

MATHS

BOOKS - KC SINHA MATHS (HINGLISH)

PROPERTIES OF TRIANGLE - FOR COMPETITION



2. The sides of a triangle are $x^2 + x + 1$, 2x + 1 and $x^2 - 1$. Prove that

the greatest angle is $120^{
m 0}$

3. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smalles one. Determine the sides of the triangle.



that $\cot heta = \cot A + \cot B + \cot C$.

6. If in a triangle of base 'a', the ratio of the other two sides is r (<1). Show that the altitude of the triangle is less than or equal to $\frac{ar}{1-r^2}$



7. Given the base of a triangle, the opposite angle A, and the product k^2 of other two sides, show that it is not possible for a to be less than $2k \sin \frac{A}{2}$

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8. In a triangle ABC, the vertices A,B,C are at distances of p,q,r fom the

orthocentre respectively. Show that aqr + brp + cpq = abc



9. Prove that a triangle ABC is equilateral if and only if $\tan A + \tan B + \tan C = 3\sqrt{3}$.

10. If
$$a, b$$
 and c be in $A. P. .prove$ that $\cos A \cot\left(\frac{A}{2}\right), \cos B \cot\left(\frac{B}{2}\right), \text{ and } \cos C \cot\left(\frac{C}{2}\right) \text{ are in } A. P.$
Value 10. Watch Video Solution

11. If the sides of triangle ABC are in G.P with common ratio r(r < 1),

show that
$$r < rac{1}{2}ig(\sqrt{5}+1ig)$$

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12. If in a triangle $r_1 = r_2 + r_3 + r$, prove that the triangle is right angled.



13. If
$$A+B+C=\pi$$
, prove that

$$\cot, rac{A}{2}+\cot, rac{B}{2}+\cot, rac{C}{2}=\cot, rac{A}{2}\cot, rac{B}{2}\cot, rac{C}{2}$$

14. Let $A_1, A_2, ..., A_n$ be the vertices of an n-sided regular polygon such that $,\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}.$ Find the value of n. Watch Video Solution

15. Prove that the sum of the radii of the radii of the circles, which are, respectively, inscribed and circumscribed about a polygon of n sides, whose side length is a, is $\frac{1}{2}a\frac{\cot \pi}{2n}$.

16. The sides of a quadrilateral are 3, 4, 5 and 6 cms. The sum of a pair of opposite angles is 120° . *Showtî heareaofthe rilateralis*3sqrt(30)` sq.cm.

17. The two adjacent sides of a cyclic quadrilateral are 2and5 and the angle between them is 60^{0} . If the area of the quadrilateral is $4\sqrt{3}$, find the remaining two sides.

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18. A cyclic quadrilateral ABCD of areal $\frac{3\sqrt{3}}{4}$ is inscribed in unit circle. If one of its side AB = 1, and the diagonal $BD = \sqrt{3}$, find the lengths of the other sides.

19. In a cyclic quadrilateral ABCD, prove that $\tan^2 \frac{B}{2} = \frac{(s-a)(s-b)}{(s-c)(s-d)}$, a, b, c, and d being the lengths of sides

ABC, CD and DA respectively and s is semi-perimeter of quadrilateral.

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20. In triangle ABC, prove that
$$\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \le \frac{1}{8}$$
 and hence, prove that $\cos \sec \frac{A}{2} + \cos \sec \frac{B}{2} + \cos \sec \frac{C}{2} \ge 6$.

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21. The sides of a triangle inscribed in a given circle subtends angles α, β, γ at the centre. Then, the minimum value of the arithmetic mean of $\cos\left(\alpha + \frac{\pi}{2}\right), \cos\left(\beta + \frac{\pi}{2}\right), \cos\left(\gamma + \frac{\pi}{2}\right)$ is

22. In a triangle ABC, prove that: $an^2, rac{A}{2}+ an^2, rac{B}{2}+ an^2, rac{C}{2}\geq 1$

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23. Let 1 < m < 3. In a triangle ABC, if 2b = (m+1) a & $\cos A = \frac{1}{2}\sqrt{\frac{(m-1)(m+3)}{m}}$ prove that the are two values to the third side, one of which is m times the other.

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24. Let A,B,C, be three angles such that $A = \frac{\pi}{4}$ and $\tan B$, $\tan C = p$. Find all possible values of p such that A, B, C are the angles of a triangle.



25. Two sides of a triangle are of lengths $\sqrt{6}$ and 4 and the angle opposite to smaller side is 30. How many such triangles are possible? Find the length of their third side and area.

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26. If the angle A, BandC of a triangle are in an arithmetic propression and if a, bandc denote the lengths of the sides opposite to A, BandCrespectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\sqrt{3}$

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27. Let ABCD be a quadrilateral with are 18, side AB parallel to the side CD, andAB = 2CD. Let AD be perpendicular to ABandCD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is 3 (b) 2 (c) $\frac{3}{2}$ (d) 1

28. One angle of an isosceles triangle is 120^0 and the radius of its incricel is $\sqrt{3}$. Then the area of the triangle in sq. units is $7 + 12\sqrt{3}$ (b) $12 - 7\sqrt{3}$ $12 + 7\sqrt{3}$ (d) 4π

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29. a triangle ABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4 \frac{\sin^2 A}{2}$. If a, bandc, denote the length of the sides of the triangle opposite to the angles A, B, andC, respectively, then b + c = 4a (b) b + c = 2a the locus of point A is an ellipse the locus of point A is a pair of straight lines

