MATHS

BOOKS - KC SINHA MATHS (HINGLISH)

PRINCIPLES OF MATHEMATICAL INDUCTION - FOR BOARDS

Solved Examples

1. Prove the following by using the principle of mathematical induction for all $n \in N$: $1^3 + 2^3 + 3^3 + \vdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

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2. Prove the following by the principle of mathematical induction: $\frac{1}{1.\,2} + \frac{1}{2.\,3} + \frac{1}{3.\,4} + + \frac{1}{n(n+1)} = \frac{n}{n+1}$

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3. Prove the following by the principle of mathematical induction:

$1.\ 2 + 2.\ 2^2 + 3.\ 2^3 + \ + n.2^n = (n-1)2^{n+1} + 2$

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4. Prove the following by using the principle of mathematical induction for all $n \in N$:

$1.\ 2.\ 3 + 2.\ 3.\ 4 + \ \text{dot} \ \text{dot} \ \text{dot} + \text{n}(\text{n}+1) \quad (\text{n}+2) = \frac{n(n+1)(n+2)(}{4}$

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5. Prove the following by the principle of mathematical induction:

$7 + 77 + 777 + \ + 777 + \ + \ddot{n} - digits7 = \frac{7}{81}\left(10^{n+1} - 9n - 10\right)$ for all $n \in NB$.

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6. Prove by the principle of mathematical induction that $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a natural number for all $n \in N$.

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7. Using principle of mathematical induction prove that $\cos\alpha \cos 2\alpha \cos 4\alpha \cos\left(2^{n-1}\alpha\right) = \frac{s \in 2^n\alpha}{2^n s \in \alpha} f \text{ or } all\, nN.$

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8. Shwo that $n^3 + (n+1)^3 + (n+2)^3$ is divisible 9 for everynatural number n.

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9. Using the Principle of mathematical induction, show that $11^{n+2} + 12^{2n-1}$, where n is a naturla number is a natural number is divisible by 133.

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10. Prove that for $n \in N$, $10^n + 3.\ 4^{n+2} + 5$ is divisible by 9 .

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11. If $xandy$ are any two distinct integers, then prove by mathematical induction that $(x^n - y^n)$ is divisible by $(x - y)$ for all $n \in N$

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12. Show by using induction that for al natural numbers $n > 2$, $2^n > 2n + 1$.

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13. Prove the following by using the principle of mathematical induction for all $n \in N$:$1 + 2 + 3 + \vdots + n < \frac{1}{8}(2n+1)^2$.

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14. Prove the following by using the principle of mathematical induction for all $n \in N$:$(2n+7) < (n+3)^2$.

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15. Prove that $(1+x)^n \geq (1+nx)$, for all natural number n, where $x \succ 1$.

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Exercise

1. Prove that :$1^2 + 2^2 + 3^2 + \ + n^2 = \frac{n(n+1)(2n+1)}{6}$

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2. if $P(n)$ be the statement $10n + 3$ is a prime number", then prove that P(1) and P(2) are true but P(3) is false.

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3. Prove by induction that $4 + 8 + 12 + \ + 4n = 2n(n+1)$ for all nN.

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4. Prove by using the principle of mathemtical induction: ` 1+2+3+...+n = (n(n+1))/2

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5. Prove by the principle of mathematical induction that for all $n \in N$:

$$1^2 + 2^2 + 3^2 + \ + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

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6. Prove the following by the principle of mathematical induction:

$$1 + 3 + 3^2 + \ + 3^{n-1} = \frac{3^n - 1}{2}$$

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7. Prove the following by using the principle of mathematical induction for all $n \in N$: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ + \vdots \frac{1}{2^n} = 1 - \frac{1}{2^n}$

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8. Prove the rule of exponents $(ab)^n = a^n b^n$ by using principle of mathematical induction for every natural number.

