MATHS

BOOKS - RS AGGARWAL MATHS (HINGLISH)

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Example

1. Evaluate :

$(i) i^{23}$ $(ii) i^{998}$ $(iii) i^{-998}$ $(iv) i^{-71}$

$(v) \left(\sqrt{-1}\right)^{91}$ $(vi) \left(i^{37} \times i^{-61}\right)$ $(vii) i^{-1}$

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2. FInd the value of $(1+i)^4 \times \left(1+\frac{1}{i}\right)^4$

A. 1

B. 16

C. 0

D. -1

Answer: B

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3. The value of $\left(i^{23}+\left(\frac{1}{i}\right)^{29}\right)^{2} is$

A. 4

B. 1

C. -2

D. -4

Answer: D

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4. Simplify :

$(i) \quad (-2i)\left(\frac{1}{6}i\right) \qquad (ii) \quad (-i)(3i)\left(\frac{-1}{6}i\right)^3 \qquad (iii) \quad 4\sqrt{-4}+5\sqrt{-9}$

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5. Show that $\left(-\sqrt{-1}\right)^{4n+3} = i$, where n is a positive integer.

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6. the sum $\left(1+i^2+i^4+\ldots.+i^{2n}\right)$ is , when n is odd

A. 1

B. 0

C. -1

D. 2

Answer: B

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7. Explain the fallacy :

$$-1 = (i \times i) = \sqrt{-1} \times \sqrt{-1} = \sqrt{(-1) \times (-1)} = \sqrt{1} = 1.$$

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8. Evaluate :

$(i) \quad \sqrt{-25} \times \sqrt{-49} \qquad (ii) \quad \sqrt{-36} \times \sqrt{16}$

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9. Evaluate : $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}.$

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10. Simplify :

$(i)\quad 3(6+6i)+i(6+6i)\qquad (ii)\quad (1-i)-(-3+6i)$

$(iii)\quad \left(\frac{1}{3}-\frac{2}{3}i\right)-\left(4+\frac{3}{2}i\right)\qquad (iv)\quad \left\{\left(\frac{1}{5}+\frac{7}{5}i\right)-\left(6+\frac{1}{5}i\right)\right\}-\left(\frac{-4}{5}+\right.$

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11. Express $\left(-3\sqrt{3}+\sqrt{-2}\right)\left(2\sqrt{3}-i\right)$ in the form $(a+ib)$.

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12. Express each of the following in the form (a + ib) :

$(i)\quad \left(3+\sqrt{-5}\right)\left(3-\sqrt{-5}\right)\qquad (ii)\quad \left(-2+\sqrt{-3}\right)\left(-3+2\sqrt{-3}\right)$

$(iii)\quad (-2+3i)^2\qquad (iv)\quad \left(\sqrt{5}-7i\right)^2$

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13. Prove that $(1-i)^4=-4$.

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14. Express $(2+3i)^3$ in the form $(a+ib)$.

A. $(6+9i)$

B. $(-4+9i)$

C. $(-46+9i)$

D. $(-6+5i)$

Answer: C

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15. Express $(3-5i)^3$ in the form (a + ib).

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16. If $z=\left(\sqrt{2}-\sqrt{-3}\right)$, find Re(z), Im(z), $\bar{z}$ and $|z|$.

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17. Write down the modulus of:

$(i) \quad 4+\sqrt{-3} \qquad (ii) \quad 2-5i \qquad (iii) \; -i \qquad (iv) \quad (-1+3i)^2$

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18. Write down the conjugate of each of the following :

$(i) \quad \left(-5+\sqrt{-1}\right) \qquad (ii) \quad \left(-6-\sqrt{-3}\right) \qquad (iii) \quad i^3 \qquad (iv) \quad (4$

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19. Find the multiplicative inverse of each of the following :

$(i) \quad \sqrt{5}+3i \qquad (ii) \quad 4-3i \qquad (iii) \quad (3i-1)^2 \qquad (iv) \quad -i$

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20. Express each of the following in the form (a + ib):

$(i) \quad \dfrac{i}{(1+i)} \qquad (ii) \quad \left(-1+\sqrt{3}i\right)^{-1} \qquad (iii) \quad \dfrac{5+\sqrt{2}i}{1-\sqrt{2}i}$

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21. Reduce $\left(\frac{1}{1+2i}+\frac{3}{1-i}\right)\left(\frac{3-2i}{1+3i}\right)$ to the form $(a+ib)$.

