MATHS

BOOKS - JEE ADVANCED PREVIOUS YEAR

JEE (ADVANCED) 2020

Section 1

1. Suppose a , b denote the distinct real roots of the quadrtic polynomial $x^2 + 20x - 2020$ and suppose c,d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$, then the value of $ac(a - c) + ad(a - d)bc(b - c)bd(b - d)$ is

A. 0

B. 8000

C. 8080

D. 16000

Answer: D

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2. If the function $F: R \to R$ is defined by $f(x) = |x|(x - \sin x)$, then which o f the following statemnts is true ?

A. f is one-one but NOT onto

B. f is onto but NOT one-one

C. fis BOT one- one and onto

D. f is NEITHER one-one NOR onto

Answer: C

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3. Let the function $: R \rightarrow R$ and $g: R \rightarrow R$ be defined by

$f = (x)^{x1} - e^{x|1}$ and $g(x) = \frac{1}{2}\left(e^{x-1} + e^{1x}\right)$

Then the area f the region in the first quarant bounded by the curves $g = f(x), y = g(x)$ and x=0n is

A. $(2 - \sqrt{3}) + \frac{1}{2}\left(e - e^{1}\right)$

B. $(2 + \sqrt{3}) + \frac{1}{2}\left(e - e^{1}\right)$

C. $(2 - \sqrt{3}) + \frac{1}{2}\left(e + e^{1}\right)$

D. $(2+\sqrt{3})+\frac{1}{2}(e+e^{1})$

Answer: A

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4. Let a, b and λ positive real numbers. Suppoose P is an end point of the latus rectum of the parabola $y^2=4\lambda x$ and suppoose the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes thorugh the point. P. if the tangents to the parabola and the ellipse at the point P are perpendicular to each other, then the eccentricity of the ellipse is

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{2}{5}$

Answer: A

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5. Let C_1 and C_2 be two biased coins such that the probabilities of getting heat in a single toss are $\frac{2}{3}$ and $\frac{1}{3}$, respectively. Suppose α is the number of hed that appear when C_1 is tossed twice. Independently. Then the probability that the roots of the quadratic polynomil $x^2 - ax + \beta$ are real equals, is

A. $\frac{40}{81}$

B. $\frac{20}{81}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: B

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6. consider all rectangles lying in the region $\left\{(x, y) \in R \times R: 0 \leq x \leq \frac{\pi}{2} \text{ and } 0 \leq 2\sin(2x)\right\}$ and having one side on the x-axis. The area of the rectangle which has the maximum perimeter among all such rectangles. Is

A. $\frac{3\pi}{2}$

B. π

C. $\frac{\pi}{2\sqrt{3}}$

D. $\frac{\sqrt{3}}{2}$

Answer: C

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Section 2

1. Let the function $f: R \to R$ be defined by $f(x) = x^3 - x^2 + (x-1)\sin x$ and let$g: R \to R$ be an arbitrary function Let $fg: R \to R$ be the function defined by $(fg)(x) = f(x)g(x)$. Then which of the folloiwng statements is/are TRUE ?

A. If g is continuous at x=1, then f g is differentiable at x=1

B. If g is differentiable at x=1, then f g is continuous at x=1

C. if g is differentiable at x=1, then f g is differentiable at x=1

D. If f g is diffrentiable at x=1, then g is diffrentiable at x=1

Answer: A::C

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2. Let M be a 3×3 invertible matrix with real entries and let I denote the $3 \times$ matrix. If M^{-1} adj(adjM). Then which of the following statements, is/are ALWAYS TRUE ?

A. $M = I$

B. $\det M = 1$

C. $M^2 = I$

D. $(adjM)^2 = I$

Answer: B:C:D

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3. Let s be the set of all complex numbers Z satisfying $|z^2 + z + 1| = 1$. Then which of the following statements is/are TRUE?

A. $\left|z + \frac{1}{2}\right| \leq \frac{1}{2} \text{for} a \mid \ \mid z \in S$

B. $|Z| \leq 2 \text{for all} z \in S$

C. $\left|z + \frac{1}{2}\right| \geq \frac{1}{2} \text{for all} z \in S$

D. The set S has exactly four elements.

Answer: B::C

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4. Let x,y an z be positive real numbers. Suppose x, y and z are the length of the sides of triangle opposite ot its angles, X,Y and Z respectively.If $\tan\frac{X}{2} + \tan\frac{Z}{2} = \frac{2y}{x+y+x}$ then which of the following statement is/are TURE ?

