MATHS

BOOKS - KC SINHA MATHS (HINGLISH)

TRIGONOMTERY - JEE MAINS AND ADVANCED QUESTIONS - FOR COMPETITION

Exercise

1. Period of $\sin^2\theta$ is(A) π^2 (B) π (C) 2π (D) $\frac{\pi}{2}$

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2. the period of the $f(x) = \sin^4 x + \cos^4 x$ is

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3. $\sin^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if (A) $x - y \neq 0$ (B) $x = -y$ (C) $x + y \neq 0$ (D) $x \neq 0, y \neq 0$

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4. The value of $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ} =$ (A) 1 (B) $\sqrt{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) 2

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5. If $\tan\theta = -\frac{4}{3}, then \sin\theta$ is $-\frac{4}{5} but \neg \frac{4}{5}$ (b) $-\frac{4}{5}$ or $\frac{4}{5}$ $\frac{4}{5} but \neg -\frac{4}{5}$ (d) none of these

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6. if $\sin(\alpha + \beta) = 1$ and $\sin(\alpha - \beta) = \frac{1}{2}$ $0 \leq \alpha, \beta, \leq \frac{\pi}{2}, then$ find $\tan(\alpha + 2\beta)$ and $\tan(2\alpha + \beta)$

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7. If $y = \sin^2\theta + \cos ec^2\theta, \theta \neq 0,$ then (A) $y = 0$ (B) $y \leq 2$ (C) $y \geq -2$ (D) $y \geq 2$

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8. The equation $a\sin x + b\cos x = c$ and $|c| > \sqrt{a^2 + b^2}$ has

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9. If $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x,$ then $\sin x$ is $\frac{\tan^2\alpha}{2}$ (b) $\frac{\cot^2\alpha}{2}$ (c) $\tan^2\alpha$ (d) $\frac{\cot\alpha}{2}$

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10. Prove that: $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right).$

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11. In a $\Delta ABC, 2ac\sin\left(\frac{A-B+C}{2}\right)$ is equal to (a) $a^2+b^2-c^2$ (b) $c^2+a^2-b^2$ (c) $b^2-c^2-a^2$ (d) $c^2-a^2-b^2$

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12. In a triangle $ABC, a=4, b=3, \angle A=60^0$ then c is root of the equation $c^2-3c-7=0$ (b) $c^2+3c+7=0$ (c) $c^2-3c+7=0$ (d) $c^2+3c-7=0$

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13. In a $\triangle\ ABC, \tan\frac{A}{2}=\frac{5}{6}$ and $\tan\frac{C}{2}=\frac{2}{5}$ then (A) a,c,b are in A.P. (B) a,b,c are in A.P. (C) b,a,c are in A.P. (D) a,b,c are in G.P.

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14. The trigonometric equation $\sin^{-1} x = 2\sin^{-1} a$ has a solution for all real values (b) $|a| < \frac{1}{a}$ $|a| \leq \frac{1}{\sqrt{2}}$ (d) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$

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15. If in a triangle ABC, $a\cos^2\left(\frac{C}{2}\right)os^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides $a, b, andc$ are in A.P. b. are in G.P. c. are in H.P. d. satisfy $a + b = \cdot$

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16. In a ΔABC, medians AD and BE are drawn. If $AD = 4$, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$ then the area of ΔABC is

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17. 10. The upper $\frac{3}{4}$ portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at the point in the horizontal plane through its foot. The tangent of the angle subtended by the pole at the same point is

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18. The sum of radii of inscribed and circumscribed circles of an n sided regular polygon of side a is

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19. $\pi < \alpha - \beta < 3\pi, \sin\alpha + \sin\beta = -\frac{21}{65}, \cos\alpha + \cos\beta = -\frac{27}{65}$,then $\cos\left(\frac{\alpha - \beta}{2}\right) =$ (A) $-\frac{6}{65}$ (B) $-\frac{3}{\sqrt{130}}$ (C) $\frac{3}{\sqrt{130}}$ (D) $\frac{6}{65}$

