



MATHS

BOOKS - NAGEEN MATHS (HINGLISH)

RELATIONS AND FUNCTIONS

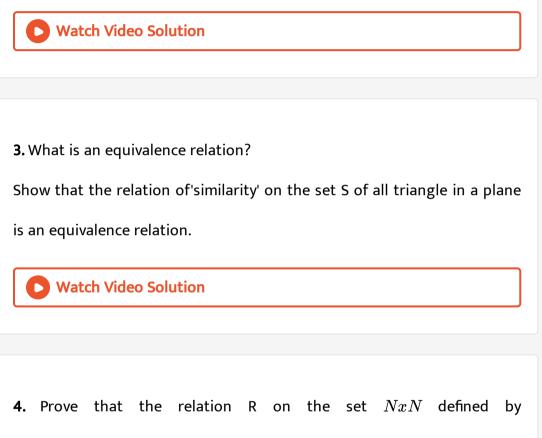
Solved Examples

- **1.** If A = {1, 2, 3, 4}, define relations on A which have properties of being
- (i) reflexive, transitive but not symmetric.
- (ii) symmetric but neither reflexive nor transitive.
- (iii) reflexive, symmetric and transitive.



2. A relation R is defined on the set of integers as follows : $aRb \Leftrightarrow (a - b)$, is divisible by 6 where a, b, \in I. prove that R is an

equivalence relation.

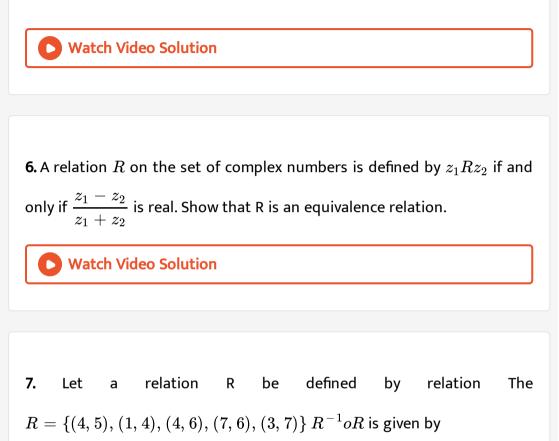


(a,b)R(c,d)a+d=b+c for all $(a,b), (c,d)\in NxN$ is an equivalence

relation.

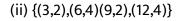
5. In the set of straight lines in a plane, for the relation 'perpendicular'

check whether it is reflexive, symmetric and transitive.



8. Which types of the following functions are ?

(i) {(a,1),(b,1),(c,1),(d,1),(e,1)}



(iii) {(a,1),(b,2),(c,3),(d,4)}.

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9. prove that a function f = $\{(x, 2x + 1) : x \in N\}$ defined on the set of

natural numbers NxxN is one - one function.

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10. Prove that the function $f\!:\!N o N$, defined by $f(x)=x^2+x+1$ is

one-one but not onto.

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11. The function $f\!:\!R o R\!:\!f(x)=\sin x$ is

A. One One

B. Onto

C. Into

D. None Of these

Answer: C

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12. show that the function

 $(i)f\!:\!N o N\!:\!f(x)=x^2$ is one-one into

(ii) $f\!:\!Z o Z\!:\!f(x)=x^2$ is many -one into .

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13. If f:R o R and g:R o R are two mappings such that f(x)=2x and $g(x)=x^2+2$ then find fog and gog.

14. If $f: R \to R, g: R \to R$ defined as $f(x) = \sin x$ and $g(x) = x^2$, then find the value of (gof)(x) and (fog)(x)and also prove that $gof \neq fog$.

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15. If f and g two functions are defined as :

 $f = \{(1,2),(3,6),(4,5)\}$ and $g = \{(2,3),(6,7),(5,8)\}$, then find gof.

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16. A function $f\colon R o R$ is defined as $f(x)=x^2+2$, then evaluate each

of the following :

(i) $f^{\,-1}(\,-6)$ (ii) $f^{\,-1}(18)$

17. Prove that the function $f : R \to R$ where R is the set of all real numbers, defined as f (x) = 3x + 4 is one-one and onto . Also find the inverse function of f.



18. Show that addition, subtraction and multiplication are binary operations on R, but division is not a binary operation on R. Further, show that division is a binary operation on the set R of nonzero real numbers.

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19. Show that subtraction and division are not binary operations on N.



20. Let P be a set of all subset of set X. Prove that the functions defined by $\cup : P \times P \to P, (A, B) \to A \cup B \text{ and } \cap : P \times P \to P, (A, B) \to A \cap B$ are binary on P.

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21. Show that the operation \lor and \land on R defined as $a \lor b =$ Maximum of a and b; $a \land b =$ Minimum of a and b are binary operations of R.

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22. Show that $+: R \times R \to R$ and $\times : R \times R \to R$ are commutative binary operations, but $: R \times R \to R$ and $\div : R \cdot \times R \cdot \to R$ are not commutative.

