MATHS

BOOKS - RS AGGARWAL MATHS (HINGLISH)

DETERMINANTS

Solved Examples

1. Evaluate:

$(i)\begin{vmatrix} 6 & -3 \\ 7 & -2 \end{vmatrix}$ $(ii)\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

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2. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x.

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3. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, find the value of x.

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4. Show that $\begin{vmatrix} \sin 10^\circ & -\cos 10^\circ \\ \sin 80^\circ & \cos 80^\circ \end{vmatrix} = 1.$

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5. Find the minors and cofactors of the elements of the determinant $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

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6. Write the minor and cofactor of each element of the following determinants and also evaluate the determinant in each case:

$\begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$

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7. Evaluate $\Delta = \begin{vmatrix} 3 & 4 & 5 \\ -6 & 2 & -3 \\ 8 & 1 & 7 \end{vmatrix}$

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8. If $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ is equal to x then value of $\begin{vmatrix} h & 2b & f \\ a & 2h & g \\ g & 2f & c \end{vmatrix}$

A. 2x

B. -x

C. -2x

D. x

Answer: C

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9. Evaluate $\begin{vmatrix} 2 & 7 & 24 \\ 6 & 16 & 72 \\ 5 & 9 & 60 \end{vmatrix}$

A. 0

B. 1024

C. 1

D. None of these

Answer: A

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10. Using the property of determinants and without expanding in questions 1 to 7 prove that ,

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

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11. prove that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$

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12. $\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix} = 0$

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13. Without expanding prove that $\begin{vmatrix} x+y & z & 1 \\ y+z & x & 1 \\ z+x & y & 1 \end{vmatrix} = 0$

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14. Evaluate $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & \omega \end{vmatrix}$ where ω is cube root of unity.

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15. Find value of $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

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16. Find the value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$.

A. xy

B. x

C. y

D. x+y

Answer: A

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17. Prove that the determinant $[x \sin\theta \cos\theta - \sin\theta - x1 \cos\theta 1x]$ is independent of 0.

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18. Let $A = \begin{bmatrix} 0 & \sin\alpha & \sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha\cos\beta \\ -\sin\alpha\sin\beta & -\cos\alpha\cos\beta & 0 \end{bmatrix}$ then

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19. Show that $|abca + 2xb + 2yc + 2zxyz| = 0$

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20. Without expanding the determinant, prove that

$$\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix} = 0.$$

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21. Evaluate $\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix}$

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22. $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$

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23. Using properties of determinants, prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

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24. Prove that :

(i) $\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix} = 2abc$

(ii) Prove that : $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

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25. Using properties of determinants, prove that

$\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix} = 2a^3b^3c^3$

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26. Q. $\begin{vmatrix} x+y & x & x \\ 15x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

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27. Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

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28. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

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29. Prove that: $\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$

$= (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$.

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30. Using properties of determinants, prove that

$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$

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31. Using properties of determinants, prove the following:

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \left(1 - x^3\right)^2$$

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32. Using properties of determinants, prove that

$$|2yy - z - x2y2z2zz - x - yx - y - zzx2x| = (x + y + z)^3$$

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33. Using properties of determinants, prove the following:

$$|3\mathrm{a} - \mathrm{a} + \mathrm{b} - \mathrm{a} + \mathrm{ca} - \mathrm{b}3\mathrm{bc} - \mathrm{ba} - \mathrm{cb} - \mathrm{c}3\mathrm{c}| = 3(\mathrm{a} + \mathrm{b} + \mathrm{c})(\mathrm{a}\,\mathrm{b} + \mathrm{b}\,\mathrm{c} +$$

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34. Prove that

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2(a+b+c)\left(ab+bc+ca-a^2-b^2-c^2\right).$$

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35. Solve the determinant using properties

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

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36. Using properties of determinants, prove that following:

$$|a+b+2cabcb+c+2abcac+a+2b| = 2(a+b+c)^3$$

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37. Using properties of determinants, prove the following :

$$|aa^2bc \hat{} 2ca \hat{} 2ab| = (a-b)(b-c)(c-a)(bc+ca+ab)$$

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38. Using properties of determinants, Find the value of $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$ if the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a^3+b^3+c^3-3abc$