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20. If $f: R \rightarrow S$ defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$ is onto , then the interval of S is :

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21. The sides of a triangle are $\sin\alpha, \cos\alpha$ and $\sqrt{1 + \sin\alpha\cos\alpha}$ for some $\alpha, 0 < \alpha < \frac{\pi}{2}$.Then the greatest angle of the triangle is

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22. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and where he retires 40 meters away from the tree the angle of elevation becomes 30°. The breadth of the river is

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23. If $\cos^{-1}x - \cos^{-1}\left(\frac{y}{2}\right) = \alpha$ then $4x^2 - 4xy\cos\alpha + y^2 =$

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24. In triangle ABC, let $\angle c = \frac{\pi}{2}$. If r is the inradius and R is circumradius of the triangle, then $2(r + R)$ is equal to $a + b$ (b) $b + c$ $c + a$ (d) $a + b + c$

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25. If in a ΔABC, the altitudes from the vertices A, B, C on opposite sides are in H.P, then sin A, sin B, sin C are in

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26. A triangle PQR, $\angle R = 90^\circ$ and $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ roots of the $ax^2 + bx + c = 0$ then prove that $a + b = c$

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27. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$ respectively then the value of $2 + q - p$ is

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28. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is (1) $\frac{4 - \sqrt{7}}{3}$ (2) $-\frac{4 + \sqrt{7}}{3}$ (3) $\frac{1 + \sqrt{7}}{4}$ (4) $\frac{1 - \sqrt{7}}{4}$

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29. If $\sin^{-1}\left(\frac{x}{5}\right) + \cos ec^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then a value of x is: (1) 1 (2) 3 (3) 4 (4) 5

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30. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that $AB(=a)$ subtends an angle of $60o$ at the foot of the tower, and the angle of elevation of the top of the tower from A or B is $30o$. The height of the tower is (1) $\frac{2a}{\sqrt{3}}$ (2) $2a\sqrt{3}$ (3) $\frac{a}{\sqrt{3}}$ (4) $a\sqrt{3}$

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31. The value of cot $\left(\cos ec^{-1}\frac{5}{3}+\frac{\tan^{-1}2}{3}\right)$ is: (1) $\frac{6}{17}$ (2) $\frac{3}{17}$ (2) $\frac{4}{17}$ (4) $\frac{5}{17}$

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32. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is $60o$. He moves away from the pole along the line BC to a point D such that $CD=7m$. From D the angle of elevation of the point A is $45o$. Then the height of the pole is (1) $\frac{7\sqrt{3}}{2}\frac{\dot{1}}{\sqrt{3}-1}m$ (2) $\frac{7\sqrt{3}}{2}\sqrt{3}^{\cdot}+1m$ (3) $\frac{7\sqrt{3}}{2}\sqrt{3}^{\cdot}-1m$ (4) $\frac{7\sqrt{3}}{2}\frac{\dot{1}}{\sqrt{3}+1}m$

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33. If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$, then which of the following is/are true:- (a) $\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\delta)=-\frac{3}{2}$ (b) $\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\delta)=-\frac{1}{2}$ (c) $\sum\cos 2\alpha+2\cos(\alpha+\beta)+2\cos(\beta+\gamma)+2\cos(\gamma+\alpha)=0$ (d) $\sum\sin 2\alpha+2\sin(\alpha+\beta)+2\sin(\beta+\gamma)+2\sin(\gamma+\alpha)=0$

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34. Let $\cos(\alpha+\beta)=\frac{4}{5}$ and let $s\in(\alpha\beta)=\frac{5}{13}$ where $0\leq\alpha,\beta\leq\frac{\pi}{4}$, then $tan2\alpha=$ (1) $\frac{56}{33}$ (2) $\frac{19}{12}$ (3) $\frac{20}{7}$ (4) $\frac{25}{16}$