23. Prove that *: R imes R o R defined as a * b = a + 2ab is not commutative . Watch Video Solution 24. Prove that in the set of real numbers '+' and ' \times ' are associative but '-' and ' \div ' are not associative. Watch Video Solution

25. Prove that *: R imes R o R defined as a * b = a + 2ab is not associative

26. Show that zero is the identity for addition on R and 1 is the identity for multiplication on R. But there is no identity element for the operations $\div R \times R \to R$ and $\div : R_{\cdot} \times R_{\cdot} \to R_{\cdot}$.

27. Show that a is the inverse of a for the addition operation + on R and

 $rac{1}{a}$ is the inverse of a
eq 0 for the multiplication operation imes on R.

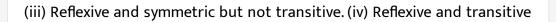
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28. Show that a is not the inverse of $a \in N$ for the addition operation + on N and $\frac{1}{a}$ is not the inverse of $a \in N$ for multiplication operation \times on N, for $a \neq 1$.

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1. Give an example of a relation. Which is (i) Symmetric but neither reflexive nor transitive. (ii) Transitive but neither reflexive nor symmetric.



but not symmetric. (v) Symm



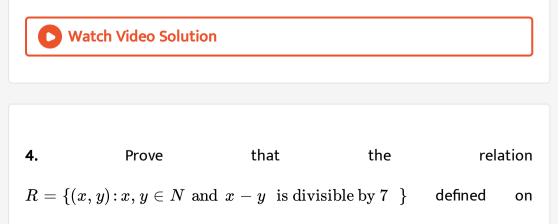
2. (i) If A= {x,y,z}, B=(1,2,3} and R= {(x,2),(y,3),(z,1),(z,2), then find R^{-1} .

(ii) If R is a relations such that R ={(4,5),(1,4),(4,6),(7,6),(3,7)}, then find $R^{-1}oR^{-1}$

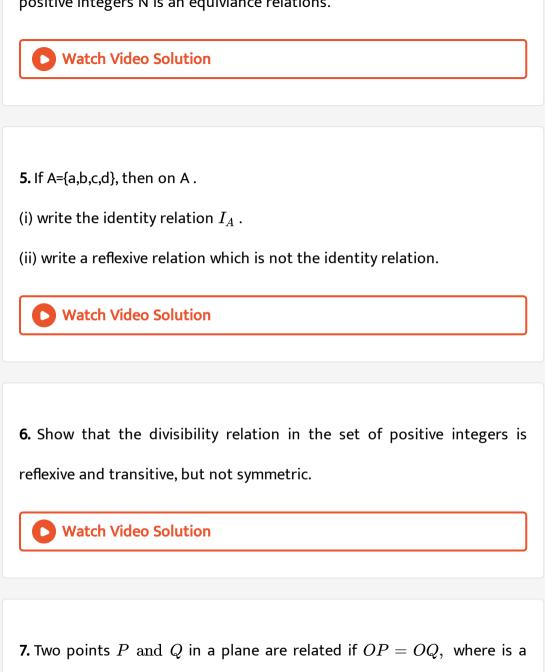
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3. Prove that the relation R on Z defined by $(a, \ b) \in R \Leftrightarrow \ a-b$ is

divisible by 5 is an equivalence relation on Z.



positive integers N is an equivlance relations.



fixed point. This relation is :

8. Let N be the set of all natural numbers and let R be a relation on $N \times N$, defined by (a, b)R(c, d)ad = bc for all $(a, b), (c, d) \in N \times N$. Show that R is an equivalence relation on $N \times N$. Also, find the equivalence class [(2,6)].

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9. (i) Show that in the set of positive integer, the relation ' is greater than ' is transitive but it is not reflexive or smmetric. (ii) Let R be a relation on the set of natural numbers N defined as $_aR_b \Rightarrow$

a divides B where a, b \in N. Is R symmetric ?

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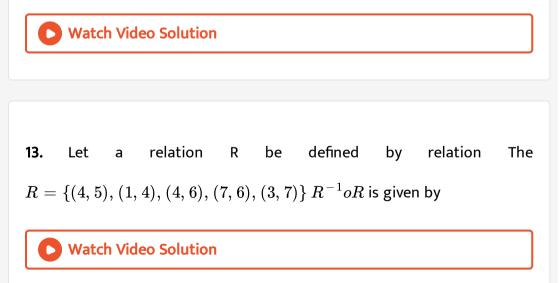
10. Show that the relation is congruent to on the set of all triangles in a

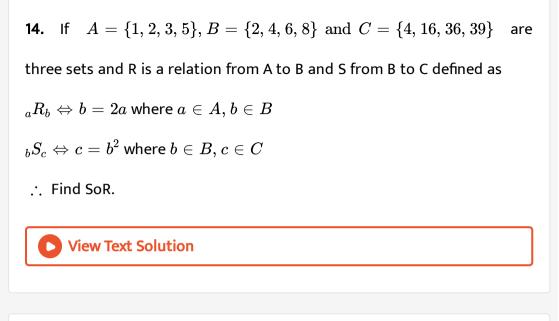
plane is an equivalence relation

11. Show that the divisibility relation in the set of positive integers is reflexive and transitive, but not symmetric.



12. Let R be a relation defined on the set of natural numbers N as $R = \{(x, y) : x, y \in N, 2x + y = 41\}$ Find the domain and range of R. Also, verify whether R is (i) reflexive, (ii) symmetric (iii) transitive.





