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## MATHS

## BOOKS - KC SINHA MATHS (HINGLISH)

## PROPERTIES OF TRIANGLE - FOR COMPETITION

## Solved Examples

1. If in a triangle $\mathrm{ABC}, \frac{\tan A}{1}=\frac{\tan B}{2}=\frac{\tan C}{3}$ then prove that $6 \sqrt{2 a}=3 \sqrt{5 b}=2 \sqrt{10} c$

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2. The sides of a triangle are $x^{2}+x+1,2 x+1$ and $x^{2}-1$. Prove that the greatest angle is $120^{0}$
3. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smalles one. Determine the sides of the triangle.

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4. In a triangle ABC if $\cos A \cos B+\sin A \sin B \sin C=1$ show that the sides are in the proportion $1: 1: \sqrt{2}$

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5. In $A B C$, if $\sin ^{3} \theta=\sin (A-\theta) \sin (B-\theta) \sin (C-\theta)$, then prove that $\cot \theta=\cot A+\cot B+\cot C$.

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6. If in a triangle of base ' $a$ ', the ratio of the other two sides is $r(<1)$.Show that the altitude of the triangle is less than or equal to $\frac{a r}{1-r^{2}}$

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7. Given the base of a triangle, the opposite angle A , and the product $k^{2}$ of other two sides, show that it is not possible for a to be less than $2 k \sin \frac{A}{2}$

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8. In a triangle $A B C$, the vertices $A, B, C$ are at distances of $p, q, r$ fom the orthocentre respectively. Show that $a q r+b r p+c p q=a b c$

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9. Prove that a triangle $A B C$ is equilateral if and only if $\tan A+\tan B+\tan C=3 \sqrt{3}$.

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10. If $a, b$ and $c$ be in $A . P$. prove that $\cos A \cot \left(\frac{A}{2}\right), \cos B \cot \left(\frac{B}{2}\right)$, and $\cos C \cot \left(\frac{C}{2}\right)$ are in $A . P$.

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11. If the sides of triangle $A B C$ are in G.P with common ratio $r(r<1)$, show that $r<\frac{1}{2}(\sqrt{5}+1)$

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12. If in a triangle $r_{1}=r_{2}+r_{3}+r$, prove that the triangle is right angled.
13. If $A+B+C=\pi$, prove that
$\cot , \frac{A}{2}+\cot , \frac{B}{2}+\cot , \frac{C}{2}=\cot , \frac{A}{2} \cot , \frac{B}{2} \cot , \frac{C}{2}$

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14. Let $A_{1}, A_{2}, \ldots . A_{n}$ be the vertices of an n -sided regular polygon such that , $\frac{1}{A_{1} A_{2}}=\frac{1}{A_{1} A_{3}}+\frac{1}{A_{1} A_{4}}$. Find the value of $n$.

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15. Prove that the sum of the radii of the radii of the circles, which are, respectively, inscribed and circumscribed about a polygon of $n$ sides, whose side length is $a$, is $\frac{1}{2} a \frac{\cot \pi}{2 n}$.

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16. The sides of a quadrilateral are $3,4,5$ and 6 cms . The sum of a pair of opposite angles is $120^{\circ}$. Showttheareaofthe rilateralis3sqrt(30) sq.cm.

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17. The two adjacent sides of a cyclic quadrilateral are $2 a n d 5$ and the angle between them is $60^{\circ}$. If the area of the quadrilateral is $4 \sqrt{3}$, find the remaining two sides.

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18. A cyclic quadrilateral $A B C D$ of areal $\frac{3 \sqrt{3}}{4}$ is inscribed in unit circle. If one of its side $A B=1$, and the diagonal $B D=\sqrt{3}$, find the lengths of the other sides.

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19. In a cyclic quadrilateral $A B C D$, prove that $\tan ^{2} \frac{B}{2}=\frac{(s-a)(s-b)}{(s-c)(s-d)}, a, b, c$, and $d$ being the lengths of sides $A B C, C D$ and $D A$ respectively and $s$ is semi-perimeter of quadrilateral.

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20. In triangle $A B C$, prove that $\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \leq \frac{1}{8}$ and hence, prove that $\operatorname{cosec} \frac{A}{2}+\operatorname{cosec} \frac{B}{2}+\operatorname{cosec} \frac{C}{2} \geq 6$.

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21. The sides of a triangle inscribed in a given circle subtends angles $\alpha, \beta, \gamma$ at the centre.Then, the minimum value of the arithmetic mean of $\cos \left(\alpha+\frac{\pi}{2}\right), \cos \left(\beta+\frac{\pi}{2}\right), \cos \left(\gamma+\frac{\pi}{2}\right)$ is

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22. In a triangle ABC , prove that: $\tan ^{2}, \frac{A}{2}+\tan ^{2}, \frac{B}{2}+\tan ^{2}, \frac{C}{2} \geq 1$

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23. Let $1<m<3$. In a triangle $A B C$, if $2 b=(m+1)$ a \& $\cos A=\frac{1}{2} \sqrt{\frac{(m-1)(m+3)}{m}}$ prove that the are two values to the third side, one of which is $m$ times the other.

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24. Let $\mathrm{A}, \mathrm{B}, \mathrm{C}$, be three angles such that $A=\frac{\pi}{4}$ and $\tan B, \tan C=p$. Find all possible values of $p$ such that $A, B, C$ are the angles of a triangle.
25. Two sides of a triangle are of lengths $\sqrt{6}$ and 4 and the angle opposite to smaller side is 30 . How many such triangles are possible? Find the length of their third side and area.

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26. If the angle $A, B a n d C$ of a triangle are in an arithmetic propression and if $a$, bandc denote the lengths of the sides opposite to $A, B a n d C$ respectively, then the value of the expression $\frac{a}{c} \sin 2 C+\frac{c}{a} \sin 2 A$ is $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\sqrt{3}$

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27. Let $A B C D$ be a quadrilateral with are 18 , side $A B$ parallel to the side $C D, \operatorname{and} A B=2 C D$. Let $A D$ be perpendicular to $A B a n d C D$. If a circle is drawn inside the quadrilateral $A B C D$ touching all the sides, then its radius is 3 (b) 2 (c) $\frac{3}{2}$ (d) 1
28. One angle of an isosceles triangle is $120^{\circ}$ and the radius of its incricel is $\sqrt{3}$. Then the area of the triangle in sq. units is $7+12 \sqrt{3}$ (b) $12-7 \sqrt{3}$ $12+7 \sqrt{3}$ (d) $4 \pi$

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29. a triangle $A B C$ with fixed base $B C$, the vertex $A$ moves such that $\cos B+\cos C=4 \frac{\sin ^{2} A}{2}$. If $a, b a n d c$, denote the length of the sides of the triangle opposite to the angles $A, B$, $a n d C$, respectively, then $b+c=4 a$ (b) $b+c=2 a$ the locus of point $A$ is an ellipse the locus of point $A$ is a pair of straight lines

