MATHS

BOOKS - KC SINHA MATHS (HINGLISH)

DIFFERENTIAL EQUATIONS - FOR COMPETITION

Solved Examples

1. The differential equation representing the family of curves $y^2=2c\left(x+\sqrt{c}\right),$ where c is a positive parameter, is of (a) order 1 (b) order 2 (c) degree 3 (d) degree 4

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2. Form the differential equation having $y=\left(\sin^{-1}x\right)^2+A\cos^{-1}x+B, whereAandB$ are arbitrary constants, as its general solution.

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3. Find the order of the differential equation of the family of curves. $y = a \sin x + b \cos(x + c)$, where a, b, c are parameters.

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4. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where $c_1 \text{and} c_2$ are arbitrary constants, is (1) $y' = y^2$ (2) $y'' = y' y$ (3) $yy'' = y'$ (4) $yy'' = (y')^2$

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5. Show that any equation of the form $yf(xy)dx + xg(xy)dy = 0$ can be converted to variable separable form by substitution $xy = v$.

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6. Solve the following differential equation: $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$

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7. Solve: $\frac{dy}{dx}=e^{x-y}+e^{2\log x-y}$

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8. Solve the differential equation $\frac{dy}{dx}=\frac{2}{x+y}$

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9. Solve: $\frac{dy}{dx}=\sin(x+y)+\cos(x+y)$

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10. Solve the differential equation $(x^2 + 4y^2 + 4xy)dy = (2x + 4y + 1)dx$

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11. Solve: $xdx + ydy = xdy - ydx$

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12. Solve $\dfrac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \dfrac{y^4}{x^2}$

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13. Solve: $\left(1 + 2e^{\frac{x}{y}}\right)dx + 2e^{\frac{x}{y}}\left(1 - \dfrac{x}{y}\right)dy = 0$

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14. Solve $\frac{dy}{dx} = \frac{s \in}{\sin 2y - x \cos y}$

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15. Solve: $\frac{dy}{dx} = \frac{y}{2y \ln y + y - x}$

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16. Solve: $\frac{dy}{dx} + xy = xy^2$

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17. Solve: $\frac{dy}{dx} + x(x+y) = x^3(x+y)^3 - 1$

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18. Solve: $\sin y \frac{dy}{dx} = \cos y(1 - x \cos y)$

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19. Solve: $\frac{dy}{dx} = \frac{yf'(x) - y^2}{f(x)}$, where $f(x)$ is a given function of x

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20. If $\phi(x)$ is a differentiable function, then the solution of the different equation $dy + \{y\phi'(x) - \phi(x)\phi'(x)\}dx = 0,$ is

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21. Let $u(x)$ and $v(x)$ satisfy the differential equation $\frac{du}{dx} + p(x)u = f(x)$ and $\frac{dv}{dx} + p(x)v = g(x)$ are continuous functions. If $u(x_1)$ for some x_1 and $f(x) > g(x)$ for all $x > x_1,$ prove that any point $(x, y),$ where $x > x_1,$ does not satisfy the equations $y = u(x)$ and $y = v(x)$.

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22. Solve $\frac{y+\sin x\cos^2(xy)}{\cos^2(xy)}dx+\left(\frac{x}{\cos^2(xy)}+\sin y\right)dy=0$

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23. Solve: $(2x\log y)dx+\left(\frac{x^2}{y}+3y^2\right)dy=0$

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24. Solve: $e^{x\frac{(y^2-1)}{y}}\{xy^2dy+y^3dx\}+\{ydx-xdy\}=0$

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25. Solve: $x^2dy-y^2dx+xy^2(x-y)dy=0$

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26. Solve: $xdy\left(y^2e^{xy}+e^{\frac{x}{y}}\right)=ydx\left(e^{\frac{x}{y}}-y^2e^{xy}\right)$

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27. Let $y=f(x)$ be a curve passing through $(1, 1)$ such that the triangle formed by the coordinate axes and the tangent at any point of the curve lies in the first quadrant and has area 2. Form the differential equation and determine all such possible curves.

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28. A curve $y=f(x)$ passes through the point $P(1, 1)$. The normal to the curve at P is $a(y-1)+(x-1)=0$. If the slope of the tangent at any point on the curve is proportional to the ordinate of the point. Determine the equation of the curve. Also obtain the area bounded by the y-axis, the curve and the normal to the curve at P.

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29. Find all the curves $y = f(x)$ such that the length of tangent intercepted between the point of contact and the x-axis is unity.

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30. Find the equation of a curve passing through the point (1.1) if the perpendicular distance of the origin from the normal at any point P(x, y) of the curve is equal to the distance of P from the x-axis.

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31. A country has a food deficit of 10%. Its population grows continuously at a rate of 3% per year. Its annual food production every year is 4% more than that of the last year. Assuming that the average food requirement per person remains constant, prove that the country will become self-sufficient in food after n years, where n is the smallest integer bigger than or equal to $\frac{\ln 10 - \ln 9}{\ln(1.04) - 0.03}$

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32. A hemi-spherical tank of radius 2 m is initially full of water and has an outlet of $12cm^2$ cross-sectional area at the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law $v(t) = \sqrt{0.62gh(t)}$, where $v(t)$ and $h(t)$ are, respectively, the velocity of the flow through the outlet and the height of water level above the outlet and the height of water level above the outlet at time t, and g is the acceleration due to gravity. Find the time it takes to empty the tank.

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33. There are 100 million litres of fluoridated water in the reservoir containing a city\'s water supply, and the water contains 700 kg of fluoride. To decrease the fluoride content, fresh water runs into the reservoir at the rate of 3 million litres per day, and the mixture of water and fluoride, kept uniform, runs-out of the reservoir at the same rate. How many kilograms of fluoride are in the reservoir 60 days after the pure water started to flow into the reservoir?