15. Show that the relation R in the set R of real numbers, defined as

 $R = ig\{(a,b) : a \leq b^2ig\}$ is neither reflexive nor symmetric nor transitive.

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16. If R and S are two equivalence relations on a set A; then $R \cap S$ is also an equivalence relation on R.



1. $f \colon R o R$ is a function where f(x)= 2x-3 . Check whether f is noe -one ?

2. On set
$$A = \{1, 2, 3\}$$
, relation R and S are given by
 $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$ and $S = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$ and $S = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$

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3. Prove that $f\!:\!N o N$ defined by $f(x)=2x,x\in N$ is one-one and

into.

4. If R is the set of real numbers and a function $f\colon R o R$ is defined as $f(x)=x^2, x\in R$, then prove that f is many-one into function.

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5. If Q is the set of rational numbers, then prove that a function $f\colon Q o Q$ defined as $f(x)=5x-3, x\in Q$ is one -one and onto function.

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6. If R is the set of real numbers then prove that a function $f\colon R o R$ defined as $f(x)=rac{1}{x},x
eq 0,x\in R,$ is one-one onto.

7. Prove that the function $f\!:\!R^+ o R$ which is defined as $f(x)=\log_e x$

is one - one .



8. If R is the set of real numbers prove that a function $f \colon R o R, \, f(x) = e^x, \, x \in R$ is one to one mapping.

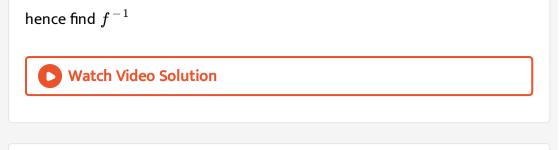
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9. A function $f\colon R o R$ is defined as $f(x)=4x-1, x\in R, ext{ then prove}$

that f is one - one.

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10. Let $A = R - \{3\}$ and B = R - [1]. Consider the function $f: A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Show that f is one-one and onto and



11. Let the function $f\colon R o R$ be defined by $f(x)=\cos x,\ orall x\in R.$

Show that f is neither one-one nor onto.

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12. If a function $f\!:\!R o R$ is defined as $f(x)=x^3+1$, then f is

A. Onto

B. Into

C. cant say

D. Not a function

Answer: A

13. Function $f: R \to R$ and $g: R \to R$ are defined as $f(x) = \sin x$ and $g(x) = e^x$.

Find (gof)(x) and (fog)(x).

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14. If $f: R \to R$ and $g: R \to R$ be two functions defined as f(x)=2x+1 and $g(x) = x^2 - 2$ respectively, then find (gof) (x) and (fog) (x) and show that (fog) (x) \neq (gof) (x).

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15. If f and g are two functions from R to R which are defined as $f(x) = x^2 + x + 1$ and g(x) = 2x - 1 for each $x \in R$, then show that (fog) (x) \neq (gof) (x).

16. If f:R o R and g:R o R be two functions defined as $f(x)=x^2$ and g(x)=5x where $x\in R$, then prove that (fog)(2) eq (gof) (2).

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17. If $f\colon R o R$ defined as f(x)=3x+ 7, then find $f^{-1}(-2)$

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18. If Q is the set of rational numbers and a function f:Q o Q is defined as $f(x)=5x-4, x\in Q$, then show that f is one-one and onto. Also define f^{-1} .

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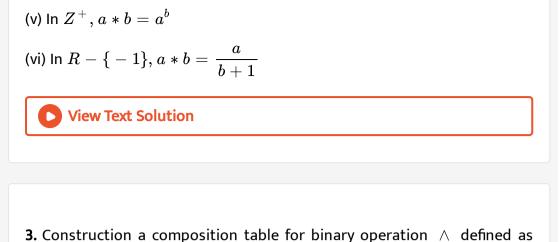
1. In the given sets, working operations * is defined, check whether * is binary or not ? Justify your answer.

(i) In Z^+ , a * b = a - b(ii) In Z^+ , a * b = ab(iii) In R, $a * b = ab^2$ (iv) In Z^+ , a * b = |a - b|(v) In Z^+ , $a \cdot b = a$ (vi) In Z^+ , a * b = a - 3b

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2. In the given sets the binary operation * is defined. Check the commutativity and associativity is each case for *:

- (i) In Z, a*b=a-b
- (ii) In Q, a * b = 1 + ab(iii) In $Q, a * b = rac{ab}{2}$ (iv) In $Z^+, a * b = 2^{ab}$



- $a \wedge b$ = minimum of {a,b} in the set {1,2,3,4,5} and
- (i) evaluate $(2 \land 3) \land 4$ and $2 \land (3 \land 4)$
- (ii) is \land commutative ?