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9. Prove the following by using the principle of mathematical induction for all $n \in N$:$1^2 + 3^2 + 5^2 + \vdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

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10. Prove by using the principle of mathemtical induction: $3.2^2 + 3.\ 2^3 + \ldots + 3^n.2^{n+1} = \frac{12}{5}(6^n - 1)$

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11. Using the principle of mathematical induction prove that : $1.\ 3 + 2.\ 3^2 + 3.\ 3^3 + \ + n.3^n = \frac{(2n+1)3^{n+1} + 3}{4}$ for all $n \in N$.

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12. Prove the following by using the principle of mathematical induction for all $n \in N$:$a + ar + ar^2 + \vdots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$

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13. Prove the following by the principle of mathematical induction: $a + (a + d) + (a + 2d) + \ + (a + (n - 1)d) = \frac{n}{2}[2a + (n - 1)d]$

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14. Prove by the principle of mathematical induction that for all nN: $\frac{1}{1.\ 3} + \frac{1}{35} + \frac{1}{57} + \ + \frac{1}{(2n - 1)(2n + 1)} = \frac{n}{2n + 1}$

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15. $\frac{1}{3.7} + \frac{1}{7.11} + \frac{1}{11.15} + \ldots\ldots + \frac{1}{(4n - 1)(4n + 3)} = \frac{n}{3(4n + 3)}$

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16. Prove the following by the principle of mathematical induction:

$$1.\,2+2.\,3+3.\,4+\ +n(n+1)=\frac{n(n+1)(n+2)}{3}$$

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17. $1.3+3.5+5.7+\ldots\ldots+(2n-1)(2n+1)=\frac{n\left(4n^2+6n-1\right)}{3}$

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18. Prove the following by the principle of mathematical induction:

$$\frac{1}{1.\,4}+\frac{1}{4.\,7}+\frac{1}{7.\,10}+\ +\frac{1}{(3n-1)(3n+2)}=\frac{n}{3n+1}$$

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19. Prove the following by the principle of mathematical induction:

$$\frac{1}{2.\,5}+\frac{1}{5.\,8}+\frac{1}{8.\,11}+\ +\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$$

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20. Prove the following by the principle of mathematical induction:

$$\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$$

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21. Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\vdots+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$

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22. Using the principle of mathematical induction prove that

$$1+\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+\ +\frac{1}{1+2+3+\ +n}=\frac{2n}{n+}$$

for all $n \in N$

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23. Prove the following by using the principle of mathematical induction for all $n \in N$: $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\vdots 1+\frac{1}{n}=(n+1)$

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24. Prove, by induction, that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ for all positive as well as negative integral values of

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25. $3.6+6.9+9.12+\ldots.\ +3n(3n+3)=$

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26. prove using mathematical induction:$-n(n+1)(n+5)$ is divisible by 6 for all natural numbers

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27. If n is as natural number, using mathematica induction show that: $n^7 - n$ is a multiple of 7.

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28. If n is a positive integer, show that $4^n - 3n - 1$ is divisible by 9

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29. Show that $9^{n+1} - 8n - 9$ is divisible by 64, where n is a positive integer.

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30. Prove the following by using the principle of mathematical induction for all $n \in N$:$10^{2n-1} + 1$is divisible by 11.

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31. Prove, by mathematical induction, that $x^n + y^n$ is divisible by $x + y$ for any positive odd integer n.

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32. Using principle of MI prove that $2.7^n + 3.5^n - 5$ is divisible by 24

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33. Using principle of mathematical induction prove that $x^{2n} - y^{2n}$ is divisible by $x + y$ for all nN.

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34. For every positive integer n, prove that $7^n - 3^n$is divisible by 4.

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35. Using the principle of mathematical induction prove that $41^n - 14^n$ s a multiple of 27 $7^n - 3^n$ s a divisible by 4 .

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36. Using the principle of mathematical induction, prove that $n < 2^n$ for all n $\in$ N

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37. Using mathematical induction, prove the following: $1 + 2 + 3 + \ldots + n < (2n + 1)^2 \forall n \in N$

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