

MATHS

BOOKS - RS AGGARWAL MATHS (HINGLISH)

CROSS,OR VECTOR, PRODUCT OF VECTORS



$$\begin{array}{ll} \textbf{1.} & \text{If} & \overrightarrow{a} = \left(3\hat{i} + \hat{j} - 4\hat{k}\right) \text{ and } \hat{b} = \left(6\hat{i} + 5\hat{j}, 2\hat{k}\right) \text{,} \\ & \text{find} \quad \left(\overrightarrow{a} \times \overrightarrow{b}\right) \text{ and } \left|\overrightarrow{a} \times \overrightarrow{b}\right|. \end{array}$$

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2. If $\overrightarrow{a} = (\hat{i} - 2\hat{j} + 3\hat{k})$ and $\overrightarrow{b} = (2\hat{i} + 3\hat{j} - 5\hat{k})$ then find $(\overrightarrow{a} \times \overrightarrow{b})$ and verify that $(\overrightarrow{a} \times \overrightarrow{b})$ is perpendicular to each one of \overrightarrow{a} and \overrightarrow{b} .

3. If
$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$$
 and $\overrightarrow{b} = \hat{j} - \hat{k}$, find a vector \overrightarrow{c} such that $\overrightarrow{a} \times \overrightarrow{c} = \overrightarrow{b}$ and $\overrightarrow{a} \cdot \overrightarrow{c} = 3$

A. $rac{1}{3} \Big(5 \hat{i} + 2 \hat{j} + 2 \hat{k} \Big)$ B. $5 \hat{i} + 2 \hat{j} + 2 \hat{k}$ C. $rac{1}{2} \Big(5 \hat{i} + 2 \hat{j} + 2 \hat{k} \Big)$

D. None of these

Answer: A

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4. Find a unit vector perpendicular to each one of the vectors $\overrightarrow{a} = \left(4\hat{i} - \hat{j} + 3\hat{k}\right)$ and $\overrightarrow{b} = \left(2\hat{i} + 2\hat{j} - \hat{k}\right)$.

5. Find a vector of magnitude 15, which is perpendicular to both the vectors $(4\hat{i} - \hat{j} + 8\hat{k})$ and $(-\hat{j} + \hat{k})$.

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$$\overrightarrow{a} = \Big(\hat{i} + 4\hat{j} + 2\hat{k} \Big), \ \overrightarrow{b} = \Big(3\hat{i} - 2\hat{j} + 7\hat{k} \Big) \ ext{and} \ \overrightarrow{c} = \Big(2\hat{i} - \hat{j} + 4\hat{k} \Big). \ ext{ fin}$$

Let

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6.

7. Find a unit vector perpendicular to each of the vectors $(\
ightarrow a + \
ightarrow b)$

and
$$(\ o a - \ o b)$$
 , where $\ o a = \hat{i} + \hat{j} + \hat{k}, \ o b = \hat{i} + 2\hat{j} + 3\hat{k}$.

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$$\textbf{8. If} \stackrel{\rightarrow}{a} = 4\hat{i} + 3\hat{j} + 2\hat{k} \, \text{ and } \stackrel{\rightarrow}{b} = 3\hat{i} + 2\hat{k}, \text{find} \Big| \stackrel{\rightarrow}{b} \times 2\stackrel{\rightarrow}{a} \Big|.$$

9. If
$$\left| \overrightarrow{a} \right| = \sqrt{26}$$
, $\left| \overrightarrow{b} \right| = 7$ and $\left| \overrightarrow{a} \times \overrightarrow{b} \right| = 35$, then $\overrightarrow{a} \cdot \overrightarrow{b} =$

10.

$$\left| \overrightarrow{a} \right| = 2, \left| \overrightarrow{b} \right| = 7 \, \, ext{and} \, \, \left(\overrightarrow{a} imes \overrightarrow{b}
ight) = \left(3 \hat{i} + 2 \hat{j} + 6 \hat{k}
ight), \ \ ext{find the angle between the set}$$

If

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11. Find the sine of the angle between the vectors
$$\overrightarrow{a} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$$
 and $\overrightarrow{b} = \left(\hat{i} + 3\hat{j} + 2\hat{k}\right)$.

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12. If vectors \overrightarrow{a} and \overrightarrow{b} are such that $\left|\overrightarrow{a}\right| = 3$, $\left|\overrightarrow{b}\right| = \frac{2}{3}$ and $\overrightarrow{a} \times \overrightarrow{b}$ is a unit vector, then write the angle between \overrightarrow{a} and \overrightarrow{b} .

13. Find the area of the parallelogram whose adjacent sides are represented by the vectors $(3\hat{i} + \hat{j} - 2\hat{k})$ and $(\hat{i} - 3\hat{j} + 4\hat{k})$.