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30. Internal bisector of $\angle A$ of triangle ABC meets side BC at D. A line drawn through D perpendicular to AD intersects the side AC at E and the

side AB at F. If a, b, c represent sides of ΔABC , then



31. If in a
$$\triangle ABC$$
, $\cos A \cdot \cos B + \sin A \cdot \sin B \cdot \sin C = 1$, then (A)
 $A = B$ (B) $C = \frac{\pi}{2}$ (C) $AC = BC$ (D) $AB = \sqrt{2}AC$

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32. In a $\triangle ABC$, if $r = r_2 + r_3 - r_1$ and $A > \frac{\pi}{3}$ then range of $\frac{s}{a}$ contains (A) $\left(\frac{1}{2}, 2\right)$ (B) [1, 2)(C) $\left(\frac{1}{2}, 3\right)$ (D) $(3, \infty)$ **Vatch Video Solution** **33.** Let us consider a triangle ABC having BC=5 cm, CA=4cm, AB=3cm, D,E are points on BC such BD = DE= EC, $\angle CAE = \theta$, then: AE^2 is equal to



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34. In triangle ABC, $R(b+c) = a\sqrt{bc}$, where R is the circumradius of the

triangle. Then the triangle is

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35. In acute angled triangle ABC, AD is the altitude. Circle drawn with

AD as its diameter cuts ABandACatPandQ, respectively. Length of

$$PQ$$
 is equal to $/\left(2R
ight)$ (b) $rac{abc}{4R^2} \, 2R\sin A \sin B \sin C$ (d) $/R$

36. Statement 1. If A is the area and 2s is the perimeter of a $\ riangle ABC$,

then
$$A \leq rac{s^2}{3} \sqrt{3}$$
,

Statement 2. A. M > G. M.

- (A) Both Statements are false
- (B) Both Statement 1 and Statement 2 are true
- (C) Statement 1 is true but Statement 2 is false.
- (D) Statement 1 is flse but Stastement 2 is true



37. Radius of circumcircle of $\ \bigtriangleup \ DEF$ is

(A)R

$$(B) \ \frac{R}{2}$$
$$(C) \ \frac{R}{4}$$

 $\left(D
ight)$ none of these

38. If $\cot A + \cot B + \cot C = k \left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} \right)$ then the value of kis (A) R^2 (B) 2R(C) $\triangle `(D)a^2 + b^2 + c^2`$ Watch Video Solution

39. Let ABCandABC' be two non-congruent triangles with sides $AB = 4, AC = AC' = 2\sqrt{2}$ and angle $B = 30^0$. The absolute value of the difference between the areas of these triangles is

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40. ABC is a triangle. Its area is 12 sq. cm. and base is 6 cm. the difference of base angle is 60^0 . If A be the angle opposite to the base, then the value of by `8sinA-6cosA is.....

41. Perpendiculars are drawn from the angles A, B and C of an acuteangled triangle on the opposite sides, and produced to meet the circumscribing circle. If these produced parts are α ., β , γ , respectively, then show that, then show that $\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma} = 2(\tan A + \tan B + \tan C).$

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42. The sides of a triangle are in AP. If the angles A and C are the greatest and smallest angle respectively, then $4(1 - \cos A)(1 - \cos C)$ is equal to

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43. The radius of the circle passing through the vertices of the triangle

ABC, is

44. Three circles touch each other externally. The tangents at their point of contact meet at a point whose distance from a point of contact is 4. Then, the ratio of their product of radii to the sum of the radii is

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45. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90o - \frac{1}{2}A$, $90o - \frac{1}{2}B$ and $90o - \frac{1}{2}C$ Watch Video Solution

Exercise

1. A ring,10 cm in diameter, is suspended from a point 12cm above its centre by 6 equal strings attached to its circumference at equal intervals.Find the cosine of the angle between consecutive strings.



4. If f,g,h are internal bisectoirs of the angles of a triangle ABC, show that

$$rac{1}{f} \cos, rac{A}{2} + rac{1}{g} \cos, rac{B}{2} + rac{1}{h} \cos, rac{C}{2} = rac{1}{a} + rac{1}{b} + rac{1}{c}$$

5. The rational number which equals the number 2. 357 with recurring decimal is
$$\frac{2355}{1001}$$
 b. $\frac{2379}{997}$ c. $\frac{2355}{999}$ d. none of these Watch Video Solution

6. A triangle side are few 7cm, $4\sqrt{3}cm$ and $\sqrt{13}cm$ then the smallest

angle is

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7. In an isosceles right angled triangle, a straight line drwan from the mid - point of one of equal sides to the opposite angle. It divides the angle into two parts, θ and $(\pi/4 - \theta)$. Then $\tan \theta$ and $\tan[(\pi/4) - \theta]$ are equal to

8. If the roots of the equation $x^3 - px^2 + qx - r = 0$ are in A.P., then

9. In any
$$!ABC$$
, $(\Sigma)\left(rac{\sin^2 A + \sin A + 1}{\sin A}
ight)$ is always greater than

10. In a
$$\triangle ABC$$
,
 $\sin^4 A + \sin^4 B + \sin^4 C = \frac{3}{2} + 2\cos A \cos B \cos C + \frac{1}{2}\cos 2A \cos 2B \cos 2$
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11. If any triangle ABC , that:
 $\frac{a\sin(B-C)}{b^2-c^2} = \frac{b\sin(C-A)}{c^2-a^2} = \frac{c\sin(A-B)}{a^2-b^2}$
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12. In any triangle ABC prove that: $\sin\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{a}\right)\frac{\cos A}{2}$

