MATHS

BOOKS - KC SINHA MATHS (HINGLISH)

BINOMIAL THEOREM - FOR COMPETITION

Solved Examples

1. Find the coefficient of $x^{-1} \in \left(1+3x^2+x^4\right)\left(1+\frac{1}{x}\right)^8$

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2. If in the expansion of $(1-x)^{2n-1}$ $a_r$denotes the coefficient of x^r then prove that $a_{r-1}+a_{2n-r}=0$

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3. If the greatest term in the expansion of $(1+x)^2 n$ has the greatest coefficient if and only if $x \varepsilon \left(\frac{10}{11}, \frac{11}{10}\right)$ and the fourth term in the expansion of $\left(kx + \frac{1}{x}\right)^m is \frac{n}{4}$ then find the value off mk.

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4. If $p+q=1$ then show that $\sum_{r=0}^{n} r^2 C_r p^r q^{n-r} = npq + n^2 p^2$

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5. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$ show that $C_1 - 2C_2 + 3C_3 - 4C_4 + \ldots + (-1)^{n-1} n.\, C_n = 0 where C_r = {}^n C_r.$

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6. If $C_r s \tan d s f$ or ^10C_r $show t\hat{2}.\, C_0 + \frac{2^2}{2}.\, C_1 + \frac{2^3}{3}.\, C_2 + \ldots + \frac{2^{11}}{11}.\, C_{10} = \frac{3^{11}-1}{11}\Big)$

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7. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$ show that $C_1 - 2C_2 + 3C_3 - 4C_4 + \ldots + (-1)^{n-1}n.\, C_n = 0 where C_r = ^n C_r.$

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8. If in the expansion of $(1-x)^{2n-1}$ $a_r$denotes the coefficient of x^r then prove that $a_{r-1} + a_{2n-r} = 0$

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9. Find the greatest term in the expansion of $(7-5x)^{11}$ when $x = \frac{2}{3}$.

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10. Let $R = \left(5\sqrt{5} + 11\right)^{2n+1} and f = R - [R] where []$ denotes the greatest integer function, prove that $Rf = 4^{2n+1}$

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11. The coefficient of $x^2y^4z^2$ in the expansion of $(2x - 3y + 4z)^9$ is

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12. Show that the roots of the equation $ax^2 + 2bx + c = 0$ are real and unequal whre a,b,c are the three consecutive coefficients in any binomial expansion with positive integral index.

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13. Find the coefficient of x^9 in $\left(1 + 3x + 3x^2 + x^3\right)^{15}$.

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14. The term independent of x in $(1+x)^m\left(1+\frac{1}{x}\right)^n$ is

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15. If the sum of the binomial coefficients in the expansion of $\left(x+\frac{1}{x}\right)^n$ is 64, then the term independent of x is equal to (A) 10 (B) 20 (C) 30 (D) 40

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16. The sum of the coefficients in the expansion of $\left(2+5x^2-7x^3\right)^{2000}=$ (A) 0 (B) 1 (C) 2 (D) none of these

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17. $(115)^{96}-(96)^{115}$ is divisible by (A) 17 (B) 19 (C) 21 (D) 23

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18. If {x} denotes the fractional part of x, then $\left\{\frac{3^{2n}}{8}\right\}, n \in N,$ is

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19. If a is the remainder when 5^{40} is divided by 11 and b is the remainder when 2^{2003} is divided by 17 then the value of b-a is (A) 1 (B) 8 (C) 7 (D) 6

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20. The sum of the series $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots.. + \frac{1}{(n-1)!1!}$ is = (A) $\frac{1}{n!2^n}$ (B) $\frac{2^n}{n}!$ (C) $\frac{2^{n-1}}{n}!$ (D) $\frac{1}{n!2^{n-1}}$

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21. the digit at the units place of the number $(32)^{32} =$ (A) 0 (B) 2 (C) 4 (D) 6

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22. In the expansion of $(x^2 + 2x + 2)^n, \neq \psi lon N$ (A) coefficient of $x = n.2^n$ (B) coefficient of x^2=n^2.2^(n-1)$(C) coefficient of$x^3=n^22^(n-2)` (D) none of these

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23. If $(1 + 2x + 3x^2)^{10} = a_0 + a_1x + a_2x^2 + \ldots\ldots. + a_{20}x^{20}$ then (A) $a_1 = 20$ (B) $a_2 = 210$ (C) $a_3 = 1500$ (D) $a_{20} = 2^{3.3} \hat{} 7$

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24. Which of the following holds true? (A) $101^{50}-100^{50}>99^{50}$ (B) $101^{50}-99^{50}<100^{50}$ (C) $(1000)^{1000}>(1000)6999$ (D) `(10001)^999lt(1000)^1000

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25. Given that $(1+x+x^2)^n=a_0+a_1x+a_2x^2+\ldots..+a_{2n}x^{2n}$ find i) $a_0+a_1+a_2\ldots..+a_{2n}$ ii) $a_0-a_1+a_2-a_3\ldots.+a_{2n}$ iii) $(a_0)^2-(a_1)^2\ldots..+(a_{2n})^2$