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30. Internal bisector of $\angle A$ of triangle ABC meets side BC at D . A line drawn through $D$ perpendicular to $A D$ intersects the side $A C$ at $E$ and the
side AB at F . If $\mathrm{a}, \mathrm{b}, \mathrm{c}$ represent sides of $\triangle A B C$, then

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31. If in a $\triangle A B C, \cos A \cdot \cos B+\sin A \cdot \sin B \cdot \sin C=1$, then (A) $A=B$ (B) $C=\frac{\pi}{2}$ (C) $A C=B C$ (D) $A B=\sqrt{2} A C$

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32. In a $\triangle A B C$, if $r=r_{2}+r_{3}-r_{1}$ and $A>\frac{\pi}{3}$ then range of $\frac{s}{a}$ contains
(A) $\left(\frac{1}{2}, 2\right)$
(B) $[1,2)$
(C) $\left(\frac{1}{2}, 3\right)$
(D) $(3, \infty)$
33. Let us consider a triangle $A B C$ having $B C=5 \mathrm{~cm}, C A=4 \mathrm{~cm}, A B=3 \mathrm{~cm}, \mathrm{D}, \mathrm{E}$ are points on BC such $\mathrm{BD}=\mathrm{DE}=\mathrm{EC}, \angle C A E=\theta$, then:
$A E^{2}$ is equal to

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34. In triangle $\mathrm{ABC}, R(b+c)=a \sqrt{b c}$, where R is the circumradius of the triangle. Then the triangle is

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35. In acute angled triangle $A B C, A D$ is the altitude. Circle drawn with $A D$ as its diameter cuts $A B a n d A C a t P a n d Q$, respectively. Length of $P Q$ is equal to $/(2 R)$ (b) $\frac{a b c}{4 R^{2}} 2 R \sin A \sin B \sin C$ (d) $/ R$

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36. Statement 1. If $A$ is the area and $2 s$ is the perimeter of a $\triangle A B C$, then $A \leq \frac{s^{2}}{3} \sqrt{3}$,

Statement 2. $A . M>G . M$.
(A) Both Statements are false
(B) Both Statement 1 and Statement 2 are true
(C) Statement 1 is true but Statement 2 is false.
(D) Statement 1 is flse but Stastement 2 is true

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37. Radius of circumcircle of $\triangle D E F$ is
(A) $R$
(B) $\frac{R}{2}$
(C) $\frac{R}{4}$
$(D)$ none of these
38. If $\cot A+\cot B+\cot C=k\left(\frac{1}{x^{2}}+\frac{1}{y^{2}}+\frac{1}{z^{2}}\right)$ then the value of $k$ is
(A) $R^{2}$
(B) $2 R$
(C) $\triangle{ }^{`}(D) a^{2}+b^{2}+c^{2 `}$

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39. Let $A B C a n d A B C^{\prime}$ be two non-congruent triangles with sides $A B=4, A C=A C^{\prime}=2 \sqrt{2}$ and angle $B=30^{\circ}$. The absolute value of the difference between the areas of these triangles is

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40. ABC is a triangle. Its area is $12 \mathrm{sq} . \mathrm{cm}$. and base is 6 cm . the difference of base angle is $60^{\circ}$. If $A$ be the angle opposite to the base, then the value of by ${ } 8 \sin A-6 \cos A$ is......
41. Perpendiculars are drawn from the angles $A, B$ and $C$ of an acuteangled triangle onthe opposite sides, and produced to meet the circumscribing circle. If these produced parts are $\alpha ., \beta, \gamma$, respectively, then show that, then show that $\frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma}=2(\tan A+\tan B+\tan C)$.

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42. The sides of a triangle are in AP. If the angles $A$ and $C$ are the greatest and smallest angle respectively, then $4(1-\cos A)(1-\cos C)$ is equal to

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43. The radius of the circle passing through the vertices of the triangle $A B C$, is
44. Three circles touch each other externally. The tangents at their point of contact meet at a point whose distance from a point of contact is 4 . Then, the ratio of their product of radii to the sum of the radii is

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45. Bisectors of angles $A, B$ and $C$ of a triangle $A B C$ intersect its circumcircle at D, E andF respectively. Prove that the angles of the triangle DEF are $90 o-\frac{1}{2} A, 90 o-\frac{1}{2} B$ and $90 o-\frac{1}{2} C$

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## Exercise

1. A ring, 10 cm in diameter, is suspended from a point 12 cm above its centre by 6 equal strings attached to its circumference at equal intervals.

Find the cosine of the angle between consecutive strings.

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2. If the angles of a triangle are in the ratio $7: 2: 1$, then prove that the ratio of smallest side to the largest side is $\sqrt{5}-1: \sqrt{5}+1$.

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3. If the base angles of triangle are $\frac{22}{12} \operatorname{and} 112 \frac{1}{2^{0}}$, then prove that the altitude of the triangle is equal to $\frac{1}{2}$ of its base.

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4. If $\mathrm{f}, \mathrm{g}, \mathrm{h}$ are internal bisectoirs of the angles of a triangle $A B C$, show that $\frac{1}{f} \cos , \frac{A}{2}+\frac{1}{g} \cos , \frac{B}{2}+\frac{1}{h} \cos , \frac{C}{2}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

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5. The rational number which equals the number 2.357 with recurring decimal is $\frac{2355}{1001}$ b. $\frac{2379}{997}$ c. $\frac{2355}{999}$ d. none of these

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6. A triangle side are few $7 \mathrm{~cm}, 4 \sqrt{3} \mathrm{~cm}$ and $\sqrt{13} \mathrm{~cm}$ then the smallest angle is

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7. In an isosceles right angled triangle , a straight line drwan from the mid

- point of one of equal sides to the opposite angle. It divides the angle into two parts, $\theta$ and $(\pi / 4-\theta)$. Then $\tan \theta$ and $\tan [(\pi / 4)-\theta]$ are equal to


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8. If the roots of the equation $x^{3}-p x^{2}+q x-r=0$ are in A.P., then

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9. In any ! $A B C,(\Sigma)\left(\frac{\sin ^{2} A+\sin A+1}{\sin A}\right)$ is always greater than

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10. 

In
a
$\triangle A B C$,
$\sin ^{4} A+\sin ^{4} B+\sin ^{4} C=\frac{3}{2}+2 \cos A \cos B \cos C+\frac{1}{2} \cos 2 A \cos 2 B \cos 2$

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11. 

