MATHS

BOOKS - KC SINHA MATHS (HINGLISH)

COMPLEX NUMBERS - FOR COMPETITION

Solved Examples

1. Express $\frac{1}{1-\cos\theta+2i\sin\theta}$ in the form $A+iB$

Watch Video Solution

2. If $z_1=x_1+iy, z_2=x_2+iy_2$ and $z_1=\frac{i(z_2+1)}{z_2-1}$, prove that $x_1^2+y_1^2-x_1=\frac{x_2^2+y_2^2+2x_2-2y_2+1}{(x_2-1)^2+y_2^2}$

Watch Video Solution

3. Find the complex number z such that $z^2 + |z| = 0$

Watch Video Solution

4. Show that the equation $\frac{A^2}{x-a} + \frac{B^2}{x-b} + \frac{C^2}{x-c} + . + \frac{H^2}{x-h} = x + 1$ where A,B,C $, \ldots \ldots, a, b, c$ and i are real cannot have imaginary roots.

Watch Video Solution

5. If α be a root of equation $x^2 + x + 1 = 0$ then find the vlaue of $\left(\alpha + \frac{1}{\alpha}\right) + \left(\alpha^2 + \frac{1}{\alpha^2}\right)^2 + \left(\alpha^3 + \frac{1}{\alpha^3}\right)^2 + \ldots + \left(\alpha^6 + \frac{1}{\alpha^6}\right)^2$

Watch Video Solution

6. If n is anodd integer greter than 3 but not a multiple of 3 prove that $\left[(x+y)^n - x^n - y^n\right]$ is divisible by $xy(x+y)\left(x^2 + xy + y^2\right)$.

Watch Video Solution

7. Prove that $\left|\frac{z_1, z_2}{1-\bar{z}_1 z_2}\right| < 1 \text{ if } |z_1| < 1, |z_2| < 1$

Watch Video Solution

8. Let z_1, z_2, z_3 be three complex numbers such that $|z_1| = 1, |z_2| = 2, |z_3| = 3 \text{ and } |z_1 + z_2 + z_3| = 1. F \in d|9z_1z_2 + 4z_1z_3 +$

.

Watch Video Solution

9. If z_1, z_2, z_3 are three complex numbers such that $|z_1| = |z_2| = |z_3| = 1,$ find the maximum value of $|z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 + z_1|^2$

Watch Video Solution

10. if $\frac{3}{2+\cos\theta+i\sin\theta}=a+ib$ then prove that $a^2+b^2=4a-3$

Watch Video Solution

11. z_1, z_2, z_3 are complex number and p,q,r are real numbers such that: $\frac{p}{|z_2-z_3|}=\frac{q}{|z_3-z_1|}=\frac{r}{|z_1-z_2|}$. Prove that $\frac{p^2}{z_2-z_3}=\frac{q^2}{z_3-z_1}=\frac{r^2}{z_1-z_2}=0$

Watch Video Solution

12. If $1, \alpha_1, \alpha_2, \ldots \alpha_{n-1}$ be n, nth roots of unity show that $(1-\alpha_1)(1-\alpha_2).(1-\alpha(n-1))=m$

Watch Video Solution

13. If the argument of $(z-a)(\bar{z}-b)$ is equal to that $\frac{(\sqrt{3}+i)(1+\sqrt{3}i)}{1+i}$ where a,b, are two real number and z is the complex conjugate of the complex number z find the locus of z in the Argand diagram. Find the value of a and b so that locus becomes a circle having its centre at $\frac{1}{2}(3+i)$

Watch Video Solution

14. Find the locus of z if arg $\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$

Watch Video Solution

15. If z_1, z_2, z_3be the vertices of an equilateral triangle, show that $\frac{1}{z_1-z_2}+\frac{1}{z_2-z_3}+\frac{1}{z_3-z_1}=0 \text{ or } z_1^2+z_2^2+z_3^2=z_1z_2+z_2z_3+z_3z_1$

.

Watch Video Solution

16. IF the vertices of a triange ABC are respresented by z_1, z_2 and z_3 respectively, show that its circumcentre is

$$\frac{z_1 \sin 2A + z_2 \sin 2B + z_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}$$

Watch Video Solution

17. Let the complex numbers z_1, z_2 and z_3 be the vertices of an equilateral triangle let z_0 be the circumcentre of the triangle. Then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$

Watch Video Solution

18. Two different non parallel lines cut the circle $|z| = r$ at points $a; b; c; d$ respectively . prove that these lines meet at a point $\left(\left(a^{-1} + \frac{b^{-1} - c^{-1} - d^{-1}}{a^{-1}b^{-1} - c^{-1}d^{-1}}\right.\right.$

Watch Video Solution

19. $(x + iy)^{\frac{1}{3}} = (a + ib)$ then prove that $\left(\frac{x}{a} + \frac{y}{b}\right) = 4\left(a^2 - b^2\right)$

Watch Video Solution

20. Point P represents the complex num,ber $z = z + iy$ and point Q the complex num,ber $z + \frac{1}{z}$. Show that if P mioves on the circle $|z| = 2$ then Q oves on the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = \frac{1}{9}$. If z is a complex such that $|z| = 2$ show that the locus of $z + \frac{1}{2}$ is an ellipse.

Watch Video Solution

21. Solve the equation $z^8 + 1 = 0$ and deduce that $\cos 4\theta = 8\left(\cos\theta - \cos\left(\frac{\pi}{8}\right)\right)\left(\cos\theta - \cos\left(\frac{3\theta}{3}\right)\right)\left(\cos\theta - \cos\left(\frac{5\pi}{8}\right)\right)\Big(\text{co}$

Watch Video Solution

22. The points, $z_1, z_2, z_3, z_4,$ in the complex plane are the vartices of a parallelogram taken in order, if and only if $z_1 + z_4 = z_2 + z_3$ $z_1 + z_3 = z_2 + z_4$ $z_1 + z_2 = z_3 + z_4$ (d) None of these

Watch Video Solution

23. for any complex nuber z maximum value of $|z|-|z-1|$ is (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$

Watch Video Solution

24. The least positive integer n for which $\left(\frac{1+i}{1-i}\right)^{n}=\frac{2}{\pi}\sin^{-1}\left(\frac{1+x^{2}}{2x}\right)$, where $x>0$ and $i=\sqrt{-1}$ is

Watch Video Solution

25. The pointo fintersection of the cures represented by the equations $art(z-3i)=\frac{3\pi}{4}$ and $arg(2z+1-2i)=\frac{\pi}{4}$ (A) $3+2i$ (B) $-\frac{1}{2}+5i$ (C) $\frac{3}{4}+\frac{9}{4}i$ (D) none of these

Watch Video Solution

26. Dividing $f(z)$ by $z - i$, we obtain the remainder i and dividing it by $z + i$, we get the remainder 1 + i, then remainder upon the division of $f(z)$ by $z^2 + 1$ is

Watch Video Solution

27. If all the roots of $z^3 - az^2 + bz + c = 0$ are of unit modulus, then (A) $|3 - 4i + b| > 8$ (B) $|c| \geq 3$ (C) $|3 - 4i + a| \leq 8$ (D) none of these

Watch Video Solution

28. Let z_1, z_2 and origin represent vertices A,B,O respectively of an isosceles triangel OAB, where OA=OB and $\angle AOB = 2\theta$. If z_1, z_2 are the roots of the equation $z^2 + 2az + b = 0$ where a,b re comlex numbers then $\cos^2\theta =$ (A) $\frac{a}{b}$ (B) $\frac{a^2}{b^2}$ (C) $\frac{a}{b^2}$ (D) $\frac{a^2}{b}$

Watch Video Solution

29. If $z = x + iy$ and $w = \frac{1 - iz}{z - i}$, then $|w| = 1$ implies that in the complex plane (A)z lies on imaginary axis (B) z lies on real axis (C)z lies on unit circle (D) None of these