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26. If n is a positive integer such that $(1+x)^n={}^nC_0+{}^nC_1+{}^nC_2x^2+\ldots\ldots+{}^nC_nx^n, f \text{ or } \varepsilon R.$ Also ^nC_r=C_r

$Onthebasisotheabove \in f \text{ or } mationanswerthefollow \in gquestionsThe$

sum_(r=1)^n r^2.C_r=(A)1(B)(-1)^(n/2).n!/(n/2!)^2(C)(n-1).^(2n)C_n+2(2n) (D)n(n+1)2^(n-2)`

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27. If n is a positive integer such that $(1+x)^n = ^n C_0 + ^n C_1 + ^n C_2 x^2 + \ldots\ldots + ^n C_n x^n, f \text{ or } \varepsilon R.$ Also ^nC_r=C_r

$Onthebasisotheabove \in f \text{ or } mationanswerthefollow \in gquestionsf \text{ or }$ aepsilon R $thevalueofthe \exp ression$a-(a-1)C_1+(a-2)C-2-(a-3)C_3+.+ (1)^n(a-n)C_n=$(A)0(B)$a^n.(-1)^n.^(2n)C_n(C)[2a-n(n+1)[.^(2n)C_n` (D) none of these

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28. If n is a positive integer such that $(1+x)^n = ^n C_0 + ^n C_1 + ^n C_2 x^2 + \ldots\ldots + ^n C_n x^n, f \text{ or } \varepsilon R.$ Also ^nC_r=C_r

$Onthebasisotheabove \in f \text{ or } mationanswerthefollow \in gquestionsthev$

^mC_r.^nC_0+^mC_(r-1).^nC_1+^mC_(r-2).^nC_2+....+^mC_1.^nC_(r-

1)+^nC_0^nC_r$wherem, n, rarepositive \int er \geq s$ and rltm,rltn=(A)

^(mn)C_r$(B)^{m+n}C_r$ (C) 0 (D) 1

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29. If in a positive integer such that If a number $a = p + f$ whre p is an integer and $0 < f < 1$. Here p is called the integral part of a and f its fractional part. Let $\neq plilonN$ and $\left(\sqrt{93}\right) + 1\right)^{2n} = p + f$, where p is the integral part and $0 < f < 1$. On the basis of bove informationi answer teh following question: The integral part p of $\left(\sqrt{3} + 1\right)^{2n}$ is (A) an even number for al $n\varepsilon N$ (B) an odd number for all $\neq \psi lonN$ (C) anodd or even number according as n is odd or even (D) an even or odd nuber according as n is odd or even

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30. $f62 + (p-1) + 4^n =$ (A) p (B) $-p$ (C) 2p (D) $-2p$

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31. Integer just greater tehn $\left(\sqrt{3}+1\right)^{2n}$ is necessarily divisible by (A) $n+2$ (B) 2^{n+3} (C) 2^n (D) 2^{n+1}

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32. In the expansion of $\left[2-2x+x^2\right]^9$ (A) Number of distinct terms is 10 (C) Sum of coefficients is 1 (B) Number of distinct terms is 55 (D) Coefficient of x^4 is 97

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33. The number of terms free from radical sign in the expansion of $\left(1+3^{\frac{1}{3}}+7^{\frac{1}{7}}\right)^{10}$ is

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34. IF $(1+x)^p = 3+\frac{8}{3}+\frac{80}{3^3}+\frac{240}{3^4}+\ldots\ldots\infty, then(1+x)^p =$

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Exercise

1. Find the value of

$$\frac{1}{81^n}-\frac{10}{\left(81^n\right)^{2n}}C_1+\frac{10^2}{\left(81^n\right)^{2n}}C_2-\frac{10^3}{\left(81^n\right)^{2n}}C_3+ +\frac{10^{2n}}{81^n}.$$

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2. With the notation $C_r={ }^n C_2=\frac{n!}{r!(n-r)!}$ when n is positive inteer let

$$S_n=C_n-\left(\frac{2}{3}\right)C_{n-1}+\left(\frac{2}{3}\right)^2 C_{n-2}\pm\ldots\ldots.+(-1)^n\left(\frac{2}{3}\right)^n.C_0$$

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3. If $kandn$ are positive integers and $s_k=1^k+2^k+3^k+ +n^k,$ then prove that $\sum_{r=1}^m \hat{} (m+1)C_r s_r=(n+1)^{m+1}-(n+1)\cdot$

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4. Let $\left(1+x^{2} \hat{} 2\right)^{2}(1+x)^{n}=\sum_{k=0}^{n+4} a_{k} x^{k} . I f a_{1}, a_{2}, a_{3}$ are in rithmetic progression find n.

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5. Findthe coefficient of $x^{2} \in\left(x+\frac{1}{x}\right)^{10} .\left(1-x+2 x^{2}\right)$

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6. Findthe coefficient of x^{4} in te expansion of $\left(1+x+2 x^{2}\right)^{6}$

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7. if $\left(1-x^{3}\right)^{n}=\sum_{r=0}^{n} a_{r} x^{r}(1-x)^{3 n-2 r}$, where n εN then find a_{r}.

