# MATHS

# BOOKS - RS AGGARWAL MATHS (HINGLISH)

# APPLICATIONS OF DERIVATIVES

## Solved Examples

**1.** The rate of change of the area of a circle with respect to its radius r at $r = 6$cm is

A. $10\pi$

B. $12\pi$

C. $8\pi$

D. $11\pi$

**Answer: B**

Watch Video Solution

**2.** A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm per second. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?

Watch Video Solution

**3.** A spherical soap bubble is expanding so that its radius is increasing at the rate of 0.02 cm/sec. At what rate is the surface area increasing when its radius is 5 cm ? (Take $\pi = 3.14$)

Watch Video Solution

**4.** 6) The volume of a spherical balloon is increasing at the rate of 20cm / sec. Find the rate of change of its surface area at the instant when its radius is 8 cm.

Watch Video Solution

**5.** The surface area of a spherical bubble is increasing at the rate of 2 $cm^2 / s$ . When the radius of the bubble is 6cm, at what rate is the volume of the bubble increasing?

Watch Video Solution

**6.** The volume of a cube is increasing at a rate of $7cm^3 / se \cdot$ How fast is the surface area increasing when the length of an edge is

12cm?

Watch Video Solution

**7.** The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle

Watch Video Solution

**8.** Water is leaking from a conical funnel at the rate of $5c\frac{m^3}{\sec}$. If the radius of the base of the funnel is 5 cm and its altitude is 10 cm, find the rate at which the water level is dropping when it is $2.5$ cm from the top.

Watch Video Solution

**9.** Sand is pouring from a pipe at the rate of 12 $cm^3/s$ . The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when t

Watch Video Solution

**10.** A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

Watch Video Solution

**11.** The two equal sides of an isosceles triangle with fixed base $b$ are decreasing at the rate of $3cm/s$. How fast is the area decreasing when the two equal sides are equal to the base?

Watch Video Solution

**12.** A point source of light along a straight road is at a height of 'a' metres. A boy 'b' metres in height is walking along the road. How fast is his shadow increasing if he is walking away from the light at the rate of 'c' metres per minute ?

Watch Video Solution

**13.** A man 160 cm tall, walks away from a source of light situated at the top of a pole 6m high at the rate of 1.1 m/sec. How fast is the length of his shadow increasing when he is 1 metre away from the pole.

Watch Video Solution

**14.** A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

Watch Video Solution

**15.** Find the point on the curve $y^2$ $\cdot$=$8x\cdot$ for which the abscissa and ordinate change at the same rate.

Watch Video Solution

**16.** The points of the ellipse $16x^2 + 9y^2 = 400$ at which the ordinate decreases at the same rate at which the abcissa increases is

Watch Video Solution

**17.** The total cost $C(x)$ of producing $x$ items in a firm is given by $C(x) = 0.0005x^3 - 0.002x^2 + 30x + 6000$ Find the marginal cost when $4$ units are produced

Watch Video Solution

**18.** The total revenue received from the sale of x units of a product is given by

$R(x) = 3x^2 + 40x + 10$

Find the marginal revenue when x = 5

Watch Video Solution

**19.** Using differentials, find the approximate value of $(82)^{\frac{1}{4}}$ upto 3 places of decimal .

Watch Video Solution

**20.** Use differentials to approximate the cube root of 127.

Watch Video Solution

**21.** Using differentials, find the approximate value of $\sqrt{26}$

Watch Video Solution

**22.** Using differentials, find the approximate value of $\sqrt{0.037}$, correct upto thre decimal places.

Watch Video Solution

**23.** Using differentials find the approximate value of $\tan 46^{0}$, if it is being given that $1^{0} = 0.01745$ radians.

Watch Video Solution

**24.** Using differentials, find the approximate value of $(\log)_{10}10.1$, it being given that $(\log)_{10}e = 0.4343$ .

Watch Video Solution

**25.** If $f(x) = 3x^2 + 15x + 5$, then find the approximate value of $f(3.02)$ using differentials

Watch Video Solution

**26.** If radius of a circle increases from 5 to 5.1 , find the increase in area.

Watch Video Solution

**27.** If $y = x^4 - 12$ and if $x$ changes from 2 to 1.99, what is the approximate change in $y$

Watch Video Solution

**28.** If there is an error of 2% in measuring the length of simple pendulum, then percentage error in its period is: 1% (b) 2% (c) 3% (d) 4%

Watch Video Solution

**29.** If the error committed in measuring the radius of a circle be $0.01\%$, find the corresponding error in calculating the area.

A. $0.02\%$

B. $0.03\%$

C. $0.04\%$

D. $0.05\%$

**Answer: A**

Watch Video Solution

**30.** If in a triangle $ABC$, the side $c$ and the angle $C$ remain constant, while the remaining elements are changed slightly, show that $\frac{da}{\cos A}+\frac{db}{\cos B}=0.$

Watch Video Solution

**31.** The area S of a triangle is calculated by measuring the sides b and c, and $\angle A$. If there be an error $\delta A$ in the measurement of $\angle A$, show that the relative error in area is given by

$$\frac{\delta S}{S}=\cot A.\,\delta A$$

Watch Video Solution

**32.** Verify Rolles theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ on the interval [1, 3].

Watch Video Solution

**33.** Verify Rolle's therorem for the function $f(x) = x(x-1)^2$ in the interval [0,1]

Watch Video Solution

**34.** Verify Rolles theorem for the function $f(x) = (x-a)^m (x-b)^n$ on the interval $[a, b]$, where $m, n$ are positive integers.

Watch Video Solution

**35.** Verify Rolle's theorem for each of the following functions:

(i) $f(x) = \sin 2x \quad \text{in} \quad \left[0, \frac{\pi}{2}\right]$

(ii) $f(x) = (\sin x + \cos x) \quad \text{in} \quad \left[0, \frac{\pi}{2}\right]$

(iii) $f(x) = \cos 2\left(x - \frac{\pi}{4}\right) \quad \text{in} \quad \left[0, \frac{\pi}{2}\right]$

(iv) $f(x) = (\sin x - \sin 2x) \quad \text{in} \quad [0, \pi]$

Watch Video Solution

**36.** Verify Rolle's theorem for each of the following functions:

(i) $f(x) = \sin^2 x \quad \text{in} \quad 0 \leq x \leq \pi$

(ii) $f(x) = e^x \cos x \quad \text{in} \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

(iii) $f(x) = \frac{\sin x}{e^x} \quad \text{in} \quad 0 \leq x \leq \pi$

Watch Video Solution

**37.** verify Rolle's theorem for the function $f(x) = x(x+3)e^{-\frac{x}{2}}$ in $[-3, 0]$

Watch Video Solution

**38.** Verify Rolles theorem for function $f(x) = \log(x^2+2) - \log 3$ on $[-1, 1]$

Watch Video Solution

**39.** Verify Rolle's theorem for the following functions

(i) $f(x) = \sqrt{4 - x^2} \text{ in } [-2, 2]$

(ii) $f(x) = \log\left(\frac{x^2+ab}{(a+b)x}\right] \text{ in } [a, b], \text{ where } 0 < a < b$

Watch Video Solution

**40.** Verify Rolle's theorem for the function

$f(x) = 2x^3 + x^2 - 4x - 2.$

Watch Video Solution

**41.** Discuss the applicability of Rolle's theorem to the functions :

(i) $f(x) = x^2 \text{in } [1, 2]$

(ii) $f(x) = x^{2/3} \text{ in } [-1, 1]$

Watch Video Solution

**42.** Discuss the applicability of Rolle's theorem on :

(i) $f(x) = |x| \text{ in } [-1, 1]$ (ii) $f(x) = \tan x \text{ in } [0, \pi]$

Watch Video Solution

**43.** Discuss the applicability of Rolle's theorem on the function

$$f(x) = \begin{cases} (x^2+1) & \text{when } 0 \le x \le 1 \\ (3-x) & \text{when } 1 < x \le 2 \end{cases}$$

Watch Video Solution

**44.** If Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + ax + 5$ on [1, 3] with $c = \left(2 + \frac{1}{\sqrt{3}}\right)$, find the value of a and b

Watch Video Solution

**45.** At what points on the curve $y = (\cos x - 1)$ in $[0, 2\pi]$, is the tangent parallel to the x-axis?

Watch Video Solution

46. Verify Lagrange's mean-value theorem for the given functions: (i) $f(x) = x(2 - x)$ in $[0, 1]$

Watch Video Solution

47. Verify the hypothesis and conclusion of Lagrange's mean-value theorem for the function $f(x) = \frac{1}{(4x - 1)}, 1 \leq x \leq 4$

Watch Video Solution

48. Find 'c' of the mean -value theorem for the functions

(i) $f(x) = 2x^2 - 10x + 29$ in $[2, 7]$

(ii) $f(x) = x(x - 1)(x - 2)$ in $\left[0, \frac{1}{2}\right]$

Watch Video Solution

**49.** Using LaGrange's mean value theorem, find a point on the curve $y = \sqrt{x-2}$ defined on the interval [2,3], where the tangent is parallel to the chord joining the end points of the curve.

Watch Video Solution

**50.** Find a point on the parabola $y = (x-3)^2$, where the tangent is parallel to the chord joining (3, 0) and (4, 1)

Watch Video Solution

**51.** Without using the derivative, find the maximum or minium values, if any of the function $f(x) = 4x^2 - 4x + 7$ for all $x \in R$

Watch Video Solution

**52.** Find the maximum and minimum values, if any, of the following functions given by (i) $f(x) = |x + 2| 1$ (ii) $g(x) = |x + 1| + 3$ (iii) $h(x) = s \in (2x) + 5$ (iv) `f" "(x)" "=" "|" "s in" "4x" "+"

Watch Video Solution

**53.** Find the local maxima or local minima, if any, of

(i) $f(x) = \dfrac{1}{(x^2 + 2)}$ (ii) $f(x) = (x^3 - 3x)$

In each case, find the local maximum or the local minimum values, as the case may be

Watch Video Solution

**54.** Find the local maxima or local minima of $f(x) = x^3 - 6x^2 + 9x + 15$ Also, find the local maximum or local minimum values as the case may be

Watch Video Solution

**55.** Find the points of local maxima, local minima and the points of inflection of the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$ . Also, find the corresponding local maximum and local minimum values

Watch Video Solution

**56.** Find all the points of local maxima and local minima as well as the corresponding local maximum and local minimum values for the function $f(x) = (x - 1)^3 (x + 1)^2$ .