Watch Video Solution

30. If $k > 1$, $|z_1|$, k and $\left|\frac{k - z_1\bar{z}_2}{z_1 - kz_2}\right| = 1$, then (A) $z_2 = 0$ (B) $|z_2| = 1$ (C) $|z_2| = 4$ (D) $|z_2| < k$

Watch Video Solution

31. Show that the area of the triangle on the Argand diagram formed by the complex number $z, izandz + iz$ is $\frac{1}{2}|z|^2$

Watch Video Solution

32. Let $z_1 = 6 + i$ and $z_2 = 4 - 3i$. If z is a complex number such thar arg $\left(\frac{z - z_1}{z_2 - z}\right) = \frac{\pi}{2}$ then (A) $|z - (5 - i) = \sqrt{5}$ (B) $|z - (5 + i) = \sqrt{5}$

(C) $|z-(5-i)|=5$ (D) $|z-(5+i)|=5$

Watch Video Solution

33. If z_1 and $\bar{z}_1$ represent adjacent vertices of a regular polygon of n sides where centre is origin and if $\frac{Im(z)}{Re(z)}=\sqrt{2}-1$, then n is equal to:

(A) 8 (B) 16 (C) 24 (D) 32

Watch Video Solution

34. Let z_1 and z_2 be complex numbers of such that $z_1 \neq z_2$ and $|z_1|=|z_2|. If z_1$ has positive real part and z_2 has negative imginary part, then which of the following statemernts are correct for te vaue of $\frac{z_1+z_2}{z_1-z_2}$ (A) 0 (B) real and positive (C) real and negative (D) purely imaginary

Watch Video Solution

35. Let the complex numbers z of the form $x + iy$ satisfy arg $\left(\frac{3z - 6 - 3i}{2z - 8 - 6i}\right) = \frac{\pi}{4}$ and $|z - 3 + i| = 3$. Then the ordered pairs (x, y) are (A) $\left(4 - \frac{4}{\sqrt{5}}, 1 + \frac{2}{\sqrt{5}}\right)$ (B) $\left(4 + \frac{5}{\sqrt{5}}, 1 - \frac{2}{\sqrt{5}}\right)$ (C) $(6 - 1)$ (D) $(0, 1)$

Watch Video Solution

36. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbes such that $|z_1| = |z_2| = 1$ and $Re(z_1 \bar{z}_2) = 0$ then the pair of complex numbers $\omega_1 = a + ic$ and $\omega_2 = b + id$ satisfy which of the following relations? (A) $|\omega_1| = 1$ (B) $|\omega_2| = 1$ (C) $Re(\omega_1 \overline{\omega}_2) = 0$ (D) $Im(\omega_1 \overline{\omega}_2) = 0$

Watch Video Solution

37. If z_1, z_2, z_3 are non zero non collinear complex number such that $\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3}, then$ (A) ponts z_1, z_2, z_3 form and equilateral triangle

(B) points z_1, z_2, z_3lies on a circle (C) z_1, z_2, z_3 and origin are concylic (D) $z_1 + z_2 + z_3 = 0$

Watch Video Solution

38. If $\sin\alpha + \sin\beta + \sin\gamma = \cos\alpha + \cos\beta + \cos\gamma = 0,$ then (A) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$ (B) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 0$ (C) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 0$ (D) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$

Watch Video Solution

39. $\frac{z_2}{z_1} =$ (A) $e^{i\theta}\cos\theta$ (B) $e^{i\theta}\cos 2\theta$ (C) $e^{-i\theta}\cos\theta$ (D) $e^{2i\theta}\cos 2\theta$

Watch Video Solution

40. $\frac{z_1}{z_2} =$ (A) $\cos 2\theta e^{2i\theta 0}$ (B) $\sec 2\theta e^{-2}i\theta)$ (C) $\cos^2\theta e^{2i\theta}$ (D) $\sec^2\theta e^{-2i\theta 0}$

Watch Video Solution

41. $\frac{z_3^2}{z_1 . z_2} =$ (A) $\sec^2 . \cos 2\theta$ (B) $\cos\theta . \sec^{22}\theta$ (C) $\cos^2\theta . \sec 2\theta$ (D) $\sec\theta . \sec^{22}\theta$

Watch Video Solution

42. Which of the following is (are) correct? (A) $\bar{a}z_1 + a\bar{z}_1 - \bar{a}z_2 - a\bar{z}_2 = 0$ (B) $\bar{a}z_1 + a\bar{z}_1 + \bar{a}z_2 + a\bar{z}_2 = -b$ (C) $\bar{a}z_1 + a\bar{z}_1 + \bar{a}z_2 + a\bar{z}_2 = 2b$ (D) $\bar{a}z_1 + a\bar{z}_1 + \bar{a}z_2 + a\bar{z}_2 = -2b$

Watch Video Solution

43. Which of the following is (are) correct? (A) $\overline{z_1 - z_2} - a(\bar{z}_1 - \bar{z}_2) = 0$ (B) $\overline{z_1 - z_2} + a(\bar{z}_1 - \bar{z}_2) = 0$ (C) $\overline{z_1 - z_2} + a(\bar{z}_1 - \bar{z}_2) = -b$ (D) $\overline{z_1 - z_2} + a(\bar{z}_1 - \bar{z}_2) = -b$

Watch Video Solution

44. Which of the following is (are) correct? (A) $\bar{z}_1 + a\bar{z}_2 = 2b$ (B) $\bar{z}_1 + a\bar{z}_2 = b$ (C) $\bar{z}_1 + a\bar{z}_2 = -b$ (D) $\bar{z}_1 + a\bar{z}_2 = -2b$

Watch Video Solution

45. If $2 + z + z^4 = 0, wherez$ is a complex number then(A)1/2 lt|z|lt1 (B)1/2lt|z|lt1/3(C)|z|ge1` (D) none of these

Watch Video Solution

46. If $|a_n| < 1f$ or $n = 1, 2, 3, \ldots$ and $1 + a_1 z + a_2 z^2 + \ldots + a_n z^n = 0$ then z lies (A) on the circle $|z| = \frac{1}{2}$ (B) inside the circle $|z| = \frac{1}{2}$ (C) $outsidethe \circ \leq$ |z|= 1/2$(D)onthech$ or $dofthe \circ \leq$ |z|=1/2 $cutoffbythel \in e$Re[(1+i)z]=0`

Watch Video Solution

47. All the roots of equation $z^n \cos tgh\eta_0 + z^{n-1}\cos\theta_1 + z^{n-2}\cos\theta_2 + \ldots + \cos\theta_n = 2,$ when $\theta_0, \theta_1, \theta_2, \ldots\ldots \theta_n \varepsilon R$lie (A) on the line $Re[(3+4i)z] = 0$ (B) inside the circel $|z| = \frac{1}{2}$ (C) outside the circle $|z| = \frac{1}{2}$ (D) on the circle $|z| = \frac{1}{2}$

Watch Video Solution

48. If $f(x) = x^4 - 8x^3 + 4x^2 + 4x + 39$ and $f(3-2i) = a + ib,$ then the vaue of a.b is

Watch Video Solution

49. if$\omega and \omega^2$ are the nonreal cube roots of unity and $[1/(a+\omega)] + [1/(b+\omega)] + [1/(c+\omega)] = 2\omega^2$ and $\left[1/(a+\omega)^2\right] + \left[1/(b+\omega)^2\right] + \left[1/(c+\omega)^2\right] = 2\omega$, then find the value of $[1/(a+1)] + [1/(b+1)] + [1/(c+1)]$.

