



MATHS

BOOKS - RS AGGARWAL MATHS (HINGLISH)

MATHEMATICAL REASONING

Examples

1. Check whether the following sentences are statements. Give reasons for your answer.

- (i) The earth is a planet.
- (ii) There are 35 days in a month.
- (iii) Mathematics is difficult.
- (iv) Every square is a rectangle.
- (v) Every set is a finite set.
- (vi) Every real number is a complex number.
- (vii) 9 is less than 7.
- (viii) The square of a natural number is an even number.



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2. Check whether the following sentences are statements.

(i) Answer this question.

(ii) Everyone in this room is rich.

(iii) There is no rain without clouds.

(iv) Mathematics is fun.

(v) She is a commerce graduate.

(vi) $\sqrt{2}$ is a rational number.

(vii) Fire is always hot.

(viii) The sides of a quadrilateral have equal length.



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3. Let p : Kolkata is a city. Express the denial of p in three different ways.



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4. Write the negation of each of the following statements in two different ways.

(i) Africa is a continent.

(ii) $\sqrt{5}$ is rational.

(iii) All integers are rational numbers.

(iv) Some prime numbers are odd numbers.

(v) Everyone in Germany speaks German.



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5. Write the negation of each of the following statements:

p: All birds have wings.

q: For every real number x , $x^2 > x$.

r: There exists a real number x such that $0 < x < 1$.

s: For every real number x , either $x > 1$ or $x < 1$.



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6. Split each of the following compound statements into simple sentences and determine whether it is true or false: (i) The grass is green and the sky is blue. (ii) Agra is in Uttar Pradesh and Shimla is in Punjab. (iii) All rational numbers are real and all real numbers are not complex numbers. (iv) $x = 5$ and $x = 2$

are the roots of the equation

$$3x^2 - x - 10 = 0$$



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7. For each of the following statements, determine whether an inclusive 'or' or exclusive 'or' is used. Give reasons for your answer.

(i) For identification you need a passport or an Adhar Card.

(ii) The school is closed if it is a holiday or a

Sunday.

(iii) $\sqrt{3}$ is a rational number or an irrational number.

(iv) Two lines intersect at a point or are parallel.

(v) Students can take Sanskrit or French as their third language.



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8. Write down the truth set of each of the following open sentences :

(i) $p(x) : x + 5 < 9, x \in N.$

(ii) $p(x) : x + 3 < 3, x \in N.$

(iii) $p(x) : x + 5 > 7, x \in R.$

(iv) $p(x) : 2x^2 + 5x - 3 = 0, x \in I,$



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9. Use quantifiers to convert each of the following open sentences defined on N , into a true statement:

(i) $x+5=8$ (ii) $x^2 > 0$ (iii) $x + 2 < 4$



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10. Let $A=\{1,2,3,4\}$. Examine whether the statements given below are true or false.

(i) $\exists x \in A$ such that $x + 3 = 8$.

(ii) $\forall x \in A, x + 2 < 7$.

(iii) $\exists x \in A$ such that $x + 1 < 3$.

(iv) $\forall x \in A, x + 3 \geq 5$.



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11. Rewrite the following statement, with 'if ... , then' in five different ways conveying the same

meaning:

If a natural number is even, then its square is even.



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12. Write each of the following statements in the form 'if ... then':

(i) A quadrilateral is a parallelogram if its diagonals bisect each other.

(ii) The banana tree will bloom if it stays warm for a month.

(iii) There is traffic jam whenever it rains.

(iv) It is necessary to have a password to log on to the server.

(v) You can access the website only if you pay a subscription fee.



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13. Write the converse and contrapositive of each of the following statements:

(i) If n is an even number, then n^2 is even.

(ii) If two integers a and b are such that

$a > b$, then $(a - b)$ is always a positive integer.

(iii) If a $\triangle ABC$ is right angled at B, then

$$AB^2 + BC^2 = AC^2.$$

(iv) If $\triangle ABC$ and $\triangle DEF$ are congruent, then they are equiangular.

(v) You cannot comprehend geometry if you do not know how to reason deductively.

(vi) Something is cold implies that it has low temperature.



