MATHS

BOOKS - NCERT MATHS (HINGLISH)

THREE DIMENSIONAL GEOMETRY

Three Dimensional Geometry

1. Find the position vector of a point A in space such that $\overrightarrow{O}A$ is inclined at $60^0 \to OX$ and at $45^0 \to OY and \left|\overrightarrow{O}A\right| = 10 units.$

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2. Find the vector equation of the line which is parallel to the vector $3\hat{i} - 2\hat{j} + 6\hat{k}$ and which passes through the point $(1, -2, 3)$.

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3. Show that the two lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{z} = z$ intersect. Find also the point of intersection of these lines.

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4. Find the angle between the lines

$\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$ and

$\vec{r} = \left(2\hat{j} - 5\hat{k}\right) + \mu\left(6\hat{i} + 3\hat{j} + 2\hat{k}\right).$

A. $\theta = \cos^{-1}\left(\frac{21}{19}\right)$

B. $\theta = \cos^{-1}\left(\frac{19}{21}\right)$

C. $\theta = \cos^{-1}\left(\frac{6}{21}\right)$

D. $\theta = \cos^{-1}\left(\frac{1}{19}\right)$

Answer: B

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5. Prove that the lines through $A(0, -1, -1) and B(4, 5, 1)$ intersecrs the line through $C(3, 9, 2) and D(-4, 4, 4)$. Also, find their point of intersection.

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6. Prove that the lines $x = py + q; z = ry + s$ and $x = p'y + q'; z = r'y + s'$ are perpendicular If $pp' + qq' + 1 = 0$

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7. Find the equation of the plane which bisects the line segment joining the points $A(2, 3, 4)$ and $B(4, 5, 8)$ at right angles.

A. $x + y + 2z = 19$

B. $x + y - 2z = 19$

C. $x + y - 2z = 38$

D. $x + y + 2z = 38$

Answer: A

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8. Find the equation of a plane which is at a distance of $3\sqrt{3}$ units from origin and the normal to which is equally inclined to the coordinate axes.

A. $x + y + z = 9$

B. $x + y - z = 9$

C. $x - y - z = 9$

D. $x - y + z = 9$

Answer: A

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9. If the line drawn from the point $(-2, -1, -3)$ meets a plane at right angle at the point $(1, -3, 3)$, find the equation of the plane.

A. $3x + 2y + 6z - 18 = 0$

B. $3x + 2y + 6z - 24 = 0$

C. $3x - 2y + 6z - 24 = 0$

D. $3x - 2y + 6z - 27 = 0$

Answer: D

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10. Find the equation of the plane through points (2,1,0),(3,-2,-2), and (3,1,7).

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11. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at angle of $\frac{\pi}{3}$ each.

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12. Find the acute angle between the two straight lines whose direction cosines are given by $l+m+n=0$

and $l^2 + m^2 - n^2 = 0$

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13. If the direction cosines of a variable line in two adjacent points be l, M, n and $l + \delta l, m + \delta m + n + \delta n$ the small angle $\delta\theta$as between the two positions is given by

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14. If O is the origin and the coordinates of A are (a, b, c) . Find the direction cosines of OA and the equation of the plane through A at right angles to OA.

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15. Two systems of rectangular axes have the same origin. If a plane cuts them at distances $a, b, canda', b', c'$ respectively, prove that $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}$

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16. Find the foot of the perpendicular from the point (2,3,-8) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Find the perpendicular distance from the given point to the line.

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17. Find the equation of the perpendicular drawn from $(2, 4, -1)$ to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{3}$.

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18. Find the length and the foot of perpendicular from the point $\left(1, \frac{3}{2}, 2\right)$ to the plane $2x - 2y + 4z + 5 = 0$.

A. $\sqrt{7} units$

B. $\sqrt{2} units$

C. $\sqrt{6} units$

D. $\sqrt{3}units$

Answer: C

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19. Find the equations of the line passing through the point $(3, 0, 1)$ parallel to the planes $x + 2y = 0$ and $3y - z = 0$.

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20. Find the shortest distance between the lines gives by

$\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$

and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu\left(3\hat{i} + 8\hat{j} - 5\hat{k}\right)$.