Watch Video Solution

50. Given that the complex numbers which satisfy the equation $\left|z\bar{z}^3\right|+\left|\bar{z}z^3\right|=350$ form a rectangle in the Argand plane with the length of its diagonal having an integral number of units, then area of rectangle is 48 sq. units if z_1, z_2, z_3, z_4 are vertices of rectangle, then $z_1+z_2+z_3+z_4=0$ rectangle is symmetrical about the real axis $arg(z_1-z_3)=\frac{\pi}{4}$ or $\frac{3\pi}{4}$

Watch Video Solution

Exercise

1. Put the following in the form $A+iB$: $\frac{(\cos x+i\sin x)(\cos y+i\sin y)}{(\cot u+i)(1+i\tan v)}$

Watch Video Solution

2. If the expression $\frac{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)-i\tan\left(\frac{x}{2}\right)}{1+2i\sin\left(\frac{x}{2}\right)}$ is real, the find the set of al possible values of x

Watch Video Solution

3. IF $a \geq 1,$ find all complex numbers z satisfying the equation $z + a|z+1| + i = 0$

Watch Video Solution

4. ω is an imaginary root of unity. Prove that If $a+b+c=0,$ then prove that $\left(a+b\omega+c\omega^2\right)^3 + \left(a+b\omega^2+c\omega^{\square}\right)^3 = 27ab\cdot$

Watch Video Solution

5. Find the integral solutions of the following equation: $(3+4i)^x = 5^{\frac{x}{2}}$

Watch Video Solution

6. Find the number of non-zero integral solutions of the equation $|1-i|^{x} = 2^{x}$.

Watch Video Solution

7. Find the integral solutions of the following equation: $(1-i)^{x} = (1+i)^{x}$

Watch Video Solution

8. Let $\left|\frac{\bar{z}_1 - 2\bar{z}_2}{2 - z_1\bar{z}_2}\right| = 1$ and $|z_2| \neq 1 where z_1$ and z_2 are complex numbers show that $|z_1| = 2$

Watch Video Solution

9. if a,b,c are complex numbers such that $a+b+c=0$ and $|a| = |b| = |c| = 1$ find the value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

Watch Video Solution

10. Show that for any two non zero complex numbers z_1, z_2

$$(|z_1| + |z_2|)\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \leq 2|z_1 + z_2|$$

Watch Video Solution

11. Prove that $\left|\frac{z-1}{1-\bar{z}}\right| = 1$ where z is as complex number.

Watch Video Solution

12. Solve the equation $x^4 - 4x^2 + 8x + 35 = 0$ gine that one of roots is $2 + \sqrt{-3}$

Watch Video Solution

13. If z_1, z_2, z_3be the vertices of an equilateral triangle, show that $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0 \text{ or } z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$

.

Watch Video Solution

14. The complex numbers $z = x + iy$ which satisfy the equation $\left|\frac{z - 5i}{z + 5i}\right| = 1$ lie on (a) The x-axis (b) The straight line $y = 5$ (c) A circle passing through the origin (d) Non of these

Watch Video Solution

15. If $\left|z^2 - 1\right| = |z|^2 + 1$ shwo that the locus of z is as straight line.

Watch Video Solution

16. If $\log_{\sqrt{3}}\left|\frac{|z|^2 - |z| + 1|}{|z| + 2}\right| < 2$ then locus of z is

Watch Video Solution

17. Three points represented by the complex numbers a,b,c lie on a circle with centre 0 and rdius r. The tangent at C cuts the chord joining the points a,b and z. Show that $z = \dfrac{a^{-1} + b^{-1} - 2c^{-1}}{a^{-1}b^{1} - c^{2}}$

Watch Video Solution

18. Show that $\left(\dfrac{1 + \cos\phi + i\sin\phi}{1 + \cos\phi - i\sin\phi}\right)^{n} = \cos\phi n\phi + i\sin n\phi$

Watch Video Solution

19. Show that the roots of equation $(1 + z)^{n} = (1 - z)^{n} arei \dfrac{\tan(r\pi)}{n}, r = 0, 1, 2, , \ldots\ldots\ldots\ldots, (n - 1)$ excluding the vlaue when n is even and $r = \dfrac{n}{2}$

Watch Video Solution

20. The least positive integer n for which $\left(\frac{i-1}{i+1}\right)^n$ is a real number is

(A) 2 (B) 3 (C) 4 (D) 5

Watch Video Solution

21. $\sum_{k=1}^{6}\left(\sin, \frac{2\pi k}{7} - i\cos, \frac{2\pi k}{7}\right) = ?$

Watch Video Solution

22. For any integer n, the argument of $\frac{\left(\sqrt{3}+i\right)^{4n+1}}{\left(1-i\sqrt{3}\right)^{4n}}$

Watch Video Solution

23. Values of $\left(1-i\sqrt{3}\right)^{\frac{1}{3}}$ is (are) (A) $2^{\frac{1}{3}}\left(\cos 20^0 + i\sin 20^0\right)$ (B) $2^{\frac{1}{3}}\left(\cos 20^0 - i\sin 20^0\right)$ (C) $2^{\frac{1}{3}}\left(\cos 100^0 + i\sin 100^0\right)(D)$2^(1/3)

(cos220^0+isin220^0)`

Watch Video Solution

24. The complex numbers z_1, z_2 and the origin form an equilateral triangle only if (A) $z_1^2 + z_2^2 - z_1 z_2 = 0$ (B) $z_1 + z_2 = z_1 z_2$ (C) $z_1^2 - z_2^2 = z_1 z_2$ (D) none of these

Watch Video Solution

25. for any complex nuber z maximum value of $|z| - |z - 1|$ is (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$

Watch Video Solution

26. $\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8$ is equal to

Watch Video Solution

27. The argument of $\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$ is 60^0 b. 120^0 c. 210^0 d. 240^0

Watch Video Solution

28. Which of the following is not correct? (A) $|7+i| > |5+i|$ (B) $|7+i| > |7-i|$ (C) $|7+2i| > |7+i|$ (D) none of these

Watch Video Solution

29. If Z is a complex number the radius of $z\bar{z} - (2+3i)z - (2-3i)\bar{z} + 9 = 0$ is equal to

Watch Video Solution

30. The polynomial $x^6 + 4x^5 + 3x64 + 2x^3 + x + 1$ is divisible by______ where w is the cube root of units $x + \omega$ b. $x + \omega^2$ c. $(x+\omega)(x+\omega^2)$ d. $(x-\omega)(x-\omega^2)$ where ω is one of the imaginary cube roots of unity.

Watch Video Solution

31. In Argand diagram, O, P, Q represent the origin, z and z+ iz respectively then $\angle OPQ$=

Watch Video Solution

32. If $z(\neq -1)$ is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is equal to

Watch Video Solution

33. The value of $(\sin\theta + i\cos\theta)^n$ is (A) $\sin n\theta + i\cos n\theta$ (B) $\cos n\theta - i\sin n\theta$ (C) $\cos\left(\frac{n\pi}{2} - n\theta\right) + is\sin\left(\frac{n\pi}{2} - n\theta\right)$ (D) none of these

Watch Video Solution

34. If $x = 2 + 5i$ (where $i^2 = -1$) and $2\left(\frac{1}{1!9!} + \frac{1}{3!7!}\right) + \frac{1}{5!5!} = \frac{2^a}{b!}$, then the value of $(x^3 - 5x^2 + 33x - 19)$ is equal to

Watch Video Solution

35. $|z - i| < |z + i|$ represents the region (A) $Re(z) > 0$ (B) $Re(z) < 0$ (C) $Im(z) > 0$ (D) $Im(z) < 0$

Watch Video Solution

36. The points representing complex numbers z for which $|z - 3| = |z - 5|$ lie on the locus given by (A) circle (B) ellipse (C) straight line (D) none of these

Watch Video Solution

37. $|z-4| < |z-2|$ represents the region given by: (a) $Re(z) > 0$ (b) $Re(z) < 0$ (c) $Re(z) > 3$ (d) None of these

Watch Video Solution

38. if $1, \omega, \omega^2, \ldots\ldots\ldots\ldots \omega^{n-1}$ are nth roots of unity , then $(1-\omega)(1-\omega^2)\ldots\ldots (1-\omega^{n-1})$ equal to

Watch Video Solution

39. If $1, \alpha_1, \alpha_2, \ldots \alpha_{n-1}$ be nth roots of unity then $(1+\alpha_1)(1+\alpha_2)\ldots\ldots\ldots (1+\alpha_{n-1}) =$ (A) 0 or 1 according as n is even or odd (B) 0 or 1 according as n is odd or even (C) n (D) $-n$

