

MATHS

BOOKS - KC SINHA MATHS (HINGLISH)

DEFINITE INTEGRALS AND PROPERTIES OF DEFINITE INTEGRALS - FOR COMPETITION

Solved Examples

1. Determine a positive integer $n \leq 5$ such that

$$\int_0^1 e^x (x-1)^n = 16 - 6e$$



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$$2. \int_0^{\pi/6} \frac{\sqrt{3 \cos 2x - 1}}{\cos x} dx$$



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3. Prove that: $\int_0^1 \tan^{-1} \cdot \left(\frac{1}{1-x+x^2} \right) dx = 2 \int_0^1 \tan^{-1} x dx$



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4. Let $f(x)$, $x \geq 0$, be a non-negative continuous function, and let $f(x) = \int_0^x f(t)dt$, $x \geq 0$, if for some $c > 0$, $f(x) \leq cF(x)$ for all $x \geq 0$, then show that $f(x) = 0$ for all $x \geq 0$.



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5.

If
 $f(x) = \frac{x-1}{x+1}$, $f^2(x) = f(f(x))$, ... $f^{k+1}(x) = f(f^k(x))$, $k = 1, 2, 3, \dots$
then $-4 \int_{\frac{1}{e}}^1 f^{2010}(x) dx = \dots$



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6. Let $f(x)$ be a continuous function defined in R such that $(f(x))^2 = \int_0^x f(t) \cdot \frac{2 \sec^2 t}{4 + \tan t} dt$ and $f(0) = 0$, If $f\left(\frac{\pi}{4}\right) = \log\left(\frac{m}{n}\right)$, where m and n are positive integers having no common factor, then $m + n$ is equal to...



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7. If $I_n = \int_0^{\infty} e^{-x} x^{n-1} \log_e x dx$, then prove that $I_{n+2} - (2n+1)I_{n+1} + n^2 I_n = 0$



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8. If $f: R \rightarrow R$ defined by $f(x) = \sin x + x$, then find the value of $\int_0^{\pi} (f^{-1}(x)) dx$



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9. If $n > 1$, evaluate: $\int_0^\infty \frac{1}{\left(x + \sqrt{1+x^2}\right)^n} dx$



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10. Evaluate: $\int_{-1}^1 \frac{x^3 + 4x^2 - 10}{(x+3)^2(x+2)^3} dx$



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11. If the value of $\int_0^\pi \left(\frac{x}{1+\sin x} \right)^2 dx = \lambda$, then find the value of the integral $= \int_0^\pi \left[\frac{2x^2 \cdot \cos^2\left(\frac{x}{2}\right)}{(1+\sin x)^2} \right] dx$



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12. Evaluate: $\int_{-4}^{-5} e^{x+5} \wedge 2 dx + 3 \int_{\frac{1}{3}}^{\frac{2}{3}} e^9 \left(x - \frac{2}{3} \right)^2 dx$



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13. Evaluate: $\int_0^{\frac{3\pi}{2}} (\ln|\sin x|) \cos(2nx) dx, n \in N$



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14. Find a function $g: R \rightarrow R$, continuous in $[0, \infty)$ and positive in $(0, \infty)$ satisfying $g(1) = 1$ and $\frac{1}{2} \int_0^x g^2(t) dt = \frac{1}{x} \left(\int_0^x g(t) dt \right)^2$



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15. Evaluate: $\int_0^{\frac{\pi}{2}} \ln(1 + \sin \alpha \sin^2 x) \cos ex^2 dx$ (where $\alpha > 0$)



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16. Evaluate: $\int_0^1 \frac{x^\alpha - 1}{\log_e x} dx$



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17. Prove that: $\int_0^{\frac{\pi}{2}} \frac{dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^2} = \frac{\pi(a^2 + b^2)}{4a^3b^3}$



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18. The value of $\int_0^{\infty} [2e^{-x}] dx$ (where, $[*]$ denotes the greatest integer function of (x)) is equal to



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19. Evaluate: $\int_0^{2\pi} [2 \sin x] dx$



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20. Evaluate: $\int_{-\left(\frac{\pi}{4}\right)}^{\frac{3\pi}{4}} \frac{\sqrt{2}\left[1 + \sin\left(x - \frac{\pi}{4}\right)\right]}{\sqrt{2} - \cos|x| + \sin|x|} dx$



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21. If $\int_{-1}^1 \left(\sin^{-1} \left[x^2 + \frac{1}{2} \right] + \cos^{-1} \left[x^2 - \frac{1}{2} \right] \right) dx = k\pi$ where $[x]$

denotes the integral part of x , then the value of k is ...



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22. Evaluate the integral:

$$\int_0^4 \frac{\{\sqrt{x}\}}{(1 + \sin\{[x]\})^5} dx + \int_0^{\frac{\pi}{4}} \sin(x - [x]) d(x - [x]^5)$$



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23. Evaluate: $\int_0^\pi \frac{x^2 \sin 2x \cdot \sin\left(\frac{\pi}{2} \cdot \cos x\right)}{2x - \pi} dx$



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24. $\int_0^\pi \frac{x \sin 2x \sin\left(\frac{\pi}{2} \cos x\right)}{2x - \pi} dx$ is equal to



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25. $\int_0^{\pi/4} \frac{x^2(\sin 2x - \cos 2x)}{(1 + \sin 2x)\cos^2 x} dx$



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26. Show that: $\int_a^b f(x)dx = \int_{a+c}^{b+c} f(x - c)dx$ and hence show that
 $\int_0^\pi \sin^{100} x \cos^{99} x dx = 0$



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27. Evaluate: $\int_{\frac{1}{2}}^2 \frac{1}{x} \tan^7 \left(x - \frac{1}{x} \right) dx$



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28. Evaluate : $\int_{-\sqrt{2}}^{\sqrt{2}} \frac{2x^7 + 3x^6 - 10x^5 - 7x^3 - 12x^2 + x + 1}{x^2 + 2} dx$



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29. Prove that $\int_0^x e^{zx} e^{-z^2} dz = e^{\frac{x^2}{4}} \int_0^x e^{\frac{z^2}{4}} dz$



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30. Evaluate the definite integral: $\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \left(\frac{x^4}{1-x^4} \right) \cos^01 \left(\frac{2x}{1+x^2} \right) dx$.



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31. Evaluate: $\int_{-\left(\frac{1}{\sqrt{3}}\right)}^{\frac{1}{\sqrt{3}}} \frac{\cos^{-1}\left(\frac{2x}{1+x^2}\right) + \tan^{-1}\left(\frac{2x}{1-x^2}\right)}{e^x + 1} dx$



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32. Find the value of $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\pi + 4x^3}{2 - \cos\left(|x|\frac{\pi}{3}\right)} dx$



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33. Evaluate $\int_0^\pi e^{|\cos x|} \left\{ 2 \sin\left(\frac{\cos x}{2}\right) + 3 \cos\left(\frac{\cos x}{2}\right) \right\} \sin x dx$



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34. Given a function $f(x)$ such that it is integrable over every interval on the real line, and $f(t+x) = f(x)$, for every x and a real t . Then show that the integral $\int_a^{a+t} f(x) dx$ is independent of a .



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35. It is known that $f(x)$ is an odd function in the interval $\left[\frac{p}{2}, \frac{p}{2}\right]$ and has a period p , Prove that $\int_q^x f(t) dt$ is also periodic function with the same period.



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36. Show that $\int_0^{n\pi + v} |\sin x| dx = 2n + 1 - \cos v$, where n is a positive integer and $v > 0$.



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37. If $f(x)$ is a function satisfying $f(x + a) + f(x) = 0$ for all $x \in R$ and positive constant a such that $\int_b^{c+b} f(x) dx$ is independent of b , then find the least positive value of a .



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38. Show that: $\left| \int_{10}^{19} \frac{\sin x}{1+x^8} dx \right| < \frac{1}{10^7}$



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39. Prove that $\int_0^1 \sqrt{(1+x)(1+x^3)} dx$ cannot exceed $\sqrt{\frac{15}{8}}$.



