



MATHS

BOOKS - RS AGGARWAL MATHS (HINGLISH)

FUNCTIONS

Example

1. Let $X = \{1, 2, 3, 4\}$ and $Y = \{1, 4, 9, 16, 25\}$

 $\mathsf{Let}\; f = \left\{(x,y)\!:\! x \in X, y \in Y \; \mathrm{and} \; y = x^2 \right\}$

(i) Show that f is a function from X to Y. Find its domain and range.

(ii) Draw a pictorial representation of the above function.

(iii) If $A = \{2, 3, 4\}$, find f(A).

2. Let $X = \{2, 3, 4, 5\}$ and $Y = \{7, 9, 11, 13, 15, 17\}$. Define a relation f from X to Y by: $f = \{(x, y) : x \in X, y \in Y \text{ and } y = 2x + 3\}$. (i) Write in roster form. (ii) Find dom (f) and range (f). (iii) Show that f is a function from X to Y.

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3. Which of the following relations are functions? Give reasons. In case of a function, find its domain and range.

(i) $f = \{(1, 3), (1, 5), (2, 3), (2, 5)\}$ (ii) $g = \{(2, 1), (5, 1), (8, 1), (11, 1)\}$

(iii) $h = \{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6)\}$

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4. Let $A = \{-2, -1, 0, 1, 2\}$ and $f: A \to Z$ be given y $f(x) = x^2 - 2x - 3$. Find: i. the range of f ii. pre images of 6,-3 and 5. 5. Let $A = \{1, 2\}, B = \{3, 6\}$ and $f: A \to B$ given by $f(x) = x^2 + 2 n d g: A \to B$ given by g(x) = 3x. Then we observe that f and g have the same domain and co-domain. Also we hve, f(1) = 3 = g(1)and f(2) = 6 = g(2). Hence f = g.

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6. Let $f\colon Z o Z\colon f(x)=x^2$ and $g\colon Z o Z\colon g(x)=|x|^2$ for all $x\in Z$. Show that f=g.

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7. Let $f \colon R \to R \colon f(x) = x+2$ and $g \colon R - |2| \to R \colon g(x) = rac{x^2-4}{x-2}$

Show that $f \neq g$. Re-define f and g such that f=g.

8. Let $f = \{(-1, -3), (0, -1), \}$ be a function, described by the formula, $f(x) = \alpha x + \beta$. Then, find the value of α and β . Also, find the formula.

A. f(x)=2xB. f(x)=1-2xC. f(x)=2x+1D. f(x)=2x-1

Answer: D

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9. Let
$$f\!:\!R o R\!:\!f(x)=x^2+3$$
.

Find the maximum possible sum of pre-images of each of the following

under f:

(i) 19 (ii) 28 (iii) 3

A. 8

B. 9

C. 10

D. None of these

Answer: B

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10. Let $f\!:\!R o R\!:\!f(x)=x^2+1$. Find (i) $f^{-1}\{-4\}$ (ii) $f^{-1}\{10\}$ (iii) $f^{-1}\{5,17\}.$

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11. Let $f: R \to R$, defined by $f(x) = \begin{cases} 1, & ext{if } x \in Q \\ -1, & ext{if } x \in Q \end{cases}$ Find (i) $\frac{f(1)}{2}$ (ii) f(0.34) (iii) $f(\sqrt{2})$ (iv) $f(\pi)$

(v) range(f) (vi)
$$f^{-1}(1)$$
 (vii) $f^{-1}\{1\}$

12. Let $f \colon R o R$, defined by

$$f(x) = \left\{egin{array}{ccc} 3x-2 & x < 0 \ 1, & x = 0 \ 4x+1, & x > 0 \end{array}
ight.$$

Find (i) f(2) (ii) f(-2) (iii) f(0) (iv) f(3.5)

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13. The relation f is defined by $f(x) = \{x^2, 0 \le x \le 33x, 3 \le x \le 10\}$ The relating g is defined by $g(x) = \{x^2, 0 \le x \le 33x, 2 \le x \le 10\}$ how that f is a function and g is not a function.

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14. The relation f is defined by $f(x) = \{x^2, 0 \le x \le 33x, 3 \le x \le 10\}$ The relating g is defined by $g(x) = \{x^2, 0 \le x \le 33x, 2 \le x \le 10\}$ how that f is a function and g is not a function.



