MATHS

BOOKS - KC SINHA MATHS (HINGLISH)

CONIC SECTIONS - FOR COMPETITION

Solved Examples

1. Find the equation of the parabola the extremities of whose latus rectum are (1, 2) and (1, -4).

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2. Find the locus of a point whose sum of the distances from the origin and the line $x = 2$ is 4 units. Sketch the path.

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3. Find the equaiton to the parabola whose axis is parallel to y-axis and which passes through the point $(0, 4), (1, 9)$ and $(-2, 6)$. Determine its latus rectum.

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4. Find the axis, tangent at the vertex, focus, directrix and latus rectum of the parabola $9y^2 - 16x - 12y - 57 = 0$.

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5. Show that the tangents at the extremities of any focal chord of a parabola intersect at right angles at the directrix.

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6. Prove that perpendicular drawn from locus upon any tangent of a parabola lies on the tangent at the vertex

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7. If S be the focus of the parabola and tangent and normal at any point P meet its axis in T and G respectively, then prove that $ST = SG = SP$.

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8. If S be the focus and SH be perpendicular to the tangent at P, then prove that H lies on the tangent at the vertex and $SH^2 = OS.\ SP$, where O is the vertex of the parabola.

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9. On the parabola $y^2 = 4ax$, three points E, F, G are taken so that their ordinates are in geometrical progression. Prove that the tangents at E and G intersect on the ordinate passing through F.

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10. Show that the length of the tangent to the parabola $y^2 = 4ax$ intercepted between its point of contact and the axis of the parabola is bisected by the tangent at the vertex.

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11. The tangents to the parabola $y^2 = 4ax$ at $P(at_1^2, 2at_1)$, and $Q(at_2^2, 2at_2)$, intersect at R. Prove that the area of the triangle PQR is $\frac{1}{2}a^2(t_1 - t_2)^3$

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12. Find the angle of intersection of the curves $y^2 = 4ax$ and $x^2 = 4by$

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13. Show that the normal at a point $(at^2 \_ 1, 2at_1)$ on the parabola $y^2 = 4ax$ cuts the curve again at the point whose parameter $t_2 = -t_1 - \frac{2}{t_1}$.

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14. Prove that through any point, three normals can be drawn to a parabola and the algebraic sum of the ordinates of the three points is zero.

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15. If a normal to a parabola $y^2 = 4ax$ makes an angle ϕ with its axis, then it will cut the curve again at an angle

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16. Three normals are drawn from the point $(14, 7)$ to the curve $y^2 - 16x - 8y = 0$. Find the coordinates of the feet of the normals.

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17. If the normals to the parabola $y^2 = 4ax$ at the ends of the latus rectum meet the parabola at $QandQ'$, then $\mathbb{Q}'$ is $10a$ (b) $4a$ (c) $20c$ (d) $12a$

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18. Normals are drawn from a point P with slopes m_1, m_2 and m_3 are drawn from the point p not from the parabola $y^2 = 4x$. For $m_1m_2 = \alpha$, if the locus of the point P is a part of the parabola itself, then the value of α is (a) 1 (b)-2 (c) 2 (d) -1

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19. Find the locus of a point which is such that, the three normals through it cut the axis in points whose distance from the vertex are in A.P.

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20. find the common tangents of the circle $x^2 + y^2 = 2a^2$ and the parabola$y^2 = 8ax$

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21. From a pt A common tangents are drawn to a circle $x^2 + y^2 = \frac{a^2}{2}$ and $y^2 = 4ax$. Find the area of the quadrilateral formed by common tangents, chord of contact of circle and chord of contact of parabola.

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22. Find the centre and radius of the smaller of the two circles that touch the parabola $75y^2 = 65(5x - 3)$ at $\left(\frac{6}{5}, \frac{8}{5}\right)$ and the X axis.

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23. Prove that the semi-latus rectum of the parabola $y^2 = 4ax$ is the harmonic mean between the segments of any focal chord of

the parabola.

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24. If the point $(at^2, 2at)$ be the extremity of a focal chord of parabola $y^2 = 4ax$ then show that the length of the focal chord is $a\left(t + \frac{1}{t}\right)^2$.

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25. Show that the focal chord, of parabola $y^2 = 4ax$, that makes an angle α with the x-axis is of length $4a \cos ec^2 \alpha$.

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26. Prove that the length of a focal chord of a parabola varies inversely as the square of its distance from the vertex.

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27. Tangents PT and QT to the parabola $y^2 = 4x$ intersect at T and the normal drawn at the point P and Q intersect at the point $R(9, 6)$ on the parabola. Find the coordinates of the point T. Show that the equation to the circle circumscribing the quadrilateral $PTQR$, is $(x-2)(x-9)+(y+3)(y-6)=0$.

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28. Show that the locus of the point of intersection of mutually perpendicular tangetns to a parabola is its directrix.

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29. The locus of the point of intersection of the tangents to the parabola $y^2 = 4ax$ which include an angle α is

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30. Two tangents to the parabola $y^2 = 4ax$ make angles θ_1, θ_2 with the x-axis. Then the locus of their point of intersection if $\cot\theta_1 + \cot\theta_2 = c$ is

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31. At any point P on the parabola $y^2 - 2y - 4x + 5 = 0$ a tangent is drawn which meets the directrix at Q. Find the locus of point R which divides QP externally in the ratio $\frac{1}{2} : 1$

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32. Find the locus of the point through which pass three normals to the parabola $y^2 = 4ax$ such that two of them make angles α and β respectively with the axis so that $\tan\alpha\tan\beta = 2$.

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33. Find the locus of points of intersection of tangents drawn at the end of all normal chords to the parabola $y^2 = 8(x-1)$.

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34. The locus of the point of intersection of those normals to the parabola $x^2 = 8y$ which are at right angles to each other, is a

parabola. Which of the following hold (s) good in respect of the loucus ?

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35. P, Q are the points $'t_1', 't_2'$ on the parabola $y^2 = 4ax$. If the normals at P, Q meet on the parabola at R, show that $T_1 t_2 = 2$. Also find the locus of the mid-point of PQ.

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36. Show that the locus of points such that two of the three normals drawn from them to the parabola $y^2 = 4ax$ coincide is $27ay^2 = 4(x - 2a)^3$.

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37. Through the vertex 'O' of parabola $y^2 = 4x$, chords OP and OQ are drawn at right angles to one another. Show that for all positions of P, PQ cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of PQ.

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38. Show that the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio $1:2$ is parabola. Find the vertex of this parabola.

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39. If tangents be drawn from points on the line $x = c$ to the parabola $y^2 = 4x$, show that the locus of point of intersection of the corresponding normals is the parabola.

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40. A ray of light is coming along the line $y = b$ from the positive direction of x-axis and striks a concave mirror whose intersection with xy-plane is a parabola $y^2 = 4ax$. Find the equation of the reflected ray and show that it passes through the focus of the parabola. Both a and b are positive.

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41. Find the equaiton of the ellipse, the extermities of whose minor axis are $(3, 1)$ and $(3, 5)$ and whose eccentricity is $\frac{1}{2}$.

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42. Find the latus rectum, the eccentricity and coordinates of the foci of the ellipse $9x^2 + 5y^2 + 30y = 0$.

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43. Tangents are drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at two points whose eccentric angles are $\alpha - \beta$ and $\alpha + \beta$ The coordinates of their point of intersection are

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44. The locus of the foot of the perpendicular from the foci an any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is

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45. Prove that the tangent and normal at any point of an ellipse bisect the external and internal angles between the focal distances of the point

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46. Prove that the product of the perpendicular from the foci on any tangent to an ellipse is equal to the square of the semi-minor axis.

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47. Find a point on the curve $x^2 + 2y^2 = 6$, whose distance from the line $x + y = 7$, is minimum.

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48. Let d be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the tangent drawn at a point P on the ellipse. If $F_1 \& F_2$ are the two foci of the ellipse, then show the $(PF_1 - PF_2)^2 = 4a^2\left[1 - \frac{b^2}{d^2}\right]$.

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49. The tangent at the point α on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the auxiliary circle in two points which subtends a right angle at the centre, then the eccentricity 'e' of the ellipse is given by the equation (A) $e^2\left(1 + \cos^2\alpha\right) = 1$ (B) $e^2\left(\cos ec^2\alpha - 1\right) = 1$ (C) $e^2\left(1 + \sin^2\alpha\right) = 1(D)$ e^2(1+tan^2alpha)=1`

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50. If the normal at any point P of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the major and minor axes at G and E respectively, and if CF is perpendicular upon this normal from the centre C of the ellipse, show that $PF.PG = b^2$ and $PF.PE = a^2$.

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51. Prove that in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix.

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52. Find the equation of the common tangent in the first quadrant of the circle $x^2 + y^2 = 16$ and the ellipse

$\frac{x^2}{25}+\frac{y^2}{4}=1$.Also find the length of the intercept of the tangent between the coordinates axes.

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53. A tangent to the ellipse $x^2+4y^2=4$ meets the ellipse $x^2+2y^2=6$ at P&Q.

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54. If any two chords be drawn through two points on the major axis of an ellipse equidistant from the centre, show that $\tan\left(\frac{\alpha}{2}\right)\cdot\tan\left(\frac{\beta}{2}\right)\cdot\tan\left(\frac{\gamma}{2}\right)\cdot\tan\left(\frac{\delta}{2}\right)=1$, where $\alpha, \beta, \gamma, \delta$ are the eccentric angles of the .extremities of the chords.