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40. Prove that $\frac{\pi}{6} \leq \int_0^1 \frac{dx}{\sqrt{4-x^2-x^3}} \leq \frac{\pi}{4\sqrt{2}}$



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41. Prove that: $4 \leq \int_1^3 \sqrt{3+x^3} dx \leq 2\sqrt{30}$



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42. prove it $2e^{-\frac{1}{4}} < \int_0^2 e^{x^2-x} dx < 2e^2$



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43. If $f(x) = \cos(\tan^{-1} x)$, then the value of the integral $\int_0^1 xf^x dx$ is
 $\frac{3 - \sqrt{2}}{2}$ (b) $\frac{3 + \sqrt{2}}{2}$ 1 (d) $1 - \frac{3}{2\sqrt{2}}$



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44. The values of the definite integral $\int_0^{\frac{3\pi}{4}} ((1+x)\sin x + (1-x)\cos x)dx$, is (A) $2\tan\left(\frac{3\pi}{8}\right)$ (B) $2\tan\left(\frac{\pi}{4}\right)$
(C) $2\tan\left(\frac{\pi}{8}\right)$ (D) 0



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45. The tangent to the graph of the function $y = f(x)$ at the point with abscissae $x = 1, x = 2, x = 3$ make angles $\frac{\pi}{6}, \frac{\pi}{3}$ and $\frac{\pi}{4}$ respectively.
The value of $\int_1^3 f'(x)f''(x)dx + \int_2^3 f''(x)dx$ is (A) $\frac{4 - 3\sqrt{3}}{3}$ (B)
 $\frac{4\sqrt{3} - 1}{3\sqrt{3}}$ (C) $\frac{4 - 3\sqrt{3}}{2}$ (D) $\frac{3\sqrt{3} - 1}{2}$



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46. Let f be a non-negative function defined on the interval $[0, 1]$. If

$$\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt, 0 \leq x \leq 1 \text{ and } f(0) = 0,$$
 then
(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$ (B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$ (C)
(D) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$ (E) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$



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47. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1 + \pi^x)\sin x} dx, n = 0, 1, 2, \dots$, then (A) $I_n = I_{n+2}$

(B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$ (C) $\sum_{m=1}^{10} I_{2m} = 0$ (D) $I_n = I_{n+1}$



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48. Let f be a function such that $f(x) \cdot f'(-x) = f(-x) \cdot f'(x)$ for

all x and $f(0) = 3$. Now answer the question: The value of $f(x)f(-x)$

for all x is (A) 9 (B) 4 (C) 16 (D) 12



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49. Let f be a function such that $f(x) \cdot f'(-x) = f(-x) \cdot f'(x)$ for all x and $f(0) = 3$. Now answer the question: $\int_{-51}^{51} \frac{dx}{3 + f(x)} =$ (A) 34 (B) 102 (C) 0 (D) 17



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50. Let f be a function such that $f(x) \cdot f'(-x) = f(-x) \cdot f'(x)$ for all x and $f(0) = 3$. Now answer the question: Number of roots of equation $f(x) = 0$ in interval $[-2, 2]$ is (A) 0 (B) 2 (C) 4 (D) 1



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51. Let f and g be differentiable functions such that: $xg(f(x))f'(g(x))g'(x) = f(g(x))g'(f(x))f'(x) \forall x \in R$ Also, $f(x) > 0$ and $g(x) > 0 \forall x \in R$ $\int_0^x f(g(t))dt = 1 - \frac{e^{-2x}}{2}, \forall x \in R$

and $g(f(0)) = 1$, $h(x) = \frac{g(f(x))}{f(g(x))} \forall x \in R$ Now answer the question:

$$f(g(0)) + g(f(0)) = \text{(A) 1 (B) 2 (C) 3 (D) 4}$$



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52. Let f and g be differentiable functions such that:

$$xg(f(x))f'(g(x))g'(x) = f(g(x))g'(f(x))f'(x) \forall x \in R \quad \text{Also,}$$

$$f(x) > 0 \text{ and } g(x) > 0 \forall x \in R \int_0^x f(g(t))dt = 1 - \frac{e^{-2x}}{2}, \forall x \in R$$

and $g(f(0)) = 1$, $h(x) = \frac{g(f(x))}{f(g(x))} \forall x \in R$ Now answer the question:

$$f(g(0)) + g(f(0)) = \text{(A) 1 (B) 2 (C) 3 (D) 4}$$



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53. Let f and g be differentiable functions such that:

$$xg(f(x))f'(g(x))g'(x) = f(g(x))g'(f(x))f'(x) \forall x \in R \quad \text{Also,}$$

$$f(x) > 0 \text{ and } g(x) > 0 \forall x \in R \int_0^x f(g(t))dt = 1 - \frac{e^{-2x}}{2}, \forall x \in R$$

and $g(f(0)) = 1$, $h(x) = \frac{g(f(x))}{f(g(x))} \forall x \in R$ Now answer the question:

$$f(g(0)) + g(f(0)) = \text{(A) 1 (B) 2 (C) 3 (D) 4}$$



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54. The value of $\frac{(5050) \int_0^1 (1 - x^{50})^{100} dx}{\int_0^1 (1 - x^{50})^{101} dx}$ is



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55. For any real number x , let $[x]$ denote the largest integer less than or equal to x . Let f be a real-valued function defined on the interval $[-10, 10]$ be

$f(x) = \begin{cases} x - [x], & \text{if } [x] \text{ is odd} \\ 1 + [x] - x, & \text{if } [x] \text{ is even} \end{cases}$ Then the value of $\frac{\pi^2}{10} \int_{-1}^{10} f(x) \cos \pi x dx$ is



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56. Let f be the function defined on $[-\pi, \pi]$ given by $f(0) = 9$ and

$f(x) = \frac{\sin\left(\frac{9x}{2}\right)}{\sin\left(\frac{x}{2}\right)}$ for $x \neq 0$. The value of $\frac{2}{\pi} \int_{-\pi}^{\pi} f(x) dx$ is



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57. Let $f: \overrightarrow{RR}$ be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$. Then the value of $f(1n5)$ is _____



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Exercise

1.
$$f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \cos ex \\ \cos^2 x & \cos^2 x & \cos ec^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$$
 then
$$\int_0^{\frac{\pi}{2}} f(x)dx = \dots \dots$$



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2.
$$\int_0^{\frac{\pi}{4}} \frac{2 \sin \theta \cos \theta}{\sin^4 \theta + \cos^4 \theta} d\theta$$



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$$3. \int_0^1 \frac{\sin^{-1} x}{\sqrt{1+x}} dx$$



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$$4. \int_0^a x \sqrt{\frac{a^2 - x^2}{a^2 + x^2}} dx$$



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$$5. \int_0^1 x^5 \sqrt{\frac{1+x^2}{1-x^2}} dx$$



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$$6. \int_1^2 \frac{dx}{x\sqrt{1+x}}$$



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$$7. \int_0^1 \frac{dx}{(x+1)\sqrt{2+x-x^2}}$$



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$$8. \int_1^2 \frac{x-1}{x+1} \cdot \frac{1}{\sqrt{x^3+x^2+x}} dx$$



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$$9. \text{Evaluate: } \int_1^{16} \tan^{-1} \sqrt{\sqrt{x}-1} dx$$



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$$10. \text{Evaluate: } \int_0^1 \cot^{-1}(1-x+x^2) dx$$



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11. If $U_n = \int_0^{\frac{\pi}{2}} \frac{\sin^2 nx}{\sin^2 x} dx$, then show that $U_1, U_2, U_3, \dots, U_n$ constitute an AP. Hence or otherwise find the value of U_n .



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12. If n is a positive integer, prove that:

$$\int_0^{2\pi} \frac{\cos(n-1)x - \cos nx}{1 - \cos x} dx = 2\pi, \text{ hence or otherwise, show that}$$
$$\int_0^{2\pi} \left(\frac{\sin\left(\frac{nx}{2}\right)}{\sin\left(\frac{x}{2}\right)} \right)^2 dx = 2n\pi.$$


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13. Let $f(x) = \int_0^{\sin^2 x} \sin^{-1}(\sqrt{t}) dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t}) dt$, then (A) $f(x)$ is a constant function (B) $f\left(\frac{\pi}{4}\right) = 0$ (C) $f\left(\frac{\pi}{3}\right) = \frac{\pi}{4}$ (D) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$



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14. Prove that for $n > 1$.

$$\int_0^1 (\cos^{-1} x)^n dx = n \left(\frac{\pi}{2}\right)^{n-1} - n(n-1) \int_0^1 (\cos^{-1} x)^{n-2} dx$$



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15. Prove that

$$\cos x + \cos 2x + \dots + \cos nx = \frac{\sin\left(n + \frac{1}{2}\right)x - \sin\left(\frac{x}{2}\right)}{2 \sin\left(\frac{x}{2}\right)} \text{ and hence}$$

prove that : $\int_0^1 \frac{\sin\left(n + \frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)} dx = \pi$



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16. If $v_n = \int_0^1 x^n \tan^{-1} x dx$, show that:

$$(n+1)u_n + (n-1)u_{n-2} = \frac{\pi}{2} - \frac{1}{n}$$



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17. Evaluate: $\int_0^1 \log(x^2 + x + 1) dx$



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18. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\left(x - \frac{\pi}{4}\right)^2 \sin x}{\sin x + \cos x} dx$



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19. Evaluate: $\int_2^3 \frac{2x^5 + x^4 - 2x^3 + 2x^2 + 1}{(x^2 + 1)(x^4 - 1)} dx$



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20. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\sec x}{1 + 2 \sin^2 x} dx$



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21. Evaluate: $\int_0^1 \frac{2 - x^2}{(1 + x)\sqrt{1 - x^2}} dx$



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22. Let $I_m = \int_0^\pi \frac{1 - \cos mx}{1 - \cos x} dx$. Show that $I_m = m\pi$.



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23. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{2 + \sin x + \cos x} dx$



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24. Show that: $\int_0^1 \frac{\tan^{-1} x}{x} dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} y \cos ecy dy$



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25. Find $\int_0^1 \frac{\log\left(2 + x^{\frac{1}{3}}\right)}{x^{\frac{1}{3}}} dx$



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26. If $[x]$ denotes the integral part of x , find $\int_0^x [t+1]^3 dt$.