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35. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is There is a regular polygon with $\frac{r}{R}=\frac{1}{\sqrt{2}}$ (17) There is a regular polygon with $\frac{r}{R}=\frac{2}{3}$ (30) There is a regular polygon with $\frac{r}{R}=\frac{\sqrt{3}}{2}$ (47) There is a regular polygon with $\frac{r}{R}=\frac{1}{2}$ (60)

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36. If $A=s\in^2 x+\cos^4 x$, then for all real x : (1) $\frac{3}{4}\leq A\leq 1$ (2) $\frac{13}{16}\leq A\leq 1$ (3) $1\leq A\leq 2$ (4) $\frac{3}{4}\leq A\leq\frac{13}{16}$

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37. The possible values of $\theta\in(0,\pi)$ such that $\sin(\theta)+\sin(4\theta)+\sin(7\theta)=0$ are (1) $\frac{2\pi}{9},\frac{i}{4},\frac{4\pi}{9},\frac{\pi}{2},\frac{3\pi}{4},\frac{8\pi}{9}$ (2)

$\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$ (3) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$ (4)
$\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

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38. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has (A) infinite number of real roots (B) no real roots (C) exactly one real root (D) exactly four real roots

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39. In $\triangle PQR$ if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$ then the angle R is equal to

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40.

$$\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = 1 + \tan A + \cot A = 1 + \sec A \cos ecA$$

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41. If x, y, z are in A.P. and $tan^{-1}x, tan^{-1}yandtan^{-1}z$ are also in A.P., then (1) $2x = 3y = 6z$ (2) $6x = 3y = 2z$ (3) $6x = 4y = 3z$ (4) $x = y = z$

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42. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta, BC = pandCD = q$, then AB is equal to (1) $\dfrac{p^2 + q^2 \cos\theta}{p\cos\theta + q\sin\theta}$ (2) $\dfrac{p^2 + q^2}{p^2\cos\theta + q^2\sin\theta}$ (3) $\dfrac{(p^2 + q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$ (4) $\dfrac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

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43. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ where $x \in \mathbb{R}$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals

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44. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is $45o$. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to $30o$. Then the speed (in m/s) of the bird is (1) $40\left(\sqrt{2} - 1\right)$ (2) $40\left(\sqrt{3} - 2\right)$ (3) $20\sqrt{2}$ (4) $20\left(\sqrt{3} - 1\right)$

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45. Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is : (1) $\frac{3x - x^3}{1 - 3x^2}$ (2) $\frac{3x + x^3}{1 - 3x^2}$ (3) $\frac{3x - x^3}{1 + 3x^2}$ (4) $\frac{3x + x^3}{1 + 3x^2}$

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46. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30^0 , 45^0 and 60^0 respectively, then the ratio, AB : BC, is : (1) $\sqrt{3}:1$ (2) $\sqrt{3}:\sqrt{2}$ (3) $1:\sqrt{3}$ (4) $2:3$

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47. A value of for which $\dfrac{2+3i\sin\theta}{1-2i\sin\theta}$ purely imaginary, is : (1) $\dfrac{\pi}{3}$ (2) $\dfrac{\pi}{6}$ (3) $\sin^{-1}\left(\dfrac{\sqrt{3}}{4}\right)$ (4) $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

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48. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30^0 . After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60^0 . Then the time taken (in minutes) by him, from B to reach the pillar, is : (1) 6 (2) 10 (3) 20 (4) 5

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49. If $0 \leq x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0,$ is : (1) 3 (2) 5 (3) 7 (4) 9

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50. If $5\left(\tan^2 x - \cos^2 x\right) = 2\cos 2x + 9$, then the value of cos4x is

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51. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP-2AB. If

BPC-β, then tan β is equal to 12. 6 (2) 4 2 4

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52. If θ lies in 3rd quadrant, then the value of the expression $\sqrt{4\sin^4\theta + \sin^2 2\theta} + 4\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$ is equal to