If
any
triangle $\quad A B C$
that:
$\frac{a \sin (B-C)}{b^{2}-c^{2}}=\frac{b \sin (C-A)}{c^{2}-a^{2}}=\frac{c \sin (A-B)}{a^{2}-b^{2}}$
12. In any triangle $A B C$ prove that: $\sin \left(\frac{B-C}{2}\right)=\left(\frac{b-c}{a}\right) \frac{\cos A}{2}$

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13. If in a ! $A B C$,
$\sin ^{3} A+\sin ^{3} B+\sin ^{3} C=3 \sin A \sin B \sin C$, then
$\left|\begin{array}{lll}a & b & c \\ b & c & a \\ c & a & b\end{array}\right|$

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14. In a triangle $A B C$, Prove that:
$\sin ^{3} A+\sin ^{3} B+\sin ^{3} C=3 \cos , \frac{A}{2} \cos , \frac{B}{2} \cos , \frac{C}{2}+\cos , \frac{3 A}{2} \cos , \frac{3 B}{2} \cos$,

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15. Prove that $\left(\frac{\cot A}{2}+\frac{\cot B}{2}\right)\left(a \frac{\sin ^{2} B}{2}+b \frac{\sin ^{2} A}{2}\right)=o t \frac{C}{2}$
16. If pandq are perpendicular from the angular points A and B of $A B C$ drawn to any line through the vertex $C$, then prove that $a^{2} b^{2} \sin ^{2} C=a^{2} p^{2}+b^{2} q^{2}-2 a b p q \cos C$.

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17. Let $O$ be $a$ point inside $a$ triangle $A B C$ such that $\angle O A B=\angle O B C=\angle O C A=\omega$, then Show that:

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18. If $x, y, z$ are respectively perpendiculars from the circumcentre on the sides of the $\triangle A B C$, the value of $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-\frac{a b c}{4 x y z}=$

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19. Prove that a triangle $A B C$ is equilateral if and only if ${ }^{\prime} \tan \mathrm{A}+\tan \mathrm{B}+\tan \mathrm{C}=3 \mathrm{sqrt}(3)$.

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20. In a triange $A B C$, if $\sin \left(\frac{A}{2}\right) \sin \left(\frac{B}{2}\right) \sin \left(\frac{C}{2}\right)=\frac{1}{8}$ prove that the triangle is equilateral.

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21. If the sides of triangle in A.P. and $L C=90+L A$ then prove that sides will be in ratio $\sqrt{7}+1: \sqrt{7}: \sqrt{7}-1$

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22. If in a triangle $A B C, \cos A+2 \cos B+\cos C=2$ prove that the sides of the triangle are in $A P$
23. In a triangle ABC , if $\frac{a-b}{b-c}=\frac{s-a}{s-c}$, then $r_{1}, r_{2}, r_{3}$ are in

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24. In a $\Delta A B C, I f \tan \left(\frac{A}{2}\right), \tan \left(\frac{B}{2}\right), \tan \left(\frac{C}{2}\right)$, are in H.P.,then a,b,c are in

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25. If the sides of triangle in A.P. and $L C=90+L A$ then prove that sides will be in ratio $\sqrt{7}+1: \sqrt{7}: \sqrt{7}-1$

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26. If the sides $a, b, c$ of a triangle are in Arithmetic progressioni then find the value of $\tan , \frac{A}{2}+\tan , \frac{C}{2}$ in terms of $\cot , \frac{B}{2}$
27. Prove that $r_{1}+r_{2}+r_{3}-r=4 R$

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28. provet $\widehat{:} \triangle A B C, \frac{1}{r_{1}}+\frac{1}{r_{2}}+\frac{1}{r_{3}}=\frac{1}{r}$

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29. provet $\widehat{:} \frac{1}{r^{2}}+\frac{1}{r_{1}^{2}}+\frac{1}{r_{2}^{2}}+\frac{1}{r_{3}^{2}}=\frac{a^{2}+b^{2}+c^{2}}{\triangle^{2}}$

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30. If $A, A_{1}, A_{2}$ and $A_{3}$ are the areas of the inscribed and escribed circles
of a triangle, prove that $\frac{1}{\sqrt{A}}=\frac{1}{\sqrt{A_{1}}}+\frac{1}{\sqrt{A_{2}}}+\frac{1}{\sqrt{A_{3}}}$
31. Prove that : $\frac{r_{1}}{b c}+\frac{r_{2}}{c a}+\frac{r_{3}}{a b}=\frac{1}{r}-\frac{1}{2 R}$

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32. $A B C$ is an isosceles triangle inscribed in a circle of radius $r$. If $A B=A C$ and $h$ is the altitude from $A$ to $B C$, then triangle $A B C$ has perimeter $P=2\left(\sqrt{2 h r-h^{2}}+\sqrt{2 h r}\right)$ and area $A=$ $\qquad$ $\ldots$ and also $(\lim )_{x \rightarrow} \frac{A}{P^{3}}={ }_{-} \quad{ }_{-}$

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33. If $p_{1}, p_{2}, p_{3}$ re the altitudes of the triangle ABC from the vertices $\mathrm{A}, \mathrm{B}$ and C respectivel. Prove that $\frac{\cos A}{p_{1}}+\frac{\cos B}{p^{2}}+\frac{\cos C}{p_{3}}=\frac{1}{R}$

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34. Three circles whose radii are $\mathrm{a}, \mathrm{b}$ and c and c touch one other externally and the tangents at their points of contact meet in a point. Prove that the distance of this point from either of their points of contact is $\left(\frac{a b c}{a+b+c}\right)^{\frac{1}{2}}$.

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35. In a triangle ABC prove that
$r_{1} r_{2} r_{3}=r^{3} \cot ^{2}\left(\frac{A}{2}\right) \cdot \cot ^{2}\left(\frac{B}{2}\right) \cdot \cot ^{2}\left(\frac{C}{2}\right)$

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36. Prove that : $\left(r_{1}+r_{2}\right) \frac{\tan (C)}{2}=\left(r_{3}-r\right) \frac{\cot (C)}{2}=c$

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37. Prove that : $4 R s \in A \sin B s \in C=a \cos A+b \cos B+\mathrm{os} C$.

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38. $\left(r_{1}-r\right)\left(r_{2}-r\right)\left(r_{3}-r\right)=4 R r^{2}$

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> 39. In a $\quad$ triangle $\quad \mathrm{ABC}$, $r^{2}+r_{1}^{2}+r_{2}^{2}+r_{3}^{2}=16 R^{2}-a^{2}-b^{2}-c^{2}$.

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40. If 1 is the incentre and $1_{1}, 1_{2}, 1_{3}$ are the centre of escribed circles of the $\triangle A B C$. Prove that

$$
I I_{1}, I I_{2}, I I I_{3}=16 R^{2} r .
$$

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41. If 1 is the incentre and $1_{1}, 1_{2}, 1_{3}$ are the centre of escribed circles of the $\triangle A B C$. Prove that
$I I_{1}, I I_{2}, I I I_{3}=16 R^{2} r$.

