MATHS

BOOKS - NAGEEN MATHS (HINGLISH)

PRINCIPLE OF MATHEMATICAL INDUCTION

Examples

1. For all natural numbers n, statement

$p(n) = 1 + 2 + 3 + 4.... + n = \frac{n(n+1)}{2}$ is true find p(n+1)

A. $\frac{n(n+1)}{2}$

B. $\frac{(n+1)(n+2)}{2}$

C. $\frac{n(n-1)}{2}$

D. $\frac{(n+1)(n+1)}{2}$

Answer: B

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2. Prove by the principle of mathematical induction that for all $n \in N$:

$$1^2 + 2^2 + 3^2 + \quad + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

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3. By the principle of mathematical induction:

$$\frac{1}{1.\,2} + \frac{1}{2.\,3} + \frac{1}{3.\,4} + \ldots + \frac{1}{n(n+1)} =$$

A. $\frac{n}{n+1}$

B. $\frac{n}{n+2}$

C. $\frac{n}{n+3}$

D. $\frac{n}{n+4}$

Answer: A

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4. $n(n^2-1)$, is divisible by (if n is an odd positive number.)

A. 20

B. 24

C. 25

D. 23

Answer: B

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Exercise 4

1. Prove the following by the principle of mathematical induction:

$2.\ 7^n + 3.\ 5^n - 5$ is divisible 25 for all $n \in N$.

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2. Prove the following by the principle of mathematical induction: $7^{2n} + 2^{3n-3}.\,3^{n-1}$ is divisible 25 for all $n \in N\cdot$

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3. Prove the following by the principle of mathematical induction: $7 + 77 + 777 + \quad + 777 + \quad + \ddot{n} - digits7 = \frac{7}{81}\left(10^{n+1} - 9n - 10\right)$ for all $n \in NB\cdot$

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4. Prove the following by using the principle of mathematical induction for all $n \in N$:$1 + 2 + 3 + \quad \vdots + n < \frac{1}{8}(2n+1)^2$.

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5. Using the principle of mathematical induction ,prove that $(1+x)^n \geq (1+nx)$ for all $n \in N$, where x>-1.

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6. Using binomial theorem, prove that $2^{3n} - 7^n - 1$ is divisible by 49 , where $n \in N$.

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7. $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+n} = \frac{2n}{n+1}$

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8. If $P(n) = 1.\ 2.3 + 2.\ 3.4 + \ldots. + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

then P(K+1) equals

A. $\frac{k(k+1)(k+2)(k+3)}{4}$

B. $\frac{k(k+1)(k+2)(k+3)(k+4)}{4}$

C. $\frac{(k+1)(k+2)(k+3)(k+4)}{4}$

D. None of these

Answer: C

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9. $1.\ 2.3 + 2.\ 3.4 + \ + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

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10. $1.3 + 2.3^2 + 3.3^3 + \ldots\ldots\ldots\ldots. + n.3^n = \frac{(2n-1)3^{n+1}+3}{4}$

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11. Prove by PMI that

$$1.2 + 2.3 + 3.4 + \ldots. + n(n+1) = \frac{(n)(n+1)(n+2)}{3}, \forall n \in N$$

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12. $1.3 + 3.5 + 5.7 + \ldots\ldots + (2n-1)(2n+1) = \frac{n(4n^2 + 6n - 1)}{3}$

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13. $1.2 + 2.2^2 + 3.2^3 + \ldots. + n.2^n = (n-1)2^{n-1} + 2$

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14. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots\ldots + \frac{1}{2^n} =$

A. $1 - \frac{1}{2^n}$

B. $1 - \frac{1}{3^n}$

C. $1 - \frac{1}{4^n}$

D. $1 - \frac{1}{5^n}$

Answer: A

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15. Prove the following by the principle of mathematical induction:

$$\frac{1}{2.\ 5} + \frac{1}{5.\ 8} + \frac{1}{8.\ 11} + \ + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$$

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16. Using the principle of mathematical induction prove that

$$\frac{1}{1.\ 2.\ 3} + \frac{1}{2.\ 3.\ 4} + \frac{1}{3.\ 4.\ 5} + \ + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

for all $n \in N$

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17. Prove the following by using the principle of mathematical induction

for all $n \in N$: $a + ar + ar^2 + \vdots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$

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18. Prove the following by using the principle of mathematical induction

for all $n \in N$: $\left(1 + \frac{3}{1}\right)\left(1 + \frac{5}{4}\right)\left(1 + \frac{7}{9}\right)1 + \frac{\vdots(2n + 1)}{n^2} = (n + 1)^2$

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19. $\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\ldots\ldots\left(1 + \frac{1}{n}\right)$

$n(n + 1)$

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20. $1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2$

$$= \frac{n(2n-1)(2n+1)}{3}$$

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21. Prove the following by the principle of mathematical induction:

$$\frac{1}{1.\ 4} + \frac{1}{4.\ 7} + \frac{1}{7.\ 10} + + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

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22. Prove the following by the principle of mathematical induction:

$$\frac{1}{3.\ 5} + \frac{1}{5.\ 7} + \frac{1}{7.\ 9} + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

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23. Prove that $1 + 2 + 3 + 4\ldots\ldots. + N < \frac{1}{8}(2n+1)^2$

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24. Prove that $n(n+1)(n+5)$ is a multiple of 3.

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25. Prove by the principle of induction that for all nN, $\left(10^{2n-1}+1\right)$ is divisible by 11.

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26. $x^{2n-1}+y^{2n-1}$ is divisible by $x+y$

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27. $3^{2n+2}-8n-9$ divisible by 8

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28. $41^n - 14^n$ is a multiple of 27

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29. Prove the following

$(2n + 7) < (n + 3)^2$

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Exercise 4 1

1. $1 + 3 + 3^2 + ... + 3^{n-1} =$

A. $\dfrac{(3^n - 1)}{2}$

B. $\dfrac{(3^{2n} - 1)}{2}$

C. $\dfrac{(3^n + 1)}{2}$

D. $\dfrac{(3^{2n} + 1)}{2}$

Answer: A

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