



MATHS

BOOKS - RS AGGARWAL MATHS (HINGLISH)

FUNCTIONS

Solved Examples

1. Let $f: \mathbb{N} \rightarrow \mathbb{N}: f(x) = 2x$ for all $x \in \mathbb{N}$

Show that f is one -one and into.

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2. Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$, defined as $f(x) = x^2$, is neither one-one nor onto.

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3. The modulus function $f: R \rightarrow R$, given by $f(x) = |x|$ is

- A. One One
- B. Many One
- C. cant say
- D. None of these

Answer: B

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4. Prove that the greatest integer function $f: R \rightarrow R$, given by $f(x) = [x]$, is neither one-one nor onto, where $[x]$ denotes the greatest integer less than or equal to x .

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5. Show that the function $f: R_0 \rightarrow R_0$, defined as $f(x) = \frac{1}{x}$, is one-one onto, where R_0 is the set of all non-zero real numbers. Is the result true, if the domain R_0 is replaced by N with co-domain being same as R_0 ?

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6. function $f: R \rightarrow R: f(x) = x^3$ is

- A. One One
- B. Many One
- C. Not a function
- D. None of these

Answer: A

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7. Show that the function $f: R \rightarrow R: f(x) = 3 - 4x$ is one-one onto and hence bijective.

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8. Show that the function $f: N \rightarrow N$ defined by

$$f(x) = \left\{ \begin{array}{ll} x - 1 & \text{if } x \text{ is even} \\ x + 1 & \text{if } x \text{ is odd} \end{array} \right\}$$

is one-one and onto.

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9. Show that $f: N \rightarrow N$ defined by

$$f(n) = \left\{ \begin{array}{ll} \frac{n}{2} & \text{if } n \text{ is even} \\ \frac{n+1}{2} & \text{if } n \text{ is odd} \end{array} \right\}$$

is a many -one onto function

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10. Show that the signum function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

is neither one-one nor onto.

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11. Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - [1]$. Consider the function $f: A \xrightarrow{\rightarrow} B$ defined by $f(x) = \left(\frac{x-2}{x-3} \right)$. Show that f is one-one and onto and hence find f^{-1}

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12. Let A and B be sets. Show that $f: A \times B \rightarrow B \times A$ such that $f(a, b) = (b, a)$ is bijective function.

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13. Find the domain and range of the real function

$$f(x) = \sqrt{9 - x^2}$$

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14. Find the domain and range of the real function defined by

$$f(x) = \frac{1}{(1 - x^2)}.$$

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15. Let $f: X \rightarrow Y$ be a function. Define a relation R in X given by

$R = \{(a, b) : f(a) = f(b)\}$. Examine whether R is an equivalence relation or not.

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16. If the functions f and g are given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$, find range of f and g . Also, write down $f \circ g$ and $g \circ f$ as sets of ordered pairs.

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17. Find $g \circ f$ and $f \circ g$, if $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$. Show that $g \circ f \neq f \circ g$.

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18. Let R be the set of all real numbers let $f: R \rightarrow R: f(x) = \sin x$ and
and
 $g: R \rightarrow R: g(x) = x^2$. Prove that $g \circ f \neq f \circ g$

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19. Find gof and gof when $f: R \rightarrow R$ and $g: R \rightarrow R$ is defined by

$$f(x) = 8x^3 \text{ and } g(x) = x^{1/3}$$

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20. If $f: R \rightarrow R$ is defined by $f(x) = x^2 - 3x + 2$, write $f\{f(x)\}$.

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21. Let $f: R \rightarrow R: f(x) = (3 - x^3)^{1/3}$. Find $f \circ f$

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22. Let $f: A \rightarrow B$, and let I_A and I_B be identity functions on A and B respectively. Prove that $(f \circ I_A) = f$ and $(I_B \circ f) = f$

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23. (Associativity) Let $f: A \rightarrow B, g: B \rightarrow C$ and $h: C \rightarrow \cdot$. Then prove that $(h \circ g) \circ f = h \circ (g \circ f)$

