MATHS

BOOKS - KC SINHA MATHS (HINGLISH)

TRIGONOMETRIC FUNCTIONS - MULTIPLE AND SUBMULTIPLE OF ANGLES - FOR BOARDS

Solved Examples

1. If $\sin A = \frac{3}{5}$ and $0^\circ < A < 90^\circ$, find the value of $\sin 2A, \cos 2A, \tan 2A,$ and $\sin 4A$.

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2. If $\tan\theta = \frac{b}{a}$, then find the value of $a\cos 2\theta + b\sin 2\theta$.

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3. If $\tan\alpha = \frac{1}{7}, \sin\beta = \frac{1}{\sqrt{10}}$, Prove that : $\alpha + 2\beta = \frac{\pi}{4}$, where $0 < \alpha < \frac{\pi}{2}$ and $0 < \beta < \frac{\pi}{2}$.

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4. Prove that : $\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = 2\sec 2\theta.$

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5. Show that $\sqrt{3}\cos ec20^\circ - \sec 20^\circ = 4$

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6. prove that : $\tan(\alpha) + 2\tan(2\alpha) + 4(\tan 4\alpha) + 8\cot(8\alpha) = \cot(\alpha)$

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7. Prove that : $\cos^2\theta + \cos^2\left(\frac{2\pi}{3} - \theta\right) + \cos^2\left(\frac{2\pi}{3} + \theta\right) = \frac{3}{2}$

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8. Prove that $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$

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9. Prove that : $2\sin^2\theta + 4\cos(\theta + \alpha)\sin\alpha\sin\theta + \cos 2(\alpha + \theta)$ is independent of θ.

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10. Prove that : $\cos^3 A\cos 3A + \sin^3 A\sin 3A = \cos^3 2A$

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11. $\tan\alpha.\ \tan(60^\circ - \alpha).\ \tan(60^\circ + \alpha) = \tan 3\alpha$

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12. Prove that $\sin^3\alpha + \sin^3\left(\frac{2\pi}{3} + \alpha\right) + \sin^3\left(\frac{4\pi}{3} + \alpha\right) = -\frac{3}{4}\sin 3\alpha$

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13. Prove that $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$

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14. If $2\tan\alpha = 3\tan\beta,$ prove that $tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}.$

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15. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$, show that $\sin(\alpha+\beta) = \dfrac{2ab}{\alpha^2+\beta^2}$

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16. If $(2^n+1)\theta = \pi$ then $2^n \cos\theta \cos 2\theta \cos 2^2\theta \ldots\ldots \cos 2^{n-1}\theta =$

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17. Prove that : $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} = \tan\dfrac{\theta}{2}$

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18. Show that : $\dfrac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} = \dfrac{1+\sin\theta}{\cos\theta} = \tan\left(\dfrac{\pi}{4}+\dfrac{\theta}{2}\right)$

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19. Prove that :$(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 4\cos^2 \frac{A-B}{2}$

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20. $\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right) =$

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21. Find the value of $\tan \frac{\pi}{8}$.

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22. if $\tan x = -\frac{4}{3}, \frac{\pi}{2} < x < \pi,$ then find the values of $\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}.$

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23. Find the value of $\sin \frac{23\pi}{24}$.

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24. If $\alpha = 112^\circ 30'$, find the value of $\sin \alpha$ and $\cos \alpha$.

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25. Prove that: $\sin^2 24^0 - \sin^2 6^0 = \frac{\sqrt{5}-1}{8}$

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26. If $A = \tan 6^\circ \tan 42^\circ$ and $B = \cot 66^\circ \cot 78^\circ$, then

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27. Show that $\sin 47^\circ + \sin 61^\circ - \sin 11^\circ - \sin 25^\circ = \cos 7^\circ$

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28. Prove that: $s \in 12^0 s \in 48^0 s \in 54^0 = \frac{1}{8}$.

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29. Prove that $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\frac{\tan\theta}{2}\right) = \cos^{-1}\left(\frac{a\cos\theta + b}{a + b\cos\theta}\right)$

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30. If α and β be two different roots of equation, $a\cos\theta + b\sin\theta = c$, prove that $\sin(\alpha+\beta) = \frac{2ab}{a^2+b^2}$.