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34. The ordinate and the normal at any point P on the curve meet the x-axis at points A and B respectively. Find the equation of the family of curves satisfying the condition , The product of abscissa of P and AB =arithmetic mean of the square of abscissa and ordinate of P.

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35. Consider a curve $y = f(x)$ in xy-plane. The curve passes through (0,0) and has the property that a segment of tangent drawn at any point $P(x, f(x))$ and the line y = 3 gets bisected by the line $x + y = 1$ then the equation of curve, is

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36. If the velocity of flow of water through a small hole is $0.6\sqrt{2gy}$, where g is the acceleration due to gravity and y is the height of water level above the hole, find the time required to empty a tank having the shape of a right circular cone of base radius a and height h filled completely with water and having a hole of area A_0 in the base.

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37. An inverted cone of height H, and radius R is pointed at bottom. It is completely filled with a volatile liquid. If the rate of evaporation is directly proportional to the surface area of the liquid in contact with air (constant of proportionality k $>$ 0). Find the time in which whole liquid evaporates.

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38. The tangent at a point 'P' of a curve meets the axis of 'y' in N, the parallel through 'P' to the axis of 'y' meets the axis of X at M, O is the origin of the area of ΔMON is constant then the curve is (A) circle C) ellipse (D) hyperbola (B) parabola

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39. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $f(x) =$

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40. The differential equation of the family of curves whose equation is $(x-h)^2 + (y-k)^2 = a^2$, where a is a constant, is

(A) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2 \frac{d^2y}{dx^2}$

(B) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2 \left(\frac{d^2y}{dx^2}\right)^2$

(C) $\left[1 + \left(\frac{dy}{dx}\right)\right]^3 = a^2 \left(\frac{d^2y}{dx^2}\right)^2$

(D) none of these

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41. Let y_1 and y_2 be two different solutions of the differential equation $\frac{dy}{dx} + P(x) \cdot y = Q(x)$.Answer the question:If $\alpha y_1 + \beta y_2$ is a solution of the given differential equation then $\alpha + \beta$ is (A) 0 (B) 1 (C) -1 (D) none of these

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42. Data could not be retrieved.

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43. Suppose we define the definite integral using the following formula $\int_a^b f(x)dx = \frac{b-a}{2}(f(a)+f(b))$, for more accurate result for $c \in (a, b), F(c) = \frac{c-a}{2}(f(a)+f(c)) + \frac{b-c}{2}(f(b)+f(c))$ and when $c = \frac{a+b}{2}, \int_a^b f(x)dx = \frac{b-a}{4}(f(a)+f(b)+2f(c))$In the above comprehension, if $f(x)$ is a polynomial and

$\lim_{t \to a} \frac{\int_a^t f(x)dx - \frac{t-a}{2}[f(t)+f(a)]}{(t-a)^3} = 0$ for all a then the degree of $f(x)$ can at most be (A) 1 (B) 2 (C) 3 (D) 4

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44. Let f be a real-valued differentiable function on R (the set of all real numbers) such that $f(1) = 1$. If the $y - \in tercept$ of the tangent at any point $P(x, y)$ on the curve $y = f(x)$ is equal to the cube of the abscissa of P, then the value of $f(-3)$ is equal to_______

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Exercise

1. Find the degree of the differential equation:

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{4}} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}$$

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2. Find order and degree: $\cos\left(\frac{dy}{dx}\right) = x + y$

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3. Find order and degree: $e^{\frac{dy}{dx}} = x^2 + 1$

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4. Find order and degree: $e^{\frac{dy}{dx}} = \left(1 + \frac{d^2y}{dx^2}\right)$

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5. Find order and degree: $\log_e\left(1 + \frac{d^2y}{dx^2}\right) = x$

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6. Find the differential equation of the family of curves $y = a \sin(bx + c)$, where a, b, c are parameters.

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7. Find the differential equation of $xy = ae^{x} + be^{-x}$.

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8. Find the differential equation of all straight lines touching the circle $x^2 + y^2 = a^2$

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9. Form the differential equation of the family of circles touching the y-axis at origin.

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10. Form the differential equation of the family of circles touching the x-axis at origin.

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11. Solve: $(1 - x^2 y^2)dx = ydx + xdy$

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12. Reduce the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$ using the transformation $y = v(x).\, e^x$. Hence solve the equation when $y = 1, \frac{dy}{dx} = 0$, for $x = 0$

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13. Solve: $2y\frac{dy}{dx} = e^{\frac{x^2+y^2}{x}} + \frac{x^2+y^2}{x} - 2x$

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14. Solve: $x\left(\frac{dy}{dx}\right)^2 + (y-x)\frac{dy}{dx} - y = 0$

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15. The slope of a curve at any point is the reciprocal of twice the ordinate at that point and it passes through the point(4,3). The equation of the curve is:

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16. A curve passes through the point $(5, 3)$ and at any point (x, y) on it, the product of its slope and the ordinate is equal to its abscissa. Find the equation of the curve and identify it.

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17. Show that the equation of the curve passing through the point $(1, 0)$ and satisfying the differential equation $(1 + y^2)dx - xydy = 0$ is $x^2 - y^2 = 1$

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18. If $p = 9, v = 3$, then find p is terms of v from the equation $\frac{dp}{dv} = v + \frac{1}{v^2}$

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19. Solve the equation $e^x dx + e^y(y + 1)dy = 0$

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20. Solve: $\left(\frac{dy}{dx}\right) + \frac{\sqrt{(x^2 - 1)(y^2 - 1)}}{xy} = 0$

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21. Solve the differential equation $\cos^2 x \frac{d^2y}{dx^2} = 1$

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22. Solve: $\tan^{-1}\left(\frac{dy}{dx}\right) = x + y$

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23. Determine the equation of the curve passing through the origin, in the form $y = f(x)$, which satisfies the differential equation $\frac{dy}{dx} = \sin(10x + 6y)$.