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55. prove that the circle drawn on any focal distance as diameter touches the auxiliary circle in an ellipse

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56. If two chords of a circle are equally inclined to the diameter through their point of intersection, prove that the chords are equal.

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57. The locus of point of intersection of the two tangents to the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ which make an angle 60° with one another is

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58. The locus of the poles of normal chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is

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59. An ellipse slides between two perpendicular straight lines. Then identify the locus of its center.

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60. A variable point P on the ellipse of eccentricity e is joined to the foci S and S'. The eccentricity of the locus of incentre of the triangle PSS' is (A) $\sqrt{\frac{2e}{1+e}}$ (B) $\sqrt{\frac{e}{1+e}}$ (C) $\sqrt{\frac{1-e}{1+e}}$ (D) $\frac{e}{2(1+e)}$

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61. Let P be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, 0<b<a$ and let the line parallel to y-axis passing through P meet the circle $x^2+y^2=a^2$ at the point Q such that P and Q are on the same side of x-axis. For two positive real numbers r and s, find the locus of the point R on PQ such that $PR:RQ=r:s$ and P varies over the ellipse.

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62. Find the condition so that the line $px+qy=r$ intersects the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in points whose eccentric angles differ by $\frac{\pi}{4}$.

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63. Point P represents the complex num,ber $z = z + iy$ and point Q the complex num,ber $z + \frac{1}{z}$. Show that if P mioves on the circle $|z| = 2$ then Q oves on the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = \frac{1}{9}$. If z is a complex such that $|z| = 2$ show that the locus of $z + \frac{1}{2}$ is an ellipse.

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64. Area of the triangle formed by the tangents at the point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentric angles are α, β, γ is

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65. Show that the tangents at the ends of conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intersect on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2.$$

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66. If a triangle is inscribed in a n ellipse and two of its sides are parallel to the given straight lines, then prove that the third side touches the fixed ellipse.

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67. A parabola is drawn to pass through A and B, the ends of a diameter of a given circle of radius a, and to have as directrix a tangent to a concentric circle of radius the axes of reference being AB and a perpendicular diameter, prove that the locus of the focus of parabola $\frac{x^2}{a^2} + \frac{y^2}{b^2 - a^2} = 1$

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68. Find the equation of the hyperbola whose directrix is $2x + y = 1$, focu$s(1, 2)$ and eccentricity $\sqrt{3}$.

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69. Find the coordinates of the foci, the eocentricity, the latus rectum, and the equations of directrices for the hyperbola $9x^2 - 16y^2 - 72x + 96y - 144 = 0$

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70. Prove that the tangent and normal at any point of an ellipse bisect the external and internal angles between the focal distances of the point

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71. If the normal at a pont P to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the x-axis at G, show that the $SG = eSP$. S being the focus of the hyperbola.

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72. Tangents are drawn to a hyperbola from a point on one of the branches of its conjugate hyperbola. Show that their chord of contact will touch the other branch of the conjugate hyperbola.

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73. If $(a\sec\theta; b\tan\theta)$ and $(a\sec\phi; b\tan\phi)$ are the ends of the focal chord of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ then prove that $\tan\left(\frac{x}{a}\right)\tan\left(\frac{\phi}{2}\right)=\frac{1-e}{1+e}$

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74. If α and β are two points on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and the chord joining these two points passes through the focus $(ae, 0)$ then $e\cos\left(\frac{\alpha-\beta}{2}\right)$

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75. If the chord joining the points $(a\sec\theta_1, b\tan\theta_1)$ and $(a\sec\theta_2, b\tan\theta_2)$ on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is a focal chord, then prove that $\tan\left(\frac{\theta_1}{2}\right)\tan\left(\frac{\theta_2}{2}\right)+\frac{ke-1}{ke+1}=0$, where $k= \pm 1$

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76. Find the locus of the-mid points of the chords of the circle $x^2+y^2=16$, which are tangent to the hyperbola $9x^2-16y^2=144$

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77. If a normal of slope m to the parabola $y^2=4ax$ touches the hyperbola $x^2-y^2=a^2$, then

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78. If $e and e'$ the eccentricities of a hyperbola and its conjugate, prove that $\frac{1}{e^2} + \frac{1}{e'^2} = 1.$

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79. If S ans S' are the foci, C is the center , and P is point on the rectangular hyperbola, show that $SP \times SP = (CP)^2$

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80. A ray emanating from the point (5, 0) is meident on the hyperbola $9x^2 - 16y^2 = 144$ at the point P with abscissa 8. Find the equation of the reflected ray after the first reflection if point P lies in the first quadrant.

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81. The normal at a point P on the ellipse $x^2+4y^2=16$ meets the x-axis at Q. If M is the midpoint of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at points. $\left(\pm\frac{(3\sqrt{5})}{2}\pm\frac{2}{7}\right)$ (b)

$\left(\pm\frac{(3\sqrt{5})}{2}\pm\frac{\sqrt{19}}{7}\right)$ $\left(\pm 2\sqrt{3},\ \pm\frac{1}{7}\right)$ (d)

$\left(\pm 2\sqrt{3}\pm\frac{4\sqrt{3}}{7}\right)$

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82. The line passing through the extremity A of the major exis and extremity B of the minor axis of the ellipse $x^2+9y^2=9$ meets is auxiliary circle at the point M. Then the area of the triangle with vertices at $A, M,$ and O (the origin) is 31/10 (b) 29/10 (c) 21/10 (d) 27/10

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83. Let $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec c\phi, b\tan\phi)$ (where $\theta+\phi=\frac{\pi}{2}$ be two points on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ If (h, k) is the point of intersection of the normals at P and Q then k is equal to (A) $\frac{a^2+b^2}{a}$ (B) $-\left(\frac{a^2+b^2}{a}\right)$ (C) $\frac{a^2+b^2}{b}$ (D) $-\left(\frac{a^2+b^2}{b}\right)$

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84. If $x=9$ is the chord of contact of the hyperbola $x^2-y^2=9$ then the equation of the corresponding pair of tangents is (A) $9x^2-8y^2+18x-9=0$ (B)

$9x^2-8y^2-18x+9=0$ (C) $9x^2-8y^2-18x-9=0$ (D)

9x^2-8y^2+18x+9=0`

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85. A hyperbola having the transverse axis of length $2\sin\theta$ is confocal with the ellipse $3x^2+4y^2=12$. Then its equation is

$x^2\cos ec^2\theta-y^2\sec^2\theta=1$ $x^2\sec^2\theta-y^2\cos ec^2\theta=1$

$x^2\sin^2\theta-y^2\cos^2\theta=1$ $x^2\cos^2\theta-y^2\sin^2\theta=1$

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86. Consider a branch of the hypebola $x^2-2y^2-2\sqrt{2}x-4\sqrt{2}y-6=0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is (A) $1-\sqrt{\frac{2}{3}}$ (B) $\sqrt{\frac{3}{2}}-1$ (C) $1+\sqrt{\frac{2}{3}}$ (D) $\sqrt{\frac{3}{2}}+1$

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87. Let $aandb$ be nonzero real numbers. Then the equation $\left(ax^2+by^2+c\right)\left(x^2-5xy+6y^2\right)=0$ represents. four straight lines, when $c=0$ and a, b are of the same sign. two straight lines and a circle, when $a=b$ and c is of sign opposite to that a two straight lines and a hyperbola, when $aandb$ are of the same sign and c is of sign opposite to that of a a circle and an ellipse, when $aandb$ are of the same sign and c is of sign opposite to that of a

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88. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be (A) $-\frac{1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

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89. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose:

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90. Let $P(x_1, y_1)$ and $Q(x_2, y_2), y_1 < 0, y_2 < 0,$ be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

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91. On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are $\left(\frac{2}{5}, \frac{1}{5}\right)$ (b) $\left(-\frac{2}{5}, \frac{1}{5}\right)$ $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ (d) $\left(\frac{2}{5}, -\frac{1}{5}\right)$

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92. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3), S(x_4, y_4)$, then : (A) $x_1 + x_2 + x_3 + x_4 = 0$ (B) $y_1 + y_2 + y_3 + y_4 = 0$ (C) $x_1x_2x_3x_4 = c^4$ (D) $y_1y_2y_3y_4 = c^4$

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93. An ellipse intersects the hyperbola $2x^2 - 2y = 1$ orthogonally. The eccentricity of the ellipse is reciprocal to that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then (b) the foci of ellipse are $(\pm 1, 0)$ (a) equation of ellipse is $x^2 + 2y^2 = 2$ (d) the foci of ellipse are $(t2, 0)$ (c) equation of ellipse is $(x^2 2y)$

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94. Statement 1 : The curve $y = -\frac{x^2}{2} + x + 1$ is symmetric with respect to the line $x = 1$. Statement 2: A parabola is symmetric about its axis (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not a correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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95. Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at point A and B. Q. The coordinates of A and B are

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96. Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B. The orthocenter of the triangle PAB is

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97. Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in first and 4th quadrant,respectively.

Tangents to the circle at P and Q intersect the x-axis at R and tangents at the parabola at P and Q intersect the x-axis at S.

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98. Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in first and 4th quadrant,respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents at the parabola at P and Q intersect the x-axis at S.

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99. Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in first and 4th quadrant,respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents at the parabola at P and Q intersect the x-axis at S.