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4. Consider the infimum binary operation \land on the set $S = \{1, 2, 3, 4, 5\}$ defined by $a \land b =$ Minimum of a and b. Write the composition table of the operation \land .

5. Let ' · ' be a binary operation on N given by $a \cdot b = L\dot{C}\dot{M}a$, b for all $a, b \in N$. Find $5 \cdot 7, 20 \cdot 16$ (ii) Is * commutative? Is * associative? Find the identity element in N Which element of N are invertible? Find them.

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6. Let * be a binary operation on the set Q of rational numbers as follows:

- (i) a * b = a b
- (ii) $a \ast b = a^2 + b^2$
- (iii) a * b = a + ab
- (iv) $a * b = (a b)^2$
- (v) $a \cdot b = rac{ab}{4}$
- (vi) $a * b = ab^2$.

Find which of the binary operations are commutative and which are associative.

7. Let $A = N \times N$ and \cdot be the binary operation on A defined by $(a, b) \cdot (c, d) = (a + c, b + d)$. Show that \cdot is commutative and associative. Find the identity element for \cdot on A, if any.

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8. State whether the following statements are true or false. Justify. (i) For

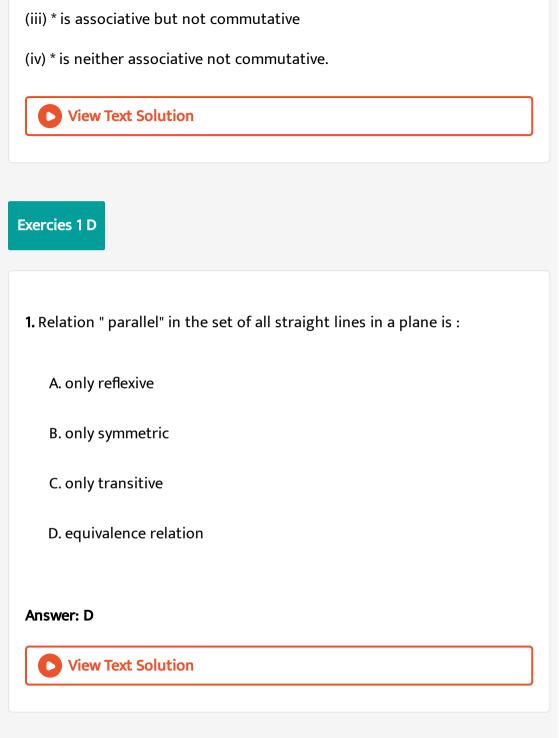
an arbitrary binary operation \cdot on a set $N, \ a \ \cdot \ a \ = \ a \, orall a \in N$. (ii) If \cdot is a commutative binary

operation on N, then `a" "*" "(b" "*" "c)"

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9. If * is a binary operation in N defined as $a^*b = a^3 + b^3$, then which of the following is true :

- (i) * is associative as well as commutative.
- (ii) * is commutative but not associative



2. In the set of straight lines in a plane, for the relation 'perpendicular' check whether it is reflexive, symmetric and transitive.

A. reflexive

B. symmetric

C. transitive

D. equivalence

Answer: B

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3. Relation " similar" in triangles in a plane is :

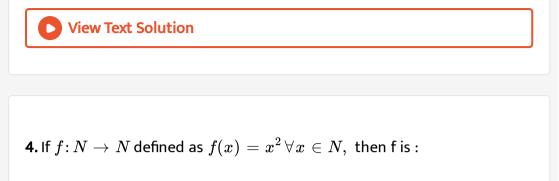
A. reflexive, symmetric , transitive

B. reflexive, transitive but not symmetric

C. symmetric , transitive but not reflexive

D. none of the above

Answer: A



A. many-one

B. one-one

C. onto

D. none of these

Answer: B

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5. The function f is defined as :

$$f(x) = egin{cases} 1, x > 0 \ 0, x = 0 \ -1, x < 0 \end{cases}$$

The range of f is :

A. {1,0}

B. {0,-1}

C. {1,-1}

D. {1,0,-1}

Answer: D

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6. If $f \colon R o A, \,$ where A =[-1,1] , is defined as $f(x) = \cos x, \,$ thn find f is

A. into

B. one-one

C. onto

D. none of these

Answer: C

7. Let $f:N \to N$ be defined as $f(n) = \frac{n+1}{2}$ if n is odd and $f(n) = \frac{n}{2}$ if n is even for all $n \in N$ State whether the function f is bijective. Justify your answer

A. one-one into

B. one-one onto

C. many-one into

D. many-one onto

Answer: B

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8. If $f \colon R o R$ is defined as f(x)=2x+5 and it is invertible , then $f^{-1}(x)$ is

A.
$$rac{x-5}{2}$$

B.
$$\frac{x-2}{5}$$

C. $\frac{x+5}{2}$

D. none of these

Answer: A

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9. Is \cdot defined on the set $\{1,2,3,4,5\}bya \cdot b = L\dot{C}\dot{M}$ of a and b a binary

operation? Justify your answer.

