix}\vec{r} & \vec{a} & \vec{b}\end{bmatrix}=1, \vec{a}\dot{\vec{b}}\neq 0, \left(\vec{a}\dot{\vec{b}}\right)^2 - |\vec{a}|^2|\vec{b}|^2=1,$

then find $\vec{r}$ in terms of $\vec{a}$ and $\vec{b}$.

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214. If $\vec{a} and \vec{b}$ are two given vectors and k is any scalar, then find the vector $\vec{r}$ satisfying $\vec{r}\times\vec{a}+k\vec{r}=\dot{\vec{b}}$

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215. $\vec{a}, \vec{b} and \vec{c}$ are three non-coplanar vectors and $\vec{r}$ is any arbitrary vector. Prove that $\left[\vec{b}\vec{c}\vec{r}\right]\vec{a}+\left[\vec{c}\vec{a}\vec{r}\right]\vec{b}+\left[\vec{a}\vec{b}\vec{r}\right]\vec{c}=\left[\vec{a}\vec{b}\vec{c}\right]\dot{\vec{r}}$

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216. If vector $\vec{x}$ satisfying $\overrightarrow{\times} x\vec{a} + \left(\vec{x}\dot{\vec{b}}\right)\vec{c} = \vec{d}$ is given

$\vec{x} = \lambda\vec{a} + \vec{a} \times \dfrac{\vec{a} \times \left(\overrightarrow{d} x x\vec{c}\right)}{\left(\vec{a}\dot{\vec{c}}\right)|\vec{a}|^2}$, then find the value of $\dot{\lambda}$

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217. Find the vector of length 3 unit which is perpendicular to $\hat{i} + \hat{j} + \hat{k}$ and lies in the plane of $\hat{i} + \hat{j} + \hat{k} and 2\hat{k}$ - $3\hat{j}$.

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218. If $\vec{b}$ is not perpendicular to $\vec{c}$, then find the vector $\vec{r}$ satisfying the equyation $\vec{r} \times \vec{b} = \vec{a} \times \vec{b} and \vec{r}\dot{\vec{c}} = 0$.

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219. If $\vec{a}, \vec{b} and \vec{c}$ are three non coplanar vectors, then prove that

$$\vec{d} = \frac{\vec{a}\dot{}\vec{d}}{\left[\vec{a}\vec{b}\vec{c}\right]}\left(\vec{b} \times \vec{c}\right) + \frac{\vec{b}\dot{}\vec{d}}{\left[\vec{a}\vec{b}\vec{c}\right]}\left(\vec{c} \times \vec{a}\right) + \frac{\vec{\cdot}\,\vec{d}}{\left[\vec{a}\vec{b}\vec{c}\right]}\left(\vec{a} \times \vec{b}\right)$$

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220. Let $\vec{a}, \vec{b} and \vec{c}$ be three non-zero vectors such that $\vec{a} + \vec{b} + \vec{c} = 0 and \lambda \vec{b} \times \vec{a} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$, then find the value of$\lambda$

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221. Prove that $\left(\vec{a}\dot{\hat{i}}\right)\left(\vec{a} \times \hat{i}\right) + \left(\vec{a}\dot{j}\right)\left(\vec{a} \times \hat{j}\right) + \left(\vec{a}\dot{\hat{k}}\right)\left(\vec{a} \times \hat{k}\right) = 0.$

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222. If $\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}\cdot\vec{b}\right)^2=144 and \left|\vec{a}\right|=4,$ then find the value of $\left|\vec{b}\right|$.

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223. A particle has an angular speed of 3 rad/s and the axis of rotation passes through the points $(1, 1, 2) and (1, 2, -2)$. Find the velocity of the particle at point $P(3, 6, 4)$.

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224. Find the moment of $\vec{F}$ about point (2, -1, 3), where force $\vec{F}=3\hat{i}+2\hat{j}-4\hat{k}$ is acting on point (1, -1, 2).

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225. Given $|\vec{a}| = |\vec{b}| = 1 and |\vec{a} + \vec{b}| = \sqrt{3}$. If $\vec{c}$ is a vector such that $\vec{c} - \vec{a} - 2\vec{b} = 3(\vec{a} \times \vec{b})$, then find the value of $\vec{c} \cdot \vec{b}$

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226. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} and \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a} and \vec{b}$. If the angle between $a and b$ is $\frac{\pi}{6}$, then prove that $|a_1 a_2 a_3 b_1 b_2 b_3 c_1 c_2 c_3| = \frac{1}{4}(a12 + a22 + a32)(b12 + b22 + b32)$

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227. If $\vec{a}, \vec{b}, \vec{c}, and \vec{d}$ are four non-coplanar unit vector such that $\vec{d}$ make equal angles with all the three vectors $\vec{a}, \vec{b} and \vec{c}$, then prove that $[\vec{d}\vec{a}\vec{b}] = [\vec{d}\vec{c}\vec{b}] = [\vec{d}\vec{c}\vec{a}]$.

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228. If the volume of a parallelepiped whose adjacent edges are $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} + \alpha\hat{k}$ is 15, then find the value of α if $(\alpha > 0)$

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229. Prove that if $\left[\vec{l}\vec{m}\vec{n}\right]$ are three non-coplanar vectors, then $\left[\vec{l}\vec{m}\vec{n}\right]\left(\vec{a} \times \vec{b}\right) = \left|\vec{l}\vec{a}\vec{l}\vec{b}\vec{l}\vec{m}\vec{a}\vec{m}\vec{b}\vec{m}\vec{n}\vec{a}\vec{n}\vec{b}\vec{n}\right|$.

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230. Using dot product of vectors, prove that a parallelogram, whose diagonals are equal, is a rectangle

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231. If $a + 2b + 3c = 4,$ then find the least value of $a^2 + b^2 + c^2$.

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232. In any triangle $ABC,$ prove the projection formula$a = b\cos C + osB$ using vector method.

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233. Prove that an angle inscribed in a semi-circle is a right angle using vector method.

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234. If $\vec{a}.\hat{i} = \vec{a}.\left(\hat{i} + \hat{j}\right) = \vec{a}.\left(\hat{i} + \hat{j} + \hat{k}\right)$, then find the unit vector $\vec{a}$

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235. Prove by vector method that $\cos(A+B)\cos A\cos B - \sin A\sin B$.

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236. If the scalar projection of vector $x\hat{i} - \hat{j} + \hat{k}$ on vector $2\hat{i} - \hat{j} + 5\hat{k} is \frac{1}{\sqrt{30}}$,

then find the value of x.

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237. If $\vec{a} = x\hat{i} + (x-1)\hat{j} + \hat{k} and \vec{b} = (x+1)\hat{i} + \hat{j} + a\hat{k}$ make an acute angle $\forall x \in R$, then find the values of a.

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238. A unit vector a makes an angle $\frac{\pi}{4}$ with z-axis. If $a + i + j$ is a unit vector, then a can be equal to

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239. if $\vec{a}, \vec{b}$ and $\vec{c}$ are there mutually perpendicular unit vectors and $\vec{a}$ ia a unit vector make equal angles which $\vec{a}, \vec{b}$ and $\vec{c}$ then find the value of $\left|\vec{a} + \vec{b} + \vec{c} + \vec{d}\right|^2$

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240. If $\vec{a}, \vec{b}, and \vec{c}$ be non-zero vectors such that no tow are collinear or $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}^{\cdot}$ If θ is the acute angle between vectors $\vec{b} and \vec{c}$, then find the value of $s \int h\eta^{\cdot}$

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241. If $\vec{p}, \vec{q}, \vec{r}$ denote vector $\vec{b} \times \vec{c}, \vec{c} \times \vec{a}, \vec{a} \times \vec{b}$, respectively, show that $\vec{a}$ is parallel to $\vec{q} \times \vec{r}, \vec{b}$ is parallel $\vec{r} \times \vec{p}, \vec{c}$ is parallel to $\vec{p} \times \vec{q}^{\cdot}$

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242. If $\vec{a}, and \vec{b}$ be two non-collinear unit vector such that $\vec{a} \times \left(\vec{a} \times \vec{b}\right) = \frac{1}{2}\vec{b}$, then find the angle between $\vec{a}, and \vec{b}$.

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243. Show that $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ if and only if $\vec{a}$ and $\vec{c}$ are collinear of $\left(\vec{a} \times \vec{c}\right) \times \vec{b} = 0$.

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244. Prove that $\left(\vec{a}\left(\vec{b} \dot{\times} \hat{i}\right)\hat{i} + \left(\vec{a}\vec{b} \dot{\times} \hat{j}\right)\hat{j} + \left(\vec{a}\vec{b} \dot{\times} \hat{k}\right)\hat{k} = \vec{a} \times \vec{b}\right.$

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245. For any four vectors, $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ prove that

$$\vec{a} \times \left(\vec{b} \times \left(\vec{c} \times \vec{d}\right)\right) = \left(\vec{b}\vec{d}\right)\left[\vec{a}\vec{c}\vec{d}\right].$$

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246. If $\vec{a}, \vec{b}, and \vec{c}$ are three vectors such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}, \vec{c} \times \vec{a} = \vec{b}$, then prove that $|\vec{a}| = |\vec{b}| = |\vec{c}|$.

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247. If $\vec{a} = \vec{p} + \vec{q}, \vec{p} \times \vec{b} = 0 and \vec{q} \cdot \vec{b} = 0$, then prove that $\dfrac{\vec{b} \times \left(\vec{a} \times \vec{b}\right)}{\vec{b} \cdot \vec{b}} = \vec{q}$.

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248. If $\vec{a} = \hat{i} + \hat{j} + \hat{k} and \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$, then find vector $\vec{c}$ such that

$\dot{\vec{a}\vec{c}} = 2 and \vec{a} \times \vec{c} = \dot{\vec{b}}$

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249. If $\vec{x}.\vec{a} = 0, \vec{x}.\vec{b} = 0$ and $\vec{x}.\vec{c} = 0$ for some non-zero vector $\vec{x}$, then prove that $\left[\vec{a}\vec{b}\vec{c}\right] = 0.$

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250. If $\vec{a}, \vec{b}, and \vec{c}$ are mutually perpendicular vectors of equal magnitudes, then find the angle between vectors $\vec{a} and \vec{a} + \vec{b} + \dot{\vec{}}$

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251. If $\vec{a}, \vec{b}, and \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0,$ then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

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252. If three unit vectors $\vec{a}, \vec{b}, and \vec{c}$ satisfy $\vec{a} + \vec{b} + \vec{c} = 0,$ then find the angle between $\vec{a} and \vec{b}^{\cdot}$

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253. If $\left|\vec{a}\right| + \left|\vec{b}\right| = \left|\vec{c}\right| and \vec{a} + \vec{b} = \vec{c},$ then find the angle between $\vec{a} and \vec{b}^{\cdot}$

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254. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k} and 3\hat{i} - 2\hat{j} + \hat{k}^{\cdot}$

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255. If $\vec{r}\dot{\hat{i}} = \vec{r}\dot{\hat{j}} = \vec{r}\dot{\hat{k}} and \left|\vec{r}\right| = 3,$ then find the vector $\dot{\vec{r}}$

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256. If $\vec{a}, \vec{b}, and \vec{c}$ are non-zero vectors such that $\vec{a}\dot{\vec{b}} = \vec{a}\dot{\vec{c}},$ then find the geometrical relation between the vectors.

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257. Find the projection of vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\dot{\hat{k}}$

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258. If θ is th angel between the unit vectors a and b , then prove that

$$\cos\left(\frac{\theta}{2}\right) = \frac{1}{2}\left|\vec{a} + \vec{b}\right|.,, \sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$$

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259. $\vec{a}, \vec{b}, and \vec{c}$ are three unit vectors and every two are inclined to each other at an angel $\cos^{-1}(3/5)^{\cdot}$ If $\vec{a} \times \vec{b} = p\vec{a} + q\vec{b} + r\vec{c}, where p, q, r$ are scalars, then find the value of $q^{\cdot}$

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260. Given unit vectors $\hat{m}, \hat{n} and \hat{p}$ such that angel between $\hat{m} and \hat{n}$ is α and angle between $\hat{p} and \left(\hat{m} \times \hat{n}\right)$ is also α, if $\left[\hat{n}\hat{p}\hat{m}\right] = 1/4$, then find the value of $\alpha^{\cdot}$

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261. Let $\vec{a}, \vec{b}, and \vec{c}$ be non-coplanar vectors and let the equation $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vector $\vec{a}, \vec{b}, \vec{c},$ then prove that $\vec{a} \times \vec{a}' + \vec{b} \times \vec{b}' + \vec{c} \times \vec{c}'$ is a null vector.

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262. Vector $\vec{O}A = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle passing through the positive x-axis on the way. Show that the vector in its new position is $\frac{4\hat{i} - \hat{j} - \hat{k}}{\sqrt{2}}$.

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263. The base of the pyramid $AOBC$ is an equilateral triangle OBC with each side equal to $4\sqrt{2}$, O is the origin of reference, AO is perpendicualar to the plane of OBC and $\left|\vec{A}O\right| = 2$. Then find the cosine of the angle between the skew straight lines, one passing though A and the midpoint of $OBand$ the other passing through O and the mid point of BC.

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264. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}and\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

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265. Let the vectors $\vec{a}and\vec{b}$ be such that $|\vec{a}| = 3|\vec{b}| = \frac{\sqrt{2}}{3}, then\vec{a} \times \vec{b}$ is a unit vector, if the angel between $\vec{a}and\vec{b}$ is?

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266. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)^{\cdot}$

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267. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}and\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}^{\cdot}$ Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}and\vec{b}and\vec{\cdot}\vec{d} = 15.$

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268. If $A, B and C$ are the vetices of a triangle ABC, then prove sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

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269. Application of cross product trigonometric proof; sin(A+B) = sinAcosB + cosAsinB

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270. Find a unit vector perpendicular to the plane determined by the points $(1, -1, 2), (2, 0, -1) and (0, 2, 1)$.

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271. If $\vec{a} and \vec{b}$ are two vectors, then prove that $\left(\vec{a} \times \vec{b}\right)^2 = \left|\vec{a}.\vec{a}\,\vec{a}.\vec{b}\,\vec{b}.\vec{a}\,\vec{b}.\vec{b}\right|$.

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272. In isosceles triangles ABC, $\left|\vec{A}B\right| = \left|\vec{B}C\right| = 8$, a point E divides AB internally in the ratio 1:3, then find the angle between $\vec{C}Eand\vec{C}A\left(where\left|\vec{C}A\right| = 12\right)^{\cdot}$

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273. Prove that in a tetrahedron if two pairs of opposite edges are perpendicular , then the third pair is also perpendicular.

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274. Let $\vec{a}, \vec{b}, and\vec{c}$ are vectors such that $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4and\left|\vec{c}\right| = 5, and\left(\vec{a} + \vec{b}\right)$ is perpendicular to $\vec{c}, \left(\vec{b} + \vec{c}\right)$ is perpendicular to $\vec{a}and\left(\vec{c} + \vec{a}\right)$ is perpendicular to $\vec{b}^{\cdot}$ Then find the value of $\left|\vec{a} + \vec{b} + \vec{c}\right|$.

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275. If $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{a} + \vec{b}\right| = 1,$ then find the value of $\left|\vec{a} - \vec{b}\right|^{.}$

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276. If $\vec{a} = 4\hat{i} + 6\hat{j} and \vec{b} = 3\hat{j} + 4\hat{k},$ then find the component of $\vec{a} and \vec{b}^{.}$

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277. A particle acted by constant forces $4\hat{i} + \hat{j} - 3\hat{k} and 3\hat{i} + \hat{9} - \hat{k}$ is displaced from point $\hat{i} + 2\hat{j} + 3\hat{k}$ to point $5\hat{i} + 4\hat{j} + \hat{k}^{.}$ find the total work done by the forces in units.

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278. If $\vec{a}$, $\vec{b}$,and $\vec{c}$ are there mutually perpendicular unit vectors and $\vec{d}$ is a unit vector which makes equal angles with $\vec{a}$, $\vec{b}$,and $\vec{c}$, then find the value of $\left|\vec{a} + \vec{b} + \vec{c} + \vec{d}\right|^2$

A. $4 + 2\sqrt{2}$

B. $4 + 2\sqrt{3}$

C. $2 + \sqrt{5}$

D. $3 + \sqrt{5}$

Answer: B

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279. Let $\vec{a} = x\hat{i} + 12\hat{j} - \hat{k}, \vec{b} = 2\hat{i} + 2x\hat{j} + \hat{k} and \vec{c} = \hat{i} + \hat{k}$. If the ordered set $\left[\vec{b}\vec{c}\vec{a}\right]$ is left handed, then find the values of x.

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280. If $\vec{a}, \vec{b}, and \vec{c}$ are three non-coplanar vectors, then find the value of

$$\frac{\vec{a}\cdot\vec{b}\times\vec{c}}{\vec{b}\cdot\vec{c}\times\vec{a}}+\frac{\vec{b}\cdot\vec{c}\times\vec{a}}{\vec{c}\cdot\left(\vec{a}\times\vec{b}\right)}+\frac{\vec{c}\cdot\left(\vec{b}\times\vec{a}\right)}{\vec{a}\cdot\vec{b}\times\vec{c}}.$$

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281. If $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are the position vectors of the vertices of a cyclic quadrilateral $ABCD,$ prove that

$$\frac{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{d}+\vec{d}x x\vec{a}\right|}{\left(\vec{b}-\vec{a}\right)\cdot\vec{d}-\vec{a}}+\frac{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{d}+\vec{d}x x\vec{b}\right|}{\left(\vec{b}-\vec{c}\right)\cdot\vec{d}-\vec{c}}=.$$

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282. The position vectors of the vertices of a quadrilateral with A as origin are $B\left(\vec{b}\right), D\left(\vec{d}\right) and C\left(l\vec{b}+m\vec{d}\right)^{\cdot}$ Prove that the area of the quadrialateral is $\frac{1}{2}(l+m)\left|\vec{b}\times\vec{d}\right|^{\cdot}$

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283. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d} and \vec{a} \times \vec{c} = \vec{b} \times \vec{d},$ then show that $\vec{a} - \vec{d},$ is paralelto $\vec{b} - \vec{c}$ provided $\vec{a} \neq \leftrightarrow d$ and $\vec{b} \neq \vec{\cdot}$

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284. Show by a numerical example and geometrically also that $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ does not imply $\vec{b} = \vec{c}$

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285. In triangle $ABC, po \in tsD, EandF$ are taken on the sides $BC, CAandAB,$ respectigvely, such that $\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = n^{\cdot}$ Prove that

$$\_ (DEF) = \frac{n^2 - n + 1}{\left((n+1)^2\right)_{ABC}}^{\cdot}$$

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286. Let A, B, C be points with position vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} + 2\hat{j} + \hat{k} and 2\hat{i} + \hat{j} + 2\hat{k}$ respectively. Find the shortest distance between point B and plane OAC.

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287. Let $\vec{a} and \vec{b}$ be unit vectors such that $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$. Then find the value of $\left(2\vec{a} + 5\vec{b}\right) \cdot 3\vec{a} + \vec{b} + \vec{a} \times \vec{b}$.

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288. $u and v$ are two non-collinear unit vectors such that $\left|\frac{\hat{u} + \hat{v}}{2} + \hat{u} \times \hat{v}\right| = 1.$ Prove that $\left|\hat{u} \times \hat{v}\right| = \left|\frac{\hat{u} - \hat{v}}{2}\right|$.

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289. A rigid body is spinning about a fixed point (3,-2,-1) with an angular velocity of 4 rad/s, the axis of rotation being in the direction of (1,2,-2). Find the velocity of the particle at point (4,1,1).

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290. $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}; \vec{r} \times \vec{b} = \vec{a} \times \vec{b}; \vec{a} \neq \vec{0}; \vec{b} \neq \vec{0}; \vec{a} \neq \lambda \vec{b}, and \vec{a}$ is not perpendicular to $\vec{b}$, then find $\vec{r}$ in terms of $\vec{a} and \vec{b}^{\cdot}$

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291. If $\left|\vec{a}\right| = 2$, then find the value of $\left|\vec{a} \times \hat{i}\right|^2 + \left|\vec{a} \times \hat{j}\right|^2 + \left|\vec{a} \times \hat{k}\right|^{2^{\cdot}}$

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292. If $\vec{a}, \vec{b} and \vec{c}$ are the position vectors of the vertices $A, B and C$ respect ively, of ABC, prove that the perpendicular distance of the vertedx A from the base BC of the triangle ABC is $\dfrac{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|.}{\left|\vec{c}-\vec{b}\right|}$

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293. $A, B, CandD$ are any four points in the space, then prove that $\left|\vec{A}B\times\vec{C}D+\vec{B}C\times\vec{A}D+\vec{C}A\times\vec{B}D\right| = 4$ (area of ABC .)

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294. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}and\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

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295. Using vectors, find the area of the triangle with vertices A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5).

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296. Let $\vec{a}, \vec{b} and \vec{c}$ be three verctors such that $\vec{a} \neq 0, \left|\vec{a}\right| = \left|\vec{c}\right| = 1, \left|\vec{b}\right| = 4$ and $\left|\vec{b} \times \vec{c}\right| = \sqrt{15}^{\cdot}$ If $\vec{b} - 2\vec{c} = \lambda\vec{a},$ then find the value of λ

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297. Find the area a parallelogram whose diagonals are $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k} and \vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}^{\cdot}$

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298. If $\vec{a} and \vec{b}$ are unit vectors such that $\left(\vec{a} + \vec{b}\right).\left(2\vec{a} + 3\vec{b}\right) \times \left(3\vec{a} - 2\vec{b}\right) = 0$, then angle between $\vec{a}$ and $\vec{b}$ is 0 b. $\pi/2$ c. π d. indeterminate

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299. If $\vec{a} and \vec{b}$ are any two unit vectors, then find the greatest positive integer in the range of $\frac{3\left|\vec{a}+\vec{b}\right|}{2}+2\left|\vec{a}-\vec{b}\right|$.

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300. If the vectors $\vec{a}, \vec{b}, and \vec{c}$ form the sides $BC, CA and AB$, respectively, of triangle $ABC, then$ $\vec{a}\dot{\vec{b}}+\vec{b}\dot{\vec{c}}+\vec{\cdot}\vec{a}=0$ b. $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ c. $\vec{a}\dot{\vec{b}}=\vec{b}\dot{\vec{c}}=\vec{\cdot}\vec{a}$ d. $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$

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301. Let $\vec{u}$ be a vector on rectangular coordinate system with sloping angle 60^{0} Suppose that $\left|\vec{u}-\hat{i}\right|$ is geometric mean of $\left|\vec{u}\right| and \left|\vec{u}-2\hat{i}\right|, where \hat{i}$ is the unit vector along the x-axis. Then find the value of $\left(\sqrt{2}+1\right)\left|\vec{u}\right|$

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302. Two adjacent sides of a parallelogram $ABCD$ are given by $\vec{A}B = 2\hat{i} + 10\hat{j} + 11\hat{k} and \vec{A}D = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angel α is given by $\frac{8}{9}$ b. $\frac{\sqrt{17}}{9}$ c. $\frac{1}{9}$ d. $\frac{4\sqrt{5}}{9}$

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303. Let $\vec{a}, \vec{b}, and \vec{c}$ be non-coplanar unit vectors, equally inclined to one another at an angle θ. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, find scalars $p, q and r$ in terms of θ.

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304. Given three vectors $\vec{a}, \vec{b}, and \vec{c}$ two of which are non-collinear. Further if $\left(\vec{a} + \vec{b}\right)$ is collinear with $\vec{c}, \left(\vec{b} + \vec{c}\right)$ is collinear with

$\vec{a}$, $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right| = \sqrt{2}$ Find the value of $\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{\cdot}\vec{a}$ 3 b. -3 c. 0 d. cannot be evaluated

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305. The value of a so that the volume of parallelepiped formed by $\hat{i} + a\hat{j} + \hat{k}, \hat{j} + a\hat{k}and a\hat{i} + \hat{k}$ is minimum is -3 b. 3 c. $1/\sqrt{3}$ d. $\sqrt{3}$

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306. $A_1, A_2, \ldots, A_n$ are the vertices of a regular plane polygon with n sides and O as its centre. Show that $\sum_{i=1}^{n} \overrightarrow{OA_i} \times \overrightarrow{OA_{i+1}} = (1 - n)\left(\overrightarrow{OA_2} \times \overrightarrow{OA_1}\right)$

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307. If $\vec{c}$ is a given non-zero scalar, and $\vec{A} and \vec{B}$ are given non-zero vector such that $\vec{A} \perp B$, then find vector $\vec{X}$ which satisfies the equation

$\vec{A}\vec{X} and \vec{A} \times \vec{X} = \vec{B}$

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308. $A, B, C and D$ are any four points in the space, then prove that

$\left|\vec{A}B \times \vec{C}D + \vec{B}C \times \vec{A}D + \vec{C}A \times \vec{B}D\right| = 4$ (area of ABC .)

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309. If vectors $\vec{a}, \vec{b}, and \vec{c}$ are coplanar, show that

$\left|\vec{a}\vec{b}\vec{c}\vec{a}\vec{a}\vec{a}\vec{b}\vec{a}\vec{c}\vec{b}\vec{a}\vec{b}\vec{b}\vec{b}\vec{c}\right| = \odot$

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310. Let $\vec{A} = 2\vec{i} + \vec{k}, \vec{B} = \vec{i} + \vec{j} + \vec{k}$ $\vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$ Determine a vector $\vec{R}$ satisfying $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R}\vec{A} = 0.$

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311. Determine the value of c so that for all real x, vectors $cx\hat{i} - 6\hat{j} - 3\hat{k} and x\hat{i} + 2\hat{j} + 2cx\hat{k}$ make an obtuse angle with each other.

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312. If $\vec{r} = x_1\left(\vec{a} \times \vec{b}\right) + x_2\left(\vec{b} \times \vec{a}\right) + x_3\left(\vec{c} \times \vec{d}\right)$ and $4\left[\vec{a}\vec{b}\vec{c}\right] = 1,$ then $x_1 + x_2 + x_3$ is equal to (A) $\frac{1}{2}\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$ (B) $\frac{1}{4}\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$ (C) $2\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$ (D) $4\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$

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313. $\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right)\left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right)\left(\vec{c} \times \vec{a}\right) \times \left(\vec{a} \times \vec{b}\right)\right]$ is equal to (where $\vec{a}, \vec{b} and \vec{c}$ are nonzero non-coplanar vector) $\left[\vec{a}\vec{b}\vec{c}\right]^2$ b. $\left[\vec{a}\vec{b}\vec{c}\right]^3$ c. $\left[\vec{a}\vec{b}\vec{c}\right]^4$ d. $\left[\vec{a}\vec{b}\vec{c}\right]$

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314. If V be the volume of a tetrahedron and V' be the volume of another tetrahedran formed by the centroids of faces of the previous tetrahedron and $V = KV'$, $thenK$ is equal to 9 b. 12 c. 27 d. 81

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315. If $\vec{a}, \vec{b} and \vec{c}$ are non coplanar vectors and $\vec{a} \times \vec{c}$ is perpendicular to $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$, then the value of $\left[a \times \left(\vec{b} \times \vec{c}\right)\right] \times \vec{c}$ is equal to $\left[\vec{a}\vec{b}\vec{c}\right]$ b. $2\left[\vec{a}\vec{b}\vec{c}\right]\vec{b}$ c. $\vec{0}$ d. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{a}$

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316. $A(\vec{a}), B\left(\vec{b}\right), C(\vec{c})$ are the vertices of the triangle ABC and $R(\vec{r})$ is any point in the plane of triangle ABC , then $r.\left(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right)$ is always equal to

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317. Let $\vec{a}, \vec{b} and \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q} and \vec{r}$ the vectors defined by the relation $\vec{p} = \dfrac{\vec{b} \times \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{q} = \dfrac{\vec{c} \times \vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]} and \vec{r} = \dfrac{\vec{a} \times \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$. Then the value of the expression $\left(\vec{a} + \vec{b}\right)\vec{p} + \left(\vec{b} + \vec{c}\right)\vec{q} + \left(\vec{c} + \vec{a}\right)\vec{r}$ is a.0 b. 1 c. 2 d. 3

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318. Let $\vec{a}, \vec{b} and \vec{c}$ be three non-coplanar vecrors and $\vec{r}$ be any arbitrary vector. Then $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{r} \times \vec{c}\right) + \left(\vec{b} \times \vec{c}\right) \times \left(\vec{r} \times \vec{a}\right) + \left(\vec{c} \times \vec{a}\right) \times \left(\vec{r} \times \vec{b}\right)$ is always equal to $\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ b. $2\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ c. $3\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ d. none of these

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319. The position vectors of point $A, B, and C$ are $\hat{i} + \hat{j} + \hat{k}, \hat{i} + 5\hat{j} - \hat{k} and 2\hat{i} + 3\hat{j} + 5\hat{k}$, respectively. Then greatest angel of triangle ABC is 120^0 b. 90^0 c. $\cos^{-1}(3/4)$ d. none of these

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320. Let $\vec{a}(x) = (s \in x)\hat{i} + (\cos x)\hat{j} and \vec{b}(x) = (\cos 2x)\hat{i} + \left(\sin 2x\hat{j}\right)$ be two variable vectors $(x \in R)^{\cdot}$ Then $\vec{a}(x) and \vec{b}(x)$ are a. collinear for unique value of x b. perpendicular for infinite values of x c. zero vectors for unique value of x d. none of these

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321. If

$\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + \hat{j} + 2\hat{k} and (1 + \alpha)\hat{i} + \beta(1 + \alpha)\hat{j} + \gamma(1 + \alpha)(1$

are a.$-2, \ -4, \ -\frac{2}{3}$ b. $2, \ -4, \frac{2}{3}$ c. $-2, 4, \frac{2}{3}$ d. $2, 4, \ -\frac{2}{3}$

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322. If $\vec{a}, \vec{b} and \vec{c}$ are unit vectors satisfying $\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2 = 9,$ then $\left|2\vec{a} + 5\vec{b} + 5\vec{c}\right|$ is.

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323. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is non-zero vector and $\left|\left(\vec{d} \cdot \vec{c}\right)\left(\vec{a} \times \vec{b}\right) + \left(\vec{d} \cdot \vec{a}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{d} \cdot \vec{b}\right)\left(\vec{c} \times \vec{a}\right)\right| = 0,$ then a. $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$ b. $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = |d|$ c. $\vec{a}, \vec{b}, and \vec{c}$ are coplanar d. none of these

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324. The vector(s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k} and \hat{i} + 2\hat{j} + \hat{k},$ and perpendicular to vector $\hat{i} + \hat{j} + \hat{k},$ is/are a. $\hat{j} - \hat{k}$ b. $-\hat{i} + \hat{j}$ c. $\hat{i} - \hat{j}$ d. $-\hat{j} + \hat{k}$

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325. Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j} and \vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{r} \dot{\vec{a}} = 0,$ then find the value of $\dot{\vec{r}} \dot{\vec{b}}$

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326. Let $\vec{a}, \vec{b}, and \vec{c}$ be vectors forming right-hand traid. Let $\vec{p} = \frac{\vec{b} \times \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}, and \vec{r} = \frac{\vec{a} \times \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}.$ If $x \cup R^{+},$ then $x\left[\vec{a}\vec{b}\vec{c}\right] + \frac{\left[\vec{p}\vec{q}\vec{r}\right]}{x}$ b. $x^4\left[\vec{a}\vec{b}\vec{c}\right]^2 + \frac{\left[\vec{p}\vec{q}\vec{r}\right]}{x^2}$ has least value $= \left(\frac{3}{2}\right)^{2/3}$ c. $\left[\vec{p}\vec{q}\vec{r}\right] > 0$ d. none of these

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327. From a point O inside a triangle $ABC,$ perpendiculars $OD, OEandOf$ are drawn to rthe sides $BC, CAandAB,$ respecrtively. Prove that the perpendiculars from $A, B, andC$ to the sides $EF, FDandDE$ are concurrent.

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328. If $a and b$ are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}} and \vec{b} = \frac{\hat{2}i + \hat{j} + 3\hat{k}}{\sqrt{14}}$,

then find the value of $\left(2\vec{a} + \vec{b}\right)\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)$

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329. Find the work done by the force $F = 3\hat{i} - \hat{j} - 2\hat{k}$ acrting on a particle such that the particle is displaced from point $A(-3, -4, 1) \top o \in tB(-1, -1, -2)$

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330. If $\vec{a}, \vec{b}, \vec{c}$ are three given non-coplanar vectors and any arbitrary vector $\vec{r}$ in space, where $\Delta 1 = \left|\vec{r}\vec{a}\vec{b}\vec{a}\cdot\vec{a}\vec{r}\vec{b}\vec{b}\vec{b}\cdot\vec{b}\vec{r}\vec{c}\vec{b}\vec{c}\cdot\vec{c}\right|$,

$\Delta 2 = \left| \vec{a}\vec{a}\vec{r}\vec{a} \cdot \vec{a}\vec{a}\vec{b}\vec{r}\vec{b} \cdot \vec{b}\vec{a}\vec{c}\vec{r}\vec{c} \cdot \vec{c} \right|$ $\Delta 3 = \left| \vec{a}\vec{a}\vec{b}\vec{a}\vec{r}\vec{a}\vec{a}\vec{b}\vec{b}\vec{b}\vec{r}\vec{b}\vec{a}\vec{c}\vec{b}\vec{c}\vec{r}\vec{c} \right|$,

$\Delta = \left| \vec{a}\vec{a}\vec{b}\vec{a} \cdot \vec{a}\vec{a}\vec{b}\vec{b}\vec{b} \cdot \vec{b}\vec{a}\vec{c}\vec{b}\vec{c} \cdot \vec{c} \right|$, then prove that

$\vec{r} = \frac{\Delta 1}{\Delta}\vec{a} + \frac{\Delta 2}{\Delta}\vec{b} + \frac{\Delta 3}{\Delta}\vec{c}$.

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331. If $A(\vec{a}), B(\vec{b}) and C(\vec{c})$ are three non-collinear points and origin does not lie in the plane of the points $A, B and C,$ then point $P(\vec{p})$ in the plane of the ABC such that vector $\vec{OP}$ is $\perp$ to planeof ABC , show that

$\vec{OP} = \frac{\left[\vec{a}\vec{b}\vec{c}\right]\left(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right)}{4^2}$, *where* is the area of the ABC

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332. $OABC$ is regular tetrahedron in which D is the circumcentre of OAB and E is the midpoint of edge AC Prove that DE is equal to half the edge of tetrahedron.

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333. In a quadrilateral $ABCD$ it is given tghat $AB \mid \mid CD$ nad the diagonals $ACandBD$ are perpendicular to each other. Show that $\dot{A}D\dot{B}C \geq \dot{A}B\dot{C}D$

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334. If $\vec{e}_1, \vec{e}_2, \vec{e}_3 and \vec{E}_1, \vec{E}_2, \vec{E}_3$ are two sets of vectors such that $\vec{e}_i \dot{\vec{E}}_j = 1, \text{ if } i = j and \vec{e}_i \dot{\vec{E}}_j = 0 and \text{ if } i \neq j,$ then prove that $\left[\vec{e}_1 \vec{e}_2 \vec{e}_3\right]\left[\vec{E}_1 \vec{E}_2 \vec{E}_3\right] = 1.$

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335. A line l is passing through the point $\vec{b}$ and is parallel to vector $\vec{c}$ Determine the distance of point $A\left(\vec{a}\right)$ from the line l in the form

$\vec{b} - \vec{a} + \frac{\left(\vec{a} - \vec{b}\right)\vec{c}}{\left|\vec{c}\right|^2}\vec{c}$ or $\frac{\left|\left(\vec{b} - \vec{a}\right) \times \vec{c}\right|}{\left|\vec{c}\right|}$.

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336. Given the vectors $\vec{A}, \vec{B}, and \vec{C}$ form a triangle such that $\vec{A} = \vec{B} + \vec{C}^{\cdot}$ find $a, b, c, and d$ such that the area of the triangle is $5\sqrt{6}$ where $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}\ \vec{B} = d\hat{i} + 3\hat{j} + 4\hat{k}\ \vec{C} = 3\hat{i} + \hat{j} - 2\hat{k}$

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337. Let a three dimensional vector $\vec{V}$ satisfy the condition, $2\vec{V} + \vec{V} \times \left(\hat{i} + 2\hat{j}\right) = 2\hat{i} + \hat{k}^{\cdot}$ If $3\left|\vec{V}\right| = \sqrt{m}^{\cdot}$ Then find the value of $m^{\cdot}$

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338. Given that

$\vec{u} = \hat{i} - 2\hat{j} + 3\hat{k}; \vec{v} = 2\hat{i} + \hat{j} + 4\hat{k}; \vec{w} = \hat{i} + 3\hat{j} + 3\hat{k} and \left(\vec{u}\dot{R} - 15\right)\hat{i} + \left(\vec{v}\dot{R} - 30\right)\hat{j} + \left(\right.$

Then find the greatest integer less than or equal to $\left|\vec{R}\right|$.

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339. Let $\vec{OA} - \vec{a}, \hat{O}B = 10\vec{a} + 2\vec{b} and \vec{O}C = \vec{b}, where O, A and C$ are non-collinear points. Let p denotes the areaof quadrilateral $OACB,$ and let q denote the area of parallelogram with $OA and OC$ as adjacent sides. If $p = kq,$ then find$k$.

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340. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}\dot{\vec{b}} = 0 = \vec{a}\dot{\vec{c}}$ and the angel between $\vec{b} and \vec{c}$ is $\frac{\pi}{3},$ then find the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$.

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341. If $\vec{x}, \vec{y}$ are two non-zero and non-collinear vectors satisfying $\left[(a-2)\alpha^2+(b-3)\alpha+c\right]\vec{x}+\left[(a-2)\beta^2+(b-3)\beta+c\right]\vec{y}+\left[(a-2)\gamma^2+(b-3)\gamma+c\right.$

are three distinct real numbers, then find the value of $\left(a^2+b^2+c^2-4\right)$.

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342. Let $\vec{a}=\alpha\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=\alpha\hat{i}+2\alpha\hat{j}-2\hat{k}, and\vec{c}=2\hat{i}+\alpha\hat{j}+\hat{k}$. Find thevalue of 6α, such that $\left\{\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)\right\}\times\left(\vec{c}\times\vec{a}\right)=0.$

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343. Let $\vec{a}, \vec{b}and\vec{c}$ be three vectors having magnitudes 1, 5and 3, respectively, such that the angel between $\vec{a}and\vec{b}is\theta$ and $\vec{a}\times\left(\vec{a}\times\vec{b}\right)=c$. Then $tan\theta$ is equal to a. 0 b. 2/3 c. 3/5 d. 3/4

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344. Two vectors in space are equal only if they have equal component in

a. a given direction b. two given directions c. three given directions d. in any arbitrary direction

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345. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{j} - \hat{k} and \vec{c} = \hat{k} - \hat{i}$. If $\vec{d}$ is a unit vector such that $\vec{a}.\vec{d} = 0 = \left[\vec{b}\vec{c}\vec{d}\right]$, then d equals a.$\pm\dfrac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$ b. $\pm\dfrac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$ c. $\pm\dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ d. $\pm\hat{k}$

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346. If vectors $\vec{a} and \vec{b}$ are two adjacent sides of a parallelogram, then the vector respresenting the altitude of the parallelogram which is the perpendicular to a is $\vec{b} + \dfrac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$ b. $\dfrac{\vec{a}\dot{}\vec{b}}{|\vec{b}|^2}$ c. $\vec{b} - \dfrac{\vec{b}\dot{}\vec{a}}{|\vec{a}|^2}$ d. $\dfrac{\vec{a} \times \left(\vec{b} \times \vec{a}\right)}{|\vec{b}|^2}$

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347. If $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is perpendicular to $\left(\vec{a} \times \vec{b}\right) \times \vec{c}$, we may have

$\left(\vec{a}\dot{}\vec{c}\right)\left|\vec{b}\right|^2 = \left(\vec{a}\dot{}\vec{b}\right)\left(\vec{b}\dot{}\vec{c}\right)$ b. $\vec{a}\dot{}\vec{b} = 0$ c. $\vec{a}\dot{}\vec{c} = 0$ d. $\vec{b}\dot{}\vec{c} = 0$

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348. $\left[\vec{a} \times \vec{b}\vec{c} \times \vec{d}\vec{e} \times \vec{f}\right]$is equal to (a) $\left[\vec{a}\vec{b}\vec{d}\right]\left[\vec{c}\vec{e}\vec{f}\right] - \left[\vec{a}\vec{b}\vec{c}\right]\left[\vec{d}\vec{e}\vec{f}\right]$ (b) $\left[\vec{a}\vec{b}\vec{e}\right]\left[\vec{f}\vec{c}\vec{d}\right] - \left[\vec{a}\vec{b}\vec{f}\right]\left[\vec{e}\vec{c}\vec{d}\right]$ (c) $\left[\vec{c}\vec{d}\vec{a}\right]\left[\vec{b}\vec{e}\vec{f}\right] - \left[\vec{a}\vec{d}\vec{b}\right]\left[\vec{a}\vec{e}\vec{f}\right]$ (d) $\left[\vec{a}\vec{c}\vec{e}\right]\left[\vec{b}\vec{d}\vec{f}\right]$

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349. $\vec{b} and \vec{c}$ are non-collinear if

$\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \left(\vec{a}\dot{}\vec{b}\right)\vec{b} = (4 - 2x - \sin y)\vec{b} + \left(x^2 - 1\right)\vec{c}$ and $\left(\vec{c}\dot{}\vec{c}\right)\vec{a} = \vec{\cdot}$ Then

a. $x = 1$ b. $x = -1$ c. $y = (4n + 1)\pi/2, n \in I$ d. $y = (2n + 1)\pi/2, n \in I$

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350. Unit vectors $\vec{a} and \vec{b}$ are perpendicular, and unit vector $\vec{c}$ is inclined at angle θ to both $\vec{a} and \vec{b}^{\cdot}$ If $\vec{c} = \alpha\vec{a} + \beta\vec{b} + \gamma\left(\vec{a} \times \vec{b}\right),$ then (a)$\alpha = \beta$ (b) $\gamma^2 = 1 - 2\alpha^2$ (c) $\gamma^2 = -\cos 2\theta$ (d) $\beta^2 = \dfrac{1 + \cos 2\theta}{2}$

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351. If $\vec{a} \perp \vec{b},$ then vector $\vec{v}$ in terms of $\vec{a} and \vec{b}$ satisfying the equation s $\vec{v}\dot{\vec{a}} = 0 and \vec{v}\dot{\vec{b}} = 1 and \left[\vec{v}\vec{a}\vec{b}\right] = 1$ is $\dfrac{\vec{b}}{\left|\vec{b}\right|^2} + \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^2}$ b. $\dfrac{\vec{b}}{\left|\vec{b}\right|^{\square}} + \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^2}$ c. $\dfrac{\vec{b}}{\left|\vec{b}\right|^2} + \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^{\square}}$ d. none of these

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352. If $\vec{a}' = \hat{i} + \hat{j}, \vec{b}' = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c}' = 2\hat{i} + \hat{j} - \hat{k}$, then the altitude of the parallelepiped formed by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ having base formed by $\vec{b}$ and $\vec{c}$ is (where $\vec{a}'$ is reciprocal vector $\vec{a}$)

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353. If $\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{j} + \hat{k}, \vec{c} = \hat{k} + \hat{i}$, then in the reciprocal system of vectors $\vec{a}, \vec{b}, \vec{c}$ reciprocal $\vec{a}$ of vector $\vec{a}$ is a. $\frac{\hat{i} + \hat{j} + \hat{k}}{2}$ b. $\frac{\hat{i} - \hat{j} + \hat{k}}{2}$ c. $\frac{-\hat{i} - \hat{j} + \hat{k}}{2}$ d. $\frac{\hat{i} + \hat{j} - \hat{k}}{2}$

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354. If unit vectors $\vec{a} and \vec{b}$ are inclined at angle 2θ such that $\left|\vec{a} - \vec{b}\right| < 1 and 0 \leq \theta \leq \pi, then \theta$ lies in interval a. $[0, \pi/6]$ b. $[5\pi/6, \pi]$ c. $[\pi/6, \pi/2]$ d. $[\pi/2, 5\pi/6]$

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355. Let $\vec{a}, \vec{b} and \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q} and \vec{r}$ the vectors defined by the relation $\vec{p} = \frac{\vec{b} \times \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]} and \vec{r} = \frac{\vec{a} \times \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$. Then the value of the expression $\left(\vec{a} + \vec{b}\right)\vec{p} + \left(\vec{b} + \vec{c}\right)\vec{q} + \left(\vec{c} + \vec{a}\right)\vec{r}$ is a.0 b. 1 c. 2 d. 3

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356. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_2\hat{k}, \vec{b} = b_1\hat{i} + a_2\hat{j} + b_2\hat{k}, and \vec{c} = c_1\hat{i} + c_2\hat{j} + c_2\hat{k},$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both vectors $\vec{a} and \vec{b}$. If the angle between $a and b$ is $\pi/6,$ then $\left|a_1a_2a_3b_1b_2b_3c_1c_2c_3\right|^2$ is equal to 0 1 $\frac{1}{4}(a12 + a22 + a32)(b12 + b22 + b32)$ $\frac{3}{4}(a12 + a22 + a32)(b12 + b22 + b32)(c12 + c22 + c32)$

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357. $A, B, CandD$ are four points such that $\vec{A}B = m\left(2\hat{i} - 6\hat{j} + 2\hat{k}\right), \vec{B}C = \left(\hat{i} - 2\hat{j}\right)and\vec{C}D = n\left(-6\hat{i} + 15\hat{j} - 3\hat{k}\right)$. If CD intersects AB at some point E, then a. $m \geq 1/2$ b.$n \geq 1/3$ c. $m = n$ d. $m < n$

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358. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}and\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}and\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$ is given by a. $\hat{i} - 3\hat{j} + 3\hat{k}$ b. $-3\hat{i} - 3\hat{j} + 3\hat{k}$ c. $3\hat{i} - \hat{j} + 3\hat{k}$ d. $\hat{i} + 3\hat{j} - 3\hat{k}$

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359. If $\hat{a}, \hat{b}, and\hat{c}$ are unit vectors, then $\left|\hat{a} - \hat{b}\right|^2 + \left|\hat{b} - \hat{c}\right|^2 + \left|\hat{c} - \hat{a}\right|^2$ does not exceed

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360. Which of the following expressions are meaningful? $\vec{u}\dot{\vec{v}} \times \vec{w}$ b. $\left(\vec{u}\dot{\vec{v}}\right)\dot{\vec{w}}$ c. $\left(\vec{u}\dot{\vec{v}}\right)\dot{\vec{w}}$ d. $\vec{u} \times \left(\vec{v}\dot{\vec{w}}\right)$

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361. Find the value of λ if the volume of a tetrashedron whose vertices are with position vectors $\hat{i} - 6\hat{j} + 10\hat{k},\ -\hat{i} - 3\hat{j} + 3\hat{k}, 5\hat{i} - \hat{j} + \lambda\hat{k} and 7\hat{i} - 4\hat{j} + 7\hat{k}$ is 11 cubic unit.

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362. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} = \hat{k} and \vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b} and \vec{c}$, whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$, is a.$2\hat{i} + 3\hat{j} - 3\hat{k}$ b. $2\hat{i} - 3\hat{j} + 3\hat{k}$ c. $-2\hat{i} - \hat{j} + 5\hat{k}$ d. $2\hat{i} + \hat{j} + 5\hat{k}$

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363. If $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)\vec{a}\times\vec{d}=0$, then which of the following may be true? $\vec{a},\vec{b},\vec{c}and\vec{d}$ are necessarily coplanar b.$\vec{a}$ lies in the plane of $\vec{c}and\vec{d}$ c. $\vec{b}$ lies in the plane of $\vec{a}and\vec{d}$ d. $\vec{c}$ lies in the plane of $\vec{a}and\vec{d}$

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364. Vector $\frac{1}{3}(2i-2j+k)$ is (A) a unit vector (B) makes an angle $\pi/3$ with vector $\left(2\hat{i}-4\hat{j}+3\hat{k}\right)$ (C) parallel to vector $\left(-\hat{i}+\hat{j}-\frac{1}{2}\hat{k}\right)$ (D) perpendicular to vector $3\hat{i}+2\hat{j}-2\hat{k}$

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365. Let $\vec{u}and\vec{v}$ be unit vectors such that $\vec{u}\times\vec{v}+\vec{u}=\vec{w}$ and $\vec{w}\times\vec{u}=\vec{v}$. Find the value of $\left[\vec{u}\vec{v}\vec{w}\right]$.

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366. The scalars $l and m$ such that $l\vec{a} + m\vec{b} = \vec{c}, where \vec{a}, \vec{b} and \vec{c}$ are given vectors, are equal to

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367. If $OABC$ is a tetrahedron where O is the orogin anf $A, B, and C$ are the other three vertices with position vectors, $\vec{a}, \vec{b}, and \vec{c}$ respectively, then prove that the centre of the sphere circumscribing the tetrahedron is given by position vector $\dfrac{a^2(\vec{b} \times \vec{c}) + b^2(\vec{c} \times \vec{a}) + c^2(\vec{a} \times \vec{b})}{2[\vec{a}\vec{b}\vec{c}]}$.

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368. Let k be the length of any edge of a regular tetrahedron (a tetrahedron whose edges are equal in length is called a regular tetrahedron). Show that the angle between any edge and a face not containing the edge is $\cos^{-1}(1/\sqrt{3})$.

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369. In ABC, a point P is taken on AB such that $AP/BP = 1/3$ and point Q is taken on BC such that $CQ/BQ = 3/1$. If R is the point of intersection of the lines $AQandCP$, ising vedctor method, find the are of ABC if the area of BRC is 1 unit

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370. Let $ABCD$ be a p[arallelogram whose diagonals intersect at P and let O be the origin. Then prove that $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D = 4\vec{O}P^{\cdot}$

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371. If $\left|(a-x)^2(a-y)^2(a-z)^2(b-x)^2(b-y)^2(b-z)^2(c-x)^2(c-y)^2(c-a)^2\right| = 0$ and vectors $\vec{A}, \vec{B}, and\vec{C}, where\vec{A} = a^2\hat{i} + a\hat{j} + \hat{k}$, etc, are non-coplanar, then prove that vectors $\vec{X}, \vec{Y}and\vec{Z}, where\vec{X} = x^2\hat{i} + x\hat{j} + \hat{k}$, etc. may be coplanar.

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372. If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}, \vec{b} = y\hat{i} + z\hat{j} + x\hat{k} and \vec{c} = z\hat{i} + x\hat{j} + y\hat{k}, then \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is (a)parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$ (b)orthogonal to $\hat{i} + \hat{j} + \hat{k}$ (c)orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$ (d)orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

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373. The lengths of two opposite edges of a tetrahedron are a and $b;$ the shortest distane between these edges is $d,$ and the angel between them is θ Prove using vectors that the volume of the tetrahedron is $\frac{abd\sin\theta}{6}$.

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374. Find the volume of a parallelepiped having three vectors of equal magnitude $|\vec{a}|$ and equal inclination θ with each other.

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375. If vectors $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{B} = \hat{i} + \hat{j} + 5\hat{k} and \vec{C}$ form a left-handed system, then $\vec{C}$ is a.$11\hat{i} - 6\hat{j} - \hat{k}$ b.$-11\hat{i} + 6\hat{j} + \hat{k}$ c. $11\hat{i} - 6\hat{j} + \hat{k}$ d. $-11\hat{i} + 6\hat{j} - \hat{k}$

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376. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} and \vec{c} = 3\hat{j} - 2\hat{k}$ Let $\vec{x}, \vec{y}, and \vec{z}$ be thre vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{c}; \vec{c}, \vec{a}$, respectively. Then $\vec{x} \cdot \vec{d} = -1$ b. $\vec{y} \cdot \vec{d} = 1$ c. $\vec{z} \cdot \vec{d} = 0$ d. $\vec{r} \cdot \vec{d} = 0, where \vec{r} = \lambda\vec{x} + \mu\vec{y} + \delta\vec{z}$

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377. Vectors $\vec{A} and \vec{B}$ satisfying the vector equation $\vec{A} + \vec{B} = \vec{a}, \vec{A} \times \vec{B} = \vec{b} and \vec{A} \cdot \vec{a} = 1, where \vec{a} and \vec{b}$ are given vectors, are a.

$\vec{A} = \dfrac{\left(\vec{a} \times \vec{b}\right) - \vec{a}}{a^2}$ b. $\vec{B} = \dfrac{\left(\vec{b} \times \vec{a}\right) + \vec{a}\left(a^2 - 1\right)}{a^2}$ c. $\vec{A} = \dfrac{\left(\vec{a} \times \vec{b}\right) + \vec{a}}{a^2}$ d. $\vec{B} = \dfrac{\left(\vec{b} \times \vec{a}\right) - \vec{a}\left(a^2 - 1\right)}{a^2}$

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378. if $\vec{\alpha} \mid \mid \left(\vec{\beta} \times \vec{\gamma}\right)$, then $\left(\vec{\alpha} \times \beta\right)\dot{\vec{\alpha}} \times \vec{\gamma}$ equals to $\left|\vec{\alpha}\right|^2\left(\vec{\beta}\dot{\vec{\gamma}}\right)$ b. $\left|\vec{\beta}\right|^2\left(\vec{\gamma}\dot{\vec{\alpha}}\right)$ c. $\left|\vec{\gamma}\right|^2\left(\vec{\alpha}\dot{\vec{\beta}}\right)$ d. $\left|\vec{\alpha}\right|\left|\vec{\beta}\right|\left|\vec{\gamma}\right|$

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379. Let $\vec{\alpha} = a\hat{i} + b\hat{j} + c\hat{k}, \vec{\beta} = b\hat{i} + c\hat{j} + a\hat{k} and \vec{\gamma} = c\hat{i} + a\hat{j} + b\hat{k}$ are three coplanar vectors with $a \neq b, and \vec{v} = \hat{i} + \hat{j} + \dot{\hat{k}}$ Then v is perpendicular to $\vec{\alpha}$ b.$\vec{\beta}$ c. $\vec{\gamma}$ d. none of these

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380. $a_1, a_2, a_3, \in R - \{0\}$ and $a_1 + a_2\cos 2x + a_3\sin^2 x = 0f$ or $allx \in R$, then (a)vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}and\vec{b} = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to each other (b)vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}and\vec{b} = -\hat{i} + \hat{j} + 2\hat{k}$ are parallel to each other (c)vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length$\sqrt{6}$ units, then one of the ordered triple $\left(a_1, a_2, a_3\right) = (1, -1, -2)$ (d)are perpendicular to each other if $2a_1 + 3a_2 + 6a_3 = 26, then\left|a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right|is2\sqrt{6}$

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381. If P is any arbitrary point on the circumcirlce of the equllateral trangle of side length l units, then $\left|\vec{P}A\right|^2 + \left|\vec{P}B\right|^2 + \left|\vec{P}C\right|^2$ is always equal to $2l^2$ b. $2\sqrt{3}l^2$ c. l^2 d. $3l^2$

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382. Let $\vec{a} and \vec{b}$ be two non-zero perpendicular vectors. A vecrtor $\vec{r}$ satisfying the equation $\vec{r} \times \vec{b} = \vec{a}$ can be $\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$ b. $2\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$ c. $\left|\vec{a}\right|\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$ d. $\left|\vec{b}\right|\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

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383. If $\vec{a} and \vec{b}$ are two vectors and angle between them is θ, then $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a}\dot{}\vec{b}\right)^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2$ $\left|\vec{a} \times \vec{b}\right| = \left(\vec{a}\dot{}\vec{b}\right)$, if $\theta = \pi/4$ $\vec{a} \times \vec{b} = \left(\vec{a}\dot{}\vec{b}\right)\hat{n}$, $\left(where\hat{n}\right.$ is unit vector,) if $\theta = \pi/4$ $\left(\vec{a} \times \vec{b}\right)\dot{}\vec{a} + \vec{b} = 0$

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384. Let $\vec{r}$ be a unit vector satisfying $\vec{r} \times \vec{a} = \vec{b}, where\left|\vec{a}\right| = 3and\left|\vec{b}\right| = 2.$ Then $\vec{r} = \dfrac{2}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$ b. $\vec{r} = \dfrac{1}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right.$ c. $\vec{r} = \dfrac{2}{3}\left(\vec{a} - \vec{a} \times \vec{b}\right.$ d.

$\vec{r} = \frac{1}{3}\left(- \vec{a} + \vec{a} \times \vec{b}\right.$

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385. If vector $\vec{b} = \left(\tan\alpha, -1, 2\sqrt{\sin\alpha/2}\right) and \vec{c} = \left(\tan\alpha, \tan\alpha, \frac{3}{\sqrt{\sin\alpha/2}}\right)$ are orthogonal and vector $\vec{a} = (1, 3, \sin 2\alpha)$ makes an obtuse angle with the z-axis, then the value of α is $\alpha = (4n+1)\pi + \tan^{-1}2$ b. $\alpha = (4n+1)\pi - \tan^{-1}2$ c. $\alpha = (4n+2)\pi + \tan^{-1}2$ d. $\alpha = (4n+2)\pi - \tan^{-1}2$

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386. Let $\vec{a}, \vec{b}, and \vec{c}$ be non-zero vectors and $\vec{V}_1 = \vec{a} \times \left(\vec{b} \times \vec{c}\right) and \vec{V}_2 \left(\vec{a} \times \vec{b}\right) \times \vec{\cdot}$ Vectors $\vec{V}_1 and \vec{V}_2$ are equal. Then $\vec{a} an \vec{b}$ are orthogonal b. $\vec{a} and \vec{c}$ are collinear c. $\vec{b} and \vec{c}$ are orthogonal d. $\vec{b} = \lambda\left(\vec{a} \times \vec{c}\right) when \lambda$ is a scalar

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387. $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ A vector coplanar with $\vec{b} and \vec{c}$ whose projectin on $\vec{a}$ is magnitude $\sqrt{\frac{2}{3}}$ is $2\hat{i} + 3\hat{j} - 3\hat{k}$ b. $-2\hat{i} - \hat{j} + 5\hat{k}$ c. $2\hat{i} + 3\hat{j} + 3\hat{k}$ d. $2\hat{i} + \hat{j} + 5\hat{k}$

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388. Let $\vec{P}R = 3\hat{i} + \hat{j} - 2\hat{k} and \vec{S}Q = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram $PQRS, and \vec{P}T = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determine by the vectors $\vec{P}T$, $\vec{P}Q$ and $\vec{P}S$ is 5 b. 20 c. 10 d. 30

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389. If in a right-angled triangle $ABC,$ the hypotenuse $AB = p$,then $\vec{AB}. \vec{AC} + \vec{BC}. \vec{BA} + \vec{CA}. \vec{CB}$ is equal to $2p^2$ b. $\frac{p^2}{2}$ c. p^2 d. none of these

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390. If $\vec{a} = \left(\hat{i} + \hat{j} + \hat{k}\right), \vec{a}\vec{b} = 1 and \vec{a} \times \vec{b} = \hat{j} - \hat{k}, then \hat{b}$ is $\hat{i} - \hat{j} + \hat{k}$ b. $2\hat{j} - \hat{k}$ c. $\hat{i}$ d. $2\hat{i}$

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391. If a satisfies $\vec{a} \times \left(\hat{i} + 2\hat{j} + \hat{k}\right) = \hat{i} - \hat{k}$, then $\vec{a}$ is equal to $\lambda\hat{i} + (2\lambda - 1)\hat{j} + \lambda\hat{k}, \lambda R$ b. $\lambda\hat{i} + (1 - 2\lambda)\hat{j} + \lambda\hat{k}, \lambda R$ c. $\lambda\hat{i} + (2\lambda + 1)\hat{j} + \lambda\hat{k}, \lambda R$ d. $\lambda\hat{i} - (1 + 2\lambda)\hat{j} + \lambda\hat{k}, \lambda R$

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392. If $\vec{r}\vec{a} = \vec{r}\vec{b} = \vec{r}\vec{c} = 0, where \vec{a}, \vec{b}, and \vec{c}$ are non-coplanar, then $\vec{r} \perp \left(\vec{c} \times \vec{a}\right)$ b. $\vec{r} \perp \left(\vec{a} \times \vec{b}\right)$ c. $\vec{r} \perp \left(\vec{b} \times \vec{c}\right)$ d. $\vec{r} = \vec{0}$

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393. The unit vector orthogonal to vector $-\hat{i}+2\hat{j}+2\hat{k}$ and making equal angles with the x and y-axis ,

a.$\pm\frac{1}{3}\left(2\hat{i}+2\hat{j}-\hat{k}\right)$

b. $\pm\frac{1}{3}\left(\hat{i}+\hat{j}-\hat{k}\right)$

c. $\pm\frac{1}{3}\left(2\hat{i}-2\hat{j}-\hat{k}\right)$

d. none of these

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394. Vectors $3\vec{a}-5\vec{b}and2\vec{a}+\vec{b}$ are mutually perpendicular. If $\vec{a}+4\vec{b}and\vec{b}-\vec{a}$ are also mutually perpendicular, then the cosine of the angel between $aandb$ is a.$\frac{19}{5\sqrt{43}}$ b. $\frac{19}{3\sqrt{43}}$ c. $\frac{19}{2\sqrt{45}}$ d. $\frac{19}{6\sqrt{43}}$

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395. If vectors $\vec{a}and\vec{b}$ are two adjacent sides of a parallelogram, then the vector respresenting the altitude of the parallelogram which is the perpendicular to a is $\vec{b} + \dfrac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$ b. $\dfrac{\vec{a}\vec{b}}{|\vec{b}|^2}$ c. $\vec{b}$ - $\dfrac{\vec{b}\vec{a}}{|\vec{a}|^2}$ d. $\dfrac{\vec{a} \times \left(\vec{b} \times \vec{a}\right)}{|\vec{b}|^2}$

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396. The value of x for which the angle between $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k} and \vec{b} = 7\hat{i} - 2\hat{j} + \hat{k}$ is obtuse and the angle between b and the z-axis acute and less that $\pi/6$ is `a1//2orx<0` d. none of these

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397. Let $\vec{a} \cdot \vec{b} = 0, where \vec{a} and \vec{b}$ are unit vectors and the unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a} and \vec{b}$ If $\vec{c} = m\vec{a} + n\vec{b} + p\left(\vec{a} \times \vec{b}\right), (m, n, p \in R),$ then a.- $\dfrac{\pi}{4} \leq \theta \leq \dfrac{\pi}{4}$ b. $\dfrac{\pi}{4} \leq \theta \leq \dfrac{3\pi}{4}$ c. $0 \leq \theta \leq \dfrac{\pi}{4}$ d. $0 \leq \theta \leq \dfrac{3\pi}{4}$

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398. A parallelogram is constructed on $3\vec{a} + \vec{b} and \vec{a} - 4\vec{b}, where \left|\vec{a}\right| = 6 and \left|\vec{b}\right| = 8, and \vec{a} and \vec{b}$ are anti-parallel. Then the length of the longer diagonal is 40 b. 64 c. 32 d. 48

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399. Let the position vectors of the points $P and Q$ be $4\hat{i} + \hat{j} + \lambda\hat{k} and 2\hat{i} - \hat{j} + \lambda\hat{k}$, respectively. Vector $\hat{i} - \hat{j} + 6\hat{k}$ is perpendicular to the plane containing the origin and the points $P and Q$. Then λ equals $1/2$ b. $1/2$ c. 1 d. none of these

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400. $a and c$ are unit vectors and $|b| = 4$. The angel between $a and c$ is $\cos^{-1}(1/4) and b - 2c = \lambda a$ The value of λ is $3, -4$ b. $1/4, 3/4$ c. $-3, 4$ d. $-1/4, 3/4$

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401. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is non-zero vector and $\left|\left(\vec{d} \cdot \vec{c}\right)\left(\vec{a} \times \vec{b}\right) + \left(\vec{d} \cdot \vec{a}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{d} \cdot \vec{b}\right)\left(\vec{c} \times \vec{a}\right)\right| = 0,$ then a. $|\vec{a}| = \left|\vec{b}\right| = |\vec{c}|$ b. $|\vec{a}| + \left|\vec{b}\right| + |\vec{c}| = |d|$ c. $\vec{a}, \vec{b}, and \vec{c}$ are coplanar d. none of these

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402. Let $\vec{a}, \vec{b}, and \vec{c}$ be three non-coplanar vectors and $\vec{d}$ be a non-zero vector, which is perpendicular to $\left(\vec{a} + \vec{b} + \vec{c}\right)^{\cdot}$ Now

$$\vec{d} = \left(\vec{a} \times \vec{b}\right)\sin x + \left(\vec{b} \times \vec{c}\right)\cos y + 2\left(\vec{c} \times \vec{a}\right)^{\cdot} Then$$

a. $\frac{\vec{a} + \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]} = 2$ b. $\frac{\vec{a} + \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]} = -2$ c. minimum value of $x^2 + y^2$ is $\pi^2/4$ d. minimum value of $x^2 + y^2$ is $5\pi^2/4$

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403. If $\vec{a} + 2\vec{b} + 3\vec{c} = 0, then \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} =$ $2(\vec{a} \times \vec{b})$ b.$6(\vec{b} \times \vec{c})$

c. $3(\vec{c} \times \vec{a})$ d. $\vec{0}$

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404. $\vec{a} and \vec{b}$ are two non-collinear unit vector, and

$\vec{u} = \vec{a} - \left(\vec{a} \dot{\vec{b}}\right)\vec{b} and \vec{v} = \vec{a} \times \dot{\vec{b}}$ Then $|\vec{v}|$ is $|\vec{u}|$ b.$|\vec{u}| + \left|\vec{u} \dot{\vec{b}}\right|$ c. $|\vec{u}| + \left|\vec{u} \dot{\vec{a}}\right|$ d. none of these

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405. The angles of triangle, two of whose sides are represented by vectors $\sqrt{3}\left(\hat{a} \times \vec{b} and \hat{b} - \left(\hat{a} \dot{\hat{b}}\right)\hat{a}, where \vec{b}\right)$ is a non zero vector and $\hat{a}$ is unit vector in the direction of $\vec{a}$, are $\tan^{-1}(\sqrt{3})$ b. $\tan^{-1}(1/\sqrt{3})$ c. $\cot^{-1}(0)$ d. $\tan^{-1}(1)$

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406. $\vec{a}, \vec{b}, and \vec{c}$ are unimodular and coplanar. A unit vector $\vec{d}$ is perpendicular to then. If $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = \frac{1}{6}\hat{i} - \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k}$, and the angel between $\vec{a} and \vec{b}$ is $30^0, then \vec{c}$ is a. $\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)/3$ b. $\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right)/3$ c. $\left(2\hat{i} + 2\hat{j} - \hat{k}\right)/3$ d. $\left(-2\hat{i} - 2\hat{j} + \hat{k}\right)/3$

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407. Vectors perpendicular to $\hat{i} - \hat{j} - \hat{k}$ and in the plane of $\hat{i} + \hat{j} + \hat{k} and - \hat{i} + \hat{j} + \vec{k}$ are $\hat{i} + \hat{k}$ b. $2\hat{i} + \hat{j} + \hat{k}$ c. $3\hat{i} + 2\hat{j} + \hat{k}$ d. $-4\hat{i} - 2\hat{j} - 2\hat{k}$

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408. If side $\vec{AB}$ of an equilateral trangle ABC lying in the x-y plane $3\hat{i}$, then side $\vec{CB}$ can be $-\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$ b. $-\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$ c. $-\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$ d. $\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$

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409. 36. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $\left(\vec{a}\times\vec{b}\right).\vec{c}\times\vec{d}=1$ and $\vec{a}.\vec{c}=\frac{1}{2}$ then a) $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar b) $\vec{b}, \vec{c}, \vec{d}$ are non -coplanar c)$\vec{b}, \vec{d}$ are non parallel d) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

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410. Let two non-collinear unit vector $\hat{a}$ a n d $\hat{b}$ form an acute angle. A point P moves so that at any time t, the position vector $OP(where O$ is the origin) is given by $\hat{a}\cos t+\hat{b}\sin t \dot{W}hen P$ is farthest from origin O, let M be the length of $OPand\hat{u}$ be the unit vector along $\dot{OP}$ Then (a) $\hat{u}=\frac{\hat{a}+\hat{b}}{\left|\hat{a}+\hat{b}\right|}andM=\left(1+\hat{a}\dot{\hat{b}}\right)^{1/2}$ (b) $\hat{u}=\frac{\hat{a}-\hat{b}}{\left|\hat{a}-\hat{b}\right|}andM=\left(1+\hat{a}\dot{\wedge}\right)^{1/2}$ (c) $\hat{u}=\frac{\hat{a}+\hat{b}}{\left|\hat{a}+\hat{b}\right|}andM=\left(1+2\hat{a}\dot{\hat{b}}\right)^{1/2}$ (d) $\hat{u}=\frac{\hat{a}-\hat{b}}{\left|\hat{a}-\hat{b}\right|}andM=\left(1+2\hat{a}\dot{\hat{b}}\right)^{1/2}$

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411. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k} and \vec{c} = \hat{i} + \hat{j} - \hat{k}$ A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection of c is $1/\sqrt{3}$ is a. $4\hat{i} - \hat{j} + 4\hat{k}$ b. $3\hat{i} + \hat{j} + 3\hat{k}$ c. $2\hat{i} + \hat{j} + 2\hat{k}$ d. $4\hat{i} + \hat{j} - 4\hat{k}$

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412. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero, non coplanar vector $\vec{b}_1 = \vec{b} - \frac{\vec{b}\vec{a}}{|\vec{a}|^2}\vec{a}$, $\vec{c}_1 = \vec{c} - \frac{\vec{\cdot}\,\vec{a}}{|\vec{a}|^2}\vec{a} + \frac{\vec{b}\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $, c_2 = \vec{c} - \frac{\vec{\cdot}\,\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{b}\vec{c}}{|\vec{b}_1|^2}$, $b_1, \vec{c}_3 = \vec{c} - \frac{\vec{\cdot}\,\vec{a}}{|\vec{c}|^2}\vec{a} + \frac{\vec{b}\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $\vec{c}_4 = \vec{c} - \frac{\vec{\cdot}\,\vec{a}}{|\vec{c}|^2}\vec{a} = \frac{\vec{b}\vec{c}}{|\vec{b}|^2}\vec{b}_1$ then the set of orthogonal vectors is $(\vec{a}, \vec{b}_1, \vec{c}_3)$ b. $(\vec{a}, \vec{b}_1, \vec{c}_2)$ c. $(\vec{a}, \vec{b}_1, \vec{c}_1)$ d. $(\vec{a}, \vec{b}_2, \vec{c}_2)$

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413. The unit vector which is orthogonal to the vector $5\hat{j}+2\hat{j}+6\hat{k}$ and is coplanar with vectors $2\hat{i}+\hat{j}+\hat{k} and \hat{i}-\hat{j}+\hat{k}$ is $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$ b. $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$ c. $\frac{3\hat{i}-\hat{k}}{\sqrt{10}}$ d. $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

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414. If $\vec{a} and \vec{b}$ are unequal unit vectors such that $(\vec{a}-\vec{b})\times[(\vec{b}+\vec{a})\times(2\vec{a}+\vec{b})]=\vec{a}+\vec{b},$ then angle θ between $\vec{a} and \vec{b}$ is 0 b. $\pi/2$ c. $\pi/4$ d. π

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415. If $\vec{a}, \vec{b}, \vec{c}$ are 3 unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}}{2}$then ($\vec{b}$ and $\vec{c}$ being non parallel). (a)angle between $\vec{a}\&\vec{b}$ is $\frac{\pi}{3}$ (b)angle between $\vec{a}$ and $\vec{c}$ is $\frac{\pi}{3}$ (c)angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$ (d)angle between $\vec{a}$ and $\vec{c}$ is $\frac{\pi}{2}$

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416. If in triangle $ABC, \vec{AB} = \frac{\vec{u}}{|\vec{u}|} - \frac{\vec{v}}{|\vec{v}|} and \vec{AC} = \frac{2\vec{u}}{|\vec{u}|}, where |\vec{u}| \neq |\vec{v}|,$ then $1 + \cos 2A + \cos 2B + \cos 2C = 0$ b.$\sin A = \cos C$ c. projection of AC on BC is equal to BC d. projection of AB on BC is equal to AB

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417. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} and \vec{c} = 3\hat{j} - 2\hat{k}$ Let $\vec{x}, \vec{y}, and \vec{z}$ be thre vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{c}; \vec{c}, \vec{a},$ respectively. Then $\vec{x}\cdot\vec{d} = -1$ b. $\vec{y}\cdot\vec{d} = 1$ c. $\vec{z}\cdot\vec{d} = 0$ d. $\vec{r}\cdot\vec{d} = 0, where \vec{r} = \lambda\vec{x} + \mu\vec{y} + \delta\vec{z}$

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418. If $a \times (b \times c) = (a \times b) \times c,$ then $\left(\vec{c} \times \vec{a}\right) \times \vec{b} = \vec{0}$ b.$\vec{c} \times \left(\vec{a} \times \vec{b}\right) = \vec{0}$ c. $\vec{b} \times \left(\vec{c} \times \vec{a}\right)\vec{0}$ d. $\left(\vec{c} \times \vec{a}\right) \times \vec{b} = \vec{b} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$

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419. If $\hat{a}, \hat{b}, and \hat{c}$ are three unit vectors inclined to each other at angle θ, then the minimum value of θ is $\frac{\pi}{3}$ b. $\frac{\pi}{4}$ c. $\frac{2\pi}{3}$ d. $\frac{5\pi}{6}$

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420. Let the pairs $a, b and c, d$ each determine a plane. Then the planes are parallel if $(\vec{a} \times \vec{c}) \times (\vec{b} \times \vec{d}) = \vec{0}$ b. $(\vec{a} \times \vec{c}) \cdot \vec{b} \times \vec{d} = \vec{0}$ c. $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$ d. $(\vec{a} \times \vec{b}) \cdot \vec{c} \times \vec{d} = \vec{0}$

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421. $P(\vec{p})$ and $Q(\vec{q})$ are the position vectors of two fixed points and $R(\vec{r})$ is the position vectorvariable point. If R moves such that $(\vec{r} - \vec{p}) \times (\vec{r} - \vec{q}) = 0$ then the locus of R is

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422. Two adjacent sides of a parallelogram $ABCD$ are $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$. Then the value of $|AC \times BD|$ is $20\sqrt{5}$ b. $22\sqrt{5}$ c. $24\sqrt{5}$ d. $26\sqrt{5}$

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423. If $\hat{a}, \hat{b}, and\hat{c}$ are three unit vectors, such that $\hat{a} + \hat{b} + \hat{c}$ is also a unit vector and $\theta_1, \theta_2 andth\eta_3$ are angles between the vectors $\hat{a}, \hat{b}; \hat{b}, \hat{c} and\hat{c}, \hat{a}$ respectively, then among $\theta_1, \theta_2, andth\eta_3$ a. all are acute angles b. all are right angles c. at least one is obtuse angle d. none of these

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424. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}\dot{\vec{b}} = 0 = \vec{a}\dot{\vec{c}}$ and the angle between $\vec{b} and \vec{c}$ is $\pi/3$, then the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$ is $1/2$ b. 1 c. 2 d. none of these

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425. Let $\vec{a} = \hat{i} + \hat{j}; \vec{b} = 2\hat{i} - \hat{k}$ Then vector $\vec{r}$ satisfying $\vec{r} \times \vec{a} = \vec{b} \times \vec{a} and \vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is $\hat{i} - \hat{j} + \hat{k}$ b. $3\hat{i} - \hat{j} + \hat{k}$ c. $3\hat{i} + \hat{j} - \hat{k}$ d. $\hat{i} - \hat{j} - \hat{k}$

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426. If $\vec{a} and \vec{b}$ are two vectors, such that $\vec{a}\dot{\vec{b}} < 0 and \left|\vec{a}\dot{\vec{b}}\right| = \left|\vec{a} \times \vec{b}\right|$, then the angle between vectors $\vec{a} and \vec{b}$ is π b. $7\pi/4$ c. $\pi/4$ d. $3\pi/4$

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427. $\vec{a}, \vec{b}, and \vec{c}$ are three vectors of equal magnitude. The angel between each pair of vectors is $\pi/3$ such that $\left|\vec{a} + \vec{b} + \vec{c}\right| = 6$. Then $\left|\vec{a}\right|$ is equal to 2 b. -1 c. 1 d. $\sqrt{6}/3$

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428. If $\vec{a}, \vec{b} and \vec{c}$ are three mutually perpendicular vectors, then the vector which is equally inclined to these vectors is a.$\vec{a} + \vec{b} + \vec{c}$ b. $\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|} + \frac{\vec{c}}{|\vec{c}|}$ c. $\frac{\vec{a}}{|\vec{a}|^2} + \frac{\vec{b}}{|\vec{b}|^2} + \frac{\vec{c}}{|\vec{c}|^2}$ d. $|\vec{a}|\vec{a} - |\vec{b}|\vec{b} + |\vec{c}|\vec{c}$

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429. Let $\vec{a} and \vec{b}$ be two non-collinear unit vector. If $\vec{u} = \vec{a} - \left(\vec{a} \cdot \vec{b}\right)\vec{b} and \vec{v} = \vec{a} \times \vec{b}, then |\vec{v}|$ is $|\vec{u}|$ b. $|\vec{u}| + |\vec{u} \cdot \vec{a}|$ c. $|\vec{u}| + |\vec{u} \cdot \vec{b}|$ d. $|\vec{u}| + \hat{u} \cdot |\vec{a} + \vec{b}|$

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430. The vertex A triangle ABC is on the line $\vec{r} = \hat{i} + \hat{j} + \lambda\hat{k}$ and the vertices $B and C$ have respective position vectors $\hat{i} and \hat{j}$. Let Delta be the area of the triangle and Delta$\left[3/2, \sqrt{33}/2\right]$. Then the range of values of λ

corresponding to A is a.$[-8, 4] \cup [4, 8]$ b. $[-4, 4]$ c. $[-2, 2]$ d. $[-4, -2] \cup [2, 4]$

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431. If a is real constant $A, BandC$ are variable angles and $\sqrt{a^2 - 4\tan A} + a\tan B + \sqrt{a^2 + 4\tan c} = 6a$, then the least vale of $\tan^2 A + \tan^2 b + \tan^2 C is\ 6$ b. 10 c. 12 d. 3

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432. The position vectors of the vertices $A, BandC$ of a triangle are three unit vectors $\vec{a}, \vec{b}, and\vec{c}$, respectively. A vector $\vec{d}$ is such that $\vec{d}\cdot\vec{a} = \vec{d}\cdot\vec{b} = \vec{d}\cdot\vec{c} and \vec{d} = \lambda\left(\vec{b} + \vec{c}\right)\cdot$ Then triangle ABC is a. acute angled b. obtuse angled c. right angled d. none of these

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433. Given that $\vec{a}, \vec{b}, \vec{p}, \vec{q}$ are four vectors such that $\vec{a} + \vec{b} = \mu\vec{p}, \vec{b} \cdot \vec{q} = 0 and \left|\vec{b}\right|^2 = 1, where \mu$ is a scalar. Then $\left|\left(\vec{a}\dot{\vec{q}}\right)\vec{p} - \left(\vec{p}\dot{\vec{q}}\right)\vec{a}\right|$ is equal to (a) $2\left|\vec{p}.\vec{q}\right|$ (b) $(1/2)\left|\vec{p}.\vec{q}\right|$ (c) $\left|\vec{p} \times \vec{q}\right|$ (d) $\left|\vec{p}.\vec{q}\right|$

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434. In fig. $AB, DEandGF$ are parallel to each other and $AD, BGandEF$ are parallel to each other. If $CD:CE = CG:CB = 2:1,$ then the value of area $(AEG):$ area (ABD) is equal to $7/2$ b. 3 c. 4 d. $9/2$

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435. In a quadrilateral $ABCD, \vec{A}C$ is the bisector of $\vec{A}Band\vec{A}D$, angle between $\vec{A}Band\vec{A}D$ is $2\pi/3$, $15\left|\vec{A}C\right| = 3\left|\vec{A}B\right| = 5\left|\vec{A}D\right|^{\cdot}$ Then the angle between $\vec{B}A and \vec{C}D$ is $\frac{\cos^{-1}(\sqrt{14})}{7\sqrt{2}}$ b. $\frac{\cos^{-1}(\sqrt{21})}{7\sqrt{3}}$ c. $\frac{\cos^{-1}2}{\sqrt{7}}$ d. $\frac{\cos^{-1}(2\sqrt{7})}{14}$

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436. Position vector $\hat{k}$ is rotated about the origin by angle 135^0 in such a way that the plane made by it bisects the angle between $\hat{i} and \hat{j}$. Then its new position is a.$\pm\frac{\hat{i}}{\sqrt{2}}\pm\frac{\hat{j}}{\sqrt{2}}$ b. $\pm\frac{\hat{i}}{2}\pm\frac{\hat{j}}{2}-\frac{\hat{k}}{\sqrt{2}}$ c. $\frac{\hat{i}}{\sqrt{2}}-\frac{\hat{k}}{\sqrt{2}}$ d. none of these

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437. A non-zero vector $\vec{a}$ is such that its projections along vectors $\frac{\hat{i}+\hat{j}}{\sqrt{2}}, \frac{-\hat{i}+\hat{j}}{\sqrt{2}}$ and $\hat{k}$ are equal, then unit vector along $\vec{a}$ is $\frac{\sqrt{2}\hat{j}-\hat{k}}{\sqrt{3}}$ b. $\frac{\hat{j}-\sqrt{2}\hat{k}}{\sqrt{3}}$ c. $\frac{\sqrt{2}}{\sqrt{3}}\hat{j}+\frac{\hat{k}}{\sqrt{3}}$ d. $\frac{\hat{j}-\hat{k}}{\sqrt{2}}$

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438. Let $\vec{a} = 2i + j + k, \vec{b} = i + 2j - k$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}$ is $\frac{1}{\sqrt{2}}(- j + k)$ b. $\frac{1}{\sqrt{3}}(- i - j - k)$ c. $\frac{1}{\sqrt{5}}(- k - 2j)$ d. $\frac{1}{\sqrt{3}}(i - j - k)$

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439. Let $\vec{a} = 2i + j - 2k and \vec{b} = i + \dot{j}$ If $\vec{c}$ is a vector such that $\vec{a}\dot{\vec{c}} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ between $\vec{a} \times \vec{b}$ and $\vec{c} is 30^0, then \left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right|$ I equal to $2/3$ b. $3/2$ c. 2 d. 3

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440. Let $ABCD$ be a tetrahedron such that the edges $AB, ACandAD$ are mutually perpendicular. Let the area of triangles $ABC, ACDandADB$ be 3, 4 and 5sq. units, respectively. Then the area of triangle BCD is $5\sqrt{2}$ b. 5 c. $\frac{\sqrt{5}}{2}$ d. $\frac{5}{2}$

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441. Vector $\vec{a}$ in the plane of $\vec{b} = 2\hat{i} + \hat{j} and \vec{c} = \hat{i} - \hat{j} + \hat{k}$ is such that it is equally inclined to $\vec{b} and \vec{d}$ where $\vec{d} = \hat{j} + 2\hat{k}$. The value of $\vec{a}$ is a. $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{2}}$ b. $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$ c. $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$ d. $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$

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442. If $\vec{a}, \vec{b} and \vec{c}$ are non-coplanar unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$, then the angle between $\vec{a} and \vec{b}$ is a. $3\pi/4$ b. $\pi/4$ c. $\pi/2$ d. π

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443. Let $\vec{u}, \vec{v} and \vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = 0$. If $\left|\vec{u}\right| = 3, \left|\vec{v}\right| = 4 and \left|\vec{w}\right| = 5$, then $\vec{u}\dot{}\vec{v} + \vec{v}\dot{}\vec{w} + \vec{w}\dot{}\vec{u}$ is 47 b. -25 c. 0 d. 25

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444. If $\vec{a}, \vec{b} and \vec{c}$ are three non-coplanar vectors, then $\left(\vec{a}+\vec{b}+\vec{c}\right)\cdot\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}+\vec{c}\right)$ equals 0 b. $\left[\vec{a}\vec{b}\vec{c}\right]$ c. $2\left[\vec{a}\vec{b}\vec{c}\right]$ d. $-\left[\vec{a}\vec{b}\vec{c}\right]$

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445. $\vec{p}, \vec{q}, and \vec{r}$ are three mutually perpendicular vectors of the same magnitude. If vector $\vec{x}$ satisfies the equation $\vec{p}\times\left(\left(\vec{x}-\vec{q}\right)\times\vec{p}\right)+\vec{q}\times\left(\left(\vec{x}-\vec{r}\right)\times\vec{q}\right)+\vec{r}\times\left(\left(\vec{x}-\vec{p}\right)\times\vec{r}\right)=0, then \vec{x}$ is given by a. $\frac{1}{2}\left(\vec{p}+\vec{q}-2\vec{r}\right)$ b. $\frac{1}{2}\left(\vec{p}+\vec{q}+\vec{r}\right)$ c. $\frac{1}{3}\left(\vec{p}+\vec{q}+\vec{r}\right)$ d. $\frac{1}{3}\left(2\vec{p}+\vec{q}-\vec{r}\right)$

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446. If vectors $b, candd$ are not coplanar, then prove that vector $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)+\left(\vec{a}\times\vec{c}\right)\times\left(\overrightarrow{dxx\vec{b}}\right)+\left(\vec{a}\times\vec{d}\right)\times\left(\vec{b}\times\vec{c}\right)$ is parallel to $\dot{\vec{a}}$

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447. The position vectors of the vertices A, B and C of a tetrahedron ABCD are $\hat{i}+\hat{j}+\hat{k}$, $\hat{k}$, $\hat{i}$ and $\hat{3}i$,respectively. The altitude from vertex D to the opposite face ABC meets the median line through Aof triangle ABC at a point E. If the length of the side AD is 4 and the volume of the tetrahedron is2/2/3, find the position vectors of the point E for all its possible positfons

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448. Consider three vectors $\vec{a}, \vec{b} and \vec{\cdot}$ Statement 1 $\vec{a} \times \vec{b} = \left(\left(\hat{i} \times \vec{a}\right)\dot{\vec{b}}\right)\hat{i} + \left(\left(\hat{j} \times \vec{a}\right)\dot{\vec{b}}\right)\hat{j} + \left(\left(\hat{k} \times \vec{a}\right)\dot{\vec{b}}\right)\hat{k}$ Statement 2: $\vec{c} = \left(\hat{i}\dot{\vec{c}}\right)\hat{i} + \left(\hat{j}\dot{\vec{c}}\right)\hat{j} + \left(\hat{k}\dot{\vec{c}}\right)\hat{k}$

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449. If $\vec{A}, \vec{B}$ and $\vec{C}$ are vectors such that $\left|\vec{B}\right| - \left|\vec{C}\right|$. Prove that $\left[\left(\vec{A} + \vec{B}\right) \times \left(\vec{A} + \vec{C}\right)\right] \times \left(\vec{B} + \vec{C}\right). \left(\vec{B} + \vec{C}\right) = 0$

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450. A parallelogram is constructed on $3\vec{a} + \vec{b} and \vec{a} - 4\vec{b}, where \left|\vec{a}\right| = 6 and \left|\vec{b}\right| = 8, and \vec{a} and \vec{b}$ are anti-parallel. Then the length of the longer diagonal is 40 b. 64 c. 32 d. 48

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451. Statement 1: Vector $\vec{c} = 5\hat{i} + 7\hat{j} + 2\hat{k}$ is along the bisector of angel between $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k} and \vec{b} = -8\hat{i} + \hat{j} - 4\hat{k}$. Statement 2: $\vec{c}$ is equally inclined to $\vec{a} and \vec{b}$.

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452. Statement 1: A component of vector $\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ in the direction perpendicular to the direction of vector $\vec{a} = \hat{i} + \hat{j} + \hat{k} is \hat{i} - \hat{j}$. Statement 2: A component of vector in the direction of $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ is $2\hat{i} + 2\hat{j} + 2\hat{k}$.

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453. Statement 1 : Points $A(1, 0), B(2, 3), C(5, 3), and D(6, 0)$ are concyclic. Statement 2 : Points $A, B, C, and D$ form an isosceles trapezium or $AB and CD$ meet at E. Then $EA.EB = EC.ED$.

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454. Let $\vec{r}$ be a non-zero vector satisfying $\vec{r}\dot{\vec{a}} = \vec{r}\dot{\vec{b}} = \vec{r}\dot{\vec{c}} = 0$ for given non-zero vectors $\vec{a}$, $\vec{b}and\vec{\cdot}$ Statement 1: $\left[\vec{a} - \vec{b}\vec{b} - \vec{c}\vec{c} - \vec{a}\right] = 0$ Statement 2: $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

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455. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}; \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}; \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}\&\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2 =$

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456. Statement 1: If $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}, \vec{B} = \hat{i} + \hat{j} - 2\hat{k}and\vec{C} = \hat{i} + 2\hat{j} + \hat{k}$, then $\left|\vec{A} \times \left(\vec{A} \times \left(\vec{A} \times \vec{B}\right)\right)\dot{\vec{C}}\right| = 243.$ Statement 2:

$$\left|\vec{A}\times\left(\vec{A}\times\left(\vec{A}\times\vec{B}\right)\right)\vec{C}\right| = \left|\vec{A}\right|^2\left|\left[\vec{A}\vec{B}\vec{C}\right]\right|$$

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457. Statement 1: $\vec{a}, \vec{b}, and\vec{c}$ are three mutually perpendicular unit vectors and $\vec{d}$ is a vector such that $\vec{a}, \vec{b}, \vec{c}and\vec{d}$ are non-coplanar. If $\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right] = 1, then\vec{d} = \vec{a} + \vec{b} + \vec{c}$ Statement 2: $\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right] \Rightarrow \vec{d}$ is equally inclined to $\vec{a}, \vec{b}, \vec{c}$.

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458. Let vectors $\vec{a}, \vec{b}, \vec{c}, and\vec{d}$ be such that $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right) = 0.$ Let P_1andP_2 be planes determined by the pair of vectors $\vec{a}, \vec{b}, and\vec{c}, \vec{d}$, respectively. Then the angle between P_1andP_2 is a.0 b. $\pi/4$ c. $\pi/3$ d. $\pi/2$

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459. The number of vectors of unit length perpendicular to vectors $\vec{a} = (1, 1, 0) and \vec{b} = (0, 1, 1)$ is a. one b. two c. three d. infinite

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460. For any two $\vec{a} and \vec{b}, \left(\vec{a} \times \hat{i}\right)\dot{\vec{b}} \times \hat{i} + \left(\vec{a} \times \hat{j}\right)\dot{\vec{b}} \times \hat{j} + \left(\vec{a} \times \hat{k}\right)\dot{\vec{b}} \times \hat{k}$ is always equal to $\dot{\vec{a}}\vec{b}$ b. $2\dot{\vec{a}}\vec{b}$ c. zero d. none of these

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461. Let $\vec{f(t)} = [t]\hat{i} + (t - [t])\hat{j} + [t + 1]\hat{k}, where[.]$ denotes the greatest integer function. Then the vectors $f\left(\frac{5}{4}\right) and \vec{f(t)}, 0 < t < i$ are(a) parallel to each other(b) perpendicular(c) inclined at $\cos^{-1}2\left(\sqrt{7\left(1 - t^2\right)}\right)$ (d)inclined at $\cos^{-1}\left(\frac{8 + t}{\sqrt{1 + t^2}}\right)$;

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462. If $\vec{a}$ is parallel to $\vec{b} \times \vec{c}$, then $\left(\vec{a} \times \vec{b}\right)\dot{\vec{a}} \times \vec{c}$ is equal to $|\vec{a}|^2\left(\vec{b}\dot{\vec{c}}\right)$ b. $\left|\vec{b}\right|^2\left(\vec{a}\dot{\vec{c}}\right)$ c. $|\vec{c}|^2\left(\vec{a}\dot{\vec{b}}\right)$ d. none of these

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463. The three vectors $\hat{i} + \hat{j}, \hat{j} + \hat{k}, \hat{k} + \hat{i}$ taken two at a time form three planes, The three unit vectors drawn perpendicular to these planes form a parallelopiped of volume:

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464. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is non-zero vector and $\left|\left(\vec{d} \cdot \vec{c}\right)\left(\vec{a} \times \vec{b}\right) + \left(\vec{d} \cdot \vec{a}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{d} \cdot \vec{b}\right)\left(\vec{c} \times \vec{a}\right)\right| = 0,$ then a.

$\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$ b. $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = |d|$ c. $\vec{a}, \vec{b}, and \vec{c}$ are coplanar d. none of these

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465. If $|a| = 2 and |b| = 3$ and $a\dot{}b = 0, then (a \times (a \times (a \times (a \times b))))$ is equal to $48\hat{b}$ b. $-48\hat{b}$ c. $48\hat{a}$ d. $-48\hat{a}$

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466. If the two diagonals of one its faces are $6\hat{i} + 6\hat{k} and 4\hat{j} + 2\hat{k}$ and of the edges not containing the given diagonals is $c = 4\hat{j} - 8\hat{k},$ then the volume of a parallelepiped is 60 b. 80 c. 100 d. 120

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467. The volume of a tetrahedron formed by the coterminous edges $\vec{a}, \vec{b}, and \vec{c}$ is 3. Then the volume of the parallelepiped formed by the coterminous edges $\vec{a}+\vec{b}, \vec{b}+\vec{c} and \vec{c}+\vec{a}$ is 6 b. 18 c. 36 d. 9

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468. If $\vec{a}, \vec{b}, \text{ and } \vec{c}$ are three mutually orthogonal unit vectors, then the triple product $\left[\vec{a}+\vec{b}+\vec{c}\vec{a}+\vec{b}\vec{b}+\vec{c}\right]$ equals: (a.) 0 (b.) $1 \text{ or } -1$ (c.) 1 (d.) 3

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469. Vector $\vec{c}$ is perpendicular to vectors $\vec{a}=(2,-3,1) and \vec{b}=(1,-2,3)$ and satisfies the condition $\vec{\cdot}\left(\hat{i}+2\hat{j}-7\hat{k}\right)=10.$ Then vector $\vec{c}$ is equal to a.$(7, 5, 1)$ b. $-7, -5, -1$ c. $1, 1, -1$ d. none of these

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470. Given $\vec{a}=x\hat{i}+y\hat{j}+2\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+2\hat{j}; \vec{a}\perp\vec{b}, \vec{a}\vec{c}=4.$ Then $\left[\vec{a}\vec{b}\vec{c}\right]^2=|\vec{a}|$ b. $\left[\vec{a}\vec{b}\vec{c}\right]^{=}|\vec{a}|$ c. $\left[\vec{a}\vec{b}\vec{c}\right]^{=}0$ d. $\left[\vec{a}\vec{b}\vec{c}\right]^{=}|\vec{a}|^2$

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471. $\vec{a} and \vec{b}$ are two unit vectors that are mutually perpendicular. A unit vector that is equally inclined to $\vec{a}, \vec{b} and \vec{a} \times \vec{b}$ is a. $\frac{1}{\sqrt{2}}\left(\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)$ b. $\frac{1}{2}\left(\vec{a}\times\vec{b}+\vec{a}+\vec{b}\right)$ c. $\frac{1}{\sqrt{3}}\left(\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)$ d. $\frac{1}{3}\left(\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)$

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472. If $\vec{r} and \vec{s}$ are non-zero constant vectors and the scalar b is chosen such that $\left|\vec{r}+b\vec{s}\right|$ is minimum, then the value of $\left|b\vec{s}\right|^2+\left|\vec{r}+b\vec{s}\right|^2$ is equal to a. $2\left|\vec{r}\right|^2$ b. $\left|\vec{r}\right|^2/2$ c. $3\left|\vec{r}\right|^2$ d. $|r|^2$

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473. The scalar $\vec{A}\left(\vec{B}+\vec{C}\right)\times\left(\vec{A}+\vec{B}+\vec{C}\right)$ equals a. 0 b. $\left[\vec{A}\vec{B}\vec{C}\right]+\left[\vec{B}\vec{C}\vec{A}\right]$ c. $\left[\vec{A}\vec{B}\vec{C}\right]$ d. none of these

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474. The volume of he parallelepiped whose sides are given by $\vec{O}A = 2i - 2, j, \vec{O}B = i + j - kand\vec{O}C = 3i - k$ is a. $\frac{4}{13}$ b. 4 c. $\frac{2}{7}$ d. 2

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475. For non-zero vectors $\vec{a}, \vec{b}, and\vec{c}$, $\left|\left(\vec{a} \times \vec{b}\right)\dot{\vec{c}}\right| = |\vec{a}||\vec{b}||\vec{c}|$ holds if and only if a. $\vec{a} \cdot \vec{b} = 0, \vec{b} \cdot \vec{c} = 0$ b. $\vec{b} \cdot \vec{c} = 0, \vec{c} \cdot \vec{a} = 0$ c. $\vec{c} \cdot \vec{a} = 0, \vec{a} \cdot \vec{b} = 0$ d. $\vec{a} \cdot \vec{b} = 0, \vec{b} \cdot \vec{c} = 0, \vec{c} \cdot \vec{a} = 0$

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476. For three vectors $\vec{u}, \vec{v}and\vec{w}$ which of the following expressions is not equal to any of the remaining three ? a. $\dot{\vec{u}}\vec{v} \times \vec{w}$ b. $\left(\vec{v} \times \vec{w}\right)\dot{\vec{u}}$ c. $\dot{\vec{v}}\vec{u} \times \vec{w}$ d. $\left(\vec{u} \times \vec{v}\right)\dot{\vec{w}}$

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477. Let $\vec{A}$ be a vector parallel to the line of intersection of planes $P_1 and P_2$. Plane P_1 is parallel to vectors $2\hat{j} + 3\hat{k} and 4\hat{j} - 3k and P_2$ is parallel to $\hat{j} - \hat{k} and 3\hat{i} + 3\hat{j}$. Then the angle betweenvector $\vec{A}$ and a given vector $2\hat{i} + \hat{j} - 2\hat{k}$ is a.$\pi/2$ b. $\pi/4$ c. $\pi/6$ d. $3\pi/4$

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478. If $\vec{a}.\vec{b} = \beta and \vec{a} \times \vec{b} = \vec{c}, then \vec{b}$ is $\frac{\left(\beta\vec{a} - \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$ b. $\frac{\left(\beta\vec{a} + \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$ c. $\frac{\left(\beta\vec{c} - \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$ d. $\frac{\left(\beta\vec{a} + \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$

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479. $\vec{b} and \vec{c}$ are unit vectors. Then for any arbitrary vector $\vec{a}, \left(\left(\left(\vec{a} \times \vec{b}\right) + \left(\vec{a} \times \vec{c}\right)\right) \times \left(\vec{b} \times \vec{c}\right)\right)\vec{b} - \vec{c}$ is always equal to a.$|\vec{a}|$ b. $\frac{1}{2}|\vec{a}|$

c. $\frac{1}{3}|\vec{a}|$ d. none of these

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480. Let $\vec{a} and \vec{b}$ be mutually perpendicular unit vectors. Then for any arbitrary $\vec{r}$, a. $\vec{r} = \left(\vec{r}\dot{\hat{a}}\right)\hat{a} + \left(\vec{r}\dot{\hat{b}}\right)\hat{b} + \left(\vec{r}\dot{\hat{a}} \times \hat{b}\right)\left(\hat{a} \times \hat{b}\right)$ b. $\vec{r} = \left(\vec{r}\dot{\hat{a}}\right) - \left(\vec{r}\dot{\hat{b}}\right)\hat{b} - \left(\vec{r}\dot{\hat{a}} \times \hat{b}\right)\left(\hat{a} \times \hat{b}\right)$ c. $\vec{r} = \left(\vec{r}\dot{\hat{a}}\right)\hat{a} - \left(\vec{r}\dot{\hat{b}}\right)\hat{b} + \left(\vec{r}\dot{\hat{a}} \times \hat{b}\right)\left(\hat{a} \times \hat{b}\right)$ none of these

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481. Value of $\left[\vec{a} \times \vec{b}\vec{a} \times \vec{c}\vec{d}\right]$ is always equal to $\left(\vec{a}\dot{\vec{d}}\right)\left[\vec{a}\vec{b}\vec{c}\right]$ b. $\left(\vec{a}\dot{\vec{c}}\right)\left[\vec{a}\vec{b}\vec{d}\right]$ c. $\left(\vec{a}\dot{\vec{b}}\right)\left[\vec{a}\vec{b}\vec{d}\right]$ d. none of these

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482. Let $\vec{a} and \vec{b}$ be unit vectors that are perpendicular to each other. Then $\left[\vec{a} + \left(\vec{a} \times \vec{b}\right)\vec{b} + \left(\vec{a} \times \vec{b}\right)\vec{a} \times \vec{b}\right]$ will always be equal to 1 b. 0 c. -1 d. none of these

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483. Let $\vec{r}, \vec{a}, \vec{b} and \vec{c}$ be four nonzero vectors such that $\vec{r}\dot{\vec{a}} = 0, \left|\vec{r} \times \vec{b}\right| = \left|\vec{r}\right|\left|\vec{b}\right| and \left|\vec{r} \times \vec{c}\right| = \left|\vec{r}\right|\left|\vec{c}\right|^{\cdot}$ Then $[abc]$ is equal to $|a||b||c|$ b. -$|a||b||c|$ c. 0 d. none of these

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484. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} and \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three nonzero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a} and \vec{b}^{\cdot}$ If the angle between $\vec{a} and \vec{b}$ is $\pi/6$, then the value of

$\left|a_1b_1c_1a_2b_2c_2a_3b_3c_3\right|$ is a.0 b. 1 c. $\frac{1}{4}(a12 + a22 + a32)(b12 + b22 + b32)$ d. $\frac{3}{4}(a12 + a22 + a32)(b12 + b22 + b32)$

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485. If $4\vec{a} + 5\vec{b} + 9\vec{c} = 0,$ then $\left(\vec{a} \times \vec{b}\right) \times \left[\left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right)\right]$ is equal to a vector perpendicular to the plane of a, b, c b. a scalar quantity c. $\vec{0}$ d. none of these

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486. If $\vec{a}, \vec{b}, and \vec{c}$ are such that $\left[\vec{a}\vec{b}\vec{c}\right] = 1, \vec{c} = \lambda\vec{a} \times \vec{b},$ angle, between $\vec{a} and \vec{b}$ is $\frac{2\pi}{3}, \left|\vec{a}\right| = \sqrt{2}, \left|\vec{b}\right| = \sqrt{3} and \left|\vec{c}\right| = \frac{1}{\sqrt{3}}$, then the angel between $\vec{a} and \vec{b}$ is $\frac{\pi}{6}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{3}$ d. $\frac{\pi}{2}$

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487. A vector of magnitude $\sqrt{2}$ coplanar with the vecrtor $\vec{a} = \hat{i} + \hat{j} + 2\hat{k} and \vec{b} = \hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\vec{c} = \hat{i} + \hat{j} + \hat{k}$, is a.$-\hat{j} + \hat{k}$ b. $\hat{i} - \hat{k}$ c. $\hat{i} - \hat{j}$ d. $\hat{i} - \hat{j}$

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488. Let P be a point interior to the acute triangle ABC If $PA + PB + PC$ is a null vector, then w.r.t traingel ABC, point P is its a. centroid b. orthocentre c. incentre d. circumcentre

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489. G is the centroid of triangle $ABCandA_1andB_1$ are rthe midpoints of sides $ABandAC$, respectively. If Delta_1 is the area of quadrilateral $GA_1AB_1and\mathrm{Delta}$ is the area of triangle ABC, then $\mathrm{Delta}/\mathrm{Delta}_1$ is equal to a.$\frac{3}{2}$ b. 3 c. $\frac{1}{3}$ d. none of these

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490. Points $\vec{a}, \vec{b}, \vec{c}, and \vec{d}$ are coplanar and $(\sin\alpha)\vec{a} + (2\sin2\beta)\vec{b} + (3\sin3\gamma)\vec{c} - \vec{d} = 0.$ Then the least value of $\sin^2\alpha + \sin^2 2\beta + \sin^2 3\gamma is$ a. $\frac{1}{14}$ b. 14 c. 6 d. $1/\sqrt{6}$

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491. If $\vec{a} and \vec{b}$ are any two vectors of magnitudes 1 and 2, respectively, and $\left(1 - 3\vec{a}\dot{\vec{b}}\right)^2 + \left|2\vec{a} + \vec{b} + 3\left(\vec{a} \times \vec{b}\right)\right|^2 = 47,$ then the angel between $\vec{a} and \vec{b}$ is $\pi/3$ b. $\pi - \cos^{-1}(1/4)$ c. $\frac{2\pi}{3}$ d. $\cos^{-1}(1/4)$

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492. If $\vec{a} and \vec{b}$ are any two vectors of magnitudes 2 and 3, respectively, such that $\left|2\left(\vec{a} \times \vec{b}\right)\right| + \left|3\left(\vec{a}\dot{\vec{b}}\right)\right| = k,$ then the maximum value of k is $\sqrt{13}$

b. $2\sqrt{13}$ c. $6\sqrt{13}$ d. $10\sqrt{13}$

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493. $\vec{a}, \vec{b} and \vec{c}$ are unit vectors such that $\left|\vec{a} + \vec{b} + 3\vec{c}\right| = 4.$ Angle between $\vec{a} and \vec{b} is \theta_1,$ between $\vec{b} and \vec{c}$ is θ_2 and between $\vec{a} and \vec{c}$ varies $[\pi/6, 2\pi/3]$. Then the maximum of $\cos\theta_1 + 3\cos\theta_2 is$ 3 b. 4 c. $2\sqrt{2}$ d. 6

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494. If the vector product of a constant vector $\vec{O}A$ with a variable vector $\vec{O}B$ in a fixed plane OAB be a constant vector, then the locus of B is a. a straight line perpendicular to $\vec{O}A$ b. a circle with centre O and radius equal to $\left|\vec{O}A\right|$ c. a straight line parallel to $\vec{O}A$ d. none of these

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495. Let $\vec{u}, \vec{v} and \vec{w}$ be such that $|\vec{u}| = 1, |\vec{v}| = 2 and |\vec{w}| = 3.$ If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v} and \vec{w}$ are perpendicular to each other, then $|\vec{u} - \vec{v} + \vec{w}|$ equals 2 b. $\sqrt{7}$ c. $\sqrt{14}$ d. 14

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496. If the two adjacent sides of two rectangles are represented by vectors $\vec{p} = 5\vec{a} - 3\vec{b}; \vec{q} = -\vec{a} - 2\vec{b}$ and $\vec{r} = -4\vec{a} - \vec{b}; \vec{s} = -\vec{a} + \vec{b},$ respectively, then the angel between the vector $\vec{x} = \frac{1}{3}(\vec{p} + \vec{r} + \vec{s})$ and $\vec{y} = \frac{1}{5}(\vec{r} + \vec{s})$ is a.$\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ b. $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ c. $\pi - \cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ d. cannot be evaluate

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497. Let P, Q, R and S be the points on the plane with position vectors $-2i - j, 4i, 3i + 3j and - 3j + 2j,$ respectively. The quadrilateral $PQRS$ must be a Parallelogram, which is neither a rhombus nor a rectangle Square Rectangle, but not a square Rhombus, but not a square

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498. $\vec{u}, \vec{v} and \vec{w}$ are three non-coplanar unit vecrtors anf $\alpha, \beta and \gamma$ are the angles between $\vec{u} and \vec{v}, \vec{v} and \vec{w}, and \vec{w} and \vec{u},$ respectively, and $\vec{x}, \vec{y} and \vec{z}$ are unit vectors along the bisectors of the angles $\alpha, \beta and \gamma$, respectively. Prove that $\left[\vec{\times} x\vec{y}\vec{y} \times \vec{z}\vec{z} \times \vec{x}\right] = \frac{1}{16}\left[\vec{u}\vec{v}\vec{w}\right]^2 \frac{\sec^2\alpha}{2}\frac{\sec^2\beta}{2}\frac{\sec^2\gamma}{2}\cdot$

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499. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}; \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, . \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ and $\left[3\vec{a} + \vec{b} \ \ 3\vec{b} + \vec{c} \ \ 3\vec{c} + \vec{a}\right] = 28\left[\vec{a}\vec{b}\vec{c}\right]$, then find the value of $\frac{\lambda}{4}$.

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500. Find the absolute value of parameter t for which the area of the triangle whose vertices the $A(-1, 1, 2); B(1, 2, 3) and C(t, 1, 1)$ is minimum.

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501. The condition for equations $\vec{r} \times \vec{a} = \vec{b} and \vec{r} \times \vec{c} = \vec{d}$ to be consistent is $\vec{b}.\vec{c} = \vec{a}.\vec{d}$ b. $\vec{a}.\vec{b} = \cdot \vec{d}$ c. $\vec{b}.\vec{c} + \vec{a}.\vec{d} = 0$ d. $\vec{a}.\vec{b} + \cdot \vec{d} = 0$

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502. If $a and b$ are nonzero non-collinear vectors, then $\left[\vec{a}\vec{b}\hat{i}\right]\hat{i} + \left[\vec{a}\vec{b}\hat{j}\right]\hat{j} + \left[\vec{a}\vec{b}\hat{k}\right]\hat{k}$ is equal to $\vec{a} + \vec{b}$ b. $\vec{a} \times \vec{b}$ c. $\vec{a} - \vec{b}$ d. $\vec{b} \times \vec{a}$

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503. If $\vec{r}\cdot\vec{a} = \vec{r}\cdot\vec{b} = \vec{r}\cdot\vec{c} = \frac{1}{2}$ or some nonzero vector $\vec{r}$, then the area of the triangle whose vertices are $A(\vec{a}), B(\vec{b}) and C(\vec{c}) is (\vec{a}, \vec{b}, \vec{c}$ are non-coplanar) $\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$ b. $|\vec{r}|$ c. $\left|\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}\right|$ d. none of these

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504. A vector of magnitude 10 along the normal to the curve $3x^2 + 8xy + 2y^2 - 3 = 0$ at its point $P(1, 0)$ can be (A) $6\hat{i} + 8\hat{j}$ (B) $-8\hat{i} + 3\hat{j}$ (C) $6\hat{i} - 8\hat{j}$ (D) $8\hat{i} + 6\hat{j}$

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505. If $a\left(\vec{\alpha} \times \vec{\beta}\right) + b\left(\vec{\beta} \times \vec{\gamma}\right) + c\left(\vec{\gamma} \times \vec{\alpha}\right) = 0$ and at least one of $a, b and c$ is nonzero, then vectors $\vec{\alpha}, \vec{\beta} and \vec{\gamma}$ are a. parallel b. coplanar c. mutually perpendicular d. none of these

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506. If $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)=\vec{b}, where \vec{a}, \vec{b}, and \vec{c}$ are nonzero vectors, then $\vec{a}, \vec{b}, and \vec{c}$ can be coplanar $\vec{a}, \vec{b}, and \vec{c}$ must be coplanar $\vec{a}, \vec{b}, and \vec{c}$ cannot be coplanar none of these

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507. If $\vec{a}, \vec{b}, \vec{c}$ are any three noncoplanar vector, then the equaltion $\left[\vec{b}\times\vec{c}\vec{c}\times\vec{a}\vec{a}\times\vec{b}\right]x^2+\left[\vec{a}+\vec{b}\vec{b}+\vec{c}\vec{c}+\vec{a}\right]x+1+\left[\vec{b}-\vec{c}\vec{c}-\vec{a}\vec{a}-\vec{b}\right]=0$ has roots a. real and distinct b. real c. equal d. imaginary

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508. If $\vec{x}+\vec{c}\times\vec{y}=\vec{a} and \vec{y}+\vec{c}\times\vec{x}=\vec{b}, where \vec{c}$ is a nonzero vector, then which of the following is not correct? $\vec{x}=\dfrac{\vec{b}\times\vec{c}+\vec{a}+\left(\vec{\cdot}\,\vec{a}\right)\vec{c}}{1+\vec{\cdot}\,\vec{c}}$ b. $\vec{x}=\dfrac{\vec{c}\times\vec{b}+\vec{b}+\left(\vec{\cdot}\,\vec{a}\right)\vec{c}}{1+\vec{\cdot}\,\vec{c}}$ c. $\vec{y}=\dfrac{\vec{a}\times\vec{c}+\vec{b}+\left(\vec{\cdot}\,\vec{b}\right)\vec{c}}{1+\vec{\cdot}\,\vec{c}}$ d. none of these

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509. If $\vec{a} and \vec{b}$ are two unit vectors incline at angle $\pi/3$, then

$\left\{\vec{a} \times \left(\vec{b} + \vec{a} \times \vec{b}\right)\right\}\dot{\vec{b}}$ is equal to $\frac{-3}{4}$ b. $\frac{1}{4}$ c. $\frac{3}{4}$ d. $\frac{1}{2}$

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510. If $\vec{a}$ and $\vec{b}$ are orthogonal unit vectors, then for a vector $\vec{r}$ noncoplanar with $\vec{a}$ and $\vec{b}$, vector $r \times a$ is equal to a. $\left[\vec{r}\vec{a}\vec{b}\right]\vec{b} - \left(\vec{r}.\vec{b}\right)\left(\vec{b} \times \vec{a}\right)$ b. $\left[\vec{r}\vec{a}\vec{b}\right]\left(\vec{a} + \vec{b}\right)$ c. $\left[\vec{r}\vec{a}\vec{b}\right]\vec{a} - \left(\vec{r}.\vec{a}\right)\vec{a} \times \vec{b}$ d. none of these

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511. Let V be the volume of the parallelepiped formed by the vectors $\vec{a} = a_i\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. If

a_r, b_r and c r, where r = 1, 2, 3, are non-negative real numbers and

$\sum_{r=1}^{3}\left(a_r + b_r + c_r\right) = 3L$ show that $V \le L^3$

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512. Find 3-dimensional vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ satisfying

$\vec{v}_1 \cdot \vec{v}_1 = 4, \vec{v}_1 \cdot \vec{v}_2 = -2, \vec{v}_1 \cdot \vec{v}_3 = 6, \vec{v}_2 \cdot \vec{v}_2 = 2, \vec{v}_2 \cdot \vec{v}_3 = -5, \vec{v}_3 \cdot \vec{v}_3 = 29$

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513. Let $\vec{u} and \vec{v}$ be unit vectors such that $\vec{u} \times \vec{v} + \vec{u} = \vec{w}$ and $\vec{w} \times \vec{u} = \dot{\vec{v}}$

Find the value of $\left[\vec{u}\vec{v}\vec{w}\right]^{\cdot}$

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514. For any two vectors $\vec{u} and \vec{v}$ prove that $\left(\vec{u}\dot{}\vec{v}\right)^2 + \left|\vec{u} \times \vec{v}\right|^2 = \left|\vec{u}\right|^2\left|\vec{v}\right|^2$

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515. If the incident ray on a surface is along the unit vector $\vec{v}$, the reflected ray is along the unit vector $\vec{w}$ and the normal is along the unit vector $\vec{a}$ outwards, express $\vec{w}$ in terms of $\vec{a}$ and $\vec{v}$

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516. If $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are distinct vectors such that $\vec{a} \times \vec{c} = \vec{b} \times \vec{d} and \vec{a} \times \vec{b} = \vec{c} \times \vec{d}$, prove that $\left(\vec{a} - \vec{d}\right) \cdot \vec{b} - \vec{c} \neq 0$,

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517. Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k} and \vec{W} = \hat{i} + 3\hat{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[UVW]$ is -1 b. $\sqrt{10} + \sqrt{6}$ c. $\sqrt{59}$ d. $\sqrt{60}$

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518. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar and l,m,n are distinct real numbers, then $[(l\vec{a} + m\vec{b} + n\vec{c})(l\vec{b} + m\vec{c} + n\vec{a})(l\vec{c} + m\vec{a} + n\vec{b})]$ = 0, implies (A) lm+mn+nl = 0 (B) l+m+n = 0 (C) $l^2 + m^2 + n^2 = 0$

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519. If $\vec{a}, \vec{b} and \vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a} - \vec{b} 2\vec{b} - \vec{c} 2\vec{c} - \vec{a}\right]$ is 0 b. 1 c. $-\sqrt{3}$ d. $\sqrt{3}$

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