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42. $\frac{1}{b c}+\frac{1}{c a}+\frac{1}{a b}=$

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43. $\frac{r_{1}}{(s-b)(s-c)}+\frac{r_{2}}{(s-c)(s-a)}+\frac{r_{3}}{(s-a)(s-b)}=\frac{3}{r}$

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44. If the distances of the vertices of a triangle =ABC from the points of contacts of the incercle with sides are $\alpha, \beta a n d \gamma$ then prove that $r^{2}=\frac{\alpha \beta \gamma}{\alpha=\beta+\gamma}$
45. If in a triangle $\left(1-\frac{r_{1}}{r_{2}}\right)\left(1-\frac{r_{1}}{r_{3}}\right)=2$ then the triangle is right angled (b) isosceles equilateral (d) none of these

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46. In a triangle $A B C$, prove that the ratio of the area of the incircle to that of the triangle is $\pi: \cot \left(\frac{A}{2}\right) \cot \left(\frac{B}{2}\right) \cot \left(\frac{C}{2}\right)$

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47. For a regular polygon, let $r$ and $R$ be the radii of the inscribed and the circumscribed circles. A false statement among the following is There is a regular polygon with $\frac{r}{R}=\frac{1}{\sqrt{2}}$ (17) There is a regular polygon with $\frac{r}{R}=\frac{2}{3}$ (30) There is a regular polygon with $\frac{r}{R}=\frac{\sqrt{3}}{2}$ (47) There is a regular polygon with $\frac{r}{R}=\frac{1}{2}$ (60)
48. A square whose side is 2 cm , has its corners cut away so as to form a regular octagon, find its area.

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49. An equilateral triangle and a regular hexagon has same perimeter.

Find the ratio of their areas.

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50. The ratio of the area of a regular polygon of $n$ sides inscribed in a circle to that of the polygon of same number of sides circumscribing the same is 3:4. Then the value of $n$ is 6 (b) 4 (c) 8 (d) 12

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51. A cyclic quadrilateral $A B C D$ of areal $\frac{3 \sqrt{3}}{4}$ is inscribed in unit circle. If one of its side $A B=1$, and the diagonal $B D=\sqrt{3}$, find the lengths of the other sides.

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52. If the number of sides of two regular polygons having the same prerimeter be $n$ and $2 n$ respectiely, prove that their areas are in the ratio $2 \frac{\cos \pi}{n}:\left(1+\frac{\cos \pi}{n}\right)$

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53. In a $\triangle A B C$, the median to the side $B C$ is of length $\frac{1}{\sqrt{11-6 \sqrt{3}}}$ and it divides the $\angle A$ into angles $30^{\circ}$ and $45 \circ$. Find the length of the side $B C$.
54. In an acute-angled triangle $\mathrm{ABC}, \tan A+\tan B+\tan C$

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55. If in a triamgle $\mathrm{ABC}, \theta$ is the angle determined by $\cos \theta=(a-b) / c$, then

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56. If $R$ be the circum radius and $r$ the in radius of a triangle $A B C$, show that $R \geq 2 r$

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57. If $\cos A=\tan B, \cos B=\tan C$ and $\cos C=\tan A$,

Show that $\sin A=\sin B=\sin C=2 \cdot \sin 18^{\circ}$

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58. If $A+B+C=\pi$, prove that: $\cot ^{2} A+\cot ^{2} B+\cot ^{2} C \geq 1$

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59. In acute angled $\triangle A B C$ prove that $\tan ^{2} A+\tan ^{2} B+\tan ^{2} C \geq 9$.

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60. In $\triangle A B C$, prove that $\operatorname{cosec} \frac{A}{2}+\operatorname{cosec} \frac{B}{2}+\operatorname{cosec} \frac{C}{2} \geq 6$.

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61. Prove that in $\triangle A B C, \cos A B \cos C \leq \frac{1}{8}$.

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62. Three equal circles each of radius $r$ touch one another. The radius of the circle touching all the three given circles internally is $(2+\sqrt{3}) r$ (b) $\frac{(2+\sqrt{3})}{\sqrt{3}} r \frac{(2-\sqrt{3})}{\sqrt{3}} r(\mathrm{~d})(2-\sqrt{3}) r$

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63. In a $\triangle A B C$, prove that
$\sum_{r=0}^{n}{ }^{n} C_{r} a^{r} b^{n-r} \cos (r B-(n-r) A)=c^{n}$.

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64. If $\triangle$ is the area and 2 s is the perimeter of $\triangle A B C$, then prove that $\triangle \leq \frac{s^{2}}{3 \sqrt{3}}$

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65. The sides of a triangle are $3 x+4 y, 4 x+3 y$ and $5 x+5 y$ units, where $x>0, y>0$. The triangle is

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66. In a $\triangle A B C, \cos e c A[\sin B \cdot \cos C+\cos B \cdot \sin C]=$
(A) $\frac{c}{a}$
(B) $\frac{a}{c}$
(C) 1
(D) none of these

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67. If the data given to construct a triangle $A B C$ are $a=5, b=7$, sin $A=3 / 4$, then it is possible to construct

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68. If the angles of a triangle are in the ratio $1: 2: 3$,the corresponding sides are in the ratio

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69. If three sides $a, b, c$ of a triangle $A B C$ are in arithmetic progression, then the value of $\cot , \frac{A}{2}, \cot , \frac{C}{2}$ is (A) 1 (B) 2 (C) 3 (D) None of these

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70. If $b=3, c=4, \operatorname{and} B=\frac{\pi}{3}$, then find the number of triangles that can be constructed.

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71. In a triangle $A B C, a=4, b=3, \angle A=60^{\circ}$ then $c$ is root of the equation $c^{2}-3 c-7=0$ (b) $c^{2}+3 c+7=0$ (c) $c^{2}-3 c+7=0$ (d) $c^{2}+3 c-7=0$
72. If in a triangle $A B C, 3 \sin A=6 \sin B=2 \sqrt{3} \sin C$, then the angle $A$ is

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73. The number of triangles $A B C$ that can be formed with $\sin A=\frac{5}{13}, a=3$ and $b=8$ is

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74. The lengths of the sides of a triangle are $\alpha-\beta, \alpha+\beta$ and $\sqrt{3 \alpha^{2}+\beta^{2}},(\alpha>\beta>0)$. Its largest angle is
75. In a $\triangle P Q R$ (as shown in figure) if $x: y: z=2: 3: 6$, then the value of $\angle Q P R$ is :


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76. If in $\triangle \mathrm{ABC}, \angle c=90^{\circ}$ then the maximum value of $\sin A \sin B$ is

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77. In an isosceles right angled triangle $\mathrm{ABC}, \angle B=90^{\circ}, A D$ is the median then $\frac{\sin \angle B A D}{\sin \angle C A D}$ is (A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 1 (D) none of these