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24. Solution of $\left(\frac{x+y-a}{x+y-b}\right)\left(\frac{dy}{dx}\right) = \left(\frac{x+y+a}{x+y+b}\right)$

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25. Solve: $ydx - xdy + xy^2dx = 0$

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26. Solve $x\,dy - y\,dx = \sqrt{x^2 + y^2}dx$

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27. Solve $\left(xy^2 - \frac{e^1}{x^3}\right)dx - x^2ydy = 0$

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28. Solve: $x + y\frac{dy}{dx} = \left(a^2\frac{\left(x\frac{dy}{dx} - y\right)}{x^2 + y^2}\right)$

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29. $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$

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30. solve $\frac{xdx + ydy}{xdy - ydx} = \sqrt{\frac{a^2 - x^2 - y^2}{x^2 + y^2}}$

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31. Solve: $\frac{xdy}{x^2 + y^2} = \left(\frac{y}{x^2 + y^2} - 1\right)dx$

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32. Solve the differential equation $y + x\frac{dy}{dx} = x$

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33. Solve $\frac{dy}{dx} = \frac{(x+y)^2}{(x+2)(y-2)}$

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34. $\left(xe^{\frac{y}{x}} - y\sin\left(\frac{y}{x}\right)\right)dx + x\sin\left(\frac{y}{x}\right)dy = 0$

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35. Solve the differential equation $(xdy - ydx)y\sin\left(\frac{y}{x}\right) = (ydx + xdy)x\cos\left(\frac{y}{x}\right)$.

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36. $x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} + 2y^2 - x^2 = 0$

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37. Reduce the equation $y^3 dy + (x + y^2) dx = 0$ to a homogeneous equation and then solve it.

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38. Prove that the equation of a curve whose slope at (x, y) is $-\frac{x+y}{x}$ and which passes through the point $(2, 1)$ is $x^2 + 2xy = 8$

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39. Find the equation of the curve which passes through $(1, 0)$ and the slope of whose tangent at (x, y) is $\frac{x^2 + y^2}{2xy}$

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40. Solve: $x\frac{dy}{dx} = y(\log y - \log x + 1)$

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41. Solve $xdy = \left(y + x \frac{f\left(\frac{y}{x}\right)}{f'\left(\frac{y}{x}\right)} \right) dx$

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42. Find the general solution of the differential equation $(1 + \tan y)(dx - dy) + 2xdy = 0$

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43. Solve: $(1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}$

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44. If y_1 and y_2 are the solution of the differential equation $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x alone and $y_2 = y_1 z$, then prove that $z = 1 + \cdot e^{-\int \frac{Q}{y_1} dx}$, where c is an arbitrary constant.

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45. If $y+\frac{d}{dx}(xy)=x(\sin x+\log x), f\in dy(x)\cdot$

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46. $\frac{dy}{dx}=x^3y^3-xy$

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47. If $\frac{dy}{dx}+2y\tan x=\sin x$ and $y=0$, when $x=\frac{\pi}{3}$, show that the maximum value of y is $\frac{1}{3}$

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48. $\sec^2 y\frac{dy}{dx}+2x\tan y=x^3$

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49. $x\frac{dy}{dx} + y = y^2 \log x$

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50. $(x^2y^3 + xy)dy = dx$

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51. $\frac{dy}{dx} + 2\frac{y}{x} = \frac{y^3}{x^3}$

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52. $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$

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53. Solve $\left(\frac{dy}{dx}\right) = e^{x-y}(e^x - e^y)$.

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54. $\left(y^2 e^x + 2xy\right)dx - x^2 dy = 0$

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55. Solution of $\left(2x - 10y^3\right)\frac{dy}{dx} + y = 0$ is:

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56. $xy - \frac{dy}{dx} = y^3 e^{-x^2}$

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57. $\frac{dy}{dx} = \frac{2xy}{x^2 - 1 - 2y}$

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58. $\frac{dy}{dx}+yf'(x)-f(x)f'(x)=0$

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59. $y\sin x\frac{dy}{dx}=\left(\sin x-y^2\right)\cos x$

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60. Solution of the equation `cos^2x(dy)/(dx)-(tan2x)y=cos^4x ,|x|

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61. Solve: $\left(x^2-ay\right)dx+\left(y^2-ax\right)dy=0$

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62. Solve: $ydx - xdy + (1+x^2)dx + x^2 \sin y dy = 0$

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63. The solution of the differential equation $\left\{1 + x\sqrt{(x^2+y^2)}\right\}dx + \left\{\sqrt{(x^2+y^2)} - 1\right\}ydy = 0$ is equal to (a) $(b)(c)(d)x^{(e)2(f)}(g) + (h)\frac{(i)(j)y^{(k)2(l)}(m)}{n}2(o)(p) + (q)\frac{1}{r}3(s)(t)(u)(v$

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64. Solve: $(x + \log y)dy + ydx = 0$

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65. The tangents to a curve at a point on it is perpendicular to the line joining the point with the origin. Find the equation of the curve.

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66. The tangent at a point 'P' of a curve meets the axis of 'y' in N, the parallel through 'P' to the axis of 'y' meets the axis of X at M, O is the origin of the area of ΔMON is constant then the curve is (A) circle C) ellipse (D) hyperbola (B) parabola

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67. Find the curve for which the intercept cut off by a tangent on x-axis is equal to four times the ordinate of the point of contact.