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100. The line $2x+y=1$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

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Exercise

1. Find the equaiton of the parabola with its axis parallel to x-axis and which passes through the points $(1,2), (-1,3)$ and $(-2,1)$.

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2. Find the vertex, focus, axis, directrix and latus rectum of the following parabolas: (i) $(y-2)^2 = 3(x+1), (ii) y^2 + 4x + 4y - 3 = 0$

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3. If the normal at $\left(am^2, -2am\right)$ to the parabola $y^2 = 4ax$ intersects the parabola again at $\tan^{-1}\left|\frac{m}{k}\right|$ then find k.

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4. If the normals at P, Q, R of the parabola $y^2 = 4ax$ meet in O and S be its focus, then prove that . $SP.\ SQ.\ SR = a.\ (SO)^2$.

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5. Show that the equation of the chord of the parabola $y^2 = 4ax$ through the points (x_1, y_1) and (x_2, y_2) on it is : $(y - y_1)(y - y_2) = y^2 - 4ax$

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6. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

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7. Prove that any three tangents to a parabola whose slopes are in harmonic progression enclose a triangle of constant area.

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8. The Circumcircle of the triangle formed by any three tangents to a parabola passes through

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9. Two equal parabolas have the same vertex and their axes are at right angles. Prove that their common tangent touches each at the end of their latus recta.

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10. Two equal parabola have the same vertex and their axes are at right angles. Prove that they cut again at an angle $\frac{\tan^{-1} 3}{4}$.

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11. A tangent to the parabola $y^2 = 8x$ makes an angle of 45^0 with the straight line $y = 3x + 5$. Then find one of the points of contact.

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12. The equation of the common tangent to the parabolas $y^2 = 4ax$and $x^2 = 4by$ is given by

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13. Find the locus of a point which is such that, the three normals through it cut the axis in points whose distance from the vertex are in A.P.

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14. Let P,Q,R be three points on a parabola, normals at which are concurrent, the centroid of ΔPQR must lie on

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15. The centroid of the triangle formed by the feet of three normals to the parabola $y^2 = 4ax$

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16. Find the locus of the point of intersection of two mutually perpendicular normals to the parabola $y^2 = 4ax$ and show that the abscissa of the point can never be smaller than 3a. What is the ordinate

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17. Prove that the circle described on the focal chord as a diameter touches the directrix

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18. Prove that the tangent at one extremity of the focal chord of a parabola is parallel to the normal at the other extremity.

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19. Find the locus of the middle points of the chords of the parabola $y^2 = 4ax$ which subtend a right angle at the vertex of the parabola.

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20. Show that all chords of a parabola which subtend a right angle at the vertex pass through a fixed point on the axis of the curve.

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21. Show that the equation of the circle described on the chord intercepted by the parabola $y^2 = 4ax$ on the line $y = \max + c$ as diameter is $m^2(x^2 + y^2) + 2x(mc - 2a) - 4amy + 4amc + c^2 = 0.$

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22. Tangent is drawn at any point (x_1, y_1) other than the vertex on the parabola $y^2 = 4ax$. If tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of contact pass through a fixed point (x_2, y_2), then $x_1 a, x_2$ in GP (b) $\frac{y_1}{2}, a, y_2$ are in GP $-4, \frac{y_1}{y_2}, x_1 / x_2$ are in GP (d) $x_1 x_2 + y_1 y_2 = a^2$

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23. prove that for a suitable point P on the axis of the parabola, chord AB through the point P can be drawn such that $\left[\left(\frac{1}{AP^2}\right) + \left(\frac{1}{BP^2}\right)\right]$ is same for all positions of the chord.

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24. P & Q are the points of contact of the tangents drawn from the point T to the parabola $y^2 = 4ax$. If PQ be the normal to the parabola at P, prove that TP is bisected by the directrix.

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25. A normal chord of the parabola $y^2 = 4ax$ subtends a right angle at the vertex if its slope is

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26. If P, Q, R are three points on a parabola $y^2 = 4ax$ whose ordinates are in geometrical progression, then the tangents at P and R meet on :

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27. The normals at P, Q, R on the parabola $y^2 = 4ax$ meet in a point on the line $y = c$. Prove that the sides of the triangle PQR touch the parabola $x^2 = 2cy$.

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28. The normals at P,Q and R are concurrent and PQ meets the diameter through R on the directrix x=-a . Prove that PQ touches [or PQ enveleopes] the parabola $y^2+16a(x+a)=0.$

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29. prove that the locus of the point of intersection of the tangents at the extremities of any chord of the parabola $y^2=4ax$ which subtends a right angle at the vertes is $x+4a=0.$

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30. Locus of foot of perpendicular from focus upon any tangent is tangent at vertex OR Image of focus in any tangent lies in Directrix

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31. if the locus of inter section of tangents to the parabola $y^2 = 4ax$ which intercept a fix length 'l' on the directrix is $(y^2 - \lambda ax)(x + a)^2 = l^2x^2$ then find the value of λ

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32. If the line y=mx+c touches the parabola $y^2 = 4a(x + a)$, then

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33. Two lines are drawn at right angles, one being a tangent to $y^2 = 4ax$ and the other $x^2 = 4by$. Then find the locus of their point of intersection.

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34. Find the locus of a point which is such that, Two of the normals drawn from it to the parabola $y^2 = 4ax$ are at right angle.

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35. If a chord PQ of the parabola $y^2 = 4ax$ subtends a right angle at the vertex, show that the locus of the point of intersection of the normals at P and Q is $y^2 = 16a(x - 6a)$.

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36. If tangent at P and Q to the parabola $y^2 = 4ax$ intersect at R then prove that mid point the parabola, where M is the mid point of P and Q.

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37. The locus of the point of intersection of those normals to the parabola $x^2 = 8y$ which are at right angles to each other, is a parabola. Which of the following hold (s) good in respect of the loucus ?

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38. The locus of the point of intersection of tangents drawn at the extremities of a normal chord to the parabola $y^2 = 4ax$ is the curve

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39. If $A\left(at_1^2, 2at_1\right)$ and $B\left(t_2^2, 2at_2\right)$ be the two points on the parabola $y^2 = 4ax$ and AB cuts the x-axis at C such that $AB : AC = 3 : 1$. show that $t_2 = -2t_1$ and if AB makes an angle of 90° at the vertex, then find the coordinates of A and B.

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40. Show that the locus of the middle point of all chords of the parabola $y^2 = 4ax$ passing through a fixed point (h, k) is $y^2 - ky = 2a(x - h)$.

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41. If on a given base, a triangle be described such that the sum of the tangents of the base angles is a constant, then the locus of the vertex is: (A) a circle (Bi a parabola (C) an ellipse (D) a hyperbola

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42. The locus of the middle points of the focal chord of the parabola y^2ax, is

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43. Prove that the locus of the circumcentre of the variable triangle having sides y-axis, y = 2 and lx + my = 1 where (l, m) lies on the parabola $y^2 = 4ax$, is also a parabola

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44. Prove that the portion of the tangent to an ellipse intercepted between the ellipse and the directrix subtends a right angle at the corresponding focus.

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45. Equilateral traingles are circumscribed to the parabola $y^2 = 4ax$. Prove that their angular points lie on the conic $(3x + a)(x + 3a) = y^2$

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46. If the tangents at the points P and Q on the parabola $y^2 = 4ax$ meet at R and S is its focus, prove that $SR^2 = SP. SQ$.

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47. The line $lx + my + n = 0$ is a normal to the parabola $y^2 = 4ax$ if

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48. The line $lx + my + n = 0$ is a normal to the parabola $y^2 = 4ax$ if

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49. Statement :1 If a parabola $y^2 = 4ax$ intersects a circle in three co-normal points then the circle also passes through the vertr of the parabola. Because

Statement : 2 If the parabola intersects circle in four points t_1, t_2, t_3 and t_4 then $t_1 + t_2 + t_3 + t_4 = 0$ and for co-normal points t_1, t_2, t_3 we have $t_1 + t_2 + t_3 = 0$.

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50. Find the equation to the ellipse whose one focus is $(2, 1)$, the directrix is $2x - y + 3 = 0$ and the eccentricity is $\frac{1}{\sqrt{2}}$.

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51. The eccentricity of an ellipse is $\frac{1}{2}$ and the distance between its foci is 4 units. If the major and minor axes of the ellipse are respectively along the x and y axes, find the equation of the ellipse.

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52. Find the equation of the ellipse, referred to its axes as the x, y axes respectively, which passes through the point $(-3, 1)$ and has the eccentricity $\sqrt{\frac{2}{5}}$.

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53. Find the latus rectum, eccentricity, coordinates of the foci and the length of axes of the ellipse

$4x^2 + 9y^2 - 8x - 36y + 4 = 0.$

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54. Find the eccentric angle of a point on the ellipse $x^2 + 3y^2 = 6$ at a distance 2 units from the centre of the ellipse.

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55. If the angle between the lines joining the foci of any ellipse to an extremity of the minor axis is 90^0, find the eccentricity. Also find the equation of the ellipse if the major axis is $2\sqrt{2}$.

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56. The line x cos $\alpha + y \sin \alpha + y \sin \alpha = p$ is tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. if

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57. The distance of point on the ellipse $x^2 + 3y^2 = 6$ from the centre is 2. Find the eccentric angle of the point in the first quadrant. Also find the equation of the tangent at the point.

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58. Show that the line $y = x + \sqrt{\dfrac{5}{6}}$ touches the ellipse $2x^2 + 3y^2 = 1$. Find the coordinates of the point of contact.