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14. Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{d}_1 = (2\hat{i} - \hat{j} + \hat{k})$ and $\vec{d}_2 = (3\hat{i} + 4\hat{j} - \hat{k})$.

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15. Find the area of the triangle whose adjacent sides are determined by

the vectors
$$\overrightarrow{a} = \Big(-2\hat{i} - 5\hat{k} \Big) \, ext{ and } \, \overrightarrow{b} = \Big(\hat{i} - 2\hat{j} - \hat{k} \Big).$$

16. Using vectors, find the area of the $\triangle ABC$, whose vertices are A(1, 2, 3), B(2, -1, 4) and C(4, 5, -1).



17. Show that the points whose position vectors are
$$\left(5\hat{i}+6\hat{j}+7\hat{k}\right), \left(7\hat{i}-8\hat{j}+9\hat{k}\right) \text{ and } \left(3\hat{i}+20\hat{j}+5\hat{k}\right)$$
 are collinear.

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18. Using vector method, show that the points A(2, -1, 3), B(4, 3, 1) and C(3, 1, 2) are collinear



19. Show that the points with position vectors $\overrightarrow{a} - 2\overrightarrow{b} + 3\overrightarrow{c}, -2\overrightarrow{a} + 3\overrightarrow{b} + 2\overrightarrow{c}$ and $-8\overrightarrow{a} + 13\overrightarrow{b}$ are collinear

whatever be
$$\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{\cdot}$$

20. Prove that
$$\left(\overrightarrow{a} - \overrightarrow{b}\right) \times \left(\overrightarrow{a} + \overrightarrow{b}\right) = 2\left(\overrightarrow{a} \times \overrightarrow{b}\right)$$

21. If
$$\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{c} \times \overrightarrow{d}$$
 and $\overrightarrow{a} \times \overrightarrow{c} = \overrightarrow{b} \times \overrightarrow{d}$, show that $(\overrightarrow{a} - \overrightarrow{d})$ is p

, it being given that $a \neq d \; ext{and} \; b \neq c.$

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22. For any three vectors
$$\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$$
 show that $\overrightarrow{a} \times (\overrightarrow{b} + \overrightarrow{c}) + \overrightarrow{b} \times (\overrightarrow{c} + \overrightarrow{a}) + \overrightarrow{c} \times (\overrightarrow{a} + \overrightarrow{b}) = \overrightarrow{0}$

23. If
$$\vec{a} + \vec{b} + \vec{c} = 0$$
, prove that
 $\left(\vec{a} \times \vec{b}\right) = \left(\vec{b} \times \vec{c}\right) = \left(\vec{c} \times \vec{a}\right)$
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24. Prove that the points *A*, *BandC* with position vectors \vec{a} , \vec{b} and \vec{c}
respectively are collinear if and only if
 $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$.
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25. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = 0$, prove that $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$.
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26. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that
 $\vec{a} \neq \vec{b} = \vec{a} \cdot \vec{c}, \vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} \neq \vec{0}$, then show that $\vec{b} = \vec{\cdot}$

27. Prove that
$$\left| \overrightarrow{a} \times \overrightarrow{b} \right|^2 = \left| \begin{array}{ccc} \overrightarrow{a} & \overrightarrow{a} & \overrightarrow{a} & \overrightarrow{b} \\ \overrightarrow{a} & \overrightarrow{b} & \overrightarrow{b} & \overrightarrow{b} \\ \overrightarrow{a} & \overrightarrow{b} & \overrightarrow{b} & \overrightarrow{b} \end{array} \right|$$

Exercise 24

1. Find
$$\left(\overrightarrow{a} \times \overrightarrow{b}\right)$$
 and $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$, when
(i) $\overrightarrow{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$
(ii) $\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\overrightarrow{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$
(iii) $\overrightarrow{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\overrightarrow{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$
(iv) $\overrightarrow{a} = 4\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{b} = 3\hat{i} + \hat{k}$
(v) $\overrightarrow{a} = 3\hat{i} + 4\hat{j}$ and $\overrightarrow{b} = \hat{i} + \hat{j} + \hat{k}$

2. Find
$$\lambda$$
 if $\left(2\hat{i}+6\hat{j}+14\hat{k}
ight) imes\left(\hat{i}-\lambda\hat{j}+7\hat{k}
ight)=\overrightarrow{0}$

3.

$$\overrightarrow{a} = ig(-3 \hat{i} + 4 \hat{j} - 7 \hat{k} ig) ext{ and } \overrightarrow{b} = ig(6 \hat{i} + 2 \hat{j} - 3 \hat{k} ig), ext{ find } ig(\overrightarrow{a} imes \overrightarrow{b} ig)$$

If

Verify that (i) \overrightarrow{a} and $\left(\overrightarrow{a} \times \overrightarrow{b}\right)$ are perpendicular to each other and (ii) \overrightarrow{b} and $\left(\overrightarrow{a} \times \overrightarrow{b}\right)$ are perpendicular to each other.