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22. Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$.

A. 3

B. $\frac{1}{2}$

C. 1

D. 2

Answer: D

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23. If $\sqrt{\frac{1+i}{1-i}} = (a+ib)$ then show that $\left(a^2+b^2\right) = 1.$

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24. If $\left(\frac{1+i}{1-i}\right)^m = 1,$ then find the least positive integral value of $m\cdot$

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25. Separate $\left(\frac{3+\sqrt{-1}}{2-\sqrt{-1}}\right)$ into real and imaginary parts .

A. $(1-2i)$

B. $(1-i)$

C. $(1+i)$

D. $(1+2i)$

Answer: C

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26. $\dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}} =$

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27. If $(x+iy)^{1/3} = a+ib, x, y, ab \in R.$ Show that

(i) $\dfrac{x}{a}+\dfrac{y}{b} = 4(a^2-b^2)$

(ii) $\dfrac{x}{a}-\dfrac{y}{b} = 2(a^2+b^2)$

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28. If z is a complex number such that $|z| = 1,$ prove that $\dfrac{z-1}{z+1}$ is purely imaginary, what will by your conclusion if $z = 1?$

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29. Find real q such that $\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

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30. Find nonzero integral solutions of $|1-i|^x = 2^x$.

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31. Find the real values of x and y if

(i) $(3x-7)+2iy = -5y+(5+x)i$

(ii)$(x+iy)(2-3i) = (4+i)$

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32. Find the real values of x and y for which $\left(\frac{x-1}{3+i}+\frac{y-1}{3-i}\right) = i.$

A. $-4, 6$

B. $4, -6$

C. $-2, 3$

D. $-3, 3$

Answer: A

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33. Find the complex number z for which $|z+1| = z + 2(1+i)$.

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34. Solve the equation $|z| + z = (2+i)$ for complex value of z.

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35. Solve the equation $z + \sqrt{2}|z+1| + i = 0$ for complex value of z.

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36. If $z = 3 + 2i$, prove that $z^2 - 6z + 13 = 0$ and hence deduce that $3z^3 - 13z^2 + 9z + 65 = 0$.

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37. If $z = -5 + 3i$, find the value of $\left(z^4 + 9z^3 + 26z^2 - 14z + 8\right)$.

A. 60

B. -60

C. -48

D. 48

Answer: B

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38. If z = 2 + i, prove that $z^3 + 3z^2 - 9z + 8 = (1 + 14i)$.

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39. where does z lie , if $\left|\frac{z-5i}{z+5i}\right| = 1$?

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40. If $a+ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

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41. If $\frac{(a+i)^2}{(2a-i)} = p+iq,$ show that: $p^2+q^2 = \frac{(a^2+1)^2}{(4a^2+1)}$.

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42. If $z = x+iyandw = \frac{1-iz}{z-i}$, show that $|w| = 1z$ is purely real.

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43. If z_1, z_2 are complex number such that $\frac{2z_1}{3z_2}$ is purely imaginary number, then find $\left|\frac{z_1 - z_2}{z_1 + z_2}\right|$.

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44. If $|z_1| = |z_2| = \ldots = |z_n| = 1$, prove that $|z_1 + z_2 + z_3 + \ldots + z_n| = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \ldots + \frac{1}{z_n}$.

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45. If $\frac{a + ib}{c + id} = x + iy$, prove that $\frac{a - ib}{c - id} = x - iyand\frac{a^2 + b^2}{c^2 + d^2} = x^2 + y^2$.

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46. If $x + iy = \sqrt{\frac{a + ib}{c + id}}$ prove that $\left(x^2 + y^2\right)^2 = \frac{a^2 + b^2}{c^2 + d^2}$

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47. If $(1+i)(1+2i)(1+3i)1+m)=(x+iy)$, then show that $2\times 5\times 10\times \ \times \left(1+n^2\right)=x^2+y^2.$

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48. If $(a+ib)(c+id)(e+\ \text{if}\ )(g+ih)=A+iB$, then show that $\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right)=A^2+B^2$

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49. If α and β are different complex number with $|\beta|=1$, then find $\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|$

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50. If $\left|z^2=1\right|=|z|^2+1$, then show that z lies on the imaginary axis.

