MATHS

BOOKS - NCERT MATHS (HINGLISH)

PRINCIPLE OF MATHEMATICAL INDUCTION

Short Answer Type Question

1. Given an example of a statement $P(n)$ which is true for all $n \geq 4$ but $P(1)$, $P(2) and\, P(3)$ are not true. Justify your answer.

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2. Given an example of a statement $P(n)$ such that it is true of all nN.

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3. prove that $4^n - 1$ is divisible by 3, for each natural number n.

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4. Using the principle of mathematical induction, prove that $(2^{3n} - 1)$ is divisible by 7 for all $n \in N$.

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5. Prove the following by the principle of mathematical induction: $n^3 - 7n + 3$ is divisible 3 for all $n \in N$.

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6. prove that $3^{2n} - 1$ is divisible by 8, for all natural numbers n.

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7. Prove that for any natural numbers n, $7^n - 2^n$ is divisible by 5.

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8. If $x \neq y$, then for every natural number n, $x^n - y^n$ is divisible by

A. $x + y$

B. $x - y$

C. 1

D. None of these

Answer: B

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9. If n be any natural number then by which largest number $(n^3 - n)$ is always divisible ?

A. 3

B. 6

C. 12

D. 18

Answer: B

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10. prove that $n(n^2+5)$ is divisible by 6, for each natural number n.

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11. prove that $n^2 < 2^n$, for all natural number $n \geq 5$.

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12. prove that $2n < (n+2)!$ for all natural numbers n.

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13. Using principle of mathematical induction prove that $\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{n}}$ for all natural numbers $n \geq 2$.

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14. prove that $2 + 4 + 6 + \ldots 2n = n^2 + n$, for all natural numbers n.

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15. Prove that $1+2+2^2+\ldots+2^n=2^{n+1}-1$, for all natural number n.

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16. prove that 1+5+9+ . . .+(4n-3)=n(2n-1), for all natural number n.

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Long Answer Type Question

1. A sequence $a_1, a_2, a_3, \ldots$ is defined by letting $a_1 = 3$ and $a_k = 7a_{k-1}$, for all natural numbers $k \geq 2$. Show that $a_n = 3 \cdot 7^{n-1}$ for natural numbers.

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2. A sequence $b_0, b_1, b_2, \ldots$ is defined by letting $b_0 = 5$ and $b_k = 4 + b_{k-1}$, for all natural number k. Show that $b_n = 5 + 4n$, for all natural number n using mathematical induction.

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3. A sequence $d_1, d_2, d_3 \ldots$ is defined by letting $d_1 = 2$ and $d_k = \frac{d_{k-1}}{k}$, for all natural numbers, $k \geq 2$. Show that

$d_n = \frac{2}{n!}$, for all $n \in N$.

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4. Prove that for all $n \in N$

$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots + \cos[\alpha+(n-1)\beta]$

= $\frac{\cos\left[\alpha + \left(\frac{n-1}{2}\right)\beta\right]\sin\left(\frac{n\beta}{2}\right)}{\sin\frac{\beta}{2}}$

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5. Using induction, prove that

$\cos\theta \cdot \cos 2\theta \cdot \cos 2^2\theta. \ldots \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n \sin\theta}$

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6. Prove that, $\sin\theta + \sin 2\theta + \sin 3\theta + \ldots \sin n\theta = \frac{\sin\frac{n\theta}{2}\sin\frac{n+1}{2}\theta}{\sin\frac{\theta}{2}}$ for all $n \in N$.

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7. Prove that $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a natural number.

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8. Prove that $\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n} > \frac{13}{24}$,for all natural number $n > 1$.

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9. What is the total number of proper subsets of a set containing n elements?

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Objective Type Questions

1. If $10^n + 3 \cdot 4^{n+2} + k$ is divisible by 9, for all $n \in N$, then the least positive integral value of k is

A. 5

B. 3

C. 7

D. 1

Answer: A

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2. For all n $\in N, 3 \cdot 5^{2n+1} + 2^{3n+1}$ is divisible by (A) 19 (B) 17 (C) 23 (D) 25

A. 19

B. 17

C. 23

D. 25

Answer: B::C

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3. If $x^n - 1$ is divisible by $x - k$ then the least positive integral value of k is

A. 1

B. 2

C. 3

D. 4

Answer: A

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4. If $P(n): 2n < n!, n \in N$ then P(n) is true for all $n \geq \ldots \ldots$

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5. State whether the following statement is true or false. Justify If P(n) is a statement $(n \in N)$ such that if P(k) is true, $P(k+1)$ is true for $k \in N$, then P(n) is true.

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