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31. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$ then $\tan\left(\frac{\alpha-\beta}{2}\right)$ is equal to

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32. If $\cos\theta = \cos\alpha\cos\beta$ then $\tan\left(\frac{\theta+\alpha}{2}\right)\tan\left(\frac{\theta-\alpha}{2}\right) =$ (i) $\tan^2\left(\frac{\alpha}{2}\right)$ (ii)$\tan^2\left(\frac{\beta}{2}\right)$ (iii)$\tan^2\left(\frac{\theta}{2}\right)$ (iv) $\cot^2\left(\frac{\beta}{2}\right)$

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33. If $\cos\theta = \frac{\cos\alpha\cos\beta}{1-\sin\alpha\sin\beta}$, prove that one value of $(\tan)\frac{\theta}{2} = \frac{\frac{\tan\alpha}{2} - \frac{\tan\beta}{2}}{1 - tan\frac{\alpha}{2}\frac{\tan\beta}{2}}$.

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Exercise

1. If $\tan\theta = \frac{a}{b}$, where $0 < \theta < \frac{\pi}{4}$ and $b > a > 0$, find the value of $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$.

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2. If $\tan\theta = -\frac{3}{4}$ and $\frac{\pi}{2} < \theta < \pi$, find the values of $\sin\theta, \cos\theta$ and $\cot\theta$.

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3. Let $\frac{3\pi}{4} < \theta < \pi$ and $\sqrt{2\cot\theta + \frac{1}{\sin^2\theta}} = k - \cot\theta$ then $k =$

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4. Prove that : $\cot\theta - \tan\theta = 2\cot 2\theta$

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5. Prove that : $\frac{1 - \tan^2\left(\frac{\pi}{4} - A\right)}{1 + \tan^2\left(\frac{\pi}{4} - A\right)} = \sin 2A.$

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6. Prove that : $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \sec 2\theta - \tan 2\theta$

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7. $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right) = 2\tan 2\theta$

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8. If $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$, show that $\cos 2\theta = \frac{1}{2} \cdot \left(a^2 + \frac{1}{a^2}\right)$

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9. Prove that : $\cos^2\theta + \sin^2\theta\cos 2\beta = \cos^2\beta + \sin^2\beta\cos 2\theta$

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10. Prove that : $1 + \tan\theta \tan 2\theta = \sec 2\theta.$

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11. Prove that : $\dfrac{1 + \sin 2A - \cos 2A}{1 + \sin 2A + \cos 2A} = \tan A$

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12. Prove that $\dfrac{1 + \sin 2\theta}{1 - \sin 2\theta} = \left(\dfrac{1 + \tan\theta}{1 - \tan\theta}\right)^2$

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13. Show that $\dfrac{1}{\sin 10^\circ} - \dfrac{\sqrt{3}}{\cos 10^\circ} = 4$

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14. Prove that :$\cos ecA - 2\cot 2A \cos A = 2\sin A$

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15. Prove that: $\cot^2 A - \tan^2 A = 4 \cos 2A \cos ec2A$

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16. Prove that : $\frac{1 + \sin 2A}{\cos 2A} = \frac{\cos A + \sin A}{\cos A - \sin A} = \tan\left(\frac{\pi}{4} + A\right)$

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17. Prove that : $\cos^6 A - \sin^6 A = \cos 2A\left(1 - \frac{1}{4}\sin^2 2A\right)$

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18. Prove that: $\cos^2 \theta + \cos^2\left(\frac{\pi}{3} + \theta\right) + \cos^2\left(\frac{\pi}{3} - \theta\right) = \frac{3}{2}$

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19. Prove that :

$$\frac{2\cos 2^n\theta + 1}{2\cos\theta + 1} = (2\cos\theta - 1)(2\cos 2\theta - 1)\left(2\cos 2^2\theta - 1\right)\ldots\left(2\cos 2^{n-1}\theta -\right.$$

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20. prove that :

$$\frac{\tan 2^n\theta}{\tan\theta} = (1 + \sec 2\theta)\left(1 + \sec 2^2\theta\right)\left(1 + \sec 2^3\theta\right)\ldots(1 + \sec 2^n\theta)$$

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21. Prove that : $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x.$

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22. Using induction, prove that

$$\cos\theta \cdot \cos 2\theta \cdot \cos 2^2\theta. \ldots \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n \sin\theta}$$

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23. Show that :

$3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = 13.$

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24. Show that : $2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1 = 0.$

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25. Show that : $\cos^2 \theta + \cos^2(\alpha + \theta) - 2\cos\alpha\cos\theta\cos(\alpha + \theta)$ is independent of θ.