16. Draw the graph of each of the following constant functions: (i) f(x)=2

for all $x \in R$ (ii) f(x)=0 for all $x \in R$ (iii)f(x)=-2 for all $x \in R$

17. Draw the graph of the modulus function, defined by

$$f\!:\!R o R\!:\!f(x) = |x| = egin{cases} x, ext{when} & x \geq 0 \ -x, ext{when} & x < 0 \end{cases}$$

18. Draw the graph of the greatest integer function: $f\colon R o R\colon f(x)=[x] ext{for all }x\in R.$

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19. Draw the graph of the smallest integer function $f\colon R o R\colon f(x)=[x]$ for all $x\in R.$

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20. Draw the graph of the fractional part function:

$$f\!:\!R o R\!:\!f(x)=x-[x]=[x]$$

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21. Draw the graph of the signum function, $f \colon R o R$, defined by

$$f(x)=\left\{egin{array}{c} rac{x}{|x|}, ext{when} \hspace{0.2cm} x
eq 0 \ 0, ext{when} \hspace{0.2cm} x=0 \end{array}
ight. ext{ or } f(x)=\left\{egin{array}{c} 1, ext{if} \hspace{0.2cm} x>0 \ 0, ext{if} \hspace{0.2cm} x=0 \ -1, ext{if} \hspace{0.2cm} x<0 \end{array}
ight.$$

22. Let
$$f \colon R \to R \colon f(x) = x^2$$
 for all $x \in R$.

23. Let $f \colon R o R \colon f(x) = x^3 ext{ for all } x \in R.$ Find its domain and range.

Also, draw its graph.

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24. Let
$$f\colon (R-\{0\}) o R\colon f(x)=rac{1}{x}$$
 for all values of $x\in R-\{0\}.$

Find its domain and range. Also draw its graph.

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Problems Based On Domanis And Ranges Of Real Functions Solved Examples



4. Find the domain of the real-valued function:

$$f(x) = rac{x^2 - x + 1}{x^2 - 5x + 4}.$$

5. Find the domain of each of the following real fuctions:

(i)
$$f(x) = \sqrt{x-3}$$
 (ii) $g(x) = \sqrt{4-x^2}$ (iii) $h(x) = rac{1}{\sqrt{1-x}}$

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6. Find the domain of the function,
$$f(x) = \sqrt{3-x} + rac{1}{\sqrt{x^2-1}}.$$

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7. Find the rage of each of the following functions. (i)
$$f(x) = 2$$
 $3x$, $x \in R, x > 0$ (ii) $f(x) = x^2 + 2$, x is a real number. (iii) $f(x) = x$, x is a real number.

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8. Find the domain and the range of the function, $f(x) = \frac{x-2}{x-3}$.



$$f(x)= \ -\left|x
ight|$$
 (ii) $f(x)=\sqrt{9-x^2}$





21. Show that $f(x) = rac{1}{\sqrt{x-|x|}}$ is not defined for any $x \in R.$ How will

you define dom (f) and range (f)?

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22. Find the domain and the range of the real function,
$$f(x) = rac{1}{\sqrt{x+[x]}}.$$

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23. Find the domain and range of the function given by $f(x) = rac{1}{\sqrt{x-[x]}}$

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Solved Examples

1. If
$$f(x) = 3x^3 - 5x^2 + 10$$
, find $f(x - 1)$.



2. If
$$f(x)=x+rac{1}{x}$$
 , prove that $\left[f(x)
ight]^3=fig(x^3ig)+3figg(rac{1}{x}ig)$.

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3. If
$$f(x)=rac{x-1}{x+1}, x
eq -1, \,$$
 . then show that $f(f(x))=-rac{1}{x}$, prove that $x
eq 0$.

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4. if
$$y = f(X) = \frac{ax - b}{bx - a}$$
 show that $x = f(y)$

5. If f is a real function defined by $f(x)=rac{x-1}{x+1}$, then prove that $f(2x)=rac{3f(x)+1}{f(x)+3}$

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$$f(x)=(x-3) ext{ and } g(x)= egin{cases} rac{x^2-9}{x+9} & ext{when } x
eq -3\ k, & ext{when } x=-3 \end{cases}$$

Find the value of k such that f(x) =g(x) for all $x \in R$.