Watch Video Solution

Watch Video Solution

**57.** Find all the points of local maxima and minima and the corresponding maximum and minimum values of the function $f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105.$

Watch Video Solution

**58.** Find the local maxima and local minima of the functions:

(i) $f(x) = \sin 2x, \quad \text{when} \quad 0 < x < \pi$

(ii) $f(x) = (\sin 2x - x), \quad \text{when} \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Watch Video Solution

**59.** Find the local maxima and local minima of the functions:

(i) $f(x) = (\sin x - \cos x), \quad \text{When} \quad 0 < x < \frac{\pi}{2}$

(ii) $f(x) = (2\cos x + x), \quad \text{when} \quad 0 < x < \pi$

Watch Video Solution

**60.** Find the point of local maxima or local minima of the function

$f(x) = \left(\sin^4 x + \cos^4 x\right) \quad \text{in} \quad 0 < x < \frac{\pi}{2}$

A.

$\therefore x = (\pi/4)$ is a point of local minimum.

B.

$\therefore x = (\pi/4)$ is a point of local maxima.

C.

$\therefore x = (\pi/2)$ is a point of local minimum.

D.

$\therefore x = (\pi/2)$ is a point of local mixima.

**Answer: A**

Watch Video Solution

**61.** Find the local maxima and local minima, and the corresponding local maximum and local minimum values of the following functions:

(i) $f(x) = x\sqrt{1-x}$, where $x > 0$

(ii) $f(x) = \dfrac{x}{(x-1)(x-4)}, \quad \text{where} \quad 1 < x < 4$

Watch Video Solution

**62.** prove that maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{\frac{1}{e}}$

Watch Video Solution

63. Find the point on the parabola $y^2 = 2x$ which is closest to the point (1, 4)

Watch Video Solution

64. Prove that the following functions do not have maxima or minima:(i) $f(x) = ex$ (ii) $g(x) = \log x$(iii) $h(x) = x^3 + x^2 + x + 1$

Watch Video Solution

65. Show that $s \in^p \theta \cos^q \theta$ attains a maximum, when $\theta = \tan^{-1}\sqrt{\frac{p}{q}}$.

Watch Video Solution

**66.** Find both the maximum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$on the interval [0, 3].

Watch Video Solution

**67.** Find the maximum and minimum values of $x + s \in 2x$on $[0, 2\pi]$.

Watch Video Solution

**68.** Show that $f(x) = \sin x(1 + \cos x)$ is maximum at $x = \frac{\pi}{3}$ in the interval $[0,\ \pi]$ .

Watch Video Solution

**69.** Amongst all pairs of positive number with sum 24, find those whose product is maximum

Watch Video Solution

**70.** Amongst all pairs of positive numbers with product 256, find those whose sum is the least.

Watch Video Solution

**71.** Find two positive numbers $x$ and $y$ such that $x + y = 60$ and $xy^3$ is maximum.

Watch Video Solution

**72.** Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

Watch Video Solution

**73.** Show that all the rectangles with a given perimeter, the square has the largest area.

Watch Video Solution

**74.** Show that of all the rectangles of given area, the square has the smallest perimeter.

Watch Video Solution

**75.** Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

Watch Video Solution

**76.** If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

Watch Video Solution

**77.** Two sides of a triangle are given. The angle between them such that the area is maximum, is given by

Watch Video Solution

**78.** Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

Watch Video Solution

**79.** Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

Watch Video Solution

**80.** The combined resistance R of two resistors $R_1$ and $R_2$ where $R_1, R_2 > 0$ is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ If $R_1 + R_2 = C$ (constant), show that the maximum reistance R is obtained by chossing $R_1 = R_2$

Watch Video Solution

**81.** A beam of length $l$ is supported at one end. If $W$ is the uniform load per unit length, the bending moment $M$ at a distance $x$ from the end is given by $M = \frac{1}{2}lx - \frac{1}{2}Wx^2$. Find the point on the beam at which the bending moment has the maximum value.

Watch Video Solution

**82.** A wire of length 25m is to be cut into two pieces. One of the wires is to be made into a square and the other into a circle. What should be the lengths of the two pieces so that the combined area of the square and the circle is minimum ?

Watch Video Solution

**83.** Show that a cylinder of a given volume which is open at the top has minimum total surface area, when its height is equal to the radius of its base.

Watch Video Solution

**84.** Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.

Watch Video Solution

**85.** Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\frac{\cos^{-1} 1}{\sqrt{3}}$

Watch Video Solution

**86.** Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

Watch Video Solution

**87.** Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

Watch Video Solution

**88.** Show that the height of a closed right circular cylinder of given surface and maximum volume, is equal to the diameter of its base.

Watch Video Solution

**89.** A closed cylinder has volume 2156cm3. What will be the radius of its base so that its total surface area is minimum?

Watch Video Solution

**90.** Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2\frac{R}{\sqrt{3}}$ . Also find maximum volume.

Watch Video Solution

**91.** Show that the cone of greatest volume which can be inscribed in a given sphere is such that three times its altitude is twice the diameter of the sphere. Find the volume of the largest come inscribed ina sphere of radius R.

Watch Video Solution

**92.** Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

Watch Video Solution

**93.** An open box is to be made out of a piece of cardboard measuring $(24cm \times 24cm)$ by cutting off equal square from the corners and turning up the sides. Find the height of the box when it has maximum volume.

Watch Video Solution

**94.** Show that $f(x) = 3x + 5$ is a strictly increasing function on R

Watch Video Solution

**95.** Show that the function $f(x) = e^x$ is strictly increasing on R.

Watch Video Solution

**96.** Show that $f(x) = e^{-x}$ is a strictly decreasing function on Rgt

Watch Video Solution

**97.** Show that the function $f(x) = a^x$, $a > 1$ is strictly increasing on $R$.

Watch Video Solution

**98.** If a is real number such that $0 < a < 1$, show that the function $f(x) = a^x$ is strictly decreasing on R.

Watch Video Solution

**99.** Show that the function $f(x) = \left(x^3 - 6x^2 + 12x - 18\right)$ is an increasing function on R.

Watch Video Solution

**100.** Show that the function $f(x) = e^x$ is strictly increasing on R.

Watch Video Solution

**101.** Show that $f(x) = e^{1/x}$ is a strictly decreasing function for all $x > 0$

Watch Video Solution

**102.** Show that $f(x) = (x-1)e^{x} + 1$ is an increasing function for all $x > 0$.

Watch Video Solution

**103.** Show that $f(x) = x - \sin x$ is increasing for all $xR$.

Watch Video Solution

**104.** Prove that the function $f(x) = \cos^2 x$ is strictly decreasing in $\left(0, \frac{\pi}{2}\right)$

Watch Video Solution

**105.** Show that $f(x) = \log \sin x$ is increasing on $(0, \pi/2)$ and decreasing on $(\pi/2, \pi)$ .

Watch Video Solution

**106.** Show that $f(x) = \sin x$ is increasing on $(0, \pi/2)$ and decreasing on $(\pi/2, \pi)$ and neither increasing nor decreasing in $(0, \pi)$ .

Watch Video Solution

**107.** Show that the function $x^2 - x + 1$ is neither increasing nor decreasing on $(0, 1)$ .

Watch Video Solution

**108.** Prove that the function $f(x) = 10^x$ is strictly increasing on R

Watch Video Solution

**109.** Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$is always an strictly increasing function in $\left(0, \frac{\pi}{4}\right)$.

Watch Video Solution

**110.** Find the intervals on which the function $f(x) = 10 - 6x - 2x^2$ is (a) strictly increasing (b) strictly decreasing.

Watch Video Solution

**111.** Find the intervals in which the given functions are strictly increasing decreasing: $-2x^3 - 9x^2 - 12x + 1$

Watch Video Solution

**112.** Find the intervals on which the function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is (a) increasing (b) decreasing.

Watch Video Solution

**113.** find intervals in which function $x^3 + 2x^2 - 1$ is increasing and decreasing?

Watch Video Solution

**114.** Find the intervals on which the function $f(x) = x^3 + 3x^2 - 105x + 25$ is (a) increasing (b) decreasing

Watch Video Solution

**115.** Find the intervals in which $f(x) = 5 + 36x + 3x^2 - 2x^3$ is increasing or decreasing.

A. f(x) is increasing on $[-2, 7]$ and decreasing on $[-\infty, 2] \cup [3, \infty]$

B. f(x) is increasing on $(-2, 3)$ and decreasing on $(-\infty, -2) \cup (3, \infty)$

C. f(x) is increasing on $[-8, 3]$ and decreasing on $[-\infty, -2] \cup [3, \infty]$

D. f(x) is increasing on $[-2, 3]$ and decreasing on $[-\infty, -2] \cup [3, \infty]$

**Answer: D**

Watch Video Solution

**116.** Find the intervals on which the function $f(x) = (x+1)^3(x-3)^3$ is (a) increasing (b) decreasing

Watch Video Solution

**117.** Find the intervals in which $f(x) = \frac{4x^2+1}{x}$ is increasing or decreasing.

Watch Video Solution

**118.** Find the intervals on which the function $f(x) = \frac{x}{(x^2+1)}$ is

(a) increasing (b) decreasing

Watch Video Solution

**119.** Find the intervals in which $f(x) = (x+2)e^{-x}$ is increasing

or decreasing.

Watch Video Solution

**120.** Find the intervals in which the $f(x) = \log(1+x) - \frac{x}{1+x}$

is (i) increasing (ii) decreasing

Watch Video Solution

121. Find the intervals in which $f(x) = \sin x - \cos x,$ where `0

Watch Video Solution

122. Separate the interval $[0, \pi/2]$ into sub-intervals in which $f(x) = s \in^4 x + \cos^4 x$ is increasing or decreasing.