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24. Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $f(n) = 3n$ for all $n \in \mathbb{Z}$ and $g: \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $g(n) = \begin{cases} \frac{n}{3}, & \text{if } n \text{ is a multiple of } 3 \\ 0, & \text{if } n \text{ is not a multiple of } 3 \end{cases}$ or all $n \in \mathbb{Z}$. Show that $g \circ f = I_{\mathbb{Z}}$ and $f \circ g \neq I_{\mathbb{Z}}$

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25. Let $A = \mathbb{R} - \left\{ \frac{7}{5} \right\}$ and $B = \mathbb{R} - \left\{ \frac{3}{5} \right\}$
 Let $f: A \rightarrow B: f(x) = \frac{3x + 4}{5x - 7}$ and $g: B \rightarrow A: g(x) = \frac{7x + 4}{5x - 3}$
 Show that $(g \circ f) = I_B$ and $(f \circ g) = I_A$

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26. If functions $f: A \rightarrow B$ and $g: B \rightarrow A$ satisfy $gof = I_A$, then show that f is one-one and g is onto.

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27. Let $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = 4x + 3$ for all $x \in \mathbb{R}$. Show that f is invertible and find f^{-1}

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28. Consider $f: \mathbb{R}_+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of given f by $f^{-1}(y) = \sqrt{y - 4}$ where \mathbb{R}_+ is the set of all non-negative real numbers.

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29. Let \mathbb{R}^+ be the set of all positive real numbers. Let

$f: \mathbb{R}^+ \rightarrow \mathbb{R}^+ : f(x) = e^x$ for all $x \in \mathbb{R}^+$. Show that f is invertible and hence find f^{-1}

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30. let $A = \left\{x: -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\right\}$ and $B = \{x: -1 \leq x \leq 1\}$.

Show that the function $f: A \rightarrow B$ defined by, $f(x) = \sin x$ for all $x \in A$, is bijective. Hence, find a formula that defines f^{-1}

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31. Let $f: \mathbb{N} \rightarrow Y: f(x) = x^2$ where $Y = \text{range}(f)$. Show that f is invertible and find f^{-1}

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32. Let $f: [-1, 1] \rightarrow Y: f(x) = \frac{x}{(x+2)}, x \neq -2$ and $Y = \text{range}(f)$. Show that f is invertible and find f^{-1}

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33. Let $f: N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \rightarrow S$, where, S is the range of f , is invertible. Find the inverse of f .

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34. Let $f: R \rightarrow R: f(x) = 10x + 7$. Find the function $g: R \rightarrow R: gof = fog = I_g$

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35. Let $f: W \rightarrow W: f(n) = \begin{cases} (n + 1) & \text{when } n \text{ is even} \\ (n - 1) & \text{when } n \text{ is odd} \end{cases}$

Show that f is invertible. Find f^{-1}

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36. Let $A = \{1, 2, 3\}$ and let $f: A \rightarrow A$ defined by

$$f = \{(1, 2), (2, 3), (3, 1)\}$$

Find f^{-1} if it exists .

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Objective Questions

1. Let $f: \mathbb{N} \rightarrow \mathbb{N}: f(x) = 2x$ for all $x \in \mathbb{N}$ then f is

A. one-one and onto

B. one - one and into

C. many -one and onto

D. many -one and into

Answer: B



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2. $f: N \rightarrow N: f(x) = x^2 + x + 1$ is

A. one-one and onto

B. one - one and into

C. many -one and onto

D. many -one and into

Answer: B



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3. $f: R \rightarrow R: f(x) = x^2$ is

- A. one-one and onto
- B. one-one and into
- C. many -one and onto
- D. many -one and into

Answer: D

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4. Show that the function $f: R \rightarrow R: f(x) = x^3$ is one -one and onto.

- A. one-one and onto
- B. one-one and into
- C. many - one and onto
- D. many -one and into

Answer: A



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5. $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+ : f(x) = e^x$ is

- A. many - one and into
- B. many - one and onto
- C. one - one and into
- D. one - one and onto

Answer: D



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6. $f: \left[\frac{-\pi}{2}, \frac{\pi}{2} \right] \rightarrow [-1, 1] : f(x) = \sin x$ is

- A. one-one and into
- B. one-one and onto
- C. many-one and into
- D. many-one and onto

Answer: B

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7. $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = \cos x$ is

- A. one-one and into
- B. one-one and onto
- C. many- one and into
- D. many-one and onto

Answer: C

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8. $f: C \rightarrow R: f(z) = |z|$ is

- A. one-one and into
- B. one -one and onto
- C. many -one and into
- D. many -one and onto

Answer: C

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9.