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51. Show that if $iz^3 + z^2 - z + i = 0$, then $|z| = 1$

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52. Find all non-zero complex numbers z satisfying $\bar{z} = iz^2$

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53. Solve the equation, $z^2 = \bar{z}$, where z is a complex number.

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54. For all complex numbers z_1 and z_2, prove that

$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left(|z_1|^2 + |z_2|^2\right).$

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55. Convert each of the following complex numbers into polar form:

$(i)\quad 3\quad (ii)\quad -5\quad (iii)\quad i\quad (iv)\quad -2i$

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56. Find the modulus and argument of each of the complex numbers given below:

$(i)\quad 1+i\quad (ii)\quad -\sqrt{3}+i\quad (iii)\quad -1-i\sqrt{3}$

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57. Convert the complex number $\left(1+i\sqrt{3}\right)$ into polar form.

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58. Convert the complex number $\left(2\sqrt{3}-2i\right)$ into polar form.

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59. Convert the complex number $\left(-2+2i\sqrt{3}\right)$ into polar form.

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60. Convert the following in the polar form :

(i) $\dfrac{1+7i}{(2-i)^2}$

(ii) $\dfrac{1+3i}{1-2i}$

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61. Convert the complex number $\dfrac{1}{(1+i)}$ into polar form.

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62. Convert the complex number $z=\dfrac{i-1}{\frac{\cos\pi}{3}+i\frac{\sin\pi}{3}}$ in the polar form.

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63. Convert of the complex number in the polar form: $1 \quad i$

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64. Express the complex number $\left(-\sqrt{3}-i\right)$ in polar form.

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65. Convert the complex number $\frac{1+2i}{1+3i}$ into polar form.

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66. Express $\sin\frac{\pi}{5}+i\left(1-\cos\frac{\pi}{5}\right)$ in polar form.

A. $2\sin\frac{\pi}{10}\left(\cos\frac{\pi}{10}+\mathrm{i}\sin\frac{\pi}{10}\right)$.

B. $\sin\frac{\pi}{10}\left(\cos\frac{\pi}{10}+\mathrm{i}\sin\frac{\pi}{10}\right)$.

C. $2\sin\frac{\pi}{5}\left(\cos\frac{\pi}{10}+\mathrm{i}\sin\frac{\pi}{10}\right)$.

D. None of these

Answer: A

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67. Convert $4(\cos 300^\circ + i\sin 300^\circ)$ into standard form.

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68. If $z_1, z_2.....z_n = z,$ then $argz_1 + argz_2 + ... + argz_n$ and $argz$ differ by a

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69. Solve : $x^2 + 3 = 0.$

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70. Solve the equation:$x^2 + 3x + 9 = 0$

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71. Solve : $9x^2 + 10x + 3 = 0.$

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72. Solve the equation: $\sqrt{2}x^2 + x + \sqrt{2} = 0$

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73. Solve the equation:$\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

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74. Solve : $3x^2 + 8ix + 3 = 0$.

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75. Evaluate $\sqrt{6 + 8i}$.

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76. Find the square root of the following complex number: $-5 + 12i$

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77. Find square root of $8 - 15i$

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78. Evaluate $\sqrt{24 - 10i}$.

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79. Evaluate $\sqrt{-i}$.

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80. Evaluate $\sqrt{4+3\sqrt{-20}}+\sqrt{4-3\sqrt{-20}}$.

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Exercise 5 A

1. Evaluate: $(i)\quad i^{19}\qquad (ii)\quad i^{62}\qquad (iii)\quad i^{373}$

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2. Evaluate:

$(i) \quad \left(\sqrt{-1}\right)^{192} \qquad (ii) \quad \left(\sqrt{-1}\right)^{93} \qquad (iii) \quad \left(\sqrt{-1}\right)^{30}$

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3. Evaluate: $(i) \quad i^{-50} \qquad (ii) \quad i^{-9} \qquad (iii) \quad i^{-131}$

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4. Evaluate: $(i) \quad \left(i^{41} + \frac{1}{i^{71}}\right) \qquad (ii) \quad \left(i^{53} + \frac{1}{i^{53}}\right)$

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5. $1 + i^2 + i^4 + i^6 = 0.$

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6. $6i^{50} + 5i^{33} - 2i^{15} + 6i^{48} = 7i.$

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7. Show that : $\frac{1}{i} + \frac{1}{i^2} + \frac{1}{i^3} + \frac{1}{i^4} = 0$

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8. Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number.