A. $a^3+b^3+c^3-3abc$

B. $3abc-a^3-b^3-c^3$

C. $-a^3-b^3-c^3$

D. None of these

Answer: B

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39. $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} - 4xyz$

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40. Prove that :

(i) $\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix} = 2abc$

(ii) Prove that : $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

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41. If a, b, c are positive and unequal, show that value of the determinant

$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative

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42. Evaluate $\begin{vmatrix} {}^mC_1 & {}^mC_2 & {}^mC_3 \\ {}^nC_1 & {}^nC_2 & {}^nC_3 \\ {}^pC_1 & {}^pC_2 & {}^pC_3 \end{vmatrix}$

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43. Prove that: $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

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44. Prove that $\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix} = (a+b+c)(a-c)^2$

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45. Prove that $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

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46. If $\begin{vmatrix} x & x^2 & x^3+1 \\ y & y^2 & y^3+1 \\ z & z^2 & z^3+1 \end{vmatrix}$ = 0 and x ,y and z are not equal to any other, prove that, xyz = -1

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47. Prove that $\begin{bmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{bmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$

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48. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = (abc)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right) = (bc+ca+ab+abc$

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49. $\begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix}$

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50. Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$

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51. Prove $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^2$

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52. Prove that

$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = \left(a^2+b^2+c^2\right)(a-b)(b-c)(c-a)(a+b+c)$$

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53. Evaluate $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

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54. If $A+B+C=\pi$, then the value of $\begin{vmatrix} \sin(A+B+C) & \sin(A+C) & \cos C \\ -\sin B & 0 & \tan C \\ \cos(A+B) & \tan(B+C) & 0 \end{vmatrix}$ is equal to

(a) 0 (b) 1 (c) $2\sin B\tan A\cos C$ (d) none of these

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55. If A+B+C =π, show that

$$\begin{vmatrix} \sin^2 A & \sin A\cos A & \cos^2 A \\ \sin^2 B & \sin B\cos B & \cos^2 B \\ \sin^2 C & \sin C\cos C & \cos^2 C \end{vmatrix} = -\sin(A-B)\sin(B-C)\sin(C-A)$$

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56. Without expanding evaluate the determinant

$$\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$$

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57. Prove that $\triangle \begin{vmatrix} a+bx & c+dx & p+qx \\ -ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2)\begin{bmatrix} a & c & p \\ b & d & q \\ u & v & w \end{bmatrix}$

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58. Show without expanding at any stage that:

$[a, a^2, bc], [b, b^2, ca], [c, c^2, ab]| = |[1, a^2, a^3], [1, b^2, b^3], [1, c^2, c^3]|$

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59. 5. Using the properties of determinants, prove that

$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

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60. Using properties of determinants, prove that

$$|b+cq+ry+zc+ar+pz+xc+bp+qx+y| = 2\,|apxbqycrz|$$

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61. Show that $\Delta = \Delta_1$, where

$$\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix} \text{ and } \Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$$

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62. If $\Delta_1 = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$ and $\Delta_2 \begin{vmatrix} q & -b & y \\ -p & a & -x \\ r & -c & z \end{vmatrix}$ then without expanding Δ_1 and Δ_2, prove that $\Delta_1 + \Delta_2 = 0$

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63. Using properties of determinants, solve for x: $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

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64. Using properties of determinants, solve the following for x:

$|x - 22x - 33x - 4x - 42x - 93x - 16x - 82x - 273x - 64| = 0$

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65. If $a+b+c=0$ and $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ then prove that either $x=0$ or $x = \sqrt{\frac{3}{2}(a^2+b^2+c^2)}$

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66. Find the area of the triangle whose vertices are: $(-2, -3), (3, 2), (-1, -8)$

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67. Show that points $A(a, b+c), B(b, c+a), C(c, a+b)$ are collinear.

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68. If the points (a, b), (a', b') and (a-a', b-b') are collinear, show that ab' =a'b.

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69. Find the value of k in order that the points (5, 5), (k, 1) and (10, 7) are collinear.

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70. Let A(1, 3), B(0, 0) and C(k, 0) be three points such that $ar(\triangle ABC) = 3$ sq units. Find the value of k.

