



## MATHS

# BOOKS - RS AGGARWAL MATHS (HINGLISH)

## BINARY OPERATIONS

### Solved Examples

1. Show that the operation  $*$  on  $Z$  define by

$$a * b = a + b + 1 \text{ for all } a, b \text{ in } Z$$

Satisfies (i) the closure property (ii) the associative

law and (iii) the commutative law (iv) find the identity element in  $Z$

(v) what is the inverse of an element  $a$  in  $Z$  ?



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2. Show that the operation  $*$  on  $Q - \{1\}$  defined by

$$a * b = a + b - ab \text{ for all } a, b \in Q - \{1\}$$

Satisfies (i) the closure property (ii) the associative law

(iii) the commutative law

(iv) what is the identity element ?

(v) for each  $a$  in  $Q - \{1\}$  find the inverse of  $a$



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3. On the set  $N$  of all natural numbers define the operation  $*$  on  $N$  by

$$m * n = \gcd(m, n) \text{ for all } m, n \in N$$

Show that  $*$  is commutative as well as associative



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4. Let  $A = \{1, 2, 3, 4, 5\}$  Define an operation  $\vee$  by

$$a \vee b = \max\{a, b\}$$

Prepare its composition table

Show that a is closed for the given operation and that the given operation is commutative



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### Exercise 3 A

1. Let  $*$  be a binary operation defined by  $a \cdot b = 3a + 4b - 2$ . Find  $4*5$ .

A. 20

B. 25

C. 30

D. 0

**Answer: C**



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2. The binary operation  $\cdot : R \times R \rightarrow R$  is defined as  $a \cdot b = 2a + b$ . Find  $(2 \cdot 3) \cdot 4$ .



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3. Let  $*$  be a binary operation, on the set of all-zero real numbers, given by  $a*b = \frac{a \cdot b}{5}$  for all

$a, b \in \mathbb{R} - \{0\}$ . Find the value of  $x$  given that  $2^x$

$$(x+5)=10.$$

A. 25

B. 30

C. 20

D. 15

**Answer: A**



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4. Let  $*$   $R \times R \rightarrow R$  be a binary operation given by  $a \cdot b = a + 4b^2$  Then compute  $(-5)*(2*0)$



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5. Let  $*$  be a binary operation on the set  $Q$  of all rational number given as  $a*b=(2a - b)^2$  for all  $a,b \in Q$  find  $3*5$  and  $5*3$  Is  $3*5=5*3$ ?



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6. Let  $*$  be a binary operation on  $\mathbb{N}$  given by  $a*b = \text{Lcm of } a \text{ and } b$  find the value of  $20*16$



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7. If the binary operation  $*$  on the set  $\mathbb{Z}$  of integers is defined by  $a \cdot b = a + 3b^2$ , find the value of  $2 \cdot 4$ .

A. 10

B. 30

C. 50



D. 40

**Answer: C**



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8. Show that  $*$  on  $\mathbb{Z}^+$  defined by  $a*b=|a-b|$  is not binary operation



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9. Let  $*$  be a binary operation on  $\mathbb{N}$  defined by  $a*b = a^b$  for all  $a, b \in \mathbb{N}$  show that  $*$  is neither

commutative nor associative



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**10.** Let  $a * b = 1 \text{ cm } (a, b)$  for all values of  $a, b \in \mathbb{N}$

(i) Find  $(12 * 16)$

(ii) Show that  $*$  is commutative on  $\mathbb{N}$

(iii) Find the identity element in  $\mathbb{N}$

(iv) Find all invertible elements in  $\mathbb{N}$



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11. Let  $Q^+$  be the set of all positive rational numbers

(i) show that the operation  $*$  on  $Q^+$  defined by

$$a * b = \frac{1}{2}(a+b)$$
 is binary operation

(ii) show that  $*$  is commutative

(iii) show that  $*$  is not associative



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12. Show that the set  $A = \{-1, 0, 1\}$  is not closed for addition



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**13.** Show that  $*$  on  $\mathbb{R} \setminus \{-1\}$  defined by  $(a*b) = \frac{a}{b+1}$  is neither commutative nor associative



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**14.** For all  $a, b \in \mathbb{R}$  we define  $a*b = |a-b|$  Show that  $*$  is commutative but not associative



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15. For all  $a, b \in \mathbb{N}$  we define  $a * b = a^3 + b^3$

Show that  $*$  is commutative but not associative



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16. Let  $X$  be a nonempty set and  $*$  be a binary operation on  $P(X)$  the power set of  $X$  defined by

$A * B = A \cap B$  for all  $A, B$  in  $P(X)$  (i) Find the

identity element in  $P(X)$

(ii) show that  $X$  is the only invertible element in

$P(X)$



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17. A binary operation  $*$  on the set  $\{0,1,2,3,4,5\}$  is defined as

$a*b$

=

$\{(a + b, \text{ if } a + b < 6), (a + b - 6 \text{ if } a + b \geq 6)\}$

Show that 0 is the identity for this operation and each element  $a$  has an inverse  $(6-a)$



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**Exercise 3 B**

1. Define  $*$  on  $\mathbb{N}$  by  $m*n = 1cm (m,n)$

Show that  $*$  is a binary operaitn which is commutative as well as associative



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2. Define  $*$  on  $\mathbb{Z}$  by  $a*b = a-b+ab$

show that  $*$  is a binary operation operation on  $\mathbb{z}$  which is neither commutative nor associative



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3. Define  $*$  on  $Z$  by  $a*b = a+b-ab$

Show that  $*$  is a binary operation on  $Z$  which is commutative as well as associative



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4. Consider a binary operation on  $Q - \{1\}$  define by

$$a*b = a+b-ab$$

(i) Find the identity element in  $Q - \{1\}$

(ii) Show that each  $a \in Q - \{1\}$  has its inverse



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5. Let  $Q_0$  be the set of all nonzero rational numbers. Let  $*$  be a binary operation on  $Q_0$  defined by  $a*b = \frac{ab}{4}$  for all  $a, b \in Q_0$ .

(i) Show that  $*$  is commutative and associative.

(ii) Find the identity element in  $Q_0$ .

(iii) Find the inverse of an element  $a$  in  $Q_0$ .



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6. On the set  $Q^+$  of all positive rational numbers, define an operation  $*$  on  $Q^+$  by  $a*b = \frac{ab}{2} \forall a, b \in Q^+$ .

Show that

(i)  $*$  is a binary operation on  $Q^+$

(ii)  $*$  is commutative

(iii)  $*$  is associative

Find the identify element in  $Q^+$  for  $*$

What is the inverse of  $a \in Q^+$ ?



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7. Let  $A = N \times N$  Define  $*$  on  $A$  by

$$(a,b)*(c,d)=(a+c,b+d)$$

Show that

(i)  $A$  is closed for  $*$

(ii)  $*$  is commutative

(iii)  $*$  is associative

(iv) identity element does not exist in  $A$



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8. Let  $A = \{1, -1, i, -i\}$  be the set of four 4th roots of unity prepare the composition table for multiplication on  $A$  and show that (i)  $A$  is closed for multiplication

(ii) multiplication is associative on  $A$

(iii) multiplication is commutative on  $A$

(iv) 1 is the multiplicative identity

(v) every element in  $A$  has its multiplicative inverse



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