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78. If in a $\triangle A B C, \mathrm{c}=3 \mathrm{~b}$ and $\mathrm{C}-\mathrm{B}=90^{\circ}$, then $\tan \mathrm{B}=$

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79. If the lengths of the sides of a triangle are $3,5,7$, then its largest angle of the triangle is

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80. In a $!A B C$ if $\mathrm{a}=7, \mathrm{~b}=8$ and $\mathrm{c}=9$, then the length of the line joining $B$ to the mid-points of $A C$ is
81. If H is the orthocenter of $\triangle A B C$ and if $A H=x, B H=y, C H=z$, then $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=$

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82. If the sides of a triangle are in the ratio $3: 7: 8$, then find $R: r$

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83. If in a $\triangle A B C, \sin ^{2} A+\sin ^{2} B+\sin ^{2} C=2$, then $\triangle$ is always a an
(A) isosceles triangle (B) right angled triangle (C) acute angled triangle
(D) obtuse angled triangle

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84. For a $\triangle A B C$, if $\cot A \cdot \cot B \cdot \cot C>0$, then nature of the triangle is
(A) acute angled triangle
(B) right angled triangle
(C) obtuse angled triangle
(D) none of these

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85. If the sides of a triangle are in G.P., and its largest angle is twice the smallest, then the common ratio $r$ satisfies the inequality ${ }^{\circ} 0$

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86. If in a $\triangle A B C, a^{2} \cos ^{2} A=b^{2}+c^{2}$, then angle A is (A) less than $45^{0}$ (B) more than of $45^{\circ}$ and $\leq \operatorname{ssthan} 90^{\circ}$ (C) right angled (D) obtuse angle
87. The perimeter of a $\triangle A B C$ is 6 times the A.M. of the sines of its angles. If the side 'a' is 1 , then the angle $A$ is

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88. If angle $C$ of a triangle $A B C$ be obtuse, then (A) $0<\tan A \tan B<1$
(B) $\tan A \tan B>1$ (C) $\tan A \tan B=1$ (D) none of these

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89. In an equilateral triangle, inradius $r$, circumradius $R$ and ex-radius $r_{1}$ are in
90. The ratio of the area of triangle inscribed in ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ to that of triangle formed by the corresponding points on the auxiliary circle is 0.5 . Then, find the eccentricity of the ellipse.

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91. If in a triangle ABC , the altitude AM be the bisector of $\angle B A D$, where D is the mid point of side BC , then prove that $\left(b^{2}-c^{2}\right)=\frac{a^{2}}{2}$.

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92. In a $\fallingdotseq / \_A B C, \tan , A / 2=5 / 6$ and $\tan , C / 2=2 / 5$ then $(A) a, c, b$ are in A.P. (B) $a, b, c$ are in A.P. (C) $b, a, c$ are in A.P. (D) a,b,c are in G.P.

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93. The sides of a triangle are $3 x+4 y, 4 x+3 y$ and $5 x+5 y$ units, where $x>0, y>0$. The triangle is

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94. In triangle $A B C, A D$ is the altitude from $A$. If $b>c, \angle C=23^{0}, a n d A D=\frac{a b c}{b^{2}}-c^{2}$, then $\angle B=_{-}{ }_{-}$

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95. A circle is inscribed in an equilateral triangle of side a. The area of any square inscribed in this circle is (A) $\frac{a^{2}}{12}$ (B) $\frac{a^{2}}{6}$ (C) $\frac{a^{2}}{3}$ (D) $2 a^{2}$

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96. Let $A_{0} A_{1} A_{2} A_{3} A_{4} A_{5}$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths the line segments $A_{0} A_{1}, A_{0} A_{2}$
and $A_{0} A_{4}$ is

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97. Let $f(x+y)=f(x) . f(y)$ for all x and y and $f(1)=2$. If in as triangle ABC, $a=f(3), b=f(1)+f(3), c=f(2)+f(3)$, then $2 A=$ (A) C (B) 2 C (C) 3 C (D) 4 C

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98. In a triangle $A B C$ the angle $B$ is greater than angle $C$. If the measure of angles $B$ and $C$ satisfy the equation $4 \sin ^{3} x-3 \sin x+0.75=0$ then the measure of angle A is (A) $\frac{\pi}{2}$ (B) $\frac{2 p}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{5 \pi}{6}$

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99. In a $\triangle A B C, \angle B=\frac{\pi}{3}, \angle C=\frac{\pi}{4}$ and D divides $B C$ internally in the ratio 1:3 Then $\frac{\angle B A D}{\angle C A D}=$ is equal to (a) $\frac{1}{\sqrt{6}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{\frac{2}{3}}$

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100. If $a, b, c$ be the sides foi a triangle $A B C$ and if roots of equation
$a(b-c) x^{2}+b(c-a) x+c(a-b)=0 \quad$ are equal then $\frac{\sin ^{2} A}{2}, \sin ^{2}, \frac{B}{2}, \frac{\sin ^{2} C}{2}$ are in (A) A.P. (B) G.P. (C) H.P. (D) none of these

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101. In a $\triangle A B C, b^{2}+c^{2}=1999 a^{2}$, then $\frac{\cot B+\cot C}{\cot A}=$ (A) $1 / 1999$
(B) 36161 (C) 999 (D) 1999

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102. If $(1+a x)^{n}=1+8 x+24 x^{2}+\ldots$. . then the value of a and n is

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103. If equations $a x^{2}+b x+c=0$ and $4 x^{2}+5 x+6=0$ have a comon root, where $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are the sides of $\triangle A B C$ opposite to angles $A, B, C$ respectively, then $2 A=(A) C(B) 2 C(C) 3 C(D) 4 C$

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104. If in $\triangle A B C, \frac{2 \cos A}{a}+\frac{\cos B}{b}+\frac{2 \cos C}{c}=\frac{a}{b c}+\frac{b}{c a}$, then $\angle A$ is equal to

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105. In triangle $A B C, a: b: c=4: 5: 6$. The ratio of the radius of the circumcircle to that of the incircle is $\qquad$ .

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106. In triangle $\mathrm{ABC}, \frac{\sin A+\sin B+\sin C}{\sin A+\sin B-\sin C}$ is equal to
107. If $\cos \mathrm{A}+\cos \mathrm{B}=4 \sin ^{2}\left(\frac{C}{2}\right)$, then

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108. If twice the square of the diameter of the circle is equal to half the sum of the squares of the sides of incribed triangle $A B C$,then $\sin ^{2} A+\sin ^{2} B+\sin ^{2} C$ is equal to

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109. If the base angles of triangle are $\frac{22}{12}$ and $112 \frac{1}{2^{0}}$, then prove that the altitude of the triangle is equal to $\frac{1}{2}$ of its base.