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9. $\left\{ i^{21} - \left(\frac{1}{i}\right)^{46} \right\}^2 =$

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10. Evaluate : $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^{3}$

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11. Show that $(1-i)^{n}\left(1-\frac{1}{i}\right)^{n}=2^{n}$ for all $n \in N$

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12. Evaluate : $\sqrt{-16}+3\sqrt{-25}+\sqrt{-36}-\sqrt{-625}.$

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13. $1+i^{2}+i^{4}+i^{6}+i^{8}+\ +i^{20}$

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14. $i^{53} + i^{72} + i^{93} + i^{102} = 2i.$

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15. The value of sum $\sum_{n=1}^{13} \left(i^n + i^{n+1}\right)$ where $i = \sqrt{-1}$, is equal to

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Exercise 5 B

1. Simplify each of the following and express it in the form a + ib :

(i)	$2(3+4i)+i(5-6i)$	(ii)	$\left(3+\sqrt{-16}\right)-\left(4-\sqrt{-9}\right)$
(iii)	$(-5+6i)-(2+i)$	(iv)	$(8-4i)-(-3+5i)$
(v)	$(1-i)^2(1+i)-(3-4i)^2$	(vi)	$\left(5+\sqrt{-3}\right)\left(5-\sqrt{-3}\right)$
(vii)	$(3+4i)(2-3i)$	$(viii)$	$\left(-2+\sqrt{-3}\right)\left(-3+2\sqrt{-3}\right)$

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2. Simplify and express each of the following in the form (a + ib) :

$(i)\quad \left(2+\sqrt{-3}\right)^2 \quad (ii)\quad (5-2i)^2 \quad (iii)\quad (-3+5i)^3$

$(iv)\quad \left(-2-\frac{1}{3}i\right)^3 \quad (v)\quad (4-3i)^{-1} \quad (vi)\quad \left(-2+\sqrt{-3}\right)^{-1}$

$(vii)\quad (2+i)^{-2} \quad (viii)\quad (1+2i)^{-3} \quad (ix)\quad (1+i)^3-(1-i)^3$

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3. Express each of the following in the form (a + ib) :

$(i)\quad \frac{1}{(4+3i)} \quad (ii)\quad \frac{(3+4i)}{(4-5i)} \quad (iii)\quad \frac{\left(5+\sqrt{2}i\right)}{\left(1-\sqrt{2}i\right)}$

$(iv)\quad \frac{(-2+5i)}{(3-6i)} \quad (v)\quad \frac{(3-4i)}{(4-2i)(1+i)} \quad (vi)\quad \frac{(2-2i)(2+3i)}{(1+2i)(2-i)}$

$(vii)\quad \frac{(2+3i)}{(2-i)} \quad (viii)\quad \frac{(1-i)^3}{(1-i^3)} \quad (ix)\quad \frac{(1+2i)^3}{(1+i)(2-i)}$

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4. Simplify and express each of the following in the form (a + ib) :

$(i)\quad \left(\frac{5}{-3+2i}+\frac{2}{1-i}\right)\left(\frac{4-5i}{3+2i}\right) \quad (ii)\quad \left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{1}{5}\right.$

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5. Show that

(i) $\left\{\frac{(3+2i)}{(2-3i)}+\frac{(3-2i)}{(2+3i)}\right\}$ is purely real,

(ii) $\left\{\frac{\left(\sqrt{7}+i\sqrt{3}\right)}{\left(\sqrt{7}-i\sqrt{3}\right)}+\frac{\left(\sqrt{7}-i\sqrt{3}\right)}{\left(\sqrt{7}+i\sqrt{3}\right)}\right\}$ is purely real.

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6. Find the real values of θ for which the complex number $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real.

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7. If |z + i| = |z - i|, prove that z is real.

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8. Give an example of two complex numbers z_1 and z_2 such that $z_1 \neq z_2$ and $|z_1| = |z_2|$.

