MATHS

BOOKS - KC SINHA MATHS (HINGLISH)

INTEGRAL CALCULUS - PREVIOUS YEAR QUESTIONS - FOR COMPETITION

Solved Examples

1. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is (1) 3000 (2) 3500 (3) 4500 (4) 2500

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2. The area (in square units) bounded by the curves $y = \sqrt{x}, 2y - x + 3 = 0$, x-axis, and lying in the first quadrant is

(1) 36 (2) 18 (3) $\frac{27}{4}$ (4) 9

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3. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to (1) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + C$ (2) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$ (3) $\frac{1}{3}x^3\psi(x^3) - \int x^3\psi(x^3)dx + C$ (4) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right] + C$

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4. The area under the curve $y = |\cos x - \sin x|, 0 \leq x \leq \frac{\pi}{2}$, and above x-axis is: (A) $2\sqrt{2} + 2$ (B) 0 (C) $2\sqrt{2} - 2$ (D) $2\sqrt{2}$

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5. If a curve passes through the point $\left(2, \frac{7}{2}\right)$ and has slope $\left(1 - \frac{1}{x^2}\right)$ at any point (x, y) on it, then the ordinate of the point on the curve whose abscissa is -2 is: (A) $\frac{5}{2}$ (B) $\frac{3}{2}$ (C) $-\frac{3}{2}$ (D) $-\frac{5}{2}$

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6. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + 2^x} dx$ is: (A) π (B) $\frac{\pi}{2}$ (C) 4π (D) $\frac{\pi}{4}$

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7. The integral $\int \frac{xdx}{2-x^2+\sqrt{2-x^2}}$ equals: (A) $\log\left|1+\sqrt{2+x^2}\right|+C$ (B) $x\log\left|1-\sqrt{2+x^2}\right|+C$ (C) $-\log\left|1+\sqrt{2-x^2}\right|+C$ (D) $x\log\left|1-\sqrt{2-x^2}\right|+C$

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8. The area enclosed by the curve $y=\sin x+\cos x and y=|\cos x-\sin x|$ over the interval $\left[0,\frac{\pi}{2}\right]$ is $4\left(\sqrt{2}-2\right)$ (b) $2\sqrt{2}$ ($\sqrt{2}$ -1) $2\left(\sqrt{2}\right.$ +1) (d) $2\sqrt{2}\left(\sqrt{2}+1\right)$

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9. A curve passes through the point $\left(1,\frac{\pi}{6}\right)$. Let the slope of the curve at each point (x,y) be $\frac{y}{x}+\sec\left(\frac{y}{x}\right), x>0$. Then the equation of the curve is (a) $(b)(c)\sin\left((d)(e)(f)\frac{y}{g}x(h)(i)(j)\right)=\log x+(k)\frac{1}{l}2(m)(n)(o)$

(p) (q) $(r)(s)\operatorname{cosec}\left((t)(u)(v)\frac{y}{w}x(x)(y)(z)\right) = \log x + 2(aa)$

(bb) (cc)

$(dd)(ee)\sec\left((ff)(gg)(hh)\frac{(ii)2y}{jj}x(kk)(ll)(mm)\right) = \log x + 2(nn)$

(oo) (pp) *[Math Processing Error]* (fff)

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10. Let $f: [0, 1] \to R$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x, x \in [0, 1]$Which of the following is true for $0 < x < 1$? (A) $0 < f(x) < \infty$ (B) $-\frac{1}{2} < f(x) < \frac{1}{2}$ (C) $-\frac{1}{4} < f(x) < 1$ (D) $-\infty < f(x) < 0$

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11. Let $f: [0, 1] \to R$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$

and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x, x \in [0, 1]$If the function $e^{-x}f(x)$ assumes its minimum in the interval $[0, 1]$ at $x = \frac{1}{4}$, which of the following is true? (A) $f'(x) < f(x), \frac{1}{4} < x < \frac{3}{4}$ (B) $f'(x) > f(x), 0 < x < \frac{1}{4}(C)f'(x) < f(x), 0 < x < \frac{1}{4}$ (D) $f'(x) < f(x), \frac{3}{4} < x < 1$

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12. Let S be the area of the region enclosed by $y = e^{-x}$ ^ $2, y = 0, x = 0, andx = 1.$ Then $S \geq \frac{1}{e}$ (b) $S \geq 1 = \frac{1}{e}$ $S \leq \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$ (d) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

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13. Let $I\overrightarrow{RI}R$ be defined as $f(x) = |x| + \ + x^2 - 1 \mid \cdot$ The total number of points at which f attains either a local maximum or a local minimum is_____

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14. If f(x) = 2 x t 0 e (t 2)(t 3)dt for all x (0,), then (A) f has a local maximum at x = 2 (B) f is decreasing on (2, 3) (C) there exists some c (0,) such that f (c) = 0