A. 6

B. 24

C. 36

D. none of these

Answer: C

10. On the set Z of integers, if the binary operation * is defined by $a \cdot b = a + b + 2$, then find the identity element.

A. commutative

B. associative

C. commutative and associative

D. none of above

Answer: D

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Exercies 1 E

1. A relation $R = \{(x,y) \colon x, y \in A ext{ and } x < y\}$ is defined on set A=

{1,2,3,4,5}. The relation R is :

A. reflexive

B. symmetric

C. transitive

D. equivalence

Answer: C

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2. If R and S are two non-empty relations on set A, then incorrect statement is :

A. R and S are reflexive , then $R\cap S$ is also reflexive .

B. R and S are symmetric , then $R \cup S$ is also symmetric

C. R and S are transitive , then $R\cap S$ is also transitive .

D. R and S are transitive , then $R \cup S$ is also transitive.

Answer: B

3. If
$$f(x) = \frac{x-1}{x+1}$$
 then $f(2x)$ is equal to
A. $\frac{1+f(x)}{3+f(x)}$
B. $\frac{1+3f(x)}{3+f(x)}$
C. $\frac{3+f(x)}{1+f(x)}$
D. $\frac{1+3f(x)}{3-f(x)}$

Answer: B

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4. If n(A)=3 and n(B)=4 , then no. of of one-one function from A to B is :

A. 12

B. 24

C. 36

D. none of these

Answer: B

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5. Let
$$f(x) = \frac{ax+b}{cx+d}$$
. Then the $fof(x) = x$, provided that : $(a \neq 0, b \neq 0, c \neq 0, d \neq 0)$

A. a=b=c=d=1

B. a=b=1

C. a=d

D. a=-d

Answer: D

6. Let
$$f: [-1, \infty] \rightarrow [-1,)$$
 is given by
 $f(x) = (x + 1)^2 - 1, x \ge -1$. Show that f is invertible. Also, find the
set $S = \{x: f(x) = f^{-1}(x)\}$.
A. $\left\{0, -1, \frac{-3 \pm \sqrt{3}}{2}\right\}$
B. $\{0, 1, -1\}$
C. $\{0, -1\}$
D. $\{\}$

Answer: C

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7. If n(A)=10, then no of different functions from A to A is :

- A. |10
- $B.\,10^{10}$
- $\mathsf{C}.\,2^{10}$

 $\mathsf{D.}\,2^{10}-1$

Answer: B

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8. If
$$g(f(x)) = |\sin x|$$
 and $f(g(x)) = \left(\sin \left(\sqrt{x}
ight)
ight)^2$ then

A.
$$f(x)=\sin^2 x, g(x)=\sqrt{x}$$

$$\mathsf{B.}\,f(x)=\sin xg(x)=|x|$$

C.
$$f(x)=x^2, g(x)=\sin\sqrt{x}$$

D. f(x) and g(x) cannot be determined

Answer: A



9. If the function $f: A \to B$ is one-one onto and $g: B \to A$, is the inverse of f, then fog =? A. f B. g C. I_A

D. I_B

Answer: D

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10. If $f(x) = \left(ax^2 + b
ight)^3$, then find the function g such that f(g(x)) = g(f(x))

A.
$$\left(rac{x^{1/3}-b}{a}
ight)^{1/2}$$

B. $rac{1}{\left(ax^2+b
ight)^3}$

$$\mathsf{C}.\,\frac{1}{\left(ax^2+b\right)^{1/3}}$$

D. none of these

Answer: A

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Exercise 11

1. Determine whether each of the following relations are reflexive, symmetric and transitive :

(i) Relation R in the set $A=\{1,2,3,\ldots,13,14\}$ defined as

$$R = \{(x, y) : 3x - y = 0\}$$

(ii) Relation R in the set N of natural numbers defined as

$$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$$

(iii) Relation R in the set $A=\{1,2,3,4,5,6\}$ as

 $R = \{(x, y) : y \text{ is divisible by x}\}$

(iv) Relation R in the set Z of all integers defined as

$$R = \{(x,y) \colon x-y ext{ is an integer } \}$$

(v) Relation R in the set A of human beings in a town at a particular time given by

- (a) $R = \{(x, y): \text{ and } y \text{ work at the same place } \}$
- (b) $R = \{(x, y) : x \text{ and } y \text{ live in the same locality } \}$
- (c) $R = \{(x, y) : x ext{ is exactly 7 cm taller than y}\}$
- (d) $R = \{(x,y) : x ext{ is wife of } y\}$
- (e) $R = \{(x,y) : x ext{ is father of } y\}$

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2. Give an example of a relation. Which is (i) Symmetric but neither reflexive nor transitive. (ii) Transitive but neither reflexive nor symmetric. (iii) Reflexive and symmetric but not transitive. (iv) Reflexive and transitive but not symmetric. (v) Symm



3. Show that the relation R on the set A of points in a plane, given by $R = \{(P, Q): \text{ Distance of the point } P \text{ from the origin is same as the distance of the point <math>Q$ from the origin}, is an equivalence relation. Further show that the set of all points related to a point $P \neq (0, 0)$ is the circle passing through P with origin as centre.