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27. Show that: $\int_0^x [x] dx = [x] \frac{[x] - 1}{2} + [x](x - [x])$, where $[x]$

denotes the integral part of x .



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28. If $[x]$ denotes the integral part of $[x]$, show that:

$$\int_0^{(2n-1)\pi} [\sin x] dx = (1-n)\pi, n \in N$$



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29. If $[x]$ denotes the integral part of x and $f(x) = \min(x - [x], -x - [-x])$ show that: $\int_{-2}^2 f(x)dx = 1$



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30. Show that: $\int_0^x [x]dx = [x]\frac{[x] - 1}{2} + [x](x - [x])$, where $[x]$ denotes the integral part of x .



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31. Evaluate: $\int_{-100}^{100} Sgn(x - [x])dx$, where $[x]$ denotes the integral part of x .



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32. Show that: $\int_0^x (x - [x])dx = \frac{x}{2}$, where $[x]$ denotes the integral part of x .



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33. Show that: $\frac{\int_0^x [x]dx}{\int_0^x \{x\}dx} = [x] - 1$, where $[x]$ denotes the integral part of x and $\{x\} = x - [x]$.



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34. Evaluate: $\int_{-2}^2 \frac{\sin^{100} x}{[\frac{x}{\pi}] + \frac{1}{2}} dx$



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35. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$



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36. Evaluate: $\int_0^\pi \sin^m \cos^{2m+1} x dx$



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37. Evaluate: $\int_0^\pi \frac{x}{1 + \cos^2 x} dx$



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38. Evaluate: $\int_0^\pi \frac{x}{a^2 - \cos^2 x} dx$



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39. Evaluate: $\int_0^\pi \frac{x}{(a^2 \cos^2 x + b^2 \sin^2 x)^2} dx$



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40. Evaluate: $\int_0^1 \frac{\sin^{-1} x}{x} dx$



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41. Evaluate: $\int_0^{\frac{\pi}{2}} x \cot x dx$



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42. Evaluate: $\int_0^{\pi} \log(1 + \cos x) dx$



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43. Evaluate: $\int_0^{\infty} \log\left(x + \frac{1}{x}\right) \frac{dx}{1+x^2}$



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44. Evaluate: $\int_0^{\pi} x \log \sin x dx$



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$$45. \int_0^{\pi/2} \left(\frac{\theta}{\sin \theta} \right)^2 d\theta =$$



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$$46. \text{ Evaluate: } \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\varphi}{1 - \sin \varphi} d\varphi$$



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$$47. \text{ Evaluate: } \int_0^2 \frac{dx}{(17 + 8x - 4x^2) [e^{6(1-x)} + 1]}$$



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$$48. \text{ If } f \text{ is an odd function, show that: } \int_{-a}^a \frac{f(\sin x)}{f(\cos x) + f(\sin^2 x)} dx = 0$$



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49. If $f(x)$ is a continuous function and attains only rational values in $[-3, 3]$ and its greatest value in $[-3, 3]$ is 5, then $\int_{-3}^3 f(x)dx =$ (A) 5
(B) 10 (C) 20 (D) 30



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50. If $g(x) = \int_0^x \cos^4 t dt$, then $g(x + \pi)$ equals $g(x) + g(\pi)$ (b)
 $g(x) - g(\pi)$ $g(x)g(\pi)$ (d) $\frac{g(x)}{g(\pi)}$



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51. If $I_n = \int_0^{\frac{\pi}{2}} \frac{\sin(2n-1)x}{\sin x} dx$, and $a_n = \int_0^{\frac{\pi}{2}} \left(\frac{\sin n\theta}{\sin \theta} \right)^2 d\theta$, then
 $a_{n+1} - a_n =$ (A) I_n (B) $2I_n$ (C) I_{n+1} (D) 0



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52. If $n \in N$ and $\int_0^1 e^x(x-1)^n dx = 2e - 5$, then $n =$ (A) 1 (B) 2 (C) 3
(D) none of these



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53. If $\int_0^1 \frac{e^t}{t+1} dt = a$, then $\int_{b-1}^b \frac{e^{-t}}{t-b-1} dt =$



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54. If $\int_0^x f(t) dt = x + \int_x^1 t f(t) dt$, then $f(1) =$ (A) $\frac{1}{2}$ (B) 0 (C) 1 (D)
 $-\frac{1}{2}$



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55. If $n \neq 1$, $\int_0^{\frac{\pi}{4}} (\tan^n x + \tan^{n-2} x) d(x - [x]) =$ (A) $\frac{1}{n-1}$ (B)
 $\frac{1}{n+1}$ (C) $\frac{1}{n}$ (D) $\frac{2}{n-1}$



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56. The equation $\int_0^x (t^2 - 8t + 13) dt = x \sin\left(\frac{a}{x}\right)$ has a solution if $\sin\left(\frac{a}{6}\right)$ is (A) zero (B) -1 (C) 1 (D) none of these



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57. If $f(\alpha) = f(\beta)$ and $n \in N$, then the value of $\int_{\alpha}^{\beta} (g(f(x)))^n g'(f(x)) \cdot f'(x) dx$ = (A) 1 (B) 0 (C) $\frac{\beta^{n+1} - \alpha^{n+1}}{n+1}$ (D) none of these



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58. If $l_1 = \int_e^{e^2} \frac{dx}{\log x}$ and $l_2 = \int_1^2 \frac{e^x}{x} dx$, then (A) $l_1 = 2l_2$ (B) $l_1 + l_2 = 0$ (C) $2l_1 = l_2$ (D) $l_1 = l_2$



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59. Let $f: (0, \infty) \rightarrow R$ and $F(x) = \int_0^x f(t)dt$. If $F(x^2) = x^2(1 + x)$, then $f(4) =$ (A) $\frac{5}{4}$ (B) 7 (C) 4 (D) 2



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60. $\int_0^{\frac{4}{\pi}} \left(3x^2 \sin\left(\frac{1}{x}\right) - x \cos\left(\frac{1}{x}\right) \right) dx =$ (A) $\frac{8\sqrt{2}}{\pi^3}$ (B) $\frac{32\sqrt{2}}{\pi^3}$ (C) $\frac{24\sqrt{2}}{\pi^3}$ (D) $\frac{\sqrt{2048}}{\pi^3}$



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61. $\int_0^{\pi} \frac{dx}{1 - 2a \cos x + a^2}$, $a < 1$ is equal to (A) $\frac{\pi a \log 2}{4}$ (B) $\frac{4\pi}{2 - a^2}$ (C) $\frac{\pi}{1 - a^2}$ (D) none of these



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62. $\int_0^1 \log(\sqrt{1+x} + \sqrt{1-x}) dx =$ (A) $\frac{1}{2} \left(\log 2 - \frac{\pi}{2} + 1 \right)$ (B) $\frac{1}{2} \left(\log 2 + \frac{\pi}{2} + 1 \right)$ (C) $\frac{1}{2} \left(\log 2 + \frac{\pi}{2} - 1 \right)$ (D) none of these



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63. If $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x dx$, then $[I_4 + 12I_2]$ is equal to (A) 4π (B) $3\left(\frac{\pi}{2}\right)^3$ (C) $\left(\frac{\pi}{2}\right)^2$ (D) $4\left(\frac{\pi}{2}\right)^3$



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64. If $f(x) = ae^{2x} + be^x + cx$ satisfies the conditions $f(0) = -1$, $f'(\log 2) = 28$, $\int_0^{\log 4} [f(x) - cx]dx = \frac{39}{2}$, then (A) $a = 5, b = 6, c = 3$ (B) $a = 5, b = -6, c = 0$ (C) $a = -5, b = 6, c = 3$ (D) none of these



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65. $\int_0^{\infty} \frac{\log x}{1+x^2} dx =$ (A) $\log 2$ (B) $\frac{\pi}{2}$ (C) 0 (D) none of these



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66. All the values of a for which $\int_1^2 [a^2 + (4 - 4a)x + 4x^3] dx \leq 12$ are given by (A) $a = 3$ (B) $a \leq 4$ (C) $0 \leq a < 3$ (D) none of these



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67. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be the function satisfying $f(x) + g(x) = x^2$. Then the value of integral $\int_0^1 f(x)g(x)dx$ is equal to (A) $\frac{e-2}{4}$ (B) $\frac{e-3}{2}$ (C) $\frac{e-4}{2}$ (D) none of these



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68. If $\int_0^1 \frac{\sin t}{1+t} dx = \alpha$, then the value of the integral $\int_{4\pi-2}^{4\pi} \frac{\frac{\sin t}{2}}{4\pi+2-t} dt$ is (2) 2α (3) α (d) $-\alpha$



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69. If $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\tan x)^{-n} dx$ ($n > 1$), then $I_n + I_{n+2} =$ (A) $\frac{1}{n-1}$ (B)
 $\frac{1}{n+1}$ (C) $-\frac{1}{n+1}$ (D) $\frac{1}{n} - 1$