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7. If
$$af(x) + bf\left(rac{1}{x}
ight) = rac{1}{x}$$
, where $a
eq b$ and $x
eq 0$, find f(x).

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8. Let $f(x) = x^2$ and g(x) = 2x + 1 be two real functions. Find (f + g)(x), (f - g)(x), (fg)(x), (fg)(x).

9. Let
$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$
 and $B = \{2, 3, 5, 7\}$. Find $A \cap B$ and hence show that $A \cap B = B$.

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10. Let f and g be real functions, defined by

$$f(x) = \frac{1}{(x+4)}$$
 and $g(x) = (x+4)^3$. Find (i) $(f+g)(x)$ (ii)
 $(f-g)(x)$ (iii) $(fg)(x)$ (iv) $\left(\frac{f}{g}\right)(x)$ (v) $\left(\frac{1}{f}\right)(x)$

11. Let f and g be real functions, defined by $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{x+1}$. Find (i) (f+g)(x) (ii) (f-g)(x) (iii) $(fg)(x)(iv)\left(\frac{f}{g}\right)(x)$.



12. Let f and g be real functions, defined by $f(x) = \sqrt{x+2}$ and $g(x) = \sqrt{4-x^2}$. Find (i) (f+g)(x) (ii) (f-g)(x) (iii)(fg)(x) (iv) (ff)(x)(v) (gg)(x) (vi) $\left(\frac{f}{g}\right)(x)$.

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13. Let f be the exponential function and g be the logarithumic function.

Then, find

(i) (f+g)(1) (ii) (fg)(1) (iii) (4f)(1) (iv) (3g)(1)

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14. If $f(x) = \log_e(1-x)$ and g(x) = [x] then find: (i) (f+g)(x) (ii) (fg)(x) (iii) $\left(\frac{f}{g}\right)(x)$ (iv) $\left(\frac{g}{f}\right)(x)$. Also find $(f+g)(-1), (fg)(0), \left(\frac{f}{g}\right)(-1), \left(\frac{g}{f}\right)\left(\frac{1}{2}\right)$.



17. Find the sum and product of identity function and the modulus function.



2. Define a function as a correspondence between two sets.



4. Let $A = \{1, 2, 3, 4\}, B = \{1, 5, 9, 11, 15, 16\}$ and $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$. Are the following true?(i) f is a

relation from A to B(ii) f is a function from A to B.Justify y



5. Let $X = \{-1, 0, 3, 7, 9\}$ and $f: X \to R: f(x) = x^3 + 1$.

Express the function f as set of ordered pairs.



6. Let $A = \{-1, 0, 1, 2\}$ and $B = \{2, 3, 4, 5\}$. Find which of the following are functions from A to B. Given reason. (i) $f = \{(-1, 2), (-1, 3), (0, 4), (1, 5)\}$ (ii) $g = \{(0, 2), (1, 3), (2, 4)\}$ (iii) $h = \{(-1, 2), (0, 3), (1, 4), (2, 5)\}$

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7.

$$A = \{1,2\} ext{ and } B = \{2,4,6\}. ext{ Let } f = \{x,y\} {:} x \in A, y \in B ext{ and } and y$$

. Write f as a set of ordered paris.

Show that f is a relation but not a function from A to B.

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Let

$$A = \{0, 1, 2\} \,\, ext{and} \,\, B = \{3, 5, 7, 9\}. \,\, ext{Let} \,\,\, f = \{(x, y) \colon x \in A, \, y \in B \,\, ext{and} \,\, y\}$$

. Write f as a set of ordered pairs.

Show that f is a function from A to B. Find dom (f) and range (f).

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9. Let $A = \{2, 3, 5, 7\}$ and $B = \{3, 5, 9, 13, 15\}$.

 ${\sf Let}\; f=\{(x,y)\!:\!x\in A, y\in B\; {\rm and}\; y=2x-1\}.$

Write f in roster form. Show that f is a function from A to B. Find the domain and range of f.

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10. Let $g = \{(1, 2), (2, 5), (3, 8), (4, 10), (5, 12), (6, 12)\}.$

Is g a function? If yes, find its domain and range. If no, give reason.

11. Let $f = \{(0, -5), (1-2), (2, 1), (3, 4), (4, 7)\}$ be a linear function

from Z into Z. Write an expression for f.