Let

$A = R - \{3\}$ and $B = R - \{1\}$. Then $f: A \rightarrow B: f(x) = \frac{(x - 2)}{(x - 3)}$

is

A. one-one and into

B. one -one and onto

C. many-one and into

D. many -one and onto

Answer: B

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10. Let $f: N \rightarrow N: f(n) = \begin{cases} \frac{1}{2}(n + 1), & \text{when } n \text{ is odd} \\ \frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$

Then f is

A. one-one and into

B. one-one and onto

C. many-one and into

D. many- one and into

Answer: D

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11. Let A and B be two sets. Show that $f: A \times B \rightarrow B \times A$ defined by $f(a, b) = (b, a)$ is

- A. one-one and onto
- B. one-one and into
- C. many-one and onto
- D. many -one and onto

Answer: A

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12. Let $f: Q \rightarrow Q: f(x) = (2x + 3)$. Then $f^{-1}(y) = ?$

A. $(2x - 3)$

B. $\frac{1}{(2y - 3)}$

C. $\frac{1}{2}(y - 3)$

D. none of these

Answer: C

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13. Let $f: R - \left\{ -\frac{4}{3} \right\} \rightarrow R$ be a function as $f(x) = \frac{4x}{3x + 4}$. The

inverse of f is map, $g: R \setminus \left\{ -\frac{4}{3} \right\} \rightarrow R - \left\{ -\frac{4}{3} \right\}$ given by (a)

$$g(y) = \frac{3y}{3 - 4y} \quad (b) \quad g(y) = \frac{4y}{4 - 3y} \quad (c) \quad g(y) = \frac{4y}{3 - 4y} \quad (d)$$

$$g(y) = \frac{3y}{4 - 3y}$$

A. $\frac{4y}{(4 - 3y)}$

B. $\frac{4y}{(4y + 3)}$

C. $\frac{4y}{(3y - 4)}$

D. none of these

Answer: A

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14. Let $f: N \rightarrow X: f(x) = 4x^2 + 12x + 15$. Then $f^{-1}(y) = ?$

A. $\frac{1}{2}(\sqrt{y-4} + 3)$

B. $\frac{1}{2}(\sqrt{y-6} - 3)$

C. $\frac{1}{2}(\sqrt{y-4} + 5)$

D. none of these

Answer: B

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15. If $f(x) = \frac{(4x + 3)}{(6x + 4)}$, $x \neq \frac{2}{3}$ then $(f \circ f)(x) = ?$

A. x

B. $(2x - 3)$

C. $\frac{4x - 6}{3x + 4}$

D. none of these

Answer: A

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16. If $f(x) = (x^2 - 1)$ and $g(x) = (2x + 3)$ then $(g \circ f)(x) = ?$

A. $(2x^2 + 3)$

B. $(3x^2 + 2)$

C. $(2x^2 + 1)$

D. none of these

Answer: C



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17. If $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ then $f(x) =$

A. x^2

B. $(x^2 - 1)$

C. $(x^2 - 2)$

D. none of these

Answer: C



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18. If $f(x) = \frac{1}{1-x}$, then $f(f(f(x)))$ is equal to

A. $\frac{1}{(1 - 3x)}$

B. $\frac{x}{(1 + 3x)}$

C. x

D. none of these

Answer: C



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19. Let $f: R \rightarrow R: f(x) = (3 - x^3)^{1/3}$. Find $f \circ f$

A. $x^{1/3}$

B. x

C. $\sqrt[3]{1-x^{1/3}}$

D. none of these

Answer: B

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20. If $f(x) = x^2 - 3x + 2$ then $(f \circ f)(x) = ?$

A. x^4

B. $x^4 - 6x^3$

C. $x^4 - 6x^3 + 10x^2$

D. none of these

Answer: D

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21. Find $g \circ f$ if $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$

A. x

B. $2x$

C. $\frac{x}{2}$

D. $3x^2$

Answer: B

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22. If f, g, h are real functions given by $f(x) = x^2, g(x) = \tan x$ and $h(x) = \log x$, then write the value of $(hog \text{ of}) \left(\sqrt{\frac{\pi}{4}} \right)$.

A. 0

B. 1

C. $\frac{1}{x}$

D. $\frac{1}{2} \log \frac{\pi}{4}$

Answer: A

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23. If the functions f and g are given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$, find $g \circ f$ as sets of ordered pairs.

- A. $\{(3, 1), (1, 3), (3, 4)\}$
- B. $\{(1, 3), (3, 1), (4, 3)\}$
- C. $\{(3, 4), (4, 3), (1, 3)\}$
- D. $\{(2, 5), (5, 2), (1, 5)\}$

Answer: B

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24. Let $f(x) = \sqrt{9 - x^2}$. then , domain $(f) = ?$

- A. $[-3, 3]$
- B. $(-\infty, -3]$

C. $[3, \infty)$

D. $(-\infty, -3] \cup (4, \infty)$

Answer: A

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25. Let $f(x) = \sqrt{\frac{x-1}{x-4}}$. Then $\text{dom}(f) = ?$

A. $[1, 4)$

B. $[1, 4]$

C. $(-\infty, 4]$

D. $(-\infty, 1] \cup (4, \infty)$

Answer: D

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26. Let $f(x) = e^{\sqrt{x^2-1}} \cdot \log(x-1)$. Then $\text{dom}(f) = ?$

A. $(-\infty, 1]$

B. $[-1, \infty)$

C. $(1, \infty)$

D. $(-\infty, -1] \cup (1, \infty)$

Answer: C



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27. Let $f(x) = \frac{x}{(x^2 - 1)}$ Then $\text{dom}(f) = ?$

A. \mathbb{R}

B. $\mathbb{R} - \{1\}$

C. $\mathbb{R} - \{-1\}$

D. $\mathbb{R} - \{-1, 1\}$

Answer: D



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28. Let $f(x) = \frac{\sin^{-1} x}{x}$. then dom $(f) = ?$

A. $(-1, 1)$

B. $[-1, 1]$

C. $[-1, 1] - [0]$

D. none of these

Answer: C



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29. Let $f(x) = \cos^{-1} 2x$. Then dom $(f) = ?$

A. $[-1, 1]$

B. $\left[\frac{-1}{2}, \frac{1}{2}\right]$

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

D. $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

Answer: B



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30. Let $f(x) = \cos^{-1}(3x - 1)$. Then, $\text{dom}(f) = ?$

A. $\left(0, \frac{2}{3}\right)$

B. $\left[0, \frac{2}{3}\right]$

C. $\left[\frac{-2}{3}, \frac{2}{3}\right]$

D. none of these

Answer: B

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31. Let $f(x) = \sqrt{\cos x}$. Then $\text{dom}(f) = ?$

A. $\left[0, \frac{\pi}{2}\right]$

B. $\left[\frac{3\pi}{2}, 2\pi\right]$

C. $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

D. none of these

Answer: C

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32. Let $f(x) = \sqrt{\log(2x - x^2)}$. Then $\text{dom}(f) = ?$

A. $(0, 2)$

B. $[1, 2]$

C. $(-\infty, 1]$

D. $\{1\}$

Answer: D

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33. Let $f(x) = x^2$. Then $\text{dom}(f)$ and $\text{range}(f)$ are respectively

A. \mathbb{R} and \mathbb{R}

B. \mathbb{R}^+ and \mathbb{R}^+

C. \mathbb{R} and \mathbb{R}^+

D. \mathbb{R} and $\mathbb{R} - \{0\}$

Answer: C

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34. Let $f(x) = x^3$. Then, $\text{dom}(f)$ and $\text{range}(f)$ are respectively

A. \mathbb{R} and \mathbb{R}

B. \mathbb{R}^+ and \mathbb{R}^+

C. \mathbb{R} and \mathbb{R}^+

D. \mathbb{R}^+ and \mathbb{R}

Answer: A



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35. Let $f(x) = \log(1 - x) + \sqrt{x^2 - 1}$. Then $\text{dom}(f) = ?$

A. $(1, \infty)$

B. $(-\infty, -1]$

C. $[-1, 1]$

D. $(0, 1)$

Answer: B



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36. Let $f(x) = \frac{1}{(1 - x^2)}$. Then range (f) = ?

A. $(-\infty, 1]$

B. $(-\infty, 0) \cup [1, \infty)$

C. $[-1, 1]$

D. none of these

Answer: B



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37. Let $f(x) = \frac{x^2}{(1 + x^2)}$. Then range (f) = ?

A. $[1, \infty)$

B. $[0, 1)$

C. $[-1, 1]$

D. $(0, 1]$

Answer: B



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38. The range of $f(x) = x + \frac{1}{x}$ is

A. $[-2, 2]$

B. $[2, \infty)$

C. $(-\infty, -2]$

D. $(-\infty, -2] \cup [2, \infty)$

Answer: D

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39. The range of $f(x) = a^x$, where $a > 0$ is

A. $] - \infty, 0]$

B. $] - \infty, 0)$

C. $[0, \infty)$

D. $(0, \infty)$

Answer: D

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Exercise 2 A

1. Define a functions . What do you mean by the domain and range of a function ? Give examples.

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2. Define each of the following:

- (i) injective function (ii) surjective function
(iii) bijective function (iv) many -one function
(v) into function

Give an example of each type of functions.

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3. Give an example of a function which is

- (i) one-one but not onto (ii) one-one and onto
(iii) neither one-one nor onto (iv) onto but not one-one

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4. Let $f: R \rightarrow R$ be defined by

$$f(x) = \begin{cases} 3x - 1 & \text{when } x > 3 \\ x^2 - 2 & \text{when } -2 \leq x \leq 3 \\ 2x + 3 & \text{when } x < -2 \end{cases}$$

Find (i) $f(2)$ (ii) $f(4)$ (iii) $f(-1)$ (iv) $f(-3)$

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5. show that the function $f: R \rightarrow R: f(x) = 1 + x^2$ is many -one into .

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6. show that the function $f: R \rightarrow R: f(x) = x^4$ is many -one and into

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7. show that the function $f: \mathbb{R} \rightarrow \mathbb{R} f(x) = x^5$ is one-one and onto .

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8. Consider a function $f: \left[0, \frac{\pi}{2}\right] \rightarrow \mathbb{R}$ given by $f(x) = \sin x$ and $g: \left[0, \frac{\pi}{2}\right] \rightarrow \mathbb{R}$ given by $g(x) = \cos x$. Show that f and g are one-one, but $f + g$ is not one-one.

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9. show that the function

(i) $f: \mathbb{N} \rightarrow \mathbb{N}: f(x) = x^2$ is one-one into

(ii) $f: \mathbb{Z} \rightarrow \mathbb{Z}: f(x) = x^2$ is many-one into .

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10. Show that the function (i) $f: \mathbb{N} \rightarrow \mathbb{N}: f(x) = x^3$ is one-one into
(ii) $f: \mathbb{Z} \rightarrow \mathbb{Z}: f(x) = x^3$ is one-one into

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11. Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = \sin x$ is neither one-one nor onto

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12. Prove that the function $f: \mathbb{N} \rightarrow \mathbb{N}$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

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13. show that the function $f: N \rightarrow Z$ defined by

$$f(n) = \begin{cases} \frac{1}{2}(n-1), & \text{when } n \text{ is odd} \\ -\frac{1}{2}n, & \text{when } n \text{ is even} \end{cases}$$

is both one-one and onto

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14. find the domain and range of the function

$$f: R \rightarrow R: f(x) = x^2 + 1$$

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15. Which of the following relations are functions ? Give reasons . In case of a functions find its domain and range .

(i) $f = \{(-1, 2), (1, 8), (2, 11), (3, 14)\}$

(ii) $g = \{(1, 1), (1, -1), (4, 2), (9, 3), (16, 4)\}$

(iii) $h = \{(a, b), (b, c), (c, d), (d, c)\}$



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16. Find the domain and range of the real function defined by

$$f(x) = \frac{x^2}{(1+x^2)}$$

Show that f is many-one



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17. Show that the function

$$f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = \begin{cases} -1 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational} \end{cases}$$

is many-one into.

Find (i) $f\left(\frac{1}{2}\right)$ (ii) $f(\sqrt{2})$ (iii) $f(\pi)$ (iv) $f(2 + \sqrt{3})$



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1. Let $A = \{1, 2, 3, 4\}$. Let $f: A \rightarrow A$ and $g: A \rightarrow A$

defined by $f = \{(1, 4), (2, 1), (3, 3), (4, 2)\}$ and

$g = \{(1, 3), (2, 1), (3, 2), (4, 4)\}$

Find (i) $g \circ f$ (ii) $f \circ g$ (iii) $f \circ f$.

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2. Let $f = \{(3, 1), (9, 3), (12, 4)\}$ and

$g = \{(1, 3), (3, 3), (4, 9), (5, 9)\}$. Show that $g \circ f$ and $f \circ g$ are both

defined. Also, find $f \circ g$ and $g \circ f$.

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3. Let $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = x^2$ and $g: \mathbb{R} \rightarrow \mathbb{R}: g(x) = (x + 1)$

Show that $(g \circ f) \neq (f \circ g)$

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4.

Let

$$f: R \rightarrow R: f(x) = (2x + 1) \quad \text{and} \quad g: R \rightarrow R: g(x) = (x^2 - 2)$$

Write down the formulae for

(i) $(g \circ f)$ (ii) $(f \circ g)$ (iii) $(f \circ f)$ (iv) $(g \circ g)$

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5.

Let

$$f: R \rightarrow R: f(x) = (x^2 + 3x + 1) \quad \text{and} \quad g: R \rightarrow R: g(x) = (2x - 3).$$

Write down the formulae for

(i) $g \circ f$ (ii) $f \circ g$ (iii) $g \circ g$

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6. Let $f: R \rightarrow R: f(x) = |x|$ prove that $f \circ f = f$

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7. Let $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = x^2$, $g: \mathbb{R} \rightarrow \mathbb{R}: g(x) = \tan x$, and $h: \mathbb{R} \rightarrow \mathbb{R}: h(x) = \log x$ find a formula for $h \circ (g \circ f)$ Show that $[h \circ (g \circ f)](\sqrt{\pi/4}) = 0$

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8. Let
 $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = (2x - 3)$ and $g: \mathbb{R} \rightarrow \mathbb{R}: g(x) = \frac{1}{2}(x + 3)$
show that $(f \circ g) = I_{\mathbb{R}} = (g \circ f)$

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9. If $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $f(x) = 2x$ for all $x \in \mathbb{Z}$. Find $g: \mathbb{Z} \rightarrow \mathbb{Z}$ such that $g \circ f = I_{\mathbb{Z}}$.

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10. Let $f: N \rightarrow N: f(x) = 2x$, $g: N \rightarrow N: g(y) = 3y + 4$

and $h: N \rightarrow N: h(z) = \sin z$

Show that $h \circ (g \circ f) = (h \circ g) \circ f$.

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11. If f be a greatest integer function and g be an absolute value function, find the value of $(f \circ g)\left(-\frac{3}{2}\right) + (g \circ f)\left(\frac{4}{3}\right)$.

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12.

Let

$f: R \rightarrow R: f(x) = x^2 + 2$ and $g: R \rightarrow R: g(x) = \frac{x}{x-1}, x \neq 1$.

Find $f \circ g$ and $g \circ f$ and hence find $(f \circ g)(2)$ and $(g \circ f)(-3)$

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Exercise 2 C

1. Prove that the function $f: \mathbb{R} \rightarrow \mathbb{R}$, given by $f(x) = 2x$, is one-one and onto.

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2. Prove that the function $f: \mathbb{N} \rightarrow \mathbb{N}: f(x) = 3x$ is one-one and into.

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3. Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$, defined as $f(x) = x^2$, is neither one-one nor onto.

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4. show that the function

(i) $f: N \rightarrow N: f(x) = x^2$ is one-one into

(ii) $f: Z \rightarrow Z: f(x) = x^2$ is many-one into .