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8. Find the consecutive terms in the binomial expansion oif $(3+2x)^7$ whose coefficients are equal

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9. If $a_0, a_1, a_2, \ldots\ldots a_n$ be the successive coefficients in the expnsion of $(1+x)^n$ show that $(a_0-a_2+a_4\ldots\ldots\ldots)^2+(a_1-a_3+a_5\ldots\ldots\ldots)^2=a_0+a_1+a_2+\ldots$

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10. If n is positive integer show that 9^{n+7} is divisible 8

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11. If n is a positive integer, then show tha $3^{2n+1}+2^{n+2}$ is divisible by 7 .

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12. If $\frac{nC_0}{2^n}+2.\frac{nC_1}{2^n}+3.\frac{nC_2}{2^n}+....(n+1)\frac{nC_n}{2^n}=16$ then the value of 'n' is

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13. If $a_1, a_2, \ldots\ldots\ldots, a_{n+1}$ are in A.P. prove that $\sum_{k=0}^{n}\hat{}\ nC_k.\ a_{k+1}=2^{n-1}(a_1+a_{n+1})$

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14. If $(1+x)^n=C_0+C_1x+C_2x^2+\ldots\ldots.+C_nx^n$, show that $3.\ C_0+3^2.\ \frac{C_1}{2}+3^3.\ \frac{C_2}{2}+.\ +3^{n+1}.\ \frac{C_n}{n+1}=\frac{4^{n+1}-1}{n+1}$

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15. Deduce that: $\sum_{r=0}^{n} .^n C_r(-1)^n \frac{1}{(r+1)(r+2)} = \frac{1}{n+2}$

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16. If n be a positive integer and P_n denotes the product of the binomial coefficients in the expansion of $(1+x)^n$, prove that $\frac{P_{n+1}}{P_n} = \frac{(n+1)^n}{n!}$.

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17. If n is a positive integer, prove that

$$\sum_{r=1}^{n} r^3 \left(\frac{{}^nC_r}{{}^nC_{r-1}}\right)^2 = \frac{(n)(n+1)^2(n+2)}{12}$$

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18. Find the coefficients of x^4 in the expansion of $(1+x+x^2)^3$

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19. Find the coefficient of $x^3y^4z^5$ in the expansion of $(xy + yz + zx)^6$

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20. Find the number of terms in the expansion of $(a + b + c + d + e)^{100}$

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21. If in the expansion of $\left(2a \frac{\_2}{4}\right)^9$ the sum of middle tem sis S, then the following is (are) thrue (A) $S = \left(\frac{63}{32}\right)a^{14}(a + 8)$ (B) $S = \left(\frac{63}{32}\right)a^{14}(a - 8)$ (C) $S = \left(\frac{63}{32}\right)a^{13}(a - 8)$ (D) $S = \left(\frac{63}{32}\right)a^{13}(8 - a)$

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22. If the numerical coefficient of the pth terms in the expansion of $(2x + 3)^6$ i s4860, then the following is (are) true (A) p=2 (B) p=3 (C) p=4

(D) p=5

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23. In the expansion of $(1+x)^{50}$ the sum of the coefficients of odd poer 5 to x is (A) 0 (B) 2^{50} (C) 2^{49} (D) 2^{51}

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24. If the coefficients of $x^2 and\ x^3$ in the expansion o $(3+ax)^9$ are the same, then the value of a is $-\frac{7}{9}$ b. $-\frac{9}{7}$ c. $\frac{7}{9}$ d. $\frac{9}{7}$

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25. if the rth term in the expansion of $\left(\frac{x}{3}-\frac{2}{x^2}\right)^{10}$ contains x^4 then r is equal to

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26. I the expansinof $\left(x^2+\frac{2}{x}\right)6n$ for positive integer n has 13th term independent of x, then the sum of divisors of n is (A) 36 (B) 38 (C) 39 (D) 32

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27. The expression $\left[x+\left(x^3-1\right)^{\frac{1}{5}} \hat{} 5+\left[x-\left(x^3-1\right)^{\frac{1}{2}}\right]^5\right.$ is a polynomial of degree (A) 5 (B) 6 (C) 7 (D) 8

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28. The coefficient of x^4 in the expansion of $\left(1+x+x^2+x^3\right)^n$ is (A) $\hat{} nC_4$ (B) $\hat{} nC_4+^n C_2$ (C) $\hat{} nC_4+^n C_2+^n C_4.^n C_2$ (D) $\hat{} nC_4+^n C_2+^n C_1.^n C_2$

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29. If

$\left(1-x+x^{2n}\right)^n = a_0 + a_1 x + a_2^2 + . + a_{2n}x^{2n} then a_0 + a_2 + a_4 + \ldots.. +$

equals (A) $\frac{3^n+1}{2}$ (B) $3^n - 1\frac{)}{2}$ (C) $1 - 3^n/2(D)$3^n+ 1/2`