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14. Rewrite the following statements in the form: 'p if and only if q' p: A quadrilateral is equiangular. q: A quadrilateral is a rectangle.



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15. Given below are pairs of statements. In each case, combine them using 'if and only if'.

(i) p : In ΔABC , $\angle B = \text{and } \angle C \geq \angle A$.

q : In ΔABC , $AC = AB$.

(ii) p : A and B are two sets such that $A \subseteq B$

and $B \subseteq A$.

q: $A=B$.

(iii) p: ΔABC is equilateral.

q: ΔABC is equiangular.

(iv) p: $\{a \in \mathbb{R} \text{ such that } |a| < 2\}$.

q: $\{a \in \mathbb{R} \text{ such that } (a > -2 \text{ and } a < 2)\}$.



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16. Check whether the following statement is true or not. If $x, y \in \mathbb{Z}$ are such that x and y are odd, then xy is odd.



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17. Using the words necessary and sufficient rewrite the statement The integer n is odd if and only if n^2 is odd Also check whether the statement is true.



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18. By using the method of contradiction verify that P: $\sqrt{5}$ is irrational.



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19. Prove the following statement by contradiction method

p : The sum of an irrational number and a rational number is irrational .



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20. Using contradiction method , check the validity of the following statement : if n is a real number with $n > 3$ then $n^2 > 9$



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21. Consider the statement : p : If x a real number such that $x^3 + 4x = 0$ then $x=0$ prove that p is a true statement using :

(i) direct method

(ii) method of contradiction

(iii) method of contrapositive



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1. Which of the following sentences are statements? In case of a statement mention whether it is true or false.

(i) The sun is a star.

(ii) $\sqrt{7}$ is an irrational number.

(iii) The sum of 5 and 6 is less than 10.

(iv) Go to your class.

(v) Ice is always cold.

(vi) Have you ever seen the Red Fort?

(vii) Every relation is a function.

(viii) The sum of any two sides of a triangle is

always greater than the third side.

(ix) May God bless you!



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2. Which of the following sentences are statements? In case of a statement, mention whether it is true or false.

(i) Paris is in France.

(ii) Each prime number has exactly two factors.

(iii) The equation $x^2 + 5|x| + 6 = 0$ has no real roots.

(iv) $(2 + \sqrt{3})$ is a complex number.

(v) Is 6 a positive integer?

(vi) The product of -3 and -2 is -6.

(vii) The angles opposite to the equal sides of an isosceles triangle are equal.

(viii) Oh! it is too hot.

(ix) Monika is a beautiful girl.

(x) Every quadratic equation has at least one real root.



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3. Which of the following statements are true and which are false? In each case give a valid reason or your answer.

(i) p : $\sqrt{11}$ is an irrational number.

(ii) q : Circle is a particular case of an ellipse.

(iii) r : Each radius of a circle is a chord of the circle.

(iv) s : The centre of a circle bisects each chord of the circle.

(v) t : If a and b are integers such that $a < b$, then $-a > -b$.

(vi) u: The quadratic equation $x^2 + x + 1 = 0$ has no real roots.



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4. Write the negation of each of the following statements:

(i) Every natural number is greater than 0.

(ii) Both the diagonals of a rectangle are equal.

(iii) The sum of 4 and 5 is 8.

(iv) The number 6 is greater than 4.

- (v) Every natural number is an integer.
- (vi) The number -5 is a rational number.
- (vii) All cats scratch.
- (viii) There exists a rational number x such that $x^2 = 3$.
- (ix) All students study mathematics at the elementary level.
- (x) Every student has paid the fees.
- (xi) There is some integer k for which $2k = 6$.
- (xii) None of the students of this class has passed.



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Exercise 29 B

1. Split each of the following into simple sentences and determine whether it is true or false.

(i) A line is straight and extends indefinitely in both the directions.

(ii) A point occupies a position and its location can be determined.

(iii) The sand heats up quickly in the sun and does not cool down fast at night.

(iv) 32 is divisible by 8 and 12.

(v) $x = 1$ and $x = 2$ are the roots of the equation

$$x^2 - x - 2 = 0.$$

(vi) 3 is rational and $\sqrt{3}$ is irrational.