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4. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2): T_1 \text{ is similar to } T_2\}$, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1 , T_2 and T_3 are related ?

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5. Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2): P_1(\text{ and } P)_2 have same number of sides}\}, \quad \text{is} \quad \text{an}$

equivalence relation. What is the set of all elements in A related to the right angle triangle

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6. Let L be the set of all lines in XY = plane and R be the relation in Ldefined as $R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4.

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7. Let R be the relation in the set $\{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, .)$. Choose the correct answer. (A) R is reflexive and symmetric but not transitive. (B) R is re

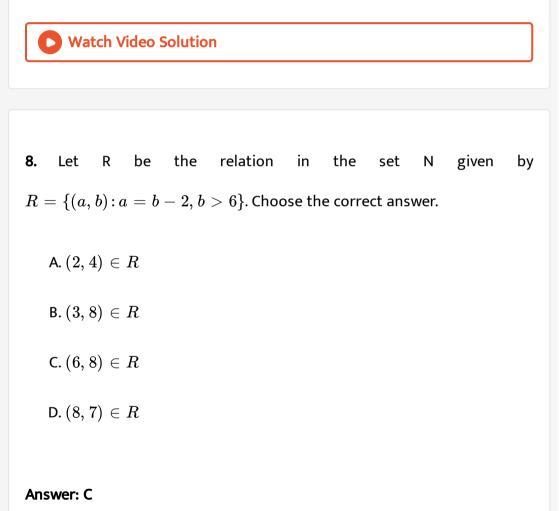
A. R is reflexive and symmetric but not transitive

B. R is reflexive and transitive but not symmetric.

C. R is symmetric and transitive but not reflexive.

D. R is an equivalence relation.

Answer: B



1. Show that the function $f: R_0 \to R_0$, defined as $f(x) = \frac{1}{x}$, is one-one onto, where R_0 is the set of all non-zero real numbers. Is the result true, if the domain R_0 is replaced by N with co-domain being same as R_0 ?

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2. Check the injectivity and surjectivity of the following functions:(i) $f: N \to N$ given by $f(x) = x^2$ (ii) $f: Z \to Z$ given by $f(x) = x^2$ (iii) $f: R \to R$ given by $f(x) = x^2$ (iv) $f: N \to N$ given by $f(x) = x^3$ (v) f: Z -

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3. Prove that the Greatest Integer Function $f: R \to R$, given by f(x) = [x], is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

4. Show that the Modulus Function $f: R \to R$, given by f(x) = |x|, is neither one-one nor onto, where |x| is x, if x is positive or 0 and |x| is x, if x is negative.

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5. Show that the Signum function $f\colon R o R$, given by $f(x)=\{1, \ ext{if} \ x>00, \ ext{if} \ x=0-1, \ ext{if} \ x<0 ext{ is neither one-one nor onto.} \}$

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6. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be

a function from A to B. Show that f is one-one.

7. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.(i) $f\colon R o R,$ defined by f(x)=34x(ii) $f\colon R o R,$ defined by $f(x)=1+x^2$



8. Let A and B be sets. Show that $f : A \times B$, $B \times A$ such that f (a, b) = (b, a) is bijective function.

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9. Let $f:N \to N$ be defined as $f(n) = \frac{n+1}{2}$ if n is odd and $f(n) = \frac{n}{2}$ if n is even for all $n \in N$ State whether the function f is bijective. Justify your answer

10. Let $A=R-\{3\}$ and $B=R-\{1\}$. Consider the function $f\colon A o B$ defined by $(x)=\left(rac{x-2}{x-3}
ight)$. Is f one-one and onto? Justify your answer.

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11. Let $f\colon R o R$ be defined as $f(x)=x^4.$ Choose the correct answer.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: D



12. Let $f \colon R o R$ be defined as f(x) = 3x. Choose the correct answer

A. f is one-one onto

- B. f is many-one onto
- C. f is one-one but not onto
- D. f is neither one-one nor onto

Answer: A

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Exercise 13

1. Let $f: \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g: \{1, 2, 5\} \rightarrow \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down

gof.

2. Let f, g and h be functions from R to R. Show that (f+g)oh = foh + goh(fg)oh = (foh)(goh)

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3. Find and , if (i) () f(x)=|x| and g(x)=|5x-2| (ii) $f(x)=8x^3$ and $g(x)=x^{1/3}$

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4. If
$$f(x)=rac{4x+3}{6x-4},\ x
eq rac{2}{3},\ ext{show that}\ fof(x)=x\ ext{for all}\ x
eq rac{2}{3}.$$

What is the inverse of f?

