MATHS

BOOKS - KC SINHA MATHS (HINGLISH)

ALGEBRA - PREVIOUS YEAR QUESTIONS - FOR COMPETITION

Exercise

1. Let $p and q$ be real numbers such that $p \neq 0, p^3 \neq q, and p^3 \neq -q$. If $\alpha and \beta$ are nonzero complex numbers satisfying $\alpha + \beta = -p and \alpha^2 + \beta^2 = q$, then a quadratic equation having $\alpha / \beta and \beta / \alpha$ as its roots is $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$

$(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

$(p^3 + q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$

$(p^3 + q)x^2 - (5p^3 + 2q)x + (p^3 + q) = 0$

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2. The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A[xyz] = [100]$ has exactly two distinct solution is

a. 0 b. $2^9 - 1$ c. 168 d. 2

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3. Let w be the complex number $\frac{\cos(2\pi)}{3} + i\frac{\sin(2\pi)}{3}$. Then the number of distinct complex numbers z satisfying $\begin{vmatrix} z+1 & w & w^2 \\ 2 & z+w^2 & 1 \\ w^2 & 1 & z+w \end{vmatrix} = 0$ is equal

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4. Let $S_k, k = 1, 2, , 100,$ denotes thesum of the infinite geometric series whose first term s $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$, then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} (k^2 - 3k + 1) S_k$ is ______.

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5. Let $S = \{1, , 2, 34\}$. The total number of unordered pairs of disjoint subsets of S is equal a.25 b. 34 c. 42 d. 41

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6. For $r = 0, 1, \ldots . . , 10,$ let $A_r, B_r,$ and $C_r$denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}, (\ + x)^{20}$ and $(1+x)^{30}$ .Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to

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7. Let a1,a2,a3 ...... a11 be real numbers satisfying $a_1 = 15, 27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots .11$ If $\frac{a1^2 + a2^2 \ldots \ldots a11^2}{11} = 90$ then find the value of $\frac{a_1 + a_2 \ldots + a_{11}}{11}$

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8. Let p be an odd prime number and T_p, be the following set of 2×2 matrices $T_p = \left\{A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots\ldots..p-1\}\right\}$ The number of A in T_p, such that A is either symmetric or skew-symmetric or both, and det (A) divisible by p is

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10. Let p be an odd prime number and T_p, be the following set of 2×2 matrices $T_p = \left\{A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots\ldots..p-1\}\right\}$ The number of A in T_p, such that A is either symmetric or skew-symmetric or both, and det (A) divisible by p is

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11. The number of 3 3 non-singular matrices, with four entries as 1 and all other entries as 0, is (1) 5 (2) 6 (3) at least 7 (4) less than 4

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12. Let A be a 2×2 matrix with non-zero entries and let A^2=I, where i is a 2×2 identity matrix, Tr(A) i= sum of diagonal elements of A and $|A|$ = determinant of matrix A. Statement 1:Tr(A)=0 Statement 2:$|A|$=1

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13. A person is to count 4500 currency notes. Let a_n, denote the number of notes he counts in the nth minute if

$a_1 = a_2 = a_3 = \ldots\ldots\ldots = a_{10} = 150$ and $a_{10}, a_{11}, \ldots\ldots\ldots$are in an AP with common difference -2, then the time taken by him to count all notes is :- (1) 24 minutes 10 11 (2) 34 minutes (3) 125 minutes (4) 135 minutes

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14. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is (1) 36 (2) 66 (3) 108 (4) 3

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15. Consider the system of linear equations: $x_1 + 2x_2 + x_3 = 3$ $2x_1 + 3x_2 + x_3 = 3$ $3x_1 + 5x_2 + 2x_3 = 1$ The system has (1) exactly 3 solutions (2) a unique solution (3) no solution (4) infinite number of solutions

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16. The number of complex numbers z such that $|z-1|=|z+1|=|z-i|$ is