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9. Find the conjugate of each of the following :

$(i)\ (-5-2i)$ $(ii)\ \frac{1}{(4+3i)}$ $(iii)\ \frac{(1+i)^2}{(3-i)}$ $(iv)\ \frac{(1+i)(2+i)}{(3+i)}$

$(v)\ \sqrt{-3}$ $(vi)\ \sqrt{2}$ $(vii)\ -\sqrt{-1}$ $(viii)\ (2-5i)^2$

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10. Find the modulus of each of the following :

$(i)\ (3+\sqrt{-5})$ $(ii)\ (-3-4i)$ $(iii)\ (7+24i)$ $(iv)\ 3i$

$(v)\ \frac{(3+2i)^2}{(4-3i)}$ $(vi)\ \frac{(2-i)(1+i)}{(1+i)}$ $(vii)\ 5$ $(viii)\ (1+2i)(i$

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11. Find the multiplicative inverse of each of the following :

$(i) \quad (1-\sqrt{3}i) \qquad (ii) \quad (2+5i) \qquad (iii) \quad \frac{(2+3i)}{(1+i)} \qquad (iv) \quad \frac{(1+}{(}$

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12. If $\left(\frac{1-i}{1+i}\right)^{100} = a+ib, \ f \in d\,(a,b)$

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13. If $\left(\frac{1+i}{1-i}\right)^{3} - \left(\frac{1-i}{1+i}\right)^{3} = x+iy, \ f \in d\,(x,y)$

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14. If $x+iy = \frac{a+ib}{a-ib}$, prove that $x^2+y^2=1$.

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15. If $a+ib=\frac{c+i}{c-i}$, where c is real, prove that: $a^2+b^2=1 and \frac{b}{a}=\frac{2c}{c^2-1}$.

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16. Show that $(1-i)^n\left(1-\frac{1}{i}\right)^n=2^n$ for all $n\in N$

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17. What is the smallest positive integer n for which $(1+i)^{2n}=(1-i)^{2n}$?

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18. Prove that: $x^4=4=(x+1+i)(x+1-i)(x-1+i(x-1-i).)$

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19. if $a = \cos\theta + i\sin\theta$, prove that $\frac{1+a}{1-a} = \cot\left(\frac{\theta}{2}\right)i$

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20. If $z_1 = 2 - i,\ z_2 = 1 + i,$ find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$

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21. Find the real values of x and y for which :

$(i)\quad (1-i)x + (1+i)y = 1 - 3i$

$(ii)\quad (x+iy)(3-2i) = (12+5i)$

$(iii)\quad x + 4yi = ix + y + 3$

$(iv)\quad (1+i)y^2 + (6+i) = (2+i)x$

$(v)\quad \frac{(x+3i)}{(2+iy)} = (1-i)$

$(vi)\quad \frac{(1+i)x-2i}{(3+i)} + \frac{(2-3i)y+i}{(3-i)} = i$

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22. Find the real numbers $x\ and\ y,$ if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.

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23. Find real values of $xandy$ for which the complex numbers $-3+ix^2yandx^2+y+4i$ are conjugate of each other.

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24. If $z=2-3i$ show that $z^2=4z+13=0$ and hence find the value of $4z^3-3z^2+169.$

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25. if $(1+i)z=(1-i)\bar{z}$ then z is

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26. If $\left(\frac{z-1}{z+1}\right)$ is purely an imaginary number and $z\neq-1$ then find the value of |z|.

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27. Solve the system of equations $Re(z^2) = 0, \ |z| = 2$

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28. Find the complex number z for which |z| = z + 1 + 2i.

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Exercise 5 C

1. Express each of the following in the form (a + ib) and find its conjugate:

$(i) \quad \frac{1}{(4+3i)} \qquad (ii) \quad (2+3i)^2 \qquad (iii) \quad \frac{(2-i)}{(1-2i)^2}$

$(iv) \quad \frac{(1+i)(1+2i)}{(1+3i)} \qquad (v) \quad \left(\frac{1+2i}{2+i}\right)^2 \qquad (vi) \quad \frac{(2+i)}{(3-i)(1+2i)}$

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2. Express each of the following in the form (a + ib) and find its multiplicative inverse :

(i) $\frac{1+2i}{1-3i}$

(ii) $\frac{(1+7i)}{(2-i)^2}$

(iii) $\frac{-4}{\left(1+i\sqrt{3}\right)}$

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3. If $(x+iy)^3 = u+iv$, then show that $\frac{u}{x}+\frac{v}{y} = 4(x^2-y^2)$.