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59. If a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose centre is C, meets the major and the minor axes at P and Q respectively then $\frac{a^2}{CP^2}+\frac{b^2}{CQ^2}$ is equal to

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60. Find the locus of the foot of the perpendicular drawn from the center upon any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$

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61. If α and β are eccentric angles of the ends of a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\frac{\tan\alpha}{2}.\frac{\tan\beta}{2}$ is (A) $\frac{1-e}{1+e}$ (B) $\frac{e+1}{e-1}$ (C) $\frac{e-1}{e+1}$ (D) none of these

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62. A point on the ellipse $x^2+3y^2=37$ where the normal is parallel to the line $6x-5y=2$ is $(5,\ -2)$ (b) (5, 2) (c) $(-5, 2)$ (d) $(-5,\ -2)$

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63. Find the coordinates of those points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, tangents at which make equal angles with the axes. Also prove that the length of the perpendicular from the centre on either of these is $\sqrt{\left(\frac{1}{2}\right)(a^2+b^2)}$

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64. A tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touches at the point P on it in the first quadrant & meets the coordinate axes in A & B respectively. If P divides AB in the ratio $3 : 1$ reckoning from the x-axis find the equation of the tangent.

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65. Show that each pair of tangents at the ends of the latera recta of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the foot of the correspoinding directrix.

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66. The locus of the poles of normal chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is

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67. A straight line PQ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = r^2 (b < r < a)$. RS is a focal chord of the ellipse. If RS is parallel to PQ and meets the circle at points R and S. Find the length of RS.

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68. The normal at a point $P(\theta)$ on the ellipse $5x^2 + 14y^2 = 70$ cuts the curve again at a point $Q(2\theta)$ then $\cos\theta$Î¸

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69. The tangent and normal at any point P of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cut its major axis in point Q and R respectively. If QR=a prove that the eccentric angle of the point P is given by $e^2\cos^2\phi+\cos\phi-1=0$

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70. Length of the focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which is inclined to the major axis at angle θ is

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71. Find the length of the chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose middle point is $\left(\frac{1}{2},\frac{2}{5}\right)$.

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72. Find the eccentricity of the ellipse if $y = x$ and $3y + 2x = 0$ are the equations of a pair of its conjugate diameters.

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73. Find the eccentric angles of the extremities of the latus recta of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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74. P and Q are corresponding points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the auxiliary circle respectively . The normal at P to the elliopse meets CQ at R. where C is the centre of the ellipse Prove that CR = a +b

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75. If the normal at an end of a latus rectaum of an ellipse passes through an extremity of the minor axis then the eccentricity of the ellispe satisfies .

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76. Prove that if any tangent to the ellipse is cut by the tangents at the endpoints of the major axis at $TandT'$, then the circle whose diameter is $\top$ ' will pass through the foci of the ellipse.

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77. Prove that the chords of contact of pairs of perpendicular tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touch another fixed

ellipse.

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78. Q. Show that the feet of normals from a point (h, K) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ lie on a conic which passes through the origin & the point (h, k)

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79. The locus of the point of intersection of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the points whose eccentric angles differ by $\pi / 2$, is

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80. The tangent at any point P on $y^2 = 4x$ meets x-axis at Q, then locus of mid point of PQ will be

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81. Any ordinate MP of an ellipse meets the auxillary circle in Q. Ptove that the locus of the point of intersection of the normals at P and Q is the circle $x^2 + y^2 = (a^2 + b^2)^2$.

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82. The locus of the poles of normal chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is

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83. The locus of the poles of normal chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is

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84. A line of fixed length $a + b$ moves so that its ends are always on two fixed perpendicular straight lines. Then the locus of the point which divides this line into portions of length $a and b$ is a/an ellipse (b) parabola straight line (d) none of these

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85. A point moves so that the sum of the squares of its distances from two intersecting straight lines is constant. Prove that its locus is an ellipse.

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86. Q. Show that the feet of normals from a point (h, K) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ lie on a conic which passes through the origin & the point (h, k)

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87. The distance of a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ from the centre is 2. Find the eccentric angle of the point.

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88. Referred to the principal axes as the axes of coordinates find the equation of the hyperbola whose foci are at $\left(0, \pm\sqrt{10}\right)$ and which passes through the point $(2, 3)$.

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89. Find the equaiton of the hyperbola with vertices at $(0, \pm 6)$ and eccentricity $\frac{5}{3}$.

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90. Show that the equation $9x^2 - 16y^2 - 18x - 64y - 199 = 0$ represents a hyperbola. For this hyperbola, find the length of axes, eccentricity, centre, foci, vertices, latus rectum and directrices.

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91. An ellipse has eccentricity $\frac{1}{2}$ and one focus at the point $P\left(\frac{1}{2}, 1\right)$. Its one directrix is the comionand tangent nearer to the point the P to the hyperbolaof $x^2 - y^2 = 1$ and the circle $x^2 + y^2 = 1$.Find the equation of the ellipse.

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92. If the line $y = mx + \sqrt{a^2m^2 - b^2}$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(a \sec \phi, b \tan \phi)$, show that $\phi = \sin^{-1}\left(\frac{b}{a}m\right)$.

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93. Find the equations of the tangents to the hyperbola $4x^2 - 9y^2 = 36$ which are parallel to the line $5x - 3y = 2$.

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94. Find the equaiton of the tangents to the hyperbola $x^2 - 2y^2 = 18$ which are perpendicular to the line $x - y = 0$.

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95. Prove that the product of the perpendiculars from the foci upon any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is b^2

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96. Find the equations to the common tangents to the two hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

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97. Normal are drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at point $\theta_1 andth\eta_2$ meeting the conjugate axis at $G_1 and G_2$, respectively. If $\theta_1 + \theta_2 = \frac{\pi}{2}$, prove that $CG_1 \dot{C} G_2 = \frac{a^2 e^4}{e^2 - 1}$, where C is the center of the hyperbola and e is the eccentricity.

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98. If the chord through the points $(a \sec \theta, b \tan \theta)$ and $(a \sec \phi, b \tan \phi)$ on the hyperbola $\frac{x^2}{a^\circ} - \frac{y^2}{b^2} = 1$ passes through a focus, prove that $\frac{\tan \theta}{2} \frac{\tan \phi}{2} + \frac{e-1}{e+1} = 0.$

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99. Show that all the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 - 2x + 4y = 0$? If yes, what is the point of concurrency and if not, give reasons.

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100. If the normal to the rectangular hyperbola $xy = c^2$ at the point $'t'$ meets the curve again at t_1 then $t^3 t_1$, has the value equal to

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101. Find the equation of the hyperbola which has $3x - 4y + 7 = 0$ and $4x + 3y + 1 = 0$ as its asymptotes and which passes through the origin.

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102. The asymptotes of a hyperbola are parallel to lines $2x + 3y = 0$ and $3x + 2y = 0$. The hyperbola has its centre at (1, 2) and it passes through (5, 3). Find its equation.

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103. For the curve $xy = c$, prove that the portion of the tangent intercepted between the coordinate axes is bisected at the point of contact.

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104. Show that the locus of the foot of the perpendicular drawn from focus to a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $x^2 + y^2 = a^2$.

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105. Show that the locus of the foot of the perpendicular drawn from focus to a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $x^2 + y^2 = a^2$.

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106. Show that the locus of the foot of the perpendicular drawn from focus to a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $x^2 + y^2 = a^2$.

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107. The locus of a point from which two tangent are drawn to $x^2 - y^2 = a^2$ which are inclined at angle $\frac{\pi}{4}$ to each other is

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108. A normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the axes in M and N and lines MP and NP are drawn perpendicular to the axes meeting at P. Prove that the locus of P is the hyperbola $a^2x^2 - b^2y^2 = (a^2 + b^2)^2$

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109. Prove that the locus of the point of intersection of the tangents at the ends of the normal chords of the hyperbola $x^2 - y^2 = a^2$ is $a^2(y^2 - x^2) = 4x^2y^2$.

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110. Chords of the hyperbola, $x^2 - y^2 = a^2$ touch the parabola, $y^2 = 4ax$. Prove that the locus of their middlepoints is the curve, $y^2(x - a) = x^3$.

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111. if the chord of contact of tangents from a point P to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ subtends a right angle at the centre, then the locus of P is

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112. Prove that the locus of the middle-points of the chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$which pass through a fixed point (α, β) is a hyperbola whose centre is $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$.

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113. Tangents are drawn from the points on a tangent of the hyperbola $x^2 - y^2 = a^2$ to the parabola $y^2 = 4ax$. If all the chords of contact pass through a fixed point Q, prove that the locus of the point Q for different tangents on the hyperbola is an ellipse.

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114. Tangents are drawn from point P on the curve $x^2 - 4y^2 = 4$ to the curve $x^2 + 4y^2 = 4$ touching it in the points Q and R . Prove that the mid -point of QR lies on $\frac{x^2}{4} - y^2 = \left(\frac{x^2}{4} + y^2\right)^2$

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115. If a hyperbola be rectangular, and its equation be $xy = c^2$, prove that the locus of the middle points of chords of constant length 2d is $(x^2 + y^2)(xy - c^2) = d^2xy$.

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116. Let P be a point on the hyperbola $x^2 - y^2 = a^2$, where a is a parameter such that P is nearest to the line $y = 2x$. Show that the locus of P is $2y - x = 0$.