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5. State with reason whether following functions have inverse

(i)
$$f: \{1, 2, 3, 4\} \rightarrow \{10\}$$
 with $f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$

(ii) $g \colon \{5, 6, 7, 8\} \to \{1, 2, 3, 4\}$ with $g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

(iii)
$$h \colon \{2, 3, 4, 5\} o \{7, 9, 11, 13\}$$
 with

 $h = \{(2,7), (3,9), (4,11), (5,13)\}$

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6. Show that $f: [-1, 1] \to R$, given by $f(x) = \frac{x}{(x+2)}$ is one-one. Find the inverse of the function $f: [-1, 1] \to \text{Range } f$. (Hint: For $y \in \text{Range } f, y = f(x) = \frac{x}{x+2}$, for some x in [-1, 1], i.e., $x = \frac{2y}{(1-y)}$)

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7. Consider $f\colon R o R$ given by f(x)=4x+3. Show that f is invertible.

Find the inverse of f.

8. Consider $f: R_{\pm} > [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of given f by $f^{-1}(y) = \sqrt{y-4}$ where R_+ is the set of all non-negative real numbers.

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9. Consider
$$f: R \to [-5, \infty)$$
 given by $f(x) = 9x^2 + 6x - 5$. Show that
 f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right)$.
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10. Let $f: X \to Y$ be an invertible function. Show that f has unique inverse. (Hint: suppose $g_1($ and $g)_2$ are two inverses of f. Then for all $y \in Y$, $fog_1(y) = I_Y(y) = fog_2(y)$ Use one oneness of f).

11. Consider $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$ given by f(1) = a, f(2) = b and f(3) = c. Find f^{-1} and show that $(f^{-1})^{-1} = f$.



12. Let $f\colon X o Y$ be an invertible function. Show that the inverse of f^{-1} is f, i.e., $\left(f^{-1}
ight)^{-1}=f.$

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13. If $f\!:\!R o R$ be given by $f(x)=\left(3-x^3
ight)^{1/3}$, then fof(x) is

A. $x^{1/3}$

 $\mathsf{B.}\,x^3$

C. *x*

D. $\left(3-x^3\right)$

Answer: C

14. Let
$$f: R - \left\{-\frac{4}{3}\right\} \to R$$
 be a function as $f(x) = \frac{4x}{3x+4}$. The inverse of f is map, $g: Ran \ge f \to R - \left\{-\frac{4}{3}\right\}$ given by.(a)
 $g(y) = \frac{3y}{3-4y}$ (b) $g(y) = \frac{4y}{4-3y}$ (c) $g(y) = \frac{4y}{3-4y}$ (d)
 $g(y) = \frac{3y}{4-3y}$

A.
$$g(y) = rac{3y}{3-4y}$$

B. $g(y) = rac{4y}{4-3y}$
C. $g(y) = rac{4y}{4-3y}$
D. $g(y) = rac{4y}{3-4y}$

Answer: b



1. Determine whether or not each of the definition of given below gives a binary operation. In the event that is not a binary operation, given justificantion for this.

(i) On Z^+ , define * by a * b = a- b (ii) On Z^+ , define * by a * b = ab (iii) On R,define * by a * $b - ab^2$ (iv) On Z^+ , define * by a *b = |a - b|(v) On Z^+ , define * by a* b = a

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2. For each operation * defined below, determine whether * is binary, commutative or associative.

(i) On Z, define a*b = a-b

(ii) On Q, define a*b = ab + 1 (iii) On Q, define a*b = $\frac{ab}{2}$ (iv) On Z⁺, define a*b = $\frac{a}{b+1}$ (v) On Z⁺, define a*b = a^b (vi) On R- {-1}, define a*b = $\frac{a}{b+1}$ **3.** Consider the binary operation \land on the set $\{1,2,3,4,5\}$ defined by

 $a \wedge b = \min \{a, b\}$. Write the operation table of the operation.

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4. Consider the binary operation \land on the set $\{1, 2, 3, 4, 5\}$ given by the

following multiplication table.

- (i) Compute (2*3)*4 and 2*(3*4)
- (ii) Is * commutative ?
- (iii) Compute (2*3) *(4*5)

(Hint : uset the following table)



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5. Let \cdot 'be the binary operation on the set $\{1, 2, 3, 4, 5\}$ defined by $a \cdot 'b = H\dot{C}\dot{F}$ of a and b. Is the operation \cdot 'same as the operation \cdot defined in Exercise 4 above? Justify your answer.



6. Let \cdot be the binary operation on N given by $a \cdot b = LCM$ of a and b. Find (i) $5 \cdot 7, 20 \cdot 16$ (ii) Is \cdot commutative? (iii) Is \cdot associative? (iv) Find the identity of \cdot in N (v) Which elements of N are invert

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7. If * defined on the set $\{1,2,3,4,5\}$ by a*b=LCM of a and b a

binary operation? Justify your answer.

8. Let * be the binary operation on N defined by a * b = HCF of a and b

. Does there exist identity for this binary operation on N?



9. Let \cdot be a binary operation on the set Q of rational numbers as follows: (i) $a \cdot b = a - b$ (ii) $a \cdot b = a^2 + b^2$ (iii) $a \cdot b = a + ab$ (iv) $a \cdot b = (a - b)^2$ (v) $a \cdot b = \frac{ab}{4}$ (vi) $a \cdot b = ab^2$. Find wh

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10. Find the which of the operations given above has identity?