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17. A polynomial of degree 2 which takes values y_0, y_1, y_2 at points x_0, x_1, x_2 respectively, is given by $p(x)=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1+\frac{(x-x_0)(x-x}{(x_2-x_0)(x_2-}$
A polynomial of degree 2 which takes values y_0, y_0, y_1 at points x_0, x_{0+t}, x_1 $t\neq 0$ is given by

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18. Let α and β be the roots of equation $x^2-6x-2=0$. If $a_n=\alpha^n-\beta^n, f$ or $n\geq 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is equal to: (1) 6 (2)-6 (3) 3 (4) -3

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19. Let $M and N$ be two 3×3 non singular skew-symmetric matrices such that $MN = NM\cdot$ If P^T denote the transpose of P, then $M^2 N^2 (M^T N^{-1})^T$ is equal to M^2 b. $-N^2$ c. $-M^2$ d. MN

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20. Let $\omega \neq 1$ be cube root of unity and S be the set of all non-singular matrices of the form $[1ab\omega 1c\omega^2\theta 1]$, $where$ each of $a, b, and c$ is either ω or $\omega^2\cdot$ Then the number of distinct matrices in the set S is a. 2 b. 6 c. 4 d. 8

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21. Let a,b, and c be three real numbers satistying $[a, b, c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0, 0, 0]$ If the point $P(a, b, c)$ with reference to (E), lies on the plane $2x+y+z=1$, the the value of $7a+b+c$ is (A) 0 (B) 12 (C) 7 (D) 6

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22. Let a,b, and c be three real numbers satistying $[a,b,c]\begin{bmatrix}1&9&7\\8&2&7\\7&3&7\end{bmatrix}=[0,0,0]$ Let ω be a solution of $x^3-1=0$ with $Im(\omega)>0. Ifa=2$ with b nd c satisfying (E) then the vlaue of $\frac{3}{\omega^a}+\frac{1}{\omega^b}+\frac{3}{\omega^c}$ is equa to (A) -2 (B) 2 (C) 3 (D) -3

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23. Let a,b, and c be three real numbers satistying $[a,b,c]\begin{bmatrix}1&9&7\\8&2&7\\7&3&7\end{bmatrix}=[0,0,0]$Let b=6, with a and c satisfying (E). If alpha and beta are the roots of the quadratic equation $ax^2+bx+c=0then\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n$ is (A) 6 (B) 7 (C) $\frac{6}{7}$ (D) oo

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24. Let $a_1, a_2, a_3, , a_{100}$ be an arithmetic progression with $a_1 = 3 and s_p = \sum_{i=1}^{p} a_i, 1 \leq p \leq 100.$ For any integer n with $1 \leq n \leq 20,$ let$m = 5n\cdot$ If $\frac{S_m}{S_n}$ does not depend on $n,$ then a_2 is________.

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25. If z is any complex number satisfying $|z - 3 - 2i| \leq 2$ then the maximum value of $|2z - 6 + 5i|$ is

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26. The minimum value of the sum of real number $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8, and a^{10} with a > 0$ is

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27. Let $\omega = e^{i\frac{\pi}{3}}$ and a, b, c, x, y, z be nonzero complex number such that $a + b + c = x, a + b\omega + c\omega^2 = y, a + b\omega^2 + c\omega = z$. Then the value of $\dfrac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

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28. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is :

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29. Let M be a 3×3 matrix satisfying $M[010] = M[1-10] = [11-1], andM[111] = [0012]$ Then the sum of the diagonal entries of M is ________.

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30. The coefficient of x7 in the expansion of $\left(1-x-x^2+x^3\right)^6$ is : (1) 144 (2) -132 (3) -144 (4) 132

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31. Let α, β be real and z be a complex number. If $z^2+\alpha z+\beta=0$ has two distinct roots on the line Re $z=1$, then it is necessary that : (1) $b \in (0, 1)$ (2) $b \in (-1, 0)$ (3) $|b|=1$ (4) $b \in (1, \infty)$

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32. 38. Assertion (A): The area of a rectangle is 630 sqem and its breadth is 15em then its lengthis 55cmReason (R): The area of a rectangle is given by A = length x breadtha) Both A and R are true and R is correct explanation of Ab) Both A and R are true and R is not correct explanation of ASed) A is false and R is true.