Watch Video Solution

40. If ω be a nth root of unity, then $1 + \omega + \omega^2 + \ldots\ldots + \omega^{n-1}$ is (a)0(B) 1 (C) -1 (D) 2

Watch Video Solution

41. If $|z| = 2$ and locus of $5z - 1$ is the circle having radius a and $z_1^2 + z_2^2 - 2z_1z_2\cos\theta = 0, then |z_1| : |z_2| =$ (A) a (B) 2a (C) $\frac{a}{10}$ (D) none of these

Watch Video Solution

42. If $|z - 4 + 3i| \leq 1$ and m and n be the least and greatest values of $|z|$ and K be the least value of $\frac{x^4 + x^2 + 4}{x}$ on the interval $(0, \infty)$, then $K =$

Watch Video Solution

43. If $a\hat{i}+b\hat{j}+c\hat{k}$ be a unit vector and z is a comple number such that $(1+a)z=b+ic, then \frac{1-iz}{1+z}$ (A) $\frac{a+ib}{1+z}$ (B) $\frac{1+c}{a+ib}$ (C) $(a+ib)(1+c)$ (D) none of these

Watch Video Solution

44. If for complex numbers z_1 and $z_2, |z_1+z_2|=|z_1|=|z_2|$ then $arg z_1 - arg z_2 =$ (A) an even multiple of π (B) an odd multiple of π (C) an odd multiple of $\frac{\pi}{2}$ (D) none of these

Watch Video Solution

45. Number of solutions of $Re(z^2)=0$ and $|z|=r\sqrt{2}$ where z is a complex number and $r>0$ is (A) 2 (B) 4 (C) 5 (D) none of these

Watch Video Solution

46. If ω is an imaginary fifth root of unity, then find the value of $loe_2|1+\omega+\omega^2+\omega^3-1/\omega|\cdot$

Watch Video Solution

47. If z is a unimodular number $(\ne \pm i)$ then $\frac{z+i}{z-i}$ is (A) purely real (B) purely imaginary (C) an imaginary number which is not purely imaginary (D) both purely real and purely imaginary

Watch Video Solution

48. The locus of the complex number z satisfying the inequaliyt $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z-1|+6}{2|z-1|-1}\right)>1\left(2where|z-1|\ne\frac{1}{2}\right)$ is (A) a circle (B) interior of a circle (C) exterior of circle (D) none of these

Watch Video Solution

49. The number of complex numbers z satisfying $|z-3-i|=|z-9-i| and |z-3+3i|=3$ are a. one b. two c. four d. none of these

Watch Video Solution

50. If |z|= maximum $\{|z+2|,|z-2|\}$, then(A)|z-$\bar{z}|=$ 1/2(B)|z+$\bar{z}$|=4(C) |z+$\bar{z}$|= 1/2(D)|$z-\bar{z}$=2

Watch Video Solution

51. If z_1 and z_2 are complex numbers such that $|z_1-z_2|=|z_1+z_2|$ and A and B re the points representing z_1 and z_2 then the orthocentre of $\triangle OAB$, where O is the origin is (A) $\frac{z_1+z_2}{2}$ (B) 0 (C) $\frac{z_1-z_2}{2}$ (D) none of these

Watch Video Solution

52. If α is an imaginary root of $z^n - 1 = 0$ then $1 + \alpha + \alpha^2 + \dots\dots\dots + \alpha^{n-1} =$ (A) 1 (B) -1 (C) 0 (D) 2

Watch Video Solution

53. If $|z^2 - 3| = 3|z|$, then the maximum value of $|z|$ is 1 b. $\dfrac{3+\sqrt{21}}{2}$ c. $\dfrac{\sqrt{21}-3}{2}$ d. none of these

Watch Video Solution

54. If the maximum value of $|3z + 9 - 7i|$ if $|z + 2 - i| = 5$ is 5K, then find k

Watch Video Solution

55. Let $P \equiv \sqrt{3}e^{i\frac{\pi}{3}}, Q \equiv \sqrt{3}e^{-\frac{\pi}{3}}$ and $R \equiv \sqrt{3}e^{-i\pi}$. If P,Q,R form a triangle PQR in the Argand plane, then $\triangle PQR$ is (A) isosceles (B) equilateral (C) scalene (D) none of these

Watch Video Solution

56. I $|z| \geq 5$ then the least value of $\left|z + \frac{2}{z}\right|$ is (A) $\frac{23}{5}$ (B) $\frac{24}{5}$ (C) 5 (D) none of these

Watch Video Solution

57. If $Re\left(\frac{2z+1}{iz+1}\right) = 1$, the the locus of the point representing z in the complex plane is a (A) straight line (B) circle (C) parabola (D) none of these

Watch Video Solution

58. $|z-4| + |z+4| = 16$ where z is as complex number ,tehn locus of z is (A) a circle (B) a straight line (C) a parabola (D) none of these

Watch Video Solution

59. A, B, C are the point representing the complex numbers z_1, z_2, z_3 respectively on the complex plane and the circumcentre of the triangle ABC lies at the origin. If the altitude of the triangle through the vertex A meets the circumcircel again at P, then prove that P represents the complex number $-\frac{z_2z_3}{z_1}$

Watch Video Solution

60. The points, $z_1, z_2, z_3, z_4,$ in the complex plane are the vartices of a parallelogram taken in order, if and only if $z_1 + z_4 = z_2 + z_3$ $z_1 + z_3 = z_2 + z_4$ $z_1 + z_2 = z_3 + z_4$ (d) None of these

Watch Video Solution

61. If all the roots of $z^3 - az^2 + bz + c = 0$ are of unit modulus, then (A) $|3 - 4i + b| > 8$ (B) $|c| \geq 3$ (C) $|3 - 4i + a| \leq 8$ (D) none of these

Watch Video Solution

62. If $a = z_1 + z_2 + z_3,\, b = z_1 + \omega z_2 + \omega^2 z_3,\, c = z_1 + \omega^2 z_2 + \omega z_3 \left(1, \omega, \omega^2\right.$ are cube roots of unity), then the value of z_2 in terms of a,b, and c is (A) $\frac{a\omega^2 + b\omega + c}{3}$ (B) $\frac{a\omega^2 + b\omega^2 + c}{3}$ (C) $\frac{a + b + c}{3}$ (D) $\frac{a + b\omega^2 + c\omega}{3}$

Watch Video Solution

63. $z = x + iy$ satisfies $arg(z + 2) = arg(z + i)$ then (A) $x + 2y + 1 = 0$ (B) $x + 2y + 2 = 0$ (C) $x - 2y + 1 = 0$ (D) $x - 2y - 2 = 0$

Watch Video Solution

64. The points $A(z_1), B(z_2)$ and $C(z_3)$ form an isosceles triangle in the Argand plane right angled at B, then $\frac{z_1 - z_2}{z_3 - z_2}$ can be (A) 1 (B) -1 (C) $-i$ (D) none of these

Watch Video Solution

65. The number of solutions of $\sqrt{2}|z-1| = z - i, wherez = x + iy$ is

(A) 0 (B) 1 (C) 2 (D) 3

Watch Video Solution

66. If $|2z-1| = |z-2|$ and z_1, z_2, z_3 are complex numbrs such that $|z_1 - \alpha| < \alpha, |z_2 - \beta| < \beta$. Then $\frac{z_1 + z_2}{\alpha + \beta} | \; =$ (A) $< |z|$ (B) $< 2|z|$ (C) $> |z|$ (D) $> 2|z|$

A. <|z|

B. null

C. null

D. null

Answer: null

Watch Video Solution

67. if $1, \alpha_1, \alpha_2, \ldots\ldots\alpha_{3n}$ be the roots of equation $x^{3n+1} - 1 = 0$ and omega be an imaginary cube root of unilty then $\frac{(\omega^2 - \alpha_1)(\omega^2 - \alpha).(\omega^2 - \alpha(3n))}{(\omega - \alpha_1)(\omega - \alpha_2)\ldots\ldots(\omega - \alpha_{3n})} =$ (A) ω (B) $-\omega$ (C) 1 (D) ω^2