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110. Let $A B C$ be an isosceles triangle with base $B C$. If $r$ is the radius of the circle inscribsed in $\triangle A B C$ and $r_{1}$ is the radius of the circle ecribed opposite to the angle A , then the product $r_{1} r$ can be equal to (where R is the radius of the circumcircle of $\Delta A B C$ )

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111. If represents the area of acute angled triangle $A B C$, then $\sqrt{a^{2} b^{2}-4^{2}}+\sqrt{b^{2} c^{2}-4^{2}}+\sqrt{c^{2} a^{2}-4^{2}}=a^{2}+b^{2}+c^{2} \frac{a^{2}+b^{2}+c^{2}}{2}$ $a b \cos C+b o s A+c a \cos B a b \sin C+b c \sin A+c a \sin B$

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112. If in a $\triangle A B C, a=6, b=3$ and $c(A-B)=\frac{4}{5}$ then (A) $C=\frac{\pi}{4}$
(B) $A=\frac{\sin ^{-1} 2}{\sqrt{5}}$ (C) $\operatorname{ar}(\triangle A B C)=9$ (D) none of these

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113. In a triangle the lengths of the two larger are 10 and 9 respectively.lf the angles are in A.P., the , length of the third side can be (A) $5-\sqrt{6}$ (B) $3 \sqrt{3}$ (C) 5 (D) $5+\sqrt{6}$

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114. In a triangle $A B C$, points $D$ and $E$ are taken on side $B C$ such that $B D=$ $\mathrm{DE}=\mathrm{EC}$. If angle $\mathrm{ADE}=$ angle $\mathrm{AED}=\theta$, then: $(\mathrm{A}) \tan \theta=3 \tan \mathrm{~B}(\mathrm{~B}) 3 \tan \theta=$ $\tan \mathrm{C}$

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115. Which of the following holds goods for any tiangle $A B C, a, b, c$ are the lengths of the sides $R$ is circumradius
$\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{a^{2}+b^{2}+c^{2}}{2 a b c}$
$\frac{\sin A}{a}+\frac{\sin B}{b}+\frac{\sin C}{c}=\frac{3}{2 R 0} \quad$ (C) $\frac{\cos A}{a}=\frac{\cos B}{b}=\frac{\cos C}{c} \quad$ (D)
$\frac{\sin 2 A}{a} 62=\frac{\sin 2 B}{b^{2}}=\frac{\sin 2 C}{c^{2}}$
116. If the vertices $P, Q, R$ of a triangle $P Q R$ are rational points, which of the following points of thetriangle $P Q R$ is/are always rational point(s) ?(A) centroid(B) incentre(C) circumcentre(D) orthocentreAgrawn Korouteden36

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117. The value of $L t_{x \rightarrow 0}\left\{\frac{\int_{0}^{x^{2}} \sec ^{2} t d t}{x \sin x}\right\}$ is (A) 0 (B) 3 (C) 2 (D) 1

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118. If $a$ and $b$ be the length of the sides and $c$ the length of hypotenuse of a right anlged triangle then

$$
\begin{equation*}
\text { (A) } a+b>c \text { (B) } a^{2}+b^{2}=c^{2} \tag{C}
\end{equation*}
$$

$$
a^{3}+b^{3}<c^{3} \text { (D) } a^{n}+b^{n}<c^{n} \text { for } n \geq 3, n=Z
$$

119. If in $\triangle A B C, \angle A=90^{\circ}$ and c , $\sin \mathrm{B} \cos \mathrm{B}$ are rational numbers, then show a and b are rational .

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120. In triangle $A B C$, the value of $\left|\begin{array}{ccc}e^{-i 2 A} & e^{i C} & e^{i B} \\ e^{i C} & e^{-i 2 B} & e^{i A} \\ e^{i B} & e^{i A} & e^{-i 2 C}\end{array}\right|$

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121. If $a, b, c, d$ and $p$ are different real numbers such that $\left(a^{2}+b^{2}+c^{2}\right) p^{2}-2(a b+b c+c d) p+\left(b^{2}+c^{2}+d^{2}\right) \leq 0$, then show that $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and d are in G.P.

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122. If in a triangle $A B C, a^{2}+b^{2}+c^{2}=c a+a b \sqrt{3}$, then the triangle is
123. If all the vertices of a triangle have integral coordinates, then the triangle may be right-angled (b) equilateral isosceles (d) none of these

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124. In a triangle, the lengths of the two larger sides are 10 and 9 , respectively. If the angles are in A.P., then the length of the third side can be $5-\sqrt{6}$ (b) $3 \sqrt{3}$ (c) 5 (d) $5+\sqrt{6}$

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125. If the tangents of the angles $\mathrm{A}, \mathrm{B}$ of a $\triangle A B C$...satisfy the equation $a b x^{2}-c^{2} x+a b=0$, then

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126. In a triangle $A B C$, points $D$ and $E$ are taken on side $B C$ such that $B D=$ $\mathrm{DE}=\mathrm{EC}$. If angle $\mathrm{ADE}=$ angle $\mathrm{AED}=\theta$, then: $(\mathrm{A}) \tan \theta=3 \tan \mathrm{~B}(\mathrm{~B}) 3 \tan \theta=$ $\tan \mathrm{C}$

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127. In a triangle ABC if $a^{4}+b^{4}+c^{4}=2 c^{2}\left(a^{2}+b^{2}\right)$, then angle C is equal to (A) $60^{0}$ (B) $120^{0}$ (C) $45^{0}$ (D) $135^{0}$

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128. Statement-1: If the measures of two angles of a triangle are $45^{\circ}$ and $60^{\circ}$, then the ratio of the smallest and the greatest sides are $(\sqrt{3}-1): 1$

Statement-2: The greatest side of a triangle is opposite to its greatest angle.
129. Statement 1. In a triangle $A B C$, if $a: b: c=4: 5: 6$, then $R: r=16: 7$, Statement 2. In a triangle ABC, $R: r=a b c: 4 s$
(A) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1
(B) Both Statement 1 and Statement 2 are true and Statement 2 is not the correct explanatioin of Statement 1
(C) Statement 1 is true but Statement 2 is false.
(D) Statement 1 is false but Stastement 2 is true

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130. Area of circle inscribed in the equilateral $\triangle A B C$ is (A) $\frac{2}{3} \pi R^{2}$ $\frac{1}{4} \pi R^{2}$ (C) $\frac{1}{3} \pi R^{2}$ (D) none of these

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131. $\sin \left\{2 \cos ^{-1}\left(-\frac{3}{5}\right)\right\}$ is equal to $6 / 25$ (b) $24 / 25$ (c) $4 / 5$

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132. Three circles touch one-another externally. The tangents at their point of contact meet at a point whose distance from a point contact is 4 . Then, the ratio of the product of the radii of the sum of the radii of circles is

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133. If $A+B+C=\pi$, prove that
$\cot , \frac{A}{2}+\cot , \frac{B}{2}+\cot , \frac{C}{2}=\cot , \frac{A}{2} \cot , \frac{B}{2} \cot , \frac{C}{2}$

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134. Given the base of a triangle, the opposite angle A , and the product $k^{2}$ of other two sides, show that it is not possible for a to be less than $2 k \sin \frac{A}{2}$
135. In a triangle $A B C$ the sides $b$ and $c$ are the roots of the equation $x^{2}-61 x+820=0$ and $A=\tan ^{-1}\left(\frac{4}{3}\right)$ thena ${ }^{2}+3$ is equal to

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136. v37

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137. If in a triangle $A B C, \operatorname{Rr}(\sin A+\sin B+\sin C)=96$ then the square of the area of the triangle $A B C$ is.......