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71. (i) Find equation of line joining (1,2) and (3,6) using determinants, (ii) Find equation of line joining (3, 1) and (9,3) using determinants.

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Exercise 6 A

1. If A is a 2×2 matrix such that $|A| \neq 0$ and $|A| = 5$, write the value of |4A|.

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2. If A is 3×3 matrix, $|A| \neq 0$ and $|3A| = k|A|$, then write the value of k.

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3. Let A be a square matrix of order $3x3$ Write the value of $|2A|$ where $|A| = 4$

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4. If A_{ij} is the cofactor of the element a_{ij} of the determinant $[2 - 3 - 7604157]$, then write the value of $a_{32} \cdot A_{32}$.

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5. $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

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6. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

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7. Solve : $\frac{3x+4}{7} - \frac{x+5}{14} = \frac{x}{28} + \frac{x+1}{14}$

The following steps are involved in solving the above problem. Arrange them in sequential order

(A) $5x+3 = \frac{3x+2}{2} \Rightarrow 10x+6 = 3x+2$

(B) $\Rightarrow 7x = -4$

(C) Given $\frac{3x+4}{7} - \frac{x+5}{14} = \frac{x}{28} + \frac{x+1}{14}$
$\Rightarrow \frac{6x+8-x-5}{14} = \frac{x+2x+2}{28}$

(D) $\Rightarrow x = -\frac{4}{7}$

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8. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ then the value of x is

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9. If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, find the value of x.

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10. If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, find the value of 3|A|

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11. Evaluate $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$

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12. Evaluate '|[sqrt(6), sqrt(5)], [sqrt(20), sqrt(24)]|`

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13. Evaluate $\begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

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14. Evaluate $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$

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15. Evaluate $\begin{vmatrix} \sin 60^\circ & \cos 60^\circ \\ \sin 30^\circ & \cos 30^\circ \end{vmatrix}$

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16. Evaluate $\begin{vmatrix} \cos 65^\circ & \sin 65^\circ \\ \sin 25^\circ & \cos 25^\circ \end{vmatrix}$

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17. Evaluate $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$

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18. Evaluate $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$

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19. Without expanding the determinant, prove that $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix} = 0$

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20. For what value of x, the given matrix $A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ is a singular matrix?

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21. Evaluate $\begin{vmatrix} 14 & 9 \\ -8 & -7 \end{vmatrix}$

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22. Evaluate $\begin{vmatrix} \sqrt{3} & \sqrt{5} \\ -\sqrt{5} & 3\sqrt{3} \end{vmatrix}$

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Exercise 6 B

1. Find the Determinant :- $\begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$

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2. $\begin{vmatrix} 29 & 26 & 22 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix}$

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3. Write the value of the following determinant: $|10218361341736|$

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4. Evaluate the following: $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$

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5. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$

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6. $\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

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7. Show that

$|11+p1+p+q23+2p1+3p+2q36+3p106p+3q|=1.$

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8. Using properties of determinants, prove that

$|a+xyzxa+yzxya+z|=a^2(a+x+y+z)$

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9. $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x+2)(x-a)^2$

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10. $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(x-4)^2$

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11. $\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} = (5x+\lambda)(\lambda-x)^2$

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12. Prove that : $|a^2 2a + 112a + 1a + 21331| = (a-1)^3$

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13. Using properties of determinants, prove the following:

$|\times + yx + 2y\, x + 2y \times + yx + yx + 2yx| = 9y^2(x+y)$

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14. Using properties of determinant, prove that

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx)$$

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15. Prove that : $Det \begin{bmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{bmatrix} = xyz(x-y)(y-z)(z-x)$

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16. $\begin{bmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{bmatrix} = 3abc - a^3 - b^3 - c^3$

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17. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

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18. $\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} = ?$

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19. Prove that

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

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20. $\begin{bmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{bmatrix} = \left(b^2-ac\right)\left(ax^2+2bxy+cy^2\right)$

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21. $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = -4(a-b)(b-c)(c-a)$

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22. $\begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix} = -8.$

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23.

