

MATHS

BOOKS - KC SINHA MATHS (HINGLISH)

BINOMIAL THEOREM - FOR BOARDS

Solved Examples

1. Expand of the expression
$$:\left(x+rac{1}{x}
ight)^6$$

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2. Expand:
$$\left(x-rac{1}{y}
ight)^{11}, y
eq 0$$

3. Expand $(2x-3y)^4$ by binomial theorem.



6. Expand:
$$\left(1-x+x^2
ight)^4$$

7. Expand $(a+b)^6-(a-b)^6$. Hence find the value of $\left(\sqrt{2}+1
ight)^6-\left(\sqrt{2}-1
ight)^6.$

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8. Simplify:
$$\left(x+\sqrt{x-1}
ight)^6+\left(x-\sqrt{x-1}
ight)^6$$

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9. Find the expansion of $\left(3x^2-2ax+3a^2
ight)^3$ using binomial

theorem.

10. If n be a positive integer and the sums of the odd terms and even terms in the expansion of $(a + x)^n$ be A and B respectively, prove that, $A^2-B^2=\left(a^2-x^2
ight)^n$

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11. If *o* be the sum of odd terms and *E* that of even terms in the expansion of $(x + a)^n$ prove that: $O^2 - E^2 = (x^2 - a^2)^n$ (ii) $4OE = (x + a)^{2n} - (x - a)^{2n}$ (iii) $2(O^2 + E^2) = (x + a)^{2n} + (x - a)^{2n}$

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12. The first three terms in the expansion of a binomial are 1, 10 and 40. Find the expansion.

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13. Using binomial theorem compute the following: $(99)^5$



17. (iv) Using binomial theorem prove that $(101)^{50}(100)^{50+}$



21. Find the 5 th term from the end in the expansion of

$$\left(\frac{x^3}{2}-\frac{2}{x^2}\right)^9$$

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22. Write the general term in the expansion of $\left(x^2-y
ight)^6$ \cdot

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23. If the third term in the expansion of
$$\left(\frac{1}{x} + {}_{\mathrm{x}}(\log)_{10x}\right)^5$$
 is 1000,

then find x_{\cdot}



24. The 21st and 22nd terms in the expansion of $(1 + x)^{44}$ are equal. Then x:





 $(1.3.5...(2n-1)/(n! 2^n))$

32. Show that the greatest coefficient in the expansion of

$$\left(x+rac{1}{x}
ight)^2nisrac{1.3.5\ldots(2n-1).2^n}{n!}$$

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33. Show that the coefficient of the middle term in the expansion of $(1 + x)^{2n}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{2n-1}$.

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34. Find the coefficient of
$$rac{1}{y^2}$$
 in $\left(y+rac{c^3}{y^2}
ight)^{10}$

35. Find the coefficient of x^9 in $(1 + 3x + 3x^2 + x^3)^{15}$.



39. If the fourth term in the expansion of $\left(px+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then

(n, p) =

40. Find the value of a so that the term independent of x in

$$\left(\sqrt{x}+rac{a}{x^2}
ight)^{10}$$
 is 405

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41. If $(1 + ax)^n = 1 + 8x + 24x^2 + \dots$ then the value of a and n

is

42. In the binomial expansion of $(1 + a)^{m+n}$, prove that the coefficient of $a^m and a^n$ are equal.



x and x^2 are 3 and -6 respectively, then m is:

45. If the coefficients of a^{r-1} , a^r and a^{r+1} in the binomial expansion of $(1+a)^n$ are in A.P., prove that $n^2 - on(4r+1) + 4r^2 - 2 = 0.$



46. Find the coefficient of x^5 in the expansion of the product $(1+2x)^6(1-x)^7$.

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47. Find the coefficient of x^{-1} in $(1+3x^2+x^4)\left(1+rac{1}{x}
ight)^8$

48. If in the expansion of $\left(1-x
ight)^{2n-1}a_r$ denotes the coefficient of

 x^r then prove that $a_{r-1}+a_{2n-r}=0$

49. Show that there will be no term containing x^{2r} in the expansion of $(x + x^{-2})^{n-3}$ if n - 2r is a positive integer but not a multiple of 3.

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50. Show that there will be a term independent of x in the expansion of $(x^a + x^{-b})^n$ only if an is a multiple of (a + b).