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13. If in a !*ABC*, $\sin^3 A + \sin^3 B + \sin^3 C = 3 \sin A \sin B \sin C$, then $\begin{array}{cccc}
a & b & c \\
b & c & a \\
c & a & b
\end{array}$ Watch Video Solution a triangle ABC, Prove that: In 14. $\sin^3 A + \sin^3 B + \sin^3 C = 3\cos, \frac{A}{2}\cos, \frac{B}{2}\cos, \frac{C}{2} + \cos, \frac{3A}{2}\cos, \frac{3B}{2}\cos, \frac{3B}{2}\cos, \frac{A}{2}\cos, \frac{A}{2}\cos,$ Watch Video Solution 15. Prove that $\left(\frac{\cot A}{2}+\frac{\cot B}{2}\right)\left(a\frac{\sin^2 B}{2}+b\frac{\sin^2 A}{2}\right)=\mathrm{ot}\frac{C}{2}$

16. If pandq are perpendicular from the angular points A and B of ABCdrawn to any line through the vertex C, then prove that $a^2b^2\sin^2 C = a^2p^2 + b^2q^2 - 2abpq\cos C$.

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17. Let O be a point inside a triangle ABC such that $\angle OAB = \angle OBC = \angle OCA = \omega$, then Show that:

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18. If x, y, z are respectively perpendiculars from the circumcentre on the

sides of the
$$\Delta ABC$$
, the value of $rac{a}{x}+rac{b}{y}+rac{c}{z}-rac{abc}{4xyz}=$

19. Prove that a triangle ABC is equilateral if and only if `tanA+tanB+tanC=3sqrt(3).

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20. In a triange ABC, if
$$\sin{\left(\frac{A}{2}\right)}\sin{\left(\frac{B}{2}\right)}\sin{\left(\frac{C}{2}\right)}=\frac{1}{8}$$
 prove that the

triangle is equilateral.

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21. If the sides of triangle in A.P. and LC=90+LA then prove that sides will be in ratio $\sqrt{7}+1$: $\sqrt{7}$: $\sqrt{7}-1$

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22. If in a triangle ABC, $\cos A + 2\cos B + \cos C = 2$ prove that the

sides of the triangle are in ${\cal AP}$

23. In a triangle ABC, if
$$rac{a-b}{b-c}=rac{s-a}{s-c}$$
, then r_1,r_2,r_3 are in

24. In a
$$\triangle ABC$$
, $If \tan\left(\frac{A}{2}\right)$, $\tan\left(\frac{B}{2}\right)$, $\tan\left(\frac{C}{2}\right)$, are in H.P.,then a,b,c

are in

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25. If the sides of triangle in A.P. and LC=90+LA then prove that sides will be in ratio $\sqrt{7}+1$: $\sqrt{7}$: $\sqrt{7}-1$

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26. If the sides a,b,c of a triangle are in Arithmetic progressioni then find the value of $\tan, \frac{A}{2} + \tan, \frac{C}{2}$ in terms of $\cot, \frac{B}{2}$

27. Prove that
$$r_1+r_2+r_3-r=4R$$

28. provet
$$\widehat{:} riangle ABC, rac{1}{r_1}+rac{1}{r_2}+rac{1}{r_3}=rac{1}{r}$$

29.
$$provet$$
 : $rac{1}{r^2} + rac{1}{r_1^2} + rac{1}{r_2^2} + rac{1}{r_3^2} = rac{a^2 + b^2 + c^2}{ riangle^2}$

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30. If A, A_1, A_2 and A_3 are the areas of the inscribed and escribed circles of a triangle, prove that $\frac{1}{\sqrt{A}} = \frac{1}{\sqrt{A_1}} + \frac{1}{\sqrt{A_2}} + \frac{1}{\sqrt{A_3}}$

31. Prove that :
$$rac{r_1}{bc}+rac{r_2}{ca}+rac{r_3}{ab}=rac{1}{r}-rac{1}{2R}$$

32. ABC is an isosceles triangle inscribed in a circle of radius r. If AB = AC and h is the altitude from A to BC, then triangle ABC has perimeter $P = 2\left(\sqrt{2hr - h^2} + \sqrt{2hr}\right)$ and area A= _____ and = _____ and also $(\lim)_{x \to 0} \frac{A}{P^3} =_{----}$

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33. If p_1, p_2, p_3 re the altitudes of the triangle ABC from the vertices A, B

and C respectivel. Prove that
$$rac{\cos A}{p_1} + rac{\cos B}{p^2} + rac{\cos C}{p_3} = rac{1}{R}$$

34. Three circles whose radii are a,b and c and c touch one other externally and the tangents at their points of contact meet in a point. Prove that the distance of this point from either of their points of contact

is
$$\left(\frac{abc}{a+b+c}\right)^{\frac{1}{2}}$$



36. Prove that :
$$(r_1+r_2)rac{ an(C)}{2}=(r_3-r)rac{ an(C)}{2}=c$$

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37. Prove that : $4\,R\,s \in A\sin B\,s \in C = a\cos A + b\cos B + \mathrm{o}sC$.