Watch Video Solution

123. Separate $\left[0, \frac{\pi}{2}\right]$ into subintervals in which $f(x) = \sin 3x$ is (a) increasing (b) decreasing

Watch Video Solution

124. Prove that $\tan x > x$ for all $x \in \left[0, \frac{\pi}{2}\right]$

Watch Video Solution

**125.** Determine the values of $x$ for which $f(x) = x^x, \ x > 0$ is increasing or decreasing.

Watch Video Solution

**126.** Find the intervals for which $f(x) = x^4 - 2x^2$ is increasing or decreasing.

Watch Video Solution

**127.** Prove that $\dfrac{x}{(1+x)} < \log(1+x) < x \quad \text{for} \quad x > 0$

Watch Video Solution

**128.** Find the equations of the tangent and the normal to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at the point (1, 3)

Watch Video Solution

**129.** Find the equation of the tangent $y = x^2 + 4x + 1$ and the normal to the curve $y = x^2 + 4x + 1$ at the point where $x = 3$

A. equation of the tangent $y - 10x + 8 = 0$ and equation of the normal $15x + 10y - 223 = 0$

B. equation of the tangent $y - 10x + 8 = 0$ and equation of the normal $x + 10y - 223 = 0$

C. equation of the tangent $y - 10x - 8 = 0$ and equation of the normal $x - 10y - 223 = 0$

D. equation of the tangent $y - 15x + 8 = 0$ and equation of the normal $x + 10y - 223 = 0$

**Answer: B**

Watch Video Solution

**130.** Show that the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(x_1, y_1)$ is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

Watch Video Solution

**131.** Find the equation of the tangent line to the curve $y = \sqrt{5x - 3} - 2$ which is parallel to the line $4x - 2y + 3 = 0$

Watch Video Solution

**132.** Find the equation(s) of normal(s) to the curve $3x^2 - y^2 = 8$ which is (are) parallel to the line $x + 3y = 4$.

Watch Video Solution

**133.** Prove that $\left(\frac{x}{a}\right)^n = \left(\frac{y}{b}\right)^N = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ for all $nN$, at the point $(a, b)$.

Watch Video Solution

**134.** At what point will be tangents to the curve $y = 2x^3 - 15x^2 + 36x - 21$ by parallel to x=axis? Also, find the equations of the tangents to the curve at these points.

Watch Video Solution

**135.** Prove that points of the curve $y^2 = 4a\left\{x + a\sin\left(\frac{x}{a}\right)\right\}$ at which tangents are parallel to x-axis lie on the parabola.

Watch Video Solution

**136.** Tangents are drawn from the origin to the curve $y = \sin x$. Prove that their points of contact lie on the curve $x^2y^2 = (x^2 - y^2)$

Watch Video Solution

**137.** Determine the points on the curve $2y = (3 - x^2)$ at which the tangent is parallel to the line $x + y = 0$

Watch Video Solution

**138.** Find the points on the curve $4x^2 + 9y^2 = 1$ , where the tangents are perpendicular to the line $2y + x = 0$ .

Watch Video Solution

**139.** Find the coordinates of the points on the curve $y = x^2 + 3x + 4,$ the tangents at which pass through the origin.

Watch Video Solution

**140.** If the straight line $x \cos \alpha + y \sin \alpha = p$ touches the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , then prove that $a^2 \cos^2 \alpha + b^2 \sin^2 \alpha = p^2$.

Watch Video Solution

**141.** if the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $x^m y^n = a^{m+n}$ prove that $p^{m+n} m^m n^n = (m+n)^{m+n} a^{m+n} \sin^n \alpha \cos^m \alpha$

Watch Video Solution

**142.** Find the equation of the normal to the curve $y = 2\sin^2 3x \text{at} \quad x = \frac{\pi}{6}$

Watch Video Solution

**143.** Find the equations of the tangent and the normal to the curve $y(x-2)(x-3) - x + 7 = 0$ at the point where it cuts the x-axis

Watch Video Solution

**144.** Show that the line $\frac{d}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-\frac{x}{a}}$ at the point where it crosses the y-axis.

Watch Video Solution

**145.** Find the equation of the tangent and the normal at the point 't, on the curve $x = a\sin^3 t, y = b\cos^3 t.$

Watch Video Solution

**146.** Find the equations of the tangent and normal to the curve $x = a\sin 3t, y = \cos 2t \text{ at } t = \frac{\pi}{4}$

Watch Video Solution

**147.** For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangents pass through the origin.

Watch Video Solution

## Exercise 11 A

**1.** The side of a square in increasing at the rate of 0.2 cm/sec. Find the rate of increase of the perimeter of the square.

Watch Video Solution

**2.** The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

Watch Video Solution

**3.** The radius of a circle is increasing uniformly at the rate of 0.3 centimetre per second. At what rate is the area increasing when the radius is 10 cm ? (Take $\pi = 3.14$)

Watch Video Solution

**4.** The side of a square sheet of metal is increasing at 3 centimetres per minute. At what rate is the area increasing when the side is 10 cm long ?

Watch Video Solution

**5.** The radius of a spherical soap bubble is increasing at the rate of 0.2 cm/sec. Find the rate of increase of its surface area, when the radius is 7 cm.

Watch Video Solution

**6.** The radius of an air bubble is increasing at the rate of 0.5 cm/sec. At what rate is the volume of the bubble increasing when the radius is 1 cm?

Watch Video Solution

**7.** The volume of a spherical balloon is increasing at a rate of $25cm^3 / \sec$. Find the rate of increase of its curved surface when the radius of balloon is 5 cm.

Watch Video Solution

**8.** A balloon which always remains spherical, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15 cm.

Watch Video Solution

**9.** The bottom of a rectangular swimming tank is 25 m by 40m. Water is pumped into the tank at the rate of 500 cubic metres per minute. Find the rate at which the level of water in the tank is rising.

Watch Video Solution

**10.** A stone is dropped into a quiet lake and waves move in circles at a speed of 3.5 cm per second. At the instant when the radius of the circular wave is 7.5 cm, how fast is the enclosed area increasing ? (Take $\pi = 22/7$)

Watch Video Solution

**11.** A man 2 metres high walks at a uniform speed of 5 km/hr away from a lamp-post 6 metres high. Find the rate at which the length of his shadow increases.

Watch Video Solution

**12.** An inverted cone has a depth of 40 cm and a base of radius 5 cm. Water is poured into it at a rate of 1.5 cubic centimetres per minutes. Find the rate at which the level of water in the cone is rising when the depth is 4 cm

Watch Video Solution

**13.** Sand is pouring from a pipe at the rate of $18cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is one-sixth of the radius of the base. How fast is the height of the sand cone increasing when its height is 3 cm?

Watch Video Solution

**14.** Water is dripping out from a conical funnel at a uniform rate of $4cm^3/cm$ through a tiny hole at the vertex in the bottom. When the slant height of the water is 3cm, find the rate of decrease of the slant height of the water-cone. Given that the vertical angle of the funnel is $120^0$.

Watch Video Solution

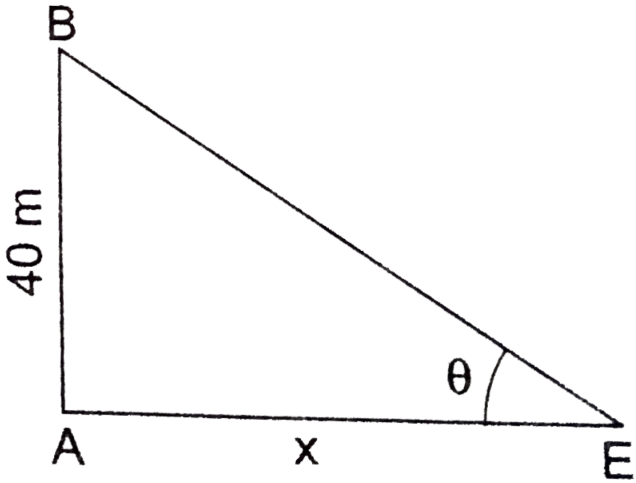
**15.** Oil is leaking at the rate of 16 mL/s from a vertically kept cylindrical drum containing oil. If the radius of the drum is 7 cm and its height is 60 cm, find the rate at which the level of the oil is changing when the oil level is 18 cm

Watch Video Solution

**16.** A 13-m long ladder is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 m/s. How fast is its height on the wall decreasing when the foot of the ladder is 5m aways from the wall ?

Watch Video Solution

**17.** A man is moving away from a 40-m high tower at a speed of 2 m/s. Find the rate at which the angle of elevation of the top of the tower is changing when he is at a distance of 30 metres from the foot of the tower. Assume that the eye level of the man is 1.6 m from the ground.




Watch Video Solution

**18.** Find an angle `theta,0

Watch Video Solution

**19.** The radius of a balloon is increasing at the rate of 10 cm/sec. At what rate is the surface area of the balloon increasing when the radius is 15 cm?

Watch Video Solution

**20.** An edge of a variable cube is increasing at the rate of 5 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long ?

Watch Video Solution

**21.** The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which the area increases, when the side is 10 cm.

Watch Video Solution

Watch Video Solution

## Exercise 11 B

**1.** Using differentials, find the approximate value of $\sqrt{37}$

Watch Video Solution

**2.** Use differentials and find approximate value of $(29)^{1/3}$

Watch Video Solution

**3.** Using differentials, find the approximate value of : $\sqrt[3]{127}$

Watch Video Solution

**4.** Using differentials, find the approximate values of the following:

$\sqrt{0.24}$

Watch Video Solution

**5.** Using differentials, find the approximate value of $\sqrt{49.5}$

Watch Video Solution

**6.** Using differentials, find the approximate value of $(15)^{1/4}$

Watch Video Solution

**7.** Using differentials, find the approximate value of $\frac{1}{(2.002)^2}$

Watch Video Solution

**8.** $\log_e 10.02$, given that $\log_e 10 = 2.3026$

Watch Video Solution

**9.** Find the value of $\log_{10}(4.04)$, it being given that $\log_{10} 4 = 0.6021$ and $\log_{10} e = 0.4343$

Watch Video Solution

**10.** $\cos 61^\circ$, it being given that $\sin 60^\circ = 0.86603$ and $1^\circ = 0.01745$ radian.

Watch Video Solution

**11.** If $y = \sin x$ and $x$ changes from $\pi/2$ to $22/14$, what is the approximate change in $y$?

Watch Video Solution

**12.** A circular metal plate expands under heating so that its radius increases by 2%. Find the approximate increase in the area of the plate if the radius of the plate before heating is 10 cm.

Watch Video Solution

**13.** If the length of a simple pendulum is decreased by 2%, find the percentage decrease in its period T, where $T = 2\pi\sqrt{\frac{l}{g}}$

Watch Video Solution

**14.** The pressure p and the volume V of a gas are connected by the relation, $pV^{1/4} = k$, where k is a contant. Find the percentage increase in the pressure, corresponding to a diminution of 0.5% in the volume

Watch Video Solution

**15.** The radius of a sphere shrinks from 10 cm to 9.8 cm. Find approximately the decrease in (i) surface area

Watch Video Solution

**16.** If there is an error of 0.1% in the measurement of the radius of a sphere, find approximately the percentage error in the calculation of the volume of the sphere.