$\begin{vmatrix} (m+n)^2 & l^2 & mn \\ (n+l)^2 & m^2 & \ln \\ (l+m)^2 & n^2 & lm \end{vmatrix} = \left(l^2+m^2+n^2\right)(l-m)(m-n)(n-l)(l+m+$

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24. Prove that

$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$$

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25. Using properties of determinants, prove that:

$$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

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26. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$

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27. Prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a+b+c)\left(a^2+b^2+c^2\right)$

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28. If a, b, c are non-zero real numbers then $D = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} =$

(A) abc (B) $a^2b^2c^2$ (C) bc+ca+ab (D) 0

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29. Using properties of determinants, show the following:

$\left|(a+C)^2 abcaab(a+c)^2 bcacbc(a+b)^2\right| = 2abc(a+b+c)^3$

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30. The determinat $\Delta = \begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix}$ equals

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31. Show that

$$\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-C^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3$$

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32. $\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0$, where α, β, γ are in AP

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33. $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$

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34. Prove that

$$\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ca & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)\left(a^2+b^2+c^2\right)$$

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35. Prove that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

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36. $\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

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37. Show that $x = 2$ is a root of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix} = 0$

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38. $\begin{vmatrix} 1 & x & x^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$=0; $b \neq c$

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39. $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ b & b & x+c \end{vmatrix} = 0$

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40. One root of the equation $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0si(A)8/3$

$(B)2/3(C)1/3(D)16/3`$

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41. $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$

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42. The solution set of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ is

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43. The number of real roots of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x-2 \end{vmatrix} = 0$ is

(i) 0 (ii) 1 (iii) 2 (iv) 3

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44. Prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a+b+c)\left(a^2+b^2+c^2\right)$

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Exercise 6 C

1. Find the area of the triangle whose vertices are:

(i) A(3, 8), B(-4, 2) and C(5, -1)

(ii) A(-2, 4), B(2, -6) and C(5, 4)

(iii) A(-8, -2), B(-4, -6) and C(-1, 5)

(iv) P(0, 0), Q(6, 0) and R(4, 3)

(v) P(1, 1), Q(2, 7) and R(10, 8)

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2. Use determinants to show that the following points are collinear.

(i) A(2, 3), B(-1, -2) and C(5, 8)

(ii) A(3, 8), B(-4, 2) and C(10, 14)

(iii) P(-2, 5), Q(-6, -7) and R(-5, -4)

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3. If the points A(3, -2), B(k, 2) and C(8, 8) are collinear then the value of k is

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4. Find the value of k for which the points P(5, 5), Q(k, 1) and R(11, 7) are collinear

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5. Find the value of k for which for points A(1, -1), B(2, k) and C(4, 5) are collinear

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6. If area of triangle is 35 sq units with vertices $(2, -6), (5, 4) and (k, 4)$.

Then k is(A) 12 (B) -2 (C) $12, 2$ (D) $12, 2$

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7. Find the values of k if area of triangle is 4 sq. units and vertices are:

$(-2, 0), (0, 4), (0, k)$

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8. Prove that the points A(a, 0), B(0, b) and C(1, 1) are collinear, if $\frac{1}{a} + \frac{1}{b} = 1.$

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Objective Questions

1. $\begin{vmatrix} \cos 70^\circ & \sin 20^\circ \\ \sin 70^\circ & \cos 20^\circ \end{vmatrix} = ?$

A. 1

B. 0

C. $\cos 50^\circ$

D. $\sin 50^\circ$

Answer: B

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2. $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix} = ?$

A. 1

B. $\frac{1}{2}$

C. $\frac{\sqrt{3}}{2}$

D. none of these

Answer: C

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3. $\begin{vmatrix} \sin 23^\circ & -\sin 7^\circ \\ \cos 23^\circ & \cos 7^\circ \end{vmatrix} = ?$

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{2}$

C. $\sin 16^\circ$

D. $\cos 16^\circ$

Answer: B

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4. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

A. $(a^2+b^2-c^2-d^2)$

B. $(a^2 - b^2 + c^2 - d^2)$

C. $(a^2 + b^2 + c^2 + d^2)$

D. none of these

Answer: C

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5. Evaluate $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & \omega \end{vmatrix}$ where ω is cube root of unity.