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68. Show that equation to the curve such that the y-intercept cut off by the tangent at an arbitrary point is proportional to the square of the ordinate of the point of tangency is of the form $\frac{a}{x} + \frac{b}{y} = 1$.

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69. Find the equation of the curve which is such that the area of the rectangle constructed on the abscissa of and the initial ordinate of the tangent at this point is a constanta $= a^2$.

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70. The differential equation of family of curves whose tangent form an angle of $\frac{\pi}{4}$ with the hyperbola $xy = C^2$ is

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71. Find the family of curves for which subnormal is a constant in a parabola.

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72. Find the equation of the curve whose slope at $x = 0$ is 3 and which passes through the point $(0, 1)$ satisfying the differential equation $(x^2 + 1)\frac{d^2y}{dx^2} = 2x\frac{dy}{dx}$.

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73. The curve in the first quadrant for which the normal at any point (x, y) and the line joining the origin to that point form an isosceles triangle with the x-axis as base is (a) an ellipse (b) a rectangular hyperbola (c) a circle (d) None of these

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74. Show that the curve for which the normal at every point passes through a fixed point is a circle.

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75. Find the curve in which the subtangent is always bisected at the origin.

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76. A curve is such that the length of perpendicular from origin on the tangent at any point P of the curve is equal to the abscissa of P. Prove that the differential equation of the curve is $y^2 - 2xy\frac{dy}{dx} - x^2 = 0$ and hence find the curve.

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77. The normal PG to a curve meets the x-axis in G. If the distance of G from the origin is twice the abscissa of P, prove that the curve is a rectangular hyperbola.

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78. Find the curve for which area of triangle formed by x-axis, tangent drawn at any point on the curve and radius vector of point of tangency is constant, equal to a^2

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79. The curve for which the ratio of the length of the segment intercepted by any tangent on the Y-axis to the length of the radius vector is constant $(k),$ is

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80. Given two curves: $y = f(x)$ passing through the point $(0, 1)$ and $g(x) = \int_{-\infty}^{x} f(t)dt$ passing through the point $\left(0, \frac{1}{n}\right)$. The tangents drawn to both the curves at the points with equal abscissas intersect on the x-axis. Find the curve $y = f(x)$.

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81. A student studying a foreign language has 50 verbs to meemorize, the rate at which the student can memorize these verbs is proportional to the number of verbs remaining to be memorized, that is, if the student memorizes y verbs in t minutes, $\frac{dy}{dt} = k(50 - y)$Assume that initially no verbs are memorized, and suppose that 20 verbs are memorized in the first minutes. How many verbs will the student memorize in t min.

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82. The differential equation satisfied by $\sqrt{1+x^2} + \sqrt{1+y^2} = k\left(x\sqrt{1+y^2} - y\sqrt{1+x^2}\right), k \in R$ is (A)

$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ (B) $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$ (C) $\frac{dy}{dx} = (1+x^2)(1+y^2)$ (D) none of these

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83. $x = f(t)$ satisfies $\frac{d^2x}{dt^2} = 2t + 3$ and for $t = 0, x = 0, \frac{dx}{dt} = 0$, then $f(t)$ is given by (A) $t^3 + \frac{t^2}{2} + t$ (B) $\frac{2t^3}{3} + \frac{3t^2}{2} + t$ (C) $\frac{t^3}{3} + \frac{3t^2}{2}$ (D) none of these

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84. The degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is (A) 1 (B) 2 (C) 3 (D) 4

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85. The general solution of the differential equation $\frac{d^2y}{dx^2} = e^{-3x}$ is (A) $y = 9e^{-3x} + C_1x + C_2$ (B) $y = -3e^{-3x} + C_1x + C_2$ (C) $y = 3e^{-3x} + C_1x + C_2$ (D) $y = \frac{e^{-3x}}{9} + C_1x + C_2$

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86. The general solution of the differential equation $x^2(1 + y^3)dx = y^2(1 + x^3)dy$ is (A) $(1 + x^2)(1 + y^2) = C$ (B) $1 + x^3 = C(1 + y^3)$ (C) $(x + y)(1 + x^2 + x^3) = C$ (D) $x(1 + y^2) = Cy(1 + x^2)$

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87. Differential equation of all tangents to the parabola $y^2 = 4ax$ is (A) $y = mx + \frac{a}{m}, m \neq 0$ (B) $y_1(yy_1 - x) = a$ (C) $xy_1^2 - yy_1 + a = 0$ (D) none of these

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88. The equation of the curve passing through origin, whose slope at any point is $\frac{x(1+y)}{1+x^2}$, is (A) $(1+y)^2 - x^2 = 1$ (B) $x^2 + (y+1)^2 = 1$ (C) $(x+y)y = 1 - x^2$ (D) $x = ye^{(1+y)}$

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89. The solution of $\cos(x+y)dy = dx$ is (A) $y = \cos^{-1}\left(\frac{y}{x}\right) + C$ (B) $y = x\sec\left(\frac{y}{x}\right) + C$ (C) $y = \tan\left(\frac{x+y}{2}\right) + C$ (D) none of these

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90. Solution of $x\frac{dy}{dx} + y = xe^x$ is (A) $xy = e^x(x+1) + C$ (B) $xy = e^x(x-1) + C$ (C) $xy = e^x(1-x) + C$ (D) $xy = e^y(y-1) + C$

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91. If $\frac{dx}{dy} = 2^{\tan y} \sec^2 y$, then x is equal to (A) $\frac{2^{\tan y}}{\log 2} + C$ (B) $2^{\tan y} + C$ (C) $\tan y + C$ (D) none of these