12. If
$$f(x)=x^3$$
, find the value of $\displaystyle rac{\{f(5)-f(1)\}}{(5-1)}$

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13. If
$$f(x)=x^2$$
 then the value of $\displaystyle rac{f(1.1)-f(1)}{1.1-1}$

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14. Let $A = \{12, 13, 14, 15, 16, 17\}$ and $f: A \overrightarrow{Z}$ be function given by f(x) = highest prime factor of x. Find range of f.

15. Let $f\colon R^+ o R$ is a function defined as $f(x)=\log x.$ Find (i) Image of domain of f, (ii) $(x\colon f(x)=-2)$ (iii) f(xy)=f(x)+f(y)

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16. Let $f:R\overrightarrow{R}$ be such that $f(x)=2^x$. Determine: Range of f (ii) $\{x:f(x)=1\}$ (iii) Whether f(x+y)=f(x)f(y) holds.

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17. Let $f\colon o R o R$: $f(x) = x^2 \, ext{ and } g \colon C o C \colon g(x) = x^2$, where C is

the set of all complex numbers.

Show that $f \neq g$.

18. f, g and h are three functions defined from R to R as follows:

(i)
$$f(x)=x^2$$
 (ii) $g(x)=x^2+1$ (iii) $h(x)=\sin x$

Then, find the range of each function.

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19. Let $f\colon R o R\colon f(x)=x^2+1.$ Find (i) $f^{-1}\{10\}$ (ii) $f^{-1}\{-3\}$

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20. The function $F(x)=rac{9x}{5}+32$ is the formula to convent $x^\circ C$ to

Fahrenheit units. Find

(i) F(0), (ii) F(-10), (iii) the value of x when f(x)=212.

Interpret the result in each case.





1. If $f(x) = x^2 - 3x + 4$, then find the values of x satisfying the equation f(x) = f(2x + 1).

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2. If
$$f(x) = \frac{x-1}{x+1}$$
 , then show that $f\left(\frac{1}{x}\right) = -f(x)$ (ii) $f\left(-\frac{1}{x}\right) = \frac{1}{f(x)}$

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3. If
$$f(x) = x^3 - rac{1}{x^3}$$
 show that $f(x) + figg(rac{1}{x}igg) = 0$

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4. If
$$f(x) = rac{x+1}{x-1}$$
 , show that $f[f\{f(x)\}] = x$

5. If
$$f(x)=rac{1}{2x+1},\ x
eq-rac{1}{2},\ ext{then show that}\ f(f(x))=rac{2x+1}{2x+3}$$
 , provided that $x
eq-rac{3}{2}.$

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6. If
$$f(x)=rac{1}{1-x}$$
 , show that $f[f\{f(x)\}]=x$.

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7. If
$$f(x)=rac{2x}{1+x^2}$$
 , show that $f(an heta)=\sin 2 heta)$

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8. If
$$y=f(x)=rac{3x+1}{5x-3}$$
, prove that x=f(y).

1. Find the domain of the following real functions:

(i)
$$f(x) = \frac{3x+5}{x^2-9}$$
 (ii) $f(x) = \frac{2x-3}{x^2+x-2}$
(iii) $f(x) = \frac{x^2-2x+1}{x^2-8x+12}$ (iv) $f(x) = \frac{x^3-8}{x^2-1}$

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2. Find the domain of the range of each of the following real functions:

$$f(x) = rac{1}{x}$$

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3. Find the domain of the range of each of the following real functions:

$$f(x) = rac{1}{x-5}$$

4. Find the domain of the range of each of the following real functions:

$$f(x)=rac{x-3}{2-x}$$

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5. Find the domain of the range of each of the following real functions:

$$f(x)=rac{3x-2}{x+2}$$

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6. Find the domain of the range of each of the following real functions:

$$f(x)=rac{x^2-16}{x-4}$$

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7. Find the domain of the range of each of the following real functions:

$$f(x)=rac{1}{\sqrt{2x-3}}$$

8. Find the domain and range of each of the following real valued function: $f(x) = \frac{ax - b}{cx - d}$.

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9. Find the domain of the range of each of the following real functions:

$$f(x) = \sqrt{3x - 5}$$

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10. Find the domain of the range of each of the following real functions:

$$f(x)=\sqrt{rac{x-5}{3-x}}$$

11. Find the domain and range of each of the following real functions:

$$f(x)=rac{1}{\sqrt{x^2-1}}$$

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12. Find the domain of the range of each of the following real functions:

 $f(x)=1-\left|x-2\right|$

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13. Find the domain of the range of each of the following real functions:

$$f(x)=rac{|x-4|}{x-4}$$

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14. Find the domain of the range of each of the following real functions:

$$f(x)=\frac{x^2-9}{x-3}$$



3. Find the domain and the range of the square root function,

 $f\!:\!R^+\cup\{0\} o R\!:\!f(x)=\sqrt{x}$ fol all non-negative real numbers. Also,

draw its graph.