38.
$$(r_1 - r)(r_2 - r)(r_3 - r) = 4Rr^2$$



40. If 1 is the incentre and $1_1, 1_2, 1_3$ are the centre of escribed circles of

the riangle ABC. Prove that

 $II_1, II_2, III_3 = 16R^2r.$

41. If 1 is the incentre and $1_1, 1_2, 1_3$ are the centre of escribed circles of

the $\ riangle ABC$. Prove that

 $II_1, II_2, III_3 = 16R^2r.$

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42.
$$\frac{1}{bc} + \frac{1}{ca} + \frac{1}{ab} =$$

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43.
$$\frac{r_1}{(s-b)(s-c)} + \frac{r_2}{(s-c)(s-a)} + \frac{r_3}{(s-a)(s-b)} = \frac{3}{r}$$

44. If the distances of the vertices of a triangle =ABC from the points of contacts of the incercle with sides are α , $\beta and\gamma$ then prove that $r^2 = \frac{\alpha\beta\gamma}{\alpha = \beta + \gamma}$

45. If in a triangle $\left(1-rac{r_1}{r_2}
ight)\left(1-rac{r_1}{r_3}
ight)=2$ then the triangle is right

angled (b) isosceles equilateral (d) none of these

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46. In a triangle ABC, prove that the ratio of the area of the incircle to

that of the triangle is
$$\pi : \cot\left(\frac{A}{2}\right) \cot\left(\frac{B}{2}\right) \cot\left(\frac{C}{2}\right)$$

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47. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$ (17) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$ (30) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$ (47) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$ (60)



48. A square whose side is 2 cm, has its corners cut away so as to form a regular octagon, find its area.

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49. An equilateral triangle and a regular hexagon has same perimeter. Find the ratio of their areas.

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50. The ratio of the area of a regular polygon of n sides inscribed in a circle to that of the polygon of same number of sides circumscribing the same is 3:4. Then the value of n is 6 (b) 4 (c) 8 (d) 12

51. A cyclic quadrilateral ABCD of areal $\frac{3\sqrt{3}}{4}$ is inscribed in unit circle. If one of its side AB = 1, and the diagonal $BD = \sqrt{3}$, find the lengths of the other sides.

52. If the number of sides of two regular polygons having the same prerimeter be n and 2n respectiely, prove that their areas are in the ratio $2\frac{\cos \pi}{n}:\left(1+\frac{\cos \pi}{n}\right)$ Watch Video Solution

53. In a $\triangle ABC$, the median to the side BC is of length $\frac{1}{\sqrt{11-6\sqrt{3}}}$ and

it divides the ${ot}A$ into angles 30° and $45\circ$. Find the length of the side

BC.



54. In an acute-angled triangle ABC, $\tan A + \tan B + \tan C$



55. If in a triangle ABC, θ is the angle determined by $\cos \theta = (a - b) / c$,

then

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56. If R be the circum radius and r the in radius of a triangle ABC, show that $R\geq 2r$

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57. If $\cos A = \tan B$, $\cos B = \tan C$ and $\cos C = \tan A$,

Show that $\sin A = \sin B = \sin C = 2 \cdot \sin 18^o$

58. If $A+B+C=\pi$, prove that: $\cot^2 A+\cot^2 B+\cot^2 C\geq 1$



59. In acute angled $\ riangle ABC$ prove that $an^2 A + an^2 B + an^2 C \ge 9$.

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60. In
$$\Delta ABC$$
, prove that $\mathrm{cosec} rac{A}{2} + \mathrm{cosec} rac{B}{2} + \mathrm{cosec} rac{C}{2} \geq 6$.

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61. Prove that in $\triangle ABC$, $\cos AB \cos C \leq \frac{1}{8}$.

62. Three equal circles each of radius r touch one another. The radius of

the circle touching all the three given circles internally is $(2+\sqrt{3})r$ (b)

$$rac{\left(2+\sqrt{3}
ight)}{\sqrt{3}}r\,rac{\left(2-\sqrt{3}
ight)}{\sqrt{3}}r$$
 (d) $\left(2-\sqrt{3}
ight)r$

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63. In a
$$\Delta ABC$$
, prove that

$$\sum_{r=0}^{n} C_r a^r b^{n-r} \cos(rB - (n-r)A) = c^n.$$
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64. If \triangle is the area and 2s is the perimeter of \triangle *ABC*, then prove that

$$riangle \ \leq rac{s^2}{3\sqrt{3}}$$

65. The sides of a triangle are 3x + 4y, 4x + 3y and 5x + 5y units, where

x > 0, y > 0. The triangle is



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66. In a \triangle ABC, \cos ecA[\sin B. \cos C + \cos B. \sin C] =