51. Find n if the coefficients of 4th the 13th terms in the expansion of $(a + b)^n$ are equal.



53. The value of m, for which the coefficients of the (2m+1) th

terms in the expansion of $\left(1+x
ight)^{10}$ are equal is 3 b. 1 c. 5 d. 8

54. Find the greatest term in the expansion of $(7 - 5x)^{11}$ when $x = \frac{2}{3}$.

55. Show that it the greatest term in the expansion of $(1 + x)^{2n}$ has also the greatest coefficient, then x lies between and $\frac{n}{n+1}$ and $\frac{n+1}{n}$.

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56. If n be a positive integer, then prove that $6^{2n} - 35n - 1$ is divisible by 1225.



57. Show that: $2^{4n} - 2^n(7n+1)$ is some multiple of the square of

14, where n is a positive integer.



60. Write down the binomial expansion of $(1+x)^{n+1}$, when x = 8. deduce that $9^{n+1} - 8n - 9$ is divisible

64 where n is a positive integer.



61. If and b are distinct integers, prove that a - b is a factor of

 $a^n - b^n$, whenever n is a positive integer.



62. If three consecutive coefficients in the expansion of $(1 + x)^n$ be

165,330 and 462, find n and the position of the coefficeint.



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65. If the coefficients of three consecutive terms in the expansion of

 $(1+x)^n$ are in the ratio 1:7:42, then find the value of n_{\cdot}

66. If three consecutive coefficients in the expansion of $(1+x)^n$

are in the ratio 6:33:110, find n and r.



67. If a,b,c be the three consecutive coefficients in the expansion of

a power oif (1+x), prove that the index power is $\left(2ac+brac{a+c}{b^2-ac}
ight)$

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68. If in the binomial expansion of $(1+x)^n$, the coefficient of

 $14th,\,15th$ and 16th terms are in $A.\ P.$, then find n.







9. Expand the following by binomial theorem: $\left(1-2x
ight)^5$



13. Express:
$$\left(\left(x+\sqrt{x^2+1}
ight)^6+\left(x-\sqrt{x^2+1}
ight)^6
ight)$$
 as a

polynomial in x

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14. Find the value of
$$\left(a^2+\sqrt{a^2-1}
ight)^4+\left(a^2-\sqrt{a^2-1}
ight)^4.$$

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15. Using binomial theorem, expand $\left\{ (x+y)^5 + (x-y)^5 \right\}$. and hence find the value of $\left\{ \left(\sqrt{2}+1 \right)^5 + \left(\sqrt{2}-1 \right)^5 \right\}$.



20. Evaluate:
$$\left(\sqrt{3}+\sqrt{2}
ight)^6+\left(\sqrt{3}+\sqrt{2}
ight)^6$$

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21. Find the number of terms in the following expansion:

$$\left(1-2x+x^2
ight)^{30}$$

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22. Find the number of terms in the expansions of the following:

$$\left(\sqrt{x}+\sqrt{y}
ight)^{10}+\left(\sqrt{x}-\sqrt{y}
ight)^{10}$$

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23. If A be the sum of odd terms and B the sum of even terms in the expnsion of $(x + a)^n$, show that 4AB= $(x + a)^{2n} - (x - a)^{2n}$



27. Using binomial theorem, evaluate : $(102)^5$



32. Using binomial theorem evaluate each of the following: $(96)^3$



35. Find the greater of the two number: $(1.01)^{1000000}$ and 10000



36. Using binomial theorem, indicate which number is larger $(1. 1)^{10000}$ or 1000.

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37. Prove that: . n $\hat{C}_0 + 2.^n C_1 + \ldots 2^n \cdot C_n = 3^n$ for every

natural number n.

Watch Video Solution 38. Prove that: $(2n)C_0 - 3.^{2n}C_1 + 3^2.^{2n}C_2 - \ldots + (-1)^{2n}...3^{2n} (2n)C_{2n} = 4^n$ for all value of N'

39. (iii) Find an approximate value of $\left(0.99
ight)^5$ using the first three terms of its expansion. Watch Video Solution **40.** Find the 7th term in the expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$. Watch Video Solution **41.** Find the 9th term in the expansion of $\left(\frac{x}{a} - \frac{3a}{x^2}\right)^{12}$. Watch Video Solution

42. Find the 5th term of (i)
$$\left(rac{a}{3}-3b
ight)^7$$

43. Find the 5 term of
$$\left(2x^2-rac{1}{3x^3}
ight)^{10}$$

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44. Find a if the 7th and 18th terms of the expansion $\left(2+a
ight)^{50}$ are

equal.