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92. The differential equation of the family of curves $y = A(x + B)^2$ after eliminating A and B is (A) $yy'' = y'^2$ (B) $2yy'' = y' - y$ (C) $2yy'' = y' + y$ (D) $2yy'' = y'^2$

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93. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1 + y}{x}$ is (A) $\frac{x}{e^x}$ (B) $\frac{e^x}{x}$ (C) xe^x (D) e^x

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94. The solution of the equation $(e^x+1)ydy+(y+1)dx=0$, is (A) $e^{x+y}=C(y+1)e^x$ (B) $e^{x+y}=C(x+1)(y+1)$ (C) $e^{x+y}=C(y+1)(1+e^x)$ (D) $e^{xy}=C(x+y)(e^x+1)$

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95. The curve for which the normal at every point passes through a fixed point is a (A) parabola (B) hyperbola (C) ellipse (D) circle

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96. The function $f(k)=\int_0^k \frac{dx}{1-\cos k\cdot\cos x}$ satisfies the differential equation (A) $\frac{df}{dk}+2f(k)\cdot\cot k=0$ (B) $\frac{df}{dk}+2f(x)\cdot\cos k=0$ (C) $\frac{df}{dk}-2f(x)\cos^2 k$ (D) none of these

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97. If the family of curves $y = ax^2 + b$ cuts the family of curves $x^2 + 2y^2 - y = a$ orthogonally, then the value of b= (A) 1 (B) $\frac{2}{3}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$

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98. If $f(x)$ is a differentiable real valued function such that $f(0) = 0$ and $f'(x) + 2f(x) \leq 1$, then (A) $f(x) > \frac{1}{2}$ (B) $f(x) \geq 0$ (C) $f(x) \leq \frac{1}{2}$ (D) none of these

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99. Solution of the equation `cos^2x(dy)/(dx)-(tan2x)y=cos^4x ,|x|

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100. Given two curves: $y = f(x)$ passing through the point $(0, 1)$ and $g(x) = \int_{-\infty}^{x} f(t)dt$ passing through the point $\left(0, \frac{1}{n}\right)$. The tangents drawn to both the curves at the points with equal abscissas intersect on the x-axis. Find the curve $y = f(x)$.

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101. The orthogonal trajectories of the family of curves $y = a^n x^n$ are given by (A) $n^2 x^2 + y^2$ = constant (B) $n^2 y^2 + x^2$ = constant (C) $a^n x^2 + n^2 y^2$ = constant (D) none of these

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102. Find the curve for which the length of normal is equal to the radius vector.

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103. The solution of the equation $\frac{dy}{dx} = x^3y^2 + xy$ is (A) $x^2y - 2y + 1 = cye^{-\frac{x^2}{2}}$ (B) $xy^2 + 2x - y = ce^{-\frac{y}{2}}$ (C) $x^2y - 2y + x = cxe^{-\frac{y}{2}}$ (D) none of these

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104. The solution of the equation $ydx - xdy = x^2ydx$ is (A) $y^2e^{-\frac{x^2}{2}} = C^2x^2$ (B) $y = Cxe^{\frac{x^2}{2}}$ (C) $x^2 = C^2y^2e^{x^2}$ (D) $ye^{x^2} = x$

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105. If $f(x)$ is differentiable, then the solution of $dy + f'(x)(y - f(x))dx = 0$ is (A) $yf(x) = Ce^{-f(f(x))^2}$ (B) $y + 1 = f(x) + Ce^{-f(x)}$ (C) $f(x) = Cye^{-\frac{y^2}{2}}$ (D) none of these

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106. If $x\frac{dy}{dx}=y(\log y-\log x+1)$, then the solution of the differential equation is (A) $\log\left(\frac{x}{y}\right)=Cy$ (B) $\log\left(\frac{y}{x}\right)=Cy$ (C) $\log\left(\frac{x}{y}\right)=Cx$ (D) $\log\left(\frac{y}{x}\right)=Cx$

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107. The solution of $(x^2+xy)dy=(x^2+y^2)dx$ is (A) $\log x=\log(x-y)+\frac{y}{x}+C$ (B) $\log x=2\log(x-y)+\frac{y}{x}+C$ (C) $\log x=\log(x-y)+\frac{x}{y}+C$ (D) none of these

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108. The solution of the differential equation $\left(\frac{dy}{dx}\right)^2-\frac{xdy}{dx}+y=0$ is (A) $y=2$ (B) $y=2x$ (C) $y=2x-4$ (D) $y=2x^2-4$

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109. The solution of the equation $xdy - ydx = \sqrt{x^2 - y^2}dx$ subject to the condition $y(1) = 0$ is (A) $y = x\sin(\log x)$ (B) $y = x^2\sin(\log x)$ (C) $y = x^2(x-1)$ (D) none of these

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110. The differential equation of family of curves whose tangents form an angle of $\frac{\pi}{4}$ with the hyperbola $xy = k$ is (A) $\frac{dy}{dx} = \frac{x^2 + ky}{x^2 - ky}$ (B) $\frac{dy}{dx} = \frac{x+k}{x-k}$ (C) $\frac{dy}{dx} = -\frac{k}{x^2}$ (D) $\frac{dy}{dx} = \frac{x^2 - k}{x^2 + k}$

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111. The solution of $x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right)$ is (A) $\tan\left(\frac{y}{2x}\right) = C - \frac{1}{2x^2}$ (B) $\sec\left(\frac{y}{x}\right) = 1 + \frac{C}{y}$ (C) $\sin\left(\frac{y}{x}\right) = C + \frac{1}{y}$ (D) $y^2 = (C + x^2)\tan\left(\frac{y}{x}\right)$