(A) \frac{c}{a}

(B) \frac{a}{c}

(C) 1

(D) none of these
```

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67. If the data given to construct a triangle ABC are a = 5, b= 7, sin

A=3/4, then it is possible to construct



68. If the angles of a triangle are in the ratio 1:2:3, the corresponding sides

are in the ratio



69. If three sides a,b,c of a triangle ABC are in arithmetic progression, then the value of \cot , $\frac{A}{2}$, \cot , $\frac{C}{2}$ is (A) 1 (B) 2 (C) 3 (D) None of these

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70. If $b = 3, c = 4, and B = \frac{\pi}{3}$, then find the number of triangles that

can be constructed.

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71. In a triangle ABC, $a = 4, b = 3, \angle A = 60^0$ then c is root of the equation $c^2 - 3c - 7 = 0$ (b) $c^2 + 3c + 7 = 0$ (c) $c^2 - 3c + 7 = 0$ (d) $c^2 + 3c - 7 = 0$


73. The number of triangles ABC that can be formed with $\sin A = \frac{5}{13}, a = 3$ and b = 8 is

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74. The lengths of the sides of a triangle are $\alpha - \beta, \alpha + \beta$ and $\sqrt{3\alpha^2 + \beta^2}, (\alpha > \beta > 0)$. Its largest angle is

75. In a ΔPQR (as shown in figure) if $x\!:\!y\!:\!z=2\!:\!3\!:\!6$, then the value of





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76. If in $\Delta extsf{ABC}$, $\angle c = 90^0$ then the maximum value of sinAsinB is



77. In an isosceles right angled triangle ABC, $\angle B = 90^{\circ}$, AD is the median then $\frac{\sin \angle BAD}{\sin \angle CAD}$ is (A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 1 (D) none of these

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78. If in a $\triangle ABC$, c = 3b and C - B = 90°, then tanB=

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79. If the lengths of the sides of a triangle are 3, 5, 7, then its largest

angle of the triangle is

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80. In a !ABC if a = 7, b = 8 and c = 9, then the length of the line joining

B to the mid-points of AC is

81. If H is the orthocenter of ΔABC and if AH=x, BH=y, CH=z,



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82. If the sides of a triangle are in the ratio 3:7:8, then find R:r

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83. If in a $riangle ABC, \sin^2 A + \sin^2 B + \sin^2 C = 2, then riangle$ is always a an

(A) isosceles triangle (B) right angled triangle (C) acute angled triangle

(D) obtuse angled triangle

84. For a riangle ABC, if $\cot A. \cot B. \cot C > 0$, then nature of the

triangle is

(A) acute angled triangle

(B) right angled triangle

(C) obtuse angled triangle

(D) none of these

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85. If the sides of a triangle are in G.P., and its largest angle is twice the smallest, then the common ratio r satisfies the inequality `0

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86. If in a riangle ABC, $a^2 \cos^2 A = b^2 + c^2$, then angle A is (A) less than 45^0 (B) more than of 45^0 and $\leq ssthan90^0$ (C) right angled (D) obtuse angle 87. The perimeter of a ΔABC is 6 times the A.M. of the sines of its

angles. If the side 'a' is 1, then the angle A is

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88. If angle C of a triangle ABC be obtuse, then (A) $0 < \tan A \tan B < 1$

(B) an A an B > 1 (C) an A an B = 1 (D) none of these

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89. In an equilateral triangle, inradius $r, \,$ circumradius R and ex-radius r_1

are in

90. The ratio of the area of triangle inscribed in ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to that of triangle formed by the corresponding points on the auxiliary circle is 0.5. Then, find the eccentricity of the ellipse.



92. In a `/_\ABC, tan, A/2 = 5/6 and tan, C/2 = 2/5 then (A) a,c,b are in A.P. (B)

a,b,c are in A.P. (C) b,a,c are in A.P. (D) a,b,c are in G.P.

93. The sides of a triangle are 3x + 4y, 4x + 3y and 5x + 5y units, where

x > 0, y > 0. The triangle is



94. In triangle
$$ABC, AD$$
 is the altitude from A . If $b > c, \angle C = 23^0, and AD = \frac{abc}{b^2} - c^2$, then $\angle B = _$ ____

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95. A circle is inscribed in an equilateral triangle of side a. The area of any

square inscribed in this circle is (A)
$$rac{a^2}{12}$$
 (B) $rac{a^2}{6}$ (C) $rac{a^2}{3}$ (D) $2a^2$

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96. Let $A_0A_1A_2A_3A_4A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths the line segments A_0A_1, A_0A_2

and A_0A_4 is



97. Let f(x + y) = f(x). f(y) for all x and y and f(1) = 2. If in as triangle ABC, a = f(3), b = f(1) + f(3), c = f(2) + f(3), then 2A = (A) C (B) 2C (C) 3C (D) 4C

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98. In a triangle ABC the angle B is greater than angle C. If the measure of angles B and C satisfy the equation $4\sin^3 x - 3\sin x + 0.75 = 0$ then the measure of angle A is (A) $\frac{\pi}{2}$ (B) $\frac{2p}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{5\pi}{6}$

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99. In a $\triangle ABC, \angle B = \frac{\pi}{3}, \angle C = \frac{\pi}{4}$ and D divides BC internally in the ratio 1:3 Then $\frac{\angle BAD}{\angle CAD}$ = is equal to (a) $\frac{1}{\sqrt{6}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{\frac{2}{3}}$

100. If a, b,c be the sides foi a triangle ABC and if roots of equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal then $\frac{\sin^2 A}{2}$, \sin^2 , $\frac{B}{2}$, $\frac{\sin^2 C}{2}$ are in (A) A.P. (B) G.P. (C) H.P. (D) none of these