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33. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after : (1) 18months (2) 19months (3) 20months (4) 21months

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34. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A = B\omega$. Then (A, B) equals (a) $(0, 1)$ (b) $(1, 1)$ (c) $(2, 0)$ (d) $(-1, 1)$

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35. The number of values of k for which the linear equations $4x + ky + 2z = 0$ $kx + 4y + z = 0$ $2x + 2y + z = 0$ posses a non-zero solution is : (1) 3 (2) 2 (3) 1 (4) zero

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36. Statement-1 : The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is $\hat{} \ 9C_3$. Statement-2 : The number of ways of choosing any 3 places from 9 different places is $\hat{} \ 9C_3$. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1. Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. Statement-1 is true, Statement-2 is false. Statement-1 is false, Statement-2 is true.

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37. If P is a 3×3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3×3 identity matrix, then there exists a column matrix, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

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38. Let $a_1, a_2, a_3, \ldots$ be in harmonic progression with $a_1 = 5 and a_{20} = 25$. The least positive integer n for which $a_n < 0$ 22 b. 23 c. 24 d. 25

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39. If the adjoint of a 3 3 matrix P is 1 4 4 2 1 7 1 1 3 , then the possible value(s) of the determinant of P is (are) (A) 2 (B) 1 (C) 1 (D) 2

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40. Let z be a complex number such that the imaginary part of z is nonzero and a = z2 + z + 1 is real. Then a cannot take the value (A) –1 (B) 1 3 (C) 1 2 (D) 3 4

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41. The total number of ways in which 5 balls of differert colours can be distributed among 3 persons so thai each person gets at least one ball is

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42. The value of $6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots.}}}\right)$ is

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43. Let n denote the number of all n-digit positive integers formed by the digits $0, 1$ or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n= the number of such n-digit integers ending with digit 0. The value of b_6, is

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44. Let n denote the number of all n-digit positive integers formed by the digits $0, 1$ or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n= the number of such n-digit integers ending with digit 0. The value of b_6, is

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45. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has (A) infinite number of real roots (B) no real roots (C) exactly one real root (D) exactly four real roots

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46. Statement-1 The sum of the series 1 +(1+ 2+4)+ (4+ 6+ 9)+(9+12+16) ... (361 +380 +400) is 8000. Statement-2 $\sum_{k=1}^{n} \left(k^3 - (k-1)^3\right) = n^3$for any natural number n. (1) Statement-1 is true, Statement-2 is false. (2) Statement-1 is false, Statement-2 is true. (3) Statement-1 is true, Statement-2 is true Statement-2 is a correct explanation for Statement-l

(4) Statement-1 is true, Statement-2 is true Statement-2 is not a correct explanation for Statement-I.

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47. Let $A = (100210321)$ If u_1 and u_2 are column matrices such that $Au_1 = (100)$ and $Au_2 = (010)$, then $u_1 + u_2$ is equal to (1) (-110) (2) $(-11-1)$ (3) $(-1-10)$ (4) $(1-1-1)$

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48. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is (1) an irrational number (2) an odd positive integer (3) an even positive integer (4) a rational number other than positive integers

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49. If 100 times the 100^{th} term of an AP with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is (1) 150 (2) 150 times its 50^{th} term (3) 150 (4) zero

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50. Out of 10 white, 9 black, and 7 red balls, find the number of ways in which selection of one or more balls can be made (balls of the same color are identical).

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51. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies (1) either on the real axis or on a circle passing through the origin (2) on a circle with centre at the origin (3) either on the real axis or on a circle not passing through the origin (4) on the imaginary axis

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52. Let P and Q be 3×3 matrices with $P \neq Q$. If $P^3 = Q^3 and P^2 Q = Q^2 P$, then determinant of $(P^2 + Q^2)$ is equal to

(1) 2 (2) 1 (3) 0 (4) 1

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