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15. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{\frac{9}{2}}} dx$ equals to (for some arbitrary constant K) (A) $-\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}} \left\{ \frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2 \right\} + K$ (B)

$$\frac{1}{(\sec x+\tan x)^{\frac{11}{2}}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K \quad \text{(C)}$$

$$-\frac{1}{(\sec x+\tan x)^{\frac{11}{2}}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K \quad \text{(D)}$$

$$\frac{1}{(\sec x+\tan x)^{\frac{11}{2}}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$$

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16. . Which of the following is true ? (A) g is increasing on (1,) (B) g is decreasing on (1,) (C) g is increasing on (1, 2) and decreasing on (2,) (D) g is decreasing on (1, 2) and increasing on (2,)

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17. Consider the statements : P : There exists some x IR such that f(x) + 2x = 2(1+x2) Q : There exists some x IR such that 2f(x) +1 = 2x(1+x) Then (A) both P and Q are true (B) P is true and Q is false (C) P is false and Q is true (D) both P and Q are false.

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18. The value of the integral /2 2 /2 x x 1n cosxdx x is

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19. If the integral $\int \frac{5\tan x}{\tan x - 2} dx = x + a\mathrm{I}\,\mathrm{n}|\sin x - 2\cos x| + k$ then a is equal to (1) 1 (2) 2 (3) 1 (4) 2

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20. The population p(t) at time t of a certain mouse species satisfies the differential equation $\left(dp\frac{t}{dt} = 0.\,5p(t) - 450\right.$ If $p(0) = 850$, then the time at which the population becomes zero is (1) 2 ln 18 (2) ln 9 (3) $\frac{1}{2}$ In 18 (4) ln 18

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21. The area bounded between the parabolas $x^2 = \frac{y}{4} \text{and} x^2 = 9y$ and the straight line $y = 2$ is (1) $20\sqrt{2}$ (2) $\frac{10\sqrt{2}}{3}$ (3) $\frac{20\sqrt{2}}{3}$ (4) $10\sqrt{2}$

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22. If $g(x) = \int_0^x \cos^4 t dt,$ then $g(x+\pi)$ equals $g(x) + g(\pi)$ (b) $g(x) - g(\pi)$ $g(x)g(\pi)$ (d) $\frac{g(x)}{g(\pi)}$

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23. $Thevalueof \int_{\sqrt{1n2}}^{\sqrt{1n3}} \frac{x \sin x^2}{\sin x^2 + \sin(1n6 - x^2)} dxis$ $\frac{1}{4} 1n \frac{3}{2}$ (b) $\frac{1}{21} n \frac{3}{2}$ $1n \frac{3}{2}$ (d) $\frac{1}{61} n \frac{3}{2}$

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24. Let the straight line $x = b$ divide the area enclosed by $y = (1 - x)^2, y = 0$ and $x = 0$ into two parts $R_1(0 \leq x \leq b)$ and $R_2(b \leq x \leq 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals (A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

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25. Let $f: [1, \infty]$ be a differentiable function such that $f(1) = 2$. If $\int_1^x f(t)dt = 3xf(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is

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26. Let $f: [-1, 2]\overrightarrow{0, \infty}$ be a continuous function such that $f(x) = f(1 - x)f$ or $allx \in [-1, 2]\cdot$ Let $R_1 = \int_{-1}^{2} xf(x)dx,$

and R_2 be the area of the region bounded by $y = f(x), x = -1, x = 2,$ and the $x - a\xi s$. Then $R_1 = 2R_2$ (b) $R_1 = 3R_2$ $2R_1$ (d) $3R_1 = R_2$

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27. Let $y'(x) + y(x)g'(x) = g(x)g'(x), y(0), x \in R,$ where $f'(x)$ denotes $\frac{dy(x)}{dx}$, and $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0.$ Then the value of $y(2)$ is_____

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28. The area of the region enclosed by the curves $y = x, x = e, y = \frac{1}{x}$ and the positive x-axis is (A) $\frac{3}{2}$ sq. units (B) $\frac{5}{2}$ sq. units (C) $\frac{1}{2}$ sq. units (D) 1 sq. units

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29. Let I be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\left(dV\frac{t}{dt} = -k(T-t)\right.$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is : (1) $T^2 - \frac{1}{k}$ (2) $I - \frac{kT^2}{2}$ (3) $I - \frac{k(T-t)^2}{2}$ (4) e^{-kT}

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30. If $\frac{dy}{dx} = y + 3 > 0 \text{and} y(0) = 2, then y(\ln 2)$ is equal to : (1) 7 (2) 5 (3) 13 (4) -2

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31. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t} \sin t dt$ Then f has :

local maximum at π and 2π . local minimum at π and 2π local minimum at π and local maximum at 2π . local maximum at π and local minimum at 2π .

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32. The value of $\int_0^1 \left(8\frac{\log(1+x)}{1+x^2}dx\right.$ is: (1) $\pi \log 2$ (2) $\frac{\pi}{8}\log 2$ (3) $\frac{\pi}{2}\log 2$ (4) $\log 2$

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