(vii) All integers are rational numbers and all rational numbers are not real numbers.

(viii) Lucknow is in Uttar Pradesh and Kanpur is in Uttarakhand.



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2. Split each of the following into simple sentences and determine whether it is true or

false. Also, determine whether an 'inclusive or' or 'exclusive or' is used.

(i) The sum of 3 and 7 is 10 or 11.

(ii) $(1 + i)$ is a real or a complex number.

(iii) Every quadratic equation has one or two real roots.

(iv) You are wet when it rains or you are in a river.

(v) 24 is a multiple of 5 or 8.

(vi) Every integer is rational or irrational.

(vii) For getting a driving licence you should have a ration card or a passport.

(viii) 100 is a multiple of 6 or 8.

(ix) Square of an integer is positive or negative.

(x) Sun rises or Moon sets.



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3. Find the truth set in case of each of the following open sentences defined on \mathbb{N} :

(i) $x + 2 < 10$ (ii) $x + 5 < 4$ (iii) $x + 3 > 2$



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4. Let $A = \{2, 3, 5, 7\}$. Examine whether the statements given below are true or false. (i)

$\exists x \in A$ such that $x + 3 > 9$. (ii) $\exists x \in A$ such that x is even. (iii)

$\exists x \in A$ such that $x + 2 = 6$. (iv)

$\forall x \in A, x$ is prime. (v) $\forall x \in A, x + 2 < 10$

(vi) $\forall x \in A, x + 4 \geq 11$



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Exercise 29 C

1. Rewrite the following statement in five different ways conveying the same meaning.

If a given number is a multiple of 6, then it is a multiple of 3.



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2. Write each of the following statements in the form 'if then':

(i) A rhombus is a square only if each of its angles measures 90° .

(ii) When a number is a multiple of 9, it is

necessarily a multiple of 3.

(iii) You get a job implies that your credentials are good.

(iv) Atmospheric humidity increases only if it rains.

(v) If a number is not a multiple of 3, then it is not a multiple of 6.



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3. Write the converse and contrapositive of each of the following:

(i) If x is a prime number, then x is odd.

(ii) If a positive integer n is divisible by 9, then the sum of its digits is divisible by 9.

(iii) If the two lines are parallel, then they do not intersect in the same plane.

(iv) If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

(v) If A and B are subsets of X such that $A \subseteq B$, then $(X-B) \subseteq (X-A)$.

(vi) If $f(2) = 0$, then $f(x)$ is divisible by $(x-2)$.

(vii) If you were born in India, then you are a citizen of India.

(viii) If it rains, then I stay at home.



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4. Given below are some pairs of statements.

Combine each pair using if and only if:

(i) p: If a quadrilateral is equiangular, then it is a rectangle.

q: If a quadrilateral is a rectangle, then it is equiangular.

(ii) p: If the sum of the digits of a number is divisible by 3, then the number is divisible by 3.

q: If a number is divisible by 3, then the sum of its digits is divisible by 3.

(iii) p: A quadrilateral is a parallelogram if its diagonals bisect each other.

q: If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

(iv) p: If $f(a) = 0$, then $(x - a)$ is a factor of polynomial $f(x)$.

q: If $(x - a)$ is a factor of polynomial $f(x)$, then $f(a) = 0$.

(v) p: If a square matrix A is invertible, then $|A|$ is nonzero.

q: If A is a square matrix such that $|A|$ is nonzero, then A is invertible.



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5. Write each of the following using 'if and only if':

(i) In order to get A grade, it is necessary and sufficient that you do all the homework regularly.

(ii) If you watch television then your mind is free and if your mind is free then you watch television.



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Exercise 29 D

1. Using contrapositive method prove that, if n^2 is an even integer, then n is also an even integer.



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2. Show that the statement For any real numbers a and b , $a^2 = b^2$ implies that $a = b$ is not true by giving a counter example



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3. By giving a counter example, show that the following statement is false. If n is an odd integer, then n is prime.



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4. Use contradiction method to prove that:

$p: \sqrt{3}$ is irrational is a true statement.



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5. By giving a counter-example, show that the following statement is false:

p: If all the sides of a triangle are equal, then the triangle is obtuse angled.



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