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70. $\int_0^{\frac{\pi}{2}} \frac{3 + 4 \cos x}{(4 + 3 \cos x)^2} dx =$ (A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{1}{4}$



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71. If for nonzero x , $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $a \neq b$ then
 $\int_1^2 f(x) dx = 1$ _



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72. $\int_0^{\pi} \frac{\sin\left(\frac{n+1}{2}\right)x}{\sin x} dx =$ (A) 0 (B) $\frac{\pi}{2}$ (C) π (D) none of these



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73. If $f(x) = \int_0^{\sin x} \cos^{-1} t dt + \int_0^{\cos x} \sin^{-1} t dt$, $0 < x < \frac{\pi}{2}$, then
 $f\left(\frac{\pi}{4}\right) =$ (A) 0 (B) $\frac{\pi}{\sqrt{2}}$ (C) 1 (D) $\frac{\pi}{2\sqrt{2}}$



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74. Let $f(x) = \int_0^{\sin^2 x} \sin^{-1}(\sqrt{t}) dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t}) dt$, then (A)
 $f(x)$ is a constant function (B) $f\left(\frac{\pi}{4}\right) = 0$ (C) $f\left(\frac{\pi}{3}\right) = \frac{\pi}{4}$ (D)
 $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$



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75. Given a function $f(x)$ such that it is integrable over every interval on the real line, and $f(t+x) = f(x)$, for every x and a real t . Then show that the integral $\int_a^{a+t} (x) dx$ is independent of a .



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76. If $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, then $\int_0^\infty e^{-ax^2} dx$ where $a > 0$ is: (A) $\frac{\sqrt{\pi}}{2}$
(B) $\frac{\sqrt{\pi}}{2a}$ (C) $2\frac{\sqrt{\pi}}{a}$ (D) $\frac{1}{2}\left(\sqrt{\frac{\pi}{a}}\right)$



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77. $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$ equals



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78. $Lt_{n \rightarrow \infty} \sum_{r=1}^{6n} \frac{1}{n+r} =$ (A) $\log 6$ (B) $\log 7$ (C) $\log 5$ (D) 0



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79. $Lt_{n \rightarrow \infty} \sum_{r=1}^n \frac{(2r)^k}{n^{k+1}}$, $k \neq -1$, is equal to (A) $\frac{2^k}{k-1}$ (B) $\frac{2^k}{k}$ (C) $\frac{1}{k+1}$
(D) $\frac{2^k}{k+1}$



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80. $Lt_{n \rightarrow \infty} \left(\frac{n!}{n^n} \right)^{\frac{1}{n}} =$ (A) e^{-2} (B) e^{-1} (C) e^3 (D) e



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81. $Lt_{n \rightarrow \infty} \left\{ \frac{n!}{(kn)^n} \right\}^{\frac{1}{n}}$, $k \neq 0$, is equal to (A) $\frac{k}{e}$ (B) $\frac{e}{k}$ (C) $\frac{1}{ke}$ (D) none
of these



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82. $\lim_{n \rightarrow \infty} \sum_{n=1}^n \frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r} + 4\sqrt{n})^2}$



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83. If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$ then $\lim_{n \rightarrow \infty} n(I_n + I_{n-2})$ equals (A) $\frac{1}{2}$ (B) 1 (C) ∞ (D) 0



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84. Let $\frac{d}{dx}F(x) = \left(\frac{e^{\sin x}}{x}\right)$, $x > 0$. If $\int_1^4 \frac{3}{x} e^s \in x^3 dx = F(k) - F(1)$, then one of the possible values of k , is: 15 (b) 16 (c) 63 (d) 64



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85. $Lt_{n \rightarrow \infty} \frac{1 + 2^4 + 3^4 + \dots + n^4}{n^5} - Lt_{n \rightarrow \infty} \frac{1 + 2^3 + 3^3 + \dots + n^3}{n^5}$
is (A) $\frac{1}{5}$ (B) $\frac{1}{30}$ (C) 0 (D) $\frac{1}{4}$



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86. If $f(y) = e^y$, $g(y) = y$, $y > 0$, and $F(t) = \int_0^t f(t-y)g(y)dy$,
then



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87. The value of $Lt_{x \rightarrow 0} \left\{ \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x} \right\}$ is (A) 0 (B) 3 (C) 2 (D) 1



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88. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} e^{\frac{r}{n}}$ is (A) $1 - e$ (B) $e - 1$ (C) e (D) $e + 1$



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89. The value of $I = \int_0^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx$ is (A) 2 (B) 1 (C) 0 (D) 3



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90.

$$\lim_{n \rightarrow \infty} \left[\frac{1}{n^2} \cdot \sec^2 \left(\frac{1}{n^2} \right) + \frac{2}{n^2} \cdot \sec^2 \left(\frac{4}{n^2} \right) + \dots + \frac{1}{n} \cdot \sec^2 1 \right]$$



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91. Let $f: \overrightarrow{RR}$ be a differentiable function having

$f(2) = 6, f'(2) = \frac{1}{48}$. Then evaluate $(\lim)_{x \rightarrow 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$



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92. If $I_1 = \int_0^1 2^{x^2} dx, I_2 = \int_0^1 2^{x^3} dx, I_3 = \int_1^2 2^{x^2} dx$

and $I_4 = \int_1^2 2^{x^2} dx$ then



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93. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_t^x \frac{\log t}{1+t} dt$. (1) $\frac{1}{2}$ (2)

0 (3) 1 (4) 2



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94. The solution for x of the equation $\int_{\sqrt{2}}^x \frac{dt}{t\sqrt{t^2 - 1}} = \frac{\pi}{2}$ is: (1) 2 (2) π (3) $\frac{\sqrt{3}}{2}$ (4) $2\sqrt{2}$



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95. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$. Then which one of the following is true? (1) $I > \frac{2}{3}$ and $J > 2$ (2) $I < \frac{2}{3}$ and $J < 2$ (3) $I < \frac{2}{3}$ and $J > 2$ (4) $I > \frac{2}{3}$ and $J < 2$



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96. $\int_0^{\pi} [\cot x] dx$, where $[.]$ denotes the greatest integer function, is equal to (1) $\pi/2$ (2) 1 (3) 1 (4) $\pi/2$



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97. Let $f(x) = \int_1^x \sqrt{2 - t^2} dt$. Then the real roots of the equation $x^2 - f'(x) = 0$ are ± 1 (b) $\pm \frac{1}{\sqrt{2}}$ (d) 0 and 1



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98. If $I(m, n) = \int_0^1 t^m (1+t)^n dt$, then the expression for $I(m,n)$ in terms of $I(m+1,n-1)$ is:



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99. If $f(x)$ is differentiable and $\int_0^{t^2} xf(x)dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals $\frac{2}{5}$ (b) $-\frac{5}{2}$ 1 (d) $\frac{5}{2}$



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100. $\int_{-2}^0 \{x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)dx\}$ is equal to (A) -4

- (B) 0 (C) 4 (D) 6



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101. If $\int_{\sin x}^1 t^2 f(t) dt = 1 - \sin x$, then the value of $f\left(\frac{1}{\sqrt{3}}\right)$ is (A) $\frac{1}{\sqrt{3}}$
(B) $\frac{1}{3}$ (C) $\sqrt{3}$ (D) 3



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102. Consider the function $f: (-\infty, \infty) \rightarrow (-\infty, \infty)$ defined by $f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}$, $0 < a < 2$, and let $g(x) = \int_0^{e^x} \frac{f'(t)dt}{1+t^2}$. Which of the following is true? (A) $g'(x)$ is positive on $(-\infty, 0)$ and negative on $(0, \infty)$ (B) $g'(x)$ is negative on $(-\infty, 0)$ and positive on $(0, \infty)$ (C) $g'(x)$ changes sign on both $(-\infty, 0)$ and $(0, \infty)$ (D) $g'(x)$ does not change sign on $(-\infty, \infty)$



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103. The value of $\int_0^{\log 5} \frac{e^x \sqrt{e^x - 1}}{e^x + 3} dx$ is (A) $3 + 2\pi$ (B) $4 - \pi$ (C) $2 + \pi$ (D) none of these



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104. $I_{10} = \int_0^{\frac{\pi}{2}} x^{10} \sin x dx$ then $I_{10} + 90I_8$ is (A) $10\left(\frac{\pi}{2}\right)^6$ (B) $10\left(\frac{\pi}{2}\right)^9$ (C) $10\left(\frac{\pi}{2}\right)^8$ (D) $10\left(\frac{\pi}{2}\right)^7$



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105. $\int_0^a [f(x) + f(-x)] dx =$ (A) 0 (B) $2 \int_0^a f(x) dx$ (C) $\int_{-a}^a f(x) dx$ (D) none of these



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106. If $f(x) = \begin{cases} e^{\cos x} \sin x & |x| \leq 2 \\ 2 & otherwise \end{cases}$ then $\int_{-2}^3 f(x)dx =$ (A) 0 (B) 1 (C) 2 (D) 3