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4. Find the value of :

(i)
$$\left(\hat{i} imes \hat{j}\right) \cdot \hat{k} + \hat{i} \cdot \hat{j}$$
 (ii) $\left(\hat{k} imes \hat{j}\right) \cdot \hat{i} + \hat{j} \cdot \hat{k}$
 $\hat{i} imes \left(\hat{j} + \hat{k}\right) + \hat{j} imes \left(\hat{k} + \hat{i}\right) + \hat{k} imes \left(\hat{i} + \hat{j}\right)$

5. Find the unit vectors perpendicular to both
$$\overrightarrow{a}$$
 and \overrightarrow{b} when

(i)
$$\overrightarrow{a} = 3\hat{i} + \hat{j} - 2\hat{k}$$
 and $\overrightarrow{b} = 2\hat{i} + 3\hat{j} - \hat{k}$
(ii) $\overrightarrow{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\overrightarrow{b} = \hat{i} + 2\hat{j} - \hat{k}$
(iii) $\overrightarrow{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\overrightarrow{b} = -\hat{i} + 3\hat{k}$
(iv) $\overrightarrow{a} = 4\hat{i} + 2\hat{j} - \hat{k}$ and $\overrightarrow{b} = \hat{i} + 4\hat{j} - \hat{k}$

6. Find the unit vectors perpendicular to the plane of the vectors

$$\overrightarrow{a}=2\hat{i}-6\hat{j}-3\hat{k}$$
 and $\overrightarrow{b}=4\hat{i}+3\hat{j}-\hat{k}.$

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7. Find a vector of magnitude 6 which is perpendicular to both the vectors

$$\overrightarrow{a}=4\hat{i}-\hat{j}+3\hat{k}$$
 and $\overrightarrow{b}=-2\hat{i}+\hat{j}-2\hat{k}.$

8. Find a unit vector perpendicular to each of the vectors $(\rightarrow a + \rightarrow b)$ and $(\rightarrow a - \rightarrow b)$, where $\rightarrow a = \hat{i} + \hat{j} + \hat{k}, \rightarrow b = \hat{i} + 2\hat{j} + 3\hat{k}$. Watch Video Solution

9. Find the angle between two vectors \overrightarrow{a} and \overrightarrow{b} with magnitudes 1 and 2 respectively and $\left|\overrightarrow{a} \times \overrightarrow{b}\right| = \sqrt{3}$.

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10. Let $\overrightarrow{a} = \hat{i} - \hat{j}$, $\overrightarrow{b} = 3\hat{j} - \hat{k}$ and $\overrightarrow{c} = 7\hat{i} - \hat{k}$. Find a vector \overrightarrow{d} which is perpendicular to both \overrightarrow{a} and \overrightarrow{b} , and $\overrightarrow{c} \cdot \overrightarrow{d} = 1$.

$$\begin{array}{l} \mathsf{A}.\overrightarrow{d} &= \frac{1}{4} \Big(\hat{i} - \hat{j} + 3 \hat{k} \Big) \\ \mathsf{B}.\overrightarrow{d} &= \hat{i} + \hat{j} + 3 \hat{k} \\ \mathsf{C}.\overrightarrow{d} &= \frac{1}{4} \Big(\hat{i} + \hat{j} + 3 \hat{k} \Big) \\ \mathsf{D}.\overrightarrow{d} &= \frac{1}{4} \Big(\hat{i} + \hat{j} + \hat{k} \Big) \end{array}$$

Answer: C



11. If
$$\overrightarrow{a}=\Big(4\hat{i}+5\hat{j}-\hat{k}\Big), \ \overrightarrow{b}=\Big(\hat{i}-4\hat{j}+5\hat{k}\Big)$$
, and $\overrightarrow{c}=\Big(3\hat{i}+\hat{j}-\hat{k}\Big)$

find a vector \overrightarrow{d} which is perpendicular to both \overrightarrow{a} and \overrightarrow{b} and for which $\overrightarrow{c} \cdot \overrightarrow{d} = 21.$

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12. Prove that
$$\left| \overrightarrow{a} \times \overrightarrow{b} \right| = \left(\overrightarrow{a} \cdot \overrightarrow{b} \right) \tan \theta$$
, where θ is the angle between \overrightarrow{a} and \overrightarrow{b} .