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30. If $a_n = \sum (r=0)^n \frac{1}{\hat{}} nC_r, then \sum_{r=0^n \frac{r}{\hat{}} nC_r}$ equals (A) $(n-1)a_n$ (B) na_n (C) $\frac{1}{2}na_n$ (D) none of these

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31. the term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}} + \sqrt{\frac{2}{92^2}}\right)^{10}$ is

(A) 0 (B) $\hat{} \ 10C_1$ (C) $\frac{5}{12}$ (D) none of these

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32. For integer $n > 1$, the di git at unit place in the number $\sum_{r=0}^{100} r! + 2^{2^n}$ is equal to

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33. If in the expansion f $(1+x)^m(1-x)^n, the coefficient of x$ and x^2\` are 3 and -6 respectively then (A) m=9 (B) n=12 (C) m=12 (D) n=9

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34. The term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}} + \frac{2}{2x^2}\right)^{10}$ is (A) $\frac{9}{4}$ (B) $\frac{3}{4}$ (C) $\frac{5}{4}$ (D) $\frac{7}{4}$

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35. The value of $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \dots\dots\dots$ is equal to (A) $\frac{2^n = 1}{n - 10}$ (B) $\frac{2^n}{n = 1}$ (C) $\frac{2^n + 1}{n + 1}$ (D) $\frac{2^n - 1}{n + 1}$

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36. The coefficient of x^kd in the expansion of $1+(1+x)+(1+x\square)^2+\ldots\ldots+(1+x)^n$ is (A) $\hat{}\ nC_k$ (B) $\hat{}\ (n+1)C_k$ (C) $\hat{}\ (n+1)C_{k+1}$ (D) none of these

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37. The term independent of x in the expansion of $(1+x+2x^2)\left(\frac{3}{2^2}-\frac{1}{3x}\right)^9$ is (A) $\frac{7}{18}$ (B) $\frac{2}{27}$ (C) $\frac{7}{18}+\frac{2}{27}$ (D) $\left(\frac{7}{18}\right)-\left(\frac{2}{27}\right)$

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38. If the largest interval to which x belongs so that the greatest therm in $(1+x)^{2n}$ has the greatest coefficient is $\left(\frac{10}{11},\frac{11}{10}\right)$ then n= (A) 9 (B) 10 (C) 11 (D) none of these

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39. The number of terms in te expansion of $\left(1+5x+10x^2+10x^3+x^5\right)^{20}$ is (A) 100 (B) 101 (C) 120 (D) none of these

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40. The number of terms in the expansion $\left(x^2+\frac{1}{x^2}+2\right)^{100}$ is (A) 3200 (B) ^102C_2` (C) 201 (D) none of these

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41. The number of terms ins $\left(x^3+1+\frac{1}{x^3}\right)^{100}$ is (A) 300 (B) 200 (C) 100 (D) 201

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42. The number of terms in the expansion of $\left(x+\frac{1}{x}+1\right)^{n}$ is (A) 2n (B) $2n+1$ (C) $2n-1$ (D) none of these

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43. The number of terms in the expansion of $(1+x)^{101}\left(1+x^{2}-x\right)^{100}$ in powers of x is

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44. The coefficient of $a^{3}b^{6}c$ in the expansionof $(2a-b+c)^{10}$ is (A) 6720 (B) 840 (C) 10 (D) none of these

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45. Thenumber of distinct terms in the expansion of $(x_1+x_2+\ldots\ldots\ldots+x_p)^{n} is (A)$^(n+p)C_n$(B)$n+p+1$(C)$n+1$(D)$^(n+p-

1)C_(p-1)`

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46. If $\{x\}$ denotes the fractional part of 'x' , then $82\left\{\frac{3^{1001}}{82}\right\} =$

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47. The digit at units place in $\left(2^9\right)^{100}$ is (A) 2 (B) 4 (C) 6 (D) 8

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48. If $(1 + ax)^n = 1 + 8x + 24x^2 + \dots .$ then the value of a and n is

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49. The sum of the coefficients in $\left(1 + x + 3x^2\right)^{2143}$ is (A) 2^{2143} (B) 0 (C) 1 (D) -1

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50. The coefficients of x^n in the expansion of $(1+x)6(2n)$ and $(1+x0^{2n-1}$ are in the rtio of (A) $1:2$ (B) $1:3$ (C) $3:1$ (D) $2:1$

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51. If $\frac{1}{1+2x+x^2}=1+a_1x+a_2x^2+\ldots..$ then the value of a_r is (A) 2 (B) $r+1$ (C) r (D) $2r$

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52. The coefficients of x^7 in the expansion of $(1-x^4)(1+x)69$ is (A) 27 (B) -24 (C) 48 (D) -48

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53. If $\left(1+x+x^{2}+x^{3}\right)^{n}=\sum_{r=0}^{3n} b_{r} x^{r}$ and $\sum_{r=0}^{3n} b_{r}=k, \text{ t h e n} \sum_{r=0}^{3n} r b_{r}$ is

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54. If the number of terms in $\left(x+1+\frac{1}{x}\right)^{n}$ n being a natural number is 301 the n= (A) 300 (B) 100 (C) 149 (D) 150