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26. Prove the following identity:

$4(\cos^3 10^0 + \sin^3 \ 20^0) = 3\ (\cos 10^0 + s \in 20^0)$

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27. Prove that : $\sin\theta\cos^3\theta - \cos\theta\sin^3\theta = \frac{1}{4}\sin 4\theta.$

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28. $\cos^3\theta \cdot \sin 3\theta + \sin^3\theta \cdot \cos 3\theta = \frac{3}{4}\sin 4\theta$

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29. If $\tan\theta + \tan(60^\circ + \theta) + \tan(120^\circ + \theta) = 3$ then $\theta =$

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30. Prove that : $4\sin\theta\sin\left(\theta + \frac{\pi}{3}\right)\sin\left(\theta + \frac{2\pi}{3}\right) = \sin 3\theta$

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31. Prove that : $\cot\theta + \cot(60^\circ + \theta) + \cot(120^\circ + \theta) = 3\cot 3\theta.$

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32. Prove that :$\cos 4x = 1 - 8\sin^2 x\cos^2 x.$

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33. prove $\sin(5\theta) = 16\sin^5(\theta) - 20\sin^3(\theta) + 5\sin(\theta)$

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34. Prove that $\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^4\theta - 1$

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35. Prove that

$\cos 4\theta - \cos 4\alpha = 8(\cos\theta - \cos\alpha)(\cos\theta + \cos\alpha)(\cos\theta - \sin\alpha)(\cos\theta + \sin\alpha$

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36. Prove that :$\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$.

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37. If $\tan x = \dfrac{b}{a}$, prove that $a\cos 2x + b\sin 2x = a$

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38. If $\tan^2\theta = 1 + 2\tan^2\phi$, prove that $\cos 2\phi = 1 + 2\cos 2\theta$.

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39. If α and β are acute angles and $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$. Prove that : $\tan\alpha = \sqrt{2}\tan\beta$.

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40. If $\tan\beta = 3\tan\alpha$, prove that $\tan(\alpha+\beta) = \frac{2\sin 2\beta}{1+\cos 2\beta}$

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41. If $x\sin\alpha = y\cos\alpha$, prove that : $\frac{x}{\sec 2\alpha} + \frac{y}{\cos ec2\alpha} = x$

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42. If $\tan 2\theta = \sec 2\alpha$, prove that $\sin 2\theta = \frac{1-\tan^4\alpha}{1+\tan^4\alpha}$

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43. If $\alpha = \frac{\pi}{3}$, prove that $\cos\alpha \cdot \cos 2\alpha \cdot \cos 3\alpha \cdot \cos 4\alpha \cdot \cos 5\alpha \cdot \cos 6\alpha = -\frac{1}{16}$.

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44. Prove that: $\cos\left(\frac{2\pi}{15}\right) . \cos\left(\frac{4\pi}{15}\right) . \cos\left(\frac{8\pi}{15}\right) . \cos\left(\frac{16\pi}{15}\right) = \frac{1}{16}$

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45. If $\tan A \tan B = \sqrt{\frac{a-b}{a+b}}$, prove that : $(a - b\cos 2A)(a - b\cos 2B) = a^2 - b^2$

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46. Prove that $\frac{2\sin\theta - \sin 2\theta}{2\sin\theta + \sin 2\theta} = \tan^2\frac{\theta}{2}$

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47. Prove that $\cot\left(\frac{\theta}{2}\right) - \tan\left(\frac{\theta}{2}\right) = 2\cot\theta$

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48. Prove that : $\frac{1+\sin\theta}{1-\sin\theta} = \tan^2\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

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49. Prove that $\sec\theta + \tan\theta = \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

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50. $\frac{\sin\alpha + \sin\beta - \sin(\alpha+\beta)}{\sin\alpha + \sin\beta + \sin(\alpha+\beta)} = \tan\left(\frac{\alpha}{2}\right)\tan\left(\frac{\beta}{2}\right)$

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51. Prove that :$\tan\left(\frac{\pi}{4}-\frac{A}{2}\right)=\sec A-\tan A=\sqrt{\frac{1-\sin A}{1+\sin A}}$

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52. Prove that $\cos ec\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\cos ec\left(\frac{\pi}{4}-\frac{\theta}{2}\right)=2\sec\theta$

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53. Prove that : $(\cos A-\cos B)^2+(\sin A-\sin B)^2=4\sin^2\frac{A-B}{2}$

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54. Prove that : $(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\frac{x+y}{2}$

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55. $\cos^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) + \cos^2\left(\frac{5\pi}{8}\right) + \cos^2\left(\frac{7\pi}{8}\right) = 2$

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56. Prove that : $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8} = \frac{3}{2}$

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57. If $\tan x = \frac{3}{4}$,$\pi < x < \frac{3\pi}{2}$, then find the value of $\cos\left(\frac{x}{2}\right)$

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58. If $\sin x = \frac{1}{4}, \frac{\pi}{2} < x < \pi$, find the values of $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$.