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117. If two points $P\&Q$ on the hyperbola ,$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ whose centre is C be such that CP is perpendicularal to CQ and $a < b1$,then prove that $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} - \frac{1}{b^2}$.

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118. If C is the centre of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, S, S its foci and P a point on it. Prove tha SP. S'P $= CP^2 - a^2 + b^2$

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119. Find the point on the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$ which is nearest to the line $3x + 2y + 1 = 0$ and compute the distance between the point and the line.

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120. Given the base of a triangle and the ratio of the tangent of half the base angles .Show that the vertex moves on a hyperbola whose foci are the extremities of a diameter

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121. Show that the area formed by the normals to $y^2 = 4ax$ at the points t_1, t_2, t_3 is

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122. The angle of intersection between the curves, $y = x^2$ and $y^2 = 4x$, at the point (0, 0) is (A) $\frac{\pi}{2}$ (B) 0 (C) π (D) none of these

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123. A particle moves on the parabola $y^2 = 4ax$. Its distance from the focus is minimum for the following values of x (A) -1 (B) 0 (C) 1 (D) a

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124. The line $x - y + 2 = 0$ touches the parabola $y^2 = 8x$ at the point (A) $(2, -4)$ (B) $\left(1, 2\sqrt{2}\right)$ (C) $\left(4, -4\sqrt{2}\right.$ (D) $(2, 4)$

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125. If the tangents at the points P and Q on the parabola $y^2 = 4ax$ meet at R and S is its focus, prove that $SR^2 = SP. SQ.$

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126. The normal at the point $P\left(ap^2, 2ap\right)$ meets the parabola $y^2 = 4ax$ again at $Q\left(aq^2, 2aq\right)$ such that the lines joining the origin to P and Q are at right angle. Then (A) $p^2 = 2$ (B) $q^2 = 2$ (C) $p = 2q$ (D) $q = 2p$

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127. Length of the shortest normal chord of the parabola $y^2 = 4ax$ is

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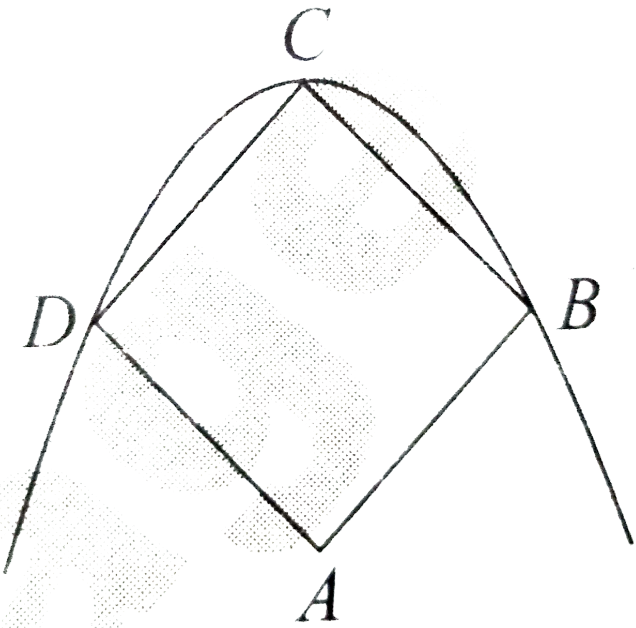
128. In the adjacent figure a parabola is drawn to pass through the vertices B,C and D of the square ABCD. If $A(2, 1), C(2, 3)$, then focus of this parabola is



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129. If the parabola $y^2 = 4ax$ passes through (3, 2). Then the length of its latusrectum, is

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130. The point of intersection of the tangents at the ends of the latus rectum of the parabola $y^2 = 4x$ is___________

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131. The line $y = mx + 1$ is a tangent to the parabola $y^2 = 4x$ if

(A) $m = 1$ (B) $m = 2$ (C) $m = 4$ (D) $m = 3$

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132. Consider the parabola $y^2 = 8x$, if the normal at a point P on the parabola meets it again at a point Q, then the least distance of Q from the tangent at the vertex of the parabola is

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133. The point on the parabola $x^2 = y + 2$, where tangent is perpendicular to the chord joining $A \equiv (14, 2)$ and $B \equiv (2, 4)$

(A) $(-3, 7)$ (B) $(3, 7)$ (C) $(4, 14)$ (D) $(2, 2)$

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134. The focal distance of a point on the parabola $y^2 = 4x$, and above its axis, is 10 units. Its coordinates are : (A) $(9, 6)$ (B) $(25, 10)$ (C) $(25, -10)$ (D) none of these

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135. P is a variable point on the curve y= f(x) and A is a fixed point in the plane not lying on the curve. If PA^2 is minimum, then the angle between PA and the tangent at P is

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136. Two parabolas $y^2 = 4a(x - \lambda_1)$ and $x^2 = 4a(y - \lambda_2)$ always touch each other (λ_1, λ_2 being variable parameters). Then their point of contact lies on a

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137. If the parabola $y = (a - b)x^2 + (b - c)x + (c - a)$ touches x- axis then the line $ax + by + c = 0$ passes through a fixed point

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138. If a normal chord at a point on the parabola $y^2 = 4ax$ subtends a right angle at the vertex, then t equals

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139. The number of chords drawn form point (a, a) on the circle $x^2 + y^2 = 2a^2$, which are bisecte by the parabola $y^2 = 4ax$, is

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140. Find the point on the axis of the parabola $3y^2 + 4y - 6x + 8 = 0$ from when three distinct normals can be drawn.

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141. The number of normals drawn from the point $(6, -8)$ to the parabola $y^2 - 12y - 4x + 4 = 0$ is

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142. The straight line $lx + my + n = 0$ will touch the parabola $y^2 = 4px$ if

(A) $lm^2 = np$

(B) $mn = pl^2$

(C) $pn^2 = lm$

(D) none of these

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143. If the parabolas $y^2 = 4ax$ and $y^2 = 4c(x-b)$ have a common normal other than the x-axis $(a, b, c$ being distinct positive real numbers), then prove that $\frac{b}{a-c} > 2.$

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144. A ray of light moving parallel to the X-axis gets reflected from a parabolic mirror whose equation is $(y-2)^2 = 4(x+1)$. After reflection , the ray must pass through the point

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145. If the chord of contact of tangents from a point P to the parabola $y^2 = 4ax$ touches the parabola $x^2 = 4by$, then find the locus of P.

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146. Thangents are drawn from $(-1, 0)$ to $y^2 = 4x$. Then the area of the triangle fromed by the tangents and the chord of the contact is : (A) 8 sq. units (B) 2 sq. units (C) 4 sq. units (D) 1 sq.units

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147. If $(a^2, a-2)$ be a point interior to the region of the parabola $y^2 = 2x$ bounded by the chord joining the points $(2, 2)$ and $(8, -4)$, then the set of all possible real values of a is

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148. If the line $y - \sqrt{3}x + 3 = 0$ cut the parabola $y^2 = x + 2$ at P and Q , then $AP\dot{A}Q$ is equal to [where $A = (\sqrt{3}, o)$]

$\frac{2(\sqrt{3}+2)}{3}$ (b) $\frac{4\sqrt{3}}{2}$ $\frac{4(2-\sqrt{2})}{3}$ (d) $\frac{4(\sqrt{3}+2)}{3}$

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149. The normal at any point $P(t^2, 2t)$ on the parabola $y^2 = 4x$ meets the curve again at Q, then the $ar(\triangle POQ)$ in m the form of $\frac{k}{|t|}(1+t^2)(2+t^2)$.

the value of k is

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150. Tangents and normal drawn to the parabola $y^2 = 4ax$ at point $P(at^2, 2at), t \neq 0,$ meet the x-axis at point $TandN,$ respectively. If S is the focus of theparabola, then $SP = ST \neq SN$ (b) $SP \neq ST = SN$ $SP = ST = SN$ (d) $SP \neq ST \neq SN$

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151. PQ is chord of contract of tangents from point T to a parabola. If PQ is normal at P, if dirctric divides PT in the ratio K : 2019 then K is ________

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152. If a chord which is normal to the parabola at one end subtend a right angle at the vertex, then angle to the axis is

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153. If the normals drawn at the end points of a variable chord PQ of the parabola $y^2 = 4ax$ intersect at parabola, then the locus of the point of intersection of the tangent drawn at the points P and Q is

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154. Find the area of the triangle whose vertices are : $(at^2\_1, 2at_1), (at^2\_2, 2at_2), (at^2\_3, 2at_3)$

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155. If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right) and g\left(\frac{5}{4} = 1,\right.$ then $(gof)(x)$ is ___________

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156. The coordinates of a point on the parabola $y^2 = 8x$ whose distance from the circle $x^2 + (y+6)^2 = 1$ is minimum is $(2, 4)$

(b) $(2, -4)$ $(18, -12)$ (d) $(8, 8)$

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157. If the normals to the parabola $y^2 = 4ax$ at three points $(ap^2, 2ap)$, and $(aq^2, 2aq)$ are concurrent, then the common root of equations $Px^2 + qx + r = 0$ and $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ is p (b) q (c) r (d) 1

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158. The point on the line $x - y + 2 = 0$ from which the tangent to the parabola $y^2 = 8x$ is perpendicular to the given line is (a, b), then the line $ax + by + c = 0$ is

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159. The equaiton $\frac{x^2}{10-a}+\frac{y^2}{4-a}=1$ represent an ellipse, if