Watch Video Solution

**17.** Show that the relative error in the volume of a sphere, due to an error in measuring the diameter, is three times the relative error in the diameter.

Watch Video Solution

## Exercise 11 C

**1.** Verify Rolle's theorem for each of the following functions :

$f(x) = x^2 \quad \text{on} \quad [-1, 1]$

Watch Video Solution

**2.** Verify Rolle's theorem for each of the following functions :

$f(x) = x^2 - x - 12 \text{ in } [-3, 4]$

Watch Video Solution

**3.** Verify Rolle's theorem for each of the following functions :

$f(x) = x^2 - 5x + 6 \text{ in } [2, 3]$

Watch Video Solution

**4.** Verify Rolle's theorem for each of the following functions :

$f(x) = x^2 - 3x - 18 \text{ in } [-3, 6]$

Watch Video Solution

**5.** Verify Rolle's theorem for each of the following functions :

$f(x) = x^2 - 4x + 3 \text{ in } [1, 3]$

Watch Video Solution

**6.** Verify Rolle's theorem for the following functions in the given intervals.

f(x) = $x(x - 4)^2$ in the interval [0,4].

Watch Video Solution

**7.** Verify Rolle's theorem for each of the following functions :

$f(x) = x^3 - 7x^2 + 16x - 12 \text{ in } [2, 3]$

Watch Video Solution

**8.** Verify Rolle's theorem for each of the following functions :

$$f(x) = x^3 + 3x^2 - 24x - 80 \quad \text{in} \quad [-4, 5]$$

Watch Video Solution

**9.** Verify Rolle's theorem for each of the following functions :

$$f(x) = (x-1)(x-2)(x-3) \quad \text{in} \quad [1, 3]$$

Watch Video Solution

**10.** Verify Rolle's theorem for each of the following functions :

$$f(x) = (x-1)(x-2)^2 \quad \text{in} \quad [1, 2]$$

Watch Video Solution

**11.** Verify Rolle's theorem for the following functions in the given intervals.

f(x) = $(x-2)^4(x-3)^3$ in the interval [2,3].

Watch Video Solution

**12.** Verify Rolle's theorem for each of the following functions :

$f(x)=\sqrt{1-x^2}\ \text{ in }\ [-1,1]$

Watch Video Solution

**13.** Verify Rolle's theorem for each of the following functions :

$f(x)=\cos x\ \text{ in }\ \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Watch Video Solution

**14.** Verify Rolle's theorem for each of the following functions :

$f(x) = \cos 2x \quad \text{in} \quad [0, \pi]$

Watch Video Solution

**15.** Verify Rolle's theorem for the following functions in the given intervals.

f(x) = sin 3x in the interval $[0, \pi]$.

Watch Video Solution

**16.** Verify Rolle's theorem for each of the following functions :

$f(x) = \sin x + \cos x \text{in} \quad \left[0, \frac{\pi}{2}\right]$

Watch Video Solution

**17.** Verify Rolle's theorem for each of the following functions :

$f(x) = e^{-x} \sin x \quad \text{in} \quad [0, \pi]$

Watch Video Solution

**18.** Verify Rolle's theorem for each of the following functions :

$f(x) = e^{-x}(\sin x - \cos x) \quad \text{in} \quad \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

Watch Video Solution

**19.** Verify Rolle's theorem for each of the following functions :

$f(x) = \sin x - \sin 2x \quad \text{in} \quad [0, 2\pi]$

Watch Video Solution

**20.** Verify Rolle's theorem for each of the following functions :

$f(x) = x(x+2)e^x \text{in } [-2, 1]$

Watch Video Solution

**21.** Verify Rolle's theorem for each of the following functions :

Show that $f(x) = x(x-5)^2$ satisfies Rolle's theorem on [0, 5] and that the value of c is $(5/3)$

Watch Video Solution

**22.** Discuss the applicability of Rolle's theorem, when :

$f(x) = (x-1)(2x-3), \quad \text{where } 1 \leq x \leq 3$

Watch Video Solution

**23.** Discuss the applicability of Rolle's theorem, when :

$$f(x) = x^{1/2} \quad \text{on} \quad [-1, 1]$$

Watch Video Solution

**24.** Discuss the applicability of Rolle's theorem, when :

$$f(x) = 2 + (x - 1)^{2/3} \quad \text{on} \quad [0, 2]$$

Watch Video Solution

**25.** Discuss the applicability of Rolle's theorem, when :

$$f(x) = \cos\frac{1}{x} on[-1, 1]$$

Watch Video Solution

**26.** Discuss the applicability of Rolle's theorem, when :

$f(x) = [x] \quad \text{on} \quad [-1, 1]$ where [x] denotes the greatest integer not exceeding x

Watch Video Solution

**27.** Using Rolle's theorem, find the point on the curve $y = x(x-4), x \in [0, 4]$

Watch Video Solution

## Exercise 11 D

**1.** Verify Lagrange's mean-value theorem for each of the following functions

$f(x) = x^2 + 2x + 3 \quad \text{on} \quad [4, 6]$

Watch Video Solution

**2.** Verify Lagrange's mean-value theorem for each of the following functions

$f(x) = x^2 + x - 1 \quad \text{on} \quad [0, 4]$

Watch Video Solution

**3.** Verify Lagrange's mean-value theorem for each of the following functions

$f(x) = 2x^2 - 3x + 1 \quad \text{on} \quad [1, 3]$

Watch Video Solution

**4.** Verify Lagrange's mean-value theorem for each of the following functions

$f(x) = x^3 + x^2 - 6x \quad \text{on} \quad [-1, 4]$

Watch Video Solution

**5.** Verify Lagrange's mean-value theorem for each of the following functions

$f(x) = (x - 4)(x - 6)(x - 8) \quad \text{on} \quad [4, 10]$

Watch Video Solution

**6.** Verify Lagrange's mean-value theorem for each of the following functions

$f(x) = e^x \quad \text{on} \quad [0, 1]$

Watch Video Solution

Watch Video Solution

**7.** Verify Lagrange's mean-value theorem for each of the following functions

$f(x) = x^{2/3} \quad \text{on} \quad [1, 0]$

Watch Video Solution

**8.** Verify Lagrange's mean-value theorem for each of the following functions

$f(x) = \log x \quad \text{on} \quad [1, e]$

Watch Video Solution

**9.** Verify Lagrange's mean-value theorem for each of the following functions

$f(x) = \tan^{-1} x \quad \text{on} \quad [0, 1]$

Watch Video Solution

**10.** Verify Lagrange's mean-value theorem for each of the following functions

$f(x) = \sin x \quad \text{on} \quad \left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$

Watch Video Solution

**11.** Verify Lagrange's mean-value theorem for each of the following functions

$f(x) = (\sin x + \cos x) \quad \text{on} \quad \left[0, \frac{\pi}{2}\right]$

Watch Video Solution

**12.** Show that Lagrange's mean-value theorem is not applicable to $f(x) = |x|$ on $[-1, 1]$

Watch Video Solution

**13.** Show that Lagrange's mean-value theorem is not applicable to $f(x) = \frac{1}{x}$ on $[-1, 1]$

Watch Video Solution

**14.** Find 'c' of Lagrange's mean-value theorem for

(i) $f(x) = (x^3 - 3x^2 + 2x)$ on $\left[0, \frac{1}{2}\right]$

(ii) $f(x) = \sqrt{25 - x^2}$ on $[1, 5]$

(iii) $f(x) = \sqrt{x + 2})$ on $[4, 6]$

Watch Video Solution

**15.** Using Lagrange's mean-value theorem, find a point on the curve $y = x^2$, where the tangent is parallel to the line joining the points (1, 1) and (2, 4)

Watch Video Solution

**16.** Find a point on the curve $y = x^3$, where the tangent to the curve is parallel to the chord joining the points (1, 1) and (3, 27)

Watch Video Solution

**17.** Find the points on the curve $y = x^3 - 3x$, where the tangent to the curve is parallel to the chord joining $(1, -2)$ and $(2, 2)$

Watch Video Solution

**18.** If $f(x) = x(1 - \log x)$, where $x > 0$, show that $(a - b)\log c = b(1 - \log b) - a(1 - \log a)$, where $0 < a < c < b$

Watch Video Solution

## Exercise 11 E

**1.** Find the maximum or minium values, if any, without using derivatives, of the functions:

$(5x - 1)^2 + 4$

Watch Video Solution

**2.** Find the maximum or minium values, if any, without using derivatives, of the functions:

$-(x-3)^2+9$

Watch Video Solution

**3.** Find the maximum or minium values, if any, without using derivatives, of the functions:

$-|x+4|+6$

Watch Video Solution

**4.** Find the maximum or minium values, if any, without using derivatives, of the functions:

$\sin 2x+5$

Watch Video Solution

**5.** Find the maximum or minium values, if any, without using derivatives, of the functions:

$|\sin 4x + 3|$

Watch Video Solution

**6.** Find the points of local maxima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$f(x) = (x - 3)^4$

Watch Video Solution

**7.** Find the points of local maxima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$f(x) = x^2$

Watch Video Solution

**8.** Find all the points of local maxima and minima and the corresponding maximum and minimum values of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$ .

Watch Video Solution

**9.** Find the points of local maxima or minima and corresponding local maximum and minimum values of $f(x) = x^3 - 6x^2 + 9x + 15$ . Also, find the points of inflection, if any:

Watch Video Solution

**10.** Find the points of local maxima or local minima and corresponding local maximum and local minimum values of each of the following functions. Also, find the points of inflection, if any : $f(x) = x^4 - 62x^2 + 120x + 9$

Watch Video Solution

**11.** Find the points of local maxima or local minima and the corresponding local maximum and minimum values of each of the following functions: $f(x) = -x^3 + 12x^2 - 5$

Watch Video Solution

**12.** Find the points of local maxima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$f(x) = (x - 1)(x + 2)^2$

Watch Video Solution

**13.** Find the points of local maxima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$f(x) = (x - 1)^3(x + 1)^2$

Watch Video Solution

**14.** Find the points of local maxima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$f(x) = \frac{x}{2} + \frac{2}{x}, x > 0$

Watch Video Solution

**15.** Find the maximum and minimum values of $2x^3 - 24x + 107$ on the interval $[-3, 3]$

Watch Video Solution

**16.** Find both the maximum and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 1$ on the interval $[1, 4]$.

Watch Video Solution

**17.** Find the maximum and minimum values of $f(x) = \sin x + \frac{1}{2}\cos 2x$ in $[0, \pi/2]$.

Watch Video Solution

**18.** The maximum value of $x^{\frac{1}{x}}, x > 0$ is $e^{\frac{1}{e}}$ (b) $\left(\frac{1}{e}\right)^{e}$ (c) 1 (d) none of these

Watch Video Solution

**19.** Show that the maximum value of $f(x) = x + \frac{1}{x}$ is less than its minimum value.

Watch Video Solution

**20.** Find the maximum profit that a company can make, if the profit function is given $P(x) = 41 + 24x - 18x^2$ .

Watch Video Solution

**21.** A jet of an enemy is flying along the curve $y = x^2 + 2$ . A soldier is placed at the point $(3, 2)$ . What is the shortest distance between the soldier and the jet?