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21. Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$

A. $51x + 18y - 50z + 173 = 0$

B. $51x + 18y - 50z = 173$

C. $51x + 18y + 50z + 173 = 0$

D. $51x - 18y + 50z - 173 = 0$

Answer: A

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22. The plane ax + by = 0 is rotated about its line of intersection with the plane z = 0 through an angle α. Prove that the equation of the plane in its new position is $ax + by \pm \left(\sqrt{a^2 + b^2}\tan\alpha\right)z = 0$

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23. Find the equation of the plane through the points (2, 1, 1) and (1, 3, 4) and perpendicular to the plane $x\ \ 2y + 4z = 10.$

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24. Find the position vector of a point A in space such that $\overrightarrow{OA}$ is inclined at $60^0 \to OX$ and at $45^0 \to OY and \left|\overrightarrow{OA}\right| = 10 units.$

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25. Find the vector equation of the line which is parallel to the vector $3\hat{i} - 2\hat{j} + 6\hat{k}$ and which passes through the point $(1, -2, 3)$.

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26. Show that the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} and \frac{x-4}{5}=\frac{y-1}{2}$ intersect. Find their point of intersection.

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27. Find the angle between the lines $\vec{r}=3\hat{i}-2\hat{j}+6\hat{k}+\lambda\left(2\hat{i}+\hat{j}+2\hat{k}\right)$ and $\vec{r}=\left(2\hat{j}-5\hat{k}\right)+\mu\left(6\hat{i}+3\hat{j}+2\hat{k}\right).$

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28. Prove that the line through $A(0, 1, 1)$ and $B(4, 5, 1)$ intersects the line through $C(3, 9, 4)$ and $D(4, 4, 4)$.

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29. Prove that the lines $x = py + q; z = ry + s$ and $x = p'y + q'; z = r'y + s'$ are perpendicular If $pp' + qq' + 1 = 0$

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30. Find the equation of the plane which bisects the line segment joining the points $A(2, 3, 4) and B(4, 5, 8)$ at right angles.

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31. Write the equation of a plane which is at a distance of $5\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axes.

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32. If he line drawn from the point $(-2, -1, -3)$ meets a plane at right angle at the point $(1, -3, 3)$, find the equation of the plane.

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33. Find the equationfo the plane through points (2,1,0),(3,-2,-2), and (3,1,7).

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34. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$

at angle of $\frac{\pi}{3}$ each.

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35. Find the acute angle between the two straight lines whose direction cosines are given by $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$

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36. If the direction cosines of a variable line in two adjacent points be l, M, n and $l + \delta l, m + \delta m + n + \delta n$ the small angle $\delta\theta$as between the two positions is given by

A. $\delta\theta^2 = \delta l^2 + \delta m^2 + \delta n^2$

B. $\delta\theta^2 = \delta l^2 - \delta m^2 + \delta n^2$

C. $\delta\theta^2 = \delta l^2 + \delta m^2 - \delta n^2$

D. $\delta\theta^2 = \delta l^2 - \delta m^2 - \delta n^2$

Answer: A

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37. If O is the origin and the coordinates of A are (a, b, c) . Find the direction cosines of OA and the equation of the plane through A at right angles to OA.

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38. Two systems of rectangular axes have the same origin. If a plane cuts them at distances $a, b, canda', b', c'$ respectively, prove that $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}$

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40. Find the distance of a point $(2, 4, -1)$ from the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}.$$

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41. Find the length and the foot of perpendicular from the point $(1, 3/2, 2)$ to the plane $2x - 2y + 4z + 5 = 0.$

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42. Find the equations of the line passing through the point $(3, 0, 1)$ parallel to the planes $x + 2y = 0$ and

$3y - z = 0.$

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43. Find the shortest distance between the lines gives by

$\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$

and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu\left(3\hat{i} + 8\hat{j} - 5\hat{k}\right).$

A. 11 units

B. 12 units

C. 13 units

D. 14 units

Answer: D

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44. Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$

A. $51x - 18y - 50z + 173 = 0$

B. $51x + 18y - 50z + 173 = 0$

C. $51x + 17y - 50z + 173 = 0$

D. $51x + 18y - 50z - 173 = 0$

Answer: B

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45. The plane ax + by = 0 is rotated about its line of intersection with the plane z = 0 through an angle α. Prove that the equation of the plane in its new position is $ax + by \pm \left(\sqrt{a^2 + b^2}\tan\alpha\right)z = 0$

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46. Find the equation of the plane through the points (2, 1, 1) and (1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$.

A. $18x + 17y + 4z = -49$

B. $18x + 17y - 4z = 49$

C. $18x + 17y + 4z = 49$

D. $18x - 17y + 4z = 49$

Answer: C

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