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138. The sides of a quadrilateral are $3,4,5$ and 6 cms . The sum of a pair of opposite angles is $120^{0}$. Showtt̂heareaofthe rilateralis $3 \mathrm{sqrt}(30)^{`}$

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139. Three circles touch one-another externally. The tangents at their point of contact meet at a point whose distance from a point contact is 4 . Then, the ratio of the product of the radii of the sum of the radii of circles is

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140. In triangle $A B C, a: b: c=4: 5: 6$. The ratio of the radius of the circumcircle to that of the incircle is $\qquad$ .

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141. If $p_{1}, p_{2}, p_{3}$ are the altitudes of a triangle from the vertices $A, B, C, \&$ denotes the area of the triangle, prove that
$\frac{1}{p_{1}}+\frac{1}{p_{2}}-\frac{1}{p_{3}}=\frac{2 a b}{(a+b+c)} \frac{\cos ^{2} C}{2}$

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142. If the sides of a quadrilateral $A B C D$ touch a circle prove that $A B+C D=B C+A D$.

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143. If in triangle $\mathrm{ABC},\left(a=(1+\sqrt{3}) c m, b=2 c m\right.$, $a n d \angle C=60^{\circ}$, then find the other two angles and the third side.

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144. If a circle is inscribed in right angled triangle $A B C$ with right angle at B , show that the diameter of the circle is equal to $A B+B C-A C$.
145. If a triangle is inscribed in a circle, then prove that the product of any two sides of the triangle is equal to the product of the diameter and the perpendicular distance of the thrid side from the opposite vertex.

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146. $A B C$ is triangle. $D$ is the middle point of $B C$. If $A D$ is perendicular to AC , then prove that
$\cos A \cos C=\frac{2\left(c^{2}-a^{2}\right)}{3 a c}$

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147. Let the angles $A, B a n d C$ of triangle $A B C$ be in $A P$. and let $b: c$ be $\sqrt{3}: \sqrt{2}$. Find angle $A$.

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148. The exradii $r_{1}, r_{2}$, and $r_{3}$ of $\triangle A B C$ are in H.P. show that its sides $\mathrm{a}, \mathrm{b}$, and c are in A.P.

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149. 112. If in a $\triangle A B C, \cos A+\cos B+\cos c=\frac{3}{2}$. Prove that $\triangle A B C$ is an equilateral triangle.

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150. With usual notion, if in triangle $A B C$, $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, thenprovethat $\frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}$

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151. $A B$ is a diameter of a circle and $C$ is any point on the circumference of the circle. Then the area of $A B C$ is maximum when it is isosceles the area
of $A B C$ is minimum when it is isosceles the perimeter of $A B C$ is minimum when it is isosceles none of these

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152. In a $\triangle A B C$, the median to the side BC is of length $\frac{1}{\sqrt{11-6 \sqrt{3}}}$ and it divides the $\angle A$ into angles $30^{\circ}$ and $45 \circ$. Find the length of the side BC.

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153. In a triangle ABC if $\cos A \cos B+\sin A \sin B \sin C=1$ show that the sides are in the proportion $1: 1: \sqrt{2}$

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154. If $\mathrm{a}, \mathrm{b}$ and c are distinct positive numbers, then the expression
$(a+b-c)(b+c-a)(c+a-b)-a b c$ is:

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155. In a triangle, the lengths of the two larger sides are 10 and 9 , respectively. If the angles are in A.P., then the length of the third side can be $5-\sqrt{6}$ (b) $3 \sqrt{3}$ (c) 5 (d) $5+\sqrt{6}$

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156. If the angles of a triangle are $30^{\circ}$ and $45^{\circ}$ and the included side is $(\sqrt{3}+1) c m$ then the area of the triangle is $\qquad$ .

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157. $A B C$ is a triangle such that $\sin (2 A+B)=\sin (C-A)=-\sin (B+2 C)=\frac{1}{2}$. If $\mathrm{A}, \mathrm{B}$, and C are in $A P$. then the value of $A, B$ and $C$ are..
158. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. Determine the sides of the triangle.

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159. If in a triangle $A B C$,
$2 \frac{\cos A}{a}+\frac{\cos B}{b}+2 \frac{\cos C}{c}=\frac{a}{b c}+\frac{b}{c a}$,
then the value of the angle $A$, is

## D Watch Video Solution

160. A circle is inscribed in an equilateral triangle of side $a$. The area of any square inscribed in this circle is $\qquad$ .

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161. Let $A_{1}, A_{2}, \ldots . A_{n}$ be the vertices of an $n$-sided regular polygon such that , $\frac{1}{A_{1} A_{2}}=\frac{1}{A_{1} A_{3}}+\frac{1}{A_{1} A_{4}}$. Find the value of $n$.

## (D) Watch Video Solution

162. Consider the following statements concerning a $\triangle A B c$
(i) The sides $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and area of triangle are rational.
(ii) $a, \tan \frac{B}{2}, \tan \frac{C}{2}$
(iii) $a, \sin A \sin B, \sin C$ are rational .

Prove that $(i) \Rightarrow(i i) \Rightarrow(i i i) \Rightarrow(i)$

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163. IF the lengths of the side of triangle are $3,5 A N D 7$, then the largest angle of the triangle is $\frac{\pi}{2}$ (b) $\frac{5 \pi}{6}$ (c) $\frac{2 \pi}{3}$ (d) $\frac{3 \pi}{4}$

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164. In triangle $A B C, a: b: c=4: 5: 6$. The ratio of the radius of the circumcircle to that of the incircle is $\qquad$ .

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165. Let $\mathrm{A}, \mathrm{B}, \mathrm{C}$, be three angles such that $A=\frac{\pi}{4}$ and $\tan B, \tan C=p$. Find all possible values of $p$ such that $A, B, C$ are the angles of a triangle.

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166. If in a triangle $P Q R ; \sin P, \sin Q, \sin R$ are in A.P; then

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167. Prove that a triangle $A B C$ is equilateral if and only if ${ }^{\prime} \tan \mathrm{A}+\tan \mathrm{B}+\tan \mathrm{C}=3 \mathrm{sqrt}(3)$.
168. Let $A B C$ be a triangle having $O$ and $I$ as its circumradius and inradis, respectively then prove that $(I O)^{2}=R^{2}-2 R r$. Further show that the triangle BIO is a right angled triangle if and only if $b$ is the rithmetic mean of $a$ and $c$.