Watch Video Solution

**22.** Find the maximum and minimum values of $f(x) = (-x + 2\sin x)$ on $[0, 2\pi]$

Watch Video Solution

## Exercise 11 F

**1.** Find two positive numbers whose product is 49 and the sum is minimum.

Watch Video Solution

Watch Video Solution

**2.** Find two positive numbers whose sum is 16 and the sum of whose squares is minimum

Watch Video Solution

**3.** Divide 15 into two parts such that product of square of one part and cube of other is maximum

Watch Video Solution

**4.** Divide 8 into two positive parts such that the sum of the square of one and the cube of the other is minimum.

Watch Video Solution

**5.** Divide a into two parts such that the product of the pth power of one part and the qth power of the second part may be maximum

Watch Video Solution

**6.** The rate of working of an engine is given by

$$R = 15v + \frac{6000}{v}, \quad \text{where} \quad 0 < v < 30$$

and v is the speed of the engine. Show that R is the least when v = 20

Watch Video Solution

**7.** Find the dimension of the rectangle of area $96cm^2$ whose perimeter is the least, Also, find the perimeter of the rectangle

Watch Video Solution

Watch Video Solution

**8.** Show that all the rectangles with a given perimeter, the square has the largest area.

Watch Video Solution

**9.** Given the perimeter of a rectangle, show that its diagonal is minimum when it is a square

Watch Video Solution

**10.** Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius $a$ is a square of side $\sqrt{2}a$ .

Watch Video Solution

**11.** Given the sum of the perimeters of a square and a circle, show that the sum of their areas is least when one side of the square is equal to diameter of the circle.

Watch Video Solution

**12.** Show that the right triangle of maximum area that can be inscribed in a circle is an isosceles triangle

Watch Video Solution

**13.** Prove that the perimeter of a right-angled triangle of given hypotenuse is maximum when the triangle is isosceles

Watch Video Solution

**14.** The perimeter of a triangle is 8 cm. If one of the sides of the triangle be 3 cm, what will be the other two sides for maximum area of the triangle ?

Watch Video Solution

**15.** A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 meters. Find the dimensions of the rectangle so as to admit maximum light through the whole opening.

Watch Video Solution

**16.** A square piece of tin of side 12 cm is to be made into a box without a lid by cutting a square from each corner and folding up the flaps to form the sides. What should be the side of the square to be cut off so that the volume of the box is maximum ?

Also, find this maximum volume

Watch Video Solution

**17.** OR An open box with a square base is to be made out of a given quantity of cardboard of area $c^2$ square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

Watch Video Solution

**18.** A cylindrical can to be made to hold 1 litres of oil. Find the dimensions which will minimize the cost of the metal to make the can.

Watch Video Solution

**19.** Show that the right-circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

Watch Video Solution

**20.** Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimetres, find the dimensions of the can which has the minimum surface area?

Watch Video Solution

**21.** The height of a closed cylinder of given volume and the minimum surface area is (a) equal to its diameter (b) half of its diameter (c) double of its diameter (d) None of these

Watch Video Solution

**22.** Prove that the volume of the largest cone, that can be inscribed in a sphere of radius $R\cdot$ is $\frac{8}{27}$ of the volume of the sphere.

Watch Video Solution

**23.** The fraction exceeds its $p^{th}$ power by the greatest number possible, where $p \geq 2$ is

Watch Video Solution

**24.** Find the point on the curve $y^2 = 4x$ which is nearest to the point $(2, \ -8)$ .

Watch Video Solution

**25.** A right circular cylinder is inscribed in a cone. Show that the curved surface area of the cylinder is maximum when the diameter of the cylinder is equal to the radius of the base of the cone.

Watch Video Solution

**26.** Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

Watch Video Solution

**27.** A rectangle is inscribed in a semi-circle of radius $r$ with one of its sides on diameter of semi-circle. Find the dimensions of the rectangle so that its area is maximum. Find also the area.

Watch Video Solution

**28.** Two sides of a triangle have lengths $'a'$ and $'b'$ and the angle between them is $\theta$ . What value of $\theta$ will maximize the area of the triangle? Find the maximum area of the triangle also.

Watch Video Solution

**29.** Show that the maximum volume of the cylinder which can be inscribed in a sphere of radius $5\sqrt{3}cm$ is $500\ \pi\ cm^3$ .

Watch Video Solution

**30.** A square-based tank of capacity 250 cu m has to bedug out. The cost of land is Rs 50 per sq m. The cost of digging increases with the depth and for the whole tank the cost is Rs $400 \times (depth)^2$. Find the dimensions of the tank for the least total cost.

Watch Video Solution

**31.** A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum? Also, find the maximum volume.

Watch Video Solution

**32.** An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be least when depth of the tank is half of its width.

Watch Video Solution

**33.** A wire of length 36cm is cut into the two pieces, one of the pieces is turned in the form of a square and other in form of an equilateral triangle. Find the length of each piece so that the sum of the areas of the two be minimum

Watch Video Solution

**34.** Find the largest possible area of a right angled triangle whose hypotenuse is 5 cm long.

Watch Video Solution

## Exercise 11 G

**1.** Show that the function $f(x) = 5x - 2$ is a strictly increasing function on R

Watch Video Solution

**2.** Show that the function $f(x) = -2x + 7$ is a strictly decreasing function on R

Watch Video Solution

**3.** Prove that $f(x) = ax + b$ , where $a$, $b$ are constants and $a > 0$ is an increasing function on $R$ .

Watch Video Solution

**4.** Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R.

Watch Video Solution

**5.** Show that the function $f(x) = x^2$ is

(a) strictly increasing on $[0, \infty]$

(b) strictly decreasing on $[-\infty, 0]$

(c) neither strictly increasing nor strictly decreasing on R

Watch Video Solution

**6.** Show that the function $f(x) = |x|$ is

(a) striclty increasing on $[0, \infty]$

(b) strictly decreasing on $[-\infty, 0]$

Watch Video Solution

**7.** Prove that function $f(x) = \log_e x$ is strictly increasing in the interval $(0, \infty)$

Watch Video Solution

**8.** 1. Prove that the function $f(x) = \log_a x$ is increasing on $(0, \infty)$ if a > 1 and decreasing on $(0, \infty)$, if `0

Watch Video Solution

**9.** Prove that $f(x) = 3^x$ is strictly increasing on R

Watch Video Solution

**10.** Show that $f(x) = x^3 - 15x^2 + 75x - 50$ is an increasing function for all $x \in R$.

Watch Video Solution

**11.** Show that $f(x) = \left(x - \frac{1}{x}\right)$ is increasing for all $x \in R$, where $x \neq 0$

Watch Video Solution

**12.** Show that $f(x) = \left(\frac{3}{x} + 5\right)$ is decreasing for all $x \in R$, where $x \neq 0$

Watch Video Solution

**13.** Show that $f(x) = \frac{1}{(1+x^2)}$ is increasing for all $x \leq 0$

Watch Video Solution

**14.** Show that $f(x) = \left(x^3 + \frac{1}{x^3}\right)$ is decreasing on $[-1, 1]$

Watch Video Solution

**15.** Show that $f(x) = \frac{x}{\sin x}$ is increasing on $\left[0, \frac{\pi}{2}\right]$

Watch Video Solution

**16.** Prove that the following functions are strictly increasing: $f(x) = \log(1+x) - \frac{2x}{2+x}$

Watch Video Solution

**17.** Let $I$ be an interval disjointed from $[-1, 1]$. Prove that the function $f(x) = x + \frac{1}{x}$ is increasing on $I$.

Watch Video Solution

**18.** Show that $f(x) = \frac{(x-2)}{(x+1)}$ is increasing for all $x \in R$, except at $x = -1$

Watch Video Solution

**19.** Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$is(a) strictly increasing (b) strictly decreasing

Watch Video Solution

**20.** Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$is (a) strictly increasing (b) strictly decreasing

Watch Video Solution

**21.** Find the intervals on which the function $f(x) = 6 - 9x - x^2$ is (a) strictly increasing (b) strictly decreasing

Watch Video Solution

**22.** Find the intervals in which the function $f(x) = x^4 - \frac{x^3}{3}$ is increasing or decreasing.

Watch Video Solution

**23.** Find the intervals in which the function $f\ (x) = \ x^3 - \ 12x^2 + \ 36x + 17\ $ is (a) increasing, (b) decreasing.

Watch Video Solution

**24.** Find the intervals on which each of the following functions is (a) increasing (b) decreasing

$f(x) = (x^3 - 6x^2 + 9x + 10)$

Watch Video Solution

**25.** Find the intervals in which $f(x) = 6 + 12x + 3x^2 - 2x^3$ is increasing or decreasing.

Watch Video Solution

**26.** Find the intervals on which each of the following functions is

(a) increasing (b) decreasing

$f(x) = 2x^3 - 24x + 5$

Watch Video Solution

**27.** Find the intervals in which $f(x) = (x-1)(x-2)^2$ is increasing or decreasing.

Watch Video Solution

**28.** Find the intervals in which $f(x) = x^4 - 4x^3 + 4x^2 + 15$ is increasing or decreasing.

Watch Video Solution

**29.** Find the intervals on which each of the following functions is

(a) increasing (b) decreasing

$f(x) = 2x^3 + 9x^2 + 12x + 15$

Watch Video Solution

**30.** Determine the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is decreasing or increasing.

Watch Video Solution

**31.** Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is (a) strictly increasing (b) strictly decreasing

Watch Video Solution

**32.** Find the intervals in which the function given by $f(x)=\frac{3}{10}x^4=\frac{4}{5}x^3-3x^2+\frac{36}{5}x+11$ is $(a)$ strictly increasing $(b)$ strictly decreasing.