A. 1

B. -1

C. 0

D. none of these

Answer: C

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6. If ω is a complex cube root of unity then the value of the determinant

$$\begin{vmatrix} 1 & \omega & \omega+1 \\ \omega+1 & 1 & \omega \\ \omega & \omega+1 & 1 \end{vmatrix}$$ is

A. 2

B. 4

C. 0

D. -3

Answer: B

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7. If $A = \begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$ then $|AdjA| =$

A. 8

B. 16

C. 64

D. 128

Answer: C

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8. $\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} = ?$

A. 2

B. 6

C. 24

D. 120

Answer: C

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9. $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = ?$

A. $(a+b+c)$

B. $3(a+b+c)$

C. $3abc$

D. 0

Answer: D

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10. find $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = .$

A. 0

B. 1

C. -1

D. none of these

Answer: B

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11. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} =$ is equal to

A. $(a-b)(b-c)(c-a)$

B. $-(a-b)(b-c)(c-a)$

C. $(a-b)(b-c)(c-a)(a+b+c)$

D. $abc(a-b)(b-c)(c-a)$

Answer: C

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12. Without expanding evaluate the determinant

$\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$

A. 0

B. 1

C. $\sin(\alpha+\delta)+\sin(\beta+\delta)+\sin(\gamma+\delta)$

D. none of these

Answer: A

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13. If a, b, c be distinct positive real numbers then the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

A. positive

B. negative

C. a perfect square

D. 0

Answer: B

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14. Q. $\begin{vmatrix} x+y & x & x \\ 15x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

A. 0

B. x^3

C. y^3

D. none of these

Answer: B

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15. Evaluate the following: $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

A. (a-1)

B. $(a-1)^2$

C. $(a-1)^3$

D. none of these

Answer: C

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16. $\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} = ?$

A. a^3

B. $-a^3$

C. 0

D. none of these

Answer: B

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17. Prove that $\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix} = (a+b+c)(a-c)^2$

A. $(a+b+c)(a-c)$

B. $(a+b+c)(b-c)$

C. $(a+b+c)(a-c)^2$

D. $(a+b+c)(b-c)^2$

Answer: C

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18. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = ?$

A. (x+y)

B. (x-y)

C. xy

D. none of these

Answer: C

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19. $\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = ?$

A. $(a-b)(b-c)(c-a)$

B. $-(a-b)(b-c)(c-a)$

C. $(a+b)(b+c)(c+a)$

D. none of these

Answer: A

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20. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = ?$

A. $4abc$

B. $2(a+b+c)$

C. $(ab+bc+ca)$

D. none of these

Answer: A

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21. $\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix} = ?$

A. a+b+c

B. 2(a+b+c)

C. 4abc

D. $a^2b^2c^2$

Answer: C

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22. $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix} =$

A. -2

B. 2

C. $x^2 - 2$

D. $x^2 + 2$

Answer: A

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23. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$ then $value of - x = ?$

A. 0

B. 6

C. -6

D. 9

Answer: C

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24. The solution set of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ is

A. $\{2, -3, 7\}$

B. $\{2, 7, -9\}$

C. $\{-2, 3, -7\}$

D. none of these

Answer: B

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25. Using properties of determinants, solve the following for x:

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

A. $\{4\}$

B. $\{2, 4\}$

C. $\{2, 8\}$

D. $\{4, 8\}$

Answer: A

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26. The solution set of the equation $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ is

A. {a, 0}

B. {3a 0}

C. {a, 3a}

D. none of these

Answer: B

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27. One root of the equation $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$ is

A. $\left\{\frac{2}{3}, \frac{8}{3}\right\}$

B. $\left\{\frac{2}{3}, \frac{11}{3}\right\}$

C. $\left\{\frac{3}{2}, \frac{8}{3}\right\}$

D. none of these

Answer: B

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28. The vertices of $\triangle$ ABC are $A(-2, 4), B(2, -6)$ and $C(5, 4)$. The area of $\triangle$ ABC is

A. 17.5 sq units

B. 35 sq units

C. 32 sq units

D. 28 sq units

Answer: B

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29. If the points $A(3, -2), B(k, 2)$ and $C(8, 8)$ are collinear then the value of k is

A. 3

B. 2

C. 5

D. 6

Answer: C