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55. The coefficient of x^{5} in the expansion of $\left(1+x^{2}\right)^{5}+(1+x)^{4}$ is (A) 30 (B) 60(C) 40(D) none of these

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56. Let the co-efficients of x^{n} In $(1+x)^{2n}$ and $(1+x)^{2n-1}$ be P & Qrespectively, then $\left(\frac{P+Q}{Q}\right)^{5}=$

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57. The sum of the coefficients of powers of x int eh expansion of the polynomial $\left(x - 3x^2 + x^3\right)^{99}$ is (A) 0 (B) 1 (C) 2 (D) -1

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58. The sixth term in the expansion of $\left(\sqrt{2^{\log(10-3^x)}} + \left(2^{(x-2)\log 3}\right)^{\frac{1}{5}}\right)^m$ is equal to 21, if it is known that the binomial coefficient of the 2nd 3rd and 4th terms in the expansion represent, respectively, the first, third and fifth terms of an A.P. (the symbol log stands for logarithm to the base 10) The value of m is

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59. A student wrote $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots\ldots f$ or $-2 < x < 2$ and got zero marks because (A)

$x was allowed \rightarrow be 0 (B) x was allowed \rightarrow be - ve (C) x was allowed \rightarrow have$

|x|` was greater than 1 for some values of x

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60. If the coefficients of mth, $(m+1)$ the and $(m+2)th$ terms in the expansion $(1+x)^n$ are in A.P., then (A) $n^2+n(4m+1)+4m^2+2=0$ (B) $(n+2m)^2=n+2$ (C) $(n-2m)^2=n+2$ (D) $n^2+4(4m+1)+nm^2-2=0$

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61. Two consecutive terms in the expansion of $(3+2x)^{74}$ have equal coefficients then term are (A) $30 \text{ and } 31$ (B) 38 and 39 (C) 31 and 32 (D) 37 and 38

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62. If the 21st and 22nd terms in the expansin of $(1+x)^{44}$ are equal then x is equal to $(A)\frac{21}{20}$ (B) $\frac{23}{24}$ (C) $\frac{8}{7}$ (D) $\frac{7}{8}$

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63. If C_r stands for ^nC_r and sum_(r=1)^n (r.C_r)/(C_(r-1)=210 then n=` (A) 19 (B) 20 (C) 21 (D) none of these

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64. If $(+x)^n = \sum_{r=0}^{n} a_r x^r \& b_r = 1 + \frac{a_r}{a_{r-1}} \& \prod_{r=1}^{n} b_r = \frac{(101)^{100}}{100!}$, then equals to: 99 (b) 100 (c) 101 (d) None of these

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65. If P_n denotes the product of all the coefficients of $(1+x)^n$ and $8!P_{n+1} = 9^8 P_n$ then n is equal to

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66. If the coefficient of x^{100} isn $1+(1+x)+1+x\Big)^2+(1+x)^3+\ldots\ldots\ldots+(1+x)^n,(n. \quad -100)is^{201}$

then n=` (A) 100 (B) 200 (C) 101 (D) none of these

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67. If ^nC_0/2- ^nC_1/3+^nC_2/4-.......+(-1)^n ^nC_n/(n+2)= 1/(1999x2000)`

then n= (A) 1998 (B) 1999 (C) 2000 (D) 2001

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68. If $\sum_{r=1}^{n} r^3\left(\frac{C(n,r)}{C(n,r-1)}\right)=14^2$ then $n=$

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69. If

$$\left(1+x+x^2+x^3\right)^n=\sum_{r=0}^{300} b_r x^r \text{ and } k=\sum_{r=0}^{300} b_r=k, then \sum_{r=0}^{300} r.\,b_r,$$ is

(A) 50.4^{100} (B) 150.4^{100} (C) 300.4^{100} (D) none of these

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70. If in the expansion of $(1+x)^n$ the coefficients of 14th, 15th and 16th terms are in A.P. then n=` (A) 12 (B) 23 (C) 27 (D) 34

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71. If the four consecutive coefficients in any binomial expansion be $a, b, c \text{ and } d$ then (A) $\frac{a+b}{a}, \frac{b+c}{b}, \frac{c+d}{c}$ are in H.P. (B) $(bc+ad)(b-c)=2\left(ac^2-b^2d\right)$ (C) $\frac{b}{a}, \frac{c}{b}, \frac{d}{c}$ are in A.P. (D) none of these

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72. In the expansion of $(a+b+c)^{10}$ (A) total number of terms in 66 (B) coefficient of $a^8bcis90$ (C) coefficient of $a^4b^5c^3$ is 0 (D) none of these

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73. Let $a_n = \frac{1000^n}{n!}$ for $n \in N$, then a_n is greatest, when

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74. If in the expansion of $(a+b)^n, n\varepsilon N$ sum of odd and even terms be α and β respectively, then (A) $(a^2-b^2)^n = \alpha^2 - \beta^2 (B)$ $(a^2-b^2)^n = (\alpha - \beta 0^n$ (C) $(a+b)^n - (a-b)^n = 4\alpha\beta (D)$ $(a+b)^{2n} - (a-b)^{2n} = 4\alpha\beta$