Watch Video Solution

## Exercise 11 H

**1.** Find the slope of the tangent of the curve

(i) $y=\left(x^3-x\right)\quad \text{at} \quad x=2$

(ii) $y=\left(2x^2+3\sin x\right)\quad \text{at} \quad x=0$

(iii) $y=(\sin 2x+\cot x+2)^2\quad \text{at} \quad x=\frac{\pi}{2}$

Watch Video Solution

2. Find the equations of the tangent and the normal to the given curve at the indicated point :

$y = x^3 - 2x + 7 \quad \text{at} \quad (1, 6)$

Watch Video Solution

3. Find the equations of the tangent and the normal to the given curve at the indicated point :

$y^2 = 4ax \quad \text{at} \quad \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

Watch Video Solution

4. Find the equations of the tangent and the normal to the given curve at the indicated point :

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{at} \quad (a\cos\theta, b\sin\theta)$

Watch Video Solution

5. Find the equations of the tangent and the normal to the given curve at the indicated point :

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1at'(a\sec\theta, b\tan\theta)$$

Watch Video Solution

6. Find the equations of the tangent and the normal to the given curve at the indicated point :

$y = x^3 \quad \text{at} \quad P(1, 1)$

Watch Video Solution

7. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$at the point $(at^2, 2at)$.

Watch Video Solution

**8.** Find the equations of the tangent and the normal to the given curve at the indicated point :

$$y = \cot^2 x - 2\cot x + 2 \quad \text{at} \quad x = \frac{\pi}{4}$$

Watch Video Solution

**9.** Find the equations of the tangent and the normal to the given curve at the indicated point :

$$16x^2 + 9y^2 = 144 \quad \text{at} \quad (2, y_1), \quad \text{where} \quad y_1 > 0$$

Watch Video Solution

10. Find the equations of the tangent and the normal to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at the point (1, 3)

Watch Video Solution

11. Find the equation of the tangent to the curve $\sqrt{x} + \sqrt{y} = a$ at the point $\left(\frac{a^2}{4}, \frac{a^2}{4}\right)$

Watch Video Solution

12. Show that the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(x_1, y_1)$ is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

Watch Video Solution

**13.** Find the equation of the tangent to the curve $y = (\sec^4 x - \tan^4 x)$ at $x = \frac{\pi}{3}$

Watch Video Solution

**14.** Find the equation of the normal to the curve $y = (\sin 2x + \cot x + 2)^2$ at $x = \frac{\pi}{2}$

Watch Video Solution

**15.** Show that the tangents to the curve $y = 2x^3 - 4$ at the points $x = 2$ and $x = -2$ are parallel

Watch Video Solution

**16.** Find the equation of the tangent to the curve $x^2 + 3y = 3$, which is parallel to the line $y - 4x + 5 = 0$

Watch Video Solution

**17.** At what points on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$ , the tangents are parallel to the $y - a\xi s$ ?

Watch Video Solution

**18.** Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$at which the tangents are parallel to the x-axis.

Watch Video Solution

**19.** Prove that the tangents to the curve $y = x^2 - 5 + 6$ at the points $(2, 0) and (3, 0)$ are at right angles.

Watch Video Solution

**20.** The co-ordinates of the points on the curve $y = x^2 + 3x + 4$ at which the tangent passes through the origin are

Watch Video Solution

**21.** Find the point on the curve $y = x^3 - 11x + 5$ at which the equation of tangent is $y = x - 11$

Watch Video Solution

**22.** Find the equation of the tangents to the curve $2x^2 + 3y^2 = 14$, parallel to the line $x + 3y = 4$

Watch Video Solution

**23.** The equation of the tangent to the curve $x^2 + 2y = 8$ which is the perpendicular to $x - 2y + 1 = 0$ is

Watch Video Solution

**24.** Find the point on the curve $y = 2x^2 - 6x - 4$ at which the tangent is parallel to the x-axis

Watch Video Solution

**25.** Find the point on the parabola $y=(x-3)^2$, where the tangent is parabola to the line joining (3,0) and (4,1

Watch Video Solution

**26.** Show that the curves $x=y^2$ and $xy=k$ cut at right angles; if $8k^2=1$

Watch Video Solution

**27.** Show that the curves $xy=a^2 and x^2+y^2=2a^2$ touch each other

Watch Video Solution

28. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$

Watch Video Solution

29. Find the equation of the tangent to the curve $x = \theta + \sin\theta, y = 1 + \cos\theta$ at $\theta = \frac{\pi}{4}$

Watch Video Solution

30. Find the equation of tangent to the curve $x = \sin 3t, y = \cos 2t$ at $t = \frac{\pi}{4}$

Watch Video Solution

## Objective Questions

**1.** If $y = 2^x$ then $\frac{dy}{dx} = ?$

A. $x\left(2^{x-1}\right)$

B. $\frac{2^x}{(\log 2)}$

C. $2^x(\log 2)$

D. none of these

**Answer: C**

Watch Video Solution

**2.** If $y = \log_{10} x$ then $\frac{dy}{dx}$ = ?

A. $\frac{1}{x}$

B. $\frac{1}{x}(\log 10)$

C. $\frac{1}{x(\log 10)}$

D. none of these

**Answer: C**

Watch Video Solution

**3.** If $y = e^{1/x}$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{x} . e^{(1/x - 1)}$

B. $\frac{-e^{1/x}}{x^2}$

C. $e^{1/x} \log x$

D. none of these

**Answer: B**

Watch Video Solution

**4.** if $y = x^x$ then $\frac{dy}{dx}$

A. $x^x \log x$

B. $x^x(1 + \log x)$

C. $x(1 + \log x)$

D. none of these

**Answer: B**

Watch Video Solution

**5.** If $y = x^{\sin x}$ then $\frac{dy}{dx} = ?$

A. $(\sin x).\, x^{(\sin x - 1)}$

B. $(\sin x \cos x).\, x^{(\sin x - 1)}$

C. $x^{\sin x}\left\{\frac{\sin x + x \log x.\, \cos x}{x}\right\}$

D. none of these

**Answer: C**

Watch Video Solution

**6.** If $y = x^{\sqrt{x}}$ then $\dfrac{dy}{dx} = ?$

A. $\sqrt{x}.\, x^{\left(\sqrt{x}-1\right)}$

B. $\dfrac{s^{\sqrt{x}}\log x}{2\sqrt{x}}$

C. $x^{\sqrt{x}}\left\{\dfrac{2+\log x}{2\sqrt{x}}\right\}$

D. none of these

**Answer: C**

Watch Video Solution

**7.** If $y = e^{\sin \sqrt{x}}$ then $\dfrac{dy}{dx} = ?$

A. $e^{\sin \sqrt{x}}. \cos \sqrt{x}$

B. $\dfrac{e^{\sin \sqrt{x}} \cos \sqrt{x}}{2\sqrt{x}}$

C. $\dfrac{e^{\sin \sqrt{x}}}{2\sqrt{x}}$

D. none of these

**Answer: B**

Watch Video Solution

**8.** If $y = (\tan x)^{\cot x}$ then $\dfrac{dy}{dx} = ?$

A. $\cot x. (\tan x)^{\cot x - 1}. \sec^2 x$

B. $-(\tan x)^{\cot x}. \operatorname{cosec}^2 c$

C. $(\tan x)^{\cot x}. \operatorname{cosec}^2 x(1 - \log \tan x)$

D. none of these

**Answer: C**

Watch Video Solution

**9.** If $y = (\sin x)^{\log x}$ then $\frac{dy}{dx} = ?$

A. $(\log x).(\sin x)^{(\log x - 1)}.\cos x$

B. $(\sin x)^{\log x}.\left\{\frac{x \log x + \log \sin x}{x}\right\}$

C. $(\sin x)^{\log x}.\left\{\frac{(x \log x)\cot x + \log \sin x}{x}\right\}$

D. none of these

**Answer: C**

Watch Video Solution

**10.** If $y = \sin(x^x) \;\; \text{then} \;\; \frac{dy}{dx} = \; ?$

A. $x^x \cos(x^x)$

B. $x^x \cos x^x (1 + \log x)$

C. $x^x \cos x^x \log x$

D. none of these

**Answer: B**

Watch Video Solution

**11.** If $y = \sqrt{x \sin x} \;\; \text{then} \;\; \frac{dy}{dx} = \; ?$

A. $\frac{(x \cos x + \sin x)}{2\sqrt{x \sin x}}$

B. $\frac{1}{2}(x \cos x + \sin x). \sqrt{x \sin x}$

C. $\frac{1}{2\sqrt{x \sin x}}$

D. none of these

**Answer: A**

Watch Video Solution

**12.** If $e^{x+y} = xy$ then $\frac{dy}{dx} = ?$

A. $\frac{x(1-y)}{y(x-1)}$

B. $\frac{y(1-x)}{x(y-1)}$

C. $\frac{(x-xy)}{xy-y}$

D. none of these

**Answer: B**

Watch Video Solution

**13.** If $(x+y)=\sin(x+y)$ then $(dy)?(dx)=?$

A. $-1$

B. 1

C. $\dfrac{1-\cos(x+y)}{\cos^2(x+y)}$

D. none of these

**Answer: A**

Watch Video Solution

**14.** If $\sqrt{x}+\sqrt{y}=\sqrt{a}$ then $\dfrac{dy}{dx}=?$

A. $\dfrac{-\sqrt{x}}{\sqrt{y}}$

B. $-\dfrac{1}{2}\cdot\dfrac{\sqrt{y}}{\sqrt{x}}$

C. $\dfrac{-\sqrt{y}}{\sqrt{x}}$

D. none of these

**Answer: C**

Watch Video Solution

**15.** If $x^y = y^x$ then $\frac{dy}{dx} = ?$

A. $\frac{(y - x \log y)}{(x - y \log x)}$

B. $\frac{y(y - x \log y)}{x(x - y \log x)}$

C. $\frac{y(y + x \log y)}{x(x + y \log x)}$

D. none of these

**Answer: B**

Watch Video Solution

16. If $x^p y^q = (x+y)^{(p+q)}$ then $\frac{dy}{dx} = ?$

A. $\frac{x}{y}$

B. $\frac{y}{x}$

C. $\frac{x^{p-1}}{y^{q-1}}$

D. none of these

**Answer: B**

Watch Video Solution

17. If $y = x^2 \sin \frac{1}{x}$ then $\frac{dy}{dx} = ?$

A. $x \sin \frac{1}{x} - \cos \frac{1}{x}$

B. $-\cos \frac{1}{x} + 2x \sin \frac{1}{x}$

C. $-x \sin \frac{1}{x} + \cos \frac{1}{x}$

D. none of these

**Answer: B**

Watch Video Solution

**18.** If $y = \cos^2 x^3 \quad \text{then} \quad \frac{dy}{dx} = \ ?$

A. $-3x^2 \sin(2x^3)$

B. $-3x^2 \sin^2 x^3$

C. $-3x^2 \cos^2(2x^3)$

D. none of these

**Answer: A**

Watch Video Solution

**19.** If $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{2\left(x + \sqrt{x^2 + a^2}\right)}$

B. $\frac{-1}{\sqrt{x^2 + a^2}}$

C. $\frac{1}{\sqrt{x^2 + a^2}}$

D. none of these

**Answer: C**

Watch Video Solution

**20.** If $y = \log\left(\frac{1 + \sqrt{x}}{1 - \sqrt{x}}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{\sqrt{x}(1 - x)}$

