MATHS

BOOKS - RS AGGARWAL MATHS (HINGLISH)

SCALAR, OR DOT, PRODUCT OF VECTORS

Solved Examples

1. Let $\vec{a}$ and $\vec{b}$ be two given vectors such that $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4$ and the angle between them is 60°. Find $\vec{a} . \vec{b}$.

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2. Find the angle between two vectors $\vec{a} and \vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively and such that $\vec{a} \dot{\vec{b}} = \sqrt{6.}$

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3. If $\vec{a}$ and $\vec{B}$ are two vectors such that $\left|\vec{a}\right| = \left|\vec{b}\right| = \sqrt{2}$ and $\vec{a} \cdot \vec{b} = -1$, find the angle between $\vec{a}$ and $\vec{b}$.

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4. Write the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$.

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5. Write the projection of $\vec{b} + \vec{c}$ on $\vec{a}$, when $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

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6. Find λ, when the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.

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7. For what value λ are the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ perpendicular to each other?

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8. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

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9. Dot products of a vector with vectors $\hat{i}-\hat{j}+\hat{k},\ 2\hat{i}+\hat{j}-3\hat{k}\ and\ \hat{i}+\hat{j}+\hat{k}$ are respectively $4, 0$ and 2. Find the vector.

A. $\left(2\hat{i}-\hat{j}+\hat{k}\right)$

B. $\left(3\hat{i}-2\hat{j}+4\hat{k}\right)$

C. $\left(3\hat{i}-5\hat{j}+\hat{k}\right)$

D. $\left(5\hat{i}-3\hat{j}+\hat{k}\right)$

Answer: A

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10. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k},\ \vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}.\vec{c}=18$.

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11. Find a vector whose magnitude is 3 units and which is perpendicular to each of the vectors $\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$.

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12. Find $\left|\vec{a} - \vec{b}\right|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $\left|\vec{a}\right| = 2$, and $\left|\vec{b}\right| = 3$ and $\left|\vec{a}.\ \vec{b}\right| = 4$

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13. If $\vec{a}$ makes equal angles with the coordinate axes and has magnitude 3,find the angle between $\vec{a}$ and each of the three coordinate axes.

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14. If a unit vector $\vec{a}$ makes angles $.^{\pi}/_{4}$ with $\hat{i}, .^{\pi}/_{3}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$ then find the value of θ. Also, find the scalar and

vector components of a along the axes.

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15. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + \vec{b}$ is also a unit vector then find the angle between $\vec{a}$ and $\vec{b}$.

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16. If sum of two unit vectors is a unit vector; prove that the magnitude of their difference is $\sqrt{3}$

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17. If ā,b,c are three vectors such that $\left|\vec{a}\right| = 5, \left|\vec{b}\right| = 12$ and $\left|\vec{c}\right| = 13$ and $\vec{a} + \vec{b} + \vec{c} = 0$ then $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

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18. Let $\vec{a}, \vec{b}, and \vec{c}$ are vectors such that $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4 and \left|\vec{c}\right| = 5, and \left(\vec{a} + \vec{b}\right)$ is perpendicular to $\vec{c}, \left(\vec{b} + \vec{c}\right)$ is perpendicular to $\vec{a} and \left(\vec{c} + \vec{a}\right)$ is perpendicular to $\vec{b}$. Then find the value of $\left|\vec{a} + \vec{b} + \vec{c}\right|$.

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19. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors of equal magniltgude, prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with vectors $\vec{a}, \vec{b}, and \vec{\cdot}$ also find the angle.

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20. If $\vec{a}, \vec{b}$, and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0,$ then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

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21. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ then show that $\vec{a} = 0$ or , $\vec{b} = c$ or $\vec{a} \perp \left(\vec{b} - \vec{c}\right)$.

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22. Let $\vec{a}$ and $\vec{b}$ be two nonzero vector. Prove that $\vec{a} \perp \vec{b} \Leftrightarrow \left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b}\right|$.

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23. express the vector $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as sum of the two vectors such that one is parallel to the vector $\vec{b} = 3\hat{i} + \hat{k}$ and other is perpendicular to $\vec{b}$.

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24. Find the values of λ for which the angle between the vectors $\vec{a} = 2\lambda^2\hat{i} + 4\lambda\hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + \lambda\hat{k}$ is obtuse.

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25. Let $A(0, 1, 1)$, $B(3, 1, 5)$ and $C(0, 3, 3)$ be the vertices of a $\triangle ABC$. Using vectors, show that $\triangle ABC$ is right angled at C.