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101. In a riangle ABC, $b^2 + c^2 = 1999a^2$, then $\frac{\cot B + \cot C}{\cot A} =$ (A) 1/1999 (B) 36161 (C) 999 (D) 1999

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102. If $(1+ax)^n = 1+8x+24x^2+...$ then the value of a and n is

103. If equations $ax^2 + bx + c = 0$ and $4x^2 + 5x + 6 = 0$ have a comon root, where a,b,c are the sides of $\triangle ABC$ opposite to angles A,B,C respectively, then 2A= (A) C (B) 2C (C) 3C (D) 4C

104. If in
$$\triangle ABC$$
, $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$, then $\angle A$ is equal to
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105. In triangle ABC, a: b: c = 4:5:6. The ratio of the radius of the

circumcircle to that of the incircle is____.

106. In triangle ABC,
$$\frac{\sin A + \sin B + \sin C}{\sin A + \sin B - \sin C}$$
 is equal to



107. If
$$\cos A + \cos B = 4 \sin^2 \left(rac{C}{2}
ight)$$
, then



108. If twice the square of the diameter of the circle is equal to half the sum of the squares of the sides of incribed triangle ABC,then $\sin^2 A + \sin^2 B + \sin^2 C$ is equal to

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109. If the base angles of triangle are $\frac{22}{12}and112\frac{1}{2^0}$, then prove that the altitude of the triangle is equal to $\frac{1}{2}$ of its base.

110. Let ABC be an isosceles triangle with base BC. If r is the radius of the circle inscribed in ΔABC and r_1 is the radius of the circle ecribed opposite to the angle A, then the product r_1r can be equal to (where R is the radius of the circumcircle of ΔABC)

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111. If represents the area of acute angled triangle ABC, then
$$\sqrt{a^2b^2 - 4^2} + \sqrt{b^2c^2 - 4^2} + \sqrt{c^2a^2 - 4^2} = a^2 + b^2 + c^2 \frac{a^2 + b^2 + c^2}{2}$$

 $ab\cos C + bos A + ca\cos B \ ab\sin C + bc\sin A + ca\sin B$

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112. If in a
$$riangle ABC$$
, $a = 6$, $b = 3$ and $c(A - B) = \frac{4}{5}$ then (A) $C = \frac{\pi}{4}$
(B) $A = \frac{\sin^{-1}2}{\sqrt{5}}$ (C) $ar(riangle ABC) = 9$ (D) none of these

113. In a triangle the lengths of the two larger are 10 and 9 respectively. If the angles are in A.P., the , length of the third side can be (A) $5 - \sqrt{6}$ (B) $3\sqrt{3}$ (C) 5 (D) $5 + \sqrt{6}$

114. In a triangle ABC, points D and E are taken on side BC such that BD= DE= EC. If angle ADE = angle AED = θ , then: (A) tan θ = 3 tan B (B) 3 tan θ =

tan C

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115. Which of the following holds goods for any tiangle ABC, a,b,c are the

lengths of the sides R is circumradius (A)

$$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$$
(B)

$$\frac{\sin A}{a} + \frac{\sin B}{b} + \frac{\sin C}{c} = \frac{3}{2R0}$$
(C)
$$\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$$
(D)

$$\frac{\sin 2A}{a} 62 = \frac{\sin 2B}{b^2} = \frac{\sin 2C}{c^2}$$

116. If the vertices P,Q,R of a triangle PQR are rational points, which of the following points of thetriangle PQR is/are always rational point(s) ?(A) centroid(B) incentre(C) circumcentre(D) orthocentreAgrawn

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117. The value of
$$Lt_{x \to 0} \left\{ \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x} \right\}$$
 is (A) 0 (B) 3 (C) 2 (D) 1

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118. If a and b be the length of the sides and c the length of hypotenuse of a right anlged triangle then (A) a+b>c (B) $a^2+b^2=c^2$ (C) $a^3+b^3< c^3$ (D) $a^n+b^n< c^n$ for $n\geq 3,n=Z$

119. If in $\Delta ABC,$ $ar{A}=90^{\circ}$ and c, sin B cos B are rational numbers, then

show a and b are rational .

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121. If a, b, c, d and p are different real numbers such that $(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)\leq 0$, then show that a, b, c and d are in G.P.

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122. If in a triangle ABC, $a^2 + b^2 + c^2 = ca + ab\sqrt{3}$, then the triangle is

123. If all the vertices of a triangle have integral coordinates, then the triangle may be right-angled (b) equilateral isosceles (d) none of these

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124. In a triangle, the lengths of the two larger sides are 10 and 9, respectively. If the angles are in A.P., then the length of the third side can be $5 - \sqrt{6}$ (b) $3\sqrt{3}$ (c) 5 (d) $5 + \sqrt{6}$



125. If the tangents of the angles A, B of a ΔABC ...satisfy the equation

 $abx^2-c^2x+ab=0,$ then

126. In a triangle ABC, points D and E are taken on side BC such that BD= DE= EC. If angle ADE = angle AED = θ , then: (A) tan θ = 3 tan B (B) 3 tan θ = tan C

Watch Video Solution 127. In a triangle ABC if
$$a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$$
, then angle C is

equal to (A) 60^0 (B) 120^0 (C) 45^0 (D) 135^0

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128. Statement-1: If the measures of two angles of a triangle are 45 ° and 60 °, then the ratio of the smallest and the greatest sides are $(\sqrt{3}-1):1$

Statement-2: The greatest side of a triangle is opposite to its greatest angle.

129. Statement 1. In a triangle ABC, if a:b:c = 4:5:6, then R:r = 16:7, Statement 2. In a triangle ABC, R:r = abc:4s(A) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1 (B) Both Statement 1 and Statement 2 are true and Statement 2 is not the

correct explanatioin of Statement 1

(C) Statement 1 is true but Statement 2 is false.

(D) Statement 1 is false but Stastement 2 is true

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130. Area of circle inscribed in the equilateral riangle ABC is (A) $rac{2}{3}\pi R^2$ (B)