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4. $(x+iy)^{\frac{1}{3}} = (a+ib)$ then prove that $\left(\frac{x}{a}+\frac{y}{b}\right) = 4(a^2-b^2)$

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5. Express $(1-2i)^{-3}$ in the standard form $a+ib$.

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6. find x and$y$ for which the inequalities hold $(x^4 + 2ix) - (3x^2 + iy) = (3 - 5i) + (1 + 2iy)$

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7. If $z^2 + |z|^2 = 0$, show that z is purely imaginary.

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8. If $z(\neq -1)$ is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is equal to

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9. If z_1 is a complex number other than -1 such that $|z_1| = 1 \, and \, z_2 = \frac{z_1 - 1}{z_1 + 1}$, then show that the real parts of z_2 is zero.

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10. For all z $\in$ C, prove that

(i) $\frac{1}{2}(z + \bar{z}) = Re(z),$

(ii) $\frac{1}{2i}(z - \bar{z}) = Im(z),$

(iii) $z\bar{z} = |z|^2,$

(iv) $z + \bar{z})$is real,

(v) $(z - \bar{z})$is 0 or imaginary.

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11. If $z_1 = (1 + i)$ and $z_2 = (-2 + 4i)$, prove that Im $\left(\frac{z_1 z_2}{\bar{z}_1}\right) = 2.$

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12. Show that a real value of x will satisfy hte equation $(1-ix)/(1+ix)=a-ib$ if $a^2+b^2=1, wherea, b$ real.

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Exercise 5 D

1. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

4

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2. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

-2

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3. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

-i

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4. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

2i

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5. Find the modulus and argument of the following complex number and hence express each of them in the polar form: $1 - i$

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6. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

-i+i

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7. Find the modulus and argument of the following complex number and hence express each of them in the polar form: $\sqrt{3}+i$

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8. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$-1+\sqrt{3}i$

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9. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$1 - \sqrt{3}i$

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10. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$2 - 2i$

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11. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$-4 + 4\sqrt{3}i$

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12. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$-3\sqrt{2}+3\sqrt{2}i$

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13. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$\frac{1+i}{1-i}$

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14. Find the modulus and argument of the following complex number and hence express each of them in the polar form: $\frac{1-i}{1+i}$

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15. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$$\frac{1+3i}{1-2i}$$

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16. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$$\frac{1-3i}{1+2i}$$

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17. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$$\frac{5-i}{2-3i}$$

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18. Find the modulus and argument of the following complex number and hence express each of them in the polar form: $\frac{-16}{1+i\sqrt{3}}$

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19. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$$\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$$

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20. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$$\sqrt{\frac{1+i}{1-i}}$$

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21. Find the modulus and argument of each of the following complex number: $-\sqrt{3}-i$

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22. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$\left(i^{25}\right)^{3}$

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23. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$$\frac{(1-i)}{\left(\cos\frac{\pi}{3}+\mathrm{i}\sin\frac{\pi}{3}\right)}$$

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24. Find the modulus and argument of each of the following complex numbers and hence express each of them in polar form:

$(\sin 120^\circ - i\cos 120^\circ)$

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Exercise 5 E

1. Solve: $x^2 + 2 = 0$

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2. Solve: $x^2 + 5 = 0$

A. $\sqrt{5}, -\sqrt{5}$

B. $-5, +5$

C. $i\sqrt{5}, -i\sqrt{5}$

D. $i\sqrt{5}$

Answer: C

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3. Solve: $2x^2 + 1 = 0$

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4. Solve: $x^2 + x + 1 = 0$

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5. Solve the equation:$x^2 - x + 2 = 0$

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6. Solve the following quadratic: $x^2 + 2x + 2 = 0$

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7. Solve: $2x^2 - 4x + 3 = 0$

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8. Solve the equation:$x^2 + 3x + 5 = 0$

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9. Solve: $\sqrt{5}x^2 + x + \sqrt{5} = 0$

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10. Solve the equation $25x^2 - 30x + 11 = 0$ by using the general expression for roots quadratic equation $ax^2 + bx + c = 0,$ we get: $a = 25, b = -30adnc - 11$.