Watch Video Solution

68. If α and β are two fixed complex numbers, then the equation $z = a\alpha + (1 - a)\beta, where a \varepsilon R$ represents in the Argand plane (A) a straight line passsing through α and β (B) a straight line passing through α but not through β (C) a striaght line passing through β but not throughα (D) a straight line passing neighter through α not or through β

Watch Video Solution

69. If $\begin{vmatrix} x^2 + x & x - 1 & x + 1 \\ x & 2x & 3x - 1 \\ 4x + 1 & x - 2 & x + 2 \end{vmatrix} = px^4 + qx^3 + rx^2 + sx + t$ be n identity in x and ω be an imaginary cube root of unity,

$$\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}=$$ (A) p (B) $2p$ (C) $-2p$ (D) $-p$

Watch Video Solution

70. If z_1, z_2, z_3, z_4 be the vertices of a quadrilaterla taken in order such that $z_1+z_2=z_2+z_3$ and $|z_1-z_3|=|z_2-z_4|$ then arg $\left(\frac{z_1-z_2}{z_3-z_2}\right)=$ (A) $\frac{\pi}{2}$ (B) $\pm\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Watch Video Solution

71. If z_1, z_2, z_3 be the vertices A,B,C respectively of triangle ABC such that $|z_1|=|z_2|=|z_3|$ and $|z_1+z_2|=|z_1-z_2|$ then C= (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

Watch Video Solution

72. If z_1, z_2, z_3 be the vertices of a triangle ABC such that $|z_1|=|z_2|=|z_3|$ and $|z_1+z_2|^2=|z_1|^2+|z_2|^2$, then

$\left|arg, \left(\frac{z_3 - z_1}{z_3 - z_2}\right)\right| =$ (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

Watch Video Solution

73. If $\sec^{-1}\left(\frac{z-2}{i}\right)$ lies between 0 and $\frac{\pi}{2}$, where $z = x + iy$ then (A) $x > 2, y > 1$ (B) $x = 2, y > 1$ (C) $x = 2, y = 1$ (D) $x < 2, y = 1$

Watch Video Solution

74. The system of equation $|z - 1 - i| = \sqrt{2}$ and $|z| = 2$ has (A) one solutions (B) two solution (C) three solutions (D) none of these

Watch Video Solution

75. If $z_1, z_2, z_3, \ldots\ldots\ldots\ldots z_{n-1}$ are the roots of the equation $1 + z + z^2 + \ldots\ldots + z^{n-1} = 0, wherene N, n > 2$ then (A) $z_1, z_2, \ldots z_{n-1}$ are terms of a G.P. (B) $z_1, z_2, \ldots\ldots, z_{n-1}$ are terms of an A.P. (C) $|z_1| = |z_2| = |z_3| = .\, |z_{n-1}| \neq 1$ (D) none of these

Watch Video Solution

76. If the greatest valueof $|z|$ such that $|z-3-4i| \leq a$ is equal to the least value of $\frac{x^4+x^2+5}{x} in(0,\infty) then a =$ (A) 1 (B) 4 (C) 3 (D) 2

Watch Video Solution

77. $|z-4|+|z+4|=16$ where z is as complex number ,then locus of z is (A) a circle (B) a straight line (C) a parabola (D) none of these

Watch Video Solution

78. Let z_1, z_2, z_3 be three distinct non zero complex numbers which form an equilateral triangle in the Argand pland. Then the complex number associated with the circumcentre of the tirangle is (A) $\frac{z_1 z_2}{z_3}$ (B) $\frac{z_1 z_3}{z_2}$ (C) $\frac{z_1+z_2}{z_3} (D) \frac{z_1+z_2+z_3}{3}$

Watch Video Solution

79. If $\sqrt{5-12i}+\sqrt{5-12i}=z$, then principal value of $argz$ can be $\frac{\pi}{4}$

b. $\frac{\pi}{4}$ c. $\frac{3\pi}{4}$ d. $-\frac{3\pi}{4}$

Watch Video Solution

80. If $z+\sqrt{2}|z+1|+i=0$, then z= (A) $2+i$ (B) $2-i$ (C) $-2-i$ (D) $-2+i$

Watch Video Solution

81. If A and B represent the complex numbers z_1 and z_2 such that $|z_1-z_2|=|z_1+z_2|$, then circumcentre of $\triangle AOB$, O being the origin is (A) $\frac{z_1+2z_2}{3}$ (B) $\frac{z_1+z_2}{3}$ (C) $\frac{z_1+z_2}{2}$ (D) $\frac{z_1-z_2}{3}$

Watch Video Solution

82. If α and β are complex numbers then the maximum value of $\dfrac{\alpha\bar{\beta}+\overline{\alpha}\beta}{|\alpha\beta|}=$

Watch Video Solution

83. If $a=\dfrac{\cos(2\pi)}{7}+i\dfrac{\sin(2\pi)}{7}$, $\alpha=a+a^2+a^4,\ \beta=a^3+a^5+a^6$ then α, β are the roots of the equation

Watch Video Solution

84. If z_1, z_2, z_3 be the vertices A,B,C respectively of an equilateral trilangle on the Argand plane and $|z_1|=|z_2|=|z_3|$ then (A) Centroid oif the triangle ABC is the complex number 0 (B) Distance between centroid and orthocentre of the triangle ABC is 0 (C) Centroid of the tirangle ABC divides the line segment joining circumcentre and orthcentre in the ratio 1:2 (D) Complex number representing the incentre of the triangle ABC is a non zero complex number

Watch Video Solution

Watch Video Solution

85. If $|z-4+3i| \leq -2,$ then the least value of $|z| =$ (A) 2 (B) 3 (C) 4 (D) 5

Watch Video Solution

86. If $|z| = 5,$ then the locus of $-1+2z$ is (A) a circle having center (2,0) (B) a circle having center $(-1, 0)$ (C) a circle having radius 5 (D) a circle having radius 9

Watch Video Solution

87. $|z+3| \leq 3$, then the greatest and least value of $|z+1|$ are

Watch Video Solution

88. If z_1 and z_2 are two complex numbers such that $|(\bar{z}_1 - 2\bar{z}_2)(2 - z_1\bar{z}_2)| = 1$ then (A) $|z_1| = 1, \quad \text{if} \quad |z_2| \neq 1$ (B) $|z_1| = 2, \quad \text{if} \quad |z_2| \neq 1$ (C) $|z_2| = 2, \quad \text{if} \quad |z_1| \neq 1$ (D) $|z_2| = 1, \quad \text{if} \quad |z_1| \neq 2$

Watch Video Solution

89. If $\left|z - \frac{4}{z}\right| = 2$ then the greatest value of $|z|$ is (A) $\sqrt{5} - 1$ (B) $\sqrt{5} + 1$ (C) $\sqrt{5}$ (D) 2

Watch Video Solution

90. If z is a complex number different form $\frac{i}{3}$ then locus of z if $\left|\frac{3z}{3z - i}\right| = 1$ is (A) a straightline paralel to x axis (B) a straight line having slope undefined (C) as straight line having slope 0 (D) a straight line passing through the point $\left(2, \frac{1}{6}\right)$

Watch Video Solution

91. If z_1 and z_2 two non zero complex numbers such that $|z_1 + z_2| = |z_1|$ then which of the following may be true (A) $argz_1 - argz_2 = 0$ (B) $argz_1 - argz_2 = \pi$ (C) $|z_1 - z_2| = ||z_1| - |z_2||$ (D) $argz_1 - argz_2 = 4\pi$

Watch Video Solution

92. The complex numbers $z_1, = 1 + 2i, z_2 = 4 - 2i$ and $z_3 = 1 - 6i$ form the vertices of a (A) a right angled triangle (B) isosceles triangle (C) equilateral triangle (D) triangle whose one of the sides is of length 8