45. Find the r^{th} term from the end in the expansion of $(x + a)^n$.



46. Find the 4th term from the end in
$$\left(rac{x^3}{2}-rac{2}{x^2}
ight)^9$$



50. Write the general term in the following expansion:

$$\left(x^2-rac{1}{x}
ight)^{12}, x
eq 0$$

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51. Find the middle term in
$$\left(\frac{2x}{3} - \frac{3y}{2}\right)^{20}$$

52. Find the middle term in the expansion of
$$\left(\frac{2x}{3} - \frac{3}{2x}\right)^6$$

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53. Find the middle term in the expansion of
$$\left(\frac{x}{y} - \frac{y}{x}\right)^7$$







58. Show that the middle term in the expansion of $(1+x)^{2n}israc{(1.\ 3.\ 5(2n-1))}{n!}2^nx^n,$ where n is a positive integer.

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59. Show that the middle term in the expansion $\left(x-rac{1}{x}
ight)^{2n}$ is

$$rac{1.\ 3.\ 5(2n-1)}{n}(\,-2)^n\,.$$

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60.) Find the coefficient of x in the expansion of $\left(2x-rac{3}{x}
ight)^9$.





69. Find the coefficient of x^6y^3 in $\left(x+y
ight)^9$



71. Prove that the coefficients of x^n in $(1+x)^{2n}$ is twice the coefficient of x^n in $(1+x)^{2n-1}$.

72. Find the coefficients of
$$x^7$$
 in $\left(ax^2+rac{1}{bx}
ight)^{11}andx^{-7}\in \left(arac{x^{-1}}{bx^2}
ight)^{11}$ and find the relation

between *aandb* so that coefficients are equal.



73. if the coefficients of x, x^2 and x^3 in the binomial expansion $(1+x)^{2n}$ are in arithmetic progression then prove that $2n^2 - 9n + 7 = 0$

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74. Find the term independent of x in $\left(x+rac{1}{x}
ight)^{2n}$

75. Find the term independent of x in the expression $\left(x-\frac{1}{x}\right)^{14}$

76. Find the term independent of x in the following binomial

expansions
$$(x
eq 0)$$
 : $\left(2x^2+rac{1}{x}
ight)^{13}$

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77. Find the term independent of x in the following binomial

expansions
$$(x
eq 0)\!:\!\left(x^2+rac{1}{x}
ight)^{12}$$

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78. Find the term independent of x in the following binomial

expansions
$$(x
eq 0)$$
 : $\left(\sqrt{rac{x}{3}}+rac{3}{2x^2}
ight)^{10}$

79. Find the term independent of x in the following binomial

expansions
$$(x
eq 0)$$
 : $\left(2x^2-rac{1}{x}
ight)^{12}$

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80. Find the term independent of x in the following binomial

expansions
$$(x
eq 0)$$
 : $\left(2x^2-rac{3}{x^3}
ight)^{25}$

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81. Find the term independent of x in the expansion off the

following expression:
$$\left(rac{3}{2}x^2-rac{1}{3x}
ight)^6$$

82. Which term in the expansion of $\left(x^3 - \frac{3}{x^2}\right)^{15}$ is independent

of x?

83. Find the term independent of x in the following binomial expansions $(x \neq 0)$: $\left(x^2 - \frac{3}{x^3}\right)^{10}$

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84. Find the term independent o f x in the expansion of the

following

expressions:

$$\left(rac{1}{2}x^{rac{1}{3}}+x^{-rac{1}{5}}
ight)^{8}$$
 and

$$ig(1+x+2x^3ig)igg(rac{3}{2}x^2-rac{1}{3x}igg)^{
m c}$$

85. Find the term independent of x in the expansion of: $\left(x-\frac{1}{x}\right)^{12}$.

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86. Find the term independent of x in the following binomial $(1)^{18}$

expansions
$$(x
eq 0)$$
 : $\left(\sqrt[3]{x}+rac{1}{2.\sqrt[3]{x}}
ight)$

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87. If there is a term independent of x in $\left(x+rac{1}{x^2}
ight)^n$, show that it

is equal to $\frac{n!}{\left(\frac{n}{3}\right)!\left(\frac{2n}{3}\right)!}$

88. If x^p occurs in the expansion of $(x^2 + 1/x)^{2n}$, prove that its coefficient is $\frac{(2n)!}{\left[\frac{1}{3}(4n-p)\right]!\left[\frac{1}{3}(2n+p)\right]!}.$

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89. Find the coefficient of a^4 in the product $(1+2a)^4(2-a)^5$ using

binomial theorem.