(A) $a>10$ (B) $a>4$ (C) $4<a<10$ (D) $a<4$

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160. If the tangents drawn through the point $(1, 2\sqrt{3}$ to the ellipse $\frac{x^2}{9}+\frac{y^2}{b^2}=1$ are at right angles, then the value of b is

(A) 1 (B) -1 (C) 2 (D) 4

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161. The equation of the ellipse whose focus is $(1, -1)$, directrix is the line $x-y-3=0$ and eccentricity is $\frac{1}{2}$, is: (A) $7x^2+2xy+7y^2+7=0$ (B)

$7x^2 + 2xy + 7y^2 + 10x - 10y - 7 = 0$ (C)

$7x^2 + 2xy + 7y^2 - 10x + 10y + 7 = 0$ (D) none of these

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162. Let P be a variable point on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with foci at S and S'. If A be the area of triangle PSS', then the maximum value of A is : (A) 12 sq. untis (B) 24 sq. units (C) 36 sq. units (D) none of these

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163. The values of 'm' for which a line with slope m is common tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and parabola $y^2 = 4ax$ can lie in interval:

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164. If the equation of the ellipse is $3x^2 + 2y^2 + 6x - 8y + 5 = 0$, then which of the following is/are true? $e = \frac{1}{\sqrt{3}}$ Center is $(-1, 2)\cdot$ Foci are $(-1, 1) and (-1, 3)\cdot$ Directrices are $y = 2 \pm \sqrt{3}$

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165. A man running around a race course notes that the sum of the distances of two flagposts from him a always 10m and the distance between the flag posts is 8m. Then the area of the path he encloses in square meters is 15π (b) 20π (c) 27π (d) 30π

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166. The number of real tangents through $(3, 5) \rightarrow 3x^2 + 5y^2 = 32$, is/are (A) 4 (B) 2 (C) 1 (D) 0

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167. The locus of point of intersection of tangents to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at two points the sum of whose eccentric angles is constant is

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168. The eccentricity of the ellipse which meets the straight line $\frac{x}{7} + \frac{y}{2} = 1$ on the x-axis and $\frac{x}{3} - \frac{y}{5} = 1$ on the y-axis and whos axis lie along the axes of coordinates is

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169. The number of maximum normals that can be drawn from any point to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is

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170. The angle between the normals of ellipse $4x^2 + y^2 = 5$, at the intersection of $2x + y = 3$ and the ellipse is (A) $\tan^{-1}\left(\frac{3}{5}\right)$ (B) $\tan^{-1}\left(\frac{3}{4}\right)$ (C) $\tan^{-1}\left(\frac{4}{3}\right)$ (D) $\tan^{-1}\left(\frac{4}{5}\right)$

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171. The equation of the tangents to the ellipse $4x^2 + 3y^2 = 5$, which are inclined at 60^0 to the X-axis are : (A) $y = \frac{x}{\sqrt{3}} \pm \sqrt{\frac{65}{12}}$ (B) $y = \sqrt{3} \pm \sqrt{\frac{65}{12}}$ (C) $y = \sqrt{3}x \pm \sqrt{\frac{12}{65}}$ (D) none of these

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172. If the tangent and normal to $xy = c^2$ at a given point on it cut off intercepts a_1, a_2 on one axis and b_1, b_2 on the other axis, then $a_1a_2 + b_1b_2 =$ (A) -1 (B) 1 (C) 0 (D) $a_1a_2b_1b_2$

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173. If from a point P, tangents $PQandPR$ are drawn to the ellipse $\frac{x^2}{2} + y^2 = 1$ so that the equation of QR is $x + 3y = 1$, then find the coordinates of P.

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174. The normal at an end of a latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through an end of the minor axis if (A)

$e^4 + e^2 = 1$ (B) $e^3 + e^2 = 1$ (C) $e^2 + e = 1$ (D) $e^3 + e = 1$

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175. If the tangent at (h, k) on $b^2x^2 - a^2y^2 = a^2b^2$ cuts the auxiliary circle in two points whose ordinates are y_1 and y_2, then $\frac{1}{y_1} + \frac{1}{y_2}$ is

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176. The eccentricity of an ellipse whose pair of a conjugate diameter are $y = x$ and $3y = -2x$ is (A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) none

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177. If α and β are eccentric angles of the ends of a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\frac{\tan\alpha}{2}.\frac{\tan\beta}{2}$ is (A) $\frac{1-e}{1+e}$ (B) $\frac{e+1}{e-1}$ (C) $\frac{e-1}{e+1}$ (D) none of these

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178. If on a given base, a triangle be described such that the sum of the tangents of the base angles is a constant, then the locus of the vertex is: (A) a circle (Bi a parabola (C) an ellipse (D) a hyperbola

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179. Two concentric hyperbolas, whose axes meet at angle of 45°, cut

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180. The eccentric angle of a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ whose distance from the centre of the ellipse is 2, is (A) $\frac{\pi}{4}$ (B) $7\frac{\pi}{6}$ (C) $3\frac{\pi}{2}$ (D) $5\frac{\pi}{3}$

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181. Find the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose latus rectum is half of its major axis.

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182. Tangents are drawn from point P on the curve $x^2 - 4y^2 = 4$ to the curve $x^2 + 4y^2 = 4$ touching it in the points Q and R .

Prove that the mid -point of QR lies on $\frac{x^2}{4} - y^2 = \left(\frac{x^2}{4} + y^2\right)^2$

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183. The equation of the chord of the ellipse $2x^2 + 5y^2 = 20$ which is bisected at the point $(2, 1)$ is

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184. A normal inclined at 45° to the axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is drawn. It meets the x-axis & the y-axis in P & Q respectively. If C is the centre of the ellipse, show that the area of triangle CPQ is $\frac{\left(a^2 - b^2\right)^2}{2(a^2 + b^2)}$ sq units

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185. An ellipse is described by using an endless string which is passed over two pins. If the axes are $6cm$ and $4cm$, the length of the string and distance between the pins are ....

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186. If a tangent having a slope of $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ intersects the major and minor axes in points A and B respectively, then the area of ΔOAB is equal to (A) 12 sq. untis (B) 24 sq. units (C) 48 sq. units (D) 64 sq. units

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187. A set of points is such that each point is three times as far away from the y-axis as it is from the point (4,0).Then locus of the points is:

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188. Length of latus rectum of the ellipse $2x^2 + y^2 - 8x + 2y + 7 = 0$ is (A) 8 (B) 4 (C) 2 (D) $\sqrt{2}$

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189. The locus of the middle points of the portions of the tangents of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ included between the axis is the curve

(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{4}$

(b) $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$

(c) $a^2x^2 + b^2y^2 = 4$

(d) $b^2x^2 + a^2y^2 = 4$

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190. The area bounded by the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ is

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191. If C is the center of the ellipse $9x^2 + 16y^2 = 144$ and S is a focus, then find the ratio of CS to the semi-major axis.

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192. If the tangent and normal to the ellipse $x^2 + 4y^2 = 4$ at point $P(\theta)$ meets the major axes in Q and R respectively, and $QR = 3$, then (A) $\cos\theta = \frac{1}{\sqrt{3}}$ (B) $\cos\theta = \frac{1}{3}$ (C) $\cos\theta = \frac{2}{3}$ (D) $\cos\theta = \left(-\frac{2}{3}\right.$

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193. iger $P(\theta)$ and $Q\left(\frac{\pi}{2}+\theta\right)$ are two points on the ellipse $\frac{x^2}{a^2}+\frac{y^{\circ}}{b^2}=1$, locus ofmid point of PQ is

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194. Prove that if any tangent to the ellipse is cut by the tangents at the endpoints of the major axis at $T and T'$, then the circle whose diameter is $\top$ ' will pass through the foci of the ellipse.

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195. At the point of intersection of the curves $y^2=4ax$ and $xy=c^2$, the tangents to the two curves make angles α and β respectively with x-axis. Then $\tan\alpha\cot\beta=$

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196. The eccentricity of ellipse $ax^2 + by^2 + 2gx + 2fy + c = 0$ if its axis is parallel to x-axis is (A) $\sqrt{\frac{a+b}{4}}$ (B) $\sqrt{\frac{a-b}{2}}$ (C) $\sqrt{\frac{b-a}{a}}$ (D) $\sqrt{\frac{b-a}{b}}$

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197. If e is the eccentricity of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and θ be the angle between its asymptotes, then $\cos\left(\frac{\theta}{2}\right)$ is equal to

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198. Two perpendicular tangents drawn to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ intersect on the curve.

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199. If e_1 and e_2 be the eccentricities of hyperbola and its conjugate, then $\frac{1}{e^2}{}_{-}1+\frac{1}{e^2}{}_{-}2=$ (A) $\frac{\sqrt{2}}{8}$ (B) $\frac{1}{4}$ (C) 1 (D) 4

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200. If the normal at any point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cuts the major and minor axes in L and M respectively and if C is the centre, then $a^2CL^2+b^2CM^2=$ (A) $(a-b)$ (B) (a^2-b^2) (C) $(a+b)$ (D) (a^2+b^2)

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201. Coordinates of the vertices B and C of ΔABC are $(2,0)$ and $(8,0)$ respectively. The vertex is hanging in such a way that $4\frac{\tan B}{2}.\frac{\tan C}{2}=1$. Then the locus of A is (A) $\frac{(x-5)^2}{25}+\frac{y^2}{9}=1$ (B) $\frac{(x-5)^2}{25}+\frac{y^2}{16}=1$ (C) $\frac{(c-5)^2}{16}+\frac{y^2}{25}=1$ (D) none of these

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202. P and Q are corresponding points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the auxiliary circle respectively . The normal at P to the elliopse meets CQ at R. where C is the centre of the ellipse Prove that CR = a +b

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203. The length of perpendicular from the centre to any tangent of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which makes equal angles with the axes, is : (A) a^2 (B) b^2 (C) $\left.\frac{\sqrt{a^2-b^2}}{2}\right)$ (D) $\sqrt{\frac{a^2+b^2}{2}}$

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204. A tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ cuts the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $PandQ$. Show that the locus of the midpoint of PQ is $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=\frac{x^2}{a^2}-\frac{y^2}{b^2}$.