14. verify that
$$\overrightarrow{a} \times \left(\overrightarrow{b} + \overrightarrow{c}\right) = \left(\overrightarrow{a} \times \overrightarrow{b}\right) + \left(\overrightarrow{a} \times \overrightarrow{c}\right)$$
, "when"
(i) $\overrightarrow{a} = \hat{i} - \hat{j} - 3\hat{k}$, $\overrightarrow{b} = 4\hat{i} - 3\hat{j} + \hat{k}$ and $\overrightarrow{c} = 2\hat{i} - \hat{j} + 2\hat{k}$
(ii) $\overrightarrow{a} = 4\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{b} = \hat{i} + \hat{j} + \hat{k}$ and $\overrightarrow{c} = \hat{i} - \hat{j} + \hat{k}$.

15. Find the area of the parallelogram whose adjacent sides are represented by the vectors (i) $\overrightarrow{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{b} = -3\hat{i} - 2\hat{j} + \hat{k}$ (ii) $\overrightarrow{a} = (3\hat{i} + \hat{j} + 4\hat{k})$ and $\overrightarrow{b} = (\hat{i} - \hat{j} + \hat{k})$ (iii) $\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\overrightarrow{b} = \hat{i} - \hat{j}$ (iv) $\overrightarrow{b} = 2\hat{i}$ and $\overrightarrow{b} = 3\hat{j}$.

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16. Find the area of the parallelogram whose diagonals are represented by the vectors

(i)
$$\overrightarrow{d}_1 = 3\hat{i} + \hat{j} - 2\hat{k}$$
 and $\overrightarrow{d}_2 = \hat{i} - 3\hat{j} + 4\hat{k}$

(ii)
$$\overrightarrow{d}_1 = 2\hat{i} - \hat{j} + \hat{k}$$
 and $\overrightarrow{d}_2 = 3\hat{i} + 4\hat{j} - \hat{k}$
(iii) $\overrightarrow{d}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$ and $\overrightarrow{d}_2 = -\hat{i} + 2\hat{j}$.

17. Find the area of the trinagle whose two adjacent sides are determined

by the vectors

(i)
$$\overrightarrow{a} = -2\hat{i} - 5\hat{k}$$
and $\overrightarrow{b} = \hat{i} - 2\hat{j} - \hat{k}$

(ii)
$$\overrightarrow{a} = 3\hat{i} + 4\hat{j}$$
 and $\overrightarrow{b} = -5\hat{i} + 7\hat{j}$.

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18. Using vectors, find the area of Δ ABC whose vertices are

(i) A(1,1,2),B(2,3,5) and C(1,5,5)

- (ii) A(1,2,3),B(2,-1,4) and C(4,5,-1)
- (iii) A(3,-1,2),B(1,-1,-3) and C(4,-3,1)
- (iv) A (1,-1,2),B(2,1,-1) and C(3,-1,2).

19. Using vector method, show that the given points A,B,C are collinear:

(i) A(3,-5,1),(-1,0,8) and C(7,-10,-6)

(ii) A(6,-7,-1),B(2,-3,1) and C(4,-5,0).

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20. Show that the points A,B,C with position vectors $(3\hat{i} - 2\hat{j} + 4\hat{k}), (\hat{i} + \hat{j} + \hat{k})$ and $(-\hat{i} + 4\hat{j} - 2\hat{k})$ respectively are

collinear.

21. Show that the points having position vectors
$$\overrightarrow{a}, \overrightarrow{b}, (\overrightarrow{c} = 3\overrightarrow{a} - 2\overrightarrow{b})$$
 are collinear, whatever be $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$.
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22. Show that the points having position vectors
$$\left(-2\overrightarrow{a}+3\overrightarrow{b}+5\overrightarrow{c}\right), \left(\overrightarrow{a}+2\overrightarrow{b}+3\overrightarrow{c}\right)$$
 and $\left(7\overrightarrow{a}-\overrightarrow{c}\right)$ are collinear, whatever be $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$.

23. Find a unit vector perpendicular to the plane ABC, where the coordinates of
$$A, B, and C$$
 are $A(3, -1, 2), B(1, -1, -3) and C(4, -3, 1)$.

$$\textbf{24. If } \overrightarrow{a} = \left(\hat{i} - 2\hat{j} + 3\hat{k} \right) \text{ and } \overrightarrow{b} = \left(\hat{i} - 3\hat{k} \right) \text{ then find } \left| \overrightarrow{b} \times 2\overrightarrow{a} \right|.$$

25. If
$$\left|\overrightarrow{a}\right| = 2$$
, $\left|b\right| = 5$ and $\left|\overrightarrow{a} x \overrightarrow{b}\right| = 8$, find $\overrightarrow{a} \overrightarrow{b}$.

26.

$$\left| \stackrel{
ightarrow}{a}
ight| = 2, \left| \stackrel{
ightarrow}{b}
ight| = 7 \, \, {
m and} \, \, \left(\stackrel{
ightarrow}{a} imes \stackrel{
ightarrow}{b}
ight) = \Big(3 \hat{i} + 2 \hat{j} + 6 \hat{k} \Big), \ \ {
m find \ the \ angle \ beta}$$