A. $2Y = X + Z$

B. $Y = X + Z$

C. $\tan\frac{X}{2} = \frac{x}{y+z}$

D. $x^2 + z^2 - y^2 = xz$

Answer: B::C

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5. Let L_1 and L_2 be the foollowing straight lines.

$L_1 : \frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{3}$ and $L_2 : \frac{x-1}{-3} = \frac{y}{-1} = \frac{z-1}{1}$

Suppose the striight line $L : \frac{x-\alpha}{l} = \frac{y-m}{m} = \frac{z-\gamma}{-2}$ lies in the plane containing L_1 and L_2 and passes throug the point of intersection of L_1 and L_2 if the L bisects the acute angle between the lines L_1 and L_2, then which of the following statements is /are TRUE ?

A. $\alpha - \gamma = 3$

B. $l + m = 2$

C. $\alpha - \gamma = 1$

D. $l + m = 0$

Answer: A::B

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6. Which of the following inequalities is/are TRUE ?

A. $\int_0^1 x \cos x dx \geq \frac{3}{8}$

B. $\int_0^1 x \sin x dx \geq \frac{3}{10}$

C. $\int_0^1 x^2 \cos x dx \geq \frac{1}{2}$

D. $\int_{0}^{1} x^2 \sin x dx \geq \frac{2}{9}$

Answer: A::B::D

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Section 3

1. Let m be the minimum possible value of $\log_3(3^{y_1} + 3^{y_2} + 3^{y_3})$. Where y_1, y_2, y_3 are real number for which $y_1 + y_2 + y_3 = 9$ Let M be the maxmum possible value of $(\log 3x_1 + \log_3 x_2 + \log_3 x_2)$ where x_1, x_2, x_3 are positive real numbers for whcih $x_1 + x_2 + x_3 = 9$ then the value of $\log_2 m^3 + \log_3(M^2)$ is ____

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2. Let $a_1, a_2, a_3 ...$ be a sequance of positive integers in arithmetic progression with common difference 2. Also let $b_1, b_2, b_3 ......$ be a sequences of posotive intergers in geometric progression with commo ratio 2. If $a_1 = b_1 = c_2.$ then the number of all possible values of c, for which the equality.

$2(a_1 + a_2. + ... + a_n) = b_1 + b_2 + ... + b_n$ holes for same positive integer n, is

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3. Let $f: [0, 2] \rightarrow \mathbb{R}$ be the function defined by $f(x) = (3 - \sin(2\pi x))\sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right)$

If $\alpha, \beta \in [0, 2]$ are such that

$\{x \in [0, 2] : f(x) \geq 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is.________

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4. In a triangle $PQR, \leq t : a\vec{a} = \vec{Q}R = \vec{b} = \vec{R}P$ and $\vec{c} = \vec{P}Q.,$

If $|\vec{a}| = 3, |\vec{b}| = 4$ and $\dfrac{\vec{a}.(\vec{c}.\vec{b})}{\vec{.}(\vec{a} - \vec{b})} = \dfrac{|\vec{a}|}{|\vec{a}| + |\vec{b}|}$

then the value of $|\vec{a} \times \vec{b}|^2$ is _____

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5. For a polynomia g(x) with real coefficients, let m_g denote the number of distinct real roots of g(x). Suppose S is the set of polynomials with real coefficients defined by

$$S = \left\{ \left(x^2 - 1\right)^2 \left(a_0 + a_1 + a_2x^2 + a_3x^3\right) : a_0a_1, a_2, a_3 = \mathbb{R} \right\}$$

For a polynomial f. let f and f denote its first and second order derivties, respectively. Then the minimum possible value of $\left(m_{f'} + m_f\right)$. where $f \in S$ is, ____

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6. Let a denote the base of the natural loganthim. The value of real number a for which the right hand limit

$\lim_{x \to} \frac{(1-x)^{1/x} - e^{-1}}{x^e}$ is equal to a nonzer real number is, _____

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