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59. If $\cos x = -\frac{1}{3}, \pi < x < \frac{3\pi}{2}$ then find the values of $\sin\left(\frac{x}{2}\right)$ and $\tan\left(\frac{x}{2}\right)$

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60. Prove that:

$$\left(1+\frac{\cos\pi}{8}\right)\left(1+\frac{\cos(3\pi)}{8}\right)\left(1+\frac{\cos(5\pi)}{8}\right)\left(1+\frac{\cos(7\pi)}{8}\right)=\frac{1}{8}$$

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61. Show that $\cot\left(142\frac{1}{2}\right)^{\circ} = \sqrt{2}+\sqrt{3}-2-\sqrt{6}$

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62. Prove that $\sin^2 48^{\circ} - \cos^2 12^{\circ} = -\frac{\sqrt{5}+1}{8}$

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63. prove that $4\sin(24^\circ + \cos 6^\circ) = \sqrt{3} + \sqrt{15}$

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64. $\cot 6^0 \cot 42^0 \cot 66^0 \cot 78^0 = 1$

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65. Prove that $\tan 12^\circ \tan 24^\circ \tan 48^\circ \tan 84^\circ = 1.$

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66. Prove that: $\sin 6^0 \cos 42^0 \cos 60^0 s \in 78^0 = \frac{1}{16}$

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67. $\sin\left(\frac{\pi}{5}\right)\sin\left(\frac{2\pi}{5}\right)\sin\left(\frac{3\pi}{5}\right)\sin\left(\frac{4\pi}{5}\right) =$

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68. Prove that : $\cos 36^\circ \cos 72^\circ \cos 108^\circ \cos 144^\circ = \frac{1}{16}$

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69.

$\cos\left(\frac{\pi}{15}\right)\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{3\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{5\pi}{15}\right)\cos\left(\frac{6\pi}{15}\right)\cos\left(\frac{7\pi}{15}\right) = \frac{1}{2^7}$

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70.

$\cos\left(\frac{\pi}{65}\right)\cos\left(\frac{2\pi}{65}\right)\cos\left(\frac{4\pi}{65}\right)\cos\left(\frac{8\pi}{65}\right)\cos\left(\frac{16\pi}{65}\right)\cos\left(\frac{32\pi}{64}\right) = \frac{1}{64}$

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71. If $\tan\left[\frac{\theta}{2}\right] = \sqrt{\frac{1-e}{1+e}}\tan\left[\frac{\phi}{2}\right]$ then prove that $\cos\phi = \frac{\cos\theta - e}{1 - e\cos\theta}$

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72. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$, show that $\sin(\alpha+\beta) = \frac{2ab}{\alpha^2+\beta^2}$

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73. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$, prove that : $\cos(\alpha-\beta) = \frac{1}{2}(a^2+b^2-2)$

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74. If α and β be the two different roots of equation $a\cos\theta + b\sin\theta = c$, prove that : $\tan(\alpha+\beta) = \frac{2ab}{a^2-b^2}$

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75. If α, β are the roots of the equation $a\cos\theta + b\sin\theta = c$, then prove that $\cos(\alpha+\beta) = \dfrac{a^2-b^2}{a^2+b^2}$.

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76. If $\cos\alpha + \cos\beta = \dfrac{1}{3}$ and $\sin\alpha + \sin\beta = \dfrac{1}{4}$ prove that $\cos\left[\dfrac{\alpha-\beta}{2}\right] = \pm\left(\dfrac{5}{24}\right)$

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77. If $2\dfrac{\tan\alpha}{2} = \dfrac{\tan\beta}{2}$, prove that $\cos\alpha = \dfrac{3+5\cos\beta}{5+3\cos\beta}$.

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78. If $\cos\alpha = \dfrac{4}{5}$ and $\cos\beta = \dfrac{5}{13}$, prove that $\dfrac{\cos(\alpha-\beta)}{2} = \dfrac{8}{\sqrt{65}}$.

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79. If $\sec(\phi+\alpha)+\sec(\phi-\alpha)=2\sec\phi$, prove that $\cos\phi=\pm\sqrt{2}\frac{\cos\alpha}{2}$

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80. If $\cos\theta=\frac{\cos\alpha\cos\beta}{1-\sin\alpha\sin\beta}$, prove that one value of $(\tan)\frac{\theta}{2}=\frac{\frac{\tan\alpha}{2}-\frac{\tan\beta}{2}}{1-tan\frac{\alpha}{2}\frac{\tan\beta}{2}}$.

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81. If $\cos\theta=\frac{\cos\alpha-\cos\beta}{1-\cos\alpha\cos\beta}$, prove that $\frac{\tan\theta}{2}=\pm\frac{\tan\alpha}{2}\frac{\cot\beta}{2}$.

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