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26. Show that the points, A, B and C having position vectors $\left(2\hat{i} - \hat{j} + \hat{k}\right)$, $\left(\hat{i} - 3\hat{j} - 5\hat{k}\right)$ and $\left(3\hat{i} - 4\hat{j} - 4\hat{k}\right)$ respectively are the vertices of a rightangled triangle. Also, find the remaining angles of the triangle.

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27. Let $\left(\hat{i}+\hat{j}+\hat{k}\right), \left(2\hat{i}+5\hat{j}\right), \left(3\hat{i}+2\hat{j}-3\hat{k}\right)$ and $\left(\hat{i}-6\hat{j}-\hat{k}\right)$ be the position vectors of points A, B, C, D respectively. Find the angle between AB and CD. Hence, show that $AB \mid\mid CD$.

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Exercise 23

1. Find $\vec{a}.\vec{b}$ when

(i) $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=3\hat{i}-4\hat{j}-2\hat{k}$

(ii) $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=-2\hat{j}+4\hat{k}$

(iii) $\vec{a}=\hat{i}-\hat{j}+5\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{k}$

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2. Find the value of λ for which $\vec{a}$ and $\vec{b}$ are perpendicular, where

(i) $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\left(\hat{i}-2\hat{j}+3\hat{k}\right)$

(ii) $\vec{a} = 3\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = -lamnda\hat{i} + 3\hat{j} + 3\hat{k}$

(iii) $\vec{A} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$

(iv) $\vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = -5\hat{j} + \lambda\hat{k}$

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3. (i) If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$, show that $\left(\vec{a} + \vec{b}\right)$ is perpendicular to $\left(\vec{a} - \vec{b}\right)$.

(ii) If $\vec{a} = \left(5\hat{i} - \hat{j} - 3\hat{k}\right)$ and $\vec{b} = \left(\hat{i} + 3\hat{j} - 5\hat{k}\right)$ then show that $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right)$ are orthogonal.

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4. If $\vec{a} = \hat{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{j} - \hat{j} + \lambda\hat{k}$, then find the value of λ, so that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors.

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5. Show that the vectors $\vec{a} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right), \vec{b} = \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right), \vec{c} = \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$ are mutually perpendicular unit vectors.

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6. Let $\vec{A} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and is such that $\vec{d} . Vec(c) = 21$.

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7. Let $\vec{a} = \left(2\hat{i} + 3\hat{j} + 2\hat{k}\right)$ and $\vec{b} = \left(\hat{i} + 2\hat{j} + \hat{k}\right)$.

Find the projection of (i) $\vec{a}$ on $\vec{b}$ and (ii) $\vec{b}$ on $\vec{a}$.

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8. Find the projection of $\left(8\hat{i}+\hat{j}\right)$ in the direction of $\left(\hat{i}+2\hat{j}-2\hat{k}\right)$

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9. Write the projection of vector $\hat{i}+\hat{j}+\hat{k}$ along the vector $\hat{j}$.

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10. (i) Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}.\vec{b}=8$ and $\vec{b}=\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$.

(ii) Write the projection of the vector $\left(\hat{i}+at(j)\right)$ on the vector $\left(\hat{i}-\hat{j}\right)$

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11. Find the angle between the vectors $\vec{a}$ and $\vec{b}$, when

(i) $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+\hat{k}$

(ii) $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

(iii) $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$.

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12. If $\vec{a} = \left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$ and $\vec{b} = \left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ then calculate the angle between $\left(2\vec{a} + \vec{b}\right)$ and $\left(\vec{a} + 2\vec{b}\right)$.

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13. if $\vec{a}$ is a unit vector and $\left(\vec{x} - \vec{a}\right).\left(\vec{x} + \vec{a}\right) = 8$ then $\left|\vec{x}\right|$

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14. Find the angles which the vector $\vec{a} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ makes with the coordinate axes.

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15. Show that the vector $\hat{i}+\hat{j}+\hat{k}$ is equally inclined with the coordinate axes.

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16. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$ making an angle $\frac{\pi}{4}$ with x-axis , $\frac{\pi}{2}$ with y-axis and an acute angle θ with z-axis

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17. Find the angle between $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$, if $\vec{a}=\left(2\hat{i}-\hat{j}+3\hat{k}\right)$ and $\vec{b}=\left(3\hat{i}+\hat{j}+2\hat{k}\right)$.