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11. Let $A = N \times N$ and \cdot be the binary operation on A defined by $(a, b) \cdot (c, d) = (a + c, b + d)$. Show that \cdot is commutative and associative. Find the identity element for \cdot on A, if any.

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12. State whether the following statements are true or false. Justify. (i) For

an arbitrary binary operation \cdot on a set $N, \ a \ \cdot \ a = \ a \ \forall a \in N$. (ii) If \cdot is a commutative binary operation on N, then `a" "*" "(b" "*" "c)"

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13. Consider a binary operation \cdot on N defined as $a \cdot b = a^3 + b^3$. Choose the correct answer. (A) Is \cdot both associative and commutative? (B) Is \cdot commutative but not associative? (C) Is \cdot associative but not commutative? (D) Is **Miscellaneous Exercise**

1. Let $f\!:\!R o R$ be defined as f(x)=10x+7. Find the function

 $g{:}R o R$ such that $gof = fog = 1_R$

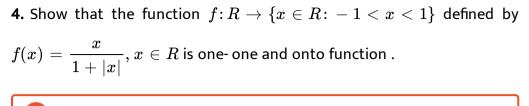
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2. Let $f \colon W o W$ be defined as f(n) = n-1, if is odd and f(n) = n+1

, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

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3. If $f\!:\!R o R$ is defined by $f(x)=x^2-3x+2$, find f(f(x)).



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5. Show that the function $f\!:\!R o R$ given by $f(x)=x^3$ is injective.

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6. Give examples of two functions $f: N \to Z$ and $g: Z \to Z$ such that o f is injective but is not injective. (Hint: Consider f(x) = x and g(x) = |x|)

7. Given examples of two functions $f\colon N o N$ and $g\colon N o N$ such that of is onto but f is not onto. (Hint: Considerf(x) = x and g(x) = |x|).



8. Given a non-empty set X, consider P(X) which is the set of all subsets of X. Define the relation R in P(X) as follows: For subsets A, B in P(X), ARB if and only if A B. Is R an equivalence relation on P(X)? Justify you answer

9. Given a non-empty set X, consider the binary operation $\cdot: P(X) \times P(X) \to P(X)$ given by $A \cdot B = A \cap B \forall A, B \in P(X)$ is the power set of X. Show that X is the identity element for this operation and X is the only invertible element i

10. Find the number of all onto functions from the set $\{1, 2, 3, , n\}$ to itself.



11. Let $S = \{a, b, c\}andT = \{1, 2, 3\}$. Find F^{-1} of the following functions F from S to T, if it exists.(i) $F = \{(a, 3), (b, 2), (c, 1)\}$ (ii) $F = \{(a, 2), (b, 1), (c, 1)\}$

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12. Consider the binary operations $\cdot : R \times R \to R$ and $o: R \times R \to R$ defined as $a \cdot b = |a - b|$ and aob = a for all $a, b \in R$. Show that \cdot is commutative but not associative, o is associative but not commutative. Further, show that \cdot is distributive over o. Dose o distribute over \cdot ? Justify your answer.

13. Given a non -empty set X, let \cdot : $P(X) \times P(X) \rightarrow P(X)$ be defined as A * B = (A B) \cup (B A), \forall A, B \in P(X) $A \cdot B = (A - B) \cup (B - A), \forall A, B \in P(X)$. Show that the empty set φ is the identity for the

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14. Define a binary operation * on the set $A = \{0, 1, 2, 3, 4, 5\}$ as $a \cdot b = a + b \pmod{6}$. Show that zero is the identity for this operation and each element a of the set is invertible with 6 - a being the inverse of a. OR A binary operation * on the set $\{0, 1, 2, 3, 4, 5\}$ is defined as $a \cdot b = \{a + b, \text{ if } a + b < 6a + b - 6, \text{ if } a + b \ge 6 \text{ Show that zero}$ is the identity for this operation and each element a of set is invertible with 6 - a, being the inverse of a.

15. Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g: A \rightarrow B$ be functions defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \left(\frac{1}{2}\right)\right| - 1, x \in A$. Are f and g equal? Justify your answer. (Hint: One may note that two functio

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16. Let $A = \{1, 2, 3\}$ Then number of relations containing (1, 2) and (1, 3)

which are reflexive and symmetric but not transitive is

A. 1

B. 2

C. 3

D. 4

Answer: A

17. Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing (1, 2) is (A) 1 (B) 2 (C) 3 (D) 4

A. 1 B. 2 C. 3 D. 4

Answer:

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18. Let $f: R \to R$ be the Signum Function defined as $f(x) = \{1, x > 00, x = 0 - 1, x < 1 \text{ and } g: R \to R$ be the Greatest Integer Function given by g(x) = [x], where [x] is greatest integer less than or equal to x. Then does fo



(D) 8
A. 10
B. 16
C. 20
D. 8

19. Number of binary operations on the set {a, b} are (A) 10 (B) 16 (C) 20

Answer: