MATHS

BOOKS - JEE ADVANCED PREVIOUS YEAR

JEE (ADVANCED) 2020

Section 1

1. For a complex z , let Re(z) denote the real part of z . Let S be the set of all complex numbers z satisfying $z^4 - |z|^4 = 4iz^2$, where $I = \sqrt{-1}$. Then the minimum possible value of $|z_1 - z_2|^2$ where$z_1, z_2 \in S$ with Re $(z_1) > 0$ and $Re(z_2) < 0$, is ....

2. The probability that a missile hits a target successfully is 0.75 . In order to destroy the target completely , at least three successful hits are required . Then the minimum number of missiles that have to be fired so that the probability of completely destroying teh target is NOT less than 0.95 , is ....

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3. Let O be the centre of the circle $x^2 + y^2 = r^2$ where $r > \frac{\sqrt{5}}{2}$. Suppose PQ is a chord of this circle and the equation of the line passing through P and Q is $2x + 4y = 5$, If the centre of the circumcircle of the triangle OPQ lies on the line $x + 2y = 4$, then the value of r is ....

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4. The trace of a square matrix is defined to be the sum of its diagonal entries . If A is 2×2 matrix such that the trace of A is 3 and the trace of A^3 is -18 , then the value of the determinant of A is . . . .

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5. Let the functions : $(-1, 1) \rightarrow R$ and $g:(-1, 1) \rightarrow (-1, 1)$ be defined by $f(x) = |2x - 1| + |2x + 1|$ and $g(x) = x - [x]$ where [x] denotes the greatest integar less than or equal to x, Let $f \circ g(-1, 1) \rightarrow R$ be the composite function defined by $(f \circ g)(x) = f(g(x))$. Suppose c is the number of points in the interval $(-1, 1)$ at which $f \circ g$ is NOT continuous , and suppose d is the number of points in the interval $(-1, 1)$ at which $f \circ g$ is NOT differentiable . Then the value of c + is _____

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6. The value of the limit

$$\lim_{x \rightarrow \frac{\pi}{2}} (4\sqrt{2}) \frac{\sin 3x + \sin x}{\left(2 \sin 2x \sin. \frac{3x}{2} + \cos. \frac{5x}{2}\right) - \left(\sqrt{2} + \sqrt{2} \cos 2x + \cos. \frac{3x}{2}\right)}$$

is ______

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Section 2

1. Let b be a nonzero real number , Suppose $f : R \to R$ is differentiable function such that f(0) = 1 . If the derivative f' of f satisfies the equation

$$f'(x) = \frac{f(x)}{b^2 + x^2}$$

for all $x \in R$, then which of the following statements is/are True ?

A. If $b > 0$ then f is an increasing function

B. If $b < 0$ then f is a decreasing function

C. $f(x)f(-x) = 1$ for all $x \in \mathbb{R}$

D. $f(x) - f(-x) = 0$ for all $x \in \mathbb{R}$

Answer: A::C

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2. Let a and b be positive real numbers such that $a > 1$ and $b < a$. Let be a point in the first quadrant that lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Suppose the tangent to the hyperbola at P passes through the oint (1,0) and suppose the normal to the hyperbola at P cuts off equal intercepts on the coordinate axes . Let Δ denote the area of the triangle formed by the tangent at P , the normal at P and the x - axis . If a denotes the eccentricity of the hyperbola , then which of the following statements is/are TRUE ?

A. $1 < e < \sqrt{2}$

B. $\sqrt{2} < e < 2$

C. $\Delta = a^4$

D. $\Delta = b^4$

Answer: A::D

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3. Le $f: R \rightarrow R$ and $g: R \rightarrow R$ be functions satisfying $f(x+y) = f(x) + f(y) + f(x)f(y)$ and $f(x) = xg(x)$ for all $x, y \in R. If \lim_{x \rightarrow 0} g(x) = 1$, then which of the following statements is/are TRUE ?

A. f is differentiable at every $x \in R$

B. If $g(0) = 1$, then g is differentiable at every $x \in R$

C. The derivative f(1) is equal to 1

D. The derivative f'(0) is equal to 1

Answer: A::B::D

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4. Let $\alpha, \beta, \gamma, \delta$ be real numbers such that $\alpha^2 + \beta^2 + \gamma^2 \neq 0$ and $\alpha + \gamma = 1$. Supose the point $(3, 2, -1)$ is the mirror image of the point $(1, 0, -1)$ with respect to the planet

$\alpha x + \beta y + \gamma z = \delta$. Then which of the following statements is/are TRUE ?

A. $\alpha + \beta = 2$

B. $\delta - \gamma = 3$

C. $\delta + \beta = 4$

D. $\alpha + \beta + \gamma = \delta$

Answer: A::B::C

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5. Let a and b be positive real numbers . Suppose $\overrightarrow{PQ} = a\hat{i} + b\hat{j}$ and $\overrightarrow{PS} = a\hat{i} - b\hat{j}$ are adjacent sides of a parallelofram PQRS . Let $\overrightarrow{u}$ and $\overrightarrow{v}$ be the vectors of $\overrightarrow{w} = \hat{i} + \hat{j}$ along $\overrightarrow{PQ}$ and $\overrightarrow{PS}$ respectively . If $\left|\overrightarrow{u}\right| + \left|\overrightarrow{v}\right| = \left|\overrightarrow{w}\right|$ and if the area of the parallelogram PQRS is 8 , then which of the following statements is/are TRUE ?

A. $a + b = 4$

B. $a - b = 2$

C. The length of the diagonal P R of the parallelogram PQRS is 4

D. $\vec{w}$ is an angle of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PS}$

Answer: A::C

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6. For nonnegative integers s and r , let

$$\binom{s}{r} = \frac{s!}{r!(s-r)!} \quad \text{if } r \leq g \quad \text{if } > s \quad 0$$

For positive integers m and n , let

$$g(m, n) = \sum_{p=0}^{m+n} \frac{f(m, n, p)}{\binom{n+p}{\text{p}}}$$

where for nay nonnegative integer p ,

$$f(m, n, p) = \sum_{i=0}^{p} \binom{n+i}{i}, \binom{n+i}{p}, \binom{p+n}{p-i}$$ Then which of the following statements is/are TRUE ?

A. g(m,n) = g(n,m) for all positive integers m,n

B. $g(m+n+1) = g(m+1, n)$ for all positive integers m,n

C. g(2m,2n) = 2g(m,n) for all positive integers m,n

D. (2m,2n) = $(g(m, n))^2$ for all positive integers m,n

Answer: A::B::D

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Section 3

1. An engineer is required to visit a factroy for exactly four days during the first 15 days of every month and it I mandatory that no two visits take on consecutive days . Then the number of all possible ways in which such visits to the factory can be made by the engineer during 1- 15 June 2021 is

______

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2. In a hotel , four rooms are available , Six persons are to be accommodated in these four rooms in such a way that each of these rooms contains at least one person and at most two persons . Then the number of all possible ways in which this can be done is _____

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3. Two fair dice , each with faces numbered 1,2,3,4,5 and 6 are rolled together and the sum of the numbers on the faces is observed . This process is repeated till sum is either a prime number or a perfect square . Suppose the sum turns out to be a parfect square before it turns out to be a prime number , If is the probability that this perfect square is an odd number , then the value of 14p , is _______

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4. Let the functions $f: [0, 1] \rightarrow R$ be defined by

$f(x) = \dfrac{4^x}{4^x + 2}$ Then the value of

$f\left(\frac{1}{40}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{40}\right)+\ldots.+\left(\frac{39}{40}\right)-f\left(\frac{1}{2}\right)$ is ______

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5. Let $R \rightarrow R$ be a differentiable functions such that its derivative f' is continuous and $f(\pi)=-6$. If F : $[0, \pi] \rightarrow R$ is defined by $F(x) \int_0^x f(t) dt, \text{ and } \int_0^x f(t) dt \text{ and } \text{ if } \int_0^\pi (f'(x)+F(x)) \cos x dx = 2$

then the value of f(0) is _______

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6. Let the function $f:(0, \pi) \rightarrow \mathbb{R}$ be defined by $f\Big(\theta=(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^4$ Suppose the function f has a local minimum at θ precisely when $\theta \in \{\lambda_1, \pi, \ldots., \lambda_r \pi\}$, where $0<\lambda_1<\ldots.<\lambda_r<1$. Then the value of $\lambda_1+\ldots.+\lambda_r$ is ______

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