Watch Video Solution

93. If the vertices of an equilateral triangle are situated at $z = 0, z = z_1$ and $z = z_2$ then which of the following is(are) true? (A) $|z_1| = |z_2|$ (B) $|z_1 + z_2| = |z_1| + |z_2|$ (C) $|z_1 - z_2| = |z_1|$ (D) |argz_1-argz_2|= $\frac{\pi}{3}$

Watch Video Solution

94. If z_1 and z_2 are two complex numbers for which $|(z_1 - z_2)(1 - z_1 z_2)| = 1$ and $|z_2| \neq 1 then$ (A) $|z_2| = 2$ (B) $|z_1| = 1$ (C) $z_1 = e^{i\theta}$ (D) $z_2 = e^{i\theta}$

Watch Video Solution

95. If $\sin x + \sin y + \sin z = \cos x + \cos y + \cos z = 0, then(A)$ sin2x+sin2y+sin2z=0(B)cos2x+cos2y+cos2z=0(C)tanx+tany+tanz=0` (D) none of these

Watch Video Solution

96. Find the complex number z satisfying the equations $\left|\frac{z-12}{z-8i}\right| = \frac{5}{3}, \left|\frac{z-4}{z-8}\right| = 1$

Watch Video Solution

97. Which of the following are correct for any two complex numbers z_1 and z_2? (A) $|z_1z_2| = |z_1||z_2|$ (B) $arg(|z_1z_2|) = (argz_1)(arg, z_2)$ (C) $|z_1 + z_2| = |z_1| + |z_2|$ (D) $|z_1 - z_2| \geq |z_1| - |z_2|$

Watch Video Solution

98. Values $(s)(-i)^{1/3}$ is/are $\frac{\sqrt{3}-i}{2}$ b. $\frac{\sqrt{3}+i}{2}$ c. $\frac{-\sqrt{3}-i}{2}$ d. $\frac{-\sqrt{3}+i}{2}$

Watch Video Solution

99. The modulus and the principal asrgumentof the complex nuber $\frac{1-i}{3+i} + 4i$ are (A) modulus $= \sqrt{3}$ (B) modulus $= 6$ (C) $arg = \tan^{-1}(18)$ (D) $arg = tn^{-1}\left(\frac{3}{4}\right)$

Watch Video Solution

100. Let $A(z_1), B(z_2)$ and $C(z_3)$ be the vertices of an equilateral triangle in the Argand plane such that $|z_1| = |z_2| = |z_3|$. Then (A) $\frac{z_2 + z_3}{2z_1 - z_2 - z_3}$ is purely real (B) $\frac{z_2 - z_3}{2z_1 - z_2 - z_3}$ is purely imaginary (C) $\left|arg\left(\frac{z_1}{z_2}\right)\right| = 2arg\left(\frac{z_3 - z_2}{z_1 - z_2}\right)\Big|$ (D) none of these

Watch Video Solution

101. If a and b are two real number lying between 0 and 1 such that $z_1 = a + i,\ z_2 = 1 + bi$ and $z_3 = 0$ form anequilateral trilangle , then (A) $a = 2 + \sqrt{3}$ (B) $b = 4 - \sqrt{3}$ (C) $a = b = 2 - \sqrt{3}$ (D) $a = 2, b = \sqrt{3}$

Watch Video Solution

102. If z_1, z_2, z_3, z_4 be the vertices of a parallelogram taken in anticlockwise direction and $|z_1 - z_2| = |z_1 - z_4|$, then $\sum_{r=1}^{4} (-1)^r z_r = 0$ (b) $z_1 + z_2 - z_3 - z_4 = 0$ $ar\frac{g(z_4 - z_2)}{z_3 - z_1} = \frac{\pi}{2}$ (d) None of these

Watch Video Solution

103. If $|z_1 + z_2| = |z_1 - z_2|$ and $|z_1| = |z_2|$, then (A) $z_1 = \pm iz_2$ (B) $z_1 = z_2$ (C) $z_= - z_2$ (D) $z_2 = \pm iz_1$

Watch Video Solution

104. If $|z| = \min(|z-1|, |z+1|\}$, where z is the complex number and f be a one -one function from $\{a, b, c\} \to \{1, 2, 3\}$ and $f(a) = 1$ is false, $f(b) \neq 1$ is false and $f(c) \neq 2$ is true then $|z + \bar{z}| =$ (A) $f(a)$ (B) $f(c)$ (C) $\frac{1}{2}f(a)$ (D) $f(b)$

Watch Video Solution

105. If z_1, z_2, z_3 are complex numbers such that $|z_1| = z_2| = |z_3| = |z_1 + z_2 + z_3| = 1, then \left(\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$ is (A) equal to 1 (B) les than (C) greater than 3 (D) equal to 3

Watch Video Solution

106. If $|z_1 = 1, |z_2| = 2, |z_3| = 3 \text{ and } |z_1 + z_2 + z_3| = 1, then |9z_1z_2 + 4z_3z_1 + z_2$

is equal to (A) 3 (B) 36 (C) 216 (D) 1296

Watch Video Solution

107. If $|z_1| = |z_2| = . = |z_n| = 1,$ then the value of $|z_1 + z_2 + z_3 + .. + z_n|$ is equal to (A) 1 (B) $|z_1| + |z_2| + z_3| + \ldots.. + |z_n|$ (C) $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \ldots\ldots\ldots + \frac{1}{z_n}\right|$ (D) n

Watch Video Solution

108. If $\left|z - \frac{4}{z}\right| = 2$ then the greatest value of $|z|$ is (A) $\sqrt{5} - 1$ (B) $\sqrt{5} + 1$ (C) $\sqrt{5}$ (D) 2

Watch Video Solution

109. If $\left|z-\frac{4}{2}z\right|=2$ then the least of $|z|$ is (A) $\sqrt{)}5\Big)=-1$ (B) $\sqrt{5}-2$ (C) $\sqrt{5}$ (D) 2

Watch Video Solution

110. If $|z-4+3i|\leq 2$ then the complex number z for which $|z|$ is minimum is (A) $\frac{12}{5}+\frac{9}{5}i$ (B) $\frac{9}{5}-\frac{12}{5}i$ (C) $\frac{12}{5}-\frac{9}{5}i$ (D) $-\frac{12}{5}+\frac{9}{5}i$

Watch Video Solution

111. Which of the gien statement(s) is (are) true? (A) $A\subseteq B$ (B) $A=B=\phi$ (C) $A\cap B\neq\phi$ (D) $B\subseteq A$

Watch Video Solution

112. Let $z_1\varepsilon A$ and $z_2\varepsilon B$ then the value of $|z_1-z_2|$ necessarily lies between (A) 3 and 15 (B) 0 and 22 (C) 2 and 22 (D) 4 and 14

Watch Video Solution

113. If $C = \{z : Re[(3+4i)z] = 0\}$ thenthe number of elements in the set $B \cap C$ is (A) 0 (B) 1 (C) 2 (D) none of these

Watch Video Solution

114. If $|z - 4 + 3i| \leq 3,$ then the least value of $|z| =$ (A) 2 (B) 3 (C) 4 (D) 5

Watch Video Solution

115. If $|z - 25i| \leq 15$ then least positive value of $argz =$ (A) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$ (B) $\tan^{-1}\left(\frac{3}{4}\right)$ (C) $\tan^{-1}\left(\frac{4}{3}\right)$ (D) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$

Watch Video Solution

116. If $|z| < 1, then 1 + 2z$ lies (A) on or inside circle having center at origin and radius 2 (B) outside the circle having center at origin and radius 2 (C) inside the circle having center at (1,0) and radius 2 (D) outside the circle having center at (1,0) and radius 2.