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112. The solution of $xdy = (2y + 2x^4 + x^2)dx,$ is (A) $y = x^4 + x\log x + C$ (B) $y = x^2 + x\log x + C$ (C) $y = x^4 + x^2\log x + C$ (D) none of these

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113. The solution of differential equation $xy^2(y_1^2 + 2) = 2y_1y^3 + x^3$, is (A) $(x + y - a)(x^2 - y^2 - bx^2) = 0$ (B) $(x^2 - y^2 - a)(x^2 - y^2 + bx^4) = 0$ (C) $(x^2 + y^2 - a)(x^2 + y^2 - bx^4) = 0$ (D) none of these

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114. The solution of $\frac{dy}{dx} + \frac{xy^2 - x^2y^3}{x^2y + 2x^3y^2} = 0$, is (A) $\log\left(\frac{y^2}{x}\right) - \frac{1}{xy} = C$ (B) $\log\left(\frac{x}{y}\right) + \frac{y^2}{x} = C$ (C) $\log(x^2y) + \frac{y^2}{x} = C$ (D) none of these

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115. The solution of $y^2dx + (x^2 - xy + y^2)dy = 0$ is (A) $y = Ce^{\tan^{-1}x}$

(B) $y = Ce^{\tan^{-1}y}$ (C) $y = Ce^{\tan^{-1}\left(\frac{y}{x}\right)}$ (D) $y = C\left[\tan^{-1}\left(\frac{y}{x}\right) + e^{x^2} + y^2\right]$

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116. The solution of $(1 - x^2)\frac{dy}{dx} + 2xy - x\sqrt{1 - x^2} = 0,$ is (A) $\frac{y}{(1 - x^2)} = \frac{1}{\sqrt{1 - x^2}} + C$ (B) $y(1 - x^2) = \sqrt{1 - x^2} + C$ (C) $y(1 - x^2)^{\frac{3}{2}} = \sqrt{1 - x^2} + C$ (D) none of these

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117. The solution of $y(2xy + e^x)dx - e^x dy = 0$ is (A) $x^2 + ye^{-x} = C$ (B) $xy^2 + e^{-x} = C$ (C) $\frac{x}{y} + \frac{e^{-x}}{x^2} = C$ (D) $x^2 + \frac{e^x}{y} = C$

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118. The largest value of c such that there exists a differentiable function $f(x)$ for $-c < x < c$ that satisfies the equation $y_1 = 1 + y^2$ with $f(0) = 0$ is (A) 1 (B) π (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

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119. If the slope of tangent to a curve $y = f(x)$ is maximum at $x = 1$ and minimum at $x = 0$, then equation of the curve which also satisfies $\frac{d^3y}{dx^3} = 4x - 3$, is (A) $y = \frac{x^4}{6} - \frac{x^3}{2} + \frac{x^2}{2} + 1$ (B) $y = \frac{x^4}{4} + x^3 - \frac{x^2}{3} + 1$ (C) $y = \frac{x^4}{4} - \frac{x^3}{7} + \frac{x^2}{3} + 3$ (D) none of these

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120. The degree and order of the differential equation of the family of all parabolas whose axis is x-axs are respectively

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121. The solution of the differential equation $(1+y^2)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$ is (A) $xe^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$ (B) $(x-2)=ke^{-\tan^{-1}y}$ (C) $xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$ (D) $xe^{\tan^{-1}y}=\tan^{-1}y+k$

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122. The differential equation for the family of curves $x^2+y^2-2ay=0$, where a is an arbitrary constant is (A) $(x^2-y^2)y'=2xy$ (B) $2(x^2+y^2)y'=xy$ (C) $2(x^2-y^2)y'=xy$ (D) $(x^2+y^2)y'=2xy$

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123. If $x\frac{dy}{dx}=y(\log y-\log x+1)$, then the solution of the differential equation is (A) $\log\left(\frac{x}{y}\right)=Cy$ (B) $\log\left(\frac{y}{x}\right)=Cy$ (C) $\log\left(\frac{x}{y}\right)=Cx$ (D) $\log\left(\frac{y}{x}\right)=Cx$

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124. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$ where $c > 0$ is a parameter, is of order and degree as follows: (A) order 1, degree 3 (B) order 2, degree 2 (C) order 1, degree 2 (D) order 1, degree 1

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125. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants, is of (a) second order and second degree (b) first order and second degree (c) first order and first degree (d) second order and first degree

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126. The differential equation of all circles passing through the origin and having their centres on the x-axis is (1) $x^2 = y^2 + xy\frac{dy}{dx}$ (2) $x^2 = y^2 + 3xy\frac{dy}{dx}$ (3) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (4) $y^2 = x^2 - 2xy\frac{dy}{dx}$

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127. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is (1) $(x2)y'^2 = 25(y2)^2$ (2) $(y2)y'^2 = 25(y2)^2$ (3) $(y2)2y'^2 = 25(y2)^2$ (4) $(x2)2y'^2 = 25(y2)^2$

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128. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is (1) $y = \ln x + x$ (2) $y = x\ln x + x^2$ (3) $y = xe(x-1)$ (4) $y = x\ln x + x$

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129. If $y(t)$ is a solution of $(1+t)\frac{dy}{dx} - ty = 1 and y(0) = -1$ then show that $y(1) = -\frac{1}{2}$.