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169. In triangle $A B C, 2 a c \sin \left(\frac{1}{2}(A-B+C)\right)$ is equal to $a^{2}+b^{2}-c^{2}$ (b) $c^{2}+a^{2}-b^{2} b^{2}-c^{2}-a^{2}$ (d) $c^{2}-a^{2}-b^{2}$

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170. In a triangle ASBC, let $\angle C=\frac{\pi}{2}$. Ifr is the in radius and R is the circumrdius of the triangle then $2(r+R)$ is equal to (A) $a+b$ (B) $b+c$
(C) $c+a$ (D) $a+b+c$

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172. Let $P Q a n d R S$ be tangent at the extremities of the diameter $P R$ of a circle of radius $r$. If $P S a n d R Q$ intersect at a point $X$ on the circumference of the circle, then prove that $2 r=\sqrt{P Q x R S}$.

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173. If $\Delta$ is the area of a triangle with side lengths $a, b, c$, then show that as $\Delta \leq \frac{1}{4} \sqrt{(a+b+c) a b c}$ Also, show that the equality occurs in the above inequality if and only if $a=b=c$.

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174. Which of the following pieces of data does NOT uniquely determine an acute-angled triangle $A B C(R$ being the radius of the circumcircle $)$ ? $a, \sin A, \sin B$ (b) $a, b, c, a, \sin B, R$ (d) $a, \sin A, R$

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175. If the angles of a triangle are in the ratio $4: 1: 1$, then the ratio of the longest side to the perimeter is- $\sqrt{3}:(2+\sqrt{3})$ b. $1: \sqrt{3}$ c. $1: 2+\sqrt{3} \mathrm{~d}$. 2: 3

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176. The ratio of the sides of a triangle $A B C$ is $1: \sqrt{3}: 2$. The ratio $A: B: C$ is

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177. In an equilateral triangle, three coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. The area of
the triangle is (fig) $4: 2 \sqrt{3}$ (b) $6+4 \sqrt{3} 12+\frac{7 \sqrt{3}}{4}$ (d) $3+\frac{7 \sqrt{3}}{4}$

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178. One angle of an isosceles triangle is $120^{\circ}$ and the radius of its incricel is $\sqrt{3}$. Then the area of the triangle in sq. units is $7+12 \sqrt{3}$ (b) $12-7 \sqrt{3}$ $12+7 \sqrt{3}$ (d) $4 \pi$

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179. Let $a, b, c$ be the sides of a triangle. No two of them are equal and $\lambda \in R \quad$ If the roots of the equation $x^{2}+2(a+b+c) x+3 \lambda(a b+b c+c a)=0$ are real, then (a) $\lambda<\frac{4}{3}$ (b)
$\lambda>\frac{5}{3}$ (c) $\lambda \in\left(\frac{1}{5}, \frac{5}{3}\right)$ (d) $\lambda \in\left(\frac{4}{3}, \frac{5}{3}\right)$

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180. Internal bisector of $\angle A$ of triangle $A B C$ meets side $B C$ at $D$. $A$ line drawn through D perpendicular to AD intersects the side AC at E and the side AB at F . If $\mathrm{a}, \mathrm{b}, \mathrm{c}$ represent sides of $\triangle A B C$, then

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181. a triangle $A B C$ with fixed base $B C$, the vertex $A$ moves such that $\cos B+\cos C=4 \frac{\sin ^{2} A}{2}$. If $a$, bandc, denote the length of the sides of the triangle opposite to the angles $A, B$, and $C$, respectively, then $b+c=4 a$ (b) $b+c=2 a$ the locus of point $A$ is an ellipse the locus of point $A$ is a pair of straight lines

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182. Consider a triangle $A B C$ and let $a$, bandc denote the lengths of the sides opposite to vertices $A, B$, and $C$, respectively. Suppose $a=6, b=10$, and the area of triangle is $15 \sqrt{3}$. If $\angle A C B$ is obtuse and
if $r$ denotes the radius of the incircle of the triangle, then the value of $r^{2}$ is

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183. If the angle $A, B a n d C$ of a triangle are in an arithmetic propression and if $a$, bandc denote the lengths of the sides opposite to $A, B a n d C$ respectively, then the value of the expression $\frac{a}{c} \sin 2 C+\frac{c}{a} \sin 2 A$ is $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\sqrt{3}$

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184. Let $A B C$ be a triangle such that $\angle A C B=\frac{\pi}{6}$ and let $a$, bandc denote the lengths of the side opposite to $A, B$, and $C$ respectively. The value(s) of $x$ for which $a=x^{2}+x+1, b=x^{2}-1$, andc $=2 x+1$ is(are) $-(2+\sqrt{3})$ (b) $1+\sqrt{3} 2+\sqrt{3}$ (d) $4 \sqrt{3}$
185. the sum of the radii of inscribed and circumscribed circle of an $n$ sides regular polygon of side a is (A) $\frac{a}{2} \cot \left(\frac{\pi}{2 n}\right)$ (B) $a \cot \left(\frac{\pi}{2 n}\right)$ $\left.\frac{a}{4} \cos , \frac{\pi}{2 n}\right)$ (D) $a \cot \left(\frac{\pi}{n}\right)$

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186. In a $\triangle A B C$, medians $A D$ and $B E$ are drawn. If $A D=4, \angle D A B=\frac{\pi}{6}$ and $\angle A B E=\frac{\pi}{3}$ then the area of $\triangle A B C$ is

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187. If in a triangle $A B C, a \cos ^{2}\left(\frac{C}{2}\right) \operatorname{os}^{2}\left(\frac{A}{2}\right)=\frac{3 b}{2}$, then the sides $a, b, a n d c$ are in A.P. b. are in G.P. c. are in H.P. d. satisfy $a+b=\cdot$

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188. The sides of a triangle are $\sin \alpha, \cos \alpha$ and $\sqrt{1+\sin \alpha \cos \alpha}$ for some $\alpha, 0<\alpha<\frac{\pi}{2}$. Then the greatest angle of the triangle is

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189. In triangle $A B C$, let $\angle c=\frac{\pi}{2}$. If $r$ is the inradius and $R$ is circumradius of the triangle, then $2(r+R)$ is equal to $a+b$ (b) $b+c$ $c+a$ (d) $a+b+c$

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190. If in a $\triangle A B C$ the altitude from the vertices $\mathrm{A}, \mathrm{B}, \mathrm{C}$ on opposite side are in H.P. then $\sin A, \sin B, \sin C$ are in (A) H.P. (B) ArithmeticoGeometric progression (C) A.P. (D) G.P.

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