Watch Video Solution

117. If the complex numbers z_1, z_2, z_3 represents the vertices of a triangle ABC, where z_1, z_2, z_3 are the roots of equation $z^3 + 3\alpha z^2 + 3\alpha z^2 + 3\beta z + \gamma = 0, \alpha, \beta, \gamma$ beng complex numbers and $\alpha^2 = \beta then \triangle ABC$ is (A) equilateral (B) right angled (C) isosceles but not equilateral (D) scalene

Watch Video Solution

118. If a and b are two real number lying between 0 and 1 such that $z_1 = a + i, z_2 = 1 + bi$ and $z_3 = 0$ form an equilateral triangle , then (A) $a = 2 + \sqrt{3}$ (B) $b = 4 - \sqrt{3}$ (C) $a = b = 2 - \sqrt{3}$ (D) $a = 2, b = \sqrt{3}$

Watch Video Solution

119. Let the complex numbers z_1, z_2 and z_3 be the vertices of a equilateral triangle. Let z_0 be the circumcentre of the tringel ,then $z_1^2 + z_2^2 + z_3^2 =$ (A) z_0^2 (B) $3z_0^2$ (C) $9z_0^2$ (D) 0

Watch Video Solution

120. If the complex number z_1, z_2 and z_3 represent the vertices of an equilateral triangle inscribed in the circle $|z| = 2$ and $z_1 = 1 + i\sqrt{3}$ then (A) $z_2 = 1, z_3 = 1 - i\sqrt{3}$ (B) $z_2 = 1 - i\sqrt{3}, z_3 = -i\sqrt{3}$ (C) $z_2 = 1 - i\sqrt{3}, z_3 = -1 + i\sqrt{3}$ (D) $z_2 = , z_3 = 1 - i\sqrt{3}$

Watch Video Solution

121. The locus of the centre of a variable circle touching circle $|z| = 5$ internally and circle `|z-4|=1 externally is (A) a parabola (B) a hyperbola (C) an ellipse (D) none of these

Watch Video Solution

122. Locus of the centre of the circle touching circles $|z| = 3$ and $|z - 4| = 1$ externally is (A) a parabola (B) a hyperbola (C) an ellipse (D) none of these

Watch Video Solution

123. Locus the centre of the variable circle touching $|z - 4| = 1$ and the line $Re(z) = 0$ when the two circles on the same side of the line is (A) a parabola (B) an ellipse (C) a hyperbola (D) none of these

Watch Video Solution

124. If $|z - 1| + |z + 3| \leq 8,$ then the maximum, value of $|z - 4| is =$

Watch Video Solution

125. If z_1, z_2, z_3 are three points lying on the circle $|z| = 2$ then the minimum value of the expression $|z_1 + z_2|^2 + |z_2 + z_3|^2 + |z_3 + z_1|^2 =$

Watch Video Solution

126. If z_1 and $\bar{z}_1$ represent adjacent vertices of a regular polygon of n sides where centre is origin and if $\frac{Im(z)}{Re(z)} = \sqrt{2} - 1$, then n is equal to:

(A) 8 (B) 16 (C) 24 (D) 32

Watch Video Solution

127. The value of the expression $2^{199} \sin\left(\frac{\pi}{199}\right) \sin\left(\frac{2\pi}{199}\right) \sin\left(\frac{3\pi}{199}\right) \ldots\ldots\ldots \sin\left(\frac{198\pi}{199}\right)\Big) =$

Watch Video Solution

128. If $\frac{1}{a+\omega}+\frac{1}{b+\omega}+\frac{1}{c+\omega}+\frac{1}{d+\omega}=\frac{1}{\omega}$ where $a, b, c, d \in R$ and ω is cube root of unity then show that $\sum \frac{1}{a^2-a+1}=1.$

Watch Video Solution

129. If $x=2+5i$ (where $i^2=-1$) and $2\left(\frac{1}{1!9!}+\frac{1}{3!7!}\right)+\frac{1}{5!5!}=\frac{2^a}{b!}$, then the value of $(x^3-5x^2+33x-19)$ is equal to

Watch Video Solution

130. Let z be a complex number lying on a circle centred at the origin having radius r. If the area of the triangle having vertices as $z, z\omega$ and $z+z\omega$, where omega is an imaginary cube root of unity is $12\sqrt{3}$ sq. units, then the radius of the circle r=

Watch Video Solution

131. Number of solutons of $Re(z^2) = 0$ and $|z| = r\sqrt{2}$ where z is a complex number and $r > 0$ is equal to.

Watch Video Solution

132. Let z_1, z_2 and origin be the vertices A,B,O respectively of an isosceles triangle OAB, where OA=OB and $\angle AOB = 2\theta. Ifz_1, z_2$ are the roots of equation $z^2 + z + 9 = 0$ then $\sec^2\theta =$

Watch Video Solution

133. Let the center of the circle represented by $z\bar{z} - (2+3i)z - (2-3i)\bar{z} + 9 = 0'be(x, y),$ then the value of $x^2 + y^2 + xy$ is

Watch Video Solution

134. Assertion (A): If $1, \omega, \omega^2$ are the cube roots of unity, then roots of equation $(x-2)^3 - 27 = 0 are 5, 2, +3\omega, 2+3\omega^2$, Reason (R): If α be one cube root of a number, then its other two cube roots are $\alpha\omega$ and $\alpha\omega^2$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

Watch Video Solution

135. Assertion (A): $arg \mid z_1 - arg z_2 = 0$, Reason: If $|z_1 + z_2| = |z_1| + |z_2|$, then origin z_1, z_2 are colinear and z_1, z_2 lie on the same side of the origin. (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

Watch Video Solution

136. Assertion (A): Circumcentre of $\triangle POQ$ is $\frac{z_1+z_2}{2}$, Reason (R): Circumcentre of a right triangle is the middle point of the hypotenuse. (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

Watch Video Solution

137. If α, β are complex numbers, then maximum value of $\frac{\alpha\bar{\beta}+\overline{\alpha}\beta}{|\alpha\beta|}$ is 2. Reason (R): For any two complex numbers z_1 and z_2, $|z_1-z_2| \geq ||z_1|-|z_2||$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

Watch Video Solution

138. Assertion (A): z_1, z_2 and origin form an equilateral triangle if $p^2=6q$ for the equation $z^2+pz+q=0$, Reason (R): Triangle having vertices z_1, z_2, z_3 in the Argand plane is equilateral if $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z3 z_1$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

Watch Video Solution

139. Assertion (A): Points representing z_1, z_2, z_3 are collinear. Reason (R): Three numbers a,b,c are in A.P., if $b - a = c - b$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

Watch Video Solution

140. Assertion (A):$arg z_1 - artg z_2 = 0$, Reason (R): If $|z_1 - z_2| = |z_1| - |z_2|$ then origin z_1 and z_2 are collinear and z_1 and z_2 lie on the same side of the origin. (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

Watch Video Solution

141. Assertion (A): $\frac{z}{4-z^2}$ lies on y-axis. $Reason(R)$: |z|^2= $z\bar{z}$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

Watch Video Solution

142. Assertion (A): $|iz+3-5i|<8$, Reason(R): For any two complex numbers $z_1 a \ne d z_2, |z_1+z_2| \geq |z_1|+|z_2|$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

Watch Video Solution

143. Assertion (A): For any non zero complex numbers $z, |z-|z|| \leq |z|argz|$ Reason (R): $|\sin\theta| \leq \theta$ for all theta` (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

Watch Video Solution

144. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals (1) (0, 1) (2) (1, 1) (3) (1, 0) (4) $(-1, 1)$

Watch Video Solution

145. Let z and ω be two non zero complex numbers such that $|z| = |\omega|$ and $argz + arg\omega = \pi,$ then z equals (A) ω (B) $-\omega$ (C) $\overline{\omega}$ (D) $-\overline{\omega}$

Watch Video Solution

146. Let $z and \omega$ be two complex numbers such that $|z| \leq 1, |\omega| \leq 1 and |z - i\omega| = |z - i\omega| = 2, then z$ equals $1 \text{ or } i$ b. $i \text{ or } -i$ c. $1 \text{ or } -1$ d. $i \text{ or } -1$

Watch Video Solution

147. If $iz^3 + z^2 - z + i = 0$, then show that $|z| = 1$

Watch Video Solution

148. If $|z| \leq 1 \text{ and } |\omega| \leq 1,$ show that $|z - \omega|^2 \leq (|z| - |\omega|)^2 + (arg z - arg \omega)^2$

Watch Video Solution

149. The value of the expression $1.(2 - \omega).(2 - \omega^2) + 2.(3 - \omega)(3 - \omega^2) + . + (n - 1)(n - \omega)(n - \omega^2),$

where ω is an imaginary cube root of unity, is.........