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107. $\sqrt{3} \int_0^\pi \frac{dx}{1 + 2 \sin^2 x} =$ (A) $-\pi$ (B) 0 (C) π (D) none of these



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108. $\int_0^\infty \frac{x \log x}{(1 + x^2)^2} dx =$ (A) e (B) 1 (C) 0 (D) none of these



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109. $\int_{-\left(\frac{\pi}{3}\right)}^{\frac{\pi}{3}} \frac{x^3 \cos x}{\sin^2 x} dx =$ (A) 0 (B) 1 (C) -1 (D) none of these



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110. $\int_{-\pi/4}^{\pi/4} \frac{e^x \sec^2 dx}{e^{2x} - 1}$ is equal to (i)0 (ii)2 (iii) e (iv)none of these



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111. Q. if $\int_0^{100} (f(x)dx = a)$, then $\sum_{r=1}^{100} \left(\int_0^1 (f(r-1+x)dx) \right) =$



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112. If $\int_0^{100\pi + a} |\sin x|dx = k - \cos \alpha$, where $0 < \alpha < \pi$, then $k =$ (A) 101
(B) 100 (C) 201 (D) none of these



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113. $\int_{-\pi}^{3\pi} \cot^{-1}(\cot x)dx =$ (A) π^2 (B) $2\pi^2$ (C) $3\pi^2$ (D) none of these



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114. The value of $\int_{-3}^3 \log\left(\frac{30-x^3}{30+x^3}\right) dx$ is (A) $2 \int_0^3 \log\left(\frac{30-x^3}{30+x^3}\right) dx$ (B)
 $\log\left(\frac{3}{57}\right)$ (C) $\log\left(\frac{57}{3}\right)$ (D) 0



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115. If $\int_0^\pi x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$, then A is (A) $\frac{\pi}{2}$ (B) π (C) 0
(D) 2π



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116. The integral $\int_{-a}^a \frac{\sin^2 x}{1-x^2} dx$, $0 < a < 1$, is equal to (A)
 $\int_{-a}^a \frac{\sin^2 x}{1+x^2} dx$ (B) $2 \int_0^a \frac{\sin^2 x}{1-x^2} dx$ (C) $\int_a^0 \frac{\sin^2 x}{1+x^2} dx$ (D) 0



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117. The value of the integral $\int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x dx$ is (A) 0 (B) $\pi - \frac{\pi^3}{3}$ (C) $2\pi - \pi^3$ (D) $\frac{\pi}{2} - 2\pi^3$



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118. $\int_{-1}^1 \frac{\sin x + x^2}{3 - |x|} dx =$ (A) 0 (B) $2 \int_0^1 \frac{\sin x}{3 - |x|} dx$ (C) $2 \int_0^1 \frac{x^2}{3 - |x|} dx$
(D) $2 \int_0^1 \frac{\sin x + x^2}{3 - |x|} dx$



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119. $\int_{-a}^a (1 + x^3)^{-1} dx =$ (A) 0 (B) $2 \int_0^a (1 - x^6)^{-1} dx$ (C)
 $2 \int_0^a (1 + x^3)^{-1} dx$ (D) $2 \int_0^a [1 + (a - x^3)]^{-1} dx$



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120. Q. $\int_0^{\pi} \left(e^{\cos^2 x} (\cos^3(2n+1)x) dx, n \in I \right)$





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121. The value of the integral $\int_0^\pi \frac{x dx}{1 + \cos \alpha \sin x}$, $0 < a < \pi$ is (A) $\frac{\pi a}{\sin \alpha}$ (B) $\frac{\pi a}{1 + \sin \alpha}$ (C) $\frac{\pi a}{\cos \alpha}$ (D) $\frac{\pi a}{1 + \cos \alpha}$



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122. $\int_0^\pi \sqrt{\frac{1 + \cos 2x}{2}} dx =$ (A) 0 (B) 1 (C) 2 (D) none of these



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123. The value of $\int_{-2}^2 (ax^3 + bx + c) dx$ depends on (A) the value of b
(B) the value of c (C) the value of a (D) the value of a and b



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124. The value of the integral $\int_0^{\frac{3}{2}} [x^2] dx$, where $[]$ denotes the greatest integer function, is (A) $2 + \sqrt{2}$ (B) $2 - \sqrt{2}$ (C) $4 + 2\sqrt{2}$ (D) $4 - 2\sqrt{2}$



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125. $\int_{-2}^2 |1 - x^2| dx =$ (A) 4 (B) 2 (C) -2 (D) 0



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126. Let f be a function defined by $f(x) = \frac{4^x}{4^x + 2}$
 $I_1 = \int_{f(1-a)}^{f(a)} xf\{x(1-x)\}dx$ and $I_2 = \int_{f(1-a)}^{f(a)} f\{x(1-x)\}dx$
where $2a - 1 > 0$ then $I_1 : I_2$ is (A) 2 (B) k (C) $\frac{1}{2}$ (D) 1



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127. $\int_4^{10} \frac{[x^2]}{[x^2 - 28x + 196] + [x^2]} dx$ is (A) 3 (B) 5 (C) 7 (D) 9



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128. The value of $\int_{-1}^1 [x[1 + \sin \pi x] + 1] dx$ is (A) 3 (B) 2 (C) 8 (D) 1



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129. The value of $\sum_{n=1}^{1000} \int_{n-1}^n e^{x - [x]} dx$, where $[x]$ is the greatest integer function, is (A) $\frac{e^{1000} - 1}{1000}$ (B) $\frac{e - 1}{1000}$ (C) $\frac{e^{1000} - 1}{e - 1}$ (D) $1000(e - 1)$



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130. If $a = \int_{\frac{1}{2}}^2 \frac{1}{x} \cot^7 \left(x - \frac{1}{x} \right) dx$, then (A) $a = 0$ (B) $0 < a < 1$ (C) $a > 0$ (D) none of these



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131. The value of the integral $\int_0^{100\pi} \sqrt{1 - \cos x} dx$ is equal to (A) $300\sqrt{2}$
(B) $200\sqrt{2}$ (C) $400\sqrt{2}$ (D) $500\sqrt{2}$



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132. $\int_0^2 x^3 \left[1 + \cos\left(\frac{\pi x}{2}\right) \right] dx$, where $[x]$ denotes the integral part of x ,
is equal to (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) none of these



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133. $\int_0^1 [x^2 - x + 1] dx$, where $[x]$ denotes the integral part of x , is (A)
1 (B) 0 (C) 2 (D) none of these



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134. If $[x]$ denotes the integral part of x and
 $a = \int_{-1}^0 \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$, $b = \int_0^1 \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} d(x - [x])$, then (A) $a = b$ (B)
 $a = -b$ (C) $a = 2b$ (D) none of these



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135. $\int_0^\pi \frac{dx}{1 + 10^{\cos x}} + \int_{-1}^1 \log\left(\frac{2-x}{2+x}\right) dx =$ (A) $\frac{\pi}{2}$ (B) $-\pi$ (C) 0 (D)
none of these



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136. Let $f(x) = \int_0^x \frac{\sin^{100} t}{\sin^{100} t + \cos^{100} t} dt$ then $f(2\pi) =$ (A) $2f\left(\frac{\pi}{2}\right)$ (B)
 $4f\left(\frac{\pi}{2}\right)$ (C) $f\left(\frac{\pi}{2}\right)$ (D) none of these



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137. Let $[x]$ denote the greatest integer less than or equal to x , then

$$\int_0^{\frac{\pi}{4}} \sin x d(x - [x]) = \text{(A) } \frac{1}{2} \text{ (B) } 1 - \frac{1}{\sqrt{2}} \text{ (C) } 1 \text{ (D) none of these}$$



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138. the value of $\int_{\frac{1}{e} \rightarrow \tan x} \frac{tdt}{1+t^2} + \int_{\frac{1}{e} \rightarrow \cot x} \frac{dt}{t \cdot (1+t^2)} =$



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139. The value of $\int_{-1}^1 \frac{\sin^2 x}{\left[\frac{x}{\sqrt{2}} \right] + \frac{1}{2}} dx$, where $[x]$ =greatest integer less

than or equal to x , is (A) 1 (B) 0 (C) 4 – sin 4 (D) none of these



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140. If $I = \int_0^1 \frac{x dx}{8+x^3}$ then the smallest interval in which I lies is (A) $\left(0, \frac{1}{8}\right)$ (B) $\left(0, \frac{1}{9}\right)$ (C) $\left(0, \frac{1}{10}\right)$ (D) $\left(0, \frac{1}{7}\right)$



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141. The value of $\int_0^1 e^{x^2} dx$ is (A) less than e (B) greater than 1 (C) less than e but greater than 1 (D) all of these



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142. The value of $\int_0^{[x]} \frac{2^x}{2^{[x]}} dx$, where $[x]$ denotes the greatest integer function is (A) $[x]\log 2$ (B) $\frac{x}{\log 2}$ (C) $\frac{1}{2} \frac{x}{\log 2}$ (D) none of these



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143. $\int_0^1 \frac{\tan^{-1} x}{x} dx - \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{t}{\sin t} dt$ has the value (A) -1 (B) 1 (C) 2 (D) 0



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144. $\int_0^\pi \cos 2x \log \sin x dx =$ (A) π (B) $-\frac{\pi}{2}$ (C) $\frac{\pi}{2}$ (D) none of these



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145. If $a, b (a < b)$ be the points of discontinuity of function $f(f(f(x)))$, where $f(x) = \frac{1}{1-x}$, $x \neq 1$, then $\int_a^b \frac{f(x)}{f(x) + f(1-x)} dx =$ (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2



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146. The value of $\int_{-1}^3 \left[\tan^{-1} \left(\frac{x}{x^2 + 1} \right) + \tan^{-1} \left(\frac{x^2 + 1}{x} \right) \right] dx =$ (A) $\frac{\pi}{2}$ (B) 2π (C) π (D) $\frac{\pi}{4}$



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147. $\int_{-1}^1 \sin^{-1} \left(\frac{x}{1+x^2} \right) dx =$ (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) 0



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148. Let $g(x) = \int_0^x f(t). dt$, where f is such that $\frac{1}{2} \leq f(t) \leq 1$ for $t \in [0, 1]$ and $0 \leq f(t) \leq \frac{1}{2}$ for $t \in [1, 2]$. Then $g(2)$ satisfies the inequality

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149. If f be decreasing continuous function satisfying $f(x + y) = f(x) + f(y) - f(x)f(y)Yx, y \in R, f'(0) = 1$ then $\int_0^1 f(x)dx$ is

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150. $\int_0^x [\sin t]dt$, where $x \in (2n\pi, (2n+1)\pi)$, $n \in N$, and $[.]$ denotes the greatest integer function is equal to (a) $-n\pi$ (b) $-(n+1)\pi$ (c) $2n\pi$ (d) $-(2n+1)\pi$

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151. Find the value of : $\int_0^{10} e^{2x - [2x]} d(x - [x])$ where $[.]$ denotes the greatest integer function).



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152. If 1 lies between the roots of equation $y^2 - my + 1 = 0$ and $[x]$ denotes the integral part of x , then $\left[\left(\frac{4|x|}{x^2 + 16} \right) \right]$ where $x \in R$ is equal to



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153. $\int_{-10}^0 \left(\left| \frac{2[x]}{3x - [x]} \right| / \frac{2[x]}{3x - [x]} \right) dx$ is equal to (where $[*]$ denotes greatest integer function.) is equal to (where $[*]$ denotes greatest integer function.)



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154. $\int_0^{2\pi} e^{\sin^2 nx} \tan nx dx =$ (A) 1 (B) π (C) 2π (D) 0



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155. $\int_a^b \frac{|x|}{x} dx, a < b,$ is equal to (A) $b - a$ (B) $b + a$ (C) $|b| - |a|$ (D)
 $|b| + |a|$



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156. The equation $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left\{ a|\sin x| + \frac{b \sin x}{1 + \cos^2 x} + c \right\} dx = 0$ where
 a, b, c are constants gives a relation between



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157. Let $f(x) = \max . \{2 - x, 2, 1 + x\}$ then $\int_{-1}^1 f(x) dx =$ (A) 0 (B) 2
(C) $\frac{9}{2}$ (D) none of these





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158. Let $f(x)$ be a continuous function such that $f(a - x) + f(x) = 0$ for all $x \in [0, a]$. Then $\int_0^a \frac{dx}{1 + e^{f(x)}} =$ (A) a (B) $\frac{a}{2}$ (C) $\frac{1}{2}f(a)$ (D) none of these



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159. Let $f: R \xrightarrow{\text{and}} g: R \xrightarrow{\text{be}}$ continuous function. Then the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x) + f(-x)][g(x) - g(-x)]dx$ is (a) π (b) 1 (c) -1 (d) 0



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160. If $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1 + \sin x} dx = k(\sqrt{2} - 1)$, then $k =$ (A) 0 (B) π (C) 2π (D) none of these



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161. $\int_{\log\left(\frac{1}{3}\right)}^{\log 3} \sin\left(\frac{e^x - 1}{e^x + 1}\right) dx =$ (A) $\log 3$ (B) $\sin 3$ (C) $2 \sin 3$ (D) 0



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162. Let $f(x)$ be a continuous function in R such that $f(x) + f(y) = f(x + y)$, then $\int_{-2}^2 f(x) dx =$ (A) $2 \int_0^2 f(x) dx$ (B) 0 (C) $2f(2)$ (D) none of these



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163. If $f(x)$ and $g(x)$ be continuous functions in $[0, a]$ such that $f(x) = f(a - x)$, $g(x) + g(a - x) = 2$ and $\int_0^a f(x) dx = k$, then $\int_0^a f(x)g(x) dx =$ (A) 0 (B) k (C) $2k$ (D) none of these



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164. Let $f(x)$ be a continuous function in R such that $f(x)$ does not vanish for all $x \in R$. If $\int_1^5 f(x)dx = \int_{-1}^5 f(x)dx$, then in R , $f(x)$ is (A) an even function (B) an odd function (C) a periodic function with period 5 (D) none of these



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165. Let $f(x)$ be an integrable odd function in $[-5, 5]$ such that $f(10+x) = f(x)$, then $\int_x^{10+x} f(t)dt =$ (A) 0 (B) $2\int_0^5 f(x)dx$ (C) > 0 (D) none of these



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166. If $\int_0^1 xe^{x^2}dx = k \int_0^1 e^{x^2}dx$, then (A) $k > 1$ (B) $0 < k < 1$ (C) $k = 1$ (D) none of these



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167. If $\int_0^1 e^x - 2(x - \alpha) dx = 0$, then 'a'



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168. Let $a = \int_0^{\frac{\pi}{2}} \frac{\sin x}{x} dx$, then (A) $0 < a < 1$ (B) $a > 2$ (C) $1 < a < \frac{\pi}{2}$
(D) none of these



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169. If $f(x) = \int_0^x \frac{e^{\cos t}}{e^{\cos t} + e^{-(\cos t)}} dt$, then $2f(\pi) =$ (A) 0 (B) π (C) $-\pi$
(D) none of these



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170. $\int_0^{\sqrt{2}} [x^2] dx$ is equal to (A) $2 - \sqrt{2}$ (B) $2 + \sqrt{2}$ (C) $\sqrt{2} - 1$ (D)
 $\sqrt{2} - 2$



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171. $\int_{\pi}^{10\pi} |\sin x| dx$ is equal to (A) 20 (B) 8 (C) 10 (D) 18



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172. If $f(a + b - x) = f(x)$, then $\int_a^b xf(x)dx$ is equal to (A) $\frac{a-b}{2} \int_a^b f(a+b-x)dx$ (B) $\frac{a+b}{2} \int_a^b f(b-x)dx$ (C) $\frac{a+b}{2} \int_a^b f(x)dx$ (D) $\frac{b-a}{2} \int_a^b f(x)dx$



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173. The value of $\int_{-2}^3 |1 - x^2| dx$ is (A) $\frac{7}{3}$ (B) $\frac{14}{3}$ (C) $\frac{28}{3}$ (D) $\frac{1}{3}$



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174. If $\int_0^\pi x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$, then A is (A) $\frac{\pi}{2}$ (B) π (C) 0 (D) 2π



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175. If $f(x) = \frac{e^x}{1 + e^x}$, $I_1 = \int_{f(-a)}^{f(a)} x g(x(1 - x)) dx$ and $I_2 = \int_{f(-a)}^{f(a)} g(x(1 - x)) dx$, then $\frac{I_2}{I_1} =$ (A) -1 (B) -3 (C) 2 (D) 1



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176. The value of the integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx$ is



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177. If $\int_0^\pi x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$, then A is (A) $\frac{\pi}{2}$ (B) π (C) 0 (D) 2π



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178. $\int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}} \left\{ (\pi + x)^3 + \cos^2(x + 3\pi) \right\} dx$ is equal to (A) $\frac{\pi}{4} - 1$ (B) $\frac{\pi^4}{32}$ (C) $\frac{\pi^4}{32} + \frac{\pi}{2}$ (D) $\frac{\pi}{2}$



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179. The value of $\int_1^a [x] f'(x) dx$, where $a > 1$, and $[x]$ denotes the greatest integer not exceeding x , is
 $af(a) - \{f(1)f(2) + \dots + f([a])\}$ $[a]f(a) - \{f(1) + f(2) + \dots + f([a])\}$
 $[a]f(a) - \{f(1) + f(2) + \dots + fA\}$ $af([a]) - \{f(1) + f(2) + \dots + fA\}$



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180. The value of the integral $\int_{e^{-1}}^{e^2} \left| \frac{(\log_e x)}{x} \right| dx$ is (a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 5



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181. If $f(x) = \begin{cases} e^{\cos x} \sin x & |x| \leq 2 \\ 2 & otherwise \end{cases}$ then $\int_{-2}^3 f(x)dx =$ (A) 0 (B) 1 (C) 2 (D) 3



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182. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x} dx, a > 0$ is



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183. The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left([x] + \log\left(\frac{1+x}{1-x}\right) \right) dx$ equals