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205. If on a given base, a triangle be described such that the sum of the tangents of the base angles is a constant, then the locus of the vertex is: (A) a circle (Bi a parabola (C) an ellipse (D) a hyperbola

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206. The coordinates of a point on the rectangular hyperbola $xy = c^2$ normal at which passes through the centre of the hyperbola are

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207. Tangents are drawn from the points on the line $x - y - 5 = 0$ to $x^2 + 4y^2 = 4$. Then all the chords of contact pass through a fixed point. Find the coordinates.

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208. The distance from the foci of $P(a, b)$ on the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ are

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209. In an ellipse the distance between the foci is 8 and the distance between the directrices is 25. The length of major axis is : (A) $5\sqrt{2}$ (B) $10\sqrt{2}$ (C) $20\sqrt{2}$ (D) none of these

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210. If a hyperbola be rectangular, and its equation be $xy = c^2$, prove that the locus of the middle points of chords of constant length 2d is $\left(x^2 + y^2\right)\left(xy - c^2\right) = d^2xy.$

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211. 2. A circle, with its centre at the focus of the parabola y? - 4ax and touching its directrix, intersects the parabola at the point

(A) (a, 2a) (B) (a,-2a) (D) a/2a 2a

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212. Through a point $P(-2, 0)$, tangerts PQ and PR are drawn to the parabola $y^2 = 8x$. Two circles each passing through the focus of the parabola and one touching at Q and other at R are drawn. Which of the following point(s) with respect to the triangle PQR lie(s) on the radical axis of the two circles?

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213. Two mutually perpendicular chords OA and OB are drawn through the vertex 'O' of a parabola $y^2 = 4ax$. Then find the locus of the circumcentre of triangle OAB.

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214. Extremities of the latera recta of the ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a > b)$ having a given major axis 2a lies on

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215. Find the number of rational points on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1.$

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216. Show that the locus of the point of intersection of mutually perpendicular tangetns to a parabola is its directrix.

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217. If $x = \dfrac{e^t + e^{-t}}{2}, y = \dfrac{e^t - e^{-t}}{2}$, then: $\dfrac{dy}{dx} =$

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218. Consider the ellipse $\dfrac{x^2}{\tan^2 \alpha} + \dfrac{y^2}{\sec^2 \alpha} = 1$ where $\alpha \in \left(0, \dfrac{\pi}{2}\right)$. Which of the following quantities would vary as α varies?

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219. The equations (s) to common tangent (s) to the two hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$ is /are

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220. The mirror image of the parabola $y^2 = 4x$ in the tangent to the parabola at the point (1, 2) is $(x-1)^2 = 4(y+1)$ (b) $(x+1)^2 = 4(y+1)$ $(x+1)^2 = 4(y-1)$ (d) $(x-1)^2 = 4(y-1)$

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221. Equation $x^2 - 2x - 2y + 5 = 0$ represents (A) a circle with centre (1, 1) (B) a parabola with vertex (1, 2) $(C)aparabolawithdirectrix$y= 5/2$(D)aparabolawithdirectrix$ y= - 1/3`

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222. The following line(s) is (are) tangent to the curve $y = x^2 - x$ (A) $x - y = 0$ (B) $x + y = 0$ (C) $x - y = 1$ (D)

$x + y = 1$

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223. The normal to the parabola $y^2 = 4ax$ from the point $(5a, 2a)$ are (A) $y = x - 3a$ (B) $y = -2x + 12a$ (C) $y = -3x + 33a$ (D) $y = x + 3a$

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224. The equation of the tangent to the parabola $y^2 = 9x$ which goes through the point $(4, 10)$ is (A) $x + 4y + 1 = 0$ (B) $9x + 4y + 4 = 0$ (C) $x - 4y + 36 = 0$ (D) $9x - 4y + 4 = 0$

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225. If $f(x+y)=f(x)f(y)$ for all $x, y \in R$, $f(1)=2$ and $a_r=f(r)$ for $r \in N$, then the co-ordinates of a point on the parabola $y^2=8x$ whose focal distance is 4 may be .

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226. If pair of tangents are drawn to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ from a point P so that the tangents are at right angles to each other, then the possibel coordinates of the point P is/are (A) $(3\sqrt{2}, \sqrt{7}$ (B) $(5, 0)$ (C) $(3, 4)$ (D) $(2\sqrt{5}, \sqrt{5}$

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227. If latus rectum of ellipse $x^2\tan^2\phi+y^2\sec^2\phi=1$ is $\frac{1}{2}$, then $(0<\phi<\pi)$ is equal to (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{12}$

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228. $\frac{x^2}{p^2 - P - 6} + \frac{y^2}{P^2 - 6P + 5} = 1$ will represent the ellipse if P lies in the interval (A) $(-\infty, -2)$ (B) $(1, \infty)$ (C) $(3, \infty)$ (D) $(5, \infty)$

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229. Equation of the tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ which are parallel to the line $x + y + 1 = 0$ are (A) $x + y + 5 = 0$ (B) $x + y - 5 = 0$ (C) $x + y + 6 = 0$ (D) $x + y - 6 = 0$

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230. If $ax + by + c = 0$ is a normal to hyperbola $xy = 1$, then

(A) $a < 0, b < 0$ (B) $a < 0, b > 0$ (C) $a > 0, b > 0$ (D) $a > 0, b < 0$

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231. Locus of the point of intersection of tangents at the end points of a focal chord is (A) $x = -\frac{a}{e}$, if $a > b$ (B) $x = \frac{a}{e}$, if $a > b$ (C) $y = -\frac{b}{e}$, if $a < b$ (D) $y = \frac{b}{e}$, if $a < b$

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232. If P is any point lying on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose foci are S and S'. Let $\angle PSS' = \alpha$ and $\angle PS'S = \beta$,then

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233. If the equation of an ellipse is $2x^2 + 3y^2 - 8x + 6y + 5 = 0$, then which of the following is/are true? (A) equation of auxiliary circle is $x^2 + y^2 - 4x + 2y + 2 = 0$ (B) equation of director circle is $x^2 + y^2 + 2y = 0$ (C) the director circle will pass through $(4, -2)$ (D) none of these

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234. Let e_1 and e_2 be the eccentricities of a hyperbola and its conjugate hyperbola respectively. Statement 1 : $e_1 e_2 > \sqrt{2}$. Statement 2 : $\frac{1}{e^2}{}_{-}1 + \frac{1}{e^2}{}_{-}2 =$

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235. Statement I The lines from the vertex to the two extremities of a focal chord of the parabola $y^2 = 4ax$ are perpendicular to each other.

Statement II If extremities of focal chord of a parabola are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$, then $t_1 t_2 = -1$.

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236. Statement I The lines from the vertex to the two extremities of a focal chord of the parabola $y^2 = 4ax$ are perpendicular to each other.

Statement II If extremities of focal chord of a parabola are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$, then $t_1 t_2 = -1$.

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237. If S be the focus and SH be perpendicular to the tangent at P, then prove that H lies on the tangent at the vertex and $SH^2 = OS.\ SP$, where O is the vertex of the parabola.

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238. If the normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ cuts the parabola again at $(aT^2, 2aT)$ then

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239. A ray of light is coming along the line $y = b$ from the positive direction of x-axis and striks a concave mirror whose intersection with xy-plane is a parabola $y^2 = 4ax$. Find the equation of the reflected ray and show that it passes through the focus of the parabola. Both a and b are positive.

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240. Statement 1 : $\left(a\cos^2\theta, 2a\cos\theta\right)$ is the parametric point of parabola $y^2 = 4ax$. Statement 2: For all value of $\theta, -1 \leq \cos\theta \leq 1$

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241. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P&Q.

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242. Statement 1: The equatio of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is $y = x + 2$. Statement 2: Curves $y^2 = 8x$ and $xy = -1$ intersect at $\left(\frac{1}{2}, -2\right)$.