Watch Video Solution

150. For positive integer n_1, n_2 the value of the expression $(1+i)^{n1} + (1+i^3)^{n1}(1+i^5)^{n2}(1+i^7)^{n_{20}}$, where $i = \sqrt{-1}$, is a real number if and only if (a) $n_1 = n_2 + 1$ (b) $n_1 = n_2 - 1$ (c) $n_1 = n_2$ (d) $n_1 > 0, n_2 > 0$

Watch Video Solution

151. Find all non zero complex numbers z satisfying $\bar{z} = iz^2$

Watch Video Solution

152. Let z_1 and z_2 be the root of the equation $z^2 + pz + q = 0$ where the coefficient p and q may be complex numbers. Let A and B represent z_1 and z_2 in the complex plane. If

$\angle AOB = \alpha \neq 0$ and 0 and $OA = OB, where O$ is the origin prove that $p^2 = 4q\cos^2\left(\frac{\alpha}{2}\right)$

Watch Video Solution

153. Let $\bar{b}z + b(\bar{z}) = c, b \neq 0$ be a line the complex plane, where $\bar{b}$ is the complex conjugate of b. If a point z_1 i the reflection of the point z_2 through the line then show that $c = \bar{z}_1 b + z_2 \bar{b}$

Watch Video Solution

154. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ is equal to 128ω (b) -128ω $128\omega^2$ (d) $-128\omega^2$

Watch Video Solution

155. The value of $sum \sum_{n=1}^{13} (i^n + i^{n+1})$, $w h e r e i = \sqrt{-1} e q u a l s$ i (b) $i - 1$ (c) $-i$ (d) 0

Watch Video Solution

156. $x + iy = \begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$, find x and y.

Watch Video Solution

157. If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ is equal to (1) $1 - i\sqrt{3}$ (2) $-1 + i\sqrt{3}$ (3) $i\sqrt{3}$ (4) $-i\sqrt{3}$

Watch Video Solution

158. For complex numbers z and w, prove that $|z|^2 w - |w|^2 z = z - w$, if and only if $z = w$ or $z\overline{w} = 1$.

Watch Video Solution

159. If $arg(z) < 0$, then $arg(-z) - \arg(z)$ equals π (b) $-\pi$ (d) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

Watch Video Solution

160. If z_1, z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1, then|z_1 + z_2 + z_3|$ is (A) equal to 1 (B) gt1 (C) gt3 (D) equal to 3

Watch Video Solution

161. Let z_1 and z_2 be nth roots of unity which subtend a right angle at the origin. Then n must be of the form (1) $4k + 1$ (2) $4k + 2$ (3) $4k + 3$ (4) $4k$

Watch Video Solution

162. The complex numbers $z_1 z_2$ and $z3$ satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of triangle which is (1) of area zero (2) right angled isosceles(3) equilateral (4) obtuse angled isosceles

Watch Video Solution

163. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $|1111 - 1 - \omega^2 \omega^2 1 \omega^2 \omega^4|$ is 3ω b. $3\omega(\omega - 1)$ c. $3\omega^2$ d. $3\omega(1 - \omega)$

Watch Video Solution

164. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$ the minimum value of $|z_1 - z_2|$ is (A) 0 (B) 2 (C) 7 (D) 17

Watch Video Solution

165. Let a complex number $\alpha, \alpha \neq 1,$ be a rootof hte euation $z^{p+q} - z^p - z^q + 1 = 0, wherep, q$ are distinct primes. Show that either $1 + \alpha + \alpha^2 + \ + \alpha^{p-1} = 0$ or $1 + \alpha + \alpha^2 + \ + \alpha^{q-1} = 0$, but not both together.

Watch Video Solution

166. If $|z| = 1$ and $w = \frac{z-1}{z+1}$ (where $z \neq \ -1$) then $Re(w)$ is (A) 0 (B) $-\frac{1}{|z+1|^2}$ (C) $\left|\frac{z}{z+1}\right| \frac{1}{|z+1|^2}$ (D) $\frac{\sqrt{2}}{|z+1|^2}$

Watch Video Solution

167. If z_1 and z_2 are two complex numbers such that $|z_1| < 1 < |z_2|$ then prove that $\left|\frac{1 - z_1\bar{z}_2}{z_1 - z_2}\right| < 1$

Watch Video Solution

168. Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{n=1}^{n} a_r z^r = 1$, where $|a_r| < 2$.

Watch Video Solution

169. If $\omega(\neq 1)$ be an imaginary cube root of unity and $(1+\omega^2) = (1+\omega^4)$, then the least positive value of n is (a) 2 (b) 3 (c) 5 (d) 6

Watch Video Solution

170. Find the centre and radius of the circle formed by all thepoints represented by $z = x + iy$ satisfying the relation $\left|\frac{z-\alpha}{z-\beta}\right| = k(k \neq 1)$, where α and β are the constant complex numbers given by $\alpha = \alpha_1 + i\alpha_2, \beta = \beta_1 + i\beta_2$.

Watch Video Solution

171. a, b, c are integers, not all simultaneously equal, and ω is cube root of unity $(\omega \neq 1)$, then minimum value of $\left|a + b\omega + c\omega^2\right|$ is 0 b. 1 c. $\frac{\sqrt{3}}{2}$ d. $\frac{1}{2}$

Watch Video Solution

172. The shaded region, where $P = (-1, 0), Q = \left(-1 + \sqrt{2}, \sqrt{2}\right) R = \left(-1 + \sqrt{2}, -\sqrt{2}, S = (1, 0)\right.$ is represented by Figure `|z+1|>2,|a r g(z+1)2,|a r g(z+1)>>pi/4|z+1|<<2,|a r g(z+1)>pi/2`

Watch Video Solution

173. If one of the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2 + \sqrt{3}\iota$. Find the other vertices of square

Watch Video Solution

174. If $w = \alpha + i\beta,$ where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{w - wz}{1 - z}\right)$ is a purely real, then the set of values of z is $|z| = 1, z \neq 2$ (b) $|z| = 1 and z \neq 1$ $z = z$ (d) None of these

Watch Video Solution

175. A man walks a distance of 3 units from the origin towards the North-East $(N45^0 E)$ direction.From there, he walks a distance of 4 units towards the North-West $(N45^0 W)$ direction to reach a point P. Then, the position of P in the Argand plane is $3e^{\frac{i\pi}{4}} + 4i$ (b) $(3 - 4i)e^{\frac{i\pi}{4}}$ $(4 + 3i)e^{\frac{i\pi}{4}}$ (d) $(3 + 4i)e^{\frac{i\pi}{4}}$

Watch Video Solution

176. If $|z| = 1 and z \neq \pm 1,$ then all the values of $\frac{z}{1 - z^2}$ lie on a line not passing through the origin $|z| = \sqrt{2}$ the x-axis (d) the y-axis

Watch Video Solution

177. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by $6 + 7i$ (b) $-7 + 6i$ $7 + 6i$ (d) $-6 + 7i$

Watch Video Solution

178. Let $z = \cos\theta + i\sin\theta$. Then , the value of $\sum_{m=1}^{15} Im\left(z^{2m-1}\right)$ at $\theta = 2^0$ is $\frac{1}{\sin 2^0}$ (b) $\frac{1}{3\sin 2^0}$ $\frac{1}{2\sin 2^0}$ (d) $\frac{1}{4\sin 2^0}$

Watch Video Solution

179. Let $z = x + iy$ be a complex number where $xandy$ are integers. Then, the area of the rectangle whose vertices are the roots of the equation $zz^3 + zz^3 = 350$ is 48 (b) 32 (c) 40 (d) 80

Watch Video Solution