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75. If 4th term in the expansion of $\left(kx+\frac{1}{x}\right)^{n} is \frac{5}{2}$ then (A) n=8 (B) n=6 (C) $k=\frac{1}{4}$ (D) $k=\frac{1}{2}$

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76. If in the expansion f $(1+x)^{m}(1-x)^{n}, the coefficient of x$ and x^2` are 3 and -6 respectively then (A) m=9 (B) n=12 (C) m=12 (D) n=9

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77. In the expansion of $\left(x^{2}+1+\frac{1}{x^{2}}\right)^{n}, n \in N,$ number of terms is $2n+1$ coefficient of constant terms is 2^{n-1} coefficient of $x^{2n-1} is n$ coefficient of x^{2} in n

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78. If n is a positive integer then $\hat{}\ nC_r +^n C_{r+1} =^{n+1}\ \hat{}\ C_{r+1}$ Also coefficient of x^r in the expansion of $(1+x)^n =^n C_r$. In an identity in x, coefficient of similar powers of x on the two sides re equal. On the basis of above information answer the following question: If n is a positive integer then $\hat{}\ nC_n +^{n+1} C_n +^{n+2} C_n + \ldots. +^{n+k} C_n =$ (A) $\hat{}\ (n+k+1)C_{n+2}$ (B) $\hat{}\ (n+k+1)C_{n+1}$ (C) $\hat{}\ (n+k+1)C_k$ (D) $\hat{}\ (n+k+1)C_{n-2}$

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79. If n is a positive integer then $\hat{}\ nC_r +^n C_{r+1} =^{n+1}\ \hat{}\ C_{r+1}$ Also coefficient of x^r in the expansion of $(1+x)^n =^n C_r$. In an identity in x, coefficient of similar powers of x on the two sides re equal. On the basis of above information answer the following question: If n is a positive integer then $\hat{}\ nC_n +^{n+1} C_n +^{n+2} C_n + \ldots. +^{n+k} C_n =$ (A) $\hat{}\ (n+k+1)C_{n+2}$ (B) $\hat{}\ (n+k+1)C_{n+1}$ (C) $\hat{}\ (n+k+1)C_k$ (D) $\hat{}\ (n+k+1)C_{n-2}$

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80. If n is a positive integer then $\hat{} nC_r +^n C_{r+1} =^{n+1} \hat{} C_{r+1}$ Also coefficient of x^r in the expansion of $(1+x)^n =^n C_r$. In an identity in x, coefficient of similar powers of x on the two sides re equal. On the basis of above information answer the following question: If n is a positive integer then $\hat{} nC_n +^{n+1} C_n +^{n+2} C_n + \ldots . +^{n+k} C_n =$ (A) $\hat{} (n+k+1)C_{n+2}$ (B) $\hat{} (n+k+1)C_{n+1}$ (C) $\hat{} (n+k+1)C_k$ (D) $\hat{} (n+k+1)C_{n-2}$

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81. If n is a positive integer then

$(1+x)^n =^n C_0 x^0 +^n C_1 x^1 +^n C_2^2 + \ldots \ldots \ldots +^n C_r x^r = \sum_{r=0}^{n} \hat{} nC_r x^r$ an

$\hat{} nC_r x^r$

On the basis of above information answer the following question: If n is a positive integer then $\frac{1}{(49)^n}$-

$\frac{8}{(49)^n}(\hat{} (2n)C_1) + \frac{8^2}{(49)^n}(\hat{} (2n)C_2) - \frac{8^3}{(49)^n}(\hat{} (2n)c_3) + \ldots\ldots + \frac{8^{2n}}{(49)}$

(A) -1 (B) 1 (C) $\left(\frac{64}{49}\right)^n$ (D) none of these

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82. If n is a positive integer then

$$(1+x)^n = {}^n C_0 x^0 + {}^n C_1 x^1 + {}^n C_2^2 + \ldots\ldots\ldots + {}^n C_r x^r = \sum_{r=0}^{n} \hat{}\ nC_r x^r \text{ an}$$

$\hat{}\ nC_r x^r$

On the basis of above information answer the following question:If n is a positive integer then

$$\lim_{n} \to \infty n \left[\hat{}\ nc_n - \frac{2}{3} \cdot {}^n C_{n-1} + \left(\frac{2}{3}\right)^2 \cdot {}^n C_{n-2-\ldots\ldots\ldots..+(-1)^n\left(\frac{2}{3}\right)^n \cdot {}^n C}\right.$$

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) $\frac{1}{3}$

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83. Prove that

$$\sum_{r=0}^{n} (-1)^r\ \hat{}\ nC_r \left[\frac{1}{2^r} + \frac{3}{2^{2r}} + \frac{7}{2^{3r}} + \frac{15}{2^{4r}} + up \to mterms\right] = \frac{2^{mn} -}{2^{mn}(2^n -}$$