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53. The number of real solutions of the equation $(\sin x - x)\left(\cos x - x^2\right) = 0$ is

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54. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} + \frac{x^4}{8} + \dots\right) = \frac{\pi}{6}$, where $|x| < 2$ then the value of x is (A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $-\frac{2}{3}$ (D) $-\frac{3}{2}$

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55. If $f:\left[0, \frac{\pi}{2}\right) \to R$ is defined as $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$

Then, the range of f is

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56. prove: $\cot^{-1}\left(\frac{1}{2}\right) - \frac{1}{2}\cot^{-1}\left(\frac{4}{3}\right) = \frac{\pi}{4}$

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57. The minimum value of $\cos\theta + \sin\theta + \frac{2}{\sin 2\theta}$ for $\theta \in \left(0, \frac{\pi}{2}\right)$ is (A) $2 + \sqrt{2}$ (B) 2 (C) $1 + \sqrt{2}$ (D) $2\sqrt{2}$

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58. In a triangle ABC, $a^2\cos^2 A = b^2 + c^2$ then triangle is

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59. The number of distinct real roots of $\left|s \in x \cos x \cos x \cos x s \in x \cos x \cos x \cos x s \in x\right| = 0$ in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is 0 b. 2 c. 1 d. 3

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60. The trigonometric equation $\sin^{-1} x = 2\sin^{-1} a$ has a solution for all real values (b) $|a| < \frac{1}{a}$ $|a| \leq \frac{1}{\sqrt{2}}$ (d) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$

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61. In a triangle the sum of two sides is x and the product of the same is y. If $x^2 - c^2 = y$ where c is the third side. Determine the ration of the in-radius and circum-radius

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62. Arithmetic mean of the non-zero solutions of the equation $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$

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63. For $x \in (0, \pi)$, the equation $\sin x + 2s \in x - \sin 3x = 3$ has infinitely many solutions three solutions one solution no solution

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64. The number of distinct solutions of the equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is

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65. In a triangle ΔXYZ, leta, bandc be the lengths of the sides opposite to the angles X, Y and Z respectively.If $2(a^2 - b^2) = c^2$ and $\lambda = \frac{\sin(X - Y)}{\sin Z}$ then possible values of n for which $\cos(n\pi\lambda) = 0$ is (are)

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66. If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is (are)

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67. Let `-pi/6beta_1 $\text{and}$ alpha_2>beta_2$then$alpha_1+beta_2` equals

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68. Let $S = \left\{x\varepsilon(-\pi, \pi) : x \neq 0, + \frac{\pi}{2}\right\}$The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \cos ecx + 2(\tan x - \cot x) = 0$ in the set S is equal to

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69. Q. The value of is equal $\sum_{k=1}^{13}\left(\frac{1}{\sin\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4} + k\frac{\pi}{6}\right)}\right)$ is equal

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70. In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z, respectively, and 2s = x + y + z. If $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ of incircle of the triangle XYZ is $\frac{8\pi}{3}$

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71. If

$f(x)|\cos(2x)\cos(2x)\sin(2x) - \cos x \cos x - \sin x \sin x \sin x \cos x|, then:$

$f'(x) = 0$ at exactly three point in $(-\pi, \pi)$ $f'(x) = 0$ at more than three point in $(-\pi, \pi)$ $f(x)$ attains its maximum at $x = 0$ $f(x)$ attains its minimum at $x = 0$

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72. Let $\alpha and \beta$ be nonzero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true? $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$ $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$ $tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$ $tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

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73. Let O be the origin, and $OXxOY$, OZ be three unit vectors in the direction of the sides QR, RP, PQ, respectively of a triangle PQR. If the triangle PQR varies, then the minimum value of $\cos(P+Q)+\cos(Q+R)+\cos(R+P)$ is: $-\frac{3}{2}$ (b) $\frac{5}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{5}{3}$

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