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184. Let $T > 0$ be a fixed real number. Suppose f is continuous function such that for all $x \in R$, $f(x+T) = f(x)$. If $I = \int_0^T f(x)dx$, then the value of $\int_3^{3+3T} f(2x)dx$ is (A) I (B) $2I$ (C) $3I$ (D) $6I$



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185. $\int_{\frac{1}{n}}^{\frac{an-1}{n}} \frac{\sqrt{x}}{\sqrt{a-x} + \sqrt{x}} dx =$ (A) $\frac{a}{2}$ (B) $\frac{na+2}{2n}$ (C) $\frac{na-2}{2n}$ (D)

none of these



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186. $\left(\sum_{n=1}^{10} \int_{-2n-1}^{-2n} \sin^{27}(x) dx + \sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27}(x) dx \right)$



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187. $\int_0^{2n\pi} \left\{ |\sin x| - \left| \frac{1}{2} \sin x \right| \right\} dx =$ (A) n (B) $2n$ (C) $-2n$ (D) $\frac{1}{2}$



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188. The value of the integral $\int_0^{\frac{\pi}{2}} (\sin^{100} x - \cos^{100} x) dx$ is (A) $\frac{1}{100}$ (B) $\frac{100!}{(100)^{100}}$ (C) $\frac{\pi}{100}$ (D) 0



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189. $\int_{-1}^1 \frac{17x^5 - x^4 + 29x^3 - 31x + 1}{x^2 + 1} dx$ is equal to (A) $\frac{4}{5}$ (B) $\frac{5}{4}$ (C) $\frac{4}{3}$ (D) $\frac{3}{4}$



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190. $\int_0^{4014} \frac{2^x}{2^x + 2^{4014-x}} dx =$ (A) 2^{2007} (B) 2^{4014} (C) 4014 (D) 2007



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191. $\int_0^3 x(3-x)^{\frac{3}{2}} dx =$ (A) $\frac{108\sqrt{3}}{35}$ (B) $-\frac{108\sqrt{3}}{35}$ (C) $\frac{54\sqrt{3}}{35}$ (D) none
of these



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192. $\int_2^4 \log[x] dx$ is (A) $\log 2$ (B) $\log 3$ (C) $\log 5$ (D) none of these



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193. If $I = \int_0^{3\pi} f(\cos^2 x) dx$ and $J = \int_0^{\pi} f(\cos^2 x) dx$, then (A) $I = 5J$
(B) $I = J$ (C) $I = 3J$ (D) none of these



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194. $\int_{\log\left(\frac{1}{2}\right)}^{\log 2} \log\left(x + \sqrt{x^2 + 1}\right) dx =$ (A) $2 \log 2$ (B) $1 - \log 2$ (C)
 $1 + \log 2$ (D) 0



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195. If $I = \int_{-\pi}^{\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx$, then I equals (A) $\frac{\pi}{2}$ (B) 2π (C) π (D) $\frac{\pi}{4}$



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196. The value of the integral $\int_{-a}^a \frac{x e^{x^2}}{1+x^2} dx$ is (A) e^{a^2} (B) 0 (C) e^{-a^2} (D) a



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197. The function f is continuous and has the property $f(f(x)) = 1 - x$ for all $x \in [0, 1]$ and $J = \int_0^1 f(x) dx$. Then which of the following is/are true? (A) $f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) = 1$ (B) the value of J equals to $\frac{1}{2}$ (C) $f\left(\frac{1}{3}\right) \cdot f\left(\frac{2}{3}\right) = 1$ (D) $\int_0^{\frac{\pi}{2}} \frac{\sin x dx}{(\sin x + \cos x)^3}$ has the same value as J



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198. A function f is defined by

$$f(x) = \int_0^{\pi} \cos t \cos(x-t) dt, 0 \leq x \leq 2\pi.$$
 Which of the following

hold(s) good? (A) $f(x)$ is continuous but not differentiable in $(0, 2\pi)$ (B)

There exists at least one $c \in (0, 2\pi)$ such that $f'(c) = 0$ (C) Maximum

value of f is $\frac{\pi}{2}$ (D) Minimum value of f is $-\frac{\pi}{2}$



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199. Let $L = \lim_{n \rightarrow \infty} \int_a^{\infty} \frac{ndx}{1+n^2x^2}$ where $a \in R$ then $\cos L$ can be (A)
-1 (B) 0 (C) 1 (D) $\frac{1}{2}$



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200. The value of $\int_0^{\infty} \frac{x}{[(1+x)(1+x^2)]} dx$ is (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) same as
 $\int_0^{\infty} \frac{dx}{[(1+x)(1+x^2)]}$ (D) cannot be evaluated



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201. Let $f(x) = \int_1^x \frac{3^t}{1+t^2} dt, x > 0$, then (A) for $0 < \alpha < \beta, f(\alpha) < f(\beta)$ (B) for $0 < \alpha < \beta, f(\alpha) > f(\beta)$ (C) $f(x) + \frac{\pi}{4} < \tan^{-1} x, \forall x \geq 1$ (D) $f(x) + \frac{\pi}{4} > \tan^{-1} x, \forall x \geq 1$



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202. Let $\phi(x, t) = \begin{cases} x(t-1) & x \leq t \\ t(x-1) & t < x \end{cases}$, where t is a continuous function of x in $[0, 1]$. Let $g(x) = \int_0^1 f(t)\phi(x, t)dt$, then $g''(x) =$ (A) $g(0) = 1$ (B) $g(0) = 0$ (C) $g(1) = 1$ (D) $g''(x) = f(x)$



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203. Let $f(x) = \int_0^{\sin^2 x} \sin^{-1}(\sqrt{t})dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t})dt$, then (A) $f(x)$ is a constant function (B) $f\left(\frac{\pi}{4}\right) = 0$ (C) $f\left(\frac{\pi}{3}\right) = \frac{\pi}{4}$ (D) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$



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204. The value of the integral $\int_0^{\frac{\pi}{4}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} =$ (A)
 $\frac{1}{ab} \tan^{-1} \left(\frac{b}{a} \right) (a, b > 0)$ (B) $\frac{1}{ab} \tan^{-1} \left(\frac{b}{a} \right) (a, b < 0)$ (C)
 $\frac{\pi}{4} (a = 1, b = 1)$ (D) none of these



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205. $\int_{\tan^{-1} a}^{\cot^{-1} a} \frac{dx}{1 + \tan x}$ cannot take the value(s) (A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$
(D) π



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206. If $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$, then A is (A) $\frac{\pi}{2}$ (B) π (C) 0
(D) 2π



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207. Let $\int_a^b \frac{f(a+b-x)}{f(a+b-x) + f(a+b-(a+b-x))} dx = 4$, then (A)
 $a = -1, b = 7$ (B) $a = 0, b = 8$ (C) $a = -10, b = 2$ (D)
 $a = 10, b = 18$



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208. Statement-1: If $\int_0^\infty e^{-ax} dx = \frac{1}{a}$, then $\int_0^\infty x^m e^{-ax} dx = \frac{\lfloor m \rfloor}{a^{m+1}}$
Statement-2: $\frac{d^n}{dx^n} (e^{kx}) = k^n e^{kx}$ and $\frac{d^n}{dx^n} \left(\frac{1}{x} \right) = \frac{(-1)^n \lfloor n \rfloor}{x^{n+1}}$ (A) Both
1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are
true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1
is false but 2 is true



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209. Statement-1: $\int_0^{[x]} 4^{x-[x]} dx = \frac{3[x]}{2 \log 2}$, Statement-2:
 $\int_0^{[x]} a^{x-[x]} dx = [x] \int_0^1 a^{x-[x]} dx$ (A) Both 1 and 2 are true and 2 is

the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct
explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true



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210. Statement-1: Let a, b, c be non zero real numbers and

$f(x) = ax^2 + bx + c$ satisfying

$\int_0^1 (1 + \cos^8 x) f(x) dx = \int_0^2 (1 + \cos^8 x) f(x) dx$ then the equation

$f(x) = 0$ has at least one root in $(0, 2)$. Statement-2: If $\int_a^b g(x) dx$

vanishes and $g(x)$ is continuous then the equation $g(x) = 0$ has at

least one real root in (a, b) . (A) Both 1 and 2 are true and 2 is the correct

explanation of 1 (B) Both 1 and 2 are true and 2 is not correct

explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true



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211. Let $I_1 = \int_0^1 \frac{e^x}{1+x} dx$ and $I_2 = \int_0^1 \frac{x^2 e^{x^2}}{2-x^3} dx$ Statement-1:
 $\frac{I_1}{I_2} = 3e$ Statement-2: $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$ (A) Both 1 and

2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true



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212. Statement-1: $\int_{-3}^3 x^8 \{x^9\} dx = 2 \times 3^7$, where $\{x\}$ denotes the fractional part of x . Statement-2: $[x] + [-x] = -1$, if x is not an integer, where $[x]$ denotes the integral part of x . (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true