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84. Sum of the series $a^n + a^{n-1}b +^{n-2} b^2 + \dots\dots\dots\dots + ab^n$ can be obtained by taking outt a^n or b^n comon and using the forumula of sum of $(n+1)$ terms of G.P. N the basis of above information answer the following question: Coefficient of $x^{50} \in (1+x)^{1000} + x(1+x)^{999} + \dots\dots. + x^{999}(1+x) + x^{1000}$ is (A) $\hat{}\ 1000C_{50}$ (B) $\hat{}\ 1002C_{50}$ (C) $\hat{}\ 1001C_{50}$ (D) $\hat{}\ 1001C_{49}$

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85. Sum of the series $a^n + a^{n-1}b +^{n-2} b^2 + \dots\dots\dots\dots + ab^n$ can be obtained by taking outt a^n or b^n comon and using the forumula of sum of $(n+1)$ terms of G.P. N the basis of above information answer the following question:Um of coeficients of x^{50} and x^{51} in $(1+x)^{199} + (1+x)^{198}x + (1+x)6197x^2 + .. + (1+x)x^{198} + x^{199}$ is euqla to the coefficient of $x^r \in$ (1+x)^200+(1+x)^199x+(1+x0^198x^2+.........+(1+x)x^199+x^200` then r may be equal to (A) 51 (B) 52 (C) 53 (D) none of these

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86. Sum of the series $a^n + a^{n-1}b +^{n-2} b^2 + \ldots\ldots\ldots\ldots + ab^n$ can be obtained by taking outt a^n or b^n comon and using the forumula of sum of $(n+1)$ terms of G.P. N the basis of above information answer the following question:Coefficientoif $xp, (0 \leq p \leq n) \in 3^{n-1} + 3^{n-2}(x+3) + 3^{n-3}(x+3)^2 + \ldots\ldots\ldots\ldots + (x +$ is (A) $\hat{}\ nC_p 3^n - p)$ (B) $\hat{}\ (n+1)C_p 3^{n-p+1}$ (C) $\hat{}\ nC_p 3^{n-p-1}$ (D) none of these

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87. In a binomial expansion $(x_y)^n$ gretest term means numericaly greatest term and therefore greatest term in $(x-y)^n$ and $(x+y)^n$ are ame. I frth therm t_r be the greatest term in the expansion of $(x+y)^n$ whose therms are all ositive, then $t_r \geq t_{r+1}$ and $t_r \geq t_=(r-1)i.e.\ \frac{t_r}{t_m} \geq 1$ and $\frac{t_r}{t_{r-}} \geq 1$ ON the basis of above information answer the following question: Greatest term in the expansion of $\left(2+3x0^{10}, whernx=\frac{3}{5}\right.$ is (A) $\hat{}\ 10C_5\left(\frac{18}{5}\right)^5$ (B) $\hat{}\ 10C_6\left(\frac{18}{5}\right)^6$ (C) $\hat{}\ 10C_4\left(\frac{18}{5}\right)^4$ (D) none of these

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88. In a binomial expansion $(x_y)^n$ gretest term means numericaly greatest term and therefore greatest term in $(x-y)^n$ and $(x+y)^n$ are ame. I frth therm t_r be the greatest term in the expansion of $(x+y)^n$ whose therms are all ositive, then $t_r \geq t_{r+1}$ and $t_r \geq t_=(r-1)i.e.\ \frac{t_r}{t_m} \geq 1$ and $\frac{t_r}{t_{r-}} \geq 1$ On the basis of above information answer the following question:If rth term is the greatest term in the expansion f $\left(2-3x0^{10}\right.$ then r= (A) 5 (B) 6 (C) 7 (D) none of these

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89. In a binomial expansion $(x_y)^n$ gretest term means numericaly greatest term and therefore greatest term in $(x-y)^n$ and $(x+y)^n$ are ame. I frth therm t_r be the greatest term in the expansion of $(x+y)^n$ whose therms are all ositive, then $t_r \geq t_{r+1}$ and $t_r \geq t_=(r-1)i.e. \frac{t_r}{t_m} \geq 1$ and $\frac{t_r}{t_{r-}} \geq 1$ On the basis of above information answer the following question:The set al values of x for which thegreatest term in teh expnsionof $(1+x)^{30}$ may have the greatest coefficient is (A) $\left(\frac{14}{15}, \frac{15}{14}\right)$ (B) $\left[\frac{15}{16}, \frac{16}{15}\right]$ (C) $\left(\frac{15}{16}, \frac{16}{15}\right)$ (D) none of these

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90. Let a,b,c,d be the foru cosecutive coefficients int eh binomial expansion $(1+x)^n$ On the basis of above information answer the following question: $\frac{a}{a+b}, \frac{b}{b+c}, \frac{c}{c+d}$ are in (A) A.P. (B) G.P. (C) H.P. (D) none of these

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91. Let a,b,c,d be the foru cosecutive coefficients int eh binomial expansion $(1+x)^n$ On the basis of above information answer the following question: ((bc+ad)(b-c))/(ac^2-b^2d)=(A)1/2` (B) 1 (C) -1 (D) 2