B. $\frac{-1}{x(1 - \sqrt{x})^2}$

C. $\dfrac{-\sqrt{x}}{2(1-\sqrt{x})}$

D. none of these

**Answer: A**

Watch Video Solution

**21.** If $y=\log\left(\dfrac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$ then $\dfrac{dy}{dx}=?$

A. $\dfrac{2}{\sqrt{1+x^2}}$

B. $\dfrac{2\sqrt{1+x^2}}{x^2}$

C. $\dfrac{-2}{\sqrt{1+x^2}}$

D. none of these

**Answer: A**

Watch Video Solution

22. If $y=\sqrt{\frac{1+\sin x}{1-\sin x}}$ then $\frac{dy}{dx}=?$

A. $\frac{1}{2}\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$

B. $\frac{1}{2}\operatorname{cosec}^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$

C. $\frac{1}{2}\operatorname{cosec}\left(\frac{\pi}{4}-\frac{x}{2}\right)\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)$

D. none of these

**Answer: B**

Watch Video Solution

23. If $y=\sqrt{\frac{\sec x-1}{\sec x+1}}$ then $\frac{dy}{dx}=?$

A. $\sec^2 x$

B. $\frac{1}{2}\sec^2\frac{x}{2}$

C. $\frac{-1}{2}\operatorname{cosec}^2\frac{x}{2}$

D. none of these

**Answer: B**

Watch Video Solution

**24.** If $y=\sqrt{\frac{1+\tan x}{1-\tan x}}$ then $\frac{dy}{dx}=$ ?

A. $\frac{1}{2}\sec^2 x.\tan\left(x+\frac{\pi}{4}\right)$

B. $\frac{\sec^2\left(x+\frac{\pi}{4}\right)}{2\sqrt{\tan\left(x+\frac{\pi}{4}\right)}}$

C. $\frac{\sec^2\left(\frac{x}{4}\right)}{\sqrt{\tan\left(x+\frac{\pi}{4}\right)}}$

D. none of these

**Answer: B**

Watch Video Solution

**25.** If $y = \tan^{-1}\left(\frac{1 - \cos x}{\sin x}\right)$ then $\frac{dy}{dx} = ?$

A. 1

B. $-1$

C. $\frac{1}{2}$

D. $\frac{-1}{2}$

**Answer: C**

Watch Video Solution

**26.** If $y = \tan^{-1}\left\{\frac{\cos x + \sin x}{\cos x - \sin x}\right\}$ then $\frac{dy}{dx} = ?$

A. 1

B. $-1$

C. $\frac{1}{2}$

D. $\frac{-1}{2}$

**Answer: A**

Watch Video Solution

**27.** If $y = \tan^{-1}\left\{\frac{\cos x}{1 + \sin x}\right\}$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. 1

D. $-1$

**Answer: B**

Watch Video Solution

**28.** If $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$, prove that $\frac{dy}{dx} = \frac{1}{2}$.

A. $\frac{-1}{2}$

B. $\frac{1}{2}$

C. $\frac{1}{(1+x^2)}$

D. none of these

**Answer: B**

Watch Video Solution

**29.** If $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{a}{b}$

B. $\frac{-b}{a}$

C. 1

D. $-1$

**Answer: D**

Watch Video Solution

**30.** If $y = \sin^{-1}(3x - 4x^3)$ then $\frac{dy}{dx} = ?$

A. $\frac{3}{\sqrt{1 - x^2}}$

B. $\frac{-4}{\sqrt{1 - x^2}}$

C. $\frac{3}{\sqrt{1 + x^3}}$

D. none of these

**Answer: A**

Watch Video Solution

**31.** If $y = \cos^{-1}(4x^3 - 3x)$ then $\frac{dy}{dx} = ?$

A. $\frac{3}{\sqrt{1-x^2}}$

B. $\frac{-3}{\sqrt{1-x^2}}$

C. $\frac{4}{\sqrt{1-x^2}}$

D. $\frac{-4}{(3x^2-1)}$

**Answer: B**

Watch Video Solution

**32.** If $y = \tan^{-1}\left(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{(1+x)}$

B. $\dfrac{1}{\sqrt{x}(1+x)}$

C. $\dfrac{2}{\sqrt{x}(1+x)}$

D. $\dfrac{1}{2\sqrt{x}(1+x)}$

**Answer: D**

Watch Video Solution

**33.** If $y = \cos^{-1}\left(\dfrac{x^2-1}{x^2+1}\right)$ then $\dfrac{dy}{dx} = ?$

A. $\dfrac{2}{(1+x^2)}$

B. $\dfrac{-2}{(1+x^2)}$

C. $\dfrac{2x}{(1+x^2)}$

D. none of these

**Answer: B**

Watch Video Solution

**34.** If $y=\tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$ then $\frac{dy}{dx}=$ ?

A. $\frac{2x}{(1+x^4)}$

B. $\frac{-2x}{(1+x^4)}$

C. $\frac{x}{(1+x^4)}$

D. none of these

**Answer: A**

Watch Video Solution

**35.** If $y=\cos^{-1}x^3$ then $\frac{dy}{dx}=$ ?

A. $\frac{-1}{(1+x)}$

B. $\frac{2}{\sqrt{(1+x)}}$

C. $\frac{-1}{2\sqrt{x}(1+x)}$

D. none of these

**Answer: C**

Watch Video Solution

**36.** If $y = \cos^{-1} x^3$ then $\frac{dy}{dx} = ?$

A. $\frac{-1}{\sqrt{1-x^6}}$

B. $\frac{-3x^2}{\sqrt{1-x^6}}$

C. $\frac{-3}{x^2\sqrt{1-x^6}}$

D. none of these

**Answer: B**

Watch Video Solution

**37.** If $y = \tan^{-1}(\sec x + \tan x)$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. 1

D. none of these

**Answer: A**

Watch Video Solution

**38.** If $y = \cot^{-1}\left(\frac{1-x}{1+x}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{-1}{(1+x^2)}$

B. $\dfrac{1}{(1+x^2)}$

C. $\dfrac{1}{(1+x^2)^{.3/2}}$

D. none of these

**Answer: B**

Watch Video Solution

**39.** If $y = \sqrt{\dfrac{1+x}{1-x}}$ then $\dfrac{dy}{dx} = ?$

A. $\dfrac{2}{(1-x)^2}$

B. $\dfrac{x}{(1-x)^{.3/2}}$

C. $\dfrac{1}{(1-x)^{.3/2}.(1+x)^{1/2}}$

D. none of these

**Answer: C**

Watch Video Solution

**40.** If $y = \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{-2}{(1+x^2)}$

B. $\frac{2}{(1+X^2)}$

C. $\frac{-1}{(1-X^2)}$

D. none of these

**Answer: A**

Watch Video Solution

**41.** If $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$ then $\frac{dy}{dx}$ = ?

A. $\frac{-2}{(1+x^2)}$

B. $\frac{-2}{(1-x^2)}$

C. $\frac{-2}{\sqrt{1-x^2}}$

D. none of these

**Answer: C**

Watch Video Solution

**42.** If $y=\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$ then $\frac{dy}{dx}=$ ?

A. $\frac{1}{(1+x^2)}$

B. $\frac{2}{(1+x^2)}$

C. $\frac{1}{2(1+x^2)}$

D. none of these

**Answer: C**

Watch Video Solution

**43.** $y = \sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}$ then $\frac{dy}{dx} = ?$

A. $\frac{-1}{2\sqrt{1-x^2}}$

B. $\frac{1}{2\sqrt{1-x^2}}$

C. $\frac{1}{2(1+x^2)}$

D. none of these

**Answer: A**

Watch Video Solution

**44.** If $x = at^2$ and $y = 2at$ then find the value of $\left(\frac{dy}{dx}\right)^2$

A. $\frac{1}{t}$

B. $\frac{-1}{t^2}$

C. $\frac{-2}{t}$

D. none of these

**Answer: A**

Watch Video Solution

**45.** If $x = a\sec\theta, y = b\tan\theta$ then $\frac{dy}{dx} = ?$

A. $\frac{b}{a}\sec\theta$

B. $\frac{b}{a}\text{cosec}\theta$

C. $\frac{b}{a}\cot\theta$

D. none of these

**Answer: B**

Watch Video Solution

**46.** If $x = a\cos^2\theta, y = b\sin^2\theta \text{ then } \frac{dy}{dx} = ?$

A. $\frac{-a}{b}$

B. $\frac{a}{b}\cot\theta$

C. $\frac{-b}{a}$

D. none of these

**Answer: C**

Watch Video Solution

**47.** Find $\frac{dy}{dx}$, when $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$

A. $\cot\theta$

B. $\tan\theta$

C. $a\cot\theta$

D. $a\tan\theta$

**Answer: B**

Watch Video Solution

**48.** If $y = x^{x} \hat{} x \wedge ((((\infty))))$, find $\frac{dy}{dx}$.

A. $\frac{y}{x(1 - \log x)}$

B. $\frac{y^2}{x(1 - \log x)}$

C. $\dfrac{y^2}{x(1 - y\log x)}$

D. none of these

**Answer: C**

Watch Video Solution

**49.** If $y = \sqrt{x + \sqrt{x + \sqrt{x} + \ldots\ldots\ldots\ldots\infty}}$, then $\dfrac{dy}{dx}$

A. $\dfrac{1}{(2y - 1)}$

B. $\dfrac{1}{(y^2 - 1)}$

C. $\dfrac{2y}{(y^2 - 1)}$

D. none of these

**Answer: A**

Watch Video Solution

**50.** If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\dot{\longrightarrow}\infty}}}$, prove that $\frac{dy}{dx}=\frac{\cos x}{2y-1}$

A. $\frac{\sin x}{(2y-1)}$

B. $\frac{\cos x}{(y-1)}$

C. $\frac{\cos x}{(2y-1)}$

D. none of these

**Answer: C**

Watch Video Solution

**51.** If $y=e^{x}+e^{x+\dots\infty}$ then $\frac{dy}{dx}=?$

A. $\frac{1}{(1-y)}$

B. $\frac{y}{(1-y)}$

C. $\frac{y}{(y-1)}$

D. none of these

**Answer: B**

Watch Video Solution

**52.** The value of k for which $f(x) = \begin{cases} \frac{\sin 5x}{3x}, & \text{if } x \neq 0 \\ \text{k}, & \text{if } x = 0 \end{cases}$ is contnuous at x = 0 is

A. $\frac{1}{3}$

B. 0

C. $\frac{3}{5}$

D. $\frac{5}{3}$

**Answer: D**

Watch Video Solution

**53.** Let $f(x) = \begin{cases} x\sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{where } x = 0 \end{cases}$

Then, which of the following is the true statement ?