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130. If $y = y(x)$ and $\left(\frac{2+\sin x}{y+1}\right)\frac{dy}{dx} = -\cos x$, $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ equals (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $-\frac{1}{3}$ (D) 1

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131. If $xdy = ydx + y^2dy$ and $y(1) = 1$, then $y(-3)$ is equal to (A) 1 (B) 5 (C) 4 (D) 3

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132. If $\left(x^2 + y^2\right)dy = xydx$ and $y(1) = 1$. If $y(x_0) = e$ then x_0 is equal to (A) $\sqrt{2}e$ (B) $\sqrt{3}e$ (C) $2e$ (D) e

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133. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circle with (a) variable radii and a fixed centre at (0, 1) (b) variable radii and a fixed centre at $(c)(d)((e)(f)0, -1(g))(h)$ (i) (j) Fixed radius 1 and variable centres along the x-axis. (k) Fixed radius 1 and variable centres along the y-axis.

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134. Solution of the differential equation $\cos x dy = y(\sin x - y)dx, 0 < x < \frac{\pi}{2}$ is (A) $\tan x = (\sec x + c)y$ (B) $\sec x = (\tan x + c)y$ (C) $y \sec x = \tan x + c$ (D) $y \tan x = \sec x + c$

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135. A curve $y = f(x)$ passes through $(1, 1)$ and tangent at $P(x, y)$ cuts the x-axis and y-axis at A and B, respectively, such that $BP : AP = 3$, then (a) equation of curve is $(b)(c)x(d)y^{(e)\,'(f)}(g) - 3y = 0(h)$ (i) (j) normal at $(k)(l)((m)(n)1, 1(o))(p)$ (q) is $(r)(s)x + 3y = 4(t)$ (u) (v) curve passes through $(w)(x)\left((y)(z)2, (aa)\frac{1}{bb}8(cc)(dd)(ee)\right)(ff)$ (gg) (hh) equation of curve is $(ii)(jj)x(kk)y^{(ll)\,'(mm)}(nn)+3y=0(oo)$

(pp)

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136. For the differential equation $(x^2+y^2)dx-2xydy=0$, which of the following are true. (A) solution is $x^2+y^2=cx$ (B) $x^2-y^2=cx$ (C) $x^2-y^2=x+c$ (D) $y(0)=0$

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137. The curve represented by the differential equation $(x^2+y^2+1)dx-2xydy=0$ satisfying $y(1)=1$ is (A) $x^2-y^2+x-1=0$ (B) $(x-1)^2+(y-2)^2=1$ (C) a hyperbola (D) a circle

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138. Which of the following are true for the differential equation $\frac{dy}{dx} - \frac{y}{x} + \frac{5x}{(x+2)(x-3)} = 0$, if the curve represented by it passes through the point $\left(5, a\log\left(\frac{7}{12}\right)\right)$ (A) Integrating factor is $\frac{1}{x}$ (B) $a = 5$ (C) $a = 4$ (D) solution is $y = x\log\left(\frac{x+2}{6(x-3)}\right)$

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139. Which of the following are true for the curve represented by the differential equation $\sec^2 y\frac{dy}{dx} + 2x\tan y = x^3$ satisfying $y(1) = 0$ (A) equation of curve is $2\tan y = x^2 - 1$ (B) equation of curve is $y^2 = x^3 - 1$ (C) curve is a parabola (D) curve is not a conic

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140. Consider the differential equation of the family of curves $y^2 = 2a\left(x + \sqrt{a}\right)$, where a is a positive parameter.Statement 1: Order of the differential equation of the family of curves is 1.Statement 2: Degree

of the differential equation of the family of curves is 2. (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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141. Statement 1: Order of the differential equation of the family of curves $y = a \sin x + b \cos(x + c)$ is 3.Statement 2: Order of the differential equation of a family of curves is equal to the number of independent arbitrary constants in the equation of family of curves.`. (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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142. Statement-1: $y(x) = \sin\left(x + \frac{\pi}{4}\right)$ Statement-2: Integrating factor of the given differential equation is $\sec x$. (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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143. Statement-1: Curve satisfying the differential equation $\frac{dy}{dx} = \frac{y}{2x}$ and passing through the point $(2, 1)$ is a parabola having focus $\left(\frac{1}{2}, 0\right)$ Statement-2: The differential equation $\frac{dy}{dx} = \frac{y}{2x}$ is homogeneous. (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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144. Statement-1: The solution of the differential equation $(x^2 + y^2)dx = 2xydy$ satisfying $y(1) = 0$ is $x^2 - y^2 = x$.Statement-2: The differential equation $(x^2 + y^2)dx = 2xydy$ can be solved by putting $y = vx$. (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B)

Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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145. Statement-1: Solution of the differential equation $\frac{dy}{dx}\tan y = \sin(x+y) + \sin(x-y)$ is $\sec y + 2\cos x = c$.Statement-2: The differential equation $\frac{dy}{dx}\tan y = \sin(x+y) + \sin(x-y)$ is (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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146. Statement-1: The differential equation of all circles in a plane must be of order 3.Statement-2: The differential equation of family of curve $y = a\sin x + b\cos(x+c)$, where a, b, c are parameters is 2. (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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147. Statement-1: The solution of differential equation $(1+x^2)\frac{dy}{dx}+2xy=4x^2$ is $3y(1+x^2)=4x^3+c$.Statement-2: The solution of a linear differential equation can be obtained by multiplying it by its integrating factor. (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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148. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis at Q such that PQ is of constant length k. Answer the question:The differential equation describing such a curve is (A) $y\frac{dy}{dx}=\pm\sqrt{k^2-x^2}$

(B) $x\frac{dy}{dx} = \pm\sqrt{k^2 - x^2}$ (C) $y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$ (D) $x\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$

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149. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis at Q such that PQ is of constant length k. Answer the question:If the curve passes through the point $(0, k)$, then its equation is (A) $x^2 - y^2 = k^2$ (B) $x^2 + y^2 = k^2$ (C) $x^2 - y^2 = 2k^2$ (D) $x^2 + y^2 = 2k^2$