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90. If the rth term in the expansion of $(1+x)^{20}$ has its coefficient

equal to that of the (r+ 4)th term, find r

91. If the co-efficient of (2r+4)th term and (r-2)th term in expansion of $(1+x)^{18}$ are equal,find r

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92. If the coefficients of $(r-5)^{th}$ and $(2r-1)^{th}$ terms in the expansion of $(1+x)^{34}$ are equal, find r.

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93. If the coefficient of (2r+5) th term and (r-6) th term in the

expansion $\left(1+x
ight)^{39}$ are equal, find. $^{n}C_{12}$

94. Given positive integers r > 1, n > 2, n being even and the coefficient of (3r)th term and (r + 2)th term in the expansion of $(1 + x)^{2n}$ are equal; find r



95. If the coefficient of (p + 1) th term in the expansion of (1+

x)⁽²ⁿ⁾ be equal to that of the (p + 3) th term, show that p=n-1



96. Find the two consecutive coefficients in the expansion of $(3x-2)^{75}$ whose values are equal

97. how that the coefficient of (r+1) th in the expansion of $(1 + x)^{n+1}$ is equal to the sum of the coefficients of the r th and (r+1) th term in the expansion of $(1 + x)^n$

Watch Video Solution 98. Find the greatest term in the $expansion(2+3x)^{10}$, when $x=rac{3}{5}$

99. Find the greastest term in the expansion of
$$(4-3x)^7$$
, when $x=\frac{2}{3}$

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100. Find the greatest term in the expansion of $(a+x)^{13}, whena = 5, x = 2$

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101. Find the limits between which x must lie in order that the greatest term in the expansion of $(1 + x)^{30}$ may have the greatest coefficient.



102. If n is a positive integer, show that: $4^n - 3n - 1$ is divisible by

9





104. Show that $9^{n+1} - 8n - 9$ is divisible by 64, where n is a

positive integer.

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105. Find remainder if, $2^{5n+6} - 31n - 32$ is divisible by 961 if n> 1

106. If n is a positive integer, show that: $3^{2n} - 1 + 24n - 32n^2$ is

divisible by 512 if n>2

107. If three consecutive coefficients in the expansion of $\left(1+x
ight)^n$

be 56, 70 and 56, find n and the position of the coefficients.

108. If three successive coefficients in the expansion of $\left(1+x ight)^n$ be

220,495 and 972, find n.

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109. If 3rd, 4th, 5th terms in the expansion of $\left(a+x
ight)^n$ be 84, 280

and 560, Find x, a and n.



110. Find a, b and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729, 7290 and 30375, respectively.



112. The coefficient of the (r-1)th, rth and (r+1)th terms in the expansion of $(x + 1)^n$ are in the ratio 1:3:5. Find both n and r

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113. If in any binomial expansion a, b, c and d be the 6th, 7th, 8th and 9th terms respectively, prove that $\frac{b^2 - ac}{c^2 - bd} = \frac{4a}{3c}$

114. If the four consecutive coefficients in any binomial expansion be a, b, c, d, then prove that (i) $\frac{a+b}{a}$, $\frac{b+c}{b}$, $\frac{c+d}{c}$ are in H.P. (ii) $(bc+ad)(b-c) = 2(ac^2 - b^2d)$

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115. If the four consecutive coefficients in any binomial expansion be a, b, c, d, then prove that (i) $\frac{a+b}{a}$, $\frac{b+c}{b}$, $\frac{c+d}{c}$ are in H.P. (ii) $(bc+ad)(b-c) = 2(ac^2 - b^2d)$

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116. If a. b, c and d are the coefficients of 2nd, 3rd, 4th and 5th terms respectively in the binomial expansion of $\left(1+x
ight)^n$, then prove that

$$rac{a}{a+b}+rac{c}{c+d}=2rac{b}{b+c}$$

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117. The coefficient of 5th, 6th and 7th terms in the expansion of

 $\left(1+x
ight)^n$ are in A.P. Find the value of n.



118. 17. If the coefficients of 2nd, 3rd and 4th terms in the expansion of $(1+x)^{2n}$ are in A.P.. Show that $2n^2-9n+7=0$

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119. In the coefficients of rth, (r+1)th, and(r+2)th terms in the binomial expansion of $(1+y)^m$ are in A.P., then prove that $m^2 - m(4r+1) + 4r^2 - 2 = 0.$



120. The coefficients of three consecutive terms in the expansion of

 $(1+x)^n$ are in the ratio 182:84:30. prove that n=18

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121. The sum of the coefficients of the first three terms in the expansion of $\left(x-rac{3}{r^2}
ight)^m, x
eq 0,$ m being a natural number, is 559.

Find the term of the expansion containing x^3 .