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18. Express the vector $\vec{a}=\left(6\hat{i}-3\hat{j}-6\hat{k}\right)$ as sum of two vectors such that one is parallel to the vector $\vec{B}=\left(\hat{i}+\hat{j}+\hat{k}\right)$ and the other is

perpendicular to $\vec{b}$.

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19. Prove that $(\rightarrow a + \rightarrow b) \rightarrow a \dot{+} \rightarrow c| \rightarrow a|^2 + | \rightarrow b|^2$, if and only if $\rightarrow a, \rightarrow b$ are perpendicular, given $\rightarrow a \neq \rightarrow 0, \rightarrow b \neq \rightarrow 0$

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20. If $\vec{a} + \vec{b} + \vec{c} = 0, \left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5, \left|\vec{c}\right| = 7,$ then find the angle between $\vec{a}$ and $\vec{b}$.

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21. Find the angle between $\vec{a}$ and $\vec{b}$, when

(i) $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 1$ and $\vec{A} . \vec{B} = \sqrt{3}$ (ii) $\left|\vec{a}\right| = \left|\vec{b}\right| = \sqrt{2}$ and $\vec{a} . \vec{b} = -1.$

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22. Find $|\rightarrow a - \rightarrow b|$, if two vector $\rightarrow a$ and $\rightarrow b$ are such that $|\rightarrow a| = 2, |\rightarrow b| = 3$ and $\rightarrow a \cdot \rightarrow b = 4$.

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23. Find $\left|\vec{a}\right| and \left|\vec{b}\right|$, if : $\left(\vec{a} + \vec{b}\right) \cdot \vec{a} - \vec{b} = 8 \, and \left|\vec{a}\right| = 8\left|\vec{b}\right|$

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24. If $\hat{a} \, and \, \hat{b}$ are unit vectors inclined at an angle θ then prove that $\frac{\cos\theta}{2} = \frac{1}{2}\left|\hat{a} + \hat{b}\right| \; \frac{\tan\theta}{2} = \frac{\left|\hat{a} - \hat{b}\right|}{\left|\hat{a} + \hat{b}\right|}$

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25. Dot product of a vector with $\hat{i}+\hat{j}-3\hat{k}, \hat{i}+3\hat{j}-2\hat{k}$ and $2\hat{i}+\hat{j}+4\hat{k}$ are $0, 5$ and 8 respectively. Find the vector. Dot products of a vector with vectors $\hat{i}-\hat{j}+\hat{k}, 2\hat{i}+\hat{j}-3\hat{k}$ and $\hat{i}+\hat{j}+\hat{k}$ are respectively $4, 0$ and 2. Find the vector.

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26. If $\overrightarrow{AB}=\left(3\hat{i}-\hat{j}+2\hat{k}\right)$ and the coordinates of A are $(0, -2, -1)$, find the coordinates of B.

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27. If $A(2, 3, 4), B(5, 4, -1), C(3, 6, 2)$ and $D(1, 2, 0)$ be four points, show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CD}$.

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28. Find the value of λ. If the vectors $2\hat{i}+\lambda\hat{j}+3\hat{k}$ and $3\hat{i}+2\hat{j}-4\hat{k}$ are perpendicular to each other.

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29. Show that the vectors $\vec{a}=3\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-3\hat{j}+5\hat{k}$, $\vec{c}=2\hat{i}+\hat{j}-4\hat{k}$ form a right angled triangle.

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30. Three vertices of a triangle are $A(0,-1,-2)$, $B(3,1,4)$ and $C(5,7,1)$. Show that it is a right-angled triangle. Also find its other two angles.

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31. If the position vectors of the vertices a, B and C of a $Tri\angle ABC$ be $(1, 2, 3), (-1, 0, 0)$ and $(0, 1, 2)$ respectively then find $\angle ABC$.

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32. If $\vec{a}$ and $\vec{b}$ are two non-collinear unit vectors such that $\left|\vec{a}+\vec{b}\right|=\sqrt{3}$, find $\left(2\vec{a}-5\vec{b}\right)3\vec{a}+\vec{b}$.

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33. If $\vec{a}$, $\vec{b}$, are two vectors such that $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}\right|$, then prove that $2\vec{a}+\vec{b}$ is perpendicular to $\vec{b}$.

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34. If $\vec{a}=3\hat{i}-\hat{j}$ and $\vec{b}=2\hat{i}+\hat{j}-3\hat{k}$, then express $\vec{b}$ in the from $\vec{b}=\vec{b}_1+\vec{b}_2$, where $\vec{b}_1 \| \vec{a}$ and $\vec{b}_2 \perp \vec{a}$.

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