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150. A tangent drawn to the curve $y = f(x)$ at $P(x, y)$ cuts the x-axis and y-axis at A and B respectively such that $BP : AP = 2 : 1$. Given that $f(1) = 1$. Answer the question:Equation of curve is (A) $y = \frac{1}{x}$ (B) $y = \frac{1}{x^2}$ (C) $y = \frac{1}{x^3}$ (D) none of these

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151. A tangent drawn to the curve $y = f(x)$ at $P(x, y)$ cuts the x-axis and y-axis at A and B respectively such that $BP : AP = 2 : 1$. Given that $f(1) = 1$. Answer the question:The curve passes through the point (A) $\left(2, \frac{1}{4}\right)$ (B) $\left(2, \frac{1}{2}\right)$ (C) $\left(2, \frac{1}{8}\right)$ (D) none of these

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152. A tangent drawn to the curve $y = f(x)$ at $P(x, y)$ cuts the x-axis and y-axis at A and B respectively such that $BP : AP = 2 : 1$. Given that $f(1) = 1$. Answer the question:Equation of normal to curve at $(1, 1)$ is (A) $x - 4y + 3 = 0$ (B) $x - 3y + 2 = 0$ (C) $x - 2y + 1 = 0$ (D) none of these

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153. A pair of curves $y = f_1(x)$ and $y = f_2(x)$ are such that following conditions are satisfied.(i) The tangents drawn at points with equal abscissae intersect on y-axis.(ii) The normals drawn at points with equal abscissae intersect on x-axis. Answer the question:Which of the following is true (A) $f'_1(x) + f'_2(x) = c$ (B) $f'_1(x) - f'_2(x) = c$ (C) $f'^2_1(x) - f'^2_1(x) = c$ (D) $f'^2_1(x) + f'^2_2(x) = c$

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154. Curves $y = f(x)$ passing through the point $(0, 1)$ and $y = \int_{-\infty}^{x} f(t)dt$ passing through the point $\left(0, \frac{1}{3}\right)$ are such that the tangents drawn to them at the point with equal abscissae intersect on x-axis. Answer the question:The equation of curve $y = f(x)$ is (A) $y = e^{3x}$ (B) $y = e^{\frac{x}{3}}$ (C) $y = 3^x$ (D) $y = \frac{1}{3^x}$

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155. Curves $y = f(x)$ passing through the point $(0, 1)$ and $y = \int_{-\infty}^{x} f(t)dt$ passing through the point $\left(0, \frac{1}{3}\right)$ are such that the tangents drawn to them at the point with equal abscissae intersect on x-axis. Answer the question:The area bounded by the curve

$y = f(x), y = x$ and ordinates $x = 0$ and $x = 1$ is (A) $\frac{e^2 - 1}{2}$ (B) $\frac{e^2 - 1}{3}$ (C) $\frac{e^3 - 1}{3}$ (D) none of these

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156. Curves $y = f(x)$ passing through the point $(0, 1)$ and $y = \int_{-\infty}^{x} f(t) dt$ passing through the point $\left(0, \frac{1}{3}\right)$ are such that the tangents drawn to them at the point with equal abscissae intersect on x-axis. Answer the question: $\lim_{x \to 0} \frac{(f(x))^2 - 1}{x} =$ (A) 3 (B) 6 (C) 4 (D) none of these

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157. A differential equation of the form $\frac{dy}{dx} + Py = Q$ is said to be a linear differential equation. Integrating factor of this differential equation is $e^{\int P dx}$ and its solution is given by $y. e^{\int P dx} = \int \left(Q e^{\int P dx}\right) dx + c.$ Answer the question:Solution of differential equation $(1 + y^2) dx + \left(x - e^{-\tan^{-1} y} dy = 0\right.$ is (A)

$y = \tan^{-1} x + c$ (B) $ye^{\tan^{-1} x} = \tan^{-1} x + c$ (C) $xe^{\tan^{-1} y} = \tan^{-1} y + c$

(D) none of these

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158. A differential equation of the form $\frac{dy}{dx} + Py = Q$ is said to be a linear differential equation. Integrating factor of this differential equation is $e^{\int} Pdx$ and its solution is given by $y.\ e^{\int Pdx} = \int \left(Qe^{\int Pdx}\right) dx + c$. Answer the question:Let $f(x)$ be a differentiable function in intervel $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for all $x > 0$. Then $f(x)$= (A) $\frac{1}{3x} + \frac{2x^2}{3}$ (B) $-\frac{1}{3x} + \frac{4x^2}{3}$$(C)$-1/x+2/x^2$(D)$1/x`

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159. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis and the y-axis in point A AND B, respectively, such that $\frac{1}{OA} + \frac{1}{OB}$ =1, where O is the origin. Find the equation of such a curve passing through $(5. 4)$

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160. For $x \in x \neq 0,$ if $y(x)$ differential function such that $x\int_1^x y(t)dt = (x+1)\int_1^x ty(t)dt,$ then $y(x)$ equals: (where C is a constant.)

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161. A curve $y = f(x)$ satisfies $\frac{d^2y}{dx^2} = 6x - 4$ and $f(x)$ has local minimum value 5 at $x = 1$. If a and b be the global maximum and global minimum values of $f(x)$ in interval $[0, 2]$, then ab is equal to...

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162. A line is drawn from a point $P(x, y)$ on the curve $y = f(x)$, making an angle with the x-axis which is supplementary to the one made by the tangent to the curve at $P(x, y)$. The line meets the x-axis at A. Another line perpendicular to it drawn from $P(x, y)$ meeting the y-axis at B. If $OA = OB$, where O is the origin, theequation of all curves which pass through $(1, 1)$ is

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