 $rac{1}{4}\pi R^2$ (C) $rac{1}{3}\pi R^2$ (D) none of these

131.
$$\sin\left\{2\cos^{-1}\left(-\frac{3}{5}\right)
ight\}$$
 is equal to $6/25$ (b) $24/25$ (c) $4/5$ (d) $-24/25$

132. Three circles touch one-another externally. The tangents at their point of contact meet at a point whose distance from a point contact is 4. Then, the ratio of the product of the radii of the sum of the radii of circles is



134. Given the base of a triangle, the opposite angle A, and the product k^2 of other two sides, show that it is not possible for a to be less than $2k\sin\frac{A}{2}$

135. In a triangle ABC the sides b and c are the roots of the equation

 $x^2-61x+820=0 ext{ and } A= an^{-1}iggl(rac{4}{3}iggr) thena^2+3$ is equal to

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136. v37



137. If in a triangle ABC, $Rr(\sin A + \sin B + \sin C) = 96$ then the

square of the area of the triangle ABC is......



138. The sides of a quadrilateral are 3, 4, 5 and 6 cms. The sum of a pair of opposite angles is 120° . Showt \hat{t} hearea of the rilateral is 3 sqrt(30)

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139. Three circles touch one-another externally. The tangents at their point of contact meet at a point whose distance from a point contact is 4. Then, the ratio of the product of the radii of the sum of the radii of circles is

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140. In triangle ABC, a:b:c = 4:5:6. The ratio of the radius of the circumcircle to that of the incircle is .

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141. If p_1 , p_2 , p_3 are the altitudes of a triangle from the vertices A, B, C, & denotes the area of the triangle, prove that

$$\frac{1}{p_1} + \frac{1}{p_2} - \frac{1}{p_3} = \frac{2ab}{(a+b+c)} \frac{\cos^2 C}{2}$$

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142. If the sides of a quadrilateral ABCD touch a circle prove that

AB + CD = BC + AD.

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143. If in triangle ABC, $ig(a=ig(1+\sqrt{3}ig)cm,b=2cm,and ot c=60^0$, then

find the other two angles and the third side.

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144. If a circle is inscribed in right angled triangle ABC with right angle at

B, show that the diameter of the circle is equal to AB + BC - AC.

145. If a triangle is inscribed in a circle, then prove that the product of any two sides of the triangle is equal to the product of the diameter and the perpendicular distance of the thrid side from the opposite vertex.

146. ABC is triangle. D is the middle point of BC. If AD is perendicular to

AC, then prove that

$$\cos A \cos C = rac{2ig(c^2-a^2ig)}{3ac}$$

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147. Let the angles A, BandC of triangle ABC be in AP and let b:c be

 $\sqrt{3}$: $\sqrt{2}$. Find angle A

148. The exradii $r_1, r_2, \text{ and } r_3 \text{ of } \Delta ABC$ are in H.P. show that its sides

a, b, and c are in A.P.



149. 112. If in a $\triangle ABC$, $\cos A + \cos B + \cos c = rac{3}{2}$. Prove that $\triangle ABC$ is an equilateral triangle.

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151. AB is a diameter of a circle and C is any point on the circumference of the circle. Then the area of ABC is maximum when it is isosceles the area



154. If a, b and c are distinct positive numbers, then the expression (a + b - c)(b + c - a)(c + a - b) - abc is:

155. In a triangle, the lengths of the two larger sides are 10 and 9, respectively. If the angles are in A.P., then the length of the third side can be $5 - \sqrt{6}$ (b) $3\sqrt{3}$ (c) 5 (d) $5 + \sqrt{6}$

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156. If the angles of a triangle are $30^0 and 45^0$ and the included side is

 $(\sqrt{3}+1)cm$ then the area of the triangle is_____.

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157. ABC is a triangle such that $\sin(2A+B) = \sin(C-A) = -\sin(B+2C) = \frac{1}{2}$. If A,B, and C are in AP. then the value of A,B and C are..

158. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. Determine the sides of the triangle.



160. A circle is inscribed in an equilateral triangle of side a. The area of

any square inscribed in this circle is _____.

161. Let
$$A_1, A_2, \dots, A_n$$
 be the vertices of an n-sided regular polygon such that $,\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}.$ Find the value of n. Watch Video Solution

162. Consider the following statements concerning a ΔABc

(i) The sides a,b,c and area of triangle are rational.

(ii) $a, \tan \frac{B}{2}, \tan \frac{C}{2}$

(iii) $a, \sin A \sin B, \sin C$ are rational .

Prove that $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (i)$

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163. IF the lengths of the side of triangle are 3, 5AND7, then the largest

angle of the triangle is
$$\frac{\pi}{2}$$
 (b) $\frac{5\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{3\pi}{4}$

164. In triangle ABC, a:b:c = 4:5:6. The ratio of the radius of the

circumcircle to that of the incircle is____.

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165. Let A,B,C, be three angles such that $A = \frac{\pi}{4}$ and $\tan B$, $\tan C = p$. Find all possible values of p such that A, B, C are the angles of a triangle.

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166. If in a triangle $PQR; \sin P, \sin Q, \sin R$ are in A.P; then

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167. Prove that a triangle ABC is equilateral if and only if `tanA+tanB+tanC=3sqrt(3).