A. f(x) is not defined at x = 0

B. $\lim_{x \to 0} f(x)$ does not exist

C. f(x) is continuous at x = 0

D. $f(x)$ is discontinuous at x = 0

**Answer: C**

Watch Video Solution

**54.** The value of k for which $f(x) = \begin{cases} \frac{3x + 4\tan x}{x}, & \text{where } x \neq 0 \\ \text{k}, & \text{where } x = 0 \end{cases}$ is continuous at x = 0, is

A. 7

B. 4

C. 3

D. none of these

**Answer: A**

Watch Video Solution

**55.** Let $f(x) = x^{3/2}$. Then, $f'(0) =$ ?

A. $\frac{3}{2}$

B. $\frac{1}{2}$

C. does not exist

D. none of these

**Answer: C**

Watch Video Solution

**56.** The function $f(x) = |x| \forall x \in R$ is

A. continuous but not differentiable at x = 0

B. differentiable but not continuous at x = 0

C. neighter continuous nor differentiable at x = 0

D. none of these

**Answer: A**

Watch Video Solution

**57.** The function $f(x)=\begin{cases}1+x, \text{ when } & x\leq 2\\ 5-x, \text{ when } & x>2\end{cases}$ is

A. continuous as well as differntiable at x= 2

B. continuous but not differntiable at x = 2

C. differentiable but not continuous at x = 2

D. none of these

**Answer: B**

Watch Video Solution

**58.** If the function $f(x)=\begin{cases}kx+5, \text{ when } & x\leq 2\\ x-1, \text{ when } & x>2\end{cases}$ is continuous at x = 2 then k = ?

A. 2

B. $-2$

C. 3

D. $-3$

**Answer: B**

Watch Video Solution

**59.** If the function $f(x)\begin{cases}\frac{1-\cos 4x}{8x^2}, & x\neq 0\\ \text{k}, & x=0\end{cases}$ is continuous at x = 0 then k = ?

A. 1

B. 2

C. $\frac{1}{2}$

D. $\frac{-1}{2}$

**Answer: C**

Watch Video Solution

**60.** If the function $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & \text{when } x \neq 0 \\ \text{k}, & \text{when } x = 0 \end{cases}$ is continuous at x = 0 then k = ?

A. a

B. $a^2$

C. $-2$

D. $-4$

**Answer: B**

Watch Video Solution

**61.** If the function $f(x)=\begin{cases}\frac{\sin^2 ax}{x^2}, & \text{when } x\neq 0\\ \text{k}, & \text{when } x=0\end{cases}$ is continuous at x = 0 then k = ?

A. 3

B. $-3$

C. $-5$

D. 6

**Answer: D**

Watch Video Solution

**62.** At $x=2$, $f(x)=[x]$ is

A. continuous but not differentiable

B. differentiable but not continuous

C. continuous as well as differentiable

D. none of these

**Answer: D**

Watch Video Solution

**63.** Let $f(x) = \begin{cases} \frac{x^2 - 2x - 3}{x+1}, & \text{when } x \neq -1 \\ k, & \text{when } x = -1 \end{cases}$ If f(x) is continuous at $x = -1$ then k = ?

A. 4

B. $-4$

C. $-3$

D. 2

**Answer: B**

Watch Video Solution

**64.** The function $f(x) = x^3 - 6x^2 + 15x - 12$ is

A. strictly decreasing on R

B. strictly increasing on R

C. increasing in $(-\infty, 2]$ and decreasing in $(2, \infty)$

D. none of these

**Answer: B**

Watch Video Solution

**65.** The function $f(x) = 4 - 3x + 3x^2 - x^3$ is

A. decreasing on R

B. increasing on R

C. strictly decreasing on R

D. strictly increasing on R

**Answer: A**

Watch Video Solution

**66.** Prove that the function $f(x) = 3x + \cos 3x$ is increasing on R

Watch Video Solution

**67.** The function $f(x) = x^3 - 6x^2 + 9x + 3$ is decreasing for

A. $1 < x < 3$

B. $x > 1$

C. $x < 1$

D. $x < 1 \text{ or } x > 3$

**Answer: A**

Watch Video Solution

**68.** The function $f(x) = x^3 - 27x + 8$ is increasing when

A. $|x| < 3$

B. $|x| > 3$

C. $-3 < x < 3$

D. none of these

**Answer: B**

Watch Video Solution

69. $f(x) = \sin x$ is increasing in

A. $\left(\frac{\pi}{2}, \pi\right)$

B. $\left(\pi, \frac{3\pi}{2}\right)$

C. $(0, \pi)$

D. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

**Answer: D**

Watch Video Solution

70. $f(x) = \frac{2x}{\log x}$ is increasing in

A. $(0, 1)$

B. $(1, e)$

C. $(e, \infty)$

D. $(-\infty, e)$

**Answer: C**

Watch Video Solution

**71.** Find the intervals in which $f(x) = \sin x - \cos x,$ where `0

A. $\left(0, \frac{3\pi}{4}\right)$

B. $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

C. $\left(\frac{7\pi}{4}, 2\pi\right)$

D. none of these

**Answer: B**

Watch Video Solution

**72.** $f(x) = \frac{x}{\sin x}$ is

A. increasing in (0, 1)

B. decreasing in (0, 1)

C. increasing in $\left(0, \frac{1}{2}\right)$ and decreasing in $\left(\frac{1}{2}, 1\right)$

D. none of these

**Answer: A**

Watch Video Solution

**73.** $f(x) = x^x$ is decreasing in the interval

A. $(0, e)$

B. $\left(0, \frac{1}{e}\right)$

C. $(0, 1)$

D. none of these

**Answer: B**

Watch Video Solution

74. $f(x) = x^2 e^{-x}$ is increasing in

A. $(-2, 0)$

B. $(0, 2)$

C. $(2, \infty)$

D. $(-\infty, \infty)$

**Answer: B**

Watch Video Solution

75. $f(x) = \sin x - kx$ is decreasing for all $x \in R$, when

A. $k < 1$

B. $k \leq 1$

C. $k > 1$

D. $k \geq 1$

**Answer: C**

Watch Video Solution

76. $f(x) = (x+1)^3(x-3)^3$ is increasing in

A. $(-\infty, 1)$

B. $(-1, 3)$

C. $(3, \infty)$

D. $(1, \infty)$

**Answer: D**

Watch Video Solution

77. $f(x) = [x(x-3)]^2$ is increasing in

A. $(0, \infty)$

B. $(-\infty, 0)$

C. $(1, 3)$

D. $\left(0, \frac{3}{2}\right) \cup (3, \infty)$

**Answer: D**

Watch Video Solution

**78.** If the function $f(x) = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in every interval, then

A. $k > 3$

B. $k \geq 3$

C. $k < 3$

D. $k \leq 3$

**Answer: A**

Watch Video Solution

**79.** $f(x) = \dfrac{x}{(x^2 + 1)}$ is increasing in

A. $(-1, 1)$

B. $(-1, \infty)$

C. $(-\infty, -1) \cup (1, \infty)$

D. none of these

**Answer: A**

Watch Video Solution

**80.** Find the least value of $k$ for which the function $x^2 + kx + 1$ is an increasing function in the interval $1 < x < 2$

A. $-2$

B. $-1$

C. 1

D. 2

**Answer: A**

Watch Video Solution

**81.** $f(x) = |x|$ has

A. minimum at x= 0

B. maximum at x = 0

C. neither a maximum nor a minimum at x= 0

D. none of these

**Answer: A**

Watch Video Solution

**82.** When x is positive, the minimum value of $x^x$ is

A. $e^{e}$

B. $\frac{e^{1}}{e}$

C. $e^{\cdot^{-1}/_{e}}$

D. $(.^{1}/_{e})$

**Answer: C**

Watch Video Solution

**83.** The maximum value of $\left(\frac{\log x}{x}\right)$ is

A. $\left(\frac{1}{e}\right)$

B. $\frac{2}{e}$

C. $e$

D. 1

**Answer: A**

Watch Video Solution

**84.** $f(x) = \text{cosec } x \text{ in } (-\pi, 0)$ has a maxima at

A. $x = 0$

B. $x = \frac{-\pi}{4}$

C. $x = \frac{-\pi}{3}$

D. $x = \frac{-\pi}{2}$

**Answer: D**

Watch Video Solution

**85.** If $x > 0 \text{ and } xy = 1$, the minimum value of $(x + y)$ is

A. $-2$

B. 1

C. 2

D. none of these

**Answer: C**

Watch Video Solution

**86.** Show that the minimum value of $\left(x^2 + \frac{250}{x}\right)$ is 75

A. 0

B.

C.

D.

**Answer:**

Watch Video Solution

**87.** Find the maximum value and the minimum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$.

A. 16

B. 25

C. $-39$

D. none of these

**Answer: C**

Watch Video Solution

**88.** The maximum value of $f(x) = (x-2)(x-3)^2$ is

A. $\frac{4}{27}$

B. $-\frac{4}{27}$

C. $\frac{7}{3}$

D. 0

**Answer: A**

Watch Video Solution

**89.** Prove that the least value of $f(x) = \left(e^x + e^{-x}\right)$ is 2

Watch Video Solution