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4. Find the domain and the range of the cube root function,

 $f\colon R^+\cup\{0\} o R\colon f(x)=x^{1/3} \;\; ext{for all}\;\; x\in R.$ Also, draw its graph.

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Exercise 3 E

1. Let f, g : R ightarrow R be defined, respectively by f(x) = x + 1,g(x) = 2x3. Find f + g, fg and $rac{f}{g}$.

2. Let f(x) = 2x +5 and $g(x) = x^2 + x$. Describe i. f + g ii. f - g iii. fg iv. f/g. Find the domain in each case.

3. Let
$$f: R \to R: f(x) = x^3 + 1$$
 and $g: R \to R: g(x) = (x+1)$. Find
(i) $(f+g)(x)$ (ii) $(f-g)(x)$ (iii) $\left(\frac{1}{f}\right)(x)$ (iv) $\left(\frac{f}{g}\right)(x)$

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4. Let
$$f: R \to R: f(x) = \frac{x}{c}$$
 where c is a constant. (i) $(cf)(x)$ (ii) $(c^2f)(x)$ (iii) $(\frac{1}{c}f)(x)$

5. Let
$$f: [2, \infty) \to R$$
 and $g: [-2, \infty) \to R$ be to two real functions
defined $f(x) = \sqrt{x - 2}andg(x) = \sqrt{x + 2}$. Find $f + gandf - g$.

Exercise 3 F Very Short Answer Questions



2. Find the values of x for which the functions $f(x)=3x^2-1$ and

g(x)=3+x are equal

3. Let $X=\{-1,0,2,5\}$ and $f\!:\!X o R\!:\!f(x)=x^3+1$. Then, write f as a set of ordered pairs.



4. Let
$$A = \{-2, -1, 0, 1, 2\}$$
 and $f: A \to Z$ be given y
 $f(x) = x^2 - 2x - 3$. Find: i. the range of f ii. pre images of 6,-3 and 5.

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5. Let $f\!:\!R o R\!:\!f(x)=x^2.$ Determine (i) range (f) (ii) $\{x\!:\!f(x)=4\}$

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6. Let $f \colon R o R$ be defined as $f(x) = x^2 + 1$. Find:

7. Let $f \colon R^+ o R$ is a function defined as $f(x) = \log x$. Find (i) Image of

domain of f, (ii) $(x\!:\!f(x)=-2)$ (iii) f(xy)=f(x)+f(y)

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8. Let $A = \{6, 10, 11, 15, 21\}$ and Let $f: A \to N: f(n)$ is the highest

prime factor of n. Find range (f).

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9. Find the range of the function $f(x) = \sin x$.

A. [-1, 1]

B.(-1,1)

C.[3, 6]

D.[-3, 6]

Answer: A



14. If
$$f(x) = 1 - rac{1}{x}$$
 then write the value of $f\left(f\left(rac{1}{x}
ight)
ight)$.

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15. If
$$f(x) = \frac{kx}{x+1}$$
, where $x \neq -1$ and $f\{f(x)\} = x$ for $x \neq -1$

then find the value of k.

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16. Find the range of the function,
$$f(x) = rac{x}{|x|}$$

17. Find the domain of the function, $f(x) = \log \lvert x \rvert$.

18. If f is a real function satisfying $f\left(x+rac{1}{x}
ight)=x^2+rac{1}{x^2}$ for all $x\in R-\{0\}$, then write the expression for f(x) .

19. Find the domain and range of each of the following real valued function: $f(x) = \frac{ax+b}{bx-a}$.

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20. Find the domain and the range of the real function f defined by

$$f(x)=\sqrt{(x-1)}.$$

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21. Write the domain and the range of the function, f(x) = -|x|.