168. Let ABC be a triangle having O and I as its circumradius and inradis, respectively then prove that $(IO)^2 = R^2 - 2Rr$. Further show that the triangle BIO is a right angled triangle if and only if b is the rithmetic mean of a and c.

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169. In triangle
$$ABC$$
, $2ac\sin\left(\frac{1}{2}(A-B+C)\right)$ is equal to $a^2+b^2-c^2$
(b) $c^2+a^2-b^2\,b^2-c^2-a^2$ (d) $c^2-a^2-b^2$

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170. In a triangle ASBC, let $\angle C = \frac{\pi}{2}$. *Ifr* is the in radius and R is the circumrdius of the triangle then 2(r + R) is equal to (A) a + b (B) b + c(C) c + a (D) a + b + c

171. In any triangle ABC prove that
$$\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) + \cot\left(\frac{C}{2}\right) = \cot\left(\frac{A}{2}\right)\cot\left(\frac{B}{2}\right)\cot\left(\frac{C}{2}\right)$$

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172. Let PQandRS be tangent at the extremities of the diameter PR of a circle of radius r. If PSandRQ intersect at a point X on the circumference of the circle, then prove that $2r = \sqrt{PQxRS}$.

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173. If Δ is the area of a triangle with side lengths a, b, c, then show that as $\Delta \leq \frac{1}{4}\sqrt{(a+b+c)abc}$ Also, show that the equality occurs in the above inequality if and only if a = b = c.

174. Which of the following pieces of data does NOT uniquely determine an acute-angled triangle ABC(R being the radius of the circumcircle)? $a, \sin A, \sin B$ (b) $a, b, c, a, \sin B, R$ (d) $a, \sin A, R$



175. If the angles of a triangle are in the ratio 4:1:1, then the ratio of the longest side to the perimeter is- $\sqrt{3}$: $(2 + \sqrt{3})$ b. $1:\sqrt{3}$ c. $1:2 + \sqrt{3}$ d. 2:3

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176. The ratio of the sides of a triangle ABC is $1: \sqrt{3}: 2$. The ratio A: B: C is



177. In an equilateral triangle, three coins of radii 1 unit each are kept so

that they touch each other and also the sides of the triangle. The area of

the triangle is (fig)
$$4\!:\!2\sqrt{3}$$
 (b) $6+4\sqrt{3}\,12+rac{7\sqrt{3}}{4}$ (d) $3+rac{7\sqrt{3}}{4}$



178. One angle of an isosceles triangle is 120^0 and the radius of its incricel is $\sqrt{3}$. Then the area of the triangle in sq. units is $7+12\sqrt{3}$ (b) $12-7\sqrt{3}$ $12+7\sqrt{3}$ (d) 4π



179. Let
$$a, b, c$$
 be the sides of a triangle. No two of them are equal and
 $\lambda \in R$ If the roots of the equation
 $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real, then (a) $\lambda < \frac{4}{3}$ (b)
 $\lambda > \frac{5}{3}$ (c) $\lambda \in \left(\frac{1}{5}, \frac{5}{3}\right)$ (d) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$

180. Internal bisector of $\angle A$ of triangle ABC meets side BC at D. A line drawn through D perpendicular to AD intersects the side AC at E and the side AB at F. If a, b, c represent sides of $\triangle ABC$, then

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181. a triangle ABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4 \frac{\sin^2 A}{2}$. If a, bandc, denote the length of the sides of the triangle opposite to the angles A, B, andC, respectively, then b + c = 4a (b) b + c = 2a the locus of point A is an ellipse the locus of point A is a pair of straight lines

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182. Consider a triangle ABC and let a, bandc denote the lengths of the sides opposite to vertices A, B, andC, respectively. Suppose a = 6, b = 10, and the area of triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and

if r denotes the radius of the incircle of the triangle, then the value of r^2

is



183. If the angle A, BandC of a triangle are in an arithmetic propression and if a, bandc denote the lengths of the sides opposite to A, BandCrespectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\sqrt{3}$

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184. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, bandc denote the lengths of the side opposite to A, B, andC respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$, andc = 2x + 1 is(are) $-(2 + \sqrt{3})$ (b) $1 + \sqrt{3} 2 + \sqrt{3}$ (d) $4\sqrt{3}$
185. the sum of the radii of inscribed and circumscribed circle of an n sides regular polygon of side a is (A) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (B) $a\cot\left(\frac{\pi}{2n}\right)$ (C) $\frac{a}{4}\cos, \frac{\pi}{2n}$ (D) $a\cot\left(\frac{\pi}{n}\right)$

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186. In a $\triangle ABC$, medians AD and BE are drawn. If $AD = 4, \angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$ then the area of $\triangle ABC$ is

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187. If in a triangle ABC, $a\cos^2\left(\frac{C}{2}\right)\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides

 $a, b, andc\,$ are in A.P. b. are in G.P. c. are in H.P. d. satisfy $a+b=\,\cdot\,$

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188. The sides of a triangle are $\sin \alpha$, $\cos \alpha$ and $\sqrt{1 + \sin \alpha \cos \alpha}$ for some α , $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is

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189. In triangle ABC, let $\angle c = \frac{\pi}{2}$. If r is the inradius and R is circumradius of the triangle, then 2(r+R) is equal to a+b (b) b+cc+a (d) a+b+c

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190. If in a $\triangle ABC$ the altitude from the vertices A,B,C on opposite side are in H.P. then $\sin A$, $\sin B$, $\sin C$ are in (A) H.P. (B) Arithmetico-Geometric progression (C) A.P. (D) G.P.

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