MATHS

BOOKS - RS AGGARWAL MATHS (HINGLISH)

FUNDAMENTAL CONCEPTS OF 3-DIMENSIONAL GEOMETRY

Solved Examples

1. Find the direction of a line whose direction ratio are $2, \ -6, 3$

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2. Find the direction cosines of each of the following vectors:

(i) $2\hat{i}+\hat{j}-2\hat{k}$ (ii) $-\hat{i}-\hat{k}$ (iii) $-\hat{j}$

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3. If a line makes angles 90^o, 135^o, 45^o with the x, y and z-axes respectively, find its direction cosines.

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4. If a line makes anles α, β, γ with the coordinate axes, porve that $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

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5. If a line makes angles α, β and γ with the coordinate axes, then prove that $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

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6. Find the direction cosines of a line which makes equal angles with the coordinate axes.

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7. A line make angle 60° and 45° with the positive direction of x-axis and y-axis repectively. What acute angle does it make with the z-axis?

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8. Find the direction cosines of the vector $\vec{r} = \left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$.

A. $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$

B. $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}$

C. $\frac{6}{7}, \frac{-2}{7}, \frac{-3}{7}$

D. $\frac{-6}{7}, \frac{-2}{7}, \frac{-3}{7}$

Answer: A

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9. Find the direction cosines of the line segment joining the points $A(7, -5, 9)$ and $B(5, -3, 8)$.

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10. Find the angles made by the vector $\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right)$ with the coordinate axes.

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11. Find the angles between the lines whose direction ratios are 3,2,-6 and 1,2,2.

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12. Find the angle between the vector $\vec{r}_1 = \left(4\hat{i} - 3\hat{j} + 5\hat{k}\right)$ and $\vec{r}_2 = \left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$.

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13. The direction cosines of the line which is perpendicular to the lines with direction cosines proportional to $(1, -2, -2), (0, 2, 1)$

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14. If $A(8, 2, 0), B(4, 6, -7), C(-3, 1, 2)$ and $D(-9, -2, 4)$ are four given point then find the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

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15. Find the angles of $\triangle ABC$ whose vertices are $A((-1, 3, 2), B(2, 3, 5)$ and $C(3, 5, -2)$.

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16. Show that the points $A(2, 3, 4)$, $B(1, 2, 3)$and $C(3, 8, 11)$ are collinear.

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17. Find the coordinates of the foot of the perpendicular drawn from the point $A(1, 2, 1)$ to the line joining $B(1, 4, 6)andC(5, 4, 4)$.

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18. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1, l_1m_2 - l_2m_1$.

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19. Find the acute angle between the two straight lines whose direction cosines are given by $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$

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20. Find the direction cosines of the two lines which are connected by th relations.

$l - 5m + 3n = 0 and 7l^2 + 5m^2 - 3n^2 = 0$

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21. If the direction cosines of a variable line in two adjacent points be l, M, n and $l+\delta l, m+\delta m+n+\delta n$ the small angle $\delta\theta$as between the two positions is given by

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22. Prove that the straight lines whose direction cosines are given by the relations $al+bm+cn=0$ and $fmn+gnl+hlm=0$ are Perpendicular to each other if $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$, and parallel if $a^2f^2+b^2g^2+c^2h^2-2bcgh-2cahf-2abfg=0.$

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23. Show that the straight lines whose direction cosines are given by the equations $al + bm + cn = 0 and \widehat{\quad} 2 + zm^2 = vn^2 + wn^2 = 0$ are parallel or perpendicular as $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$ or $a^2(v+w) + b^2(w+u) + c^2(u+v) = 0.$

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24. If the edges of a rectangular parallelepiped are a,b, c, prove that the angles between the four diagonals are given by $\cos^{-1}\left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2}\right).$

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25. Show that the angle between two diagonals of a cube is $\cos^{-1}\sqrt{\frac{1}{3}}$.

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26. A line makes angles $\alpha, \beta, \gamma$and $\delta$with the diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$

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Exercise

1. Find the direction of a line segment whose direction ratios are:

2,-6, 3

2, -1, -2

$-9, 6, -2$

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2. Find the direction ratios and the direction cosines of the line segment joining the points:

A(1,0,0) and B(0,1,1)

A(5,6,-3) and B(1,-6,3)

A(-5,7,-9) and B(-3,4,-6)

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3. Show that the line joining the point A(1,-1,2) and B(3, 4, -2) is perpendicular to the line joining the points C(0,3,2) and D(3,5,6).

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4. Show that the line joining the origin to the point $(2, 1, 1)$ is perpendicular to the line determined by the points $(3, 5, -1)$ and $(4, 3, -1)$.

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5. Find the value of $p$for which the line through the points $A(4, 1, 2)$ and $B(5, p, 0)$ is perpendicular to the line through the points $C(2, 1, 1)$and $D(3, 3, -1)$.

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6. If Obe the origin and $P(2, 3, 4)$ and $Q(1, -2, 1)$be any two points, show that $OPOQ$.

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7. Show that the line segment joining the points A(1, 2, 3) and B(4, 5, 7) is parallel to the line segment joining the points C(-4 ,3, -6) and D(2, 9, 2).

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8. If the line segment joining the points A(7, p, 2) and B(q, -2, 5) be parallel to the line segment joining the points C(2, -3, 5) and D(-6, -15, `11), find the value of p and q.

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9. Show that the points $(2, 3, 4), (-1, -2, 1), (5, 8, 7)$ are collinear.

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10. Show that the point A(-2, 4, 7), B(3, -6, -8) and C(1, -2, -2) are collinear.

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11. Find the value of p for which the points A(-1, 3, 2), B(-4, 2, -2), and C(5, 5, p) are collinear.

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12. Find the angle between the two lines whose direction cosines are:

$\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$ and $\frac{3}{7}, \frac{2}{7}, \frac{6}{7}$.

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13. Find the angle between the lines whose direction ratios are a, b, c and bc, ca, ab.

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14. Find the angle between the lines whose direction ratios are:

$2, -3, 4$ and $1, 2, 1$.

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15. Find the angle between two lines whose direction ratios are proportional to $1, 1, 2 and (\sqrt{3}-1), (-\sqrt{3}-1), 4$.

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16. Find the angle between the vectors $\vec{r}_1 = \left(3\hat{i} - 2\hat{j} + \hat{k}\right)$ and $\hat{r}_2 = \left(4\hat{i} + 5\hat{j} + 7\hat{k}\right)$.

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17. Find the angle made by the following vector with the coordinates axes:

$\left(\hat{i} + \hat{j} + \hat{k}\right)$

$\left(\hat{j} - \hat{k}\right)$

$\left(\hat{i} - 4\hat{j} + 8\hat{k}\right)$

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18. Find the coordinates of the foot of perpendicular drawn from th point $A(1, 8, 4)$ to the line joining the points

$B(0,\ -1,3)andC(2-3,\ -1)$.

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