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11. Solve: $8x^2 + 2x + 1 = 0$

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12. Solve the equation :$27x^2 - 10x + 1 = 0$

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13. Solve: $2x^2 - \sqrt{3}x + 1 = 0$

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14. Solve the following quadratic: $17x^2 - 8x + 1 = 0$

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15. Solve: $3x^2 + 5 = 7x$

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16. Solve the equation : $3x^2 - 4x + \frac{20}{3} = 0$

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17. Solve the following equation by factorization method: $3x^2 + 7ix + 6 = 0$

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18. Solve the following quadratic: $21x^2 - 28x + 10 = 0$

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19. Solve the equation $x^2 - 4x + 13 = 0$

A. $-2 + 3i, \ -2 - 3i$

B. $2 + 3i, 2 - 3i$

C. $-2 + 3i, 2 + 3i$

D. $2 - 3i, \ -2 - 3i$

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20. Solve: $x^2 + 3ix + 10 = 0$

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21. Solve: $2x^2 + 3ix + 2 = 0$

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Exercise 5 F

1. Evaluate: $\sqrt{5+12i}$

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2. Evaluate: $\sqrt{-7+24i}$

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3. Evaluate: $\sqrt{-2+2\sqrt{3}i}$

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4. Evaluate: $\sqrt{1+4\sqrt{-3}}$

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5. Evaluate: $\sqrt{i}$

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6. Evaluate: $\sqrt{4i}$

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7. Evaluate: $\sqrt{3+4\sqrt{-7}}$

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8. Evaluate: $\sqrt{16-30i}$

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9. Evaluate: $\sqrt{-4-3i}$

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10. Evaluate: $\sqrt{-15-8i}$

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11. Find the square root of $6\sqrt{2}i-7$ and $-11-60i$

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12. Evaluate:

$\sqrt{7-30\sqrt{-2}}.$

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13. Evaluate: $\sqrt{-8i}$

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14. Evaluate: $\sqrt{1-i}$

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Exercise 5 G

1. Evaluate $\frac{1}{i^{78}}$.

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2. Evaluate $\left(i^{57}+i^{70}+i^{91}+i^{101}+i^{104}\right)$.

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3. Evaluate $\left(\frac{i^{180}+i^{178}+i^{176}+i^{174}+i^{172}}{i^{170}+i^{168}+i^{166}+i^{164}+i^{162}}\right)$.

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4. Evaluate $\left(i^{4n+1}-i^{4n-1}\right)$.

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5. Evaluate $\left(\sqrt{-36}\times\sqrt{-25}\right)$.

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6. If $n\in N,$ then find the value of $i^{n}+i^{n+1}+i^{n+2}+i^{n+3}$.

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7. Find the sum $(i+i^{2}+i^{3}+i^{4}+\ldots.$ up to 400 terms).

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8. Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number.

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9. Evaluate $\left(i^{41} + \frac{1}{i^{71}}\right)$.

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10. Find the least positive integer n foe which $\left(\frac{1+i}{1-i}\right)^n = 1$.

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11. Express $(2-3i)^3$ in the form (a + ib).

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12. Express the following expression in the form of $a+ib$

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

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13. Perform the indicated operation and find the result in the form $a+ib$: $\frac{3-\sqrt{-16}}{1-\sqrt{-9}}$

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14. Solve for $x:(1-i)x+(1+i)y=1-3i.$

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15. Solve the following quadratic equations by factorization method:

$x^2-5ix-6=0$ (ii) $x^2+4ix-4=0$

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16. Find the conjugate of $\frac{1}{3+4i}$.

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17. If $z = (1-i)$, find z^{-1}.

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18. If $z = \left(\sqrt{5}+3i\right)$, find z^{-1}.

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19. $arg(\bar{z}) = -arg(z)$

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20. If |z| = 6 and arg(z) $= \frac{3\pi}{4}$, find z.

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21. Find the principal argument of (-2i).

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22. Find the principal argument of $\left(1+i\sqrt{3}\right)^2$.

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23. Write -9 in polar form.

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24. Write 2i in polar form.

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25. Write $-1+\sqrt{3}$ in polar form.

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26. Write $1-i$ in polar form.

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27. Write $-1+\sqrt{3}$ in polar form.

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28. If $|z|=2$ and arg $(z)=\frac{\pi}{4}$, find z.

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