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243. Equation of normal to parabola $y^2 = 4ax$ at $(at^2, 2at)$ is $y - 2at = -t(x - at^2) i.e. y = -tx + 2at + at^3$ Greatest and least distances between two curves occur along their common normals. Least and greatest distances of a point from a curve occur along the normal to the curve passing through that point. Now answer the following questons: The point on the parabola $y^2 = 4x$ which is nearest to the point (2, 1) is: (A) $(1, 2)$ (B) $(1, 2\sqrt{2}$ (C) $(1, -2)$ (D) none of these

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244. Equation of normal to parabola $y^2 = 4ax$ at $(at^2, 2at)$ is $y - 2at = -t(x - at^2) i.e. y = -tx + 2at + at^3$ Greatest and least distances between two curves occur along their

common normals. Least and greatest distances of a point from a curve occur along the normal to the curve passing through that point. The number of normal(s) from point $\left(\frac{7}{6}, 4\right)$ to parabola $y^2 = 2x - 1$ is (A) 1 (B) 2 (C) 3 (D) 0

A. 1

B. 2

C. 3

D. 0

Answer: 1

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245. Equation of normal to parabola $y^2 = 4ax$ at $\left(at^2, 2at\right)$ is $y - 2at = -t\left(x - at^2\right)i.e. y = -tx + 2at + at^3$ Greatest and least distances between two curves occur along their common normals. Least and greatest distances of a point from a curve occur along the normal to the curve passing through that point. Shortest distance between parabola $2y^2 - 2x + 1 = 0$ and $2x^2 - 2y + 1 = 0$ is: (A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2\sqrt{2}}$ (D) $2\sqrt{2}$

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246. A variable point P on the ellipse of eccentricity e is joined to the foci S and S'. The eccentricity of the locus of incentre of the triangle PSS' is (A) $\sqrt{\frac{2e}{1+e}}$ (B) $\sqrt{\frac{e}{1+e}}$ (C) $\sqrt{\frac{1-e}{1+e}}$ (D) $\frac{e}{2(1+e)}$

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247. A variable point P on the ellipse of eccentricity e is joined to the foci S and S'. The eccentricity of the locus of incentre of the triangle PSS' is (A) $\sqrt{\frac{2e}{1+e}}$ (B) $\sqrt{\frac{e}{1+e}}$ (C) $\sqrt{\frac{1-e}{1+e}}$ (D) $\frac{e}{2(1+e)}$

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248. A series of concentric ellipses $E_1, E_2, E_3 \ldots, E_n$ are drawn such that E touches the extremities of the major axis of E_{n-1}, and the foci of E_n coincide with the extremities of minor axis of E_{n-1} If the eccentricity of the ellipses is independent of n, then the value of the eccentricity, is (A) $\frac{\sqrt{5}}{3}$ (B) $\frac{\sqrt{5}-1}{2}$ (C) $\frac{\sqrt{5}+1}{2}$ (D) $\frac{1}{\sqrt{5}}$

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249. A parabola is drawn to pass through A and B, the ends of a diameter of a given circle of radius a, and to have as directrix a tangent to a concentric circle of radius the axes of reference being AB and a perpendicular diameter, prove that the locus of the focus of parabola $\frac{x^2}{a^2} + \frac{y^2}{b^2 - a^2} = 1$

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250. A parabola is drawn to pass through A and B, the ends of a diameter of a given circle of radius a, and to have as directrix a tangent to a concentric circle of radius the axes of reference being AB and a perpendicular diameter, prove that the locus of the focus of parabola $\frac{x^2}{a^2} + \frac{y^2}{b^2 - a^2} = 1$

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251. An ellipse passes through a focus of the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$ and its major and minor axes coincide with the transverse and conjugate axes of the hyperbola and the product of eccentricities of ellipse and hyperbola is 1. Equation of ellipse is : (A) $\frac{x^2}{16}+\frac{y^2}{9}=1$ (B) $\frac{x^2}{25}+\frac{y^2}{9}=1$ (C) $\frac{x^2}{25}+\frac{y^2}{16}=1$ (D) none of these

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252. An ellipse passes through a focus of the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$ and its major and minor axes coincide with the transverse and conjugate axes of the hyperbola and the product of eccentricities of ellipse and hyperbola is 1. Foci of the ellipse are (A) $(\pm 4, 0)$ (B) $(\pm 3, 0)$ (C) $(\pm 5, 0)$ (D) none of these

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253. An ellipse passes through a focus of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ and its major and minor axes coincide with the transverse and conjugate axes of the hyperbola and the product of eccentricities of ellipse and hyperbola is 1. If l and l' be the length of semi latera recta of ellipse and hyperbola, then $ll' =$

(A) $\frac{144}{15}$ (B) $\frac{256}{15}$ (C) $\frac{225}{12}$ (D) none of these

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254. Statement I The line $y = mx + \frac{a}{m}$ is tangent to the parabola $y^2 = 4ax$ for all values of m.

Statement II A straight line y=mx+c intersects the parabola $y^2 = 4ax$ one point is a tangent line.

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255. Let C_1 and C_2 be parabolas $x^2 = y - 1$ and $y^2 = x - 1$ respectively. Let P be any point on C_1 and Q be any point C_2. Let P_1 and Q_1 be the reflection of P and Q, respectively w.r.t the line y = x then prove that P_1 lies on C_2 and Q_1 lies on C_1 and $PQ \geq [PP_1, QQ_1]$. Hence or otherwise , determine points P_0 and Q_0 on the parabolas C_1 and C_2 respectively such that $P_0Q_0 \leq PQ$ for all pairs of points (P,Q) with P on C_1 and Q on C_2

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256. Let C_1 and C_2 be parabolas $x^2 = y - 1$ and $y^2 = x - 1$ respectively. Let P be any point on C_1 and Q be any point C_2. Let P_1 and Q_1 be the reflection of P and Q, respectively w.r.t the line y = x then prove that P_1 lies on C_2 and Q_1 lies on C_1 and $PQ \geq [PP_1, QQ_1]$. Hence or otherwise , determine points P_0 and Q_0 on the parabolas C_1 and C_2 respectively such that

$P_0Q_0 \leq PQ$ for all pairs of points (P,Q) with P on C_1 and Q on C_2

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257. Let C_1 and C_2 be parabolas $x^2 = y - 1$ and $y^2 = x - 1$ respectively. Let P be any point on C_1 and Q be any point C_2. Let P_1 and Q_1 be the reflection of P and Q, respectively w.r.t the line y = x then prove that P_1 lies on C_2 and Q_1 lies on C_1 and $PQ \geq [PP_1, QQ_1]$. Hence or otherwise , determine points P_0 and Q_0 on the parabolas C_1 and C_2 respectively such that $P_0Q_0 \leq PQ$ for all pairs of points (P,Q) with P on C_1 and Q on C_2

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258. Consider the parabola P touching x-axis at (1,0) and y-axis (0,2). Directrix of parabola P is : (A) $x - 2y = 0$ (B) $x + 2y = 0$ (C) $2x - y = 0$ (D) $2x + y = 0$

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259. Consider the parabola P touching x-axis at (1,0) and y-axis (0,2). Directrix of parabola P is : (A) $x - 2y = 0$ (B) $x + 2y = 0$ (C) $2x - y = 0$ (D) $2x + y = 0$

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260. If a hyperbola passes through foci of the ellipse $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$ and its transverse and conjugate axes coincide with the major and minor axes of the ellipse and the product of their eccentricities is 1, then the product of length of semi transverse and conjugate axes of hyperbola is...

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261. The locus of the mid-point of the portion of a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ included between the axes is the curve $\frac{a^2}{x^2}+\frac{b^2}{y^2}=k$, where $k^3=$

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262. If S_1 and S_2 are the foci of the ellipse $\frac{x^2}{8^2}+\frac{y^2}{3^2}=1$ and length of perpendicular from centre to tangent drawn at a point P on the ellipse be 4, then $(PS_1-PS_2)^2=$

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263. Radius of the largest circle passing through the focus of the parabola $y^2 = 4x$ and lying inside the parabola is...

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264. Let P be a point on the ellipse $\frac{x^2}{100} + \frac{y^2}{25} = 1$ and the length of perpendicular from centre of the ellipse to the tangent to ellipse at P be $5\sqrt{2}$ and F_1 and F_2 be the foci of the ellipse, then $PF_1 . PF_2$.

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265. if the parabola $y = x^2 + bx + c$ touches the straight line $y = x$ at (1,1), $then$1000+100b+10c=`

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266. If m be the slope of common tangent to the circle $x^2 + y^2 = 16$ and ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$ in the first quadrant, then $81m^8 =$

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267. Points A, B, C lie on the parabola $y^2 = 4ax$ The tangents to the parabola at A, B and C, taken in pair, intersect at points P, Q and R. Determine the ratio of the areas of the $\triangle ABC$ and $\triangle PQR$

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268. From a pt A common tangents are drawn to a circle $x^2 + y^2 = \frac{a^2}{2}$ and $y^2 = 4ax$. Find the area of the quadrilateral formed by common tangents, chord of contact of circle and chord of contact of parabola.

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269. Tangent is drawn at any point (x_1, y_1) other than the vertex on the parabola $y^2 = 4ax$. If tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of contact pass through a fixed point (x_2, y_2), then $x_1 a, x_2$ in GP (b) $\frac{y_1}{2}, a, y_2$ are in GP $-4, \frac{y_1}{y_2}, x_1 / x_2$ are in GP (d) $x_1 x_2 + y_1 y_2 = a^2$

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270. Tangents PT and QT to the parabola $y^2 = 4x$ intersect at T and the normal drawn at the point P and Q intersect at the

point $R(9, 6)$ on the parabola. Find the coordinates of the point T. Show that the equation to the circle circumscribing the quadrilateral $PTQR$, is $(x-2)(x-9)+(y+3)(y-6)=0$.

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271. If A be the area of the largest circle with centre $(1, 0)$ that can be inscribed in the ellipse $x^2+4y^2=16$, then $\frac{945}{\pi}A=$.

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272. The locus of the point of intersection of the tangents to $y^2=4x+y$ and $y^2=8x+16$ which are perpendicular to each other is $x=-k$. Then $k=$

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