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92. Let a,b,c,d be the foru cosecutive coefficients int eh binomial expansion $(1+x)^n$ On the basis of above information answer the following question: The value of n is (A) $\frac{2ac-b(a+c)}{b^2-ac}$ (B) $\frac{2ac+b(a+c)}{b^2-ac}$ (C) $\frac{2ac}{a+c}-b$ (D) $\frac{b^2-ac}{2ac+b(a+c)}$

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93. Assertion: 1sum_(r=0)^n(r+1).^nC_r=(n+2)2^(n-10, $Reason:$ sum_(r=0)^n(r+1).^nC_rx^r=(1+x)6n+nx(1+x)^(n-1) (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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94. Assertion: Number of rational terms in the expansion of $\left(2^{\frac{1}{3}}+3^{\frac{1}{2}}\right)^{630}$ is 6, Reason: If p is a prime number then p^{k} in rational only when k is a non negative integer (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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95. Assertion: If $(1+ax)^{n}=1+8x+24^{2}+\ldots\ldots\ldots$ then vaues of a and n are 2 and 4 Reason IN an identity in x the coefficients of similar powers of x on the two sides are equal. (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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96. Assertion: Sum of coefficient of the polynomiasl `(1+3x^2-5x^3)^2001 is -1. Reason: Sum of coefficients of a polynomial in x can be obtained by putting x=1 in the polynomial. (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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97. Assertion: 1sum_(r=0)^n(r+1).^nC_r=(n+2)2^(n-10, $Reason$: sum_(r=0)^n(r+1).^nC_rx^r=(1+x)6n+nx(1+x)^(n-1) (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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98. Assertion: If n is an even positive integer n then $\sum_{r=1}^{n-1} \frac{1}{\lfloor r \lfloor n-r} = \frac{2^{n-1}}{\lfloor n}$. Reason: $\hat{} \, nC_1 +^n C_3 + \dots\dots\dots +^n C_{n-1} = 2^{n-1}$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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99. Assertion: If n is an even positive integer n then $\sum_{r=0}^{n} \hat{} \, n \frac{C_r}{r+1} = \frac{2^{n+1}-1}{n+1}$, Reason : $\sum_{r=0}^{n} \hat{} \, n \frac{C_r}{r+1} x^r = \frac{(1+x)^{n+1}-1}{n+1}$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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100. Assertion: The coefficient of x^4 in $(1+x+x^2+x^3)$ is $\hat{} \, nC_4 +^n C_2 +^n C_1 .^n C_2$, Reason: $(1+x+x^2+x^3)^n = (1+x)^n(1+x^2)^n$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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101. Assertion: $r = 15$ Reason : $\hat{} \, nC_x =^I nC_y \rightarrow x + y = n$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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102. Assertion: $f(n)$ is divisible by 961, Reason : $2^{5n} = (1+31)^n$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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103. If $a_n = \sum (r = 0)^n \frac{1}{\hat{}} nC_r, then \sum_{r=0^n \frac{r}{\hat{}} nC_r}$ equals (A) $(n-1)a_n$ (B) na_n (C) $\frac{1}{2}na_n$ (D) none of these

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104. If in the expansion of $(1+x)^m(1-x)^n$ the coefficients of x and `x^2 and 3 and -6 respectivly then m is (A) 6 (B) 9 (C) 12 (D) 24

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105. For any positive integer (m,n) (with $n \geq m$), Let $\binom{n}{m} = .^n C_m$ Prove that

$$\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + ... + (n-m+1)\binom{m}{m}$$

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106. For $2 \leq r \leq n, \binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2}$ is equal to

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107. In the binomial expansion of $(a-b)^{\cap} \geq 5,$ the sum of the 5th and 6th term is zero. Then a/b equals $(n-5)/6$ b. $(n-4)/5$ c. $n/(n-4)$ d. $6/(n-5)$

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108. The sum $\sum_{i=0}^{m} \binom{10}{i}\binom{20}{m-1}$, where $\binom{p}{q} = 0$ if $p < q$, is maximum when m is equal to (A) 5 (B) 10 (C) 15 (D) 20

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109. The coefficient of t^{24} in $\left(1+t^2\right)^{12}\left(1+t^{12}\right)\left(1+t^{24}\right)$ is $\hat{}\ 12C_6+3$ b. $\hat{}\ 12C_6+1$ c. $\hat{}\ 12C_6$ d. $\hat{}\ 12C_6+2$

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110. Prove that $2^k(n,0)(n,k)-2^{k-1}(n,1)(n-1,k-1)+2^{k-2}(n,2)(n-2,k-2)-\ldots$

.

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111. If $\hat{}\ n-1C_r=\left(k^2-3\right)^n C_{r+1}, thenk \in\ (-\infty,\ -2]$ b. $[2,\infty)$ c. $\left[-\sqrt{3},\sqrt{3}\right]$ d. $\left(\sqrt{3},2\right]$

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112. The value of

$(300)(3010) - (301)(3011) + (302)(3012) + \; + (3020)(3030) =$

$\hat{} \; 60C20$ b. $\hat{} \; 30C10$ c. $\hat{} \; 60C30$ d. $\hat{} \; 40C30$

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