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5. Show that the function $f: R \rightarrow R: f(x) = x^4$ is neither one-one nor onto.

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6. Show that the function

(i) $f: N \rightarrow N: f(x) = x^3$ is one-one into

(ii) $f: Z \rightarrow Z: f(x) = x^3$ is one-one into

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7. Show that the function $f: R_0 \rightarrow R_0$, defined as $f(x) = \frac{1}{x}$, is one-one onto, where R_0 is the set of all non-zero real numbers. Is the result true, if the domain R_0 is replaced by N with co-domain being same as R_0 ?

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8. show that the function $f: R \rightarrow R: f(x) = 1 + x^2$ is many -one into .

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9. Let $f: R \rightarrow R: f(x) = \frac{2x - 7}{4}$ be an invertible function . Find f^{-1}

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10. Let $f: R \rightarrow R: f(x) = 10x + 3$. Find f^{-1}



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11. $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1, & \text{if } x \text{ is irrational} \end{cases}$

show that f is many-one and into.

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12. If $f(x) = x + 7$ and $g(x) = x - 7, x \in \mathbb{R}$ find $(f \circ g)(7)$.

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13. Let $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = x^2$ and $g: \mathbb{R} \rightarrow \mathbb{R}: g(x) = (x + 1)$

Show that $(g \circ f) \neq (f \circ g)$

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14. Let $f: R \rightarrow R: f(x) = (3 - x^3)^{1/3}$. Find $f \circ f$

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15. If $f: R \rightarrow R$ is defined by $f(x) = 3x + 2$, find $f(f(x))$.

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16. Let $f: \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g: \{1, 2, 5\} \rightarrow \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$.

Write down $g \circ f$.

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17. Let $A = \{1, 2, 3, 4\}$ and $f = \{(1, 4), (2, 1), (3, 3), (4, 2)\}$.

Write down $(f \circ f)$

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18. Find $g \circ f$ and $f \circ g$ when $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 8x^3$ and $g(x) = x^{1/3}$

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19. Let $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = 10x + 7$. Find the function $g: \mathbb{R} \rightarrow \mathbb{R}: g \circ f = f \circ g = I_g$

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20. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B . State whether f is one-one or not.

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Exercise 2 D

1. Let $A = \{2, 3, 4, 5\}$ and $B = \{7, 9, 11, 13\}$ and

let $f = \{(2,7), (3,9), (4,11), (5,13)\}$

Show that f is invertible and find f^{-1}



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2. show that the function $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = 2x + 3$ is invertible and

find f^{-1}



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3. Let $f: \mathbb{Q} \rightarrow \mathbb{Q}: f(x) = 3x - 4$ show that f is invertible and find

f^{-1}



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4. Let $f: R \rightarrow R: f(x) = \frac{1}{2}(3x + 1)$. Show that f is invertible and find f^{-1}

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5. If $f(x) = \frac{4x + 3}{6x - 4}$, $x \neq \frac{2}{3}$, show that $f \circ f(x) = x$ for all $x \neq \frac{2}{3}$.

What is the inverse of f ?

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6. show that the function f in $A = R - \{\frac{2}{3}\}$ defined as $f(x) = \frac{4x + 3}{6x - 4}$ is

one one and onto. Hence find f^{-1}

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7. show that the function f on $A = \mathbb{R} - \left\{ \frac{-4}{3} \right\}$ onto itself defined

by

$$f(x) = \frac{4x}{(3x + 4)}$$
 is one-one and onto. Hence find f^{-1}

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8. Consider $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 9x^2 + 6x - 5$. Show that f

is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6} - 1}{3} \right)$.

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9. Let $f: \mathbb{N} \rightarrow \mathbb{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$.

Show that $f: \mathbb{N} \rightarrow S$, where, S is the range of f , is invertible. Find the inverse of f .

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10. Let $A = \mathbb{R} - \{2\}$ and $B = \mathbb{R} - \{1\}$. If $f: A \rightarrow B$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$, show that f is bijective.

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11. Let f and g be two functions from \mathbb{R} into \mathbb{R} defined by $f(x) = |x| + x$ and $g(x) = x$ for all $x \in \mathbb{R}$. Find $f \circ g$ and $g \circ f$.

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