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213. Let a function f be even and integrable everywhere and periodic with period 2. Let $g(x) = \int_0^x f(t)dt$ and $g(t) = k$. The value of $g(x+2) - g(x)$ is equal to (A) $g(1)$ (B) 0 (C) $g(2)$ (D) $g(3)$



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214. Let a function f be even and integrable everywhere and periodic with period 2. Let $g(x) = \int_0^x f(t)dt$ and $g(2) = k$. The value of $g(2)$ in terms of k is equal to (A) k (B) $2k$ (C) $3k$ (D) $5k$



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215. If $m > 0, n > 0$, the definite integral $I = \int_0^1 x^{m-1}(1-x)^{n-1}dx$

depends upon the values of m and n is denoted by $\beta(m, n)$, called the beta function. For example,

$$\int_0^1 x^3(1-x)^4dx = \int_0^1 x^{4-1}(1-x)^{5-1}dx = \beta(4, 5) \quad \text{and}$$
$$\int_0^1 x^{\frac{3}{2}}(1-x)^{-\frac{1}{2}}dx = \int_0^1 x^{\frac{5}{2}-1}(1-x)^{\frac{1}{2}-1}dx = \beta\left(\frac{5}{2}, \frac{1}{2}\right).$$

Obviously,

$\beta(n, m) = \beta(m, n)$. Now answer the question: The integral

$$\int_0^{\frac{\pi}{2}} \cos^{2m} \theta \sin^{2n} \theta d\theta = \begin{array}{l} \text{(A) } \frac{1}{2} \beta\left(m + \frac{1}{2}, n + \frac{1}{2}\right) \\ \text{(B) } 2\beta(2m, 2n) \\ \text{(C) } \beta(2m + 1, 2n + 1) \\ \text{(D) none of these} \end{array}$$



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216. If $m > 0, n > 0$, the definite integral $I = \int_0^1 x^{m-1}(1-x)^{n-1} dx$

depends upon the values of m and n is denoted by $\beta(m, n)$, called the beta function. For example,

$$\int_0^1 x^3(1-x)^4 dx = \int_0^1 x^{4-1}(1-x)^{5-1} dx = \beta(4, 5) \quad \text{and}$$

$$\int_0^1 x^{\frac{3}{2}}(1-x)^{-\frac{1}{2}} dx = \int_0^1 x^{\frac{5}{2}-1}(1-x)^{\frac{1}{2}-1} dx = \beta\left(\frac{5}{2}, \frac{1}{2}\right). \text{ Obviously,}$$

$\beta(n, m) = \beta(m, n)$. Now answer the question: If

$$\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx = k \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx, \text{ then } k \text{ is equal to (A)}$$

- (B) 1 (C) $\frac{n}{m}$ (D) none of these



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217. If $m > 0, n > 0$, the definite integral $I = \int_0^1 x^{m-1}(1-x)^{n-1} dx$

depends upon the values of m and n is denoted by $\beta(m, n)$, called the beta function. For example,

$$\int_0^1 x^3(1-x)^4 dx = \int_0^1 x^{4-1}(1-x)^{5-1} dx = \beta(4, 5) \quad \text{and}$$

$$\int_0^1 x^{\frac{3}{2}}(1-x)^{-\frac{1}{2}} dx = \int_0^1 x^{\frac{5}{2}-1}(1-x)^{\frac{1}{2}-1} dx = \beta\left(\frac{5}{2}, \frac{1}{2}\right). \text{ Obviously,}$$

$\beta(n, m) = \beta(m, n)$. Now

answer

the

question: If

$$\int_0^2 (8 - x^3)^{-\frac{1}{3}} dx = k\beta\left(\frac{1}{3}, \frac{2}{3}\right), \text{ then } k \text{ equals to (A) 1 (B) } \frac{1}{2} \text{ (C) } \frac{1}{3} \text{ (D) } \frac{1}{4}$$



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218. Let $I = \int_0^{10\pi} \frac{\cos 6x \cos 7x \cos 8x \cos 9x}{1 + e^{2 \sin^3 4x}} dx$ Now answer the question: If $I = k \int_0^{\frac{\pi}{2}} \cos 6x \cos 7x \cos 8x \cos 9x dx$, then $k =$ (A) 5 (B) 10 (C) 20 (D) none of these



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219. Let $I = \int_0^{10\pi} \frac{\cos 6x \cos 7x \cos 8x \cos 9x}{1 + e^{2 \sin^3 4x}} dx$ Now answer the question: If $I = C \int_0^{\frac{\pi}{2}} \cos 6x \cos 8x \cos 2x dx$, then $C =$ (A) 5 (B) 10 (C) 20 (D) none of these



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220. Let $I = \int_0^{10\pi} \frac{\cos 6x \cos 7x \cos 8x \cos 9x}{1 + e^{2 \sin^3 4x}} dx$ Now answer the question: The value of I equals (A) $\frac{5\pi}{4}$ (B) $\frac{5\pi}{8}$ (C) $\frac{5\pi}{16}$ (D) $\frac{5\pi}{32}$



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221. Evaluate: if $\int f(x)dx = g(x)$, then $\int f^{-1}(x)dx$



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222. If $f(x)$ is an integrable function and $f^{-1}(x)$ exists, then $\int f^{-1}(x)dx$ can be easily evaluated by using integration by parts. Sometimes it is convenient to evaluate $\int f^{-1}(x)dx$ by putting $z = f^{-1}(x)$. Now answer the question. If $I_1 = \int_a^b [f^2(x) - f^2(a)]dx$ and $I_2 = \int_{f(a)}^{f(b)} 2x[b - f^{-1}(x)]dx \neq 0$, then $\frac{I_1}{I_2} =$ (A) 1:2 (B) 2:1 (C) 1:1 (D) none of these



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223. If $A = \int_0^1 x^{50}(2-x)^{50} dx$, $B = \int_0^1 x^{50}(1-x)^{50} dx$ and $\frac{A}{B} = 2^n$,

then the value of n is ...



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224. Let $u = \int_0^1 \frac{\ln(x+1)}{x^2+1} dx$ and $v = \int_0^{\frac{\pi}{2}} \ln(\sin 2x) dx$, then
 $u = -\frac{\pi}{2} \ln 2$ (b) $4u + v = 0$ (c) $u + 4v = 0$ (d) $u = \frac{\pi}{8} \ln 2$



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225. If $f(4-x) = f(4+x)$ and $f(8-x) = f(8+x)$ and $f(x)$ is a function for which $\int_0^8 f(x) dx = 5$. Then $\int_0^{200} f(x) dx$ is equal to



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226. Let $f(x) = \int_1^x \frac{dt}{t^3(1+t^3)^{\frac{1}{3}}}$ and $\lim_{x \rightarrow \infty} f(x) = \frac{a^2 - 1}{a^3}$, then $a^{30} =$



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227. Let $a = \int_0^{\log 2} \frac{2e^{3x} + e^{2x} - 1}{e^{3x} + e^{2x} - e^x + 1} dx$, then $4e^a =$



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228. If $\int_0^1 \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}} dx = \frac{1}{a}(2^a - 1)$, then the value of $4a^2$ is

...



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229. $\alpha = \frac{\pi}{4020}$ and $\beta = \frac{2009}{4020}\pi$ and $\int_{\alpha}^{\beta} \frac{dx}{1 + \tan x} = \frac{k\pi}{8040}$, then k is equal to



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230. If $\int_0^{\frac{\pi^2}{4}} (2 \sin \sqrt{x} + \sqrt{x} \cos \sqrt{x}) dx = \frac{\pi^2}{n}$, then $n =$



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231. If $\int_0^{4+2\sqrt{3}} \frac{16}{4+x^2} dx = \frac{3+m\pi}{12}$, then $m= \dots$



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232. If $I = \int_{-2\pi}^{5\pi} \cot^{-1}(\tan x) dx$. Then, $2\frac{I}{\pi^2}$ is



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233. The value of $I = \int_0^3 \left([x] + \left[x + \frac{1}{3} \right] + \left[x + \frac{2}{3} \right] \right) dx$ where $[.]$

denotes the greatest integer function is equal to



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- 234.** $\int_1^{10\pi} ([\sec^{-1} x] + [\cot^{-1} x]) dx$ where $[.]$ denotes the greatest integer function is (A) $10\pi - \sec 1$ (B) $10\pi + \sec 1$ (C) $10\pi - \sec 1 + \cot 1$ (D) $\sec 1 + \cot 1$



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- 235.** $f(x)$ is a continuous function for all real values of x and satisfies $\int_n^{n+1} f(x) dx = \frac{n^2}{2} \forall n \in I$. Then $\int_{-3}^5 f(|x|) dx$ is equal to (b) $\frac{35}{2}$ (c) $\frac{17}{2}$ (d) none of these



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- 236.** For any $t \in R$ and f be a continuous function, let $I_1 = \int_{\sin^2 t}^{1+\cos^2 t} x \cdot f(x(2-x)) dx$ and $I_2 = \int_{\sin^2 t}^{1+\cos^2 t} f(x(2-x)) dx$. Then $\frac{I_1}{I_2}$ is (i) 0